3 Star Chaotic System

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In this problem set, I investigate the three star pattern and attempt to recreate the results of Hut and Bachall. In their origional paper, two binary stars are orbiting for a period of time, and a third star (the green line) is shot in between the system, causing a chaotic pattern, and ejecting stars out at the end.

I was unable to perfectly model the figure produced, however, I found some more interesting results and I became curious about the angle of deviation of stars from the third star's initial condition.

Figure 1 shows my attempt to match the results found in Hut and Bachall. Note that the plot constructed with the center of mass remaining at the origin (0, 0). Note that for the non inertial frame of reference, the same pattern occurs, but the entire image is translated to the right. This is because the initial momentum is entirely to the right, seen in figure 2.

I became interested in the nature of the deviation from the center. Because the plot is relative to the center of mass at (0, 0), all ejections occur at an angle from the center of mass. Using massive parallel computing over night, I ran the simulation 1000 times (from 20.00 to 30.00 with a step of 0.01) and recorded the angle of deviation of all three stars, and produced the bifurcation diagram in figure 3.

A few interesting characteristics of this plot are present. First, the clear periodic nature of the orbiting stars. Every second repetition of the stable period switches between the two sets ((1, 2), (3)) and ((1, 3), (2)). Although I did not run the simulation longer, it appears star (1) is never isolated. This is not a conclusion, however, and more information is needed. At around 20.9, there occurs a bifurcation, with small strips in the middle of stability. Note that if the star starts at the initial condition in a stable region, there were no chaotic oscillations at the point of impact. Of course, there are a few flaws in this plot, notably the fact that in some chaotic regions, there are not star pairs, meaning I did not let the simulation run long enough. Therefore, I attempted to zoom in on the portion between [22, 25] with a smaller step (0.001) and a longer integration path to make sure both stars accurately left from the interaction.

I zoom in on one of the chaotic regions in figure 5, while connecting lines to show the nature of the pairing at the end (showing that two stars pair after ejection implies the two lines will meet).

Some portions of the biffurcation diagram in figure 5 show that two binary stars stick together for a long set of parameters, for example from 23.27 to 23.2727, the second and third star pair (oscillating between an angle above and below the equator.

