

$$H(Y|X) = \sum_{x \in X} P(x) H(Y|X=x)$$

$$P(Y|X) = \frac{P(X,Y)}{P(X)}$$

$$H(\text{root}) = \frac{4}{7} H(Y|\text{left}) + \frac{3}{7} H(Y|\text{right})$$

$$H(\text{root}) = -P \log_2 P - (1-P) \log_2 (1-P)$$

$$= -\sum_i P_i \log_2 P_i = -\frac{2}{7} \log_2 \frac{2}{7} - \frac{5}{7} \log_2 \frac{5}{7} = 0.86.$$

$$H(Y|\text{left}) = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.81.$$

$$H(Y|\text{right}) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.92.$$

$$IG(\text{split}) = H(\text{root}) - \left(\frac{4}{7} H(Y|\text{left}) + \frac{3}{7} H(Y|\text{right}) \right)$$

$$= 0.86 - \frac{4}{7} \cdot 0.81 - \frac{3}{7} \cdot 0.92 = 0.006.$$

$$H(Y|\text{left}) = 0; H(Y|\text{right}) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97.$$

$$IG(\text{split}) = 0.86 - \left(\frac{2}{7} \cdot 0 + \frac{5}{7} \cdot 0.97 \right) = 0.17.$$

