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ECE 211: Electric Circuits

Lesson 5: Circuit Theorems (Part 2)

Text Chapter 4

Prof. Jenny Marley

Lesson Objectives

- Explain the benefits of Thevenin equivalent circuits.
- Apply source transformation to find the equivalent circuit of a circuit between two terminals.
- Find the Thevenin and Norton equivalent circuits between two terminals using only source transformation.
- Find the maximum load power that can be delivered between two terminals.

Review: Branch Current Method

1. Assign currents to each branch using the fewest number of unknowns (in general, you will have *n* unknowns, where *n* = # of meshes - # current sources)
   - Eg: A circuit with 3 meshes and 1 current source has 2 unknowns (2 unknown branch currents)
2. Assign voltage polarities across resistors
3. Write *n* KVL equations
4. Solve for unknown branch currents
5. Find any other quantities (depending on the specific problem)

Review: Nodal Analysis

1. Select a reference node
2. Assign node voltages
   - # Unknowns = *n* = # Nodes – 1 reference – # Voltage sources
3. Apply KCL to write *n* equations in *n* unknowns
4. Solve for (unknown) node voltages
5. Answer any questions and/or solve for any other circuit parameters desired

Superposition

What about dependent sources? They MUST remain in the circuit!

- If a circuit has 2 or more **independent** sources, the output is the sum of the partial outputs caused by each source acting along.
- Steps to apply:

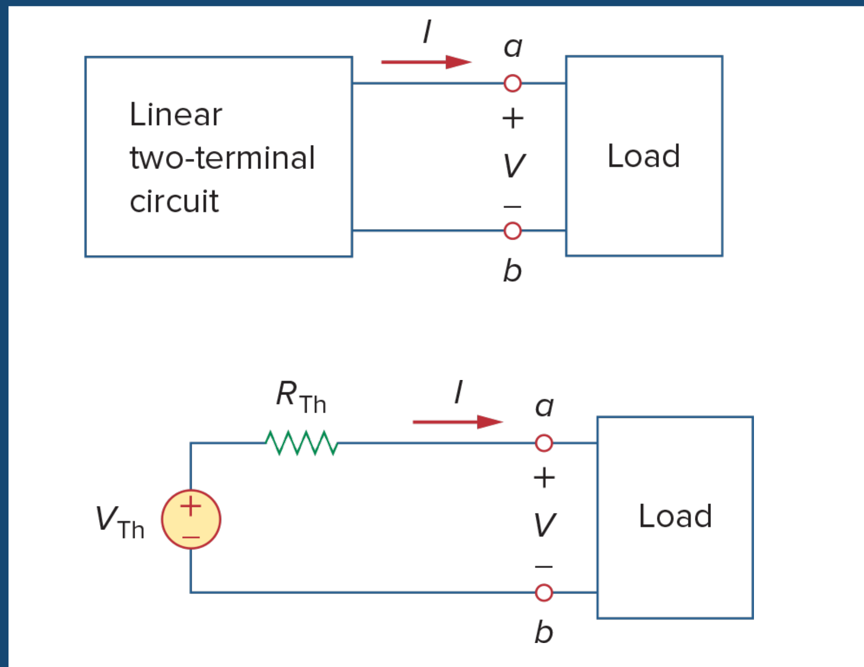
1. Turn off all but 1 **independent** source
   - Current sources look like an open circuit when turned off
   - Voltage sources look like a short circuit when turned off

