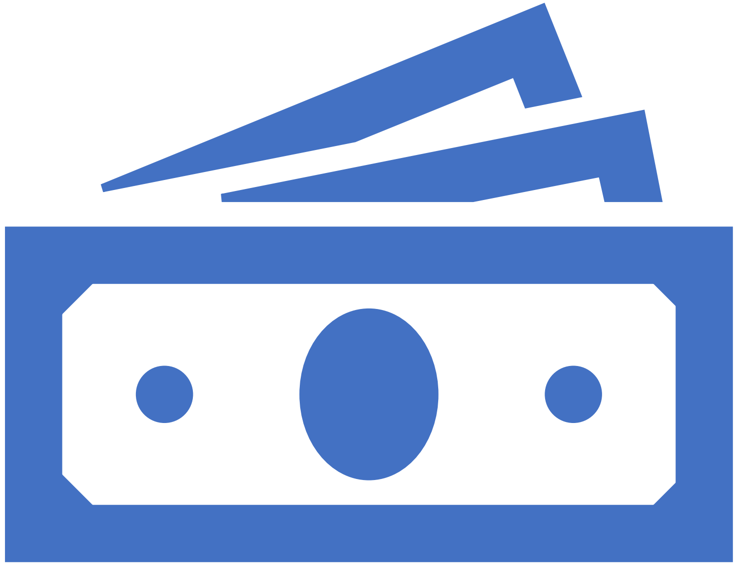


Selling to Multiple No Regret Buyers

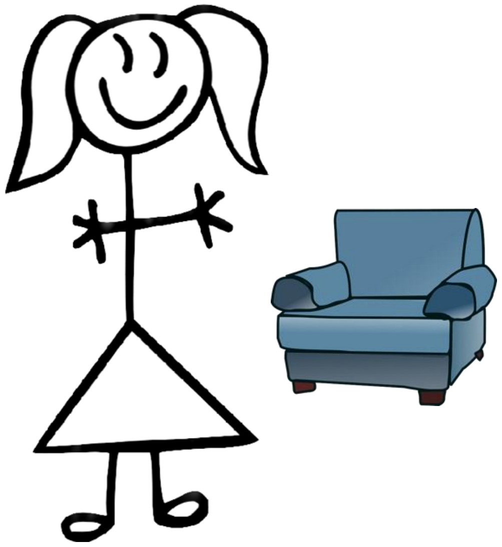
Linda Cai, S. Matthew Weinberg, Evan Wildenhain, Shirley Zhang

Princeton University

Auction: Selling an Item to n Buyers

Auction: Selling an Item to n Buyers

One seller with an item



Auction: Selling an Item to n Buyers

n independent and identical buyers

$v_1 \sim D$

$v_2 \sim D$

…

$v_n \sim D$

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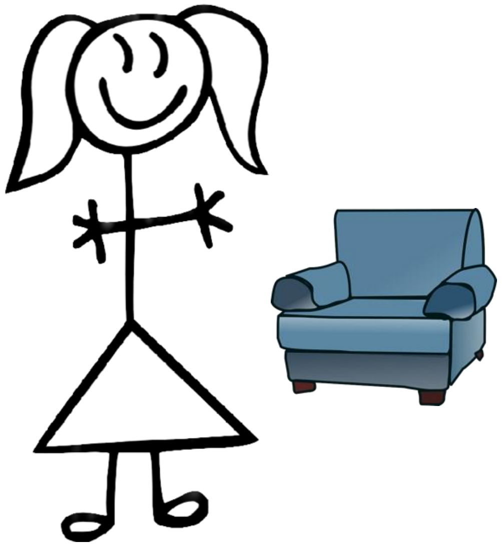

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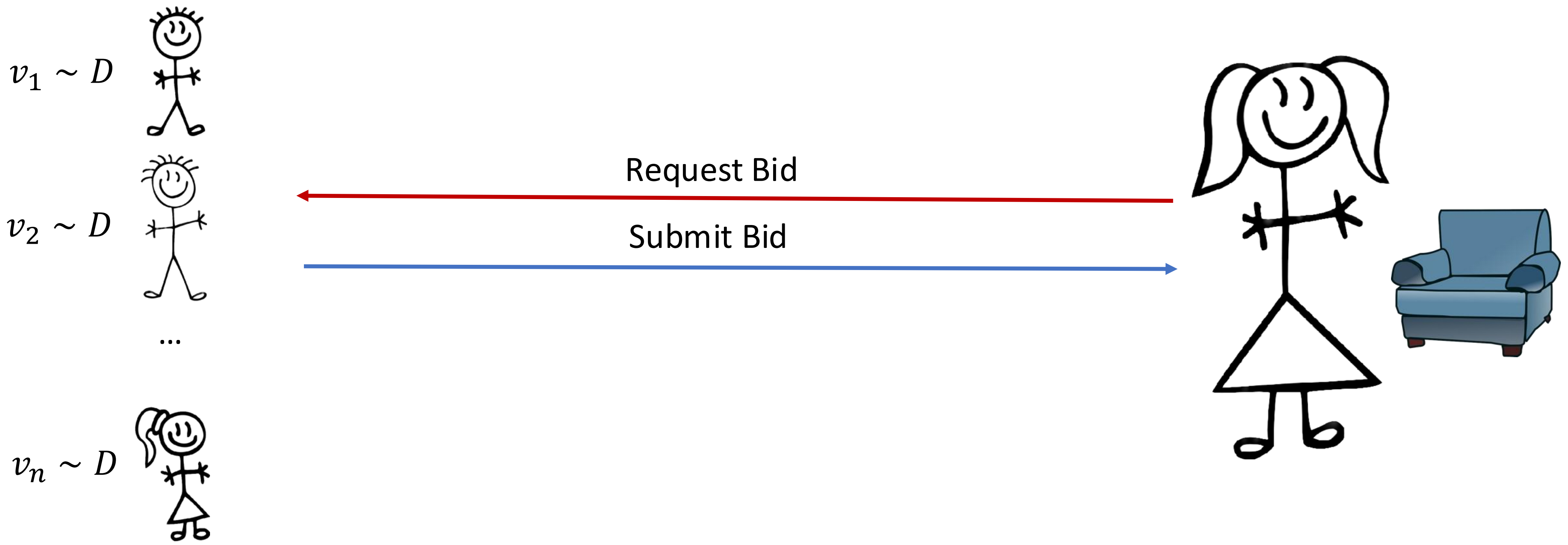

