Gaussian Process for Online Real-Time Pricing

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Abstract

Traditional machine learning is normally tackled with problems with given static datasets, many online retailing data was created, collected, and labeled through real-time interactions with customers and users. Therefore, understanding how to apply dynamic pricing in retailing under the 'Big Data' scale is of increasing importance in both practice and research. The motivation of this work is presenting real-time product preference pricing strategies from high volume and high dimensional datasets. Due to previously insufficient data samples and conventional dynamic pricing strategy was usually designed based on specific, well-defined scenarios, which cannot promise the expected revenue expectation. This research focuses on given the market's heterogeneity and high-dimensional online retailing data, introduces a data-driven non-parametric inference model to improve demand estimation for better pricing decisions. Findings from this work can help the retailer to give precise pricing strategy in revenue management and even to optimize advertising, recommendation systems, and many more applications.

Keywords:

High Dimension, Gaussian Processes, Dynamic Pricing, Multi-Armed Bandit

1 Introduction

1.1 Dynamic Pricing

Over the past few decades, dynamic pricing has been widely used in many industries, such as retail, hotels, airlines, and has achieved great success. Dynamic Pricing has proven its ability to balance supply and demand to maximize a company's revenue. With the increasing popularity of online retailing, dynamic pricing has attracted much attention. Early research of dynamic pricing was talking about customer arrival rate and customer reservation price distribution. An example of this can be tracked to its early application in the airline industry (see, e.g., Littlewood (1972), Smith et al. (1992)).

Based on the nature of the service or product provided and dynamic market change, different possibilities may introduce different scenarios, which is also the design basis of traditional pricing strategies. When designing a dynamic pricing strategy, it is important to distinguish between considering limited or unlimited inventory and time range of customer arrival. Another possible difference is that, based on a stochastic process, dynamic heterogeneity across and within the individuals' evolution over time indicates the nature of the market as well (Desarbo et al. (1997), Dew et al. (2020)). The consumer reference price also depends on the minimum value between the sales price of the last sales period and the historical sales price that has shown a peak-end effect, which affects customers' buying preferences Nasiry and Popescu (2011).

1.2 Dynamic Pricing in Online Retailing

Traditional dynamic pricing strategies were designed based on the prior knowledge of the market environment, in retailing industries like fast fashion and supermarkets, as online retail evolves and the availability of online sales data increases, facilitating the estimation of demand and the adjustment of price in real-time (see survey paper Den Boer (2015)).

The combination of operations research and statistical learning has become a new trend in dynamic pricing research. Moreover recently, non-parametric demand models attract more attestations (see, e.g., Araman and Caldentey (2009), Wang et al. (2011), Jasin et al. (2015)). Cohen et al. (2016) consider that the company did not understand the features of the product at first but can learn the value based on whether the product was sold at the recorded price in the past. This work considered the issue of dynamic pricing with the uncertainty of the demand model.

2 Research Goal(s) and Question(s)

2.1 Contribution of this Research

As the market will be unknown in a realistic online retail scenario, this means that we cannot have prior knowledge of the global optimal pricing strategy, and GPR is naturally applicable in this field. The computational cost is a critical point in machine learning model design work. CPR suffers $O\left(n^3\right))$ computation, and $O\left(n^2\right)$ storage requirements limit GPR to comparable small datasets. GPR in real-time retailing environment, purchases, and customers' features may update hundreds per second, which requires our generative model to extend to local optimal pricing.

The goal of this work is to provide a non-parametric machine learning approach based on Gaussian Process Regression (GPR) to offer a more flexible framework for approximating unknown nonlinearities in retailing pricing. I will propose a new policy base on Local Gaussian Process Regression (LGP) for real-time learning base on high-dimensional historical sales prices and product feature datasets (insights gained from Nguyen-Tuong et al. (2009)).

2.2 Gaussian Process Regression

Original GPs have natural limitations in tackling a large number of datasets. Therefore some skills were introduced.

