CMSC 315 Project #3: Binary Trees

In this project, you'll work with various types of Binary Trees, including Complete Binary Trees, Binary Heaps, and Binary Search Trees.

Let's review the similarities and differences between these tree structures.

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1. **Binary Tree (General)**

A binary tree is a tree in which each node has at most two children. There are no additional constraints.

Example:

```
      A
     / \
    B   C
       /
      D
```

- Node A has two children (B and C).
- Node C has one child (D).
- No rules about ordering or completeness are followed.

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2. **Complete Binary Tree**

A complete binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes in the last level are as far left as possible — meaning the last level is filled left to right with no gaps.

Example:

```
      A
     / \
    B   C
   / \
  D   E
```

- All levels are filled except the last.
- The last level is filled **left to right**.

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3. **Binary Heap**

A binary heap is a complete binary tree where each parent node follows the **heap property**:

- **Max-heap**: Parent ≥ children.
- **Min-heap**: Parent ≤ children.

Example (Max-Heap):

```
       90
      /  \
    70    50
   / \   /
  20 40 10
```

- Every parent is **greater than or equal to** its children.
- It is also a **complete binary tree**.

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4. **Binary Search Tree (BST)**

A binary search tree (BST) is a binary tree where, for every node, all values in its left subtree are less than the node's value, and all values in its right subtree are greater. This rule must hold recursively for every node in the tree, not just its direct children.

Example:

```
         60
        /  \
      40    80
     /        \
   30          90
     \        /  \
      35     85   100
                    \
                     102
```

Summary Table:

Tree Type	Structure	Ordering Property
Binary Tree	Any	None
Complete Binary Tree	All levels full, last left-to-right	None
Binary Heap (Max)	Complete	Parent ≥ children
Binary Search Tree	Any	Left < Parent < Right

What about an empty binary tree, i.e., a tree with 0 nodes?

An empty binary tree is considered a valid binary search tree as well as a valid max-heap. That's because the rules for both structures apply to each node in the tree — and in the case of an empty tree, there simply aren't any nodes to violate the rules!

Starter Code Info

Download project3_starter.zip and extract the files. The zip contains three classes, `Main`, `CompleteBinaryTree`, and `InvalidTreeException`. You should be able to create a new Java project and copy the classes into your project.

A complete binary tree can be efficiently represented using an array, which is why heaps are typically implemented this way. However, in this project, you'll implement the tree using a recursive data structure to gain practice with recursion.

The `CompleteBinaryTree` class defines one instance variable named `root`, which is an instance of the nested class `TreeNode`. Each `TreeNode` stores an integer value, along with references to left and right nodes that are the roots of two subtrees.

The `Main` class contains a `main` method with code to create an instance of `CompleteBinaryTree` from a sample array in integers. The tree is built using the array in level-order, meaning the nodes are filled from top to bottom, left to right, one level at a time. Here's how the positions in the array map to the tree:

If a node is at index i in the array:

- Its left child is at index 2 * i + 1
- Its right child is at index 2 * i + 2

This pattern is used recursively to create the tree. Thus, `Integer[] values = { 90, 70, 50, 20, 40 };` produces the following tree:

```
        90
       /  \
     70    50
    / \
  20 40
```

After creating the tree, the `main` method calls the `preorder` method to print the values using a preorder traversal.

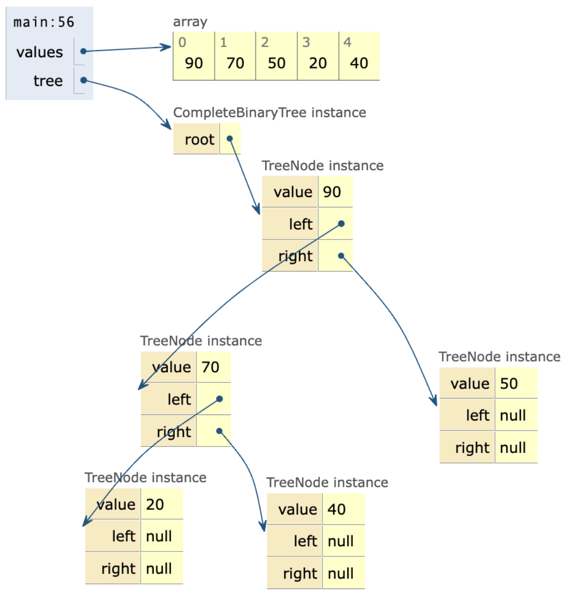
Run the `Main` class and confirm the output:

```
Preorder: 90 70 20 40 50
```