2. Analyze the circuit to find the output for only that 1 independent source; this is one partial output

3. Repeat for each source

4. Total output = ∑ partial outputs (eg $V_0 = V_{01} + V_{02} + \ldots + V_{0n}$)

Thevenin’s Theorem

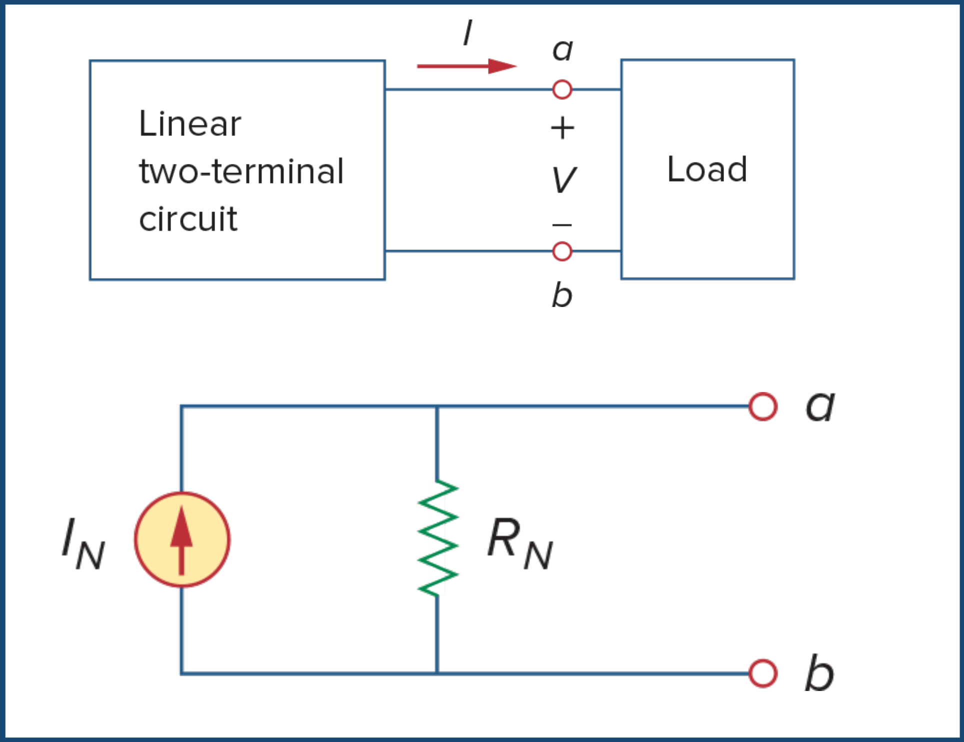


- Any linear, 2-terminal device (i.e. circuit) can be replaced with a voltage source equal to the **open circuit voltage** in series with a resistor equal to the resistance ‘looking in’ with all independent sources OFF.

Steps to find Thevenin equivalent circuit

1. Find the Thevenin voltage, V_T, by analyzing your circuit to find the voltage across the terminals of your device.
2. Find the Thevenin resistance, R_T, by turning OFF all **independent** sources and finding the equivalent resistance 'seen' between the terminals of your device.
   - If you only have independent sources, you are left with a resistor network.
   - If you have dependent sources, you must use a test source across the terminals of your device and then use Ohm's Law to find the resistance. NEVER take out dependent sources when finding R_T.
3. Draw your equivalent circuit between the terminals of your device. This is a voltage sources in series with a resistor.

Norton’s Theorem



Notice:

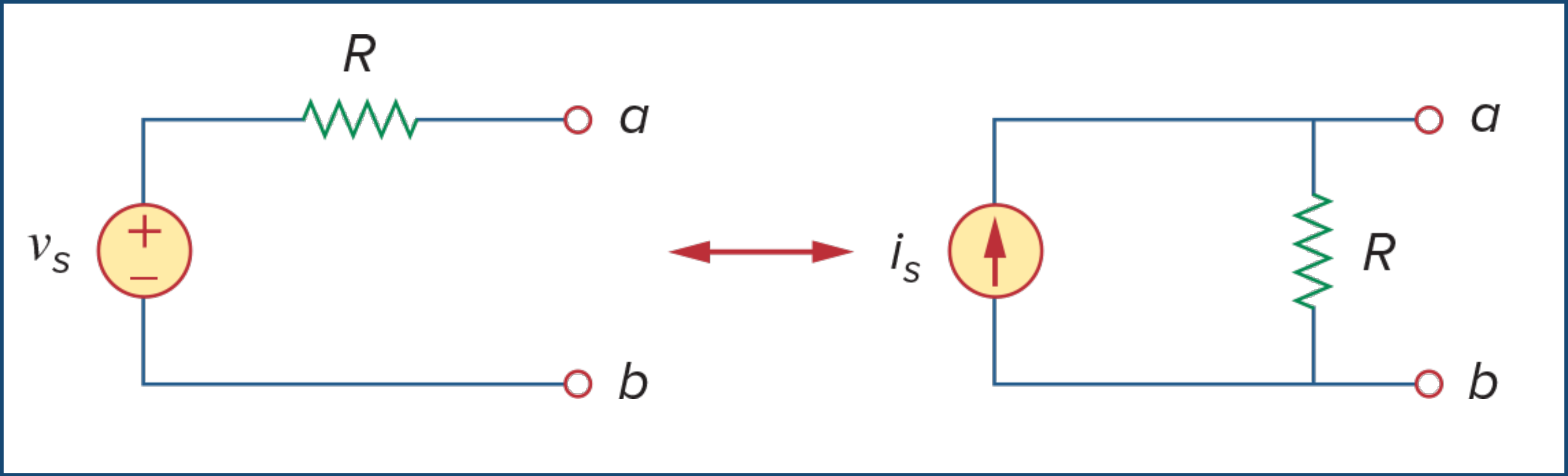
$R_T = R_N = R_{eq}$

- Any linear 2-terminal circuit can be replaced with a current source equal to the **short circuit current** in parallel with a resistor equal to the resistance looking in with all **independent** sources OFF.

Steps to find Norton equivalent circuit

1. Find the Norton current, I_N, by analyzing your circuit to find the short-circuit current between the terminals of your device.
2. Find the Norton resistance, R_N, by turning OFF all **independent** sources and finding the equivalent resistance 'seen' between the terminals of your device.
   - If you only have independent sources, you are left with a resistor network.
   - If you have dependent sources, you must use a test source across the terminals of your device and then use Ohm's Law to find the resistance. NEVER take out dependent sources when finding R_N.
3. Draw your equivalent circuit between the terminals of your device. This is a current source in parallel with a resistor.