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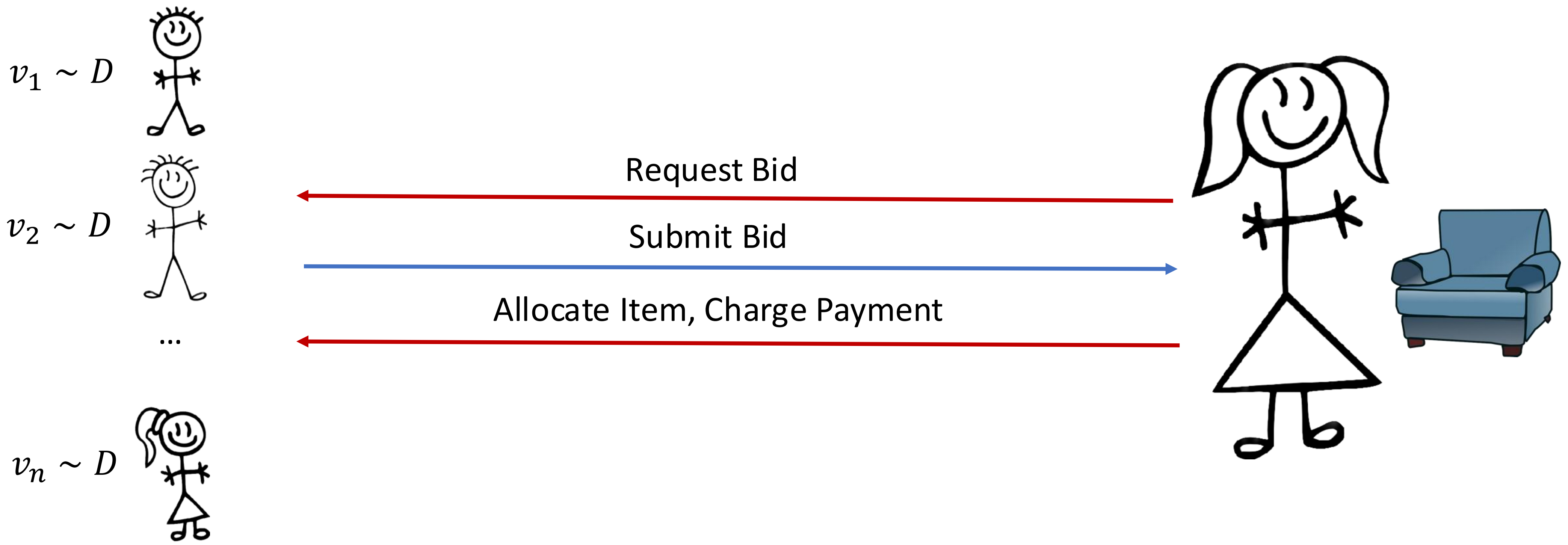

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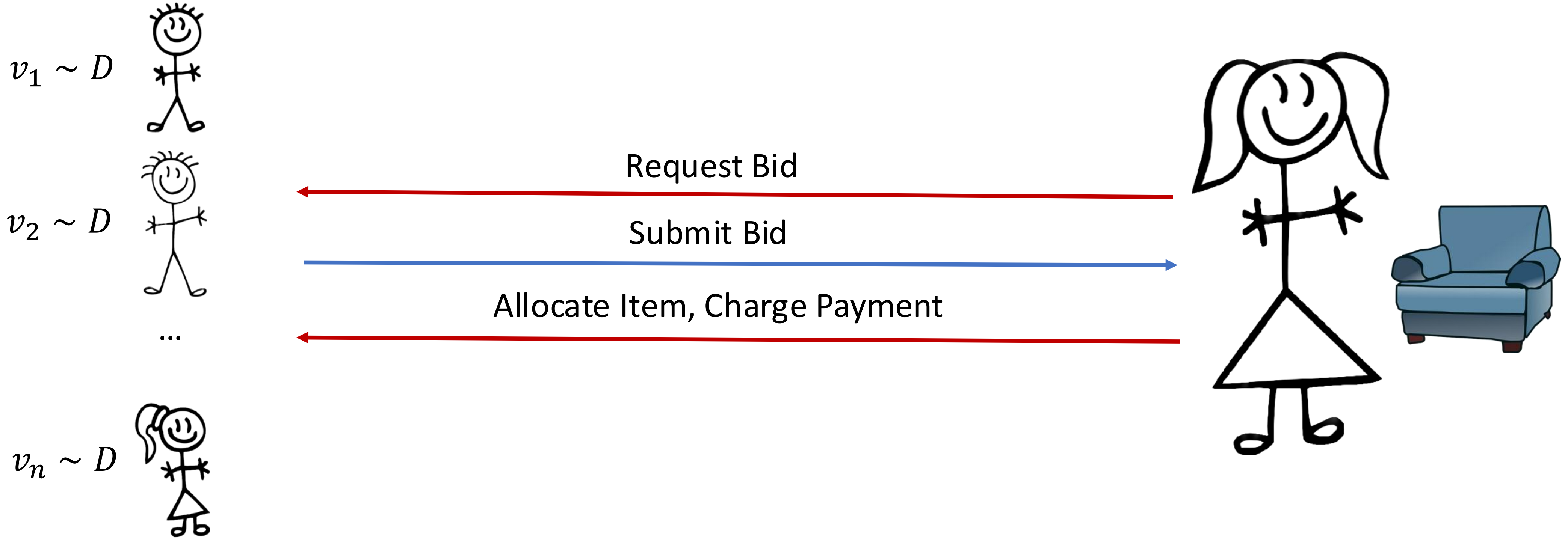

Goal: maximize their expected utility
= value of item * Pr[receiving the item] – expected payment

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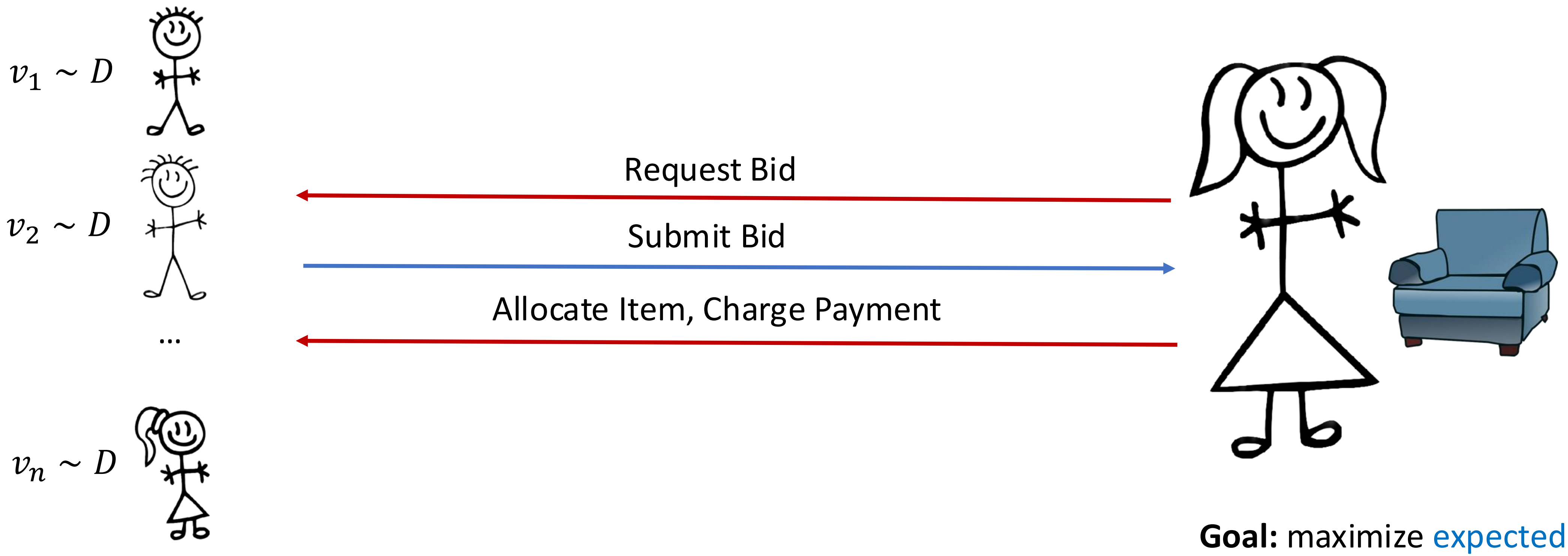