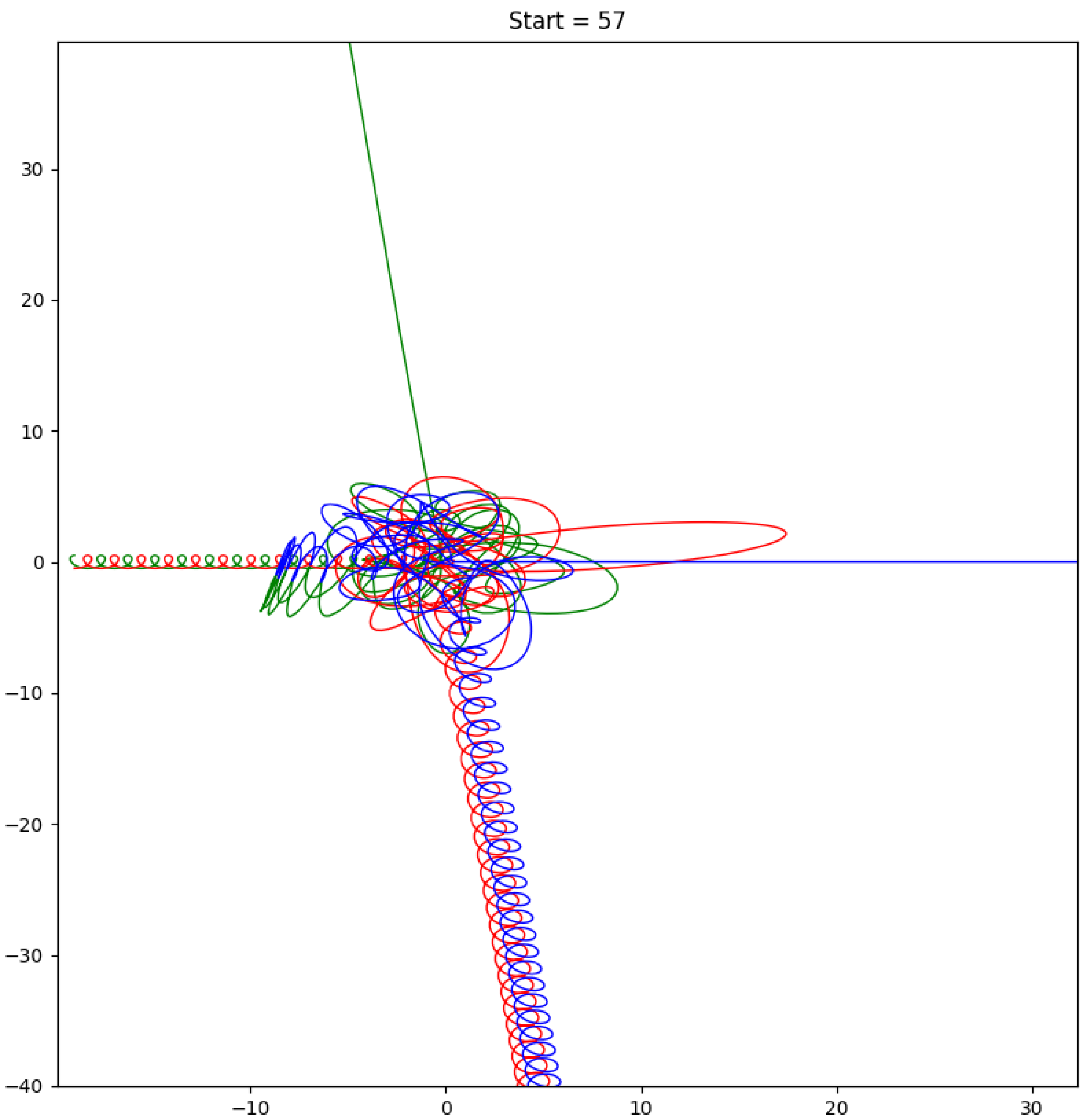

Figure 1: Attempt to recreate the work from Hut and Bachall

