

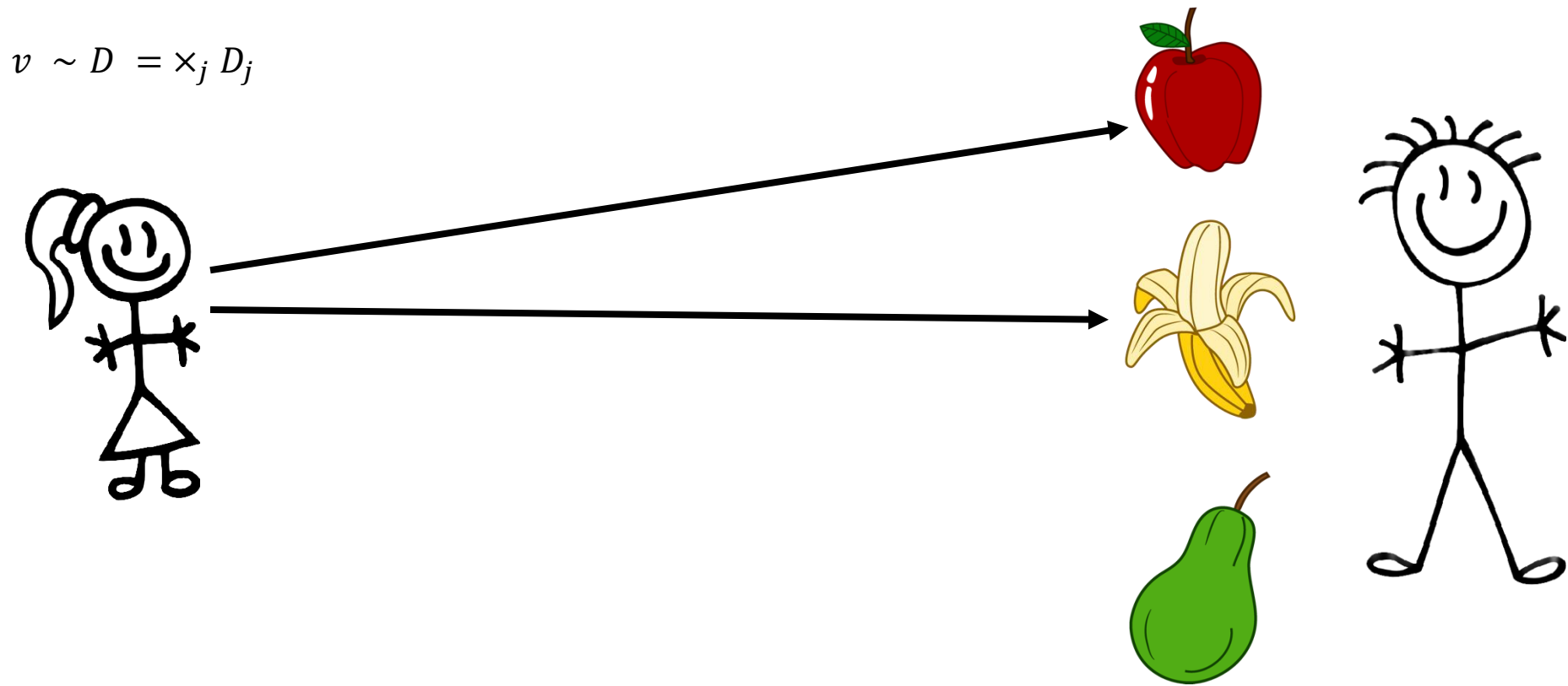
# Bundling in Oligopoly: Revenue Maximization with Single-Item Competitors

Moshe Babaioff (Hebrew University of Jerusalem),

**Linda Cai** (Princeton University), Brendan Lucier (Microsoft Research)

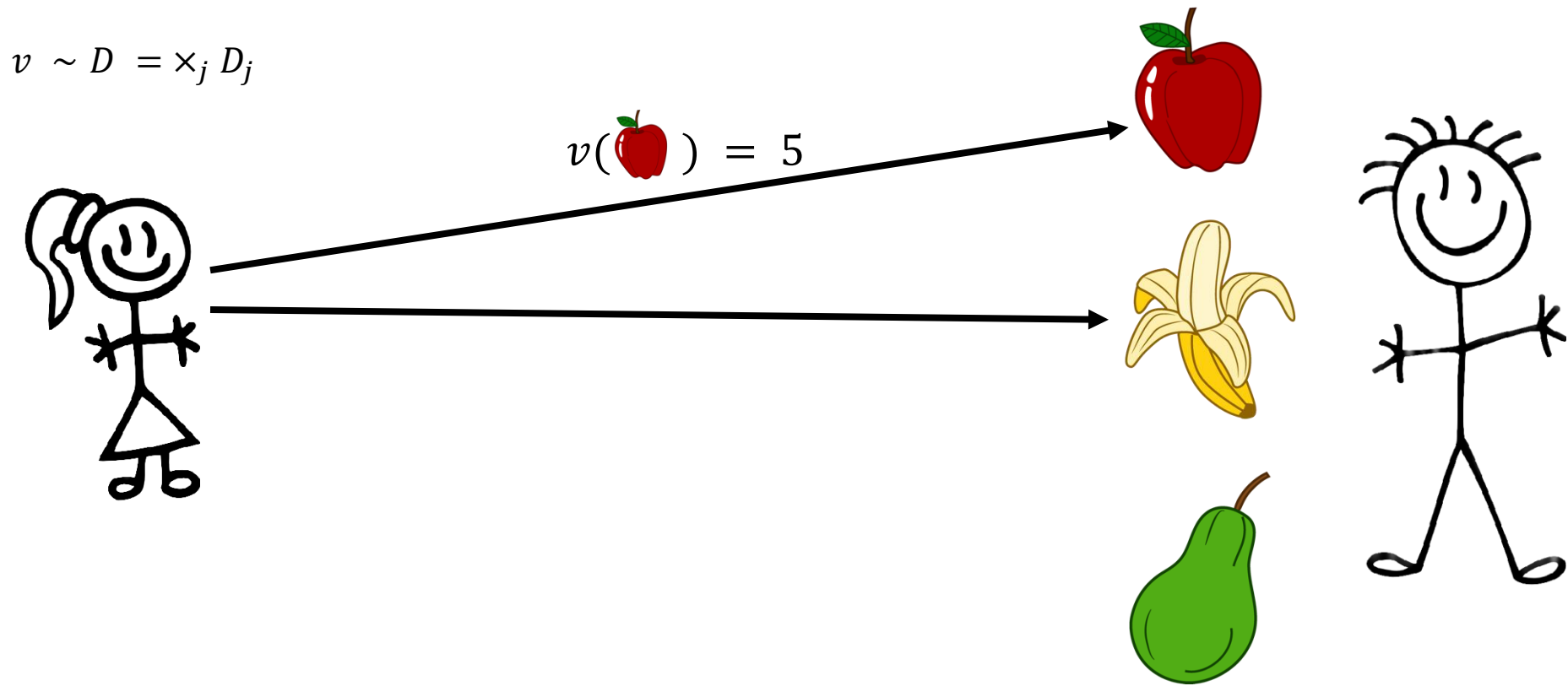


# Revenue Maximizing Auction – Monopolist Seller



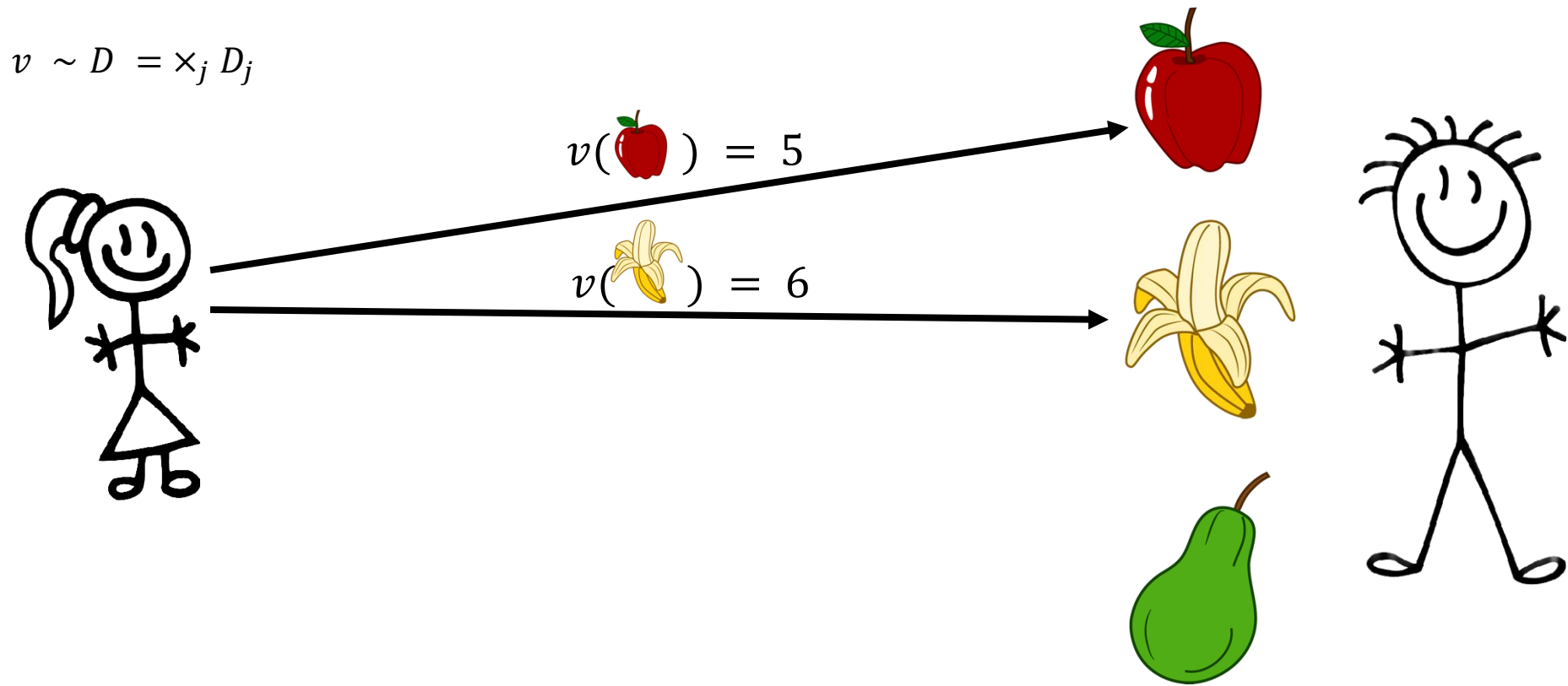
One Principal Seller with  $m$  items, One buyer with additive valuation function

