Introduction to Machine Learning

Convolutional Neural Networks

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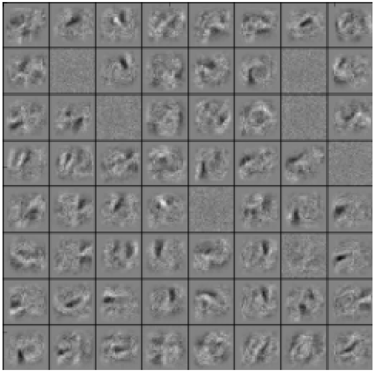
Overview

- What makes vision hard?
  - Vision needs to be robust to a lot of transformations or distortions
    - Change in pose or viewpoint
    - Change in illumination
    - Deformation
    - Occlusion (some objects are hidden behind others)
  - Many object categories can vary wildly in appearance (e.g. chairs)
  - Geoff Hinton: "Imaging a medical database in which the age of the patient sometimes hops to the input dimension which normally codes for weight!"

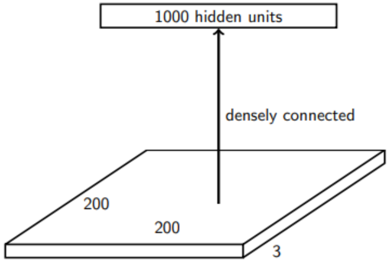
Overview

- Recall we looked at some hidden layer features for classifying handwritten digits:



Overview

- Suppose we want to train a network that takes a 200×200 RGB image as the input.



- What is the problem with having this as the first layer?
  - Too many parameters! Input size = $200 \times 200 \times 3$ = 120K. Parameters = 120K $\times$ 1000 = 120 million.
  - What happens if the object in the image shifts a little?

Overview

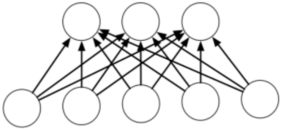
- The same sorts of features that are useful in analyzing one part of the image will probably be useful for analyzing other parts as well
  - E.g. edges, corners, contours, object parts
- We want a neural net architecture that lets us learn a set of feature detectors that are applied at all image locations

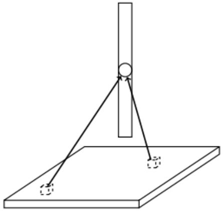
Overview

- So far, we have seen this type of layers:
    - Fully connected layers
- Different layers could be stacked together to build powerful models
- Let’s add another layer type: the convolution layer

Fully Connected Layers

- Fully connected layers:

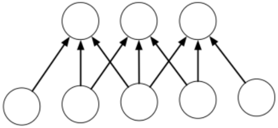


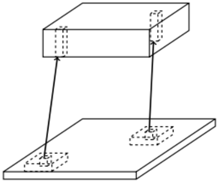


- Each hidden unit looks at the entire image

Locally Connected Layers

- Locally connected layers:

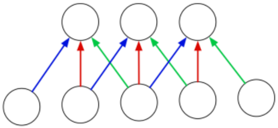


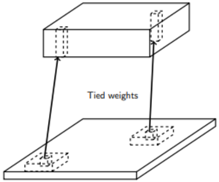


- Each column or set of hidden units looks at a small region of the image

Convolution Layers

- Convolution layers:

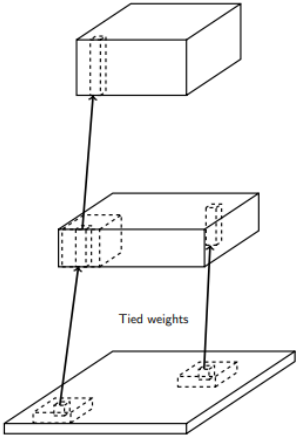




- Each column or set of hidden units looks at a small region of the image, and the weights are shared between all image locations

Going Deeply Convolutional

- Convolution layers can be stacked:

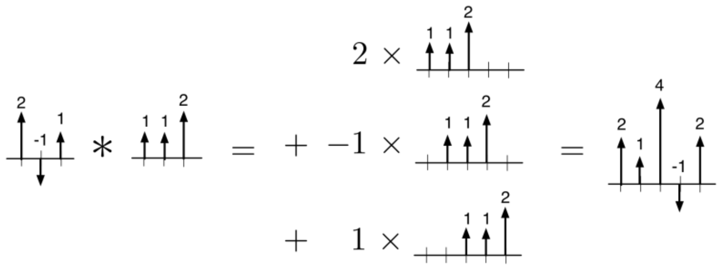


Convolution

- We have been vectorizing our computations by expressing them in terms of matrix and vector operations for computational efficiency
- Now we introduce a high-level operation, convolution.
  - The motivation is not computational efficiency
  - Rather, the motivation is to get some understanding of what convolution layers can do

Convolution

- Let’s look at the 1-D case first.



Convolution

- Convolution can also be viewed as matrix multiplication

$$(2,-1,1)*(1,1,2)=\begin{pmatrix}1 & & \\ 1 & 1 & \\ 2 & 1 & 1 \\ & 2 & 1 \\ & & 2\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix}$$

Convolution

- Some properties of convolution
  - Commutativity

$$a * b = b * a \tag{1}$$

  - Linearity

$$a * (\lambda_1 b + \lambda_2 c) = \lambda_1 a * b + \lambda_2 a * c \tag{2}$$

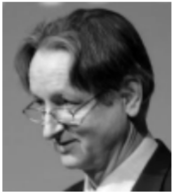
2-D Convolution

- Here is an example of 2-D convolution, which we will use in computer vision

$$\begin{array}{|c|c|c|}\hline 1&3&1\\\hline 0&-1&1\\\hline 2&2&-1\\\hline\end{array} * \begin{array}{|c|c|}\hline 1&2\\\hline 0&-1\\\hline\end{array} = \begin{array}{l} 1\times \begin{array}{|c|c|c|c|}\hline 1&3&1&\\\hline 0&-1&1&\\\hline 2&2&-1&\\\hline &&&\\\hline\end{array} \\ + \; 2\times \begin{array}{|c|c|c|c|}\hline &1&3&1\\\hline &0&-1&1\\\hline &2&2&-1\\\hline &&&\\\hline\end{array} \\ + -1\times \begin{array}{|c|c|c|c|}\hline &&&\\\hline &1&3&1\\\hline &0&-1&1\\\hline &2&2&-1\\\hline\end{array} \end{array} = \begin{array}{|c|c|c|c|}\hline 1&5&7&2\\\hline 0&-2&-4&1\\\hline 2&6&4&-3\\\hline 0&-2&-2&1\\\hline\end{array}$$

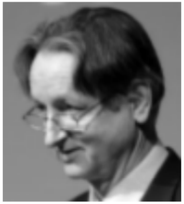
2-D Convolution

- The thing we convolve by is called a kernel, or filter
- What does this convolution kernel do?



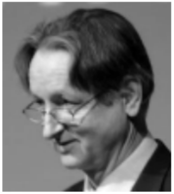
$*$

0	1	0
1	4	1
0	1	0

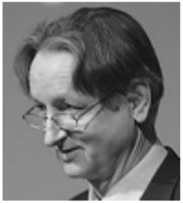


2-D Convolution

- What does this convolution kernel do?



$$*\quad \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 8 & -1 \\ \hline 0 & -1 & 0 \\ \hline\end{array}$$

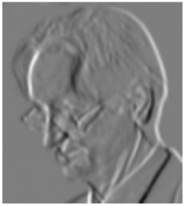


2-D Convolution

- What does this convolution kernel do?

