

Selling to Multiple No Regret Buyers

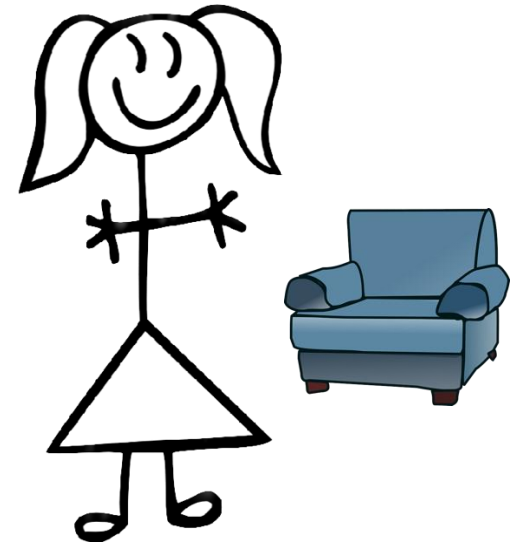
Linda Cai, S. Matthew Weinberg, Evan
Wildenhain, Shirley Zhang

Princeton University

Auction: Selling an Item to n Buyers

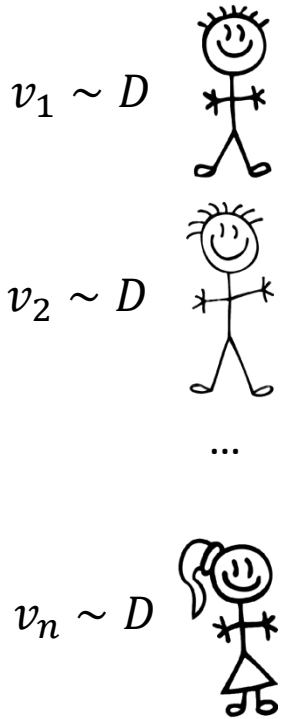
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One seller with an item

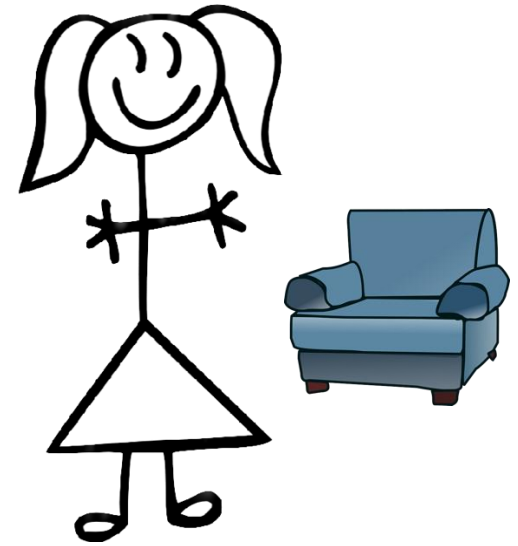


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n independent and identical buyers

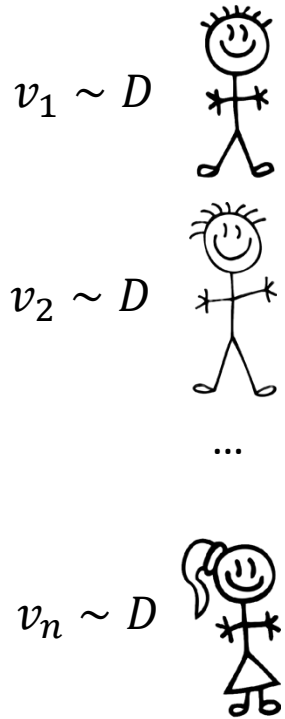


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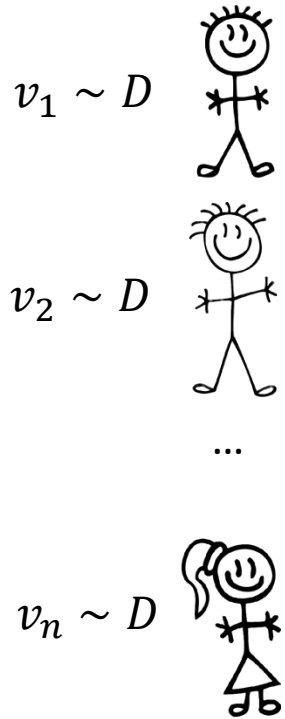


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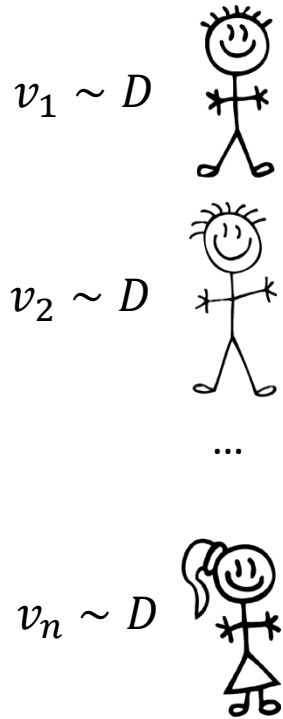


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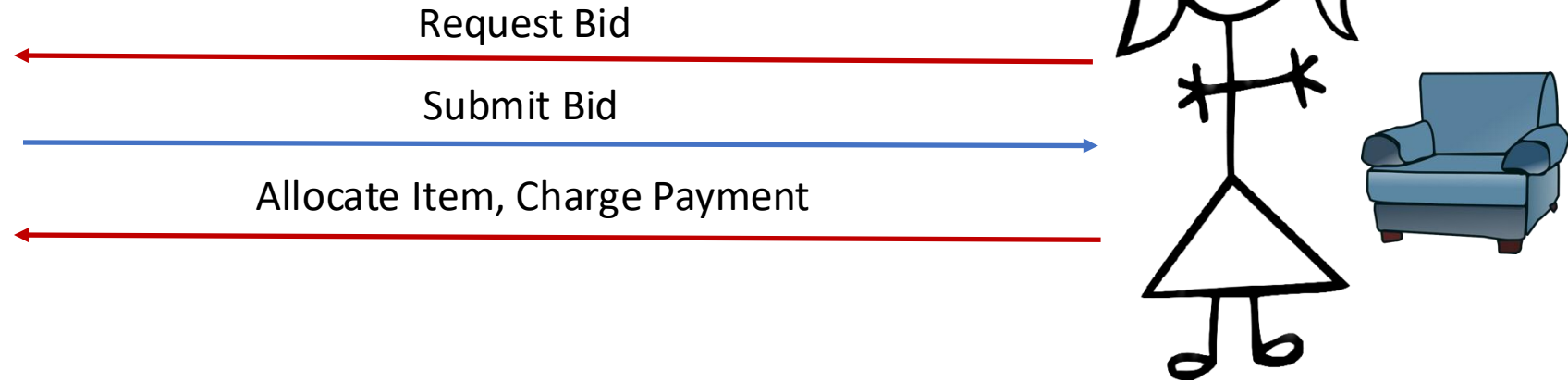


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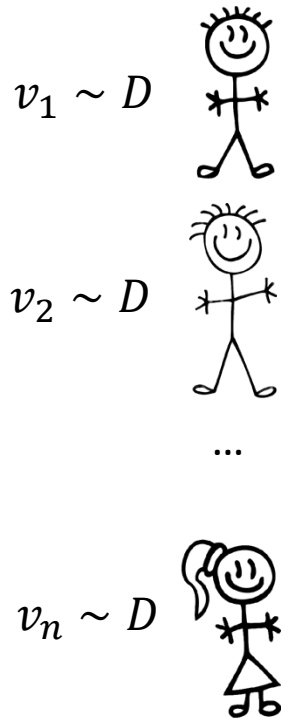