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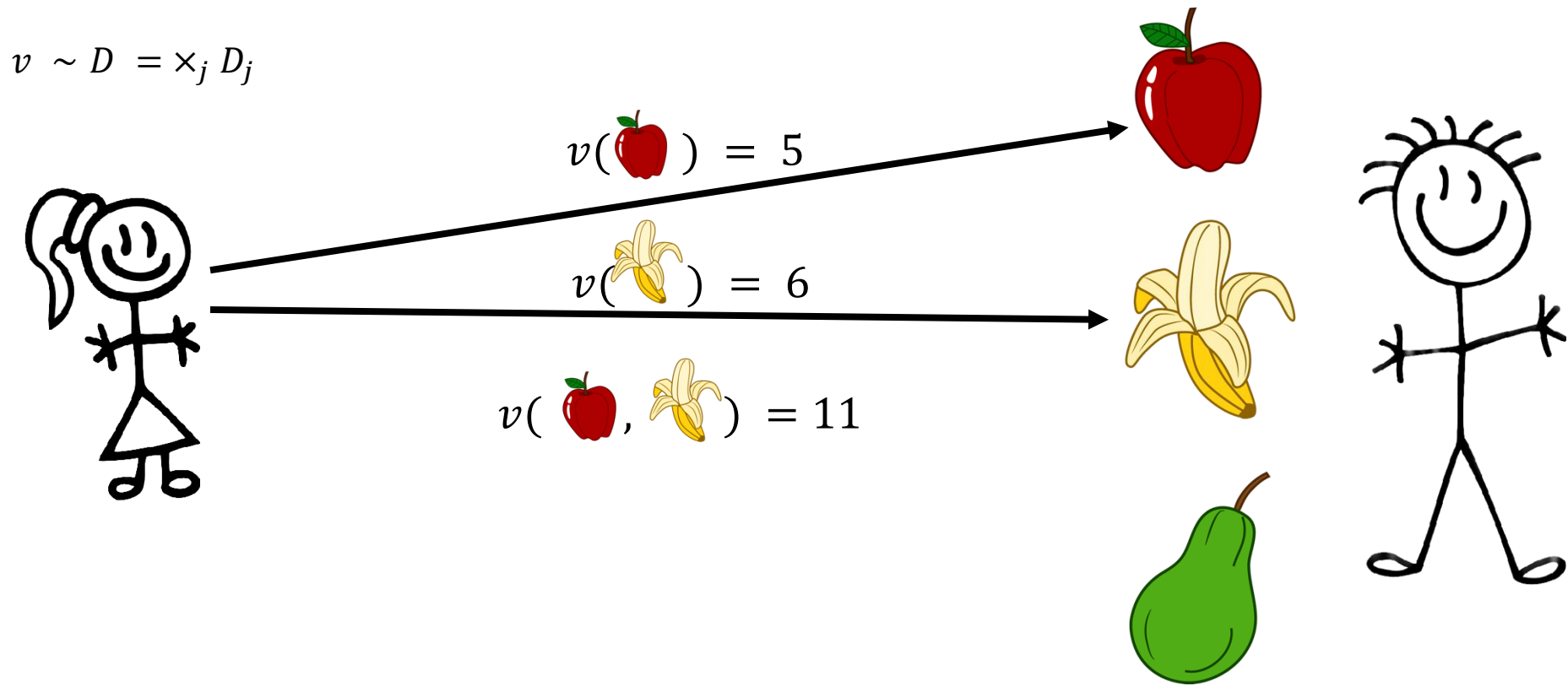
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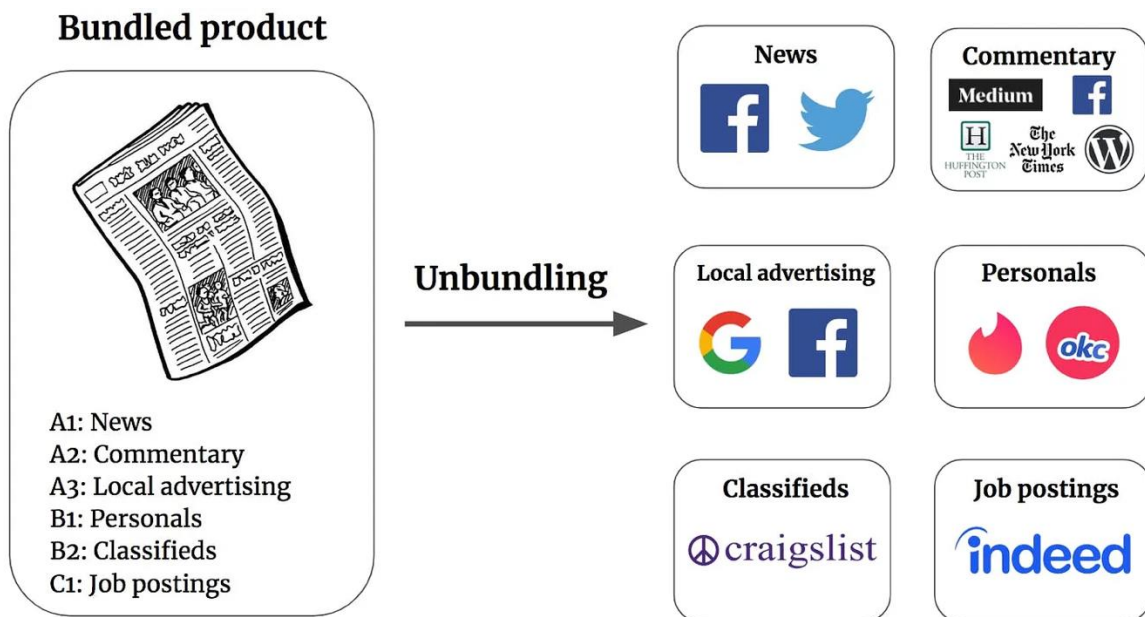


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# True Monopoly is Rare

Even dominant sellers often face sub-category competition who offer similar products.

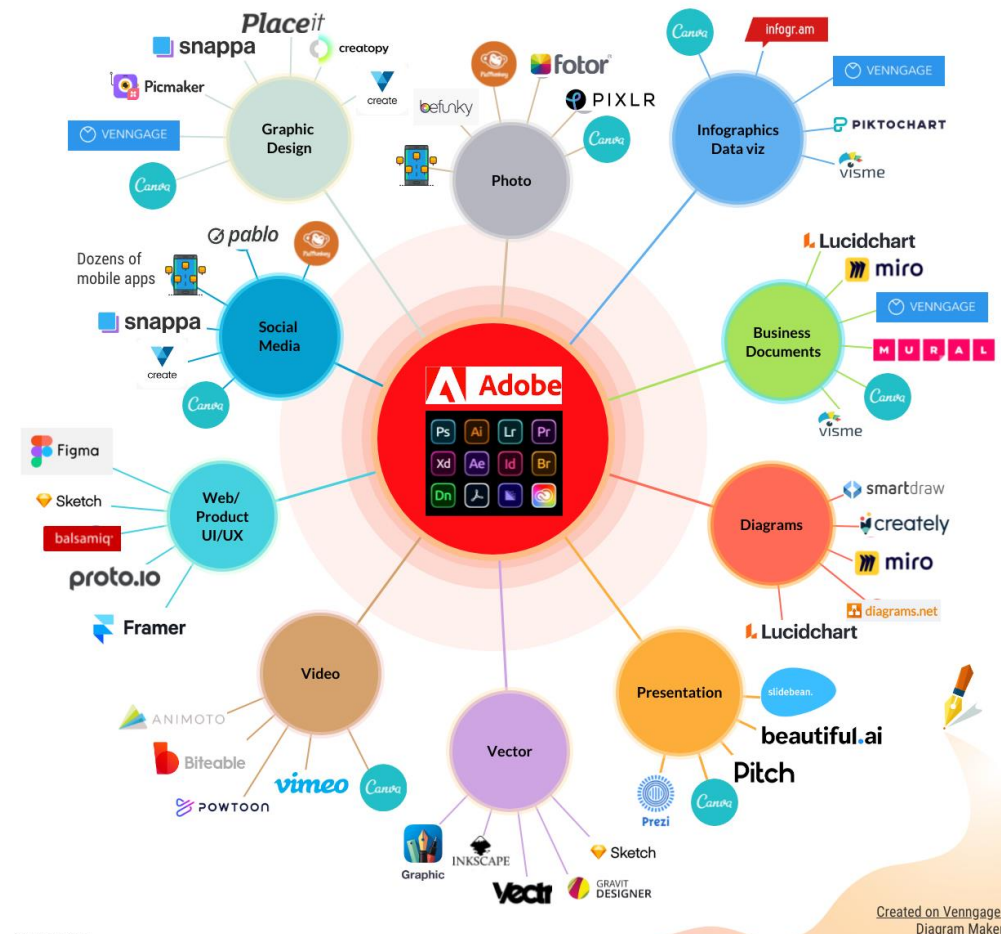
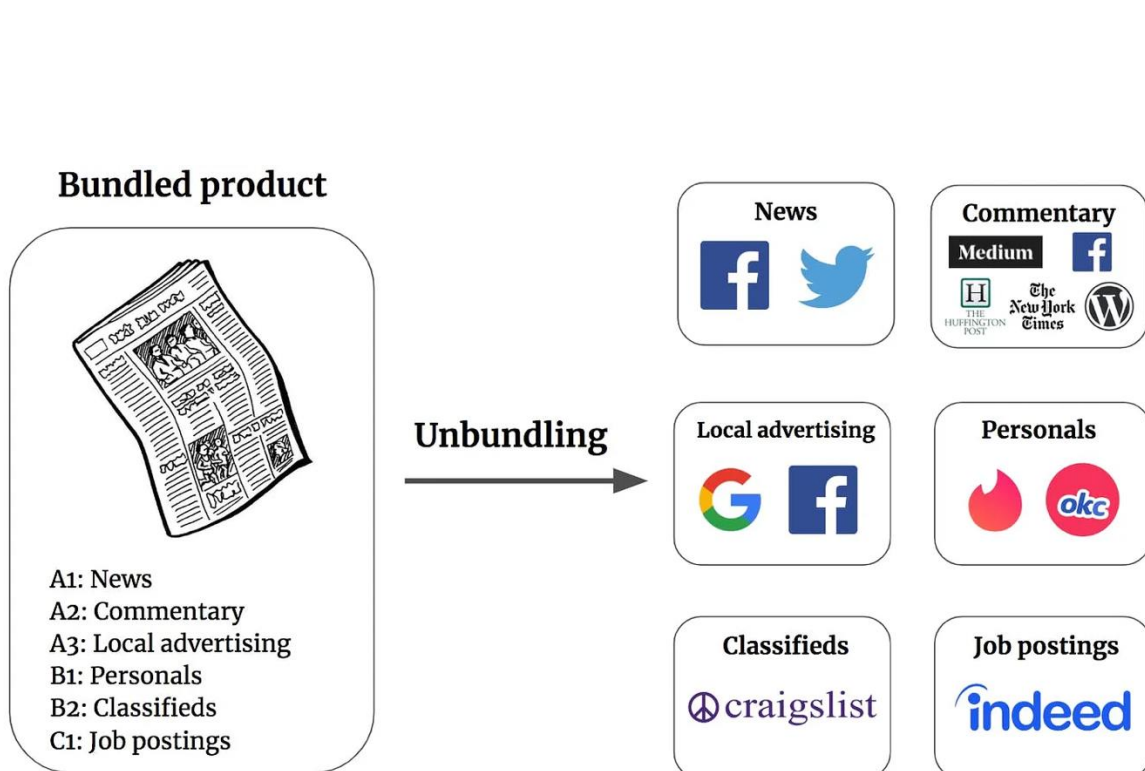
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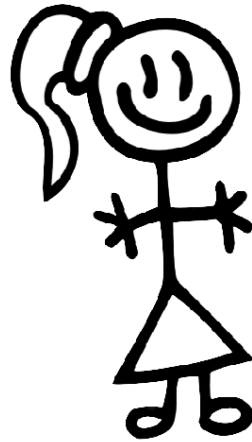


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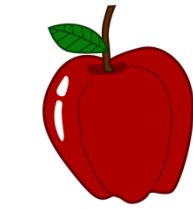


# Revenue Maximizing Auction – Our Model

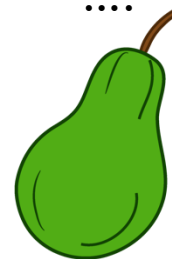
$$v \sim D = \times_j D_j$$



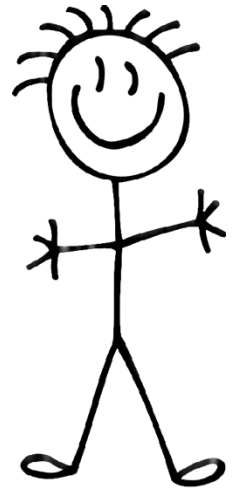
$m$  items



....