Let $\mathcal{X}$ be a set of data in $\mathbb{R}^d$. A Gaussian process is a collection of $\{f(\mathbf{x}), \mathbf{x} \in \mathcal{X}\}$, which have joint Gaussian distributions. It is specified by the mean function $m(\mathbf{x})$ and covariance (kernel) function $k(\mathbf{x}, \mathbf{x}')$ Rasmussen et al. (2004), it can be written as

$$f(\mathbf{x}) \sim \mathrm{GP}\left(0, k\left(\mathbf{x}, \mathbf{x}'\right)\right)$$

Given a set of training data with their observations $\{\mathbf{x}_i, y_i\}_{i=1}^n$, where $n \in \mathbb{N}$ and $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathcal{X}$, and $y_i = \mathbf{x}_i + \epsilon_i$, where ϵ_i is Gaussian noise with zero mean and variance σ_n, we can describe it as a Gaussian distribution

$$\mathbf{y} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I}\right)$$

where $\mathbf{X}$ denotes a whole dataset, $\mathbf{y}$ represents the label set.

And here we take the Gaussian kernel as an example

$$\mathbf{K}\left(\mathbf{x}, \mathbf{x}'\right) = \sigma^2 \exp\left(-\frac{\left\|\mathbf{x}-\mathbf{x}'\right\|^2}{2\ell^2}\right)$$

where ℓ denotes the length scale parameter of the Gaussian Kernel, which determines the relevancy of the input feature to the regression.

If we want to make predictions on the test data $\mathbf{X}_*$, we will have

$$\left[\begin{array}{c} \mathbf{y} \\ f\left(\mathbf{X}_*\right) \end{array}\right] \sim \mathcal{N}\left(0, \left[\begin{array}{cc} \mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma^2 \mathbf{I} & \mathbf{K}\left(\mathbf{X}, \mathbf{X}_*\right) \\ \mathbf{K}\left(\mathbf{X}_*, \mathbf{X}\right) & \mathbf{K}\left(\mathbf{X}_*, \mathbf{X}_*\right) \end{array}\right]\right)$$

Same as above, we have prediction posterior Gaussian distribution

$$p(\mathbf{y}_* | \mathbf{y}) \sim \mathcal{N}\left(\hat{m}(\mathbf{X}_*), \hat{\mathbf{K}}(\mathbf{X}_*, \mathbf{X}_*)\right)$$

where predicted mean:

$$\hat{m} = \mathbf{K}(\mathbf{X}_*, \mathbf{X})\left(\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{y}$$

And predicted covariance:

$$\hat{\mathbf{K}}(\mathbf{X}_*, \mathbf{X}_*) = \mathbf{K}(\mathbf{X}_*, \mathbf{X}_*) - \mathbf{K}(\mathbf{X}_*, \mathbf{X})\left(\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{K}(\mathbf{X}, \mathbf{X}_*)$$

2.3 Problems and Research Overview

My work endeavors to answer two research questions in the Gaussian Process for dynamic pricing (DP):

- How to propose a new method to speed up GPs in a real-time environment?
- How to model dynamic pricing in Gaussian Process systems.

To address the first problem, ideas gained from Bayesian inference could be a good start point, marginalizing over the dynamics of the model and infer the joint distribution directly through the specially tailored Particle Markov Chain Monte Carlo samplers.Frigola et al. (2013).

For the second problem, Multi-Armed Bandit(MAB) would be a bridge to connect DP and GPR which combine GPA with MAB as a nonparametric learning algorithm that can learn any functional relation between price and demand, which offer an optimal price without prior knowledge about demand and learn it from experimentation - a variant of the multi-armed bandit problem.