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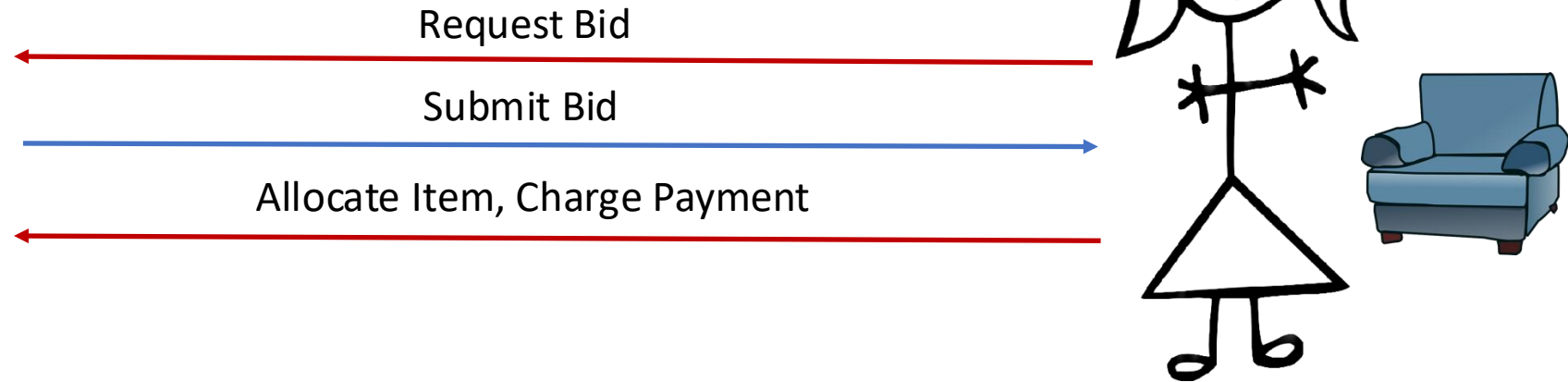


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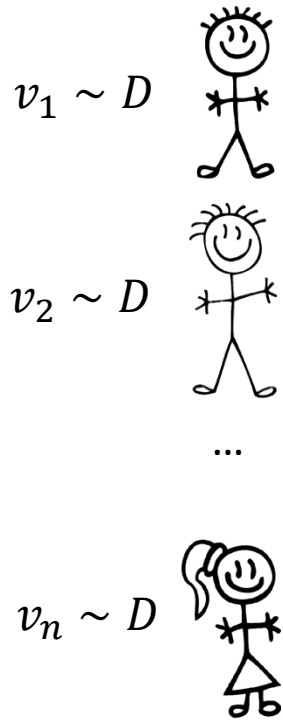


Goal: maximize their expected utility

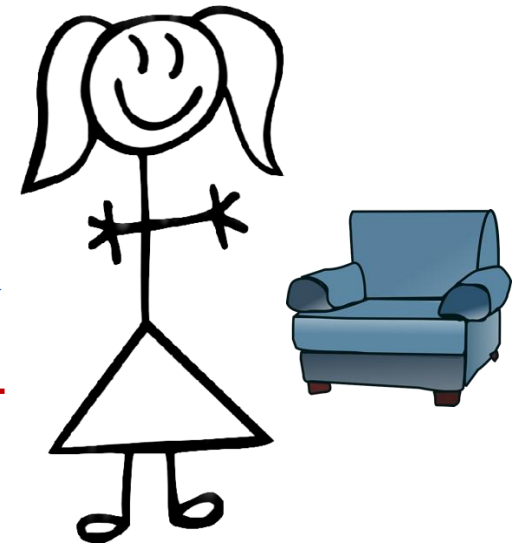
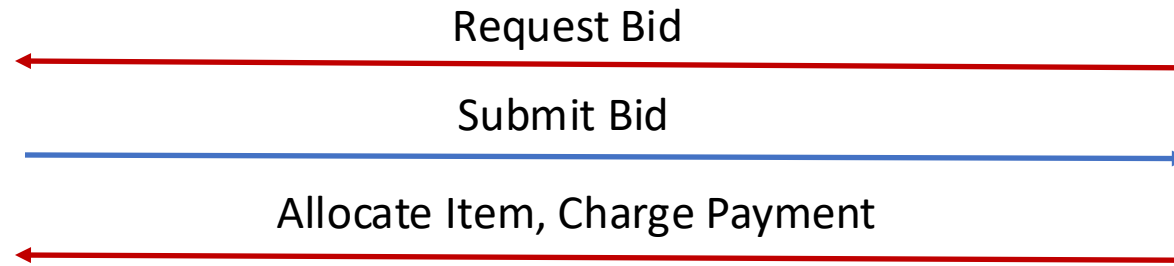
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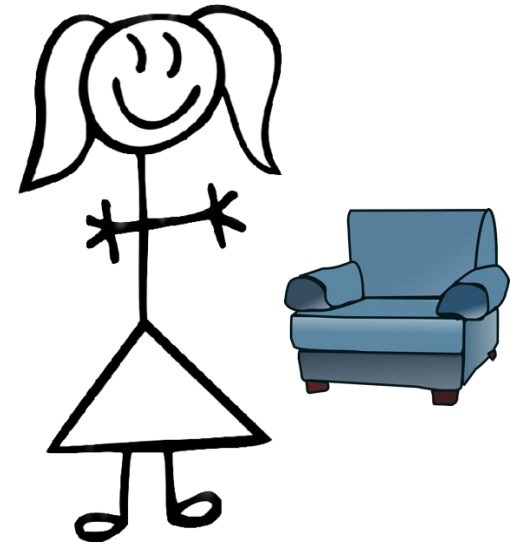
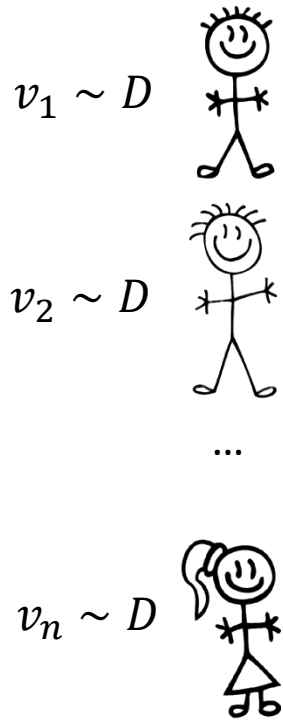


Goal: maximize **expected revenue** (payment received)

Goal: maximize their **expected utility**

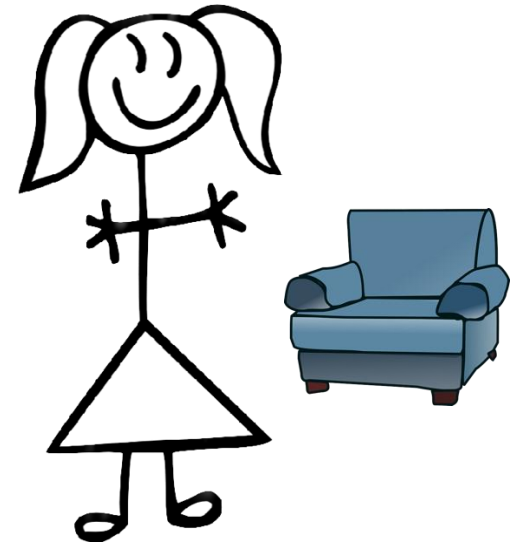
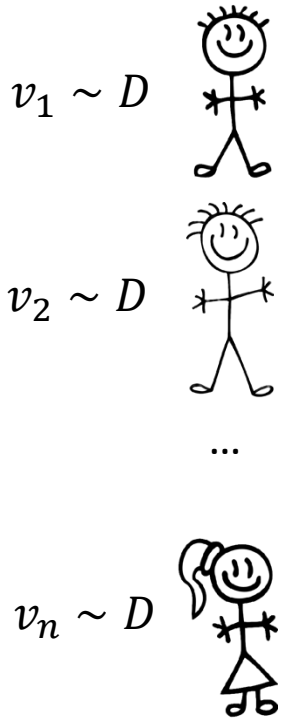
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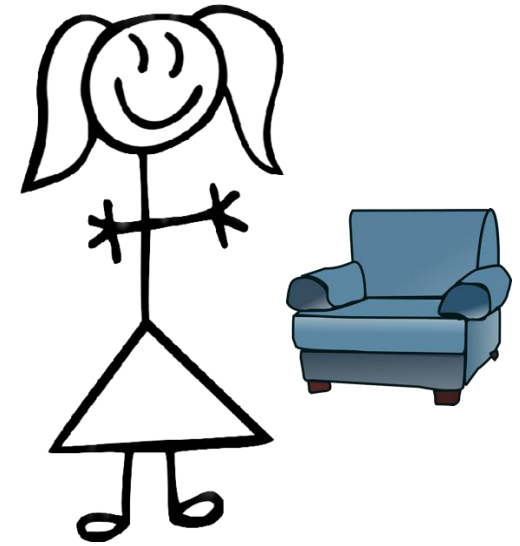
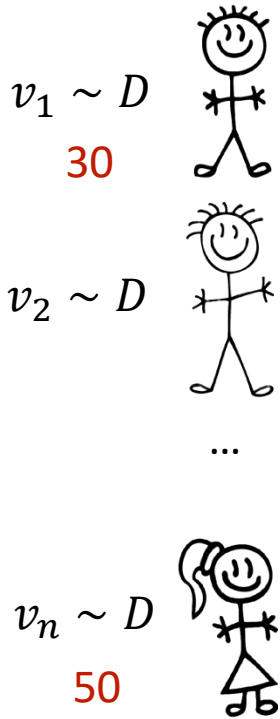
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One Shot Auction: classical Bayesian auction design study *truthful* auctions: all buyers can maximize their expected utility by reporting their value truthfully