It is important to understand the `CompleteBinaryTree` constructor and the recursive helper method named `makeNode`. Try using a debugger to step through the creation of the binary tree:

- Set a breakpoint in the `CompleteBinaryTree` constructor and use the IDE's debugger to **step into** each call to the `makeNode` method. Pay special attention to the value of the `index` parameter for each recursive call.

- Alternatively, click this link to use the Python Tutor code visualizer with a slightly adapted version of the code (a RuntimeException is thrown to reduce the code size). Keep pressing the `Next` button to execute each line of code and view the method call stack and object structures.



NOTE: Python tutor only allows one top-level public class. Other classes must either be nested or not declared as public.

Project #3 Tasks

You will adapt the `CompleteBinaryTree` and `Main` classes as described in the following tasks.

Task 1. Create a second `CompleteBinaryTree` constructor.

Update `CompleteBinaryTree` to add a **second constructor** with the following signature:

```
/**
 * Constructs a CompleteBinaryTree from a whitespace-separated string of
 * integers representing the tree in level-order.
 *
 * The string is parsed into integer tokens and used to recursively build
the
 * tree starting from index 0 via {@code makeNode}.
 *
 * If the input is null or contains only whitespace, the tree is
considered
 * empty ({@code root} is null). If any token is not a valid integer, an
 * {@code InvalidTreeException} is thrown.
 *
 * @param levelOrderValues the level-order representation of the tree as a
 *                         string
 * @throws InvalidTreeException if any token is not a valid integer
 */
public CompleteBinaryTree(String levelOrderValues) throws
InvalidTreeException
```

The constructor should create a complete binary tree from the parameter string:

1. **Check Input Validity**: If the input string is null or contains only whitespace, simply return to keep the tree empty (root is null).
2. **Trim and Split**: Trim the string to remove leading and trailing whitespace, then split the string into tokens using whitespace as the delimiter. A sequence of whitespace should be treated as a single delimiter.
3. **Parse Tokens**: Convert each token into an integer. If any token is not a valid integer, throw an `InvalidTreeException` with an appropriate message. This step should produce an Integer array.
4. **Initialize the Tree**: Use the parsed integers to recursively build the tree by calling the `makeNode` method, starting from index 0.

After implementing the new constructor, evolve the `main` method to prompt the user for a string and call the new constructor:

```
//Integer[] values = { 90, 70, 50, 20, 40 };
// CompleteBinaryTree tree = new CompleteBinaryTree(values);
System.out.print("Enter a binary tree: ");
String treeString = input.nextLine();
CompleteBinaryTree tree = new CompleteBinaryTree(treeString);
tree.preorder();
```

Test your code with valid input:

```
Enter a binary tree: 90 70 50 20 40
Preorder: 90 70 20 40 50
```

Test your code with invalid input to confirm the exception is thrown and handled:

```
Enter a binary tree: 50 30 abc 99
Node value must be an integer.
```

Test your code with empty input or input containing just whitespace. No exception should be thrown and preorder is called on the empty tree.

```
Enter a binary tree:
Preorder:
```

Task 2. Evolve the `preorder` method to use indentation to reflect the tree structure.

