2/6/14 # 8.02 Electromagnetism Fernando Trujano

Math review Tues 9-11 pm

Office hours

TR 11-12 → Prof. Jesse Thaler 6-318

MW 2:30-3:30 → TA Janos Perczel

8-310 8-308

Electricity and Magnetism

- We will learn about electric and magnetic fields: How they are created and what they affect.

Scalar and Vector Fields

scalar Field

$\phi(\underbrace{x, y, z}_{\text{Position}}, \underbrace{t}_{\text{Time}})$

Temperature map

Contours

Vector Field

magnitude and direction for every point

$\vec{A}(x,y,z,t)$

$= A_x(x,y,z,t)\hat{i}, \quad A_y(x,y,z,t)\hat{j}, \quad A_z(x,y,z,t)\hat{k}$

Coulomb's Law, Electric Fields

Two types of charge: positive, negative

Unit charge = Coulomb

$e = 1.602 \cdot 10^{-19}\,C$ — electron (negative), proton (positive)

Charge is quantized $Q = \pm Ne$

Charge is conserved

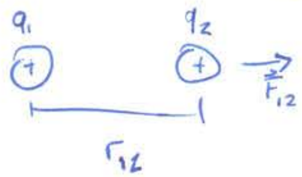
Electric Force

Opposites attract, likes repel

can charge by Friction, Transfer, touching induction

~~Coulo~~ Coulomb's Law

$$F_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$



$$k_e = \frac{1}{4\pi\varepsilon_0} \approx 9 \cdot 10^9 \ Nm^2/c^2$$

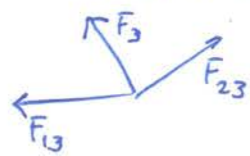
$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|r_{12}|}$$

so

$$F = K_e \frac{q_1 q_2}{r_{12}^3} \vec{r}$$

Superposition principle

Net force on any charge is vector sun of forces from other individual charges



$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$F_{12} = \underbrace{k_e}_{\text{conversion}} \underbrace{\frac{q_1 q_2}{r_{12}^2}}_{\text{magnitude}} \underbrace{\hat{r}_{12}}_{\text{direction}}$$

$$= \underbrace{\frac{k_e q_1}{r^2}}_{\text{Electric Field}} \hat{r}_{12} \, q_2$$

For Fields

Use test charge of +1.

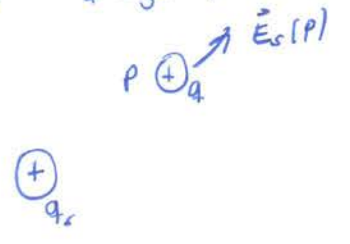
What would the proton do

2/13

Electric Field Lines

- Point away from positive charges
- Never cross each other
- Directions in which positive charges on those lines will accelerate

$F_q(P) = q\vec{E}_s(P)$



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8.02 Electromagnetism

Gravitational Field

Force

$$\vec{F}_g = -G\frac{Mm}{r^2}\hat{r}$$

Electric Field

electrostatic Force

$$\vec{F}_e = k_e\frac{Qq}{r^2}\hat{r}$$

$$k_e = 9\cdot10^9\ m^2/c^2$$

Electric Field @ at $p = \vec{E} = \lim_{q \to 0}\frac{\vec{F}_e}{q} = k_e\frac{Q}{r^2}\hat{r}$

$\therefore N/C$

2/10/14 8.02 ocw Lecture 2 Fernando Trujano

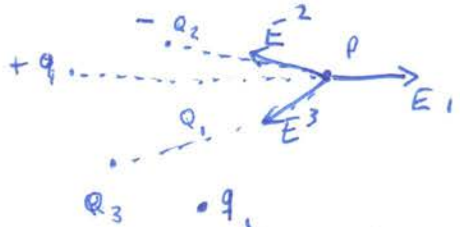
$\vec{E} = \frac{Qk}{r^2}\hat{r}$

Electric Field.