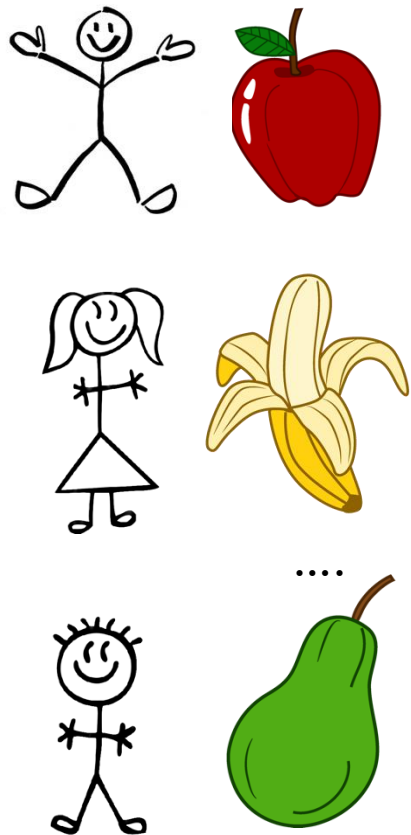


Single Multi-good  
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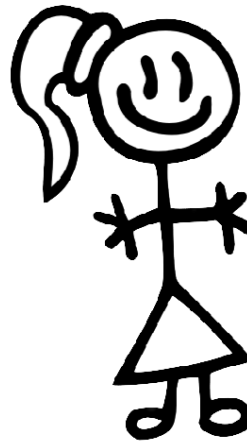


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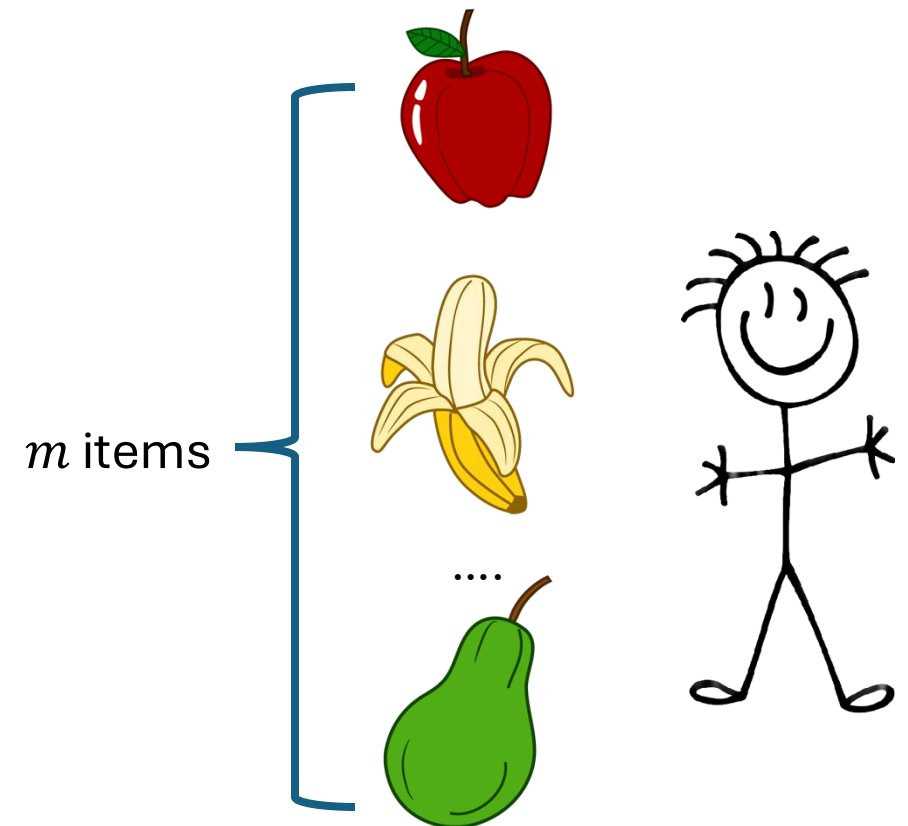
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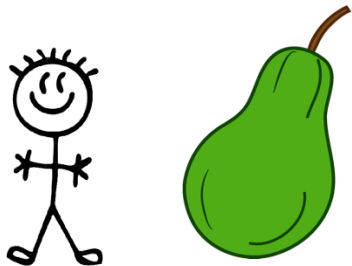
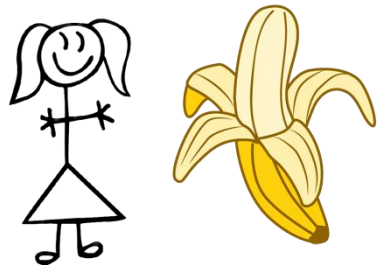
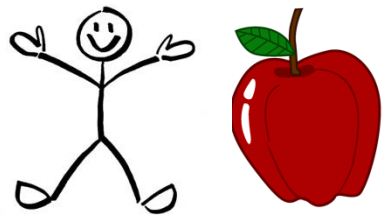


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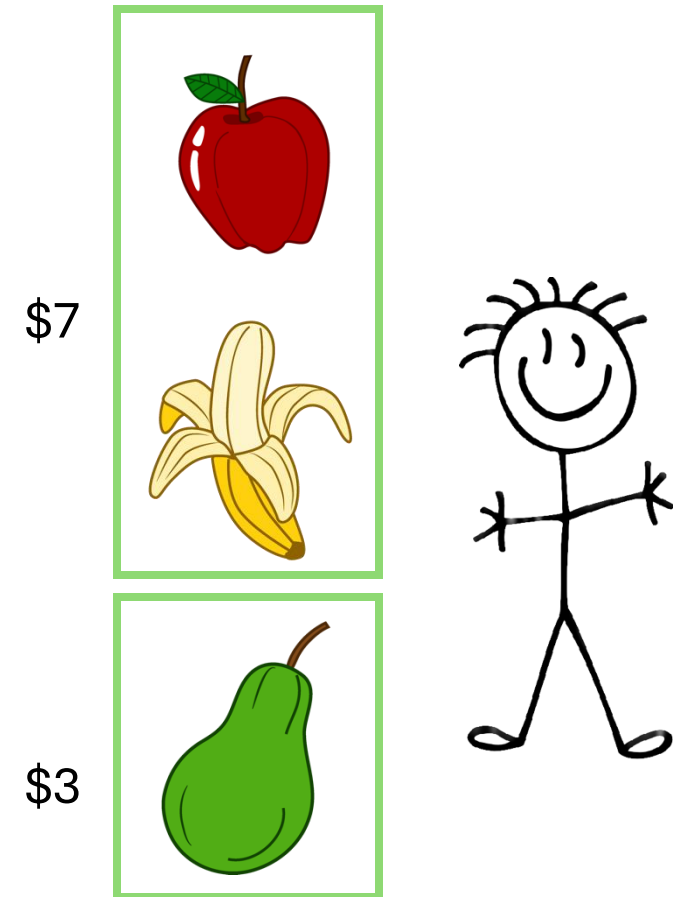


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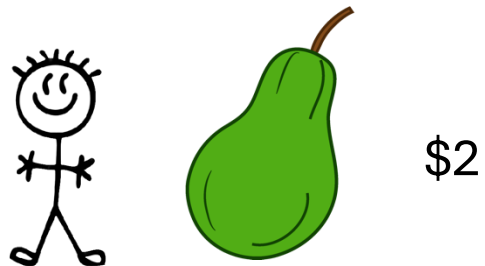
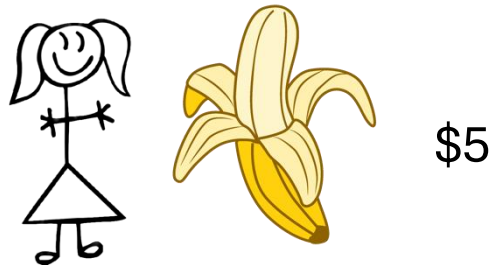
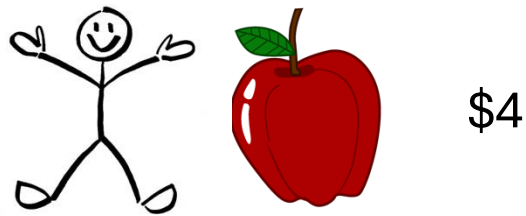
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Principal Seller Commit to a Menu

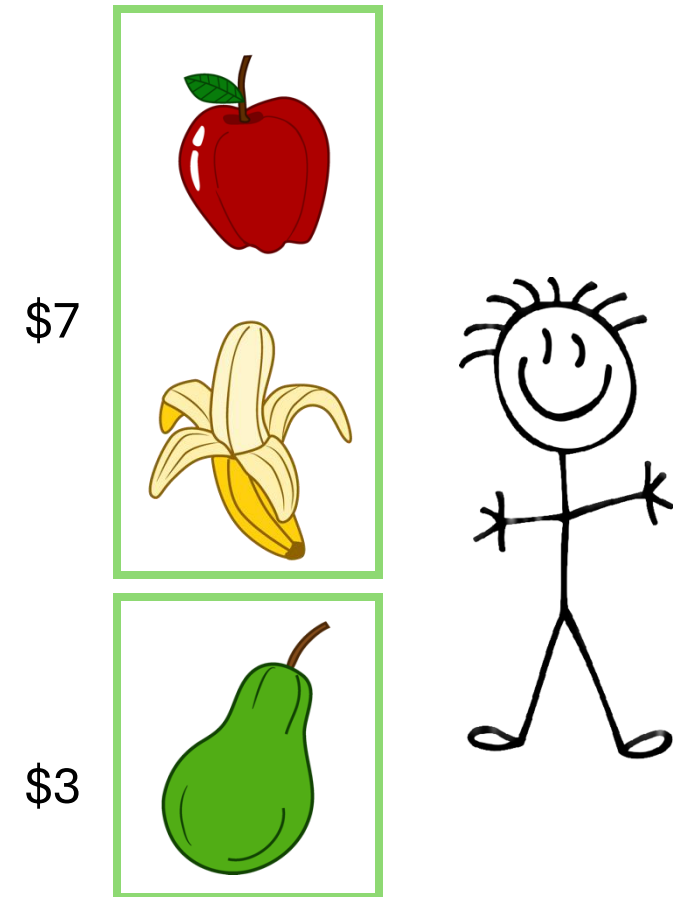
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Item Sellers Commit to a (Possibly Mixed) Pricing Strategy