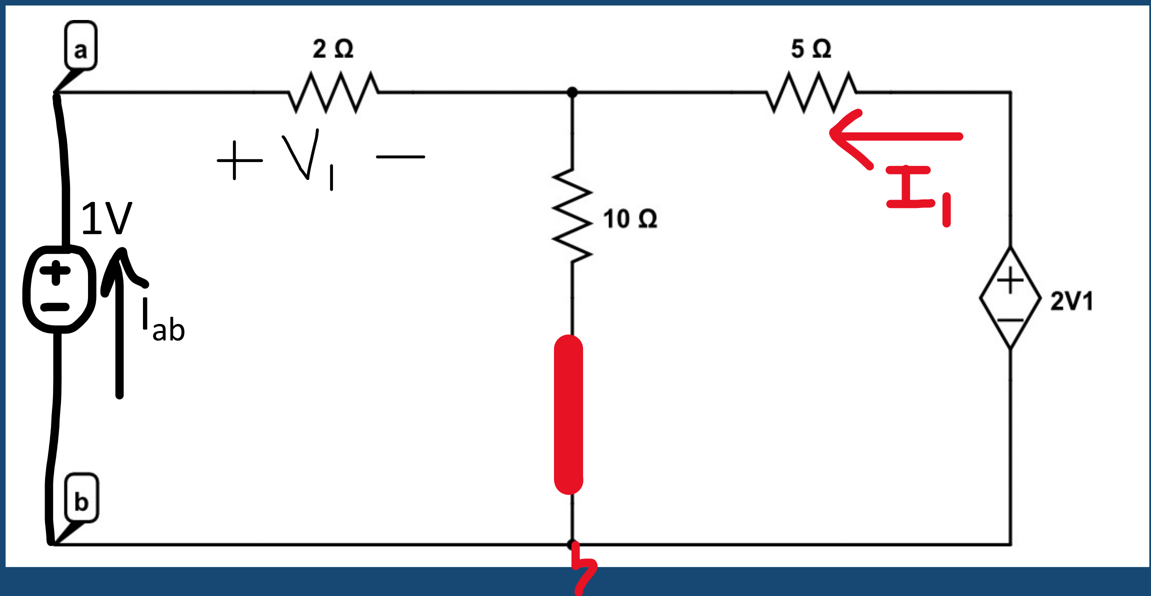
Source Transformation



- $V_S = I_s R$

Recap: Find the Thevenin resistance

- Did I have to use a current source? No! Try a voltage source:



- KVL Left: $-1+2I_{ab}+10(I_1+I_{ab})=0$
- KVL Right: $-10(I_1+I_{ab})-5I_1+4I_{ab}=0$

→ I_1 = -1/20 A, I_{ab}=1/8 A → 1 A = $I_{ab}R_T$ → **R_T = 8 Ω** ✅

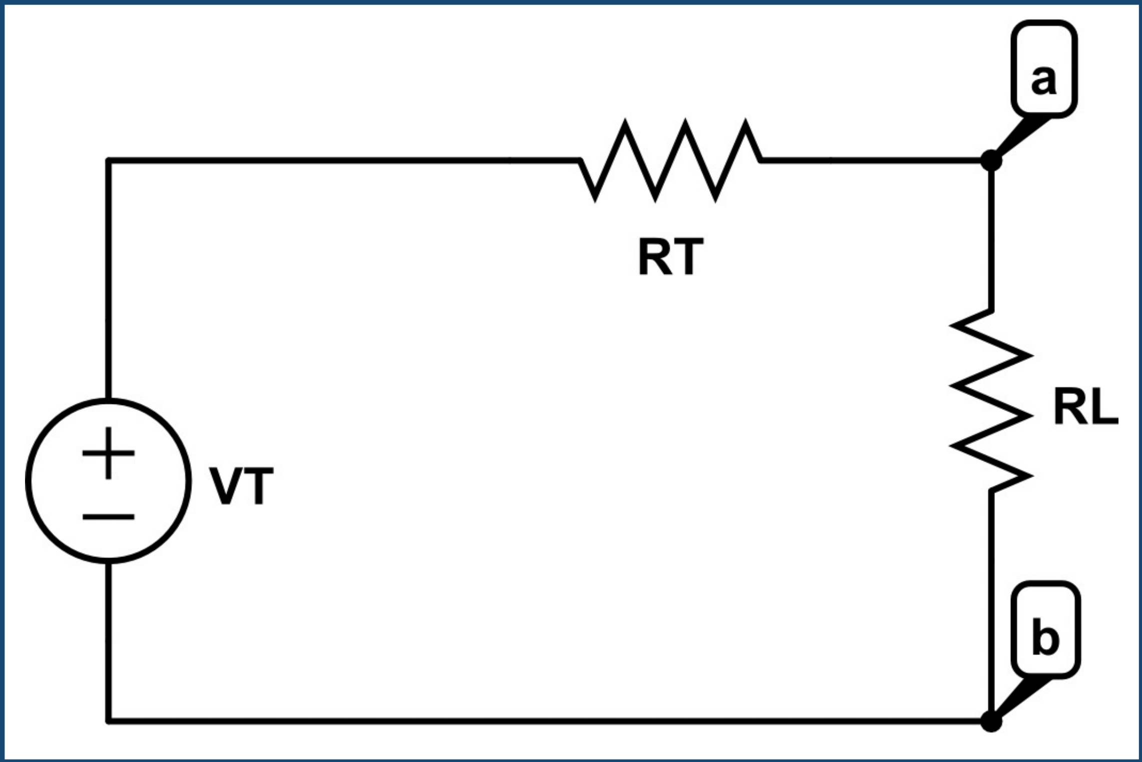
Why Thevenin?