3 Timeline

As my first research topic during my Ph.D. student, the time span will not exceed six months. Following is a sample timeline for this work:

Wed Jul 01, 2020	Research Start
Fri Jun 05, 2020	Understand the selected topic
Wed Jun 17, 2020	Explore a research question
Tue Jun 23, 2020	Design research strategy
Mon Aug 24, 2020	Read, note, and compare sources
Tue Sep 08, 2020	Write the paper statement
Mon Oct 05, 2020	Write the first draft
Wed Oct 14, 2020	Evaluate the first draft
Fri Nov 20, 2020	Revise and rewrite
Sun Nov 29, 2020	Put the paper in final form

4 Summary

The focus of my ongoing Ph.D. research is to study the application of non-parametric statistical methods in real business problems, specifically, how is influenced the dynamic environment. My future work studies Bayesian approaches in a more complex social system by incorporating the structure of the recommendation network and sequential learning framework. Broadly, this work makes contributions to the business science and data mining community.

References

Victor Araman and René Caldentey. Dynamic pricing for nonperishable products with demand learning. *Operations Research*, 57:1169–1188, 10 2009. doi: 10.1287/opre.1090.0725.

Maxime Cohen, Ilan Lobel, and Renato Leme. Feature-based dynamic pricing. pages 817–817, 07 2016. doi: 10.1145/2940716.2940728.

Arnoud V. Den Boer. Dynamic pricing and learning: Historical origins, current research, and new directions. *Surveys in Operations Research and Management Science*, 20, 06 2015. doi: 10.1016/j.sorms.2015.03.001.

Wayne Desarbo, Asim Ansari, Pradeep Chintagunta, Charles Himmelberg, Kamel Jedidi, Richard Johnson, Wagner Kamakura, Peter Lenk, Kannan Srinivasan, and Michel Wedel. Representing heterogeneity in consumer response models 1996 choice conference participants. *Marketing Letters*, 8, 07 1997. doi: 10.1023/A:1007916714911.

Ryan Dew, Asim Ansari, and Yang Li. Modeling dynamic heterogeneity using gaussian processes. *Journal of Marketing Research*, 57(1):55–77, 2020. doi: 10.1177/0022243719874047.

Roger Frigola, Fredrik Lindsten, Thomas B Schön, and Carl Edward Rasmussen. Bayesian inference and learning in gaussian process state-space models with particle mcmc. pages 3156–3164, 2013.

Stefanus Jasin, Qi Chen, and Izak Duenyas. Real time dynamic pricing with minimal and flexible price adjustment. 11 2015. doi: 10.13140/RG.2.1.4477.8962.

K. Littlewood. Forecasting and control of passenger bookings. *Airline Group of the International Federation of Operational Research Societies Symp. Proc.*, 12, 01 1972.

Javad Nasiry and Ioana Popescu. Dynamic pricing with loss averse consumers and peak-end anchoring. *Operations Research*, 59:1361–1368, 11 2011. doi: 10.1287/opre.1110.0952.

Duy Nguyen-Tuong, Matthias Seeger, and Jan Peters. Model learning with local gaussian process regression. *Advanced Robotics*, 23:2015–2034, 10 2009. doi: 10.1163/016918609X12529286896877.

Carl Rasmussen, Olivier Bousquet, Ulrike Luxburg, and Gunnar Rätsch. Gaussian processes in machine learning. *Advanced Lectures on Machine Learning: ML Summer Schools 2003, Canberra, Australia, February 2 - 14, 2003, Tübingen, Germany, August 4 - 16, 2003, Revised Lectures, 63-71 (2004)*, 3176, 09 2004. doi: 10.1007/978-3-540-28650-9_4.

Barry Smith, John Leimkuhler, and Ross Darrow. Yield management at american airlines. *Interfaces*, 22:8–31, 02 1992. doi: 10.1287/inte.22.1.8.

Zizhuo Wang, Shiming Deng, and Yinyu Ye. Close the gaps: A learning-while-doing algorithm for a class of single-product revenue management problems. *CoRR*, abs/1101.4681, 2011.