Assume a complete binary tree with the following structure:

```
      90
     /  \
   70    50
  / \   / \
 20 40 10 25
```

The current `preorder` method prints the node values on a single line:

```
Enter a binary tree: 90 70 50 20 40 10 25
Preorder: 90 70 20 40 50 10 25
```

Update the recursive `preorder` helper method so it prints each node's value on a separate line with indentation that reflects its depth in the tree. This indentation visually aligns nodes by level, producing an indented, top-down representation of the tree's structure. For example:

```
Enter a binary tree: 90 70 50 20 40 10 25
Preorder:
90
    70
        20
        40
    50
        10
        25
```

To do this, modify the recursive `preorder` method to take a second parameter representing the current node level. Use that value to print 4 spaces per level before the node's value. The root node starts at level 0, its immediate children at level 1, and so on.

```java
/**
 * Recursive helper method for preorder traversal.
 * Prints each node's value with indentation based on its depth level
 * to visually represent the tree structure.
 *
 * @param root  the current subtree root
 * @param level the indentation level, increases with depth in the tree
 */
private void preorder(TreeNode root, int level)
```

You'll also need to update the initial call to the recursive `preorder` helper method in the non-recursive `preorder` method.

Run the `main` method to confirm your implementation.

Task 3. Check if a complete binary tree is a max-heap.

Edit the `CompleteBinaryTree` class to add a method named `isMaxHeap` that checks whether the binary tree satisfies the max-heap property. In a max-heap, the value of each parent node must be greater than or equal to the values of its children. You may implement additional helper methods as needed.

```java
/**
 * Checks whether the binary tree satisfies the max-heap property.
 *
 * @return true if the tree is a max-heap, false otherwise
 */
public boolean isMaxHeap()
```

Update the `main` method to call `isMaxHeap` and print the result. For example:

```
Enter a binary tree: 90 70 50 20 40 10 25
Preorder:
90
    70
        20
        40
    50
        10
        25
Is a max-heap: true
```

Test with a binary tree that violates the max-heap property — for example, where a child node (e.g., 20) is greater than its parent node (e.g., 17).

```
Enter a binary tree: 40 35 17 22 19 20
Preorder:
40
    35
        22
        19
    17
        20
Is a max-heap: false
```

Test with an empty tree.

```
Enter a binary tree:
Preorder:
Is a max-heap: true
```

Task 4. Check if a complete binary tree is a binary search tree.

Modify the `CompleteBinaryTree` class to include a method named `isBinarySearchTree` that determines whether the binary tree satisfies the binary search tree (BST) property.

```java
/**
 * Checks whether the tree is a valid binary search tree (BST).
 * A BST is valid if, for every node:
 * - All nodes in the left subtree are strictly less than the node's value.
 * - All nodes in the right subtree are strictly greater than the node's value.
 *
 * @return true if the tree satisfies BST properties, false otherwise
 */
public boolean isBinarySearchTree()
```

You'll most likely need a helper method that takes a node and validity range:

```java
/**
 * Recursive helper method to check BST property using range limits.
 * At each node, ensures:
 * - Node's value is strictly greater than the min bound.
 * - Node's value is strictly less than the max bound.
 * Recursively checks left and right subtrees with updated bounds.
 *
 * @param node current node in the tree
 * @param min  lower bound (exclusive) for the node's value
 * @param max  upper bound (exclusive) for the node's value
 * @return true if the subtree rooted at the current node is a valid BST, false
 *         otherwise
 */
private boolean isBinarySearchTree(TreeNode node, Integer min, Integer max)
```

Hints

1. **Understand the BST Rule:**
   In a valid binary search tree (BST), every node must be greater than all nodes in its left subtree and less than all nodes in its right subtree.

2. **Think Recursively:**
   Consider how you can check this condition not just for the current node, but also for all of its descendants.

3. **Pass Down Constraints:**
   As you traverse the tree, refine lower and upper bounds that each node's value must satisfy based on its ancestors. Values in the left subtree must be less than the node's value, and values in the right subtree must be greater.

4. **What Happens at the Leaves?**
   Determine what the base case should be when you reach the end of a branch.

5. **Avoid Local Comparisons Only:**
   Be careful not to only compare a node to its immediate children as that alone isn't enough to validate the entire tree structure.

Given this sample tree:

```
        10
       /  \
      5    15
     / \   / \
    2   7 12  20
```

Let's walk through how the value bounds are determined for each node during BST validation. A `null` bound indicates no constraint on that side of the range. When we say a node is "left of" or "right of" another, we mean it appears in that node's left or right subtree.