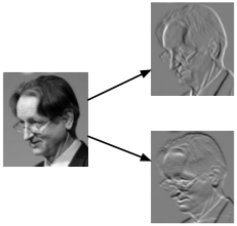
$*$

1	0	-1
2	0	-2
1	0	-1

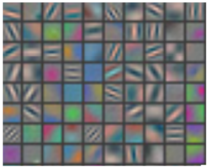


Convolutional Networks

- Let's finally turn to convolutional networks.
  - Detection layers (or convolutional layers)
  - Pooling layers
- The convolution layer has a set of filters.
  - Its output is a set of feature maps, each one obtained by convolving the image with a filter.



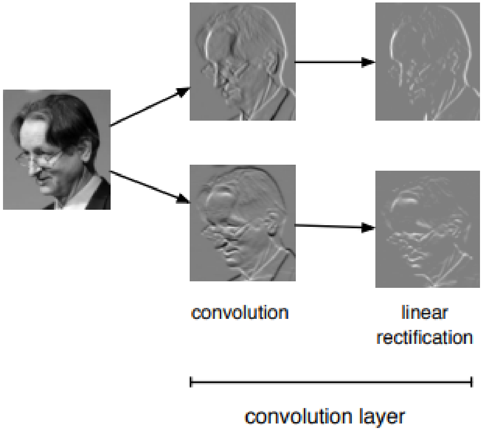
Example first-layer filters



(Zeiler and Fergus, 2013, Visualizing and understanding convolutional networks)

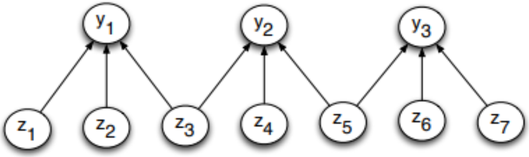
Convolutional Networks

- It’s common to apply a linear rectification nonlinearity: $y_i = \max(z_i, 0)$
  -
  - Two edges in opposite directions shouldn’t cancel



Pooling Layers

- The other type of layer is the pooling layer.
  - These layers reduce the size of the representation and build invariance to small transformations

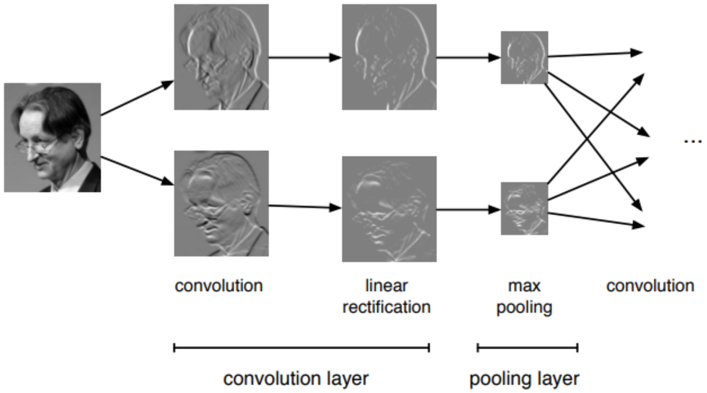


- Most commonly, we use max-pooling
  - Which computes the maximum value of the units in a pooling group

$$y_i = \max_{j \in J} z_j \tag{3}$$

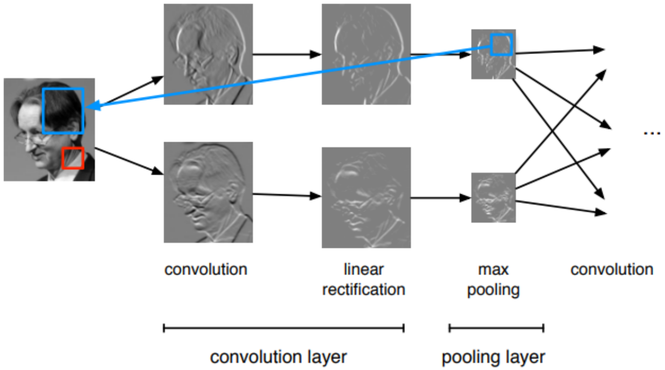
Convolutional Networks

- A more holistic view on the CNN(s):