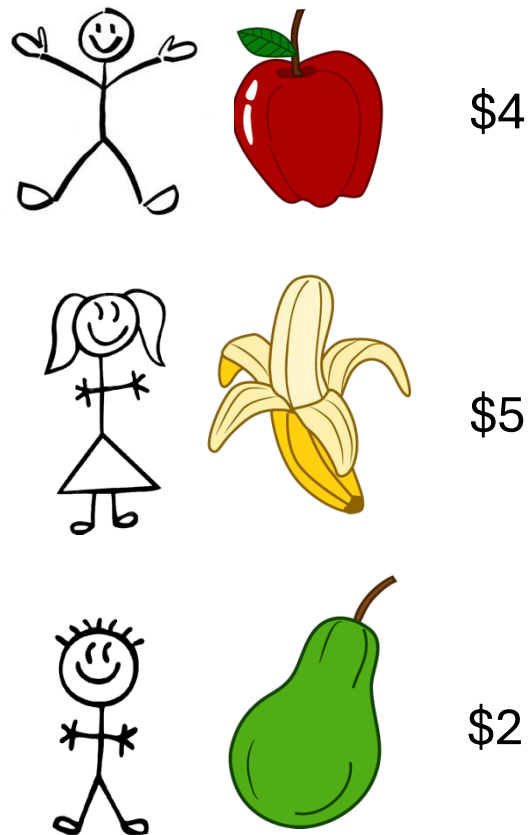
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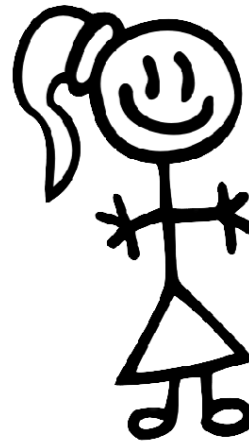
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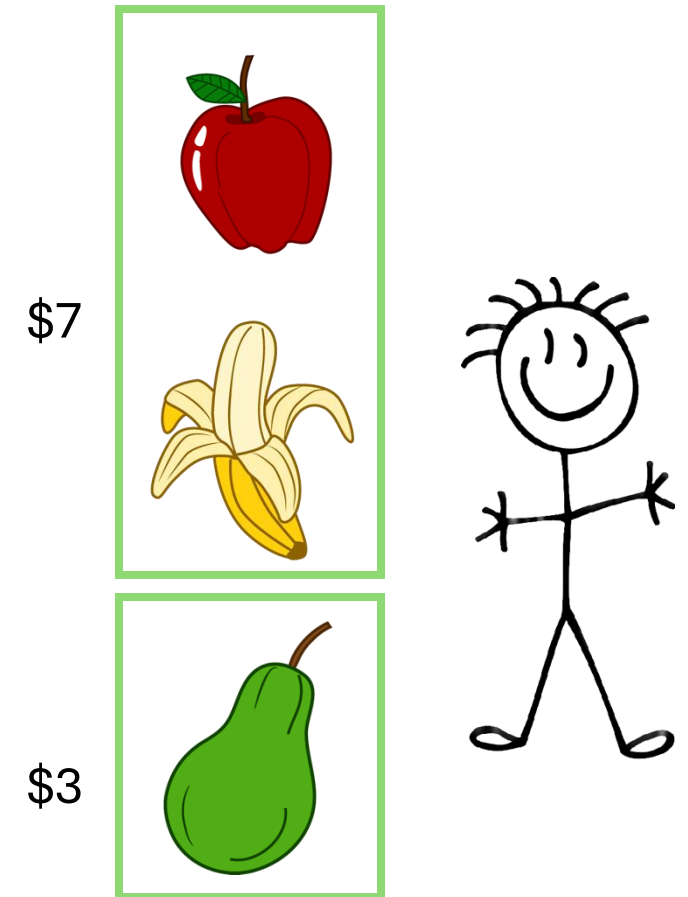
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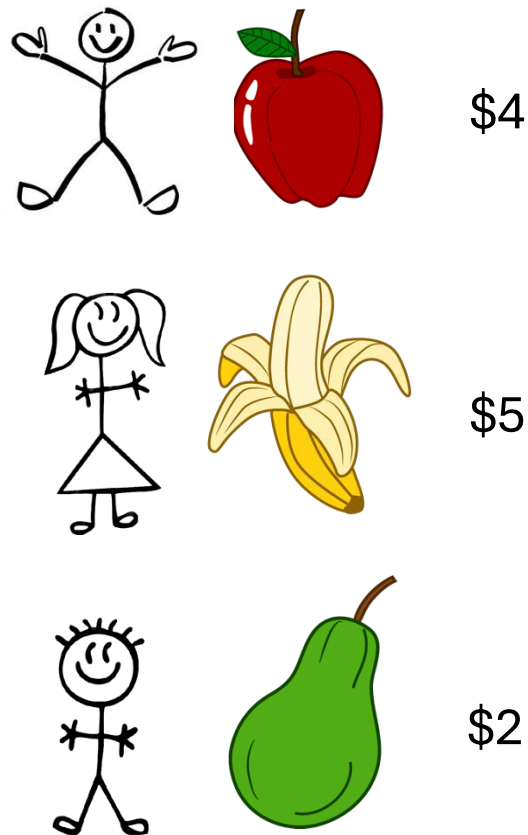
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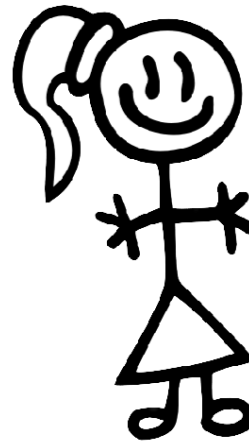
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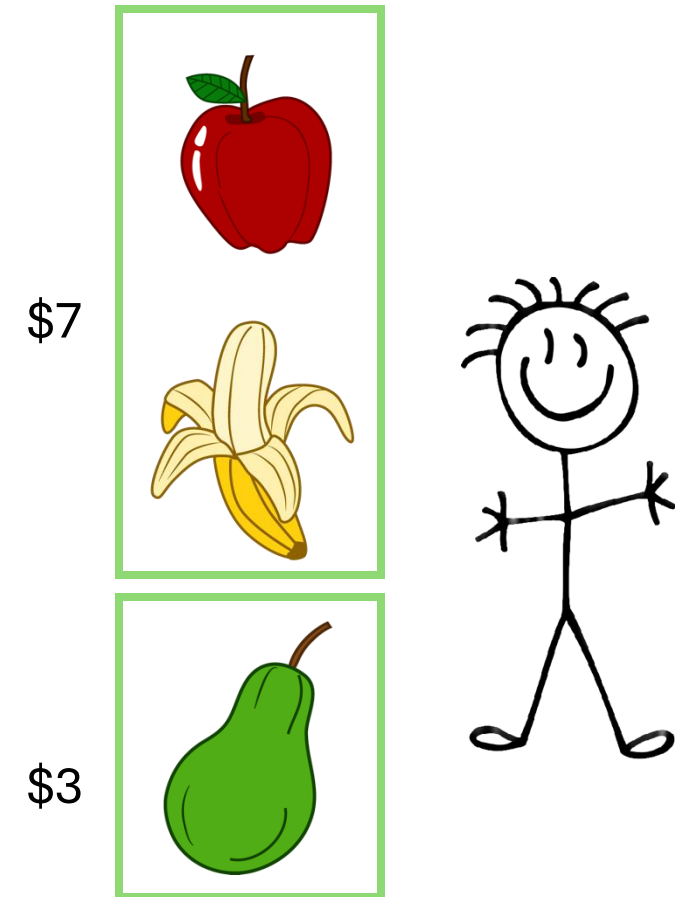
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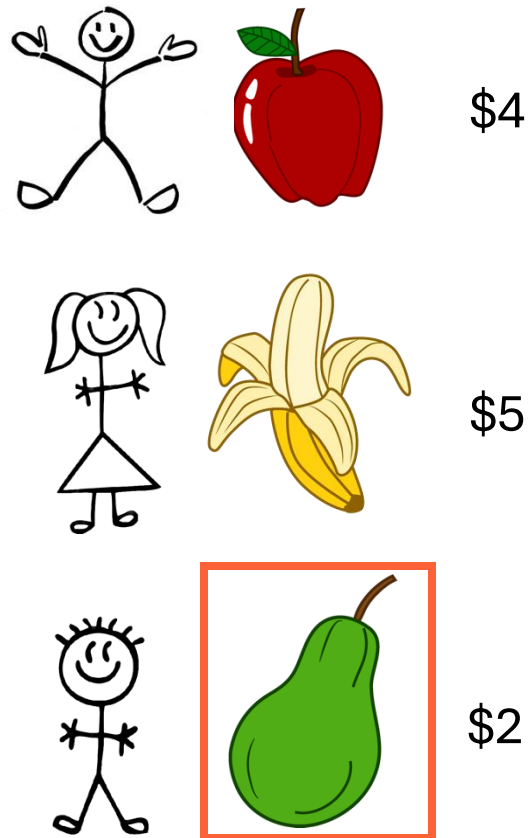


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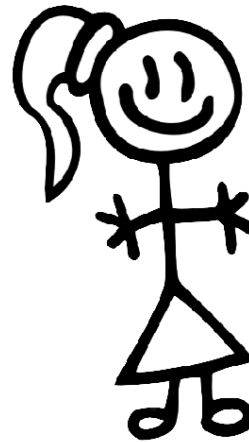
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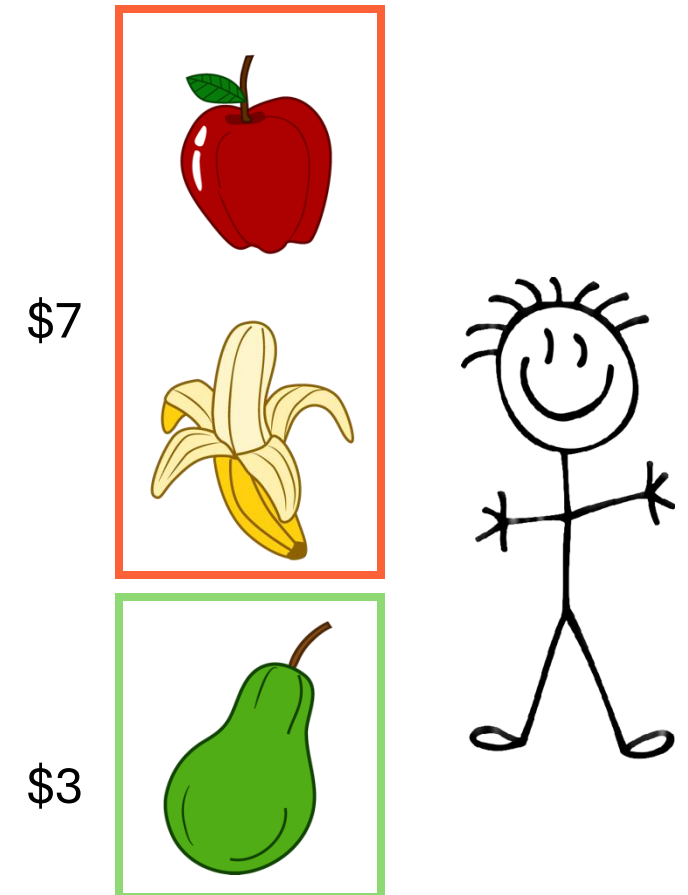
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What menu the principal should choose to maximize revenue?

- **Main Result 2:** (Under assumptions to be described later) We show that *selling the grand bundle* gives the principal a **3**-approximation to the optimal revenue.

# Revenue Benchmark - Monopolist

## **Constant Approximation To Optimal Revenue** [Babaioff, Immorlica, Lucier, Weinberg]

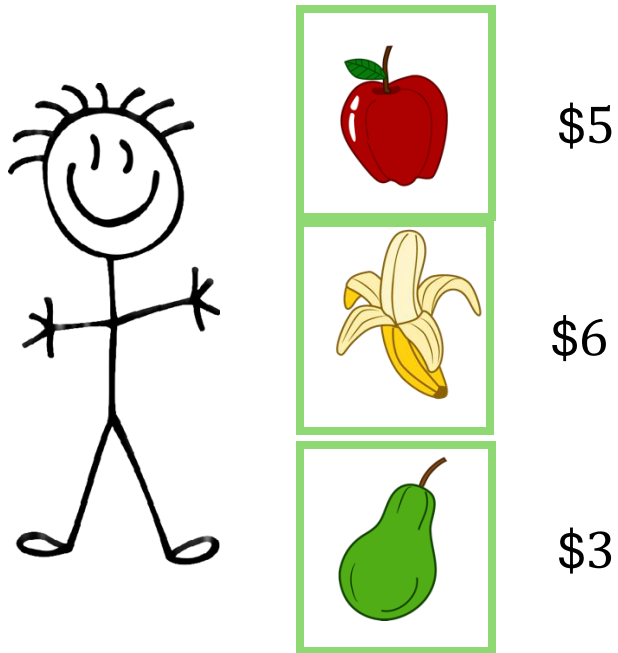
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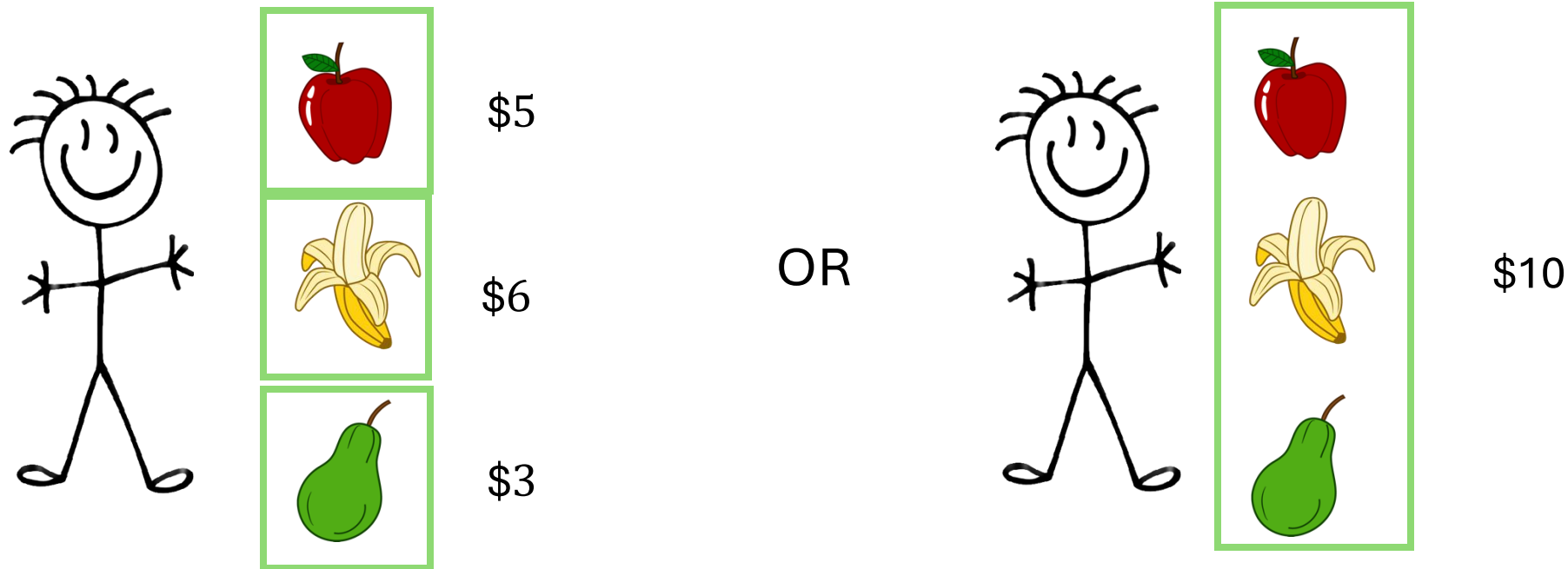
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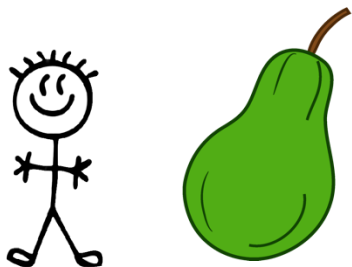
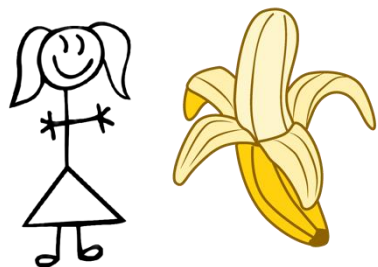
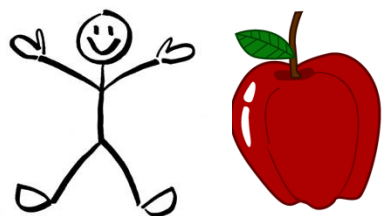
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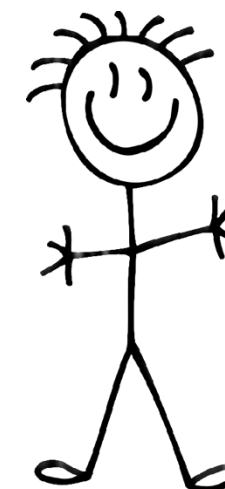
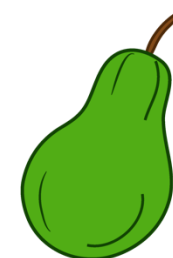