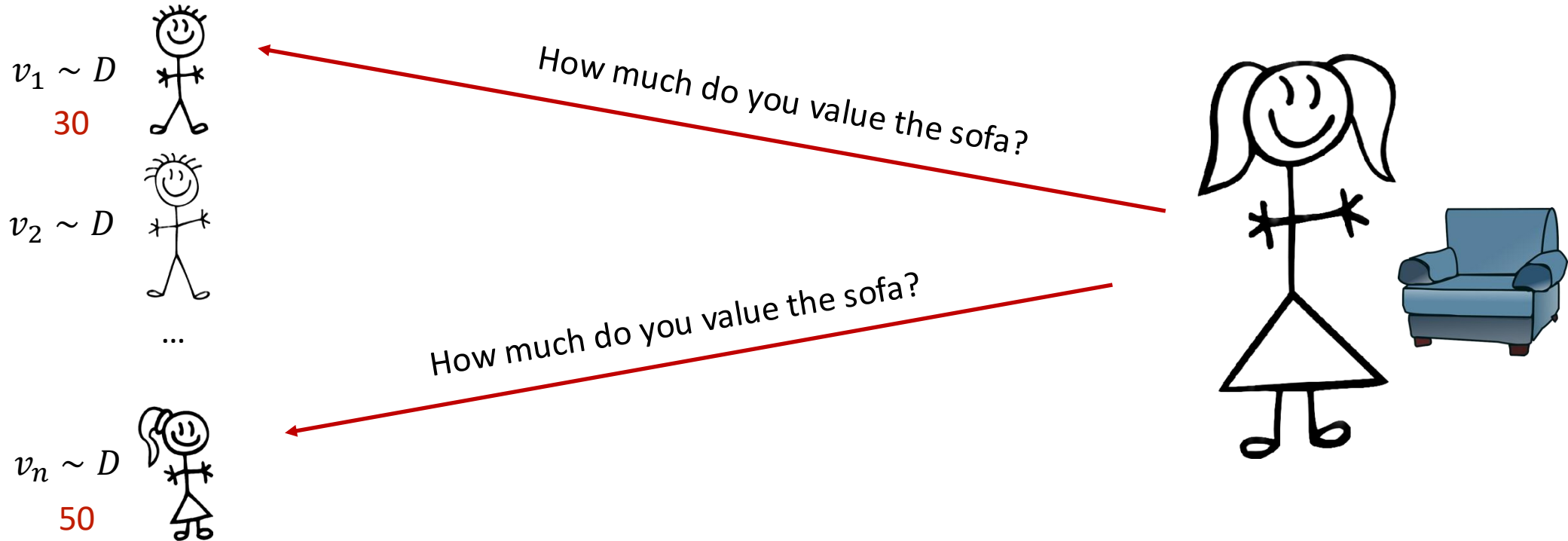
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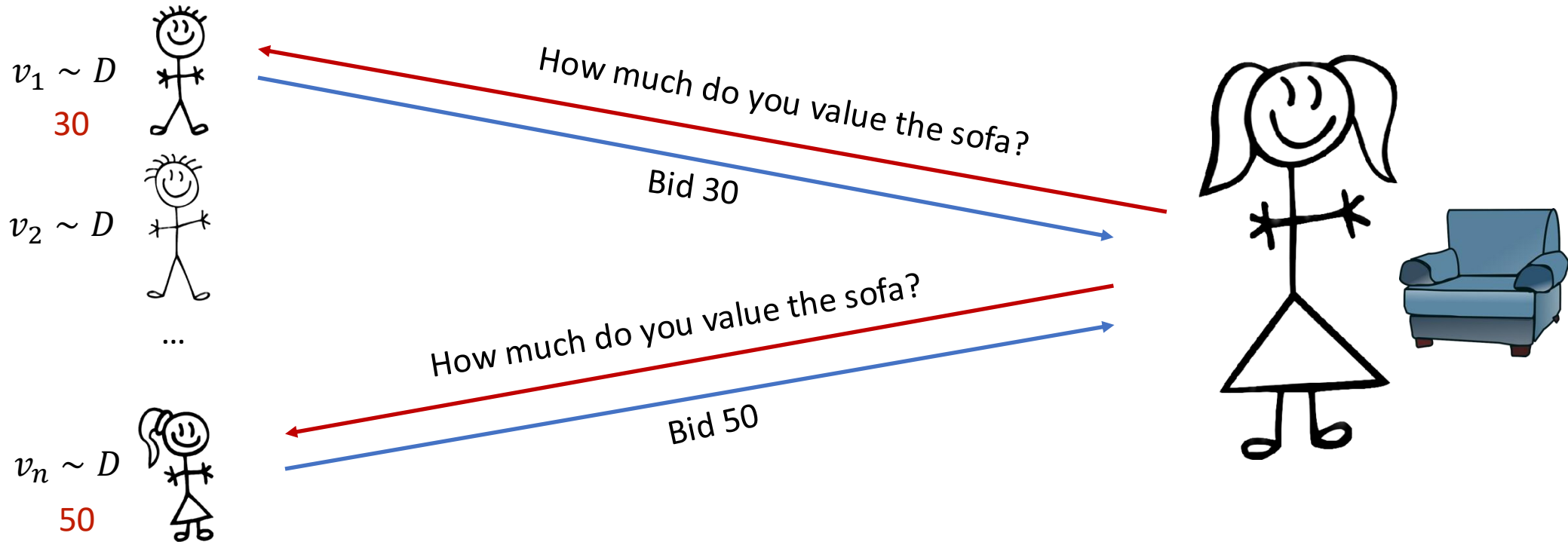
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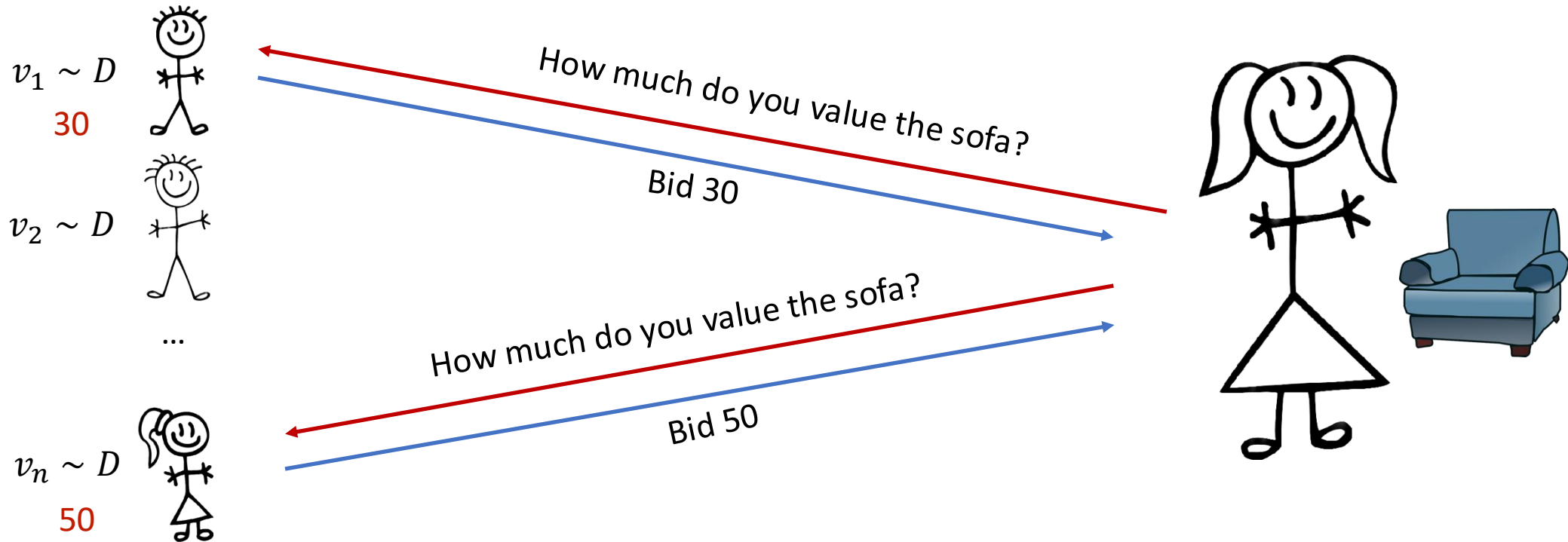
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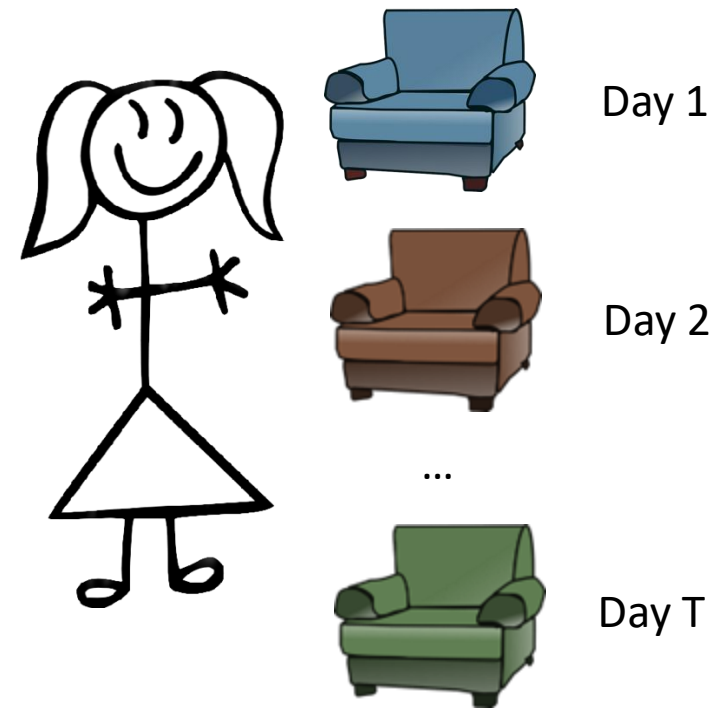
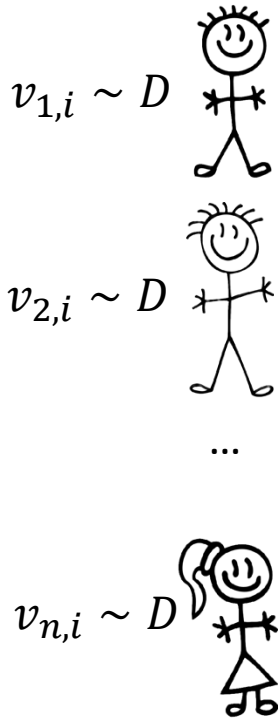
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Myerson gives a complete characterization of *revenue optimal* truthful auction