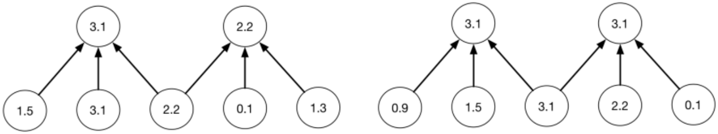
Convolutional Networks

- Because of pooling, higher-layer filters can cover a larger region of the input than equal-sized filters in the lower layers



Convolutional Networks

- We said the network's responses should be robust to the translation of the input. But this can mean two different things
    - Convolution layers are equivariant: it you translate the inputs, the outputs are translated by the same amount.
    - We'd like the network to be invariant: if you translate the inputs, the prediction should not change
    - Pooling layers provide invariance to small translation



Convolution Layers

- Each layer consists of several feature maps, each of which is an array
  - For the input layer, the feature maps are usually called channels
  - If the input layer represents a grayscale image, it has 1 channel.
- Each unit is connected to each unit within its receptive field in the previous layers.
  - This includes all of t he previous layer's feature maps.

Convolution Layers

- For simplicity, focus on 1-D signals (e.g. audio waveforms). Suppose the convolution layer's input has J feature maps and its output has I feature maps. Let t index the locations. Suppose the convolution kernels have radius R, i.e. dimension $K = 2R + 1$.
- Each unit in a convolution layer receives inputs from all the units in its receptive field in the previous layer:

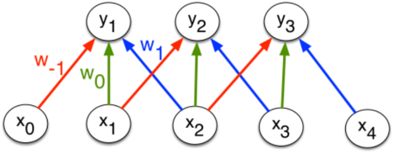
$$y_{i,t} = \sum_{j=1}^{J} \sum_{\tau=-R}^{R} w_{i,j,\tau} x_{j,t+\tau} \tag{4}$$

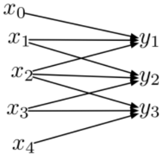
Backpropagation Update

- How do we train a conv net? With backprop, of course!
- Recall what we need to do. Backprop is a message passing procedure, where each layer knows how to pass messages backwards through the computation graph. Let's determine the updates for convolution layers.
- We assume we are given the loss derivatives $\overline{y_{i,t}}$ with respect to the output units.
- We need to compute the cost derivatives with respect to the input units and with respect to the weights.

Backpropagation Update

- Consider the computation graph for the inputs:





- Each input unit influences all the output units that have it within their receptive fields. Using the multivariate Chain Rule, we need to sum together the derivative terms for all these edges.

Backpropagation Update

- Recall the formula for the convolution layer:

$$y_{i,t} = \sum_{j=1}^{J} \sum_{\tau=-R}^{R} w_{i,j,\tau} x_{j,t+\tau} \tag{5}$$

- We compute the derivatives, which requires summing over all the output units which have the input unit in their receptive field:

$$\overline{x}_{j,t} = \sum_{\tau} \overline{y}_{i,t-\tau} \frac{\partial y_{i,t-\tau}}{\partial x_{j,t}} \tag{6}$$

$$\overline{x}_{j,t} = \sum_{\tau} \overline{y}_{i,t-\tau} w_{i,j,\tau} \tag{7}$$

- Written in terms of convolution,

$$\overline{\boldsymbol{x}}_j = \overline{\boldsymbol{y}}_i * \boldsymbol{w}_{i,j} \tag{8}$$

Image Classification

Overview

- Object recognition is the task of identifying which object category is present in an image
- It is challenging because objects can differ widely in position, size, shape, appearance, etc., and we have to deal with occlusions, lighting changes, etc.
- Why we care about it
    - Direct application to image search
    - Closely related to object detection, the task of locating all instances of an object in an image
        - e.g. a self-driving car detecting pedestrians or stop signs
- For the past 5-10 years, all the best object recognizers have been various kinds of conv nets and an architecture we called the Transformer
    - We will cover Transformers later in this class

Recognition Datasets

- In order to train and evaluate a machine learning system, we need to collect a dataset
  - The design of the dataset can have major implications
- Some questions to consider:
  - Which classes to include?
  - Where should the image come from?
  - How many images to collect?
  - How to normalize (preprocess) the images?