# Oligopoly - Selling Separately Yields No Revenue

Single-Item Sellers



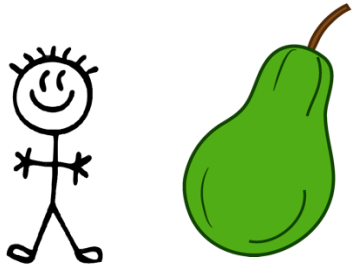
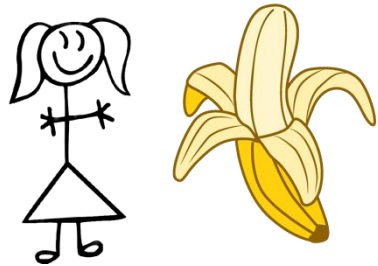
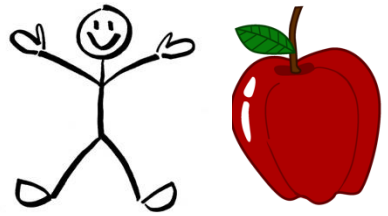
Principal Seller



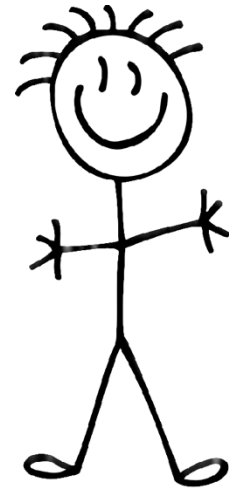
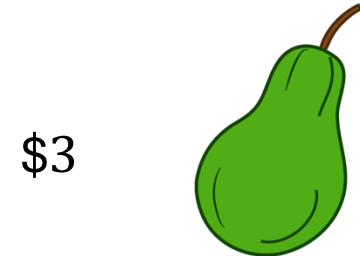
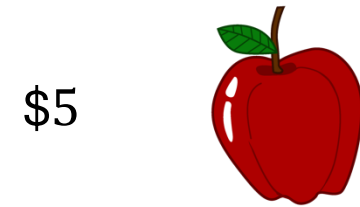
Principal seller selling items separately results in Bertrand competition

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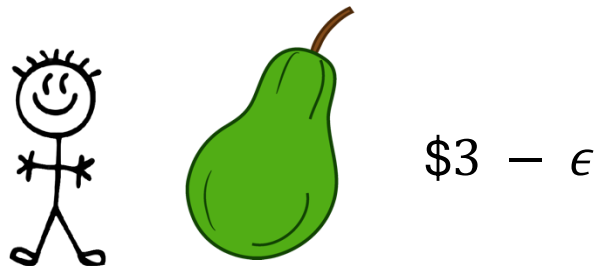
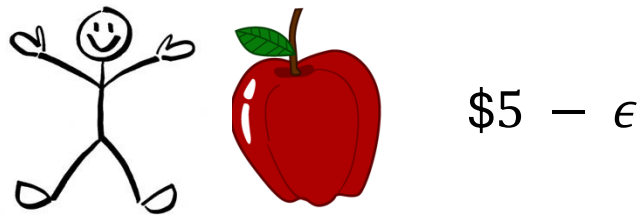
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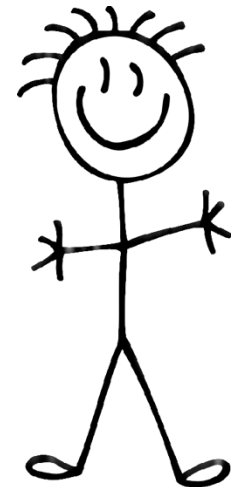
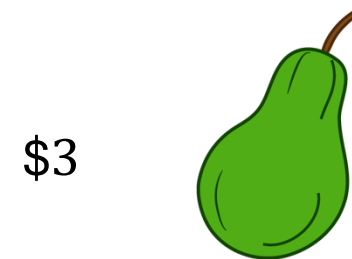
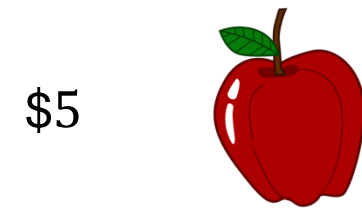
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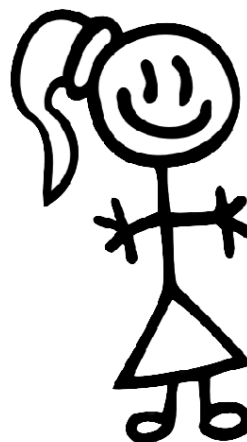
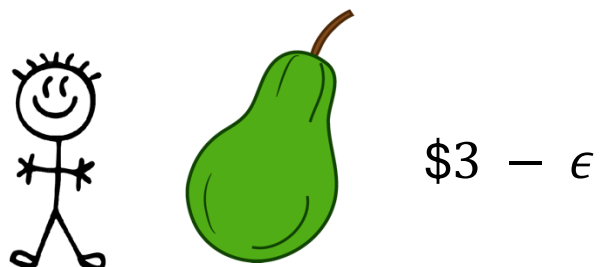
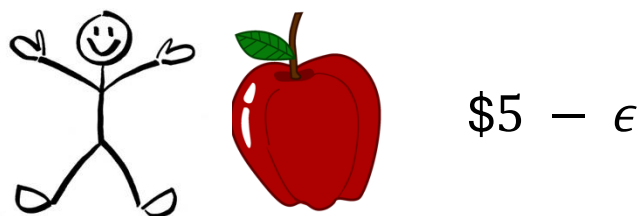
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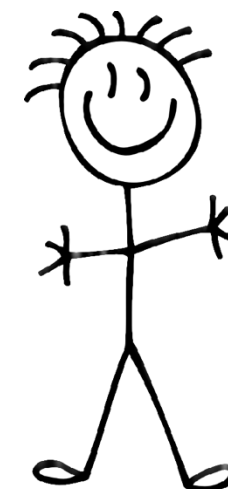
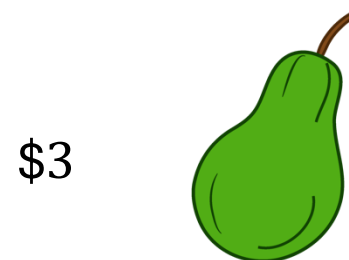


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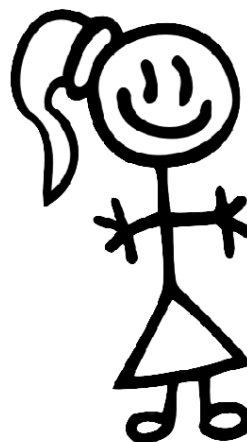
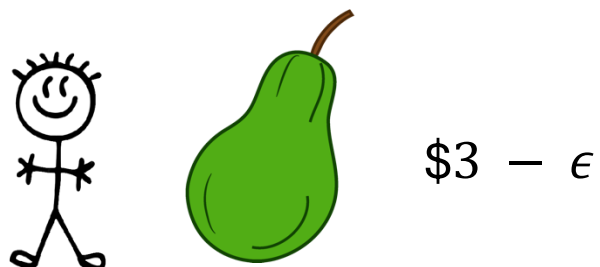
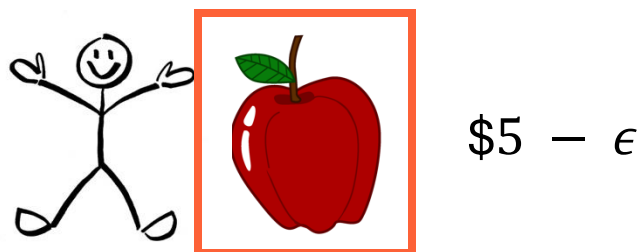


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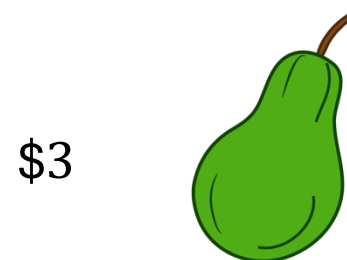
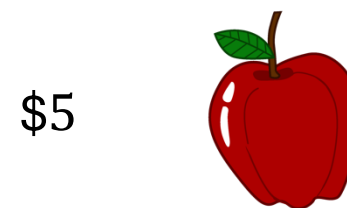