Why Thevenin?

- If we are only interested in what's happening (i.e. things like voltage, current, power, etc) between terminals *a* and *b*, then we can **greatly** simplify our analysis by replacing the rest of the circuit with its Thevenin (or Norton!) equivalent.
- We also might only be interested in a single subsystem within our circuit.
- It turns out that it also tells us something about how to transmit the **maximum** power to a load between terminals *a* and *b*.

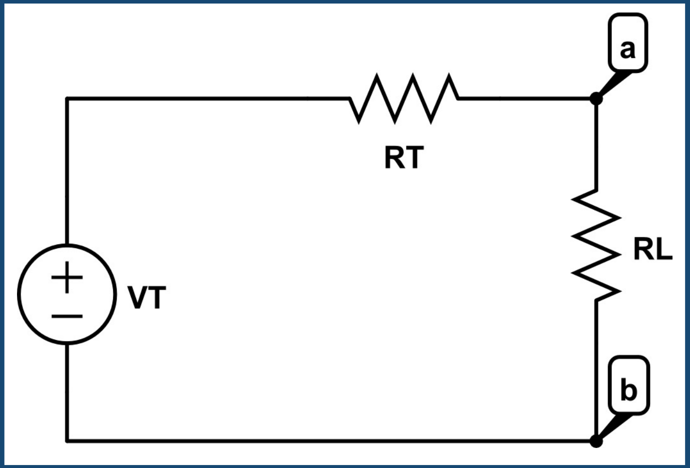
Load Power

- Consider a Thevenin equivalent circuit with an unknown resistive load R_L connected to it.



Load Power

- Consider a Thevenin equivalent circuit with an unknown resistive load R_L connected to it.
- Find an expression for R_L's power in terms of V_T , R_T , and R_L:



Load Power

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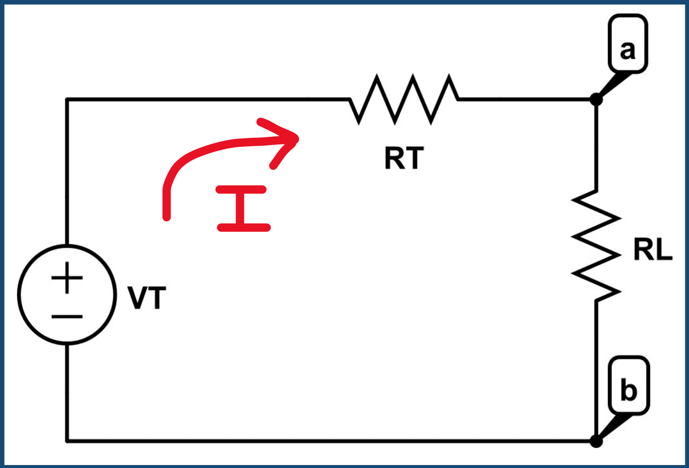
- Find an expression for R_L's power in terms of V_T , R_T , and R_L:

$$P_L = I^2 R_L = \left(\frac{V_T}{R_T+R_L}\right)^2 R_L = \frac{R_L}{(R_T+R_L)^2} V_T^2$$