Image Classification

- Conv nets are just one of many possible approaches to image classification
  - However, they have been successful and now become a classical approach in the deep learning era
- Biggest image classification “advances” of the lasst two decades
  - Datasets have gotten much larger (because of digital cameras and the Internet)
  - Computers got much faster
    - Graph processing units (GPUs) turned out to be really good at training big neural nets
    - Generally, about 30 times faster than CPUs
  - As a result, we could fit bigger and bigger neural nets

MNIST Dataset

- MNIST dataset of handwritten digits
  - Categories: 10 digit classes
  - Source: Scans of handwritten zip codes from envelopes
  - Size: 60,000 training images and 10,000 test images, grayscale, of size 28×28
  - Normalization: centered within the image, scaled to a consistent size
    - The assumption is that the digit recognizer would be part of a larger pipeline that segments and normalizes images
- In 1998, Yann LeCun and colleagues built a conv net called LeNet which was able to classify digits with 98.9% test accuracy
  - It was good enough to be used in a system for automatically reading numbers on checks

Calltech101

- Caltech101 was the first major object recognition dataset, collected in 2003
- Design decisions:
    - Categories: 101 object categories; open the dictionary to random pages and select from nouns that were associated with images
    - Source: find candidates with Google Image Search, hand-select the ones that actually represent the object category
    - Number of examples: as many as possible per category
        - Most machine learning benchmarking is done using a fixed number of training examples per category (usually between 1 and 20)
    - Normalization:
        - Scale to be 300 pixels wide
        - Flip so that the object is facing the same direction
        - Rotate certain object categories because their proposed algorithm couldn't handle vertical objects

Input Vectors

Calltech101

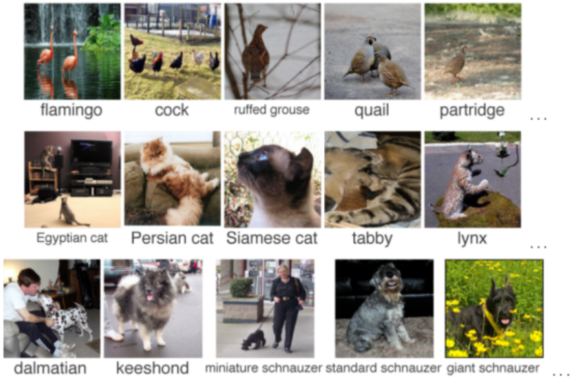
- Beware of dataset biases
  - These are idiosyncrasies of a dataset resulting from how it was collected or normalized
- An algorithm can appear to have good training and testing error, but fail to generalize if the training data doesn't resemble the real world
- E.g. in Caltech101, the sizes and locations of the images are a lot m ore regular than you would expect in the "real world"

Calltech101

- There were lots of works on Caltech101 for 5 years or so
  - It quickly became clear that dataset biases made it too gameable
- By contrast, MNIST is still a productive source of insights after 20 years of its introduction!