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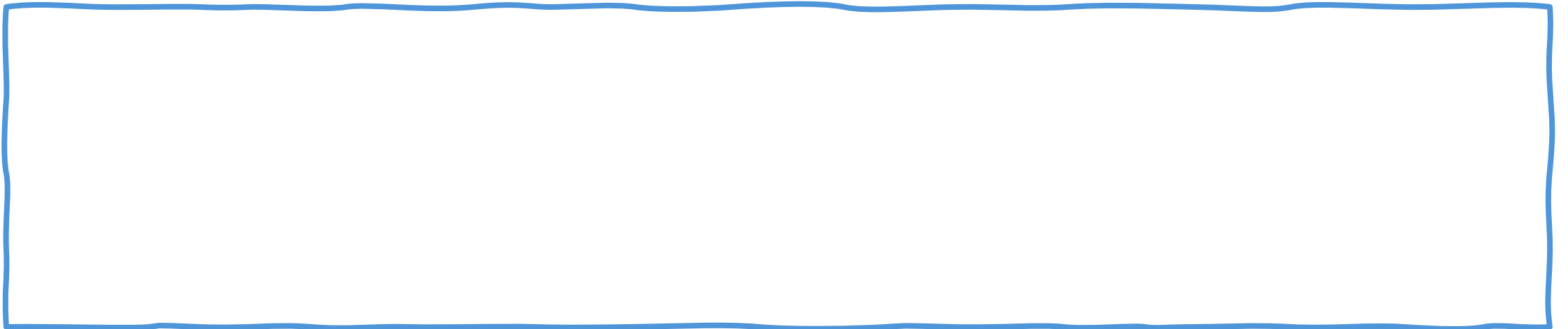
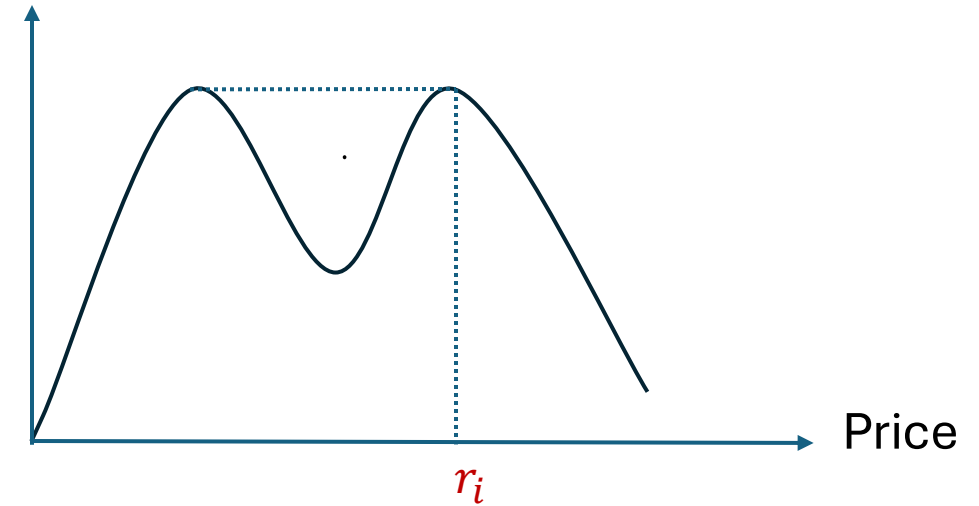


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# Oligopoly – Revenue Benchmark

For each item  $i$ : let  $r_i$  be the maximal revenue maximizing price for item  $i$  in a monopolist setting

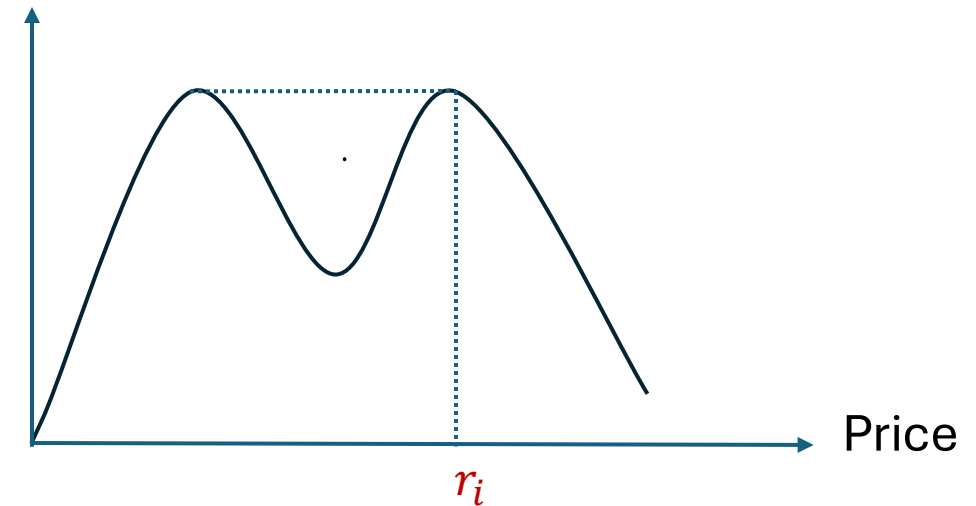
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## Main Result 1

For **any** menu of the principal seller and **any** mixed Nash equilibrium among the item sellers, the expected revenue of the principal seller is at most the buyer's expected *truncated social welfare*

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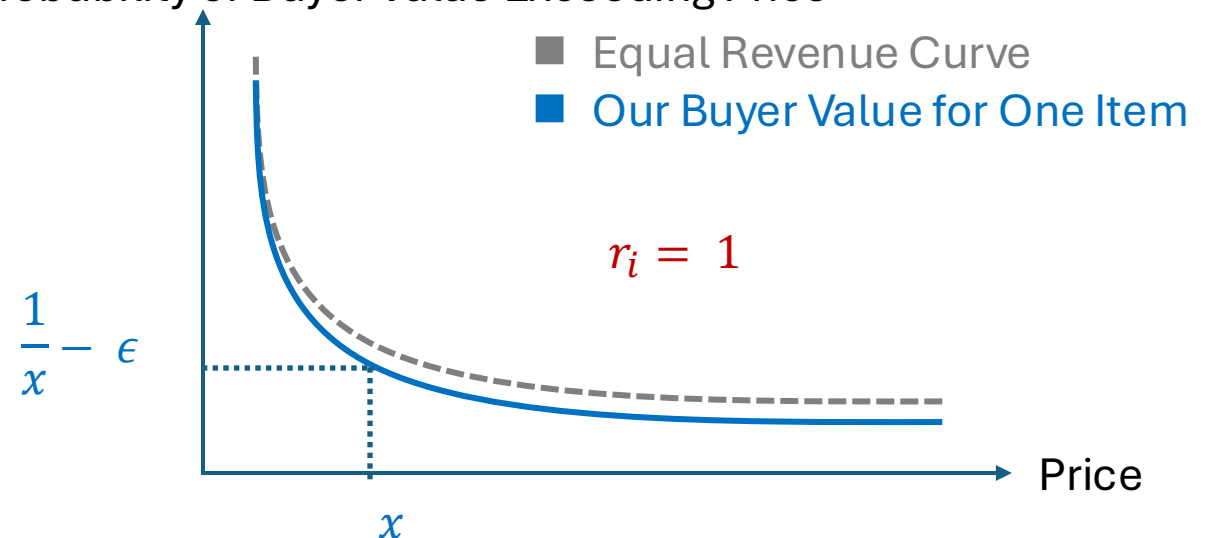
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Probability of Buyer Value Exceeding Price



# Oligopoly – Revenue Benchmark

## Main Result 1

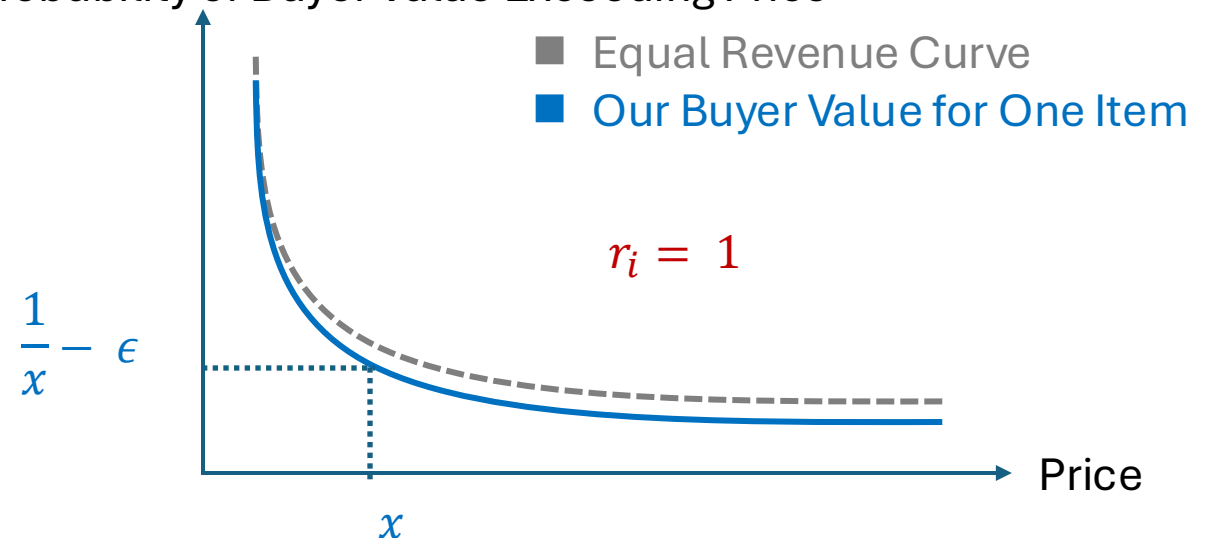
For any menu of the principal seller and any mixed Nash equilibrium among the item sellers, the expected revenue of the principal seller is at most the buyer's expected truncated social welfare

$$\mathbb{E}_{v \sim D} \left[ \sum_{i \in [m]} \min\{v_i, r_i\} \right]$$

Consider i.i.d items each with distribution:

| Optimal Principal Seller Revenue in Our Model | $O(m)$             |
|---|--------------------|
| Optimal Monopolist Revenue                    | $\Omega(m \log m)$ |

Probability of Buyer Value Exceeding Price



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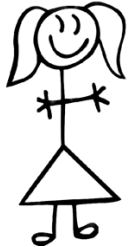
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Roughly speaking, we can assume item seller  $i$  prices in range  $[0, r_i]$  at equilibrium

# Oligopoly – Revenue Benchmark



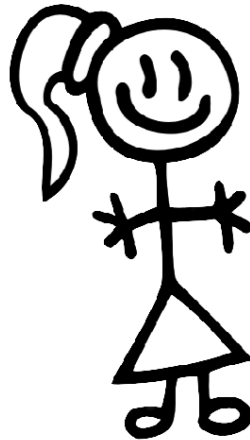
$$q_1 \leq r_1$$



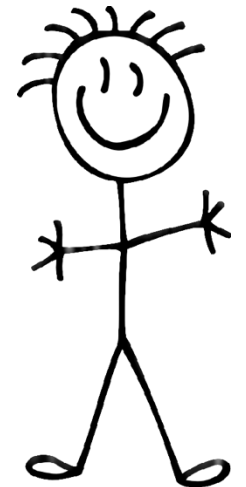
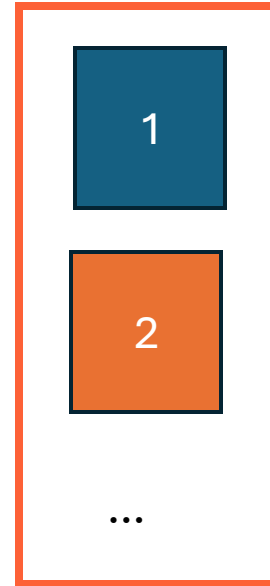
$$q_2 \leq r_2$$

...