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Goal: maximize expected revenue (payment received)

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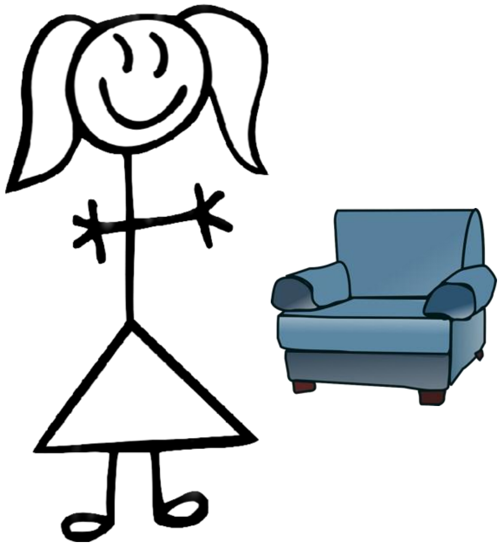

Auction: Selling an Item to n Buyers

One Shot Auction: classical Bayesian auction design study *truthful* auctions: all buyers can maximize their expected utility by reporting their value truthfully

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$v_2 \sim D$ 

…

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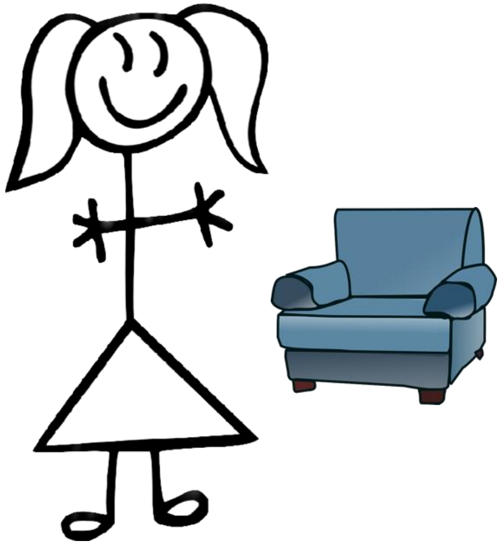

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One Shot Auction: classical Bayesian auction design study *truthful* auctions: all buyers can maximize their expected utility by reporting their value truthfully

$v_1 \sim D$

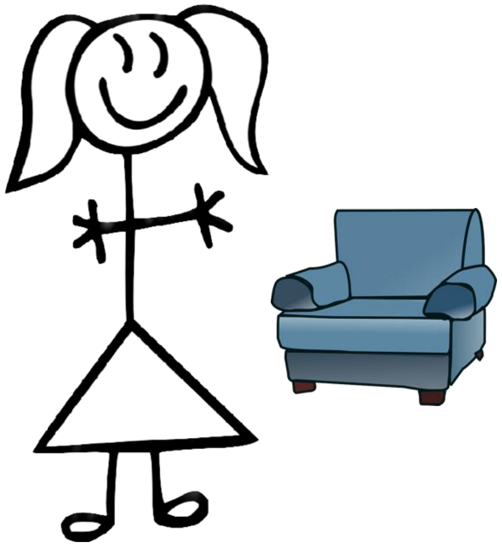
30

$v_2 \sim D$

…

$v_n \sim D$

50



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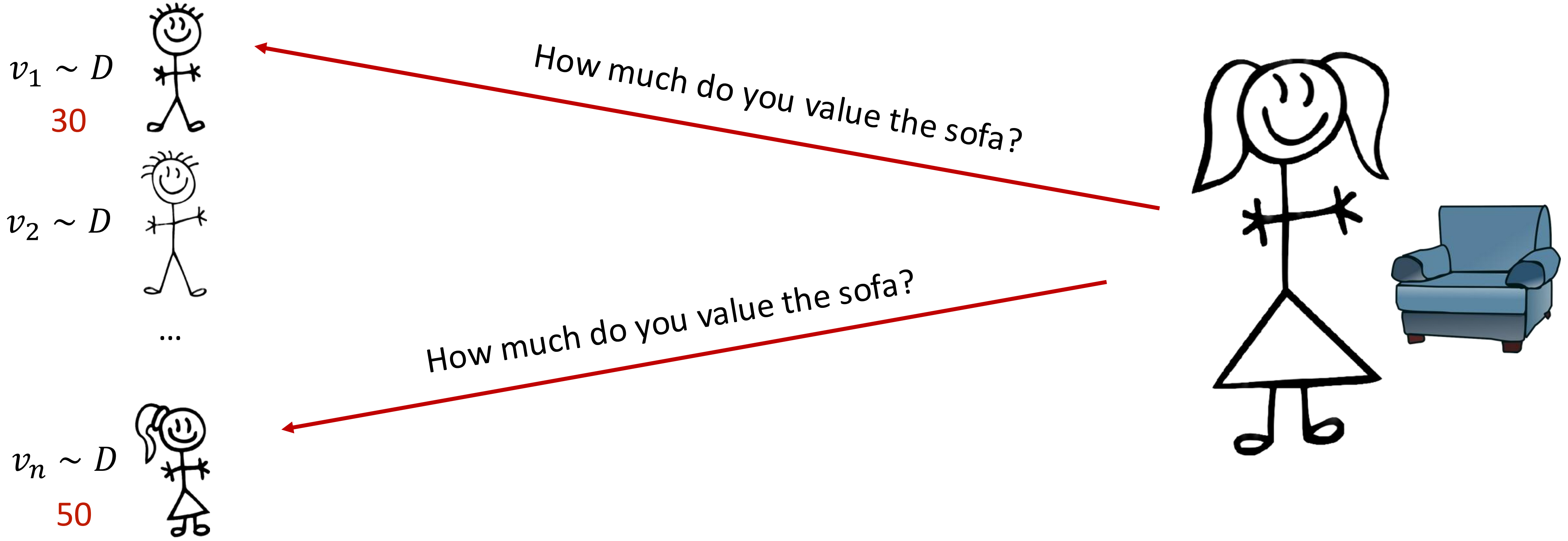