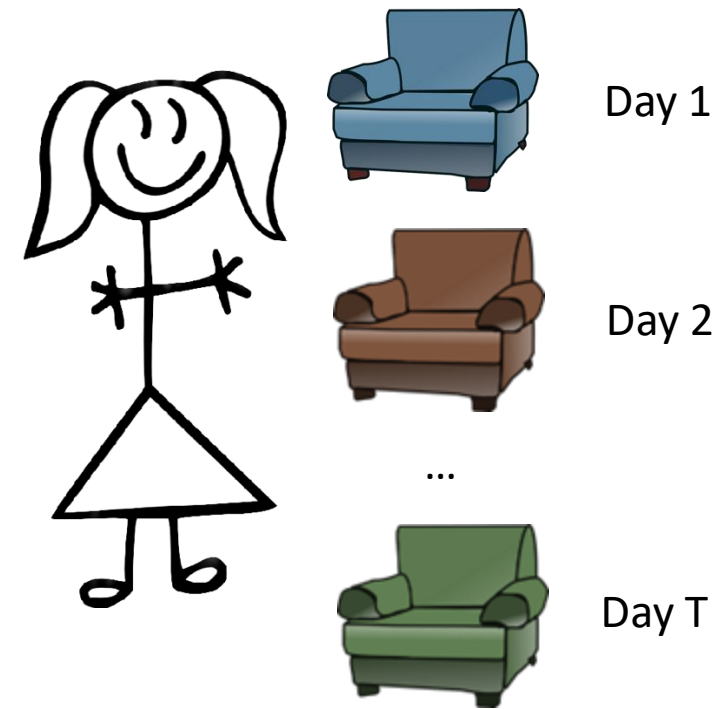
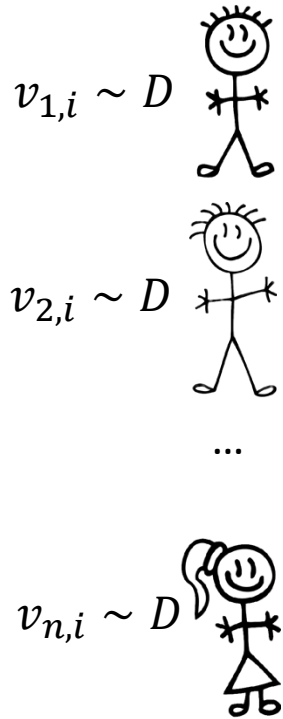
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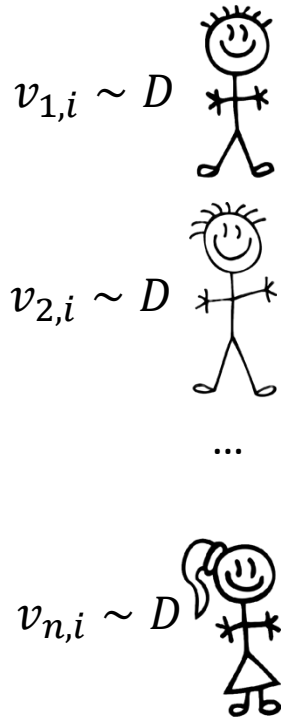