$$v \sim D = \times_j D_j$$

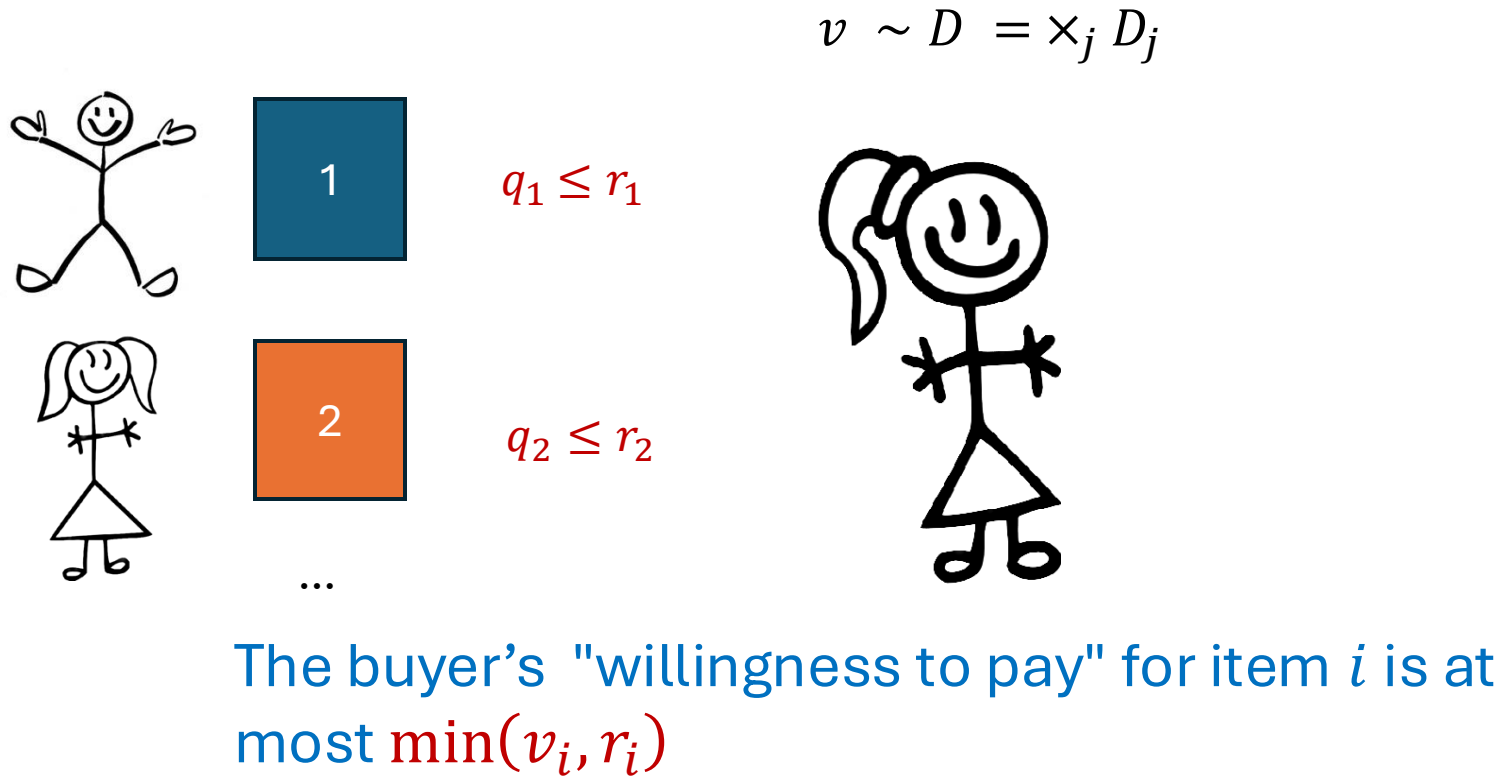


Principal Seller

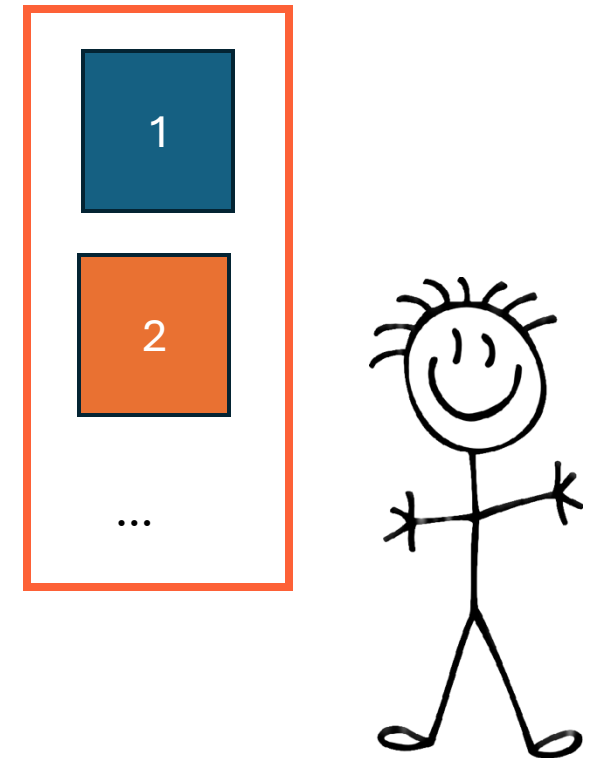


Principal menu price  
for item set  $S$ :  $p(S)$

# Oligopoly – Revenue Benchmark



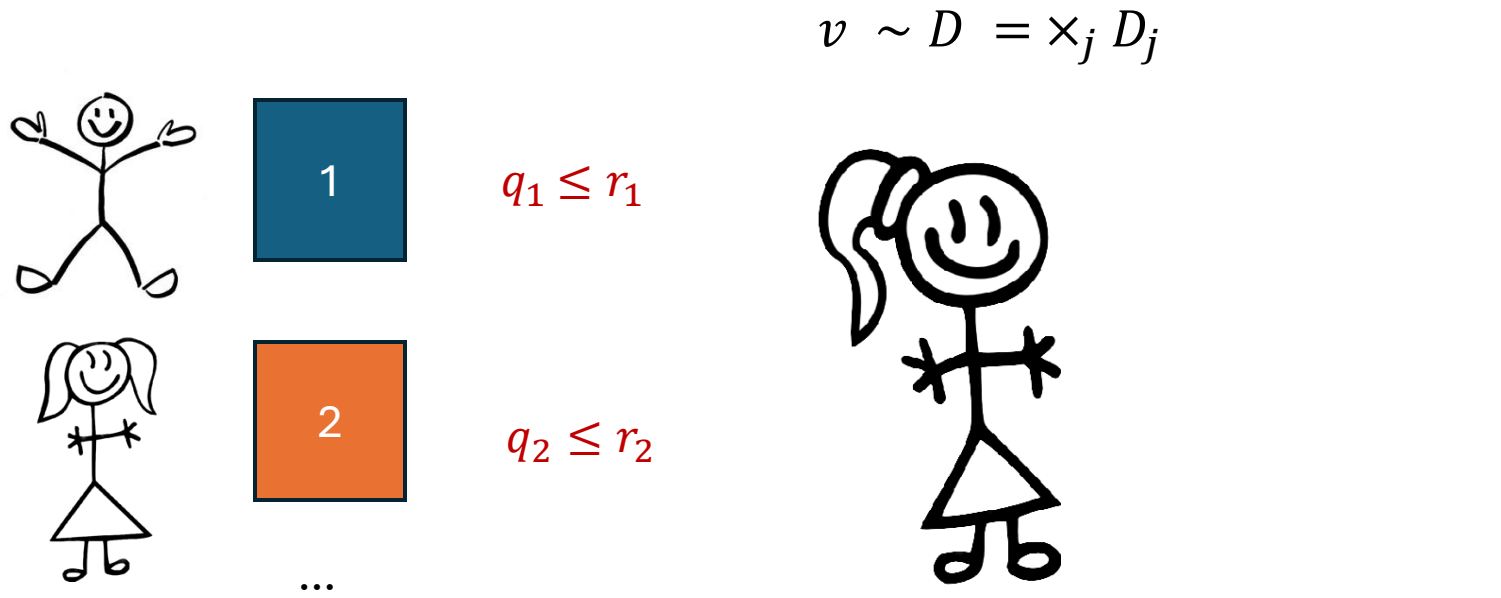
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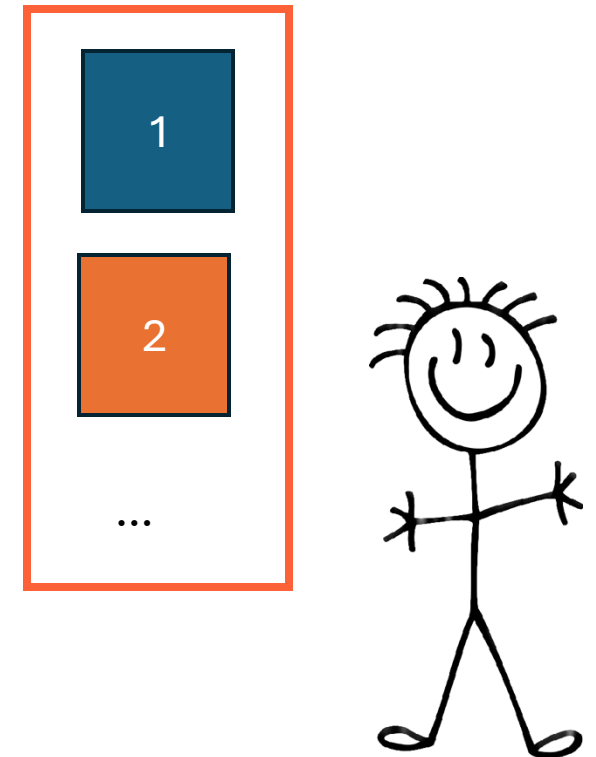
# Oligopoly – Revenue Benchmark



The buyer's "willingness to pay" for item  $i$  is at most  $\min(v_i, r_i)$

⇒ Principal revenue is upper bounded by  
*truncated social welfare*  $E_{v \sim D} [\sum_i \min\{v_i, r_i\}]$

Principal Seller



Principal menu price  
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# When is Bundling Effective?

## Main Result 2

*Under our assumptions*, there exists a price  $p$  at which the principal can sell the grand bundle of all items such that, at **any** mixed-nash equilibrium for the item sellers, the principal's revenue is at least  $1/3$  of the expected truncated social welfare.

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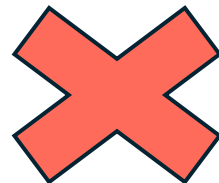
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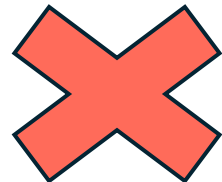
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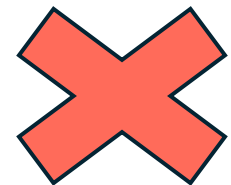
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All but one item has value distribution  $U[0, 1]$   
One item has value distribution  $U[0, 200]$



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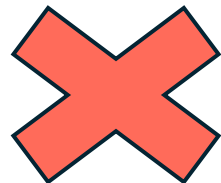
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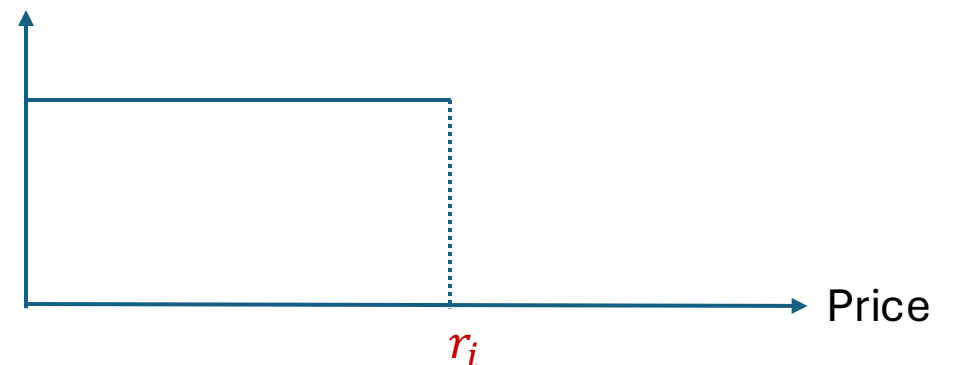
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Every item has equal revenue curve value distribution