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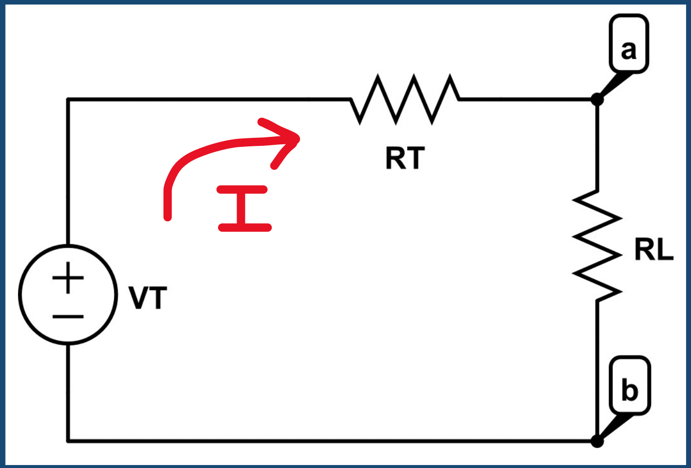
- What if $R_L = 0\ \Omega$?



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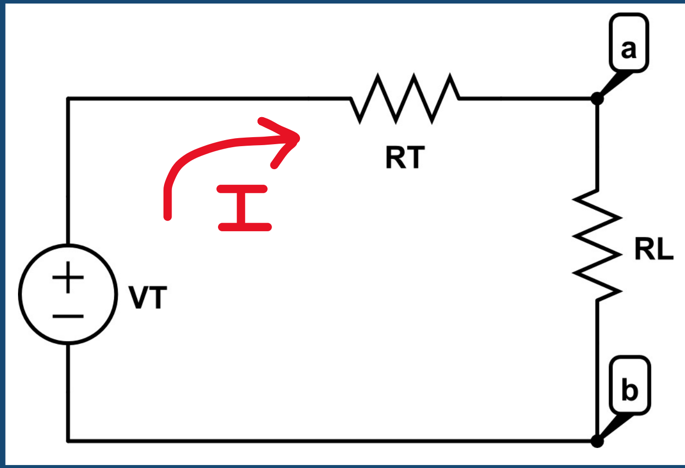
- What if $R_L = 0\ \Omega$? $\quad P_L = 0$ W
- What if $R_L \rightarrow \infty\ \Omega$?