Goal: maximize their **expected total utility**

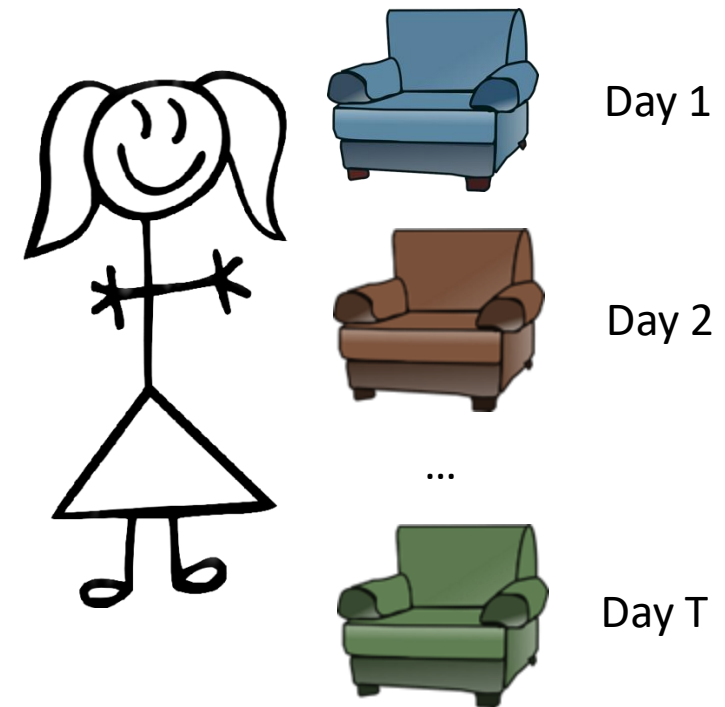
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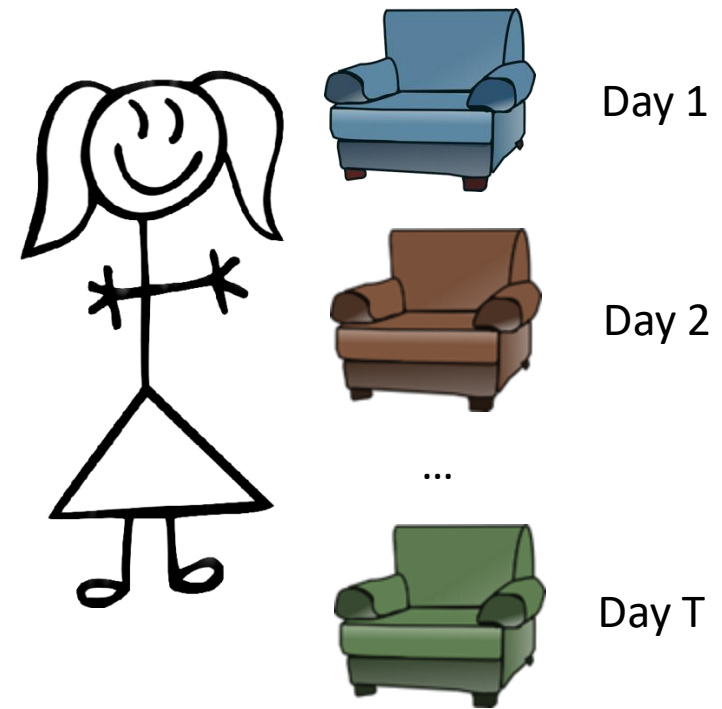
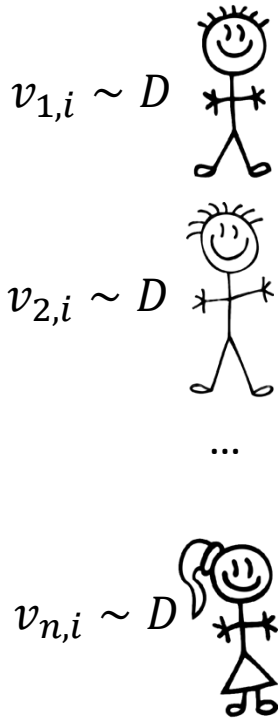
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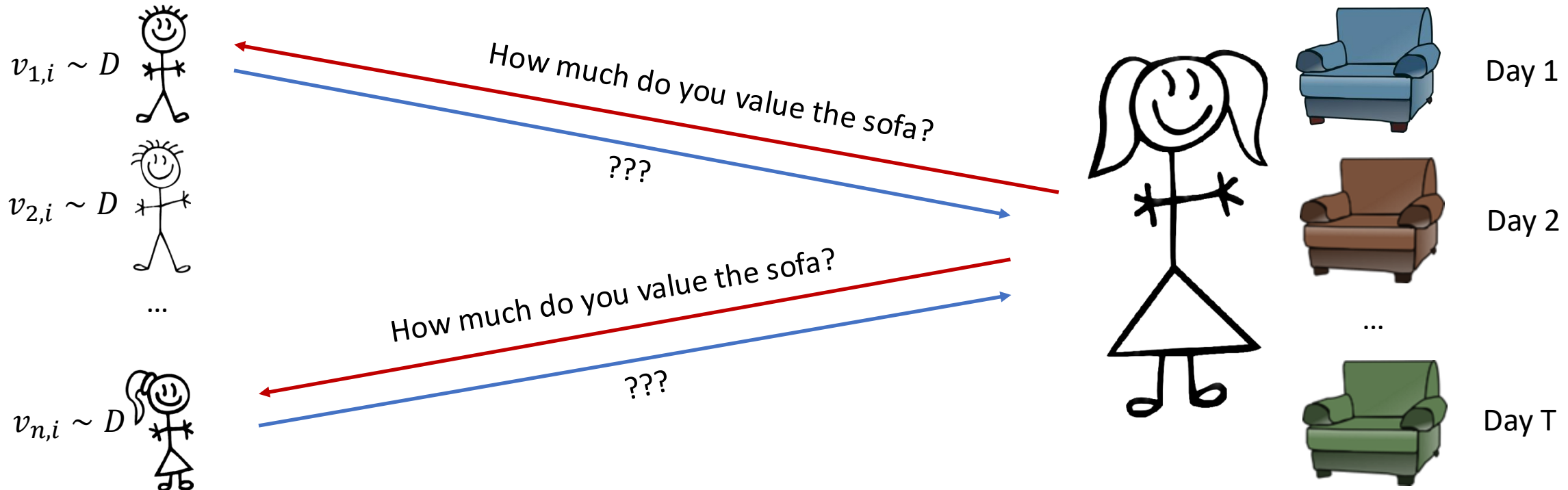
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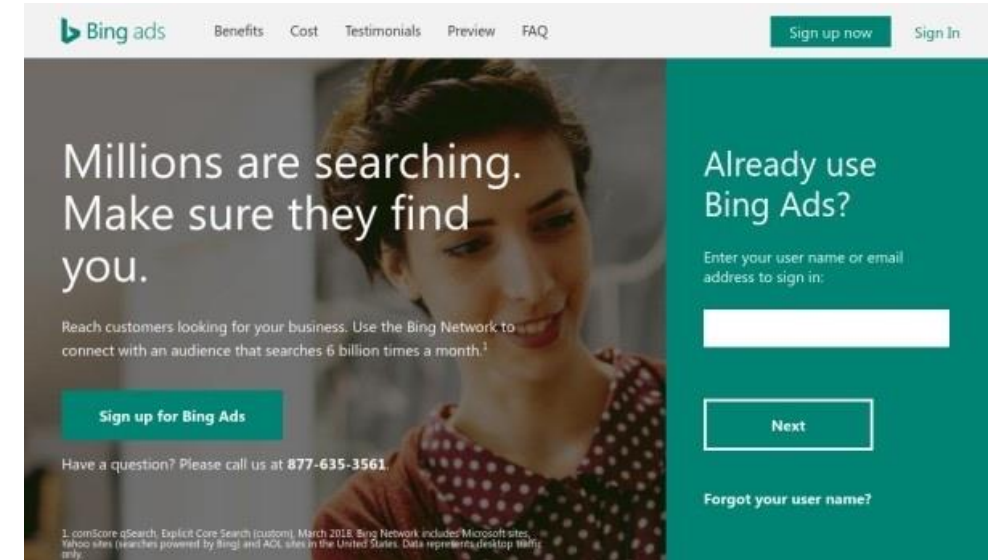
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Question: not clear how to incentivize truthfulness, how do we model buyer behavior?

Learning in Repeated Auction



The image shows the Bing Ads landing page. At the top, there is a navigation bar with the Bing Ads logo, links for Benefits, Cost, Testimonials, Preview, and FAQ, and buttons for Sign up now and Sign In. The main content area features a large background image of a smiling woman. The headline reads "Millions are searching. Make sure they find you." Below this, a sub-headline states: "Reach customers looking for your business. Use the Bing Network to connect with an audience that searches 6 billion times a month.¹" A green button labeled "Sign up for Bing Ads" is positioned below the sub-headline. To the right, a green sidebar contains the text "Already use Bing Ads?" followed by a form to "Enter your user name or email address to sign in:" with a text input field and a "Next" button. Below the form is a link for "Forgot your user name?". At the bottom left, a small footnote reads: "1. comScore qSearch, Explicit Core Search (custom), March 2016. Bing Network includes Microsoft sites, Yahoo sites (searches powered by Bing) and AOL sites in the United States. Data represents desktop traffic only."

Bing ads Benefits Cost Testimonials Preview FAQ Sign up now Sign In

Millions are searching. Make sure they find you.

Reach customers looking for your business. Use the Bing Network to connect with an audience that searches 6 billion times a month.¹

Sign up for Bing Ads

Have a question? Please call us at 877-635-3561

Already use Bing Ads?

Enter your user name or email address to sign in:

Next

Forgot your user name?

1. comScore qSearch, Explicit Core Search (custom), March 2016. Bing Network includes Microsoft sites, Yahoo sites (searches powered by Bing) and AOL sites in the United States. Data represents desktop traffic only.



The image is a Google Ads advertisement for PhotoKing. It features a dark blue background with two white smartphones. The left smartphone displays a photo of a person, and the right smartphone displays a photo of a person with a heart icon and the number 498. The text "GOOGLE ADS" is prominently displayed in the center, with "How to Easily promote your Business online" below it. The PhotoKing logo, which includes a chess knight icon and the text "PHOTO KING", is located at the bottom right.