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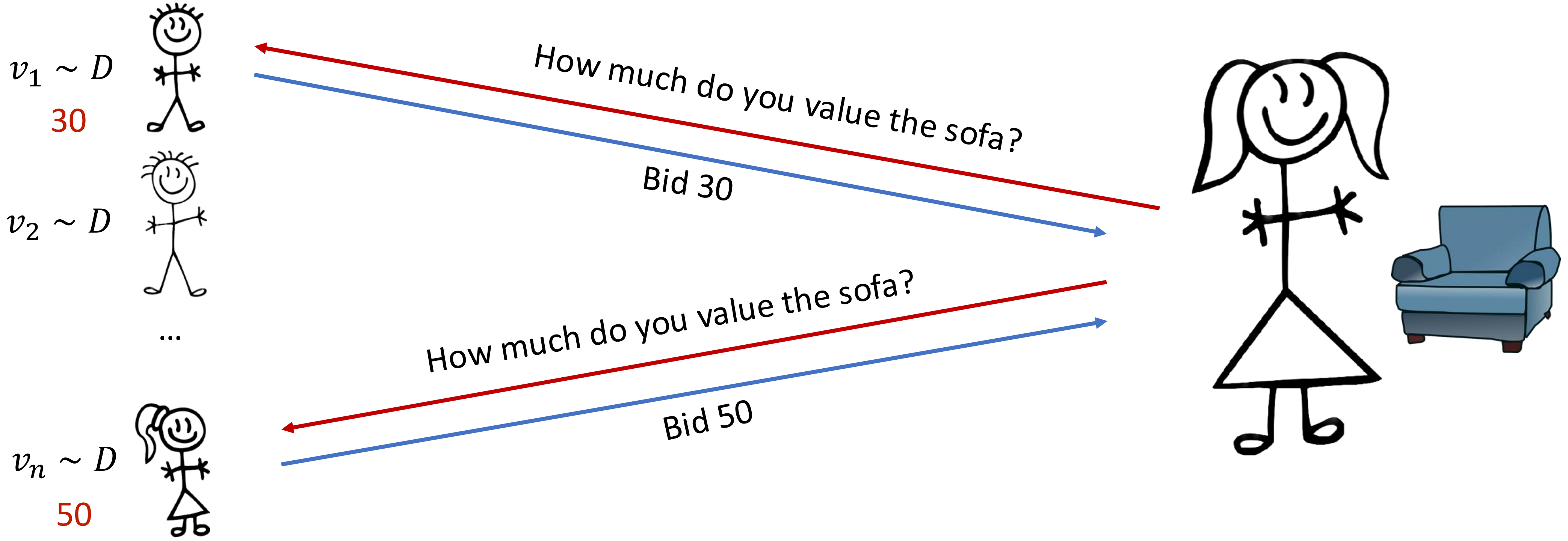

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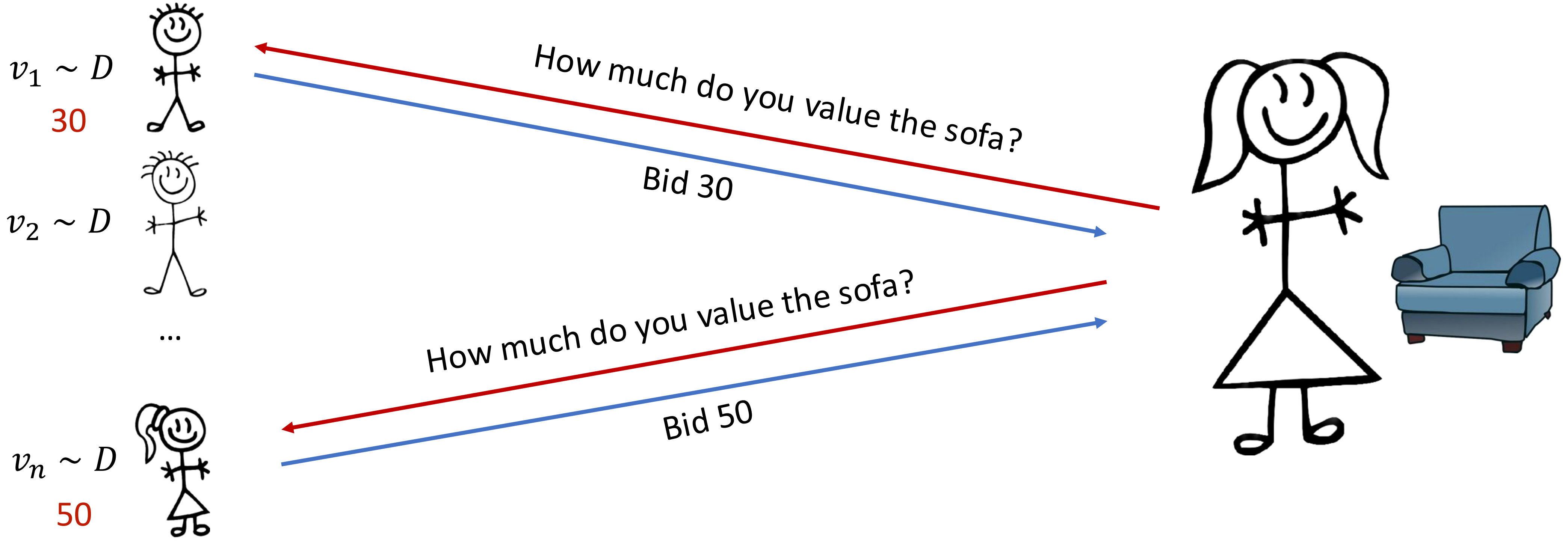

Myerson gives a complete characterization of *revenue optimal* truthful auction

Auction: Selling an Item to n Buyers

Repeated Auction: there are T rounds, in each round the buyers' values are drawn independently from the prior distribution

$v_{1,i} \sim D$

$v_{2,i} \sim D$

…

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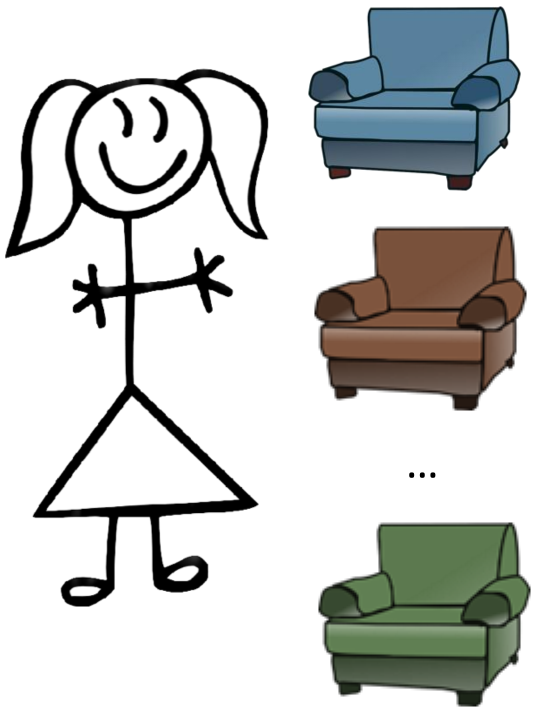

Day 1

Day 2

…

Day T

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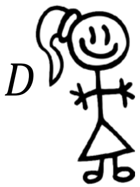

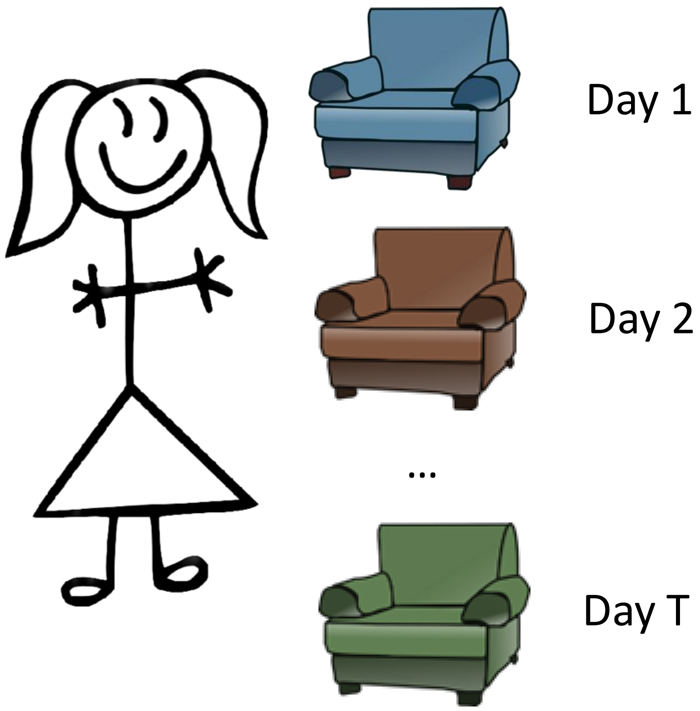

Goal: maximize their expected **total** utility

= $\sum_{i=1}^{T}$ (value of item in round i)$*$ Pr[receiving the item in round i] – expected payment in round i

