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U09M11109: VisNav - Visual Navigation for UAV

Lecture **6-2**: RANSAC and 3D-3D correspondences

Today

- Recap on 2-view
- RANSAC
- 3D-3D correspondences

SRI International

RANDOM SAMPLE CONSENSUS: A PARADIGM FOR MODEL FITTING WITH APPLICATIONS TO IMAGE ANALYSIS AND AUTOMATED CARTOGRAPHY

Technical Note 213

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Least-Squares Fitting of Two 3-D Point Sets

K. S. ARUN, T. S. HUANG, AND S. D. BLOSTEIN

Abstract—Two point sets $\{p_i\}$ and $\{p_i'\}$ $i = 1, 2, \cdots, N$ are related by $p_i' = Rp_i + T + N_i$, where R is a rotation matrix, T a translation vector, and N_i a noise vector. Given $\{p_i\}$ and $\{p_i'\}$, we present an algorithm for finding the least-squares solution of R and T, which is based on the singular value decomposition (SVD) of a 3×3 matrix. This new algorithm is compared to two earlier algorithms with respect to computer time requirements.

Index Terms—Computer vision, least-squares, motion estimation, quaternion, singular value decomposition.

I. Introduction

In many computer vision applications, notably the estimation of motion parameters of a rigid object using 3-D point correspondences [1] and the determination of the relative attitude of a rigid object with respect to a reference [2], we encounter the following mathematical problem. We are given two 3-D point sets $\{p_i\}$, $i = 1, 2, \cdots, N$ (here, p_i and p_i' are considered as 3×1 column matrices)

$$p_i' = Rp_i + T + N_i \tag{1}$$

where R is a 3×3 rotation matrix, T is a translation vector (3×1 column matrix), and N_i a noise vector. (We assume that the rotation is around an axis passing through the origin). We want to find R and T so minimize

$$\Sigma^2 = \sum_{i=1}^{N} \|p_i' - (Rp_i + T)\|^2. \tag{2}$$

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As iterative algorithm for finding the solution was described in Huang, Blostein, and Margerum [3]; a noniterative algorithm based on quaternions in Faugeras and Hebert [4]. In this correspondence, we describe a new noniterative algorithm which involves the singular value decomposition (SVD) of a 3×3 matrix. The computer time requirements of the three algorithms are compared.

After the submission of our correspondence, it was brought to our attention that an algorithm similar to ours had been developed independently by Professor B. K. P. Horn, M.I.T., but not published.

II. Decoupling Translation and Rotation

It was shown in [3] that if the least-squares solution to (1) is $\hat{R}$ and $\hat{T}$, then $\{p_i'\}$ and $\{p_i' \triangleq \hat{R}p_i = \hat{T}\}$ have the same centroid, i.e.,

$$p' = p^* \tag{3}$$

where

$$p' \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i' \tag{4}$$

$$p^* \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i^* = \hat{R}p + \hat{T} \tag{5}$$

$$p \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i. \tag{6}$$

Let

$$q_i \triangleq p_i - p \tag{7}$$

$$q_i' \triangleq p_i' - p'. \tag{8}$$

We have

$$\Sigma^2 = \sum_{i=1}^{N} \|q_i' - Rq_i\|^2. \tag{9}$$

Therefore, the original least-squares problem is reduced to two parts:

(i) Find $\hat{R}$ to minimize Σ^2 in (9).
(ii) Then, the translation is found by

$$\hat{T} = p' - \hat{R}p. \tag{10}$$

In the next section, we describe an algorithm for (i) which involves the SVD of a 3×3 matrix.

III. An SVD Algorithm for Finding $\hat{R}$

A. Algorithm

Step 1: From $\{p_i\}$, $\{p_i'\}$ calculate p, p'; and then $\{q_i\}$, $\{q_i'\}$.

Step 2: Calculate the 3×3 matrix

$$H \triangleq \sum_{i=1}^{N} q_i q_i'^{t} \tag{11}$$

where the superscript t denotes matrix transposition.

Step 3: Find the SVD of H,

$$H = UAV^{t}. \tag{12}$$

Step 4: Calculate

$$X = VU^{t}. \tag{13}$$

Step 5: Calculate, $\det(x)$, the determinant of X.
If $\det(x) = +1$, then $\hat{R} = X$.
If $\det(x) = -1$, the algorithm fails. (This case usually does not occur. See Sections IV and V.)

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[2]

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2-view Geometry

$$\tilde{\boldsymbol{y}}_{2,k}^{\mathsf{T}} \, \boldsymbol{E} \, \tilde{\boldsymbol{y}}_{1,k} = 0$$

Essential matrix encodes relative pose (up to scale) between C_1 and C_2

2-view Geometry

Last week's assumptions:

- no wrong correspondences (outliers)
- 3D point is not moving
- camera calibration is known

Estimating Poses from Correspondences

Given *N* calibrated pixel correspondences:

$$(\tilde{\boldsymbol{y}}_{1,k}, \tilde{\boldsymbol{y}}_{2,k}) \text{ for } k = 1, \ldots, N$$

1. leverage the epipolar constraints to estimate the essential matrix ***E***

$$\tilde{\boldsymbol{y}}_{2,k}^{\mathsf{T}} \boldsymbol{E} \, \tilde{\boldsymbol{y}}_{1,k} = 0$$

For 8 points: $\boldsymbol{A}\boldsymbol{e} = 0$ N>8 points: $\arg\min_{\|\boldsymbol{e}\|=1} \|\boldsymbol{A}\boldsymbol{e}\|^2$

2. Retrieve the rotation and translation (up to scale) from the ***E***

$$\boldsymbol{E} = [\boldsymbol{t}]_\times \boldsymbol{R}$$

2-view Geometry

In practice:

- Many wrong correspondences (outliers)
- Some 3D points might be moving

RANSAC

RANdom **SA**mple **C**onsensus

Problem: estimate model *P* from N data points, possibly corrupted with outliers.