Node	min	max	Explanation
10	null	null	Root node — no bounds yet
5	null	10	Left of 10 → must be < 10
2	null	5	Left of 5 → must be < 5
7	5	10	Right of 5 → must be > 5 and < 10
15	10	null	Right of 10 → must be > 10
12	10	15	Left of 15 → must be > 10 and < 15
20	15	null	Right of 15 → must be > 15

Modify the `main` method to call the `isBinarySearchTree` method and print the result.

Let's test with a BST that is not a Max-Heap, a Max-Heap that is not a BST, a tree that is neither a max-heap nor a BST, an empty tree, and a tree with a single node.

BST (Not a Max-Heap)

This is a valid Binary Search Tree, but violates the max-heap property because `15 > 10` (the root):

```
        10
       /  \
      5    15
     / \   /
    2   7 12
```

Run the program to test this tree:

```
Enter a binary tree: 10 5 15 2 7 12
Preorder:
10
    5
        2
        7
    15
        12
Is a max-heap: false
Is a binary search tree: true
```

Max-Heap (Not a BST)

This is a valid Max-Heap (each parent is ≥ children), but not a BST:

```
        20
       /  \
     18    15
    /
   10
```

Not a BST: 15 is in right subtree of 20

Run the program to test this tree:

```
Enter a binary tree: 20 18 15 10
Preorder:
20
    18
        10
    15
Is a max-heap: true
Is a binary search tree: false
```

Not a Max-Heap and Not a BST

This is neither a max-heap nor a BST.

```
        10
       /  \
      5    15
     / \   / \
    2  12 11 20
```

It's not a max-heap because nodes 12, 15, and 20 violate the max-heap property (they are larger than their parent).

It's not a BST because node 12 is in the left-subtree of node 10 but 12 is not less than 10.

Run the program to confirm:

```
Enter a binary tree: 10 5 15 2 12 11 20
Preorder:
10
    5
        2
        12
    15
        11
        20
Is a max-heap: false
Is a binary search tree: false
```

Empty Tree

An empty tree is both a max-heap and a BST.

```
Enter a binary tree:
Preorder:
Is a max-heap: true
Is a binary search tree: true
```

Single Node

A tree with a single node is both a max-heap and a BST.

```
Enter a binary tree: 6
Preorder:
6
Is a max-heap: true
Is a binary search tree: true
```

Task 5. Create an in-order list of values.

Update `CompleteBinaryTree` to add a method that returns an ArrayList containing the values of the tree nodes in in-order traversal. You may implement additional recursive helper methods as needed.

```
/**
 * Returns an ArrayList containing the values of the tree nodes in in-order
 * traversal, visiting nodes in the following order:
 * - Left subtree
 * - Current node
 * - Right subtree
 *
 * @return an ArrayList containing the values of the nodes in in-order traversal
 */
public ArrayList<Integer> inorderList()
```

Update the `main` method to call the new method and print the resulting list. Notice the in-order traversal produces a sorted list when the tree is a BST, but not for a max-heap.

```
Enter a binary tree: 10 5 15 2 7 12
Preorder:
10
    5
        2
        7
    15
        12
Is a max-heap: false
Is a binary search tree: true
Inorder List: [2, 5, 7, 10, 12, 15]
```

```
Enter a binary tree: 20 18 15 10
Preorder:
20
    18
        10
    15
Is a max-heap: true
Is a binary search tree: false
Inorder List: [10, 18, 20, 15]
```

Submitting your solution

You are to submit two files.

1. The first is a `.zip` file that contains all the source code for the project. The `.zip` file should contain only source code and nothing else, which means only the `.java` files. If you elect to use a package the `.java` files should be in a folder whose name is the package name. Every outer class should be in a separate `.java` file with the same name as the class name. Each file should include a comment block at the top containing your name, the project name, the date, and a short description of the class contained in that file.

2. The second is a Word document (PDF or RTF is also acceptable) that contains the documentation for the project, which should include the following:

    - A UML class diagram that includes all classes.
    - A test plan that includes test cases that you have created indicating what aspects of the program each one is testing.
    - A short paragraph on lessons learned from the project.