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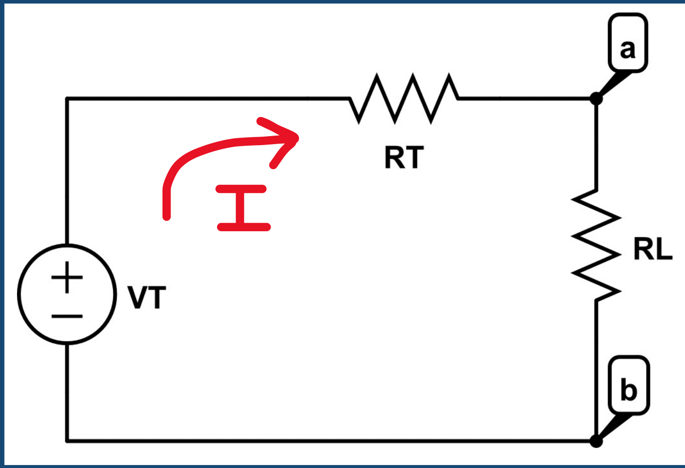
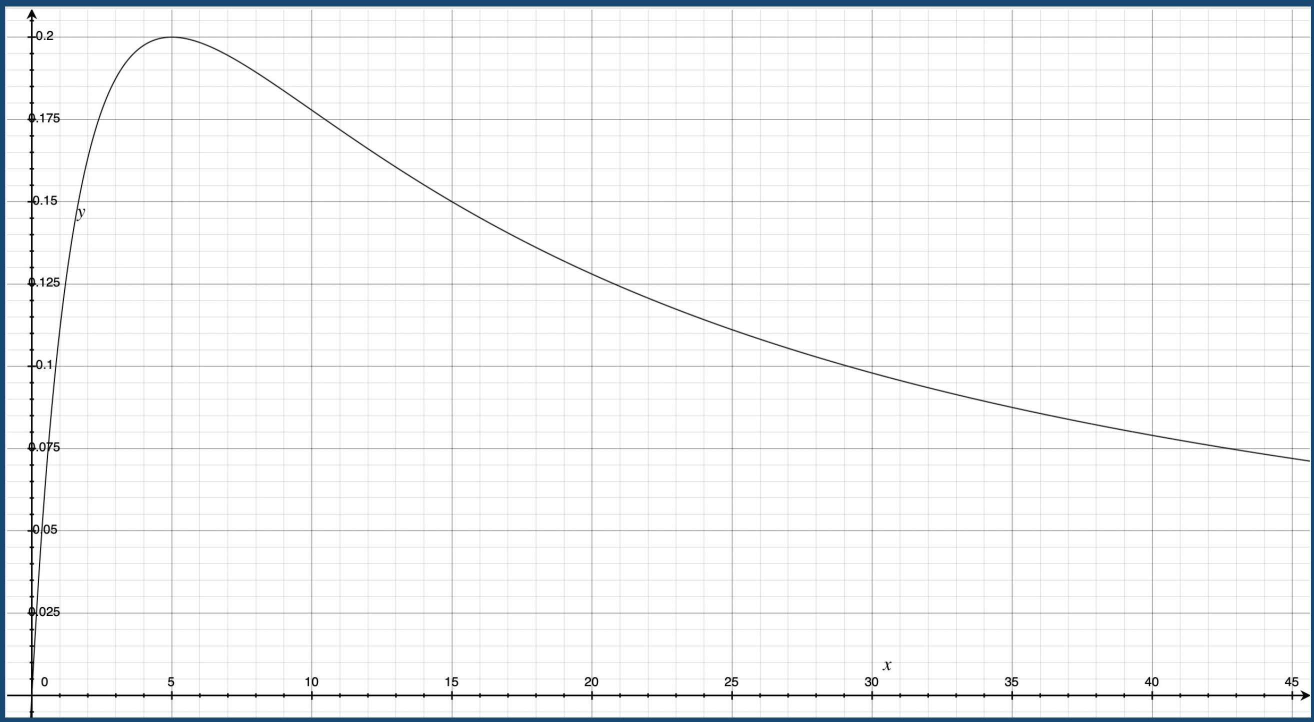


- What if R_L = 0 Ω? P_L = 0 W
- What if $R_L \rightarrow \infty$ Ω? P_L = 0 W
- What about in between?

Maximum Power Graphically: Zoomed In

- Let V_T = 2 V, R_T = 5 Ω and graph P_{RL} vs R_L:

P_{RL} (W)



R_L (Ω)

At what load resistance does the maximum power occur?

R_L = 5 Ω = R_T !

But Why?

- Recall that we found the relationship $P_L = \frac{R_L}{(R_T+R_L)^2} V_T^2$, which is a function of R_L with known parameters R_T and $V_{T.}$
- How do we find the maximum of a function?

But Why?

- Recall that we found the relationship $P_L = \frac{R_L}{(R_T+R_L)^2} V_T^2$, which is a function of R_L with known parameters R_T and V_T.
- How do we find the maximum of a function?

$$\frac{dP_L}{dR_L} = V_T^2 \left[\frac{(R_T+R_L)^2 - R_L(2(R_T+R_L))}{(R_T+R_L)^4}\right]=0$$

$$\frac{dP_L}{dR_L} = V_T^2 \left[\frac{(R_T+R_L) \quad -R_L(2)}{(R_T+R_L)^3}\right]=0 \rightarrow \frac{(R_T+R_L) \quad -R_L(2)}{(R_T+R_L)^3} = 0 \rightarrow \frac{(R_T-R_L)}{(R_T+R_L)^3} = 0$$

→ **$R_T = R_L$**

Summary

- Thevenin equivalent circuits allow us to simplify circuit analysis and/or easily calculate the maximum load power that can be delivered between two terminals.
- The maximum transferred power to a load occurs when $R_L = R_T$.
- Source transformation allows us to convert a current source in parallel with a resistor to an equivalent voltage source in series with the same resistance.