Dosage Calculator

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1 Introduction

This is a document recording how to calculate dosage for medication given several different doses during different parts of the day.

2 Discreet Derivation

The first step is to define some parameters. Lets say during a period of T (which is usually 24 hours) a person receives n doses with an exponential half-life of h. The doses are received at $(t_0, t_1, ..., t_{n-1})$ where $t_i \in [0, T)\ \forall i$. The corresponding dose amounts are $(A_0, A_1, ..., A_{n-1})$. Considering that every period of T, the user repeats the doses then we can calculate the amount of medication in the patient's system as

$$\sum_{i=0}^{n-1} \Theta(t - t_i) \sum_{j=0}^{j_i} A_i (\frac{1}{2})^{\frac{t-t_i-Tj}{h}} \tag{1}$$

where $j_i = \frac{t-t'}{T}$ where $t' \cong t \bmod T$ in its remainder form and $t - t' \geq t_i$ or $j_i = \frac{t-t'}{T} - 1$ if $t - t' < t_i$. $\Theta(t - t_i)$ is a heavy step side function.

$$\begin{aligned}
\sum_{j=0}^{j_i} A_i (\frac{1}{2})^{\frac{t-t_i-Tj}{h}} &= A_i (\frac{1}{2})^{\frac{t-t_i}{h}} \frac{1 - (\frac{1}{2})^{-\frac{T}{h}(j_i+1)}}{1 - (\frac{1}{2})^{-\frac{T}{h}}} \\
&= \left\{ \begin{array}{ll} A_i (\frac{1}{2})^{\frac{-t_i}{h}} \frac{(\frac{1}{2})^{\frac{t+T}{h}} - (\frac{1}{2})^{\frac{t'}{h}}}{-1+(\frac{1}{2})^{\frac{T}{h}}}, & \text{if } t - t' \geq t_i \\ A_i (\frac{1}{2})^{\frac{-t_i}{h}} \frac{(\frac{1}{2})^{\frac{t+T}{h}} - (\frac{1}{2})^{\frac{t'+T}{h}}}{-1+(\frac{1}{2})^{\frac{T}{h}}}, & \text{if } t - t' < t_i \end{array} \right\}
\end{aligned} \tag{2}$$

Equilibrium occurs when $t \to \infty$ which for this formula's sake would be

$$\sum_{j=0}^{j_i} A_i (\frac{1}{2})^{\frac{t-t_i-Tj}{h}} \to \begin{array}{ll} A_i (\frac{1}{2})^{\frac{-t_i}{h}} \frac{(\frac{1}{2})^{\frac{t'}{h}}}{1-(\frac{1}{2})^{\frac{T}{h}}}, & \text{if } t - t' \geq t_i \\ A_i (\frac{1}{2})^{\frac{-t_i}{h}} \frac{(\frac{1}{2})^{\frac{t'+T}{h}}}{1-(\frac{1}{2})^{\frac{T}{h}}}, & \text{if } t - t' < t_i \end{array} \tag{3}$$

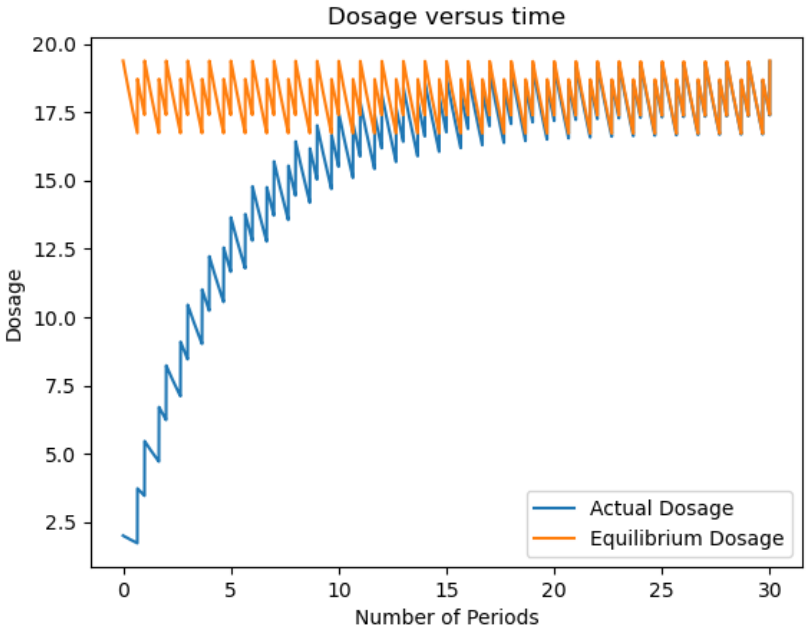

Figure 1: This Figure is an example of the dosage calculation inside a person with the parameters. Period = 24 (hours). half-life = 75 (hours), both doses are 2 (mg), and dose 1 is taken at 0 (hours) while dose 2 is taken at 16 (hours). Blue graph is the dose in the body. Orange graph is the dose at equilibrium

We can combine eq. 1 with eq. 2, we will get the blue graph that is in fig. 1. If we combine eq. 1 with eq. 3 we will get the equilibrium dosage which is the orange graph in fig. 1