Test charge = proton

Superposition Principle

$E_p = E_1 + E_2 + E_3 + \dots$



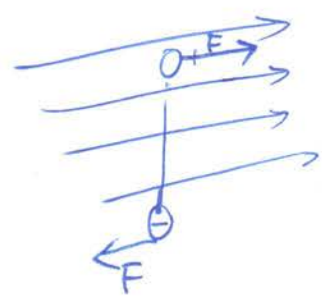
$\vec{F} = qE$

can know the Force that acts at any charge

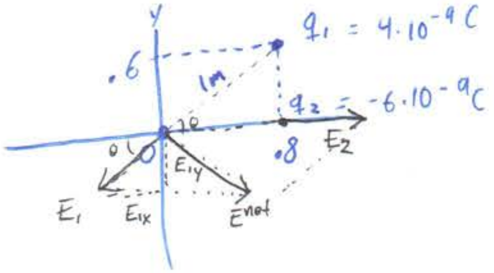
Field Lines:

- Force is tangental to field lines
  - + in direction
  - − opposite

Dipole will rotate in an electric Field



Electric Field Example



$$|E_1| = \frac{q_1 k_e}{1^2} = 36\ N/C$$

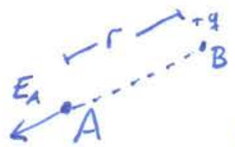
$$E_{1x} = E_1 \cos\theta = -28.8\ N/C$$

$$E_{1y} = E_1 \sin\theta = -21.6\ N/C$$

$$E_{2_x} = \frac{q_2 k_e}{.8^2} =$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

Electric Field Review



$$|E_A| = \frac{k_e q}{r^2}$$

units N/C

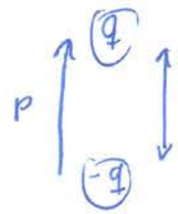
If I place charge Q at A

$$\boxed{F_A = E_A Q}$$

8.02 Lecture

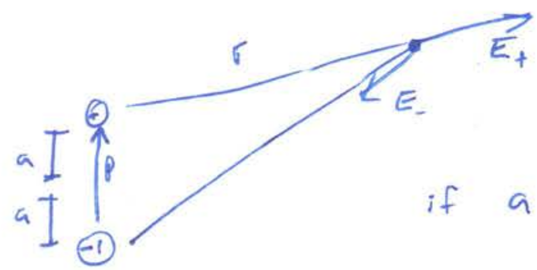
Electric Dipole

Two equal but opposite charges +q and -q



Dipole moment.

$$\vec{p} = q2a\hat{j} = 2qa\hat{j}$$



if $a << r$

$$E_x(r,\theta) = \frac{3p}{4\pi\varepsilon_0 r^3}\sin\theta\cos\theta$$

$$E_y(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}(3\cos^2\theta - 1)$$

Torque on Dipole: $T = \vec{r}\times\vec{F} = \vec{p}\times\vec{E}$

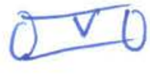
Continious charge distibutions

E Field at P due to Δq

$$\Delta\vec{E} = k_e\frac{\Delta q}{r^2}\hat{r} \rightarrow d\vec{E} = k_e\frac{dq}{r^2}\hat{r}$$

Superposition

$$\vec{E} = \sum\Delta\vec{E} \rightarrow \int d\vec{E}$$



$\rho = Q/V$

13/2/14

8.02 Lecture

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Table 5

Gauss's Law

~~Dipole:~~
Point Charge:

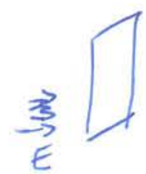
$$\phi_E = \oiint E \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

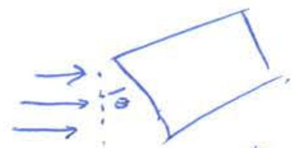
Flux - amount of flow over some Field through A

Electric Flux

$$\phi_E = \iint \vec{E} \cdot d\vec{A}$$

$$\phi_E = EA$$





$\phi_E = EA\cos\theta$

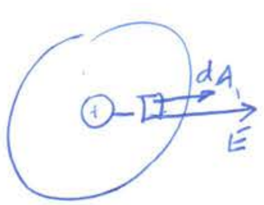
Open Surface - rectangle in space ← no volume
Closed Surface - sphere ← volume