ImageNet

- ImageNet is the modern object recognition benchmark dataset
  - It was introduced in 2009 and has led to amazing progress in object recognition since then

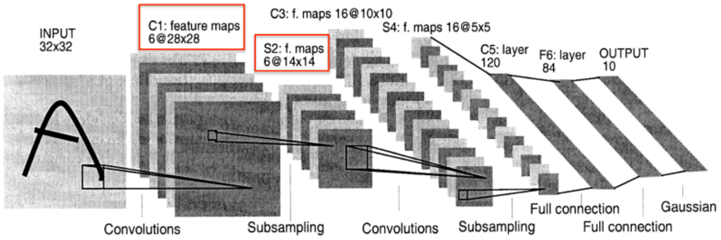


ImageNet

- Used for the ImageNet Large Scale Visual Recognition Challenge (ILSVRC). an annual benchmark competition for object recognition algorithms
- Design decisions:
  - Categories: Taken from a lexical database called WordNet
    - WordNet consists of “synsets” or sets of synonymous words
    - They tried to use as many of these as possible; almost 22,000 as of 2010
    - Of these, they chose the 1000 most common for the ILSVRC
    - The categories are really specific, e.g. hundreds of kinds of dogs
  - Size: 1.2 million full-sized images for the ILSVRC
  - Source: Results from image search engines, hand-labeled
    - Labeling such specific categories was challenging; annotators had to be given the WordNet hierarchy, Wikipedia, etc.
  - Normalization: none, although the contestants are free to do preprocessing

LeNet

- Here's the LeNet architecture, which was applied to handwritten digit recognition on MNIST in 1998
  - In the image, subsampling equates to pooling



Size of CNNs

	LeNet (1989)	LeNet (1998)	AlexNet (2012)
classification task	digits	digits	objects
categories	10	10	1,000
image size	16×16	28×28	$256 \times 256 \times 3$
training examples	7,291	60,000	1.2 million
units	1,256	8,084	658,000
parameters	9,760	60,000	60 million
connections	65,000	344,000	652 million
total operations	11 billion	412 billion	200 quadrillion (est.)

AlexNet

- AlexNet, 2012. 9 Weight layers. 16.4% top-5 error (i.e. the network gets 5 tries to guess the right category)

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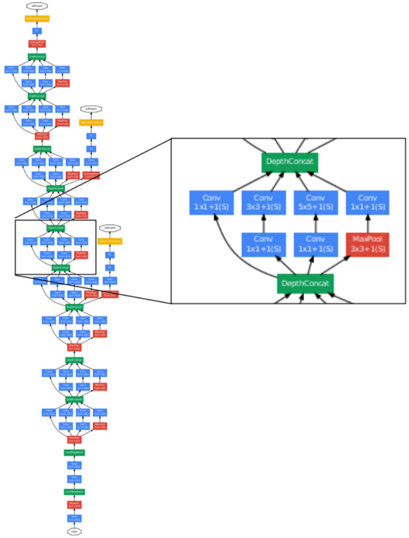
total operations	11 billion	412 billion	200 quadrillion (est.)

- They used lots of tricks to train NN
  - ReLU units, weight decay, data augmentation, SGD, dropout
- AlexNet's stunning performance on the ILSVRC is what set off the deep learning boom of the last 5-10 years

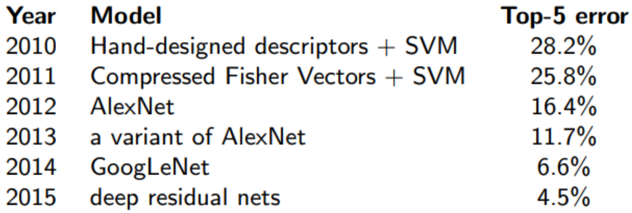
GoogLeNet

- GoogLeNet, 2014
- 22 weight layers
- Fully convolutional (no fully connected layers)
- Convolutions are broken down into a bunch of smaller convolutions
- 6.6% test error on ImageNet



Classification

- ImageNet results over the years. Note that errors are top-5 errors



Year	Model	Top-5 error
2010	Hand-designed descriptors + SVM	28.2%
2011	Compressed Fisher Vectors + SVM	25.8%
2012	AlexNet	16.4%
2013	a variant of AlexNet	11.7%
2014	GoogLeNet	6.6%
2015	deep residual nets	4.5%

- Human performance is around 5.1%
- They stopped running the object recognition competition because the performance is already so good