Optimal Stopping with Multi-Dimensional Comparative Loss Aversion

Linda Cai, Joshua Gardner, Matt Weinberg

Princeton University

Optimal Stopping Problem

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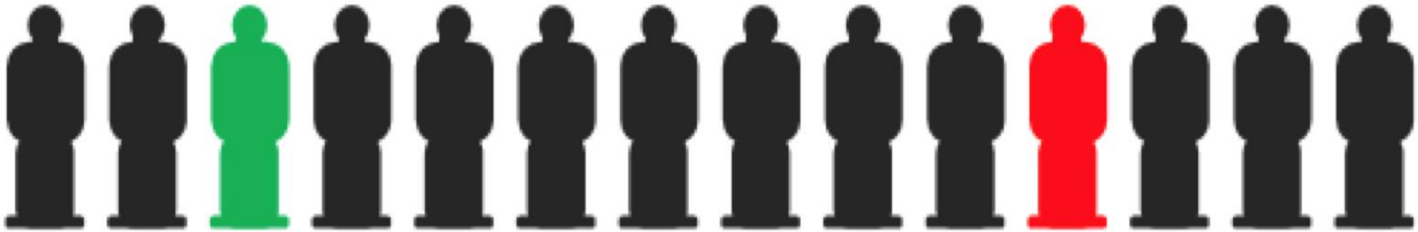
- n candidates, each value V_i of candidate i is drawn from distribution D_i



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- Decision maker goal: maximize their utility

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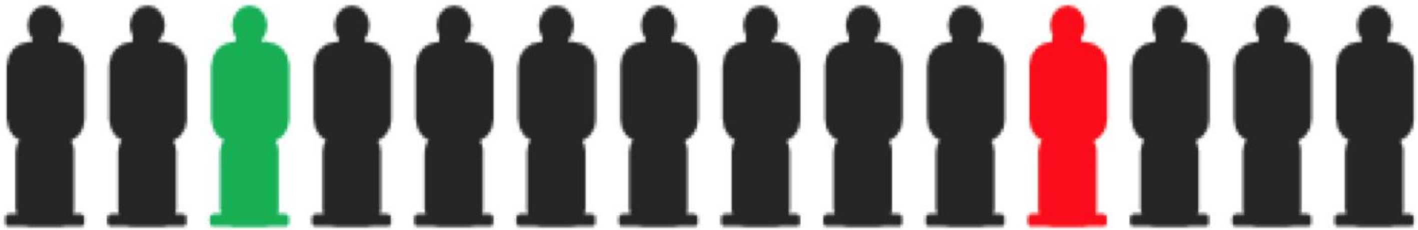

Candidate selected
value = V

Utility $U = V$

Best candidate
value = S

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Random Order


Optimal Order

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The Prophet
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Optimal Stopping with Behaviorally Biased Agents

Jon Kleinberg　　Robert Kleinberg　　Sigal Oren

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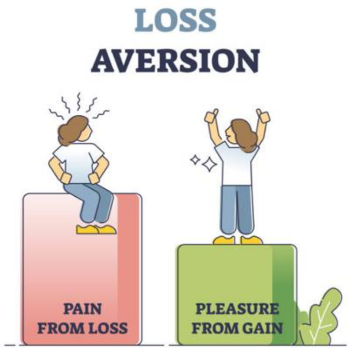

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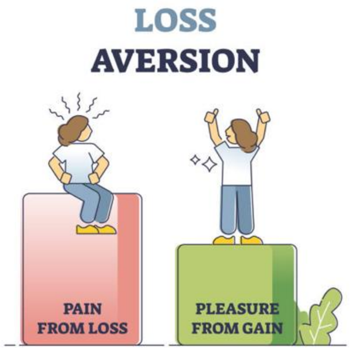

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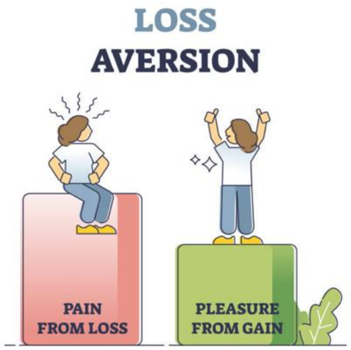

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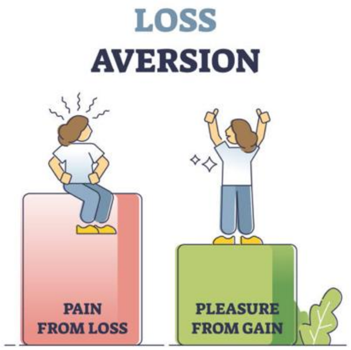