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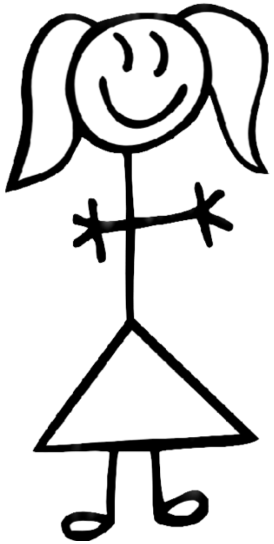

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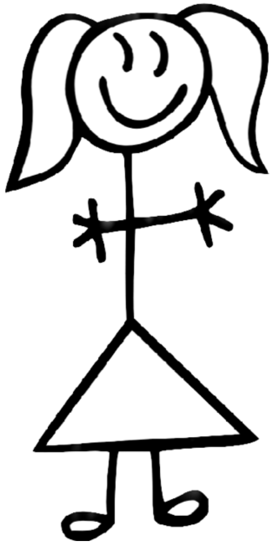

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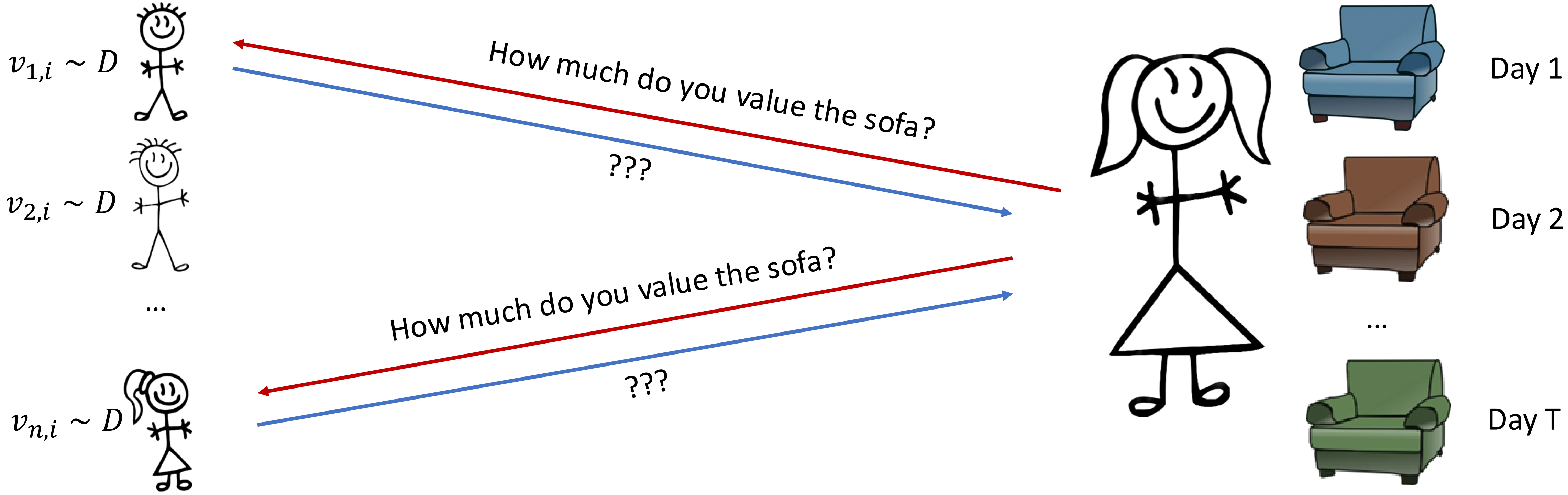
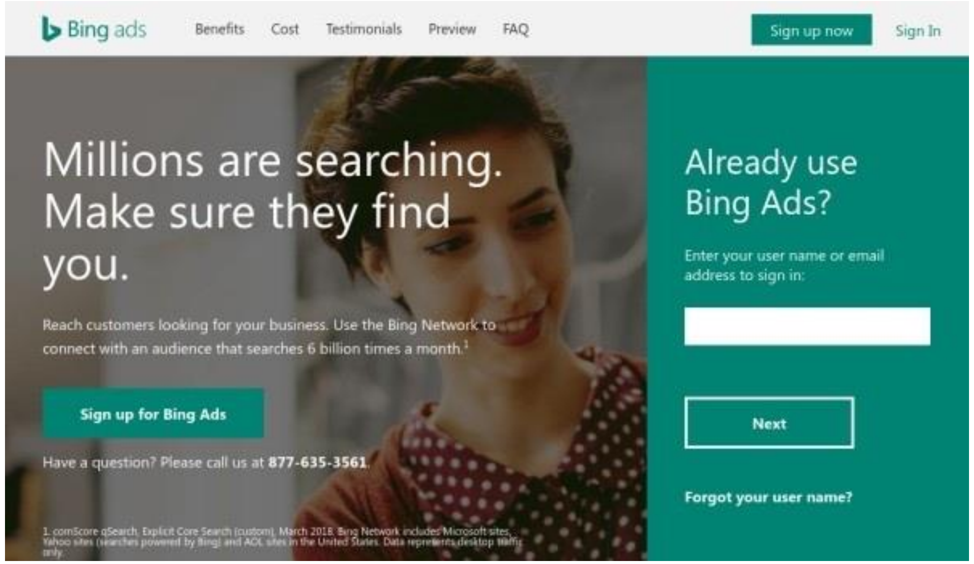


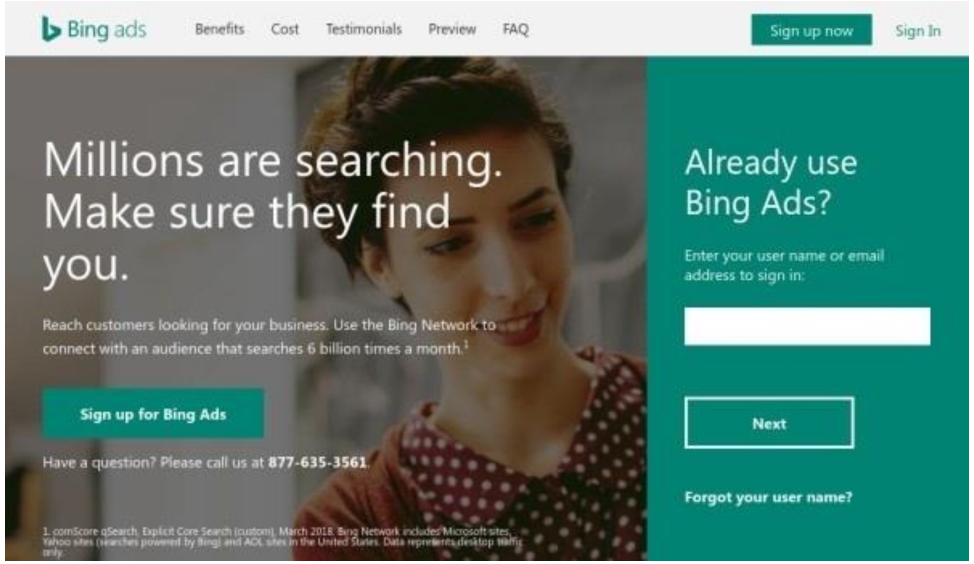
Question: not clear how to incentivize truthfulness, how do we model buyer behavior?

