Slide 6

OR

$w^T x + b \Rightarrow \underline{\underline{w^T x}} = z$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix}$$

$x_0 \to 1$

$z = w_0 x_0 + w_1 x_1$

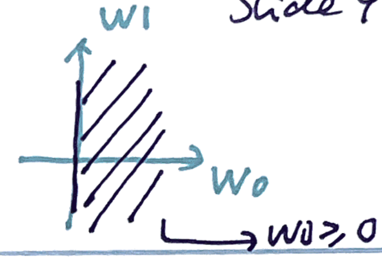
when $x_1 = 0$, $z \Rightarrow w_0 \times 1 + 0 \times w_1 = \underline{w_0 \geq 0}$ $(t=1)$

$x_1 = 1$, $z \Rightarrow w_0 + 1 \times w_1 = w_0 + w_1 < 0$.

poss. solution $= w_0 = 1, w_1 = -2$

$w_0 = 1, w_2 = -3$

Slide 9



Slide 7 $\quad D \Rightarrow D+1$

AND

$z = w_0 x_0 + w_1 x_2 + w_2 x_2$. $\quad$ Example sol: $w_0 = -1.5,\ w_1 = 1,\ w_2 = 1$

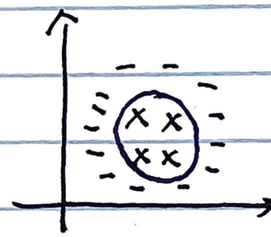
$w_0 + 0 + 0 \leq 0$

$w_0 + w_2 < 0$

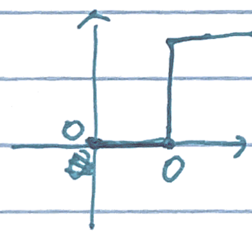
$w_0 + w_1 < 0$

$w_0 + w_1 + w_2 \geq 0$

~~$z = w_0$~~



$$\frac{\partial \mathcal{L}_{0,1}}{\partial w_j,} = \boxed{\frac{\partial \mathcal{L}_{0,1}}{\partial z}} \cdot \frac{\partial z}{\partial w_j}.$$



Slide 14.

j = index

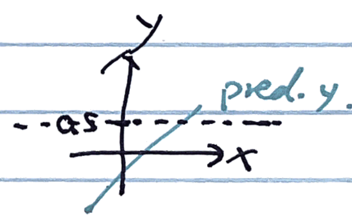
$\boxed{z = w^T x}$

$\mathcal{L}_{SE} = \frac{1}{2}(z - t)^2$

if $z \geq 0.5$, predict +

otherwise, predict −

Slide 17.



Slide 18

$\sigma(z) = \frac{1}{1+e^{-z}}$ $\quad$ $\mathcal{L}_{SE} = \frac{1}{2}\left(\frac{1}{1+e^{-z}} - t\right)^2$

$$\frac{\partial \mathcal{L}}{\partial w_j} = \frac{\partial \mathcal{L}}{\partial z} \cdot \frac{\partial z}{\partial w_j}$$

slide 24

$z = w^T x,$

$y = \sigma(z) = \dfrac{1}{1+e^{-z}}$

$\mathcal{L}_{CE} = -t \log y - (1-t)\log(1-y).$ $\qquad \dfrac{1+e^{-z}-1}{1+e^{-z}} = \dfrac{e^{-z}}{1+e^{-z}}$

$= -t \log\left(\dfrac{1}{1+e^{-z}}\right) - (1-t)\log\left(1 - \dfrac{1}{1+e^{-z}}\right)$ $\qquad = \dfrac{1}{e^{z}} \times \dfrac{1}{1+e^{-z}}$

$= -t\left[\log(1) - \log(1+e^{-z})\right] - (1-t)\left(\log \dfrac{1}{e^{z}+1}\right)$ $\qquad = \dfrac{1}{e^{z}+1}$

$= -t(0 - \log(1+e^{-z})) - (1-t)(\log(1) - \log(e^{z}+1))$

$= t\log(1+e^{-z}) - (1-t)(\cancel{0} - \log(e^{z}+1))$

$= t\log(1+e^{-z}) + (1-t)\log(e^{z}+1)$

$\mathcal{L}_{CE} = t\log(1+e^{-z}) - (1-t)(-z - \log(1+e^{-z})).$

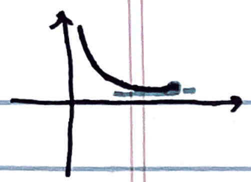
$= t\log(1+e^{-z}) - \left[-z(1-t) - (1-t)\log(1+e^{-z})\right]$

$= t\log(1+e^{-z}) - \left[-z + tz - \log(1+e^{-z}) + t\log(1+e^{-z})\right]$

$= \cancel{t\log(1+e^{-z})} + z - tz + \log(1+e^{-z}) - \cancel{t\log(1+e^{-z})}$

$= z - tz + \log(1+e^{-z})$

Slide 26.



Convex

nonconvex

$\cancel{L_{CE} = -t \log(1+}$ $\qquad (\log x)' = \frac{1}{x}$

$$\frac{\partial L_{CE}}{\partial w_j} = \frac{\partial L_{CE}}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial w_j} = \left(-\frac{t}{y} + \frac{(1-t)}{1-y}\right) \cdot y(1-y) \cdot x_j$$

$$\frac{\partial L_{CE}}{\partial y} = \frac{\partial}{\partial y} \; -t \log\Big(\underbrace{\frac{1}{1+e^{-z}}}_{y}\Big) - (1-t)\log\Big(1 - \underbrace{\frac{1}{1+e^{-z}}}_{y}\Big)$$

$$g(m) = \frac{u(m)}{v(m)} \; ; \; g' = \frac{u'v - v'u}{v^2}$$

$$= -t \cdot \frac{1}{y} + \frac{(1-t)}{1-y}$$

$u = 1 \; ; \; v = 1+e^{-z}$

$$\frac{\partial y}{\partial z} = \frac{\partial}{\partial z} \frac{1}{1+e^{-z}} = \frac{0 \cdot v - (-e^{-z}) \times 1}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2} = y(1-y)$$

$$y = \frac{1}{1+e^{-z}} \; ; \; y - y^2 = \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2} = \frac{1+e^{-z}-1}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2}$$

$$\frac{\partial z}{\partial w_j} = \frac{\partial}{\partial w_j} w^T x = \frac{\partial}{\partial w_j} \; w_0 x_0 + w_1 x_1 + \dots + \boxed{w_j} x_j + \dots + w_D x_D$$

$$= x_j.$$

$w_0, w_1, \dots w_j, \dots w_D$

$x_0, x_1, \dots x_j \dots x_D$