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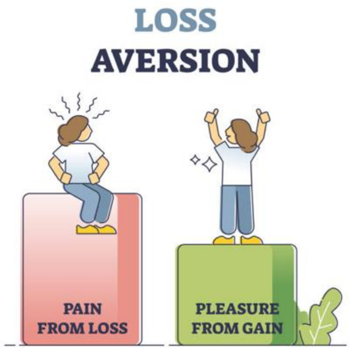

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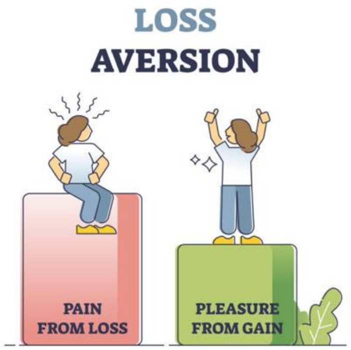

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- Utility Function of the Biased Agent:

$$U = V - \lambda \cdot (S - V)$$

Best candidate so far

Candidate selected

Loss aversion factor λ

Optimal Stopping with Behaviorally Biased Agents

- How do we evaluation the utility of a biased agent?

Best candidate so far

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Loss aversion factor $\lambda = 0$

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The Rational Prophet
(Optimal Offline Algorithm)

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	Rational Prophet	Rational Gambler
Adversarial Order	$\frac{1}{2+\lambda}$	$\frac{1}{1+\lambda}$
Random Order	$\frac{1}{\Theta(\log(\lambda))}$	$\frac{1}{\Theta(\log(\lambda))}$

Comparative Loss Aversion

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Best Location

Most Beautiful

Most Space

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How do we model human agent utility when candidates have various pros and cons along different dimensions?

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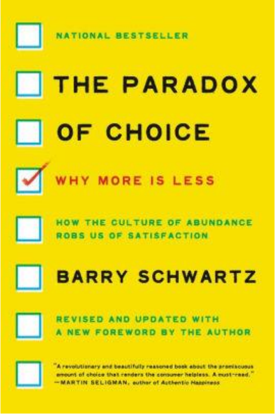

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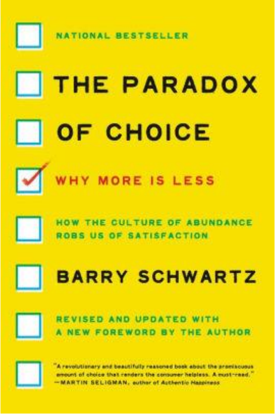

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- Supported by empirical research in choice overload
- **[Sagi and Friedland 07]** "regret is related to the comparison between the alternative chosen and the union of the positive attributes of the alternatives rejected"

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$V^{(1)} = (1, 0.5, 0.5)$ $V^{(2)} = (0.5, 1, 0.5)$ $V^{(3)} = (0.5, 0.5, 1)$

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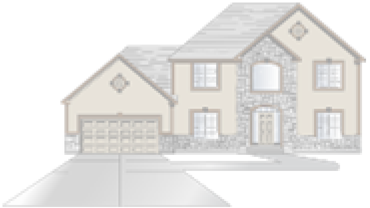

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$$U^{(i)} = \|V^{(i)}\|_1 - \lambda \cdot \left\|S^{(i)} - V^{(i)}\right\|_1$$

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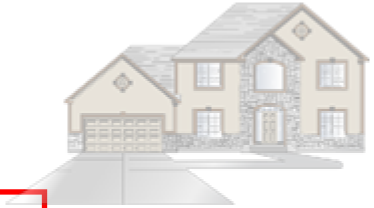

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$$= (0.5 + 0.5 + 1) - \lambda \cdot \big((1 - 0.5) + (1 - 0.5) + (1 - 1)\big) = 2 - \lambda$$

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- How does the utility of a biased agent with comparative loss aversion compare with that of the rational agents?

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Loss aversion bias $\lambda = 2$

Number of features $k = 2$

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Regret: $\lambda \cdot 2^{i-1} = 2 \cdot 2^{i-1} = 2^i$

Utility of Biased Agent: value – regret $= 2^i - 2^i = 0$

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Regret: 0

Utility of Biased Agent: value – regret $= 1 - 0 = 1$

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Optimal biased agent selection,

Utility = 1

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Optimal biased agent selection,
Utility = 1

Rational prophet selection,
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Unbounded gap (growing with number of candidates) between utility of biased agent and rational prophet!

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What if the candidates are not adversarially ordered?

E.g. Candidate values are drawn i.i.d from the following distribution.