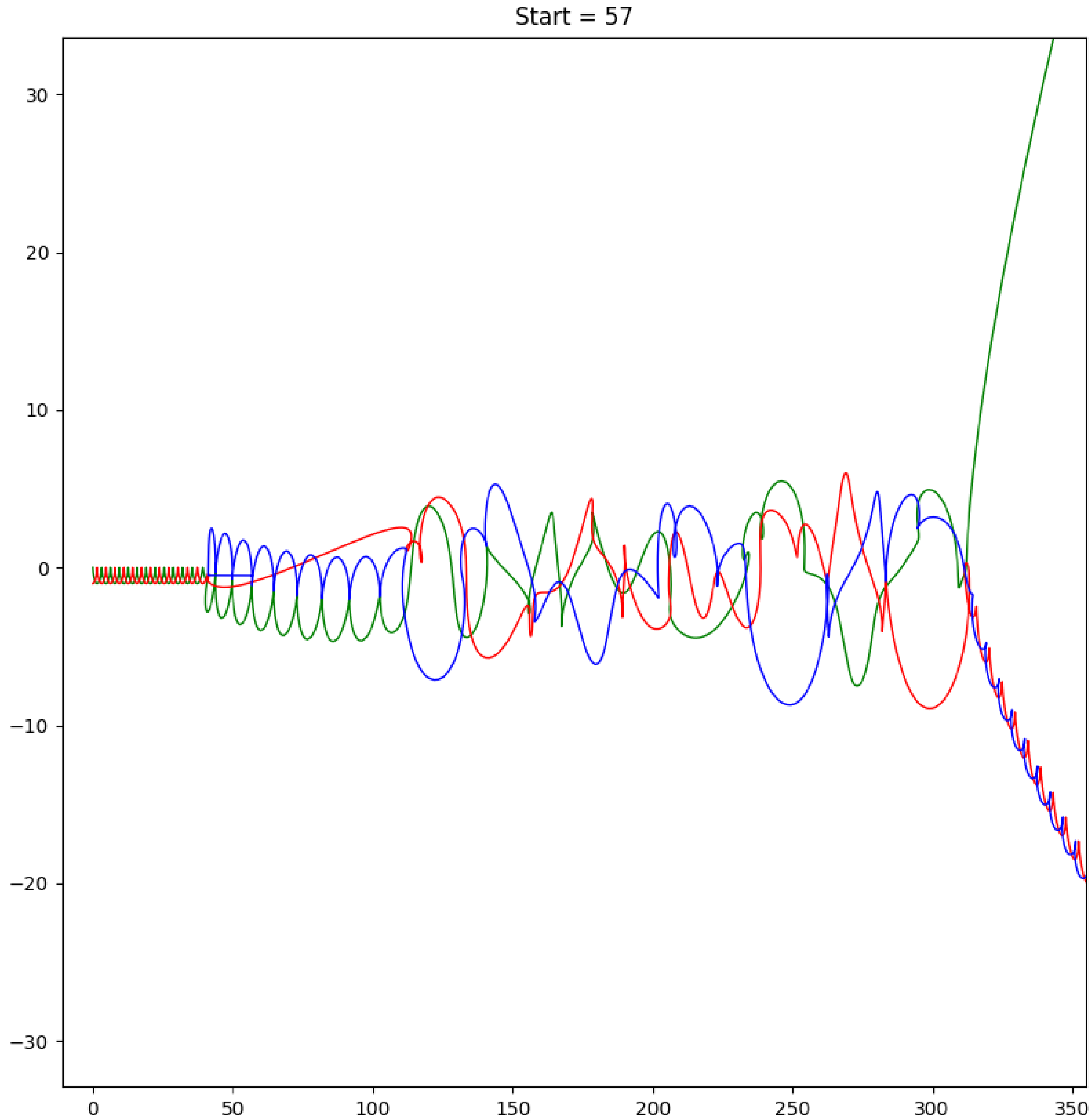

Figure 2: Recreation of Bachall and Hut with non inertial reference frame.