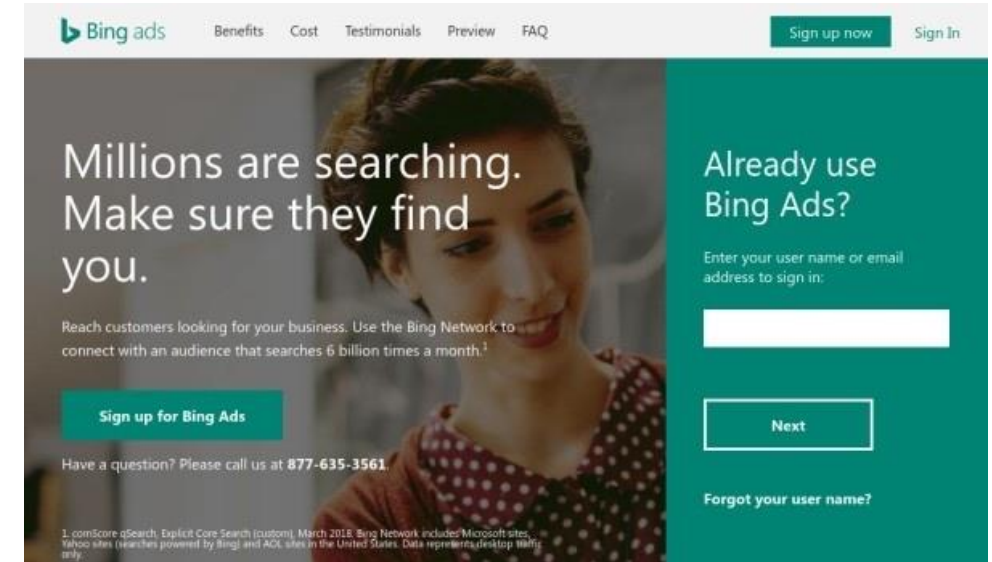
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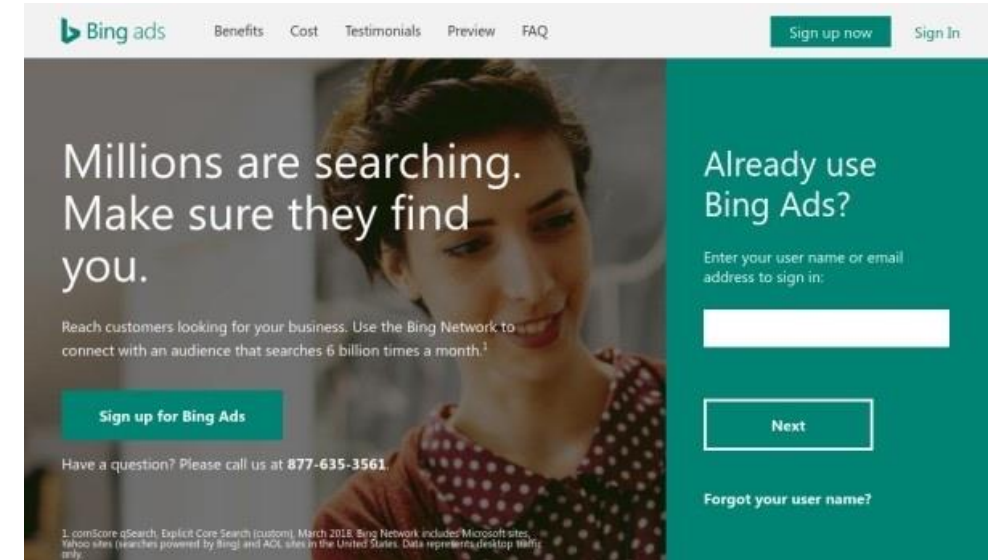
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- **[Nekipelov, Syrgkanis, and Tardos 2015]** Bidder behavior is more consistent with no-regret learning in repeated auctions (e.g. in Bing ad auction)



No Regret Learning with Context



Action 1



Action 2



Action 3

...



Action m

No Regret Learning with Context



Action 1



Action 2



Action 3

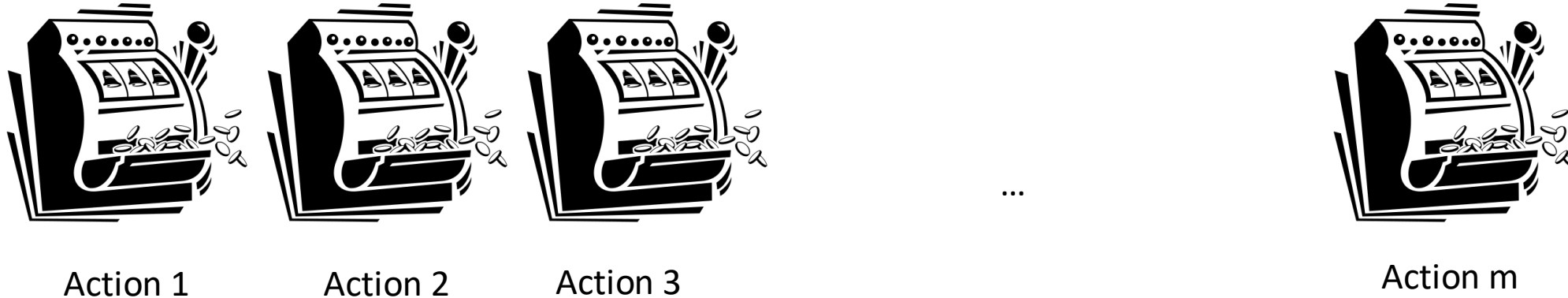
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In a repeated game with T rounds with context space V and action space A :

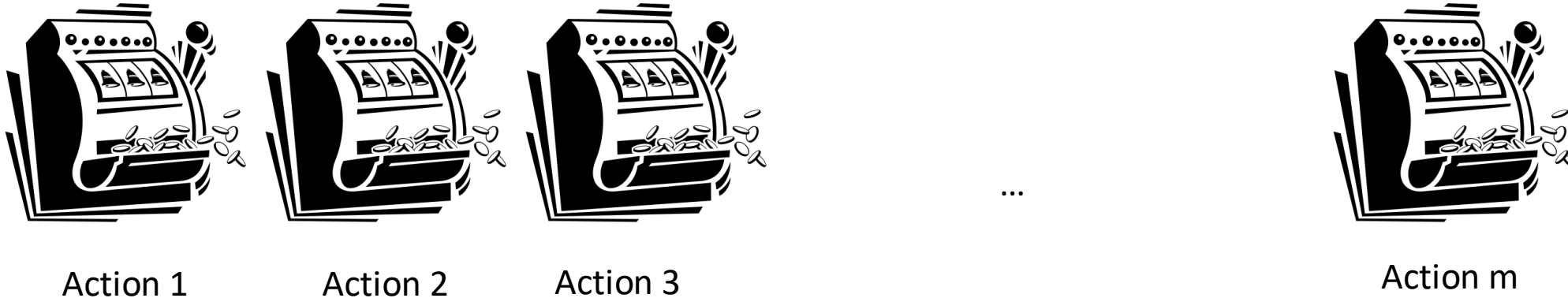
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- In each round t , player i observes a context $v_{i,t} \in V$, then takes an action $a_{i,t} \in A$ and observe a reward $r_t(a_{i,t})$ (the reward function may be correlated with the context)

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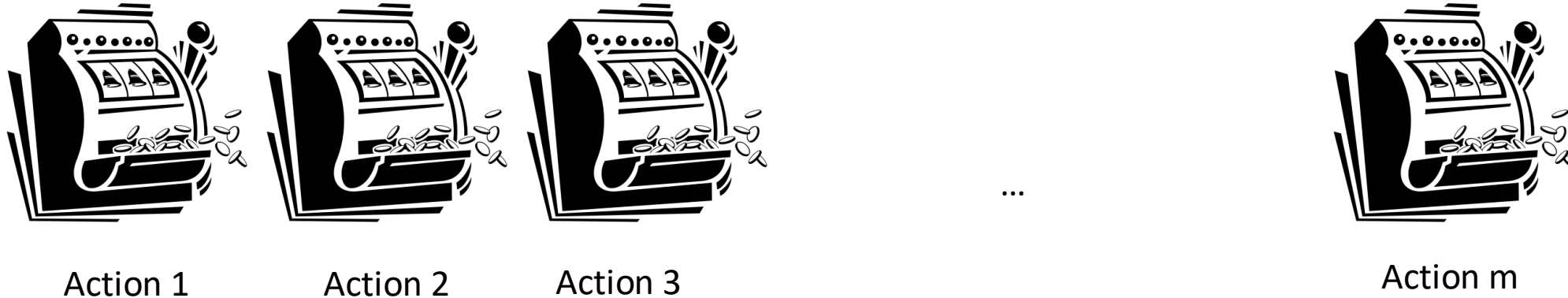
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$$\sum_{t=1}^T r_t(a_{i,t}) \geq \max_{\text{fixed action strategy } s} \sum_{t=1}^T r_t(\text{action } s \text{ takes given context } v_{i,t}) - o(T)$$