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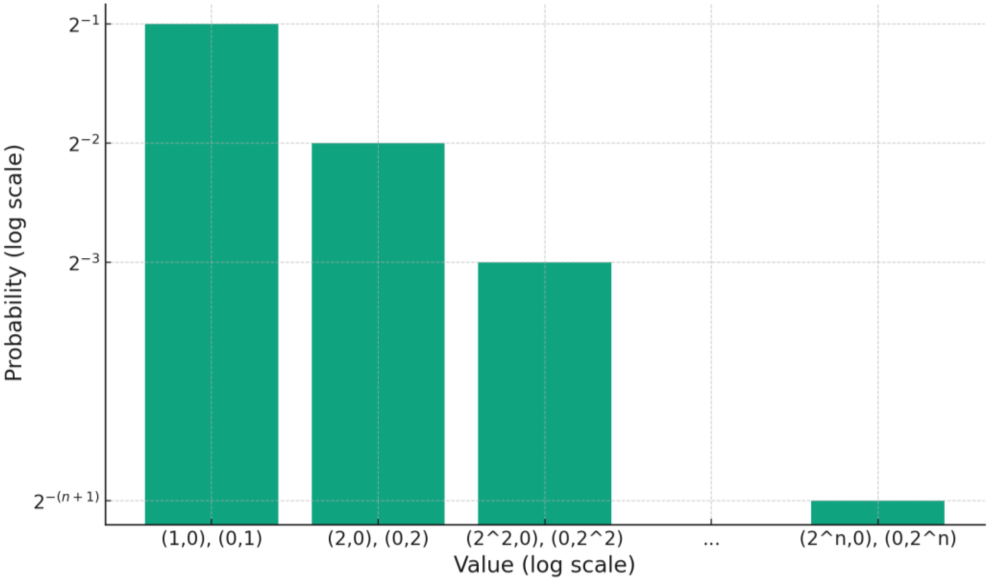

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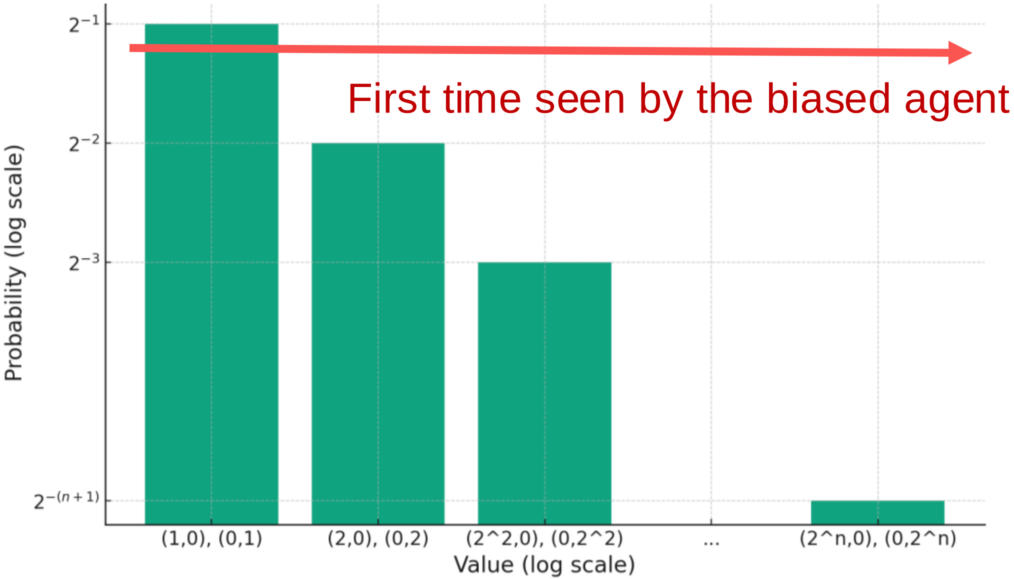

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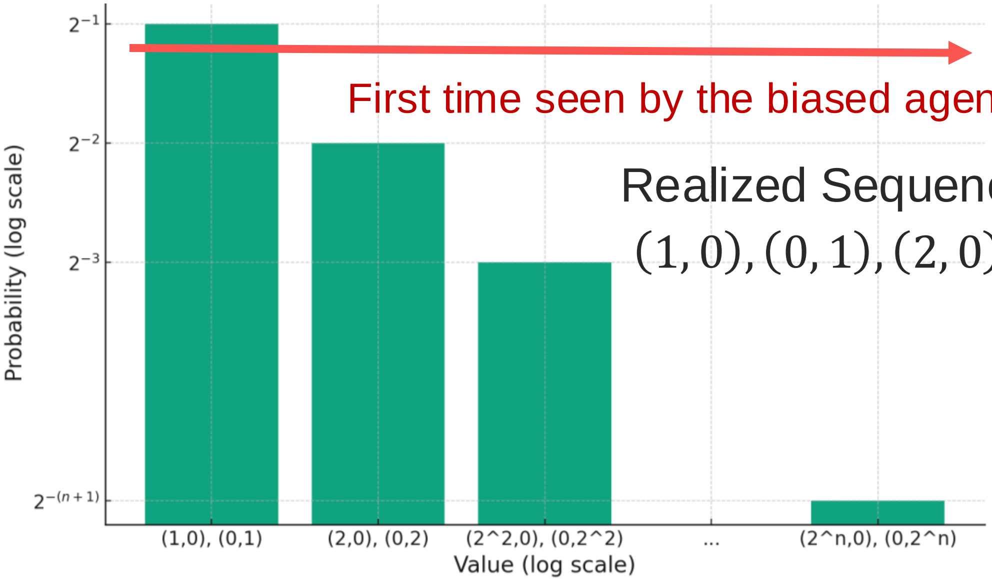

