$$\mathcal{H} = -\frac{N}{2}\sum_\mu m_\mu^2 = \qquad m_\mu = \frac{1}{N}\sum_i \xi_i^\mu \sigma_i$$

$$= -\frac{1}{2N}\sum_\mu\left(\sum_i \xi_i^\mu \sigma_i\right)^2$$

$$Z = \sum_{\{\sigma\}} \exp\left(\frac{\beta}{2N}\sum_\mu\left(\sum_i \xi_i^\mu \sigma_i\right)^2\right) =$$

$$= \int \frac{dz_\mu}{(\sqrt{2\pi})^P} \sum_{\{\sigma\}} \exp\left(-\sum_\mu \frac{z_\mu^2}{2} + \right.$$

$$\left. + \sqrt{\frac{\beta}{N}}\sum_\mu z_\mu \sum_i \xi_i^\mu \sigma_i\right)$$

$$z_\mu \longrightarrow \sqrt{\beta}\, z_\mu \quad :$$

$$Z = \int \frac{dz}{(2\pi/\beta)^{P/2}} \sum_{\{\sigma\}} \exp\left(-\frac{\beta}{2}\sum_\mu z_\mu^2\right.$$

$$\left. + \frac{\beta}{\sqrt{N}}\sum_{\mu, i} \xi_i^\mu z_\mu \sigma_i\right)$$

$$P(\sigma_i = +1 | z) = \frac{\exp\left(\frac{\beta}{\sqrt{N}}\sum_\mu \xi_i^\mu z_\mu\right)}{\exp(- \dots) + \exp(+ \dots)} =$$

$$= \frac{1}{1 + \exp\left(-\frac{2\beta}{\sqrt{N}}\sum_\mu \xi_i^\mu z_\mu\right)} \qquad (X)$$

$$P(z_\mu|\sigma) \propto \exp\left(-\frac{\beta}{2}z_\mu^2 + \frac{\beta}{\sqrt{N}} z_\mu \sum_i \xi_i^\mu \sigma_i\right) =$$

$$= \exp\left[-\frac{\beta}{2}\left(z_\mu^2 - 2 z_\mu \frac{1}{\sqrt{N}}\sum_i \xi_i^\mu \sigma_i\right)\right]$$

$$\propto \exp\left[-\frac{\beta}{2}\left(z_\mu - \frac{1}{\sqrt{N}}\sum_i \xi_i^\mu \sigma_i\right)^2\right]$$

$$\Rightarrow z_\mu \sim \frac{1}{\sqrt{N}}\sum_i \xi_\mu^i \sigma_i + \frac{1}{\sqrt{\beta}}\varepsilon \qquad (X)$$

$$\varepsilon \sim \mathcal{N}(0,1)$$

$$E \equiv -\beta \hat{\mathcal{H}}$$

$$\mathcal{L} = \prod_{d \in D} P(\sigma = \sigma_d | \xi)$$

le Z vanno dentro [illegible]

$$l = \log \mathcal{L} = \sum_{d \in D}\left(E(\sigma_d|\xi) - \log Z(\xi)\right)$$

$$\xi_i^\mu \to \xi_i^\mu + \varepsilon \frac{\partial l}{\partial \xi_i^\mu}$$

$$\frac{\partial E}{\partial \xi_i^\mu} = \frac{\partial}{\partial \xi_i^\mu}\left(\frac{\beta}{2N}\sum_{\mu'}\left(\sum_i \xi_i^{\mu'} \sigma_i\right)^2\right) =$$

$$= \frac{\beta}{N}\left(\sum_j \xi_j^\mu \sigma_j\right)\sigma_i =$$

$$\frac{\partial \log Z}{\partial \xi_i^\mu} = \left\langle \frac{\beta}{\sqrt{N}} z_\mu \sigma_i \right\rangle_{\text{model}}$$