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Definition (Mean-Based No Regret): An algorithm is *mean-based no regret* if:
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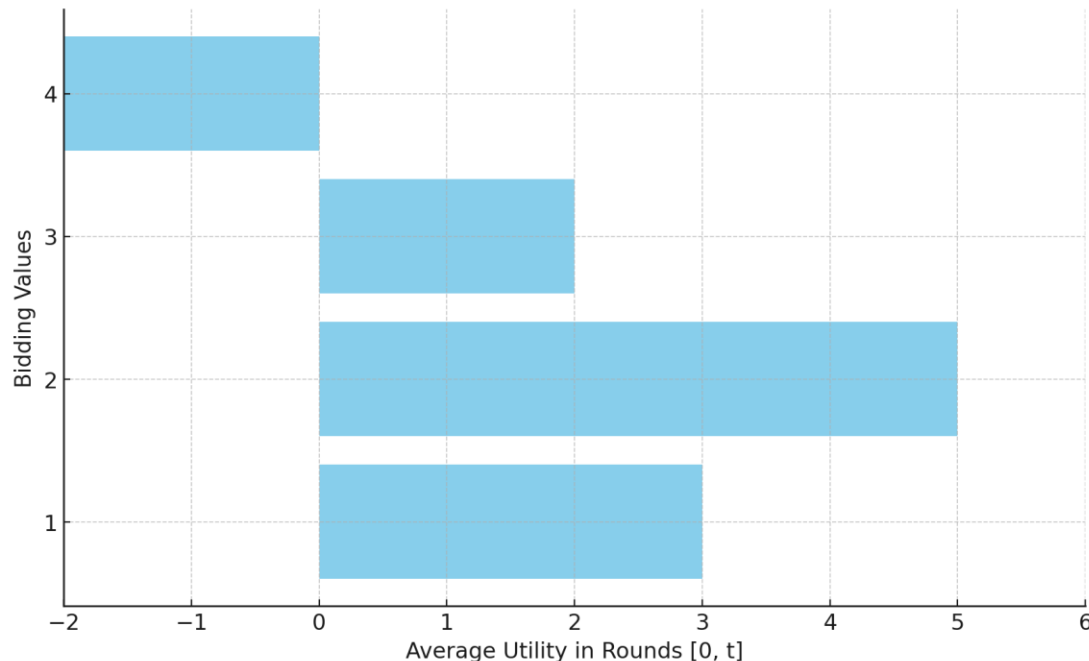
- Intuition: if an action seems much better historically it would be played with much higher probability

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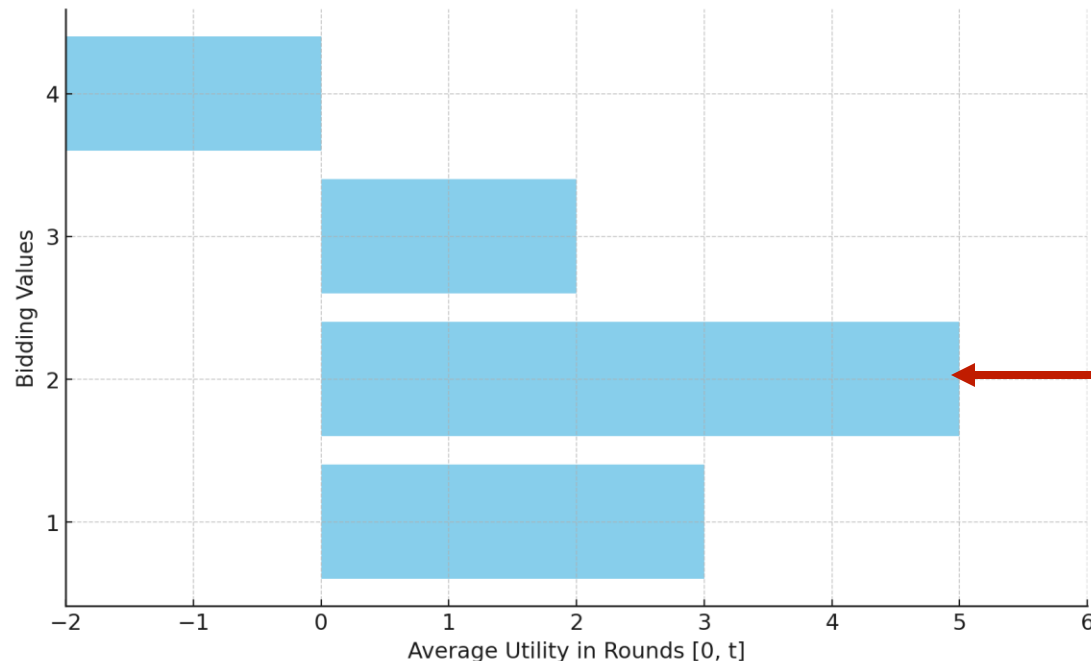
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A mean-based no regret algorithm will choose to bid **2** with high probability

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Once the difference between *cumulative utility* of two actions exceed the initial cost, the inferior action will never be chosen.

Exploitability of Learning Agents

- **Question:** How much revenue can the seller extract *knowing* that the buyers are using *mean-based no regret* learning algorithms (but not the specific algorithm they are using)?
- [Braverman, Mao, Schneider, and Weinberg'18] For the one seller one buyer setting:
 - **For naïve mean-based no regret buyer:** Full surplus extraction (seller gets revenue \approx optimal social welfare and buyer gets \approx zero utility)
 - **For non overbidding mean-based no regret buyer:** Optimal revenue can be unboundedly worse than for naïve learners, yet still unboundedly better than a seller who runs the same auction in each round.

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- **[Our Result]** For the one seller ~~one~~ *multiple* buyer setting:
 - For **naïve mean-based no regret buyers**: Full surplus extraction (seller gets revenue \approx optimal social welfare and buyer gets \approx zero utility)
 - For **non-overbidding mean-based no regret buyers**: *much harder to understand seller's optimal revenue than the single buyer setting (formal barriers)*

Exploitability of a Naïve learner [BMSW]

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Idea: lure the buyer into bidding high by offering the item for cheap at first, then start charging the item with a high price. It will take a while for the **mean-based no regret** buyer to realize they should not bid high anymore.

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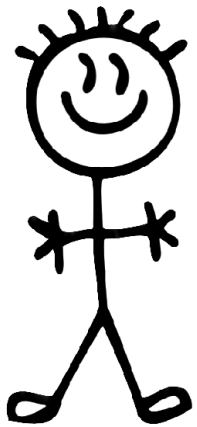


$$v_{1,t} = 1/2$$

Action space: bid **1** → get the item
bid **<1** → get nothing

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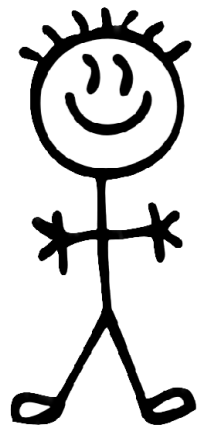
First $T/2$ rounds charge 0 Last $T/2$ rounds charge 1

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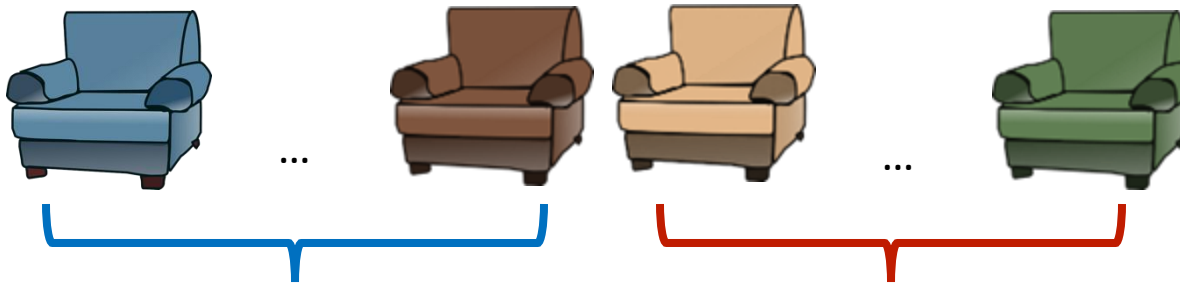
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bid < 1 → get nothing

Exploitability of a Naïve learner [BMSW]

Idea: lure the buyer into bidding high by offering the item for cheap at first, then start charging the item with a high price. It will take a while for the **mean-based no regret** buyer to realize they should not bid high anymore.