if $\vec{E}$ not uniform

$$d\phi_E = \vec{E} \cdot dA$$

$d\phi_E$ positive = E points out
" neg = E points in

$$\phi_E = \oiint d\phi_E = \oiint E \cdot dA$$



$$\Phi_E = \int \frac{kQ}{r^2} dA$$

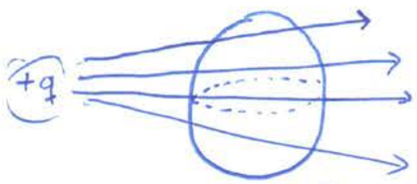
← E the same throughout

$$= \frac{kQ}{r^2} \underbrace{\int dA}_{4\pi r^2}$$

$$= 4\pi kQ = \boxed{\frac{Q}{\varepsilon_0}}$$

Gaussian Surfaces

$$\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$



Flux through surface = 0

+Q uniformly distributed throughout non-conducting solid sphere of radius a. Find E everywhere

Enclosed	Flux
Q	$E \cdot 4\pi r^2$

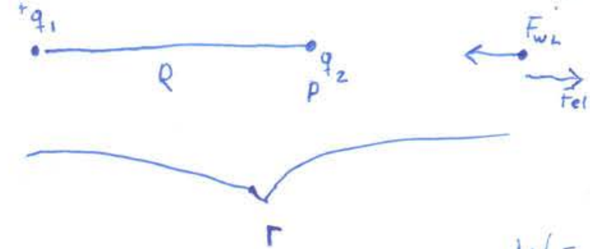
E outside $= \frac{Q}{4\pi r^2 \varepsilon_0}$

E inside

8.02 Electrostatic Potential and Electric Energy

Electrostatic Potential Energy U
Electric Potential V

as we move charge from infinity to P



$$W = \int_{\infty}^{R} \vec{F}_{WL} \cdot d\vec{r} = \int_{R}^{\infty} \vec{F}_{el} \cdot d\vec{r}$$

$$= kq_1q_2 \int_{R}^{\infty} \frac{dr}{r^2} = -\frac{1}{r}\Big|_{R}^{\infty}$$

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 R} \text{ Joules} = \frac{kq_1q_2}{R}$$

Electric Forces are conservative!
-Path independent

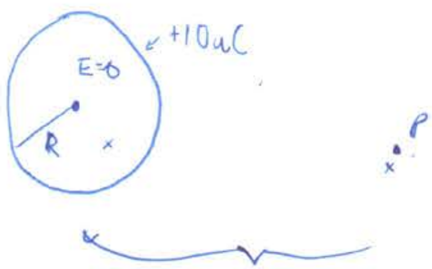
Electric Potential

test charge

$$U = \frac{qQ}{4\pi\varepsilon_0 R}$$

Electric Potential. - work per unit charge $\infty \to P$ w/q

$$V = \frac{\cancel{q}Q}{4\pi\varepsilon_0 R\cancel{q}} = \text{J/C Volts}$$
$$= \frac{kQ}{R}$$



Find Electric Potential at P

$r > R$

$$\vec{V}_P = \int_r^{\infty} \frac{F_{elec}}{q} \cdot dr$$

$$= \int_r^{\infty} E \cdot d\vec{r}$$

$$= \frac{Q}{4\pi\varepsilon_0 R}$$

$+Q_1$

$\cdot P$

$-Q_2$

$$V_P = V_{P_{Q_1 \text{ alone}}} + V_{P_{Q_2 \text{ alone}}}$$

Electric Dipole

$$\vec{p} = \sum_i q_i \vec{r}_i$$

$$P_{(A)} = P_{(B)}$$

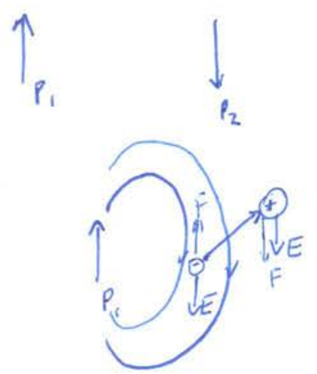
$$\vec{p} = 0 + Qa\hat{j}\,\cancel{(a\hat{i} + a\hat{n})} - 2Q(a\hat{j} + \hat{a}) + 2Q(a\hat{i})$$