Learning in Repeated Auction



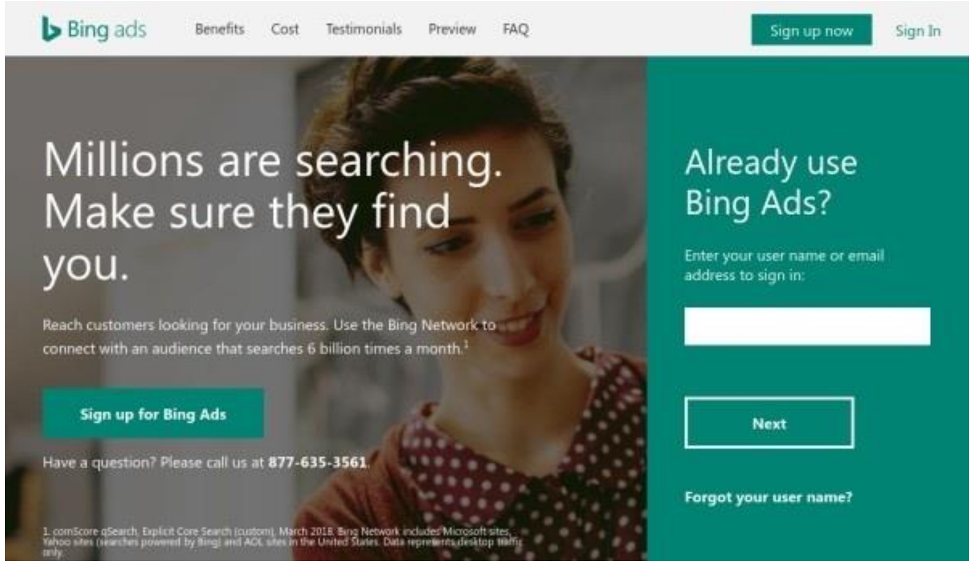
Learning in Repeated Auction

- In classical auction theory, agents are typically assumed to be rational/best responding



Learning in Repeated Auction

- In classical auction theory, agents are typically assumed to be rational/best responding
- **[Nekipelov, Syrgkanis, and Tardos 2015]** Bidder behavior is more consistent with no-regret learning in repeated auctions (e.g. in Bing ad auction)



No Regret Learning with Context

Action 1

Action 2

Action 3

…

Action m

No Regret Learning with Context

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Action 3

…

Action m

In a repeated game with T rounds with context space V and action space A:

No Regret Learning with Context

Action 1

Action 2

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In a repeated game with T rounds with context space V and action space A:

- In each round t, player i observes a context $v_{i,t} \in V$, then takes an action $a_{i,t} \in A$ and observe a reward $r_t(a_{i,t})$ (the reward function may be correlated with the context)

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$$\sum_{t=1}^{T} r_t(a_{i,t}) \geq \max_{fixed\, action\, strategy\, s} \sum_{t=1}^{T} r_t(\text{action } s \text{ takes given context } v_{i,t}) - o(T)$$

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- Buyers may be using common off-the-shelf no regret learning algorithms such as Exp3, Multiplicative Weights Update, Follow-the-Perturbed-Leader, etc

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Definition (Mean-Based No Regret): An algorithm is *mean-based no regret* if:
cumulative utility for action i is less than that of another action j by at least $f(T) = o(T)$
$\Rightarrow$ the probability that the algorithm plays action i on round t is at most $o(1)$.

- Intuition: if an action seems much better historically it would be played with much higher probability

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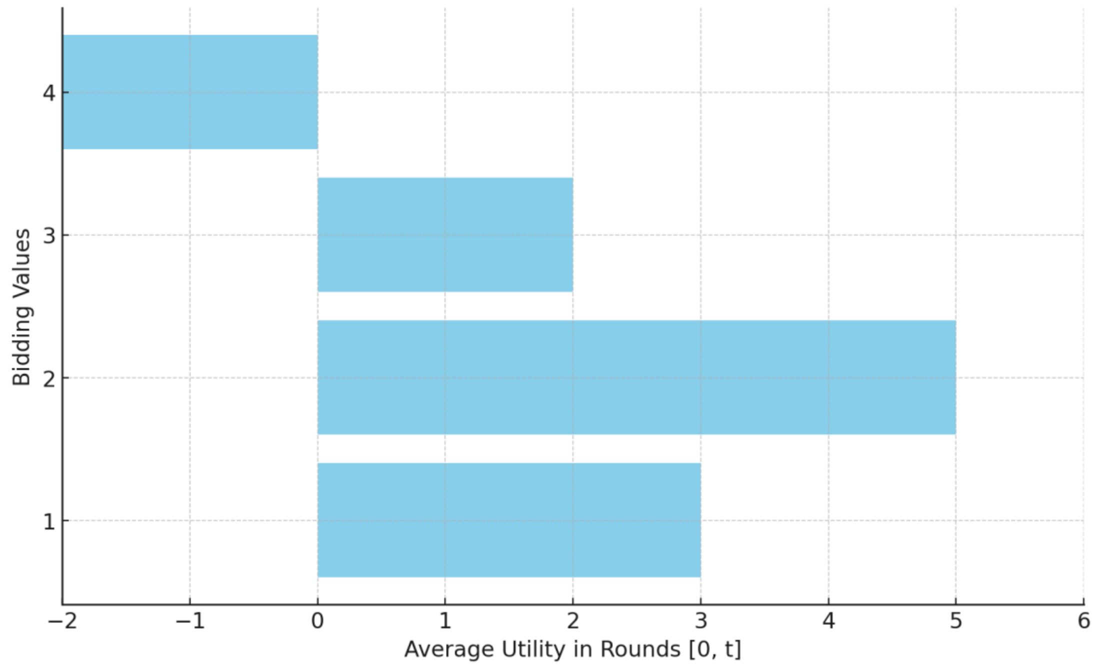

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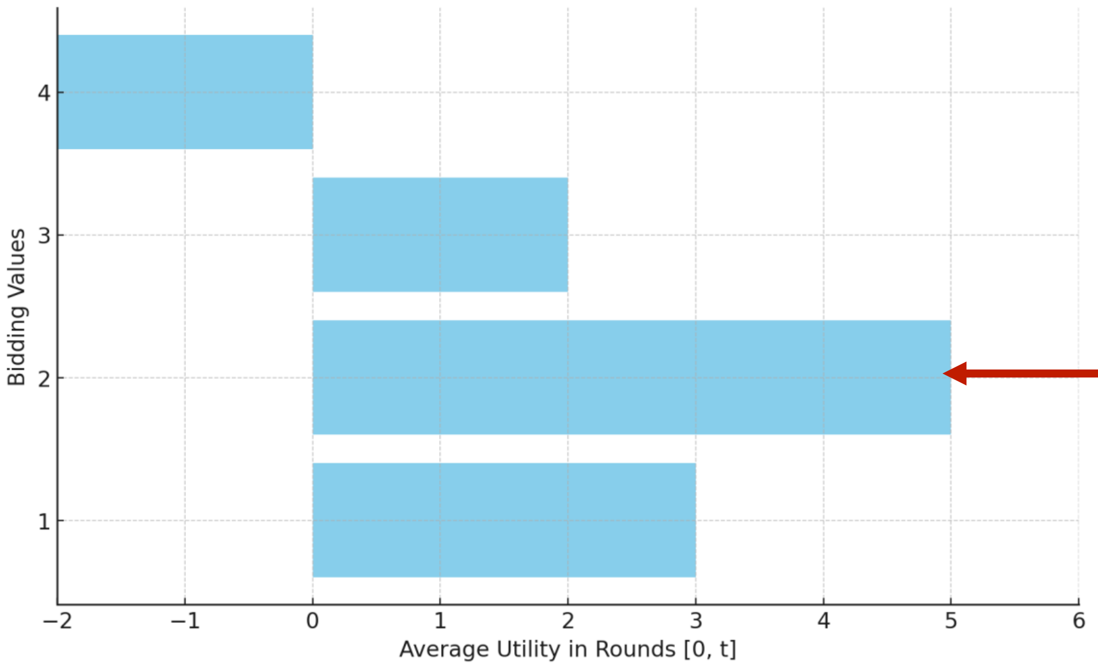