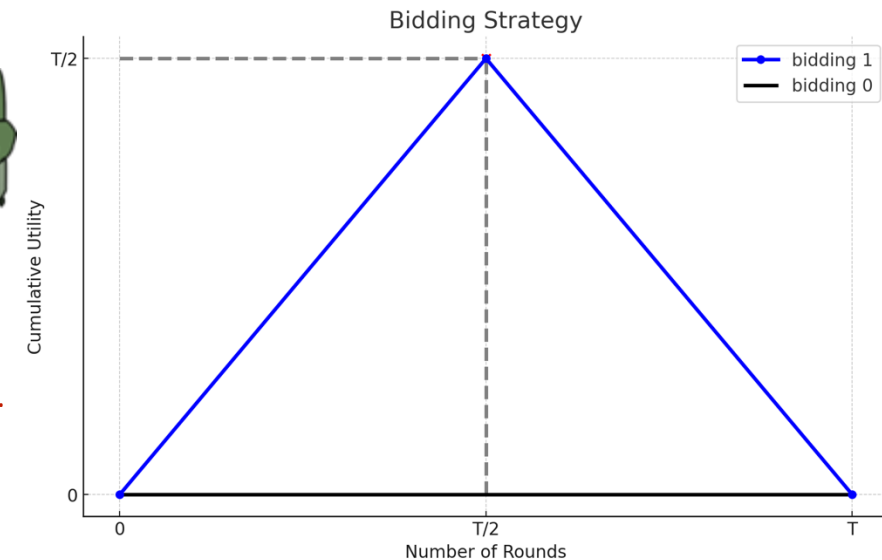


$$v_{1,t} = 1/2$$



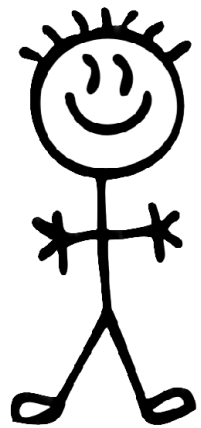
First $T/2$ rounds charge 0 Last $T/2$ rounds charge 1

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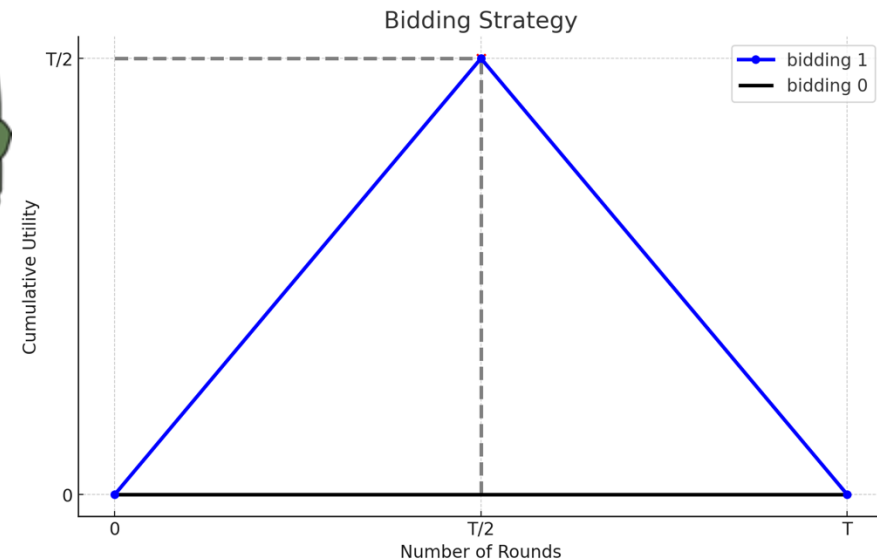
...



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- Naïve **mean-based no regret** buyer: bid 1 for all but $o(T)$ rounds, average utility ≈ 0
- Seller revenue : $1 - o(1)$

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- Then, the auction is conducted in several phases.
- In each phase, the seller lures the buyer with the highest value into bidding high an intended value by offering the item for a fair price during first half of the phase, then start charging surcharge in the second half of the phase.

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- In each phase, the seller lures the buyer with the highest value into bidding high an intended value by offering the item for a fair price during first half of the phase, then start charging surcharge in the second half of the phase.
- Once the no-regret buyers learn to switch to other actions than their intended value at the end of the phase, repeat above process for a different set of intended values.

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In a Phase with **R** rounds:

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In a Phase with **R** rounds:

Buyer value to bid mapping:

$$v_1 \rightarrow b_1$$

$$v_2 \rightarrow b_2$$

...

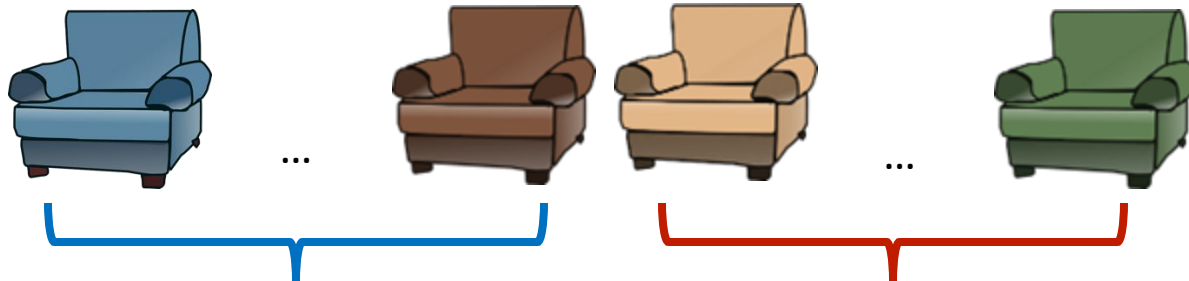
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In a Phase with R rounds:



First $R/2$ rounds, run second price auction on underlying value based on bids

Last $R/2$ rounds, run second price auction with surcharge

Buyer value to bid mapping:

$$v_1 \rightarrow b_1$$

$$v_2 \rightarrow b_2$$

...

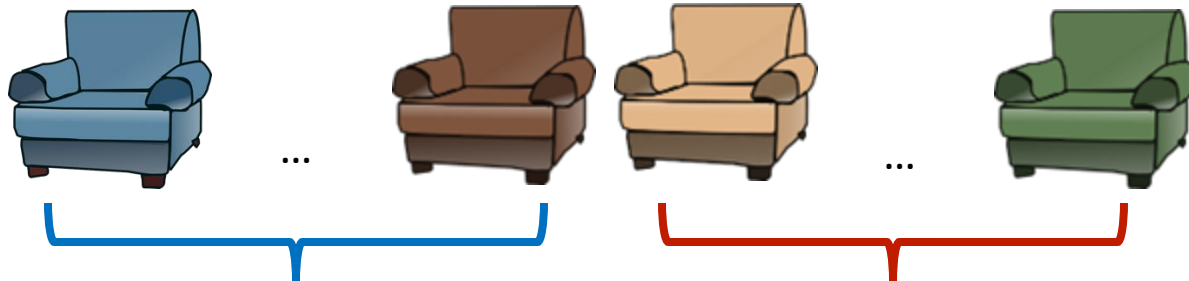
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bids $(b_1, b_2) \Rightarrow$ charge v_1
buyer with v_2

Last $R/2$ rounds, run second price auction with surcharge

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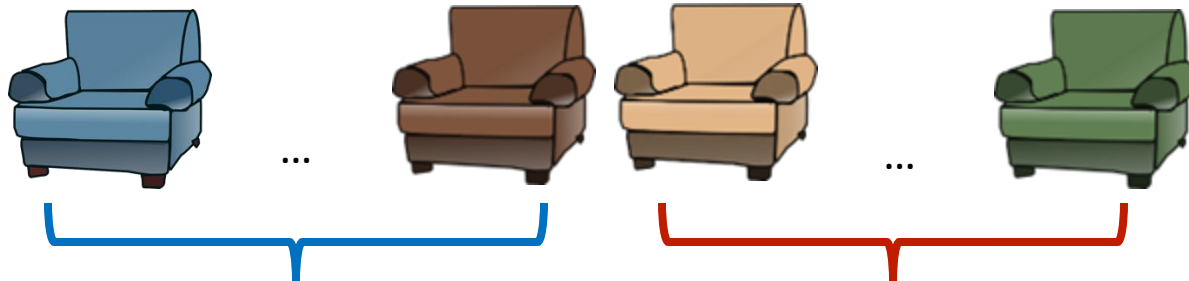
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bids $(b_1, b_2) \Rightarrow$ charge v_1
buyer with $v_2 + 2(v_1 - v_2)$

Buyer value to bid mapping:

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- What would happen if the buyers are a mixture over different types of learning (some more naïve than others)?
- In the Ads auction setting, the sellers are typically bidding for multiple different keywords simultaneously. What would happen in the multiple item multiple buyer case?

THANK YOU!