Assume: we have an algorithm to estimate *P* from *n* data points (n << N)

Basic idea:

1. sample *n* points
2. compute an estimate *P'* of *P*
3. count how many other points agree with *P'*
4. repeat until you get a *P'* that agrees with many points

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Consensus Set

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Example: Linear Regression

Fit a line through
N 2D points, possibly
corrupted with outliers.

Note: we have an algorithm
to estimate a line from n=2 points

RANSAC:

1. sample *2* points
2. compute a line estimate *P'* of *P*
3. count how many points are within a **tolerance** from *P'*
4. repeat until you get a *P'* that agrees with many points

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Consensus
set size = 13

RANSAC:

1. sample *2* points
2. compute a line estimate *P'* of *P*
3. count how many points are within a **tolerance** from *P'*
4. repeat until you get a *P'* that agrees with many points

RANSAC: Parameter Tuning

1. **Error Tolerance** ϵ :
   depends on the noise

2. **Acceptable consensus set**:
   - from the paper: n+5
   - rule of thumb: >50% of points

3. **Maximum number of iterations**

Example: RANSAC for Essential Matrix estimation

RANSAC:

1. sample *n* point correspondences
2. compute an estimate E*'* of the essential matrix E
3. count how many points are within a **tolerance** from *E'*
4. repeat until you get a *E'* that agrees with many points

Example: RANSAC for Essential Matrix estimation

RANSAC
- essentially selects the set of inliers
- provides **geometric verification** for the correspondences

Beyond Motion Estimation

The tools we discussed (feature matching, essential matrix estimation, RANSAC) can be used also for **object detection and localization**

So far: pixel correspondences,
a.k.a., **2D-2D correspondences**

3D-3D Point Correspondences

Structured Light Cameras

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RGB-D cameras can measure depth (D) and image (RBG)

How can we use the depth information to estimate the relative pose between two RGB-D cameras observing the same scene?

3D-3D Point Correspondences

1. We can use camera images to establish 2D-2D correspondences:

$$(\tilde{\boldsymbol{y}}_{1,k}, \tilde{\boldsymbol{y}}_{2,k}) \text{ for } k = 1, \ldots, N$$

2. For each camera we can compute the set of 3D points corresponding to pixels

We obtain 3D-3D correspondences: $(\boldsymbol{p}_{1,k}, \boldsymbol{p}_{2,k}) \quad k = 1, \ldots, N$

2-view Geometry from 3D-3D Correspondences

How to estimate the relative pose between the cameras from 3D-3D correspondences $(\boldsymbol{p}_{1,k}, \boldsymbol{p}_{2,k})$ **with** $k = 1, \ldots, N$ **?**

Few More Comments:

3 points are sufficient to compute the relative pose from 3D-3D correspondences

We can use the solver seen today as a 3-point minimal solver within a **RANSAC** method

Also useful for 3D objects localization:

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Other names: vector registration, point cloud alignment, ..

Backup

Other Matrices in 2-view Geometry

Homography matrix **H**

$$\lambda_2 \boldsymbol{x}_2 = H \lambda_1 \boldsymbol{x}_1$$

Section 5.3

Fundamental matrix **F**

$$\boldsymbol{F} = \boldsymbol{K}_2^{-\top} \; [\boldsymbol{t}]_\times \boldsymbol{R} \; \boldsymbol{K}_1^{-1}$$

Chapter 6

Essential Matrix Properties

- A matrix is an essential matrix *if and only* if it has singular values $\{\sigma, \sigma, 0\}$

- The space of the essential matrices is called the *Essential space* $\mathcal{S}_E$ (i.e., the space of 3×3 matrices that can be written as $[\boldsymbol{t}]_\times \boldsymbol{R}$ for some $\boldsymbol{R} \in \mathrm{SO}(3)$ and $\boldsymbol{t} \in \mathbb{R}^3$). The projection of a matrix $\boldsymbol{M}$ onto the Essential space can be computed as prescribed in [1, Thm 5.9]:

$$\underset{\boldsymbol{E} \in \mathcal{S}_E}{\arg\min} \|\boldsymbol{E} - \boldsymbol{M}\|_F^2 = \boldsymbol{U} \begin{bmatrix} \frac{\lambda_1+\lambda_2}{2} & 0 & 0 \\ 0 & \frac{\lambda_1+\lambda_2}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{V}^\mathsf{T}$$

where $\boldsymbol{M} = \boldsymbol{U} \mathrm{diag}\,(\lambda_1, \lambda_2, \lambda_3)\, \boldsymbol{V}^\mathsf{T}$ is a singular value decomposition of $\boldsymbol{M}$.

[1]