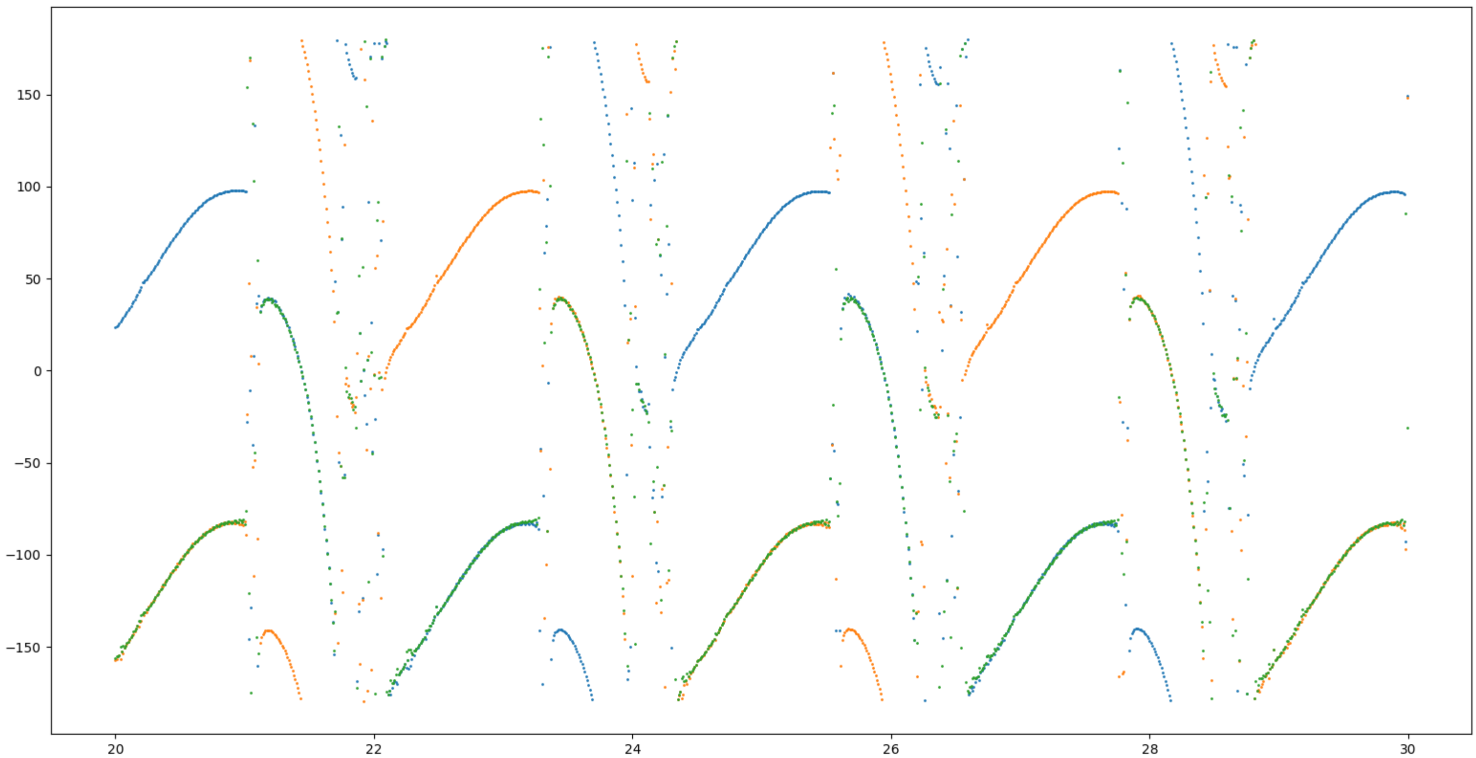

Figure 3: A Plot of the phase angles after leaving the center.

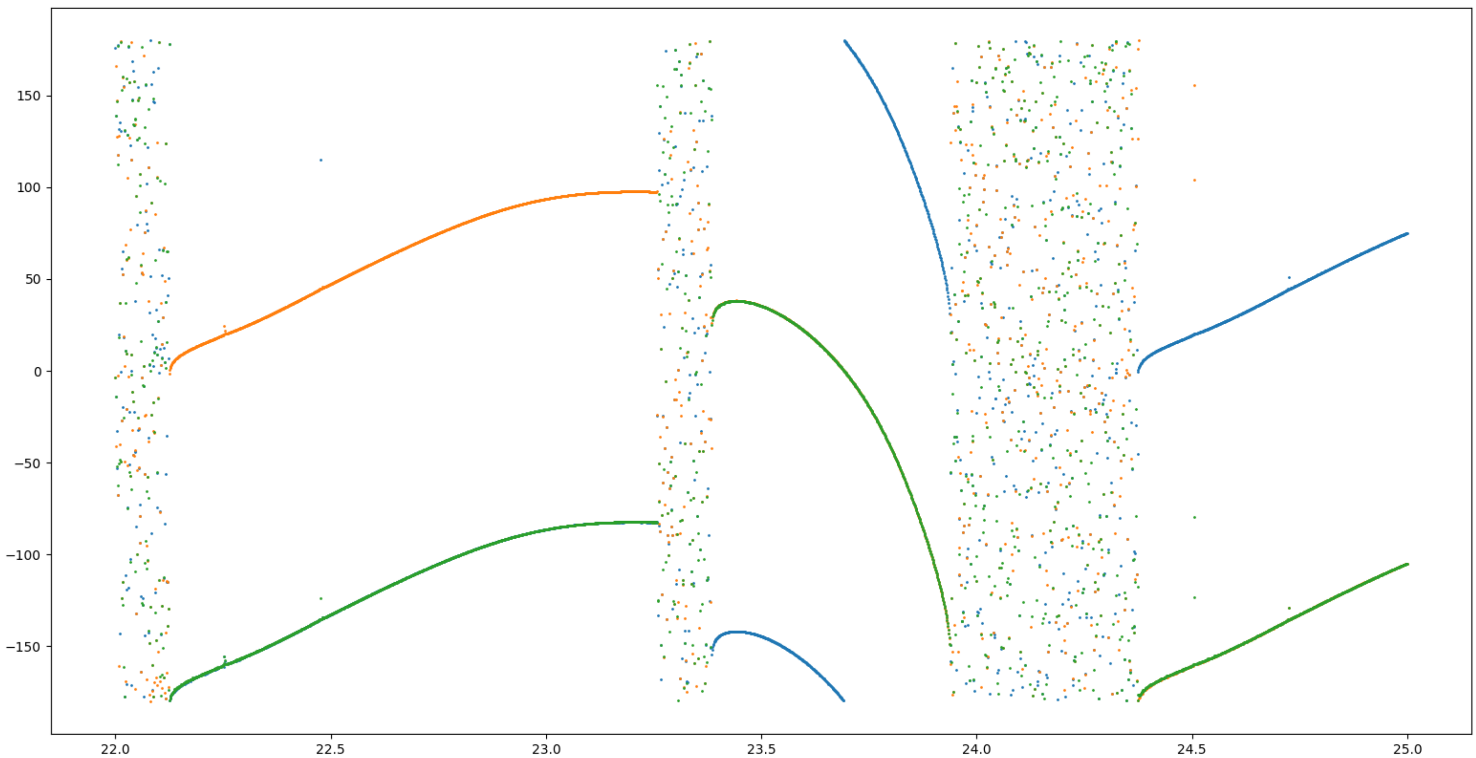

Figure 4: A Plot of the zoomed in portion of the binary star system angle of deviation from 22 to 25 with steps of 0.001