Monopolist Revenue



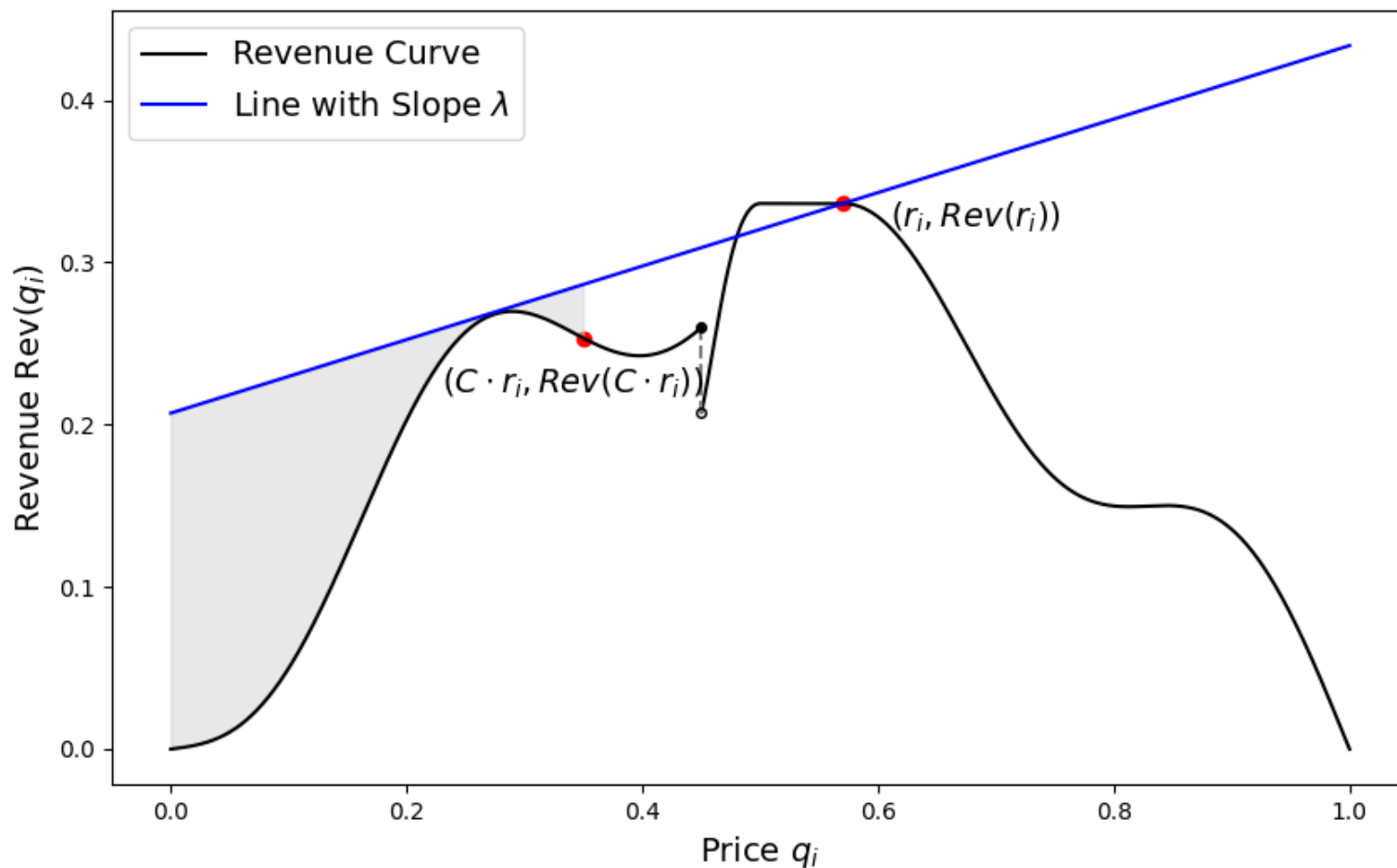
# When is Bundling Effective?

## Main Result

Under certain conditions, bundling is more effective than selling items individually.

## Our Assumptions

1. Each item has a unique value to each consumer (perfect substitutes).
2. Each consumer has a unique willingness to pay for each item (perfect complements).



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Note: If only some subset  $S$  of the items satisfy our assumptions, then the principal's revenue can approximate the expected truncated welfare of the items in  $S$ .

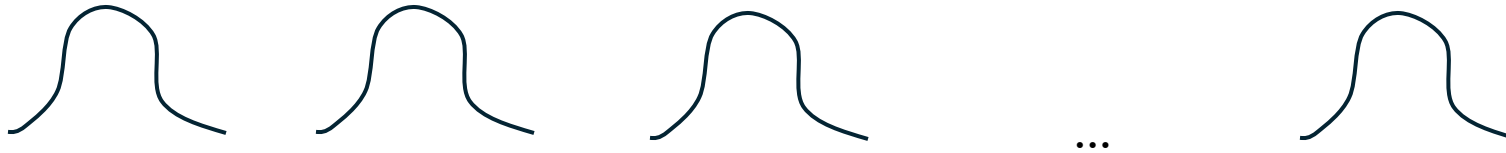
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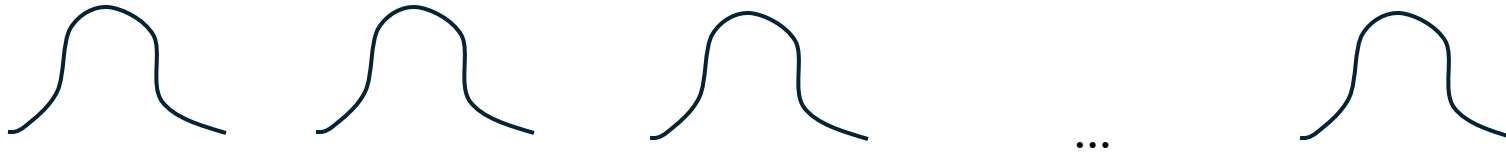
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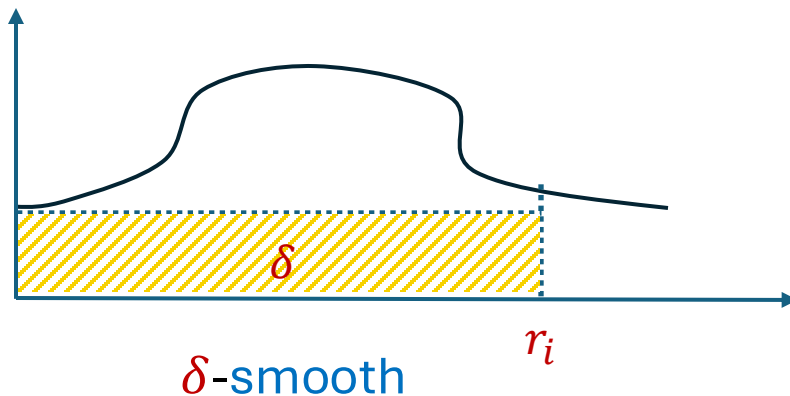
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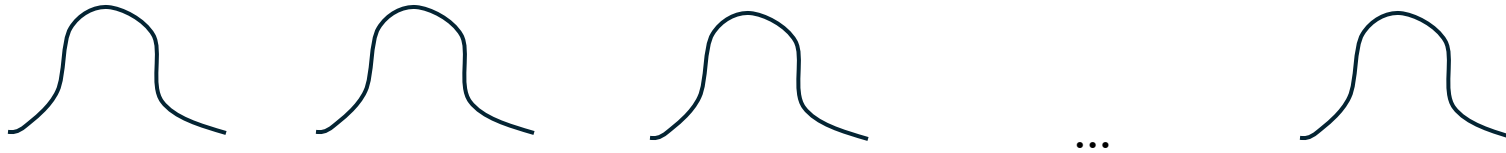
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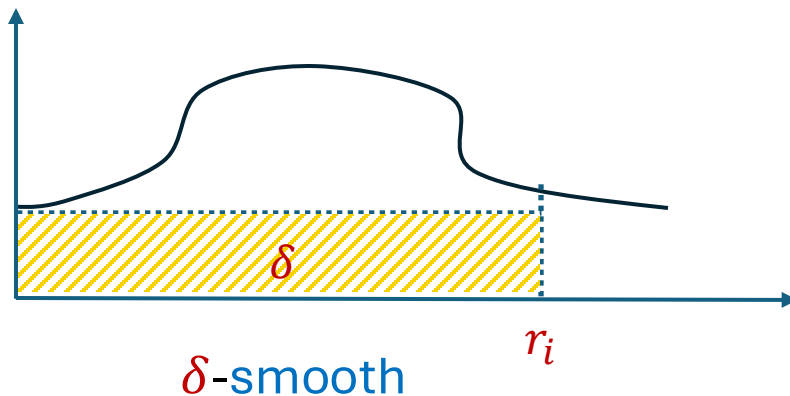
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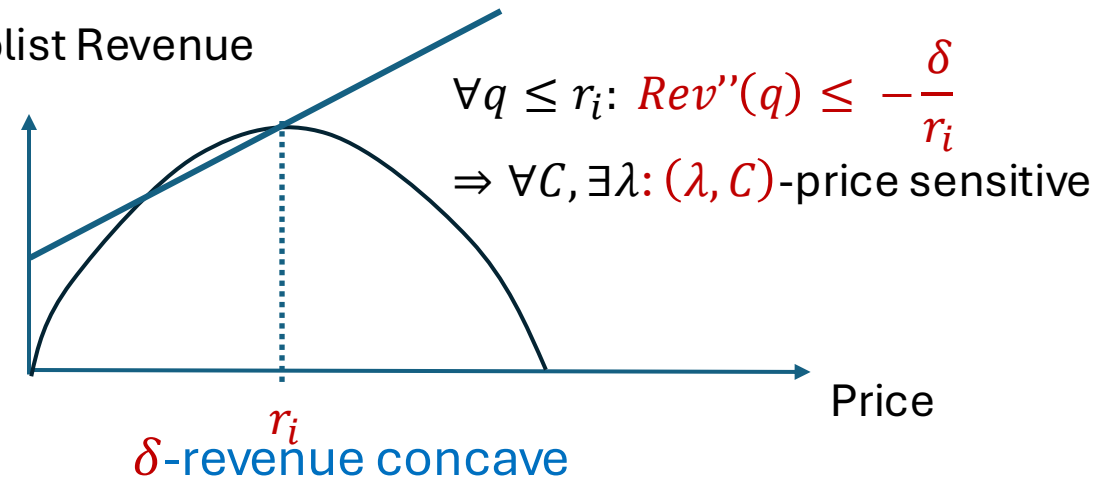


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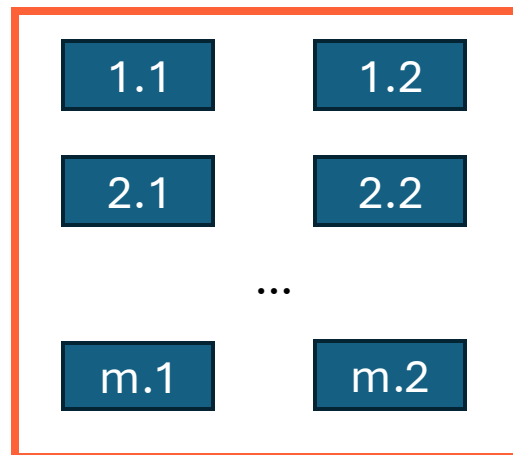
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The Grand Bundle



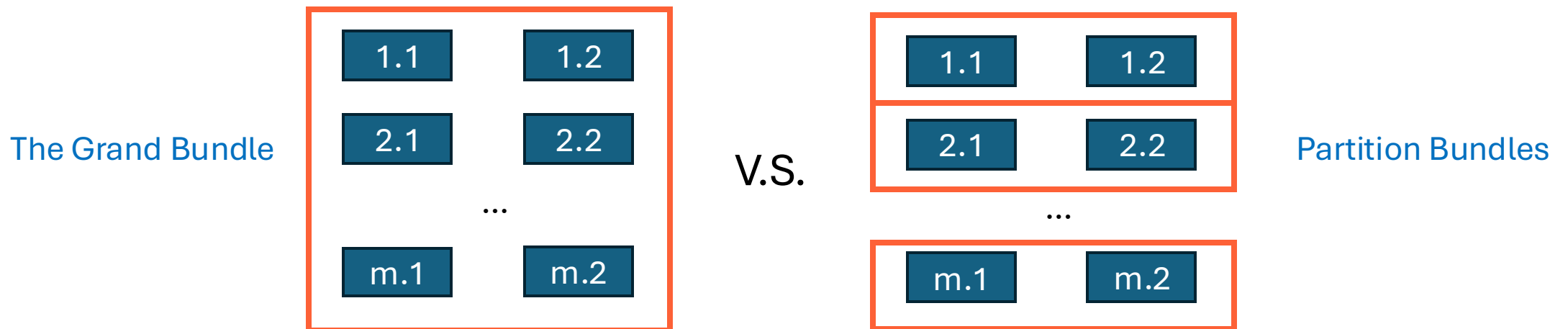
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- Any burning question in your mind!



# THANK YOU!

Questions?