A mean-based no regret algorithm will choose to bid 2 with high probability

Learning in Repeated Auction

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1) Allocates random costs to each expert at the initial time;
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Once the difference between cumulative utility of two actions exceed the initial cost, the inferior action will never be chosen.

Exploitability of Learning Agents

- **Question:** How much revenue can the seller extract *knowing* that the buyers are using mean-based no regret learning algorithms (but not the specific algorithm they are using)?

- [Braverman, Mao, Schneider, and Weinberg'18] For the one seller one buyer setting:
  - **For naïve mean-based no regret buyer**: Full surplus extraction (seller gets revenue ≈ optimal social welfare and buyer gets ≈ zero utility)
  - **For non overbidding mean-based no regret buyer**: Optimal revenue can be unboundedly worse than for naïve learners, yet still unboundedly better than a seller who runs the same auction in each round.

Exploitability of Learning Agents

- **Question:** How much revenue can the seller extract *knowing* that the buyers are using mean-based no regret learning algorithms (but not the specific algorithm they are using)?

- **[Our Result]** For the one seller ~~one~~ *multiple* buyer setting:
  - **For naïve mean-based no regret buyers**: Full surplus extraction (seller gets revenue ≈ optimal social welfare and buyer gets ≈ zero utility)
  - **For non-overbidding mean-based no regret buyers**: *much harder to understand seller's optimal revenue than the single buyer setting (formal barriers)*

Exploitability of a Naïve learner [BMSW]

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Idea: lure the buyer into bidding high by offering the item for cheap at first, then start charging the item with a high price. It will take a while for the mean-based no regret buyer to realize they should not bid high anymore.

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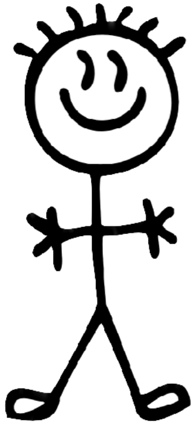

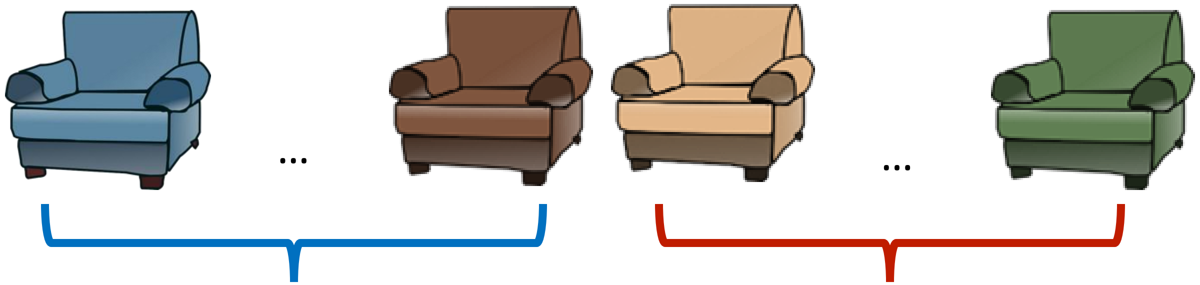

First $T/2$ rounds charge 0 Last $T/2$ rounds charge 1

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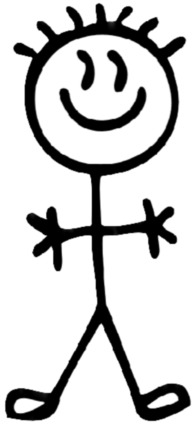

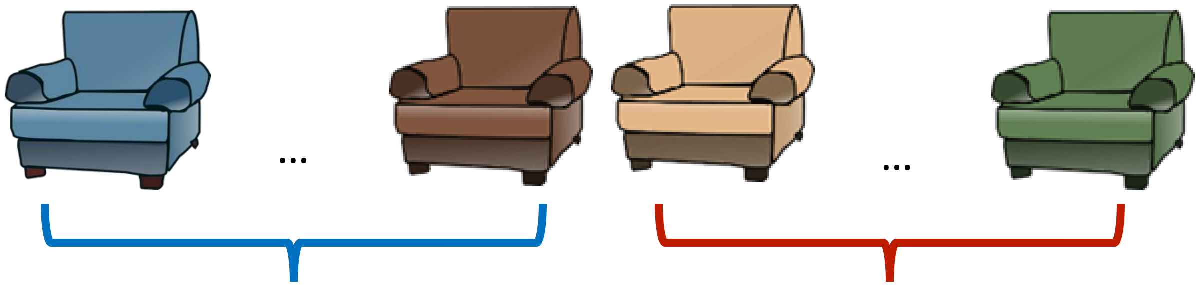

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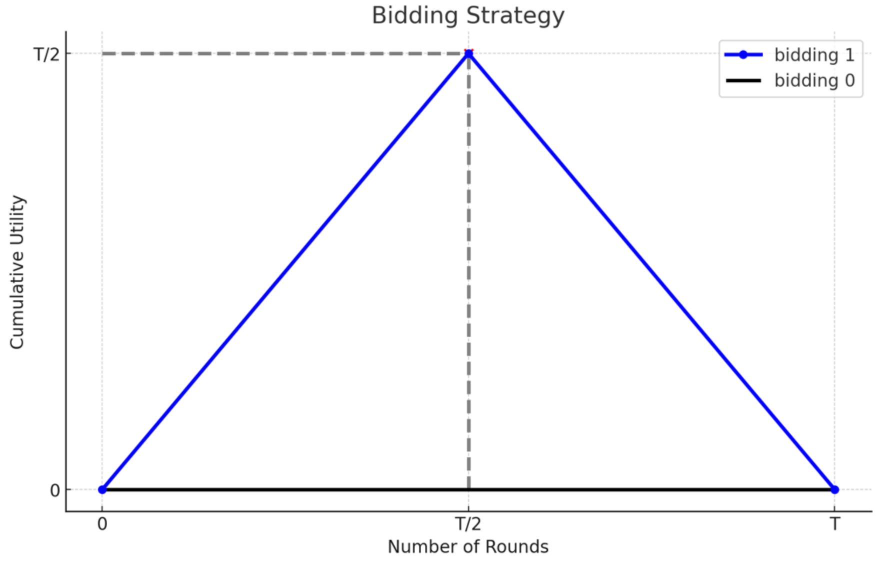