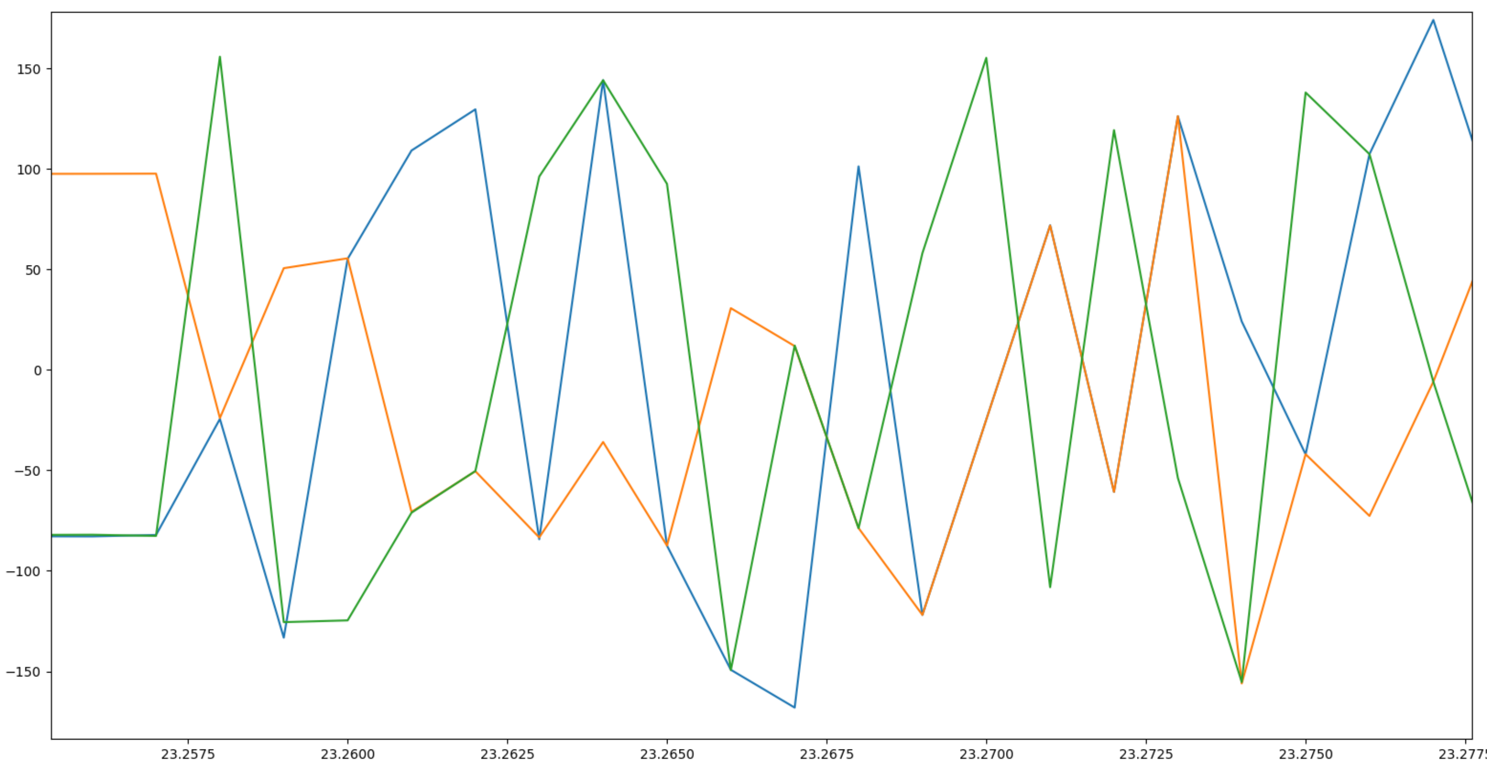

Figure 5: A Zoomed in portion of the chaotic region