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$(1,0),(0,1),(2,0),(0,1),(0,2),(1,0),(0,2^2)$ ...

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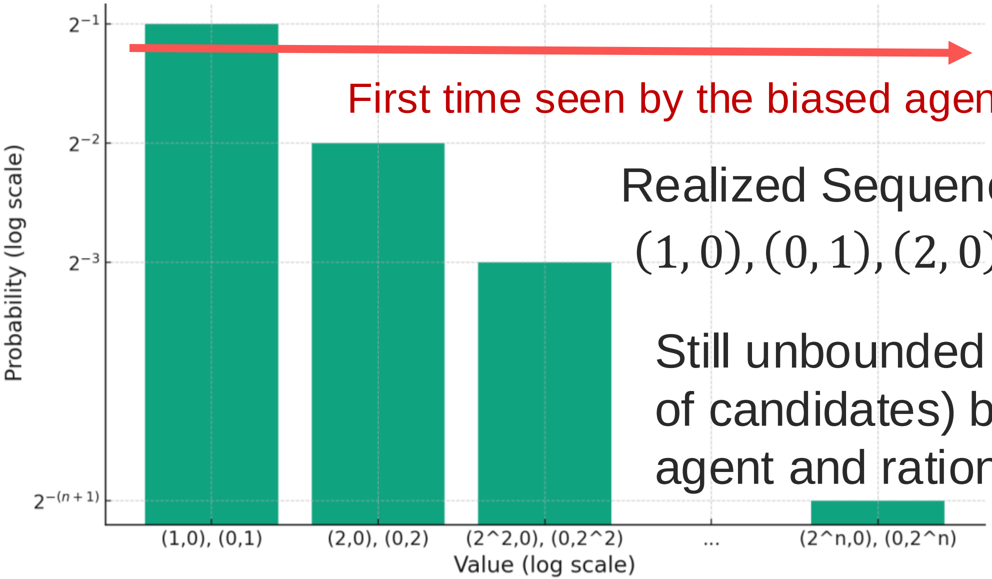

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Still unbounded gap (growing with number of candidates) between utility of biased agent and ration agents.

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Competitive ratio of the biased agent against rational agents:

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Competitive ratio of the biased agent against rational agents:

	Arrival Order	Rational Prophet	Rational Gambler
Our result # features $k \geq 2$	Adversarial Order, $\lambda(k-1) \geq 1$	$\to 0^*$	$\to 0^*$
	Adversarial Order, $\lambda(k-1) < 1$	$\dfrac{1-\lambda(k-1)}{2+\lambda}$	$\dfrac{1-\lambda(k-1)}{1+\lambda}$
	Optimal Order, $\lambda(k-1) \geq 1$	$\to 0^*$	$\to 0^*$
	Optimal Order, $\lambda(k-1) < 1$	$\dfrac{1-\lambda(k-1)}{3(2+\lambda)}$	$\dfrac{1-\lambda(k-1)}{4(1+\lambda)}$

* Competitive ratio approaches 0 as the number of candidate increases.

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[KKO21] # features $k = 1$	Adversarial Order	$\frac{1}{2+\lambda}$	$\frac{1}{1+\lambda}$
	Random Order	$\boldsymbol{\frac{1}{\Theta(\log(\lambda))}}$	$\boldsymbol{\frac{1}{\Theta(\log(\lambda))}}$

Conclusion and Future Work

We consider an online decision-maker who suffers from comparative loss aversion and show significant utility loss (possibly unbounded and unmitigated by assumption on arrival orders) from using multi-dimensional reference points.

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Future Work:

- Incorporating additional phenomena of behavioral game theory in the study of optimal stopping problems
- Design interventions to mitigate psychological utility loss

If the ability to choose enables you to get a better car, house, job, vacation, or coffeemaker, but the process of choice makes you feel worse about what you’ve chosen, you really haven’t gained anything from the opportunity to choose. And much of the time, better objective results and worse subjective results are exactly what our overabundance of options provides.

-Barry Schwartz, The Paradox of Choice

THANK YOU