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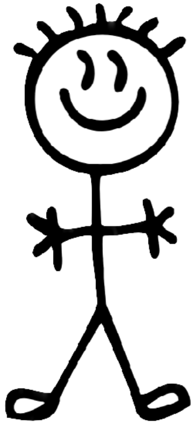

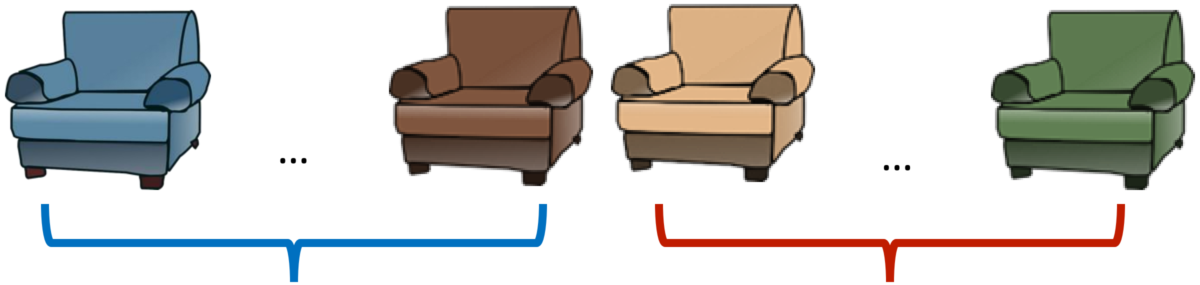

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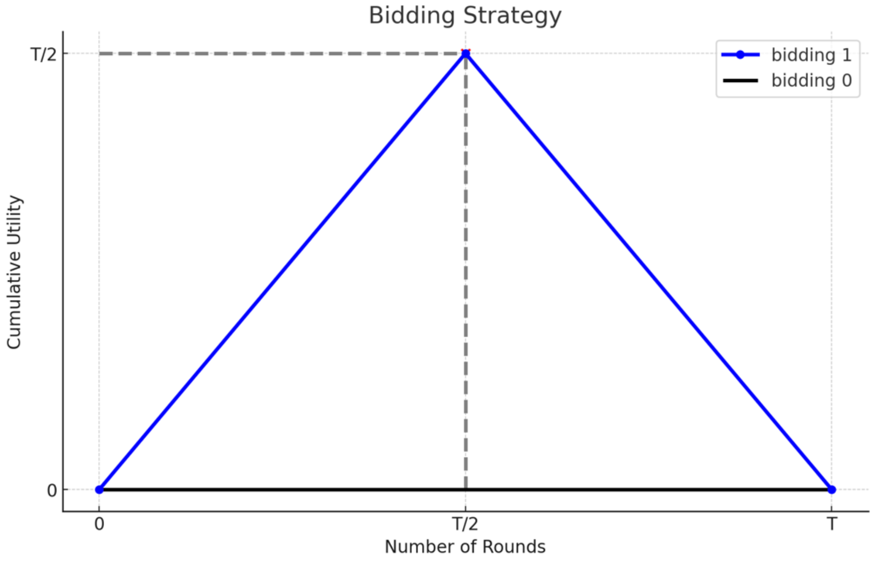

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- Naïve mean-based no regret buyer: bid 1 for all but $o(T)$ rounds, average utility ≈ 0
- Seller revenue : $1 - o(1)$

Exploitability of Multiple Naïve learners

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Our Idea:

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- Then, the auction is conducted in several phases.
- In each phase, the seller lures the buyer with the highest value into bidding ~~high~~ an intended value by offering the item for a fair price during first half of the phase, then start charging surcharge in the second half of the phase.

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- In each phase, the seller lures the buyer with the highest value into bidding ~~high~~ an intended value by offering the item for a fair price during first half of the phase, then start charging surcharge in the second half of the phase.
- Once the no-regret buyers learn to switch to other actions than their intended value at the end of the phase, repeat above process for a different set of intended values.

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In a Phase with R rounds:

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In a Phase with R rounds:

Buyer value to bid mapping:

$v_1 \rightarrow b_1$

$v_2 \rightarrow b_2$

…

$v_m \rightarrow b_m$

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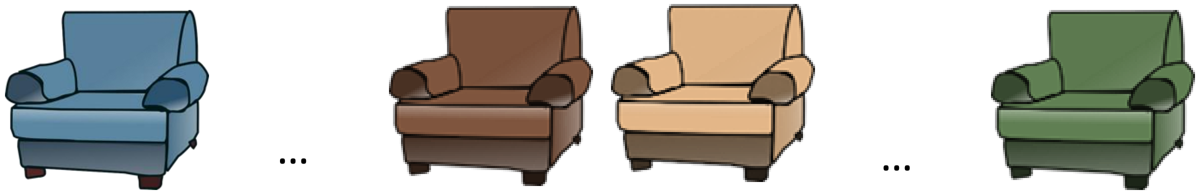

First $R/2$ rounds, run second price auction on underlying value based on bids

Last $R/2$ rounds, run second price auction with surcharge

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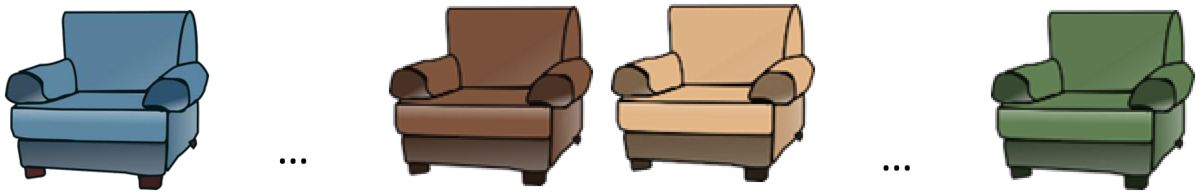

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bids $(b_1, b_2) \Rightarrow$ charge v_1 buyer with v_2

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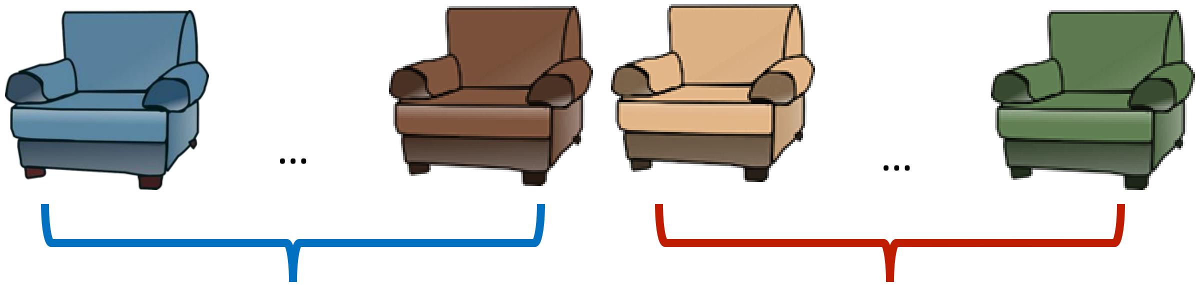

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bids $(b_1, b_2) \Rightarrow$ charge v_1 buyer with $v_2 + 2(v_1 - v_2)$

Buyer value to bid mapping:

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…

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- What would happen if the buyers are a mixture over different types of learning (some more naïve than others)?
- In the Ads auction setting, the sellers are typically bidding for multiple different keywords simultaneously. What would happen in the multiple item multiple buyer case?

THANK YOU!