Torque on Dipoles

$$T = r \times F = \vec{p} \times \vec{E}$$

$$F_{net} = F_+ + F_- = qE + (-q)E = 0$$

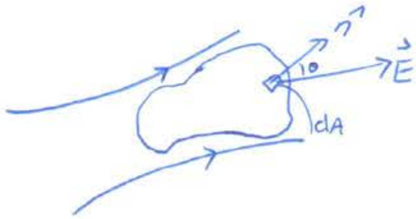
$$T = pE\sin\theta$$



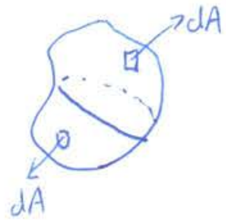
Torque. will rotate

$$F = qE = ma$$

8.02 Electric Flux and Gauss's Law



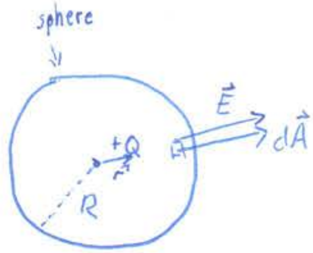
$$\underbrace{d\Phi}_{\text{Flux } N/C\,m^2} = \vec{E}\cdot\hat{n}\,dA = E\cdot dA = E\,dA\cos\theta$$



total Flux through surface

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

closed surface



$$\Phi = 4\pi R^2 E \qquad E_R = \frac{kQ}{R^2}\hat{r} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$$

$$\phi = \frac{Q}{\varepsilon_0}$$ ← independent of R and shape! ← can pick any shape!

$$\boxed{\text{Gauss's Law} \quad \Phi = \oint \vec{E}\cdot d\vec{A} = \frac{\sum Q_{ins}}{\varepsilon_0}}$$

Spherical Symmetry

Find E everywhere

$r < R$ r_1: ① Choose Gaussian Surface ← sphere containing radius r_1

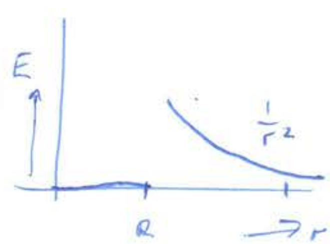
$$E\,\underbrace{4\pi r_1^2}_{\text{surface area}} = \frac{Q_{enclosed}}{\varepsilon_0}$$ ← no Q enclosed

$$\therefore E = 0$$

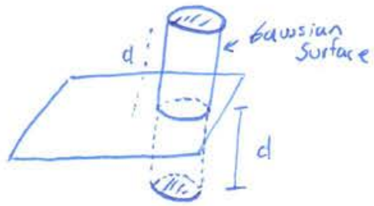
$r > R$

$$4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{\varepsilon_0 4\pi r^2} = \frac{kQ}{r^2}$$



same $\vec{E}$ as if all Q was in a point



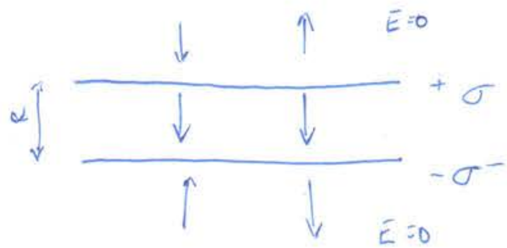
$\sigma = \frac{Q}{A}$ $\frac{C}{m^2}$

charge density

Find E anywhere.

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

independent of distance to plane b/c surface is infinately long



$E = \frac{\sigma}{\varepsilon_0}$

~~What is the potential difference $V(P) - V(\infty)$ between~~

Infinite Sheet

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

slab

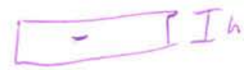
Inside:

$$\boxed{E = \frac{h\rho}{\varepsilon_0}}$$

h from center

Outside

$$\boxed{E = \frac{h\rho}{2\varepsilon_0}}$$



cylinder → cylinder, rod — $dA = 2\pi r$

sphere → sphere — $dA = 4\pi r^2$

pillbox → infinite sheet, slab — A

$$F = \frac{k q_1 q_2}{r^2}$$

$$V = \frac{kq}{r}$$

$\sigma = Q/_{magic}$

$$U = \frac{k q_1 q}{r}$$

$$U = qV$$

$$\int E \cdot dA = \frac{1}{\varepsilon_0} \int \rho_0 \, dA \, dr$$

8.02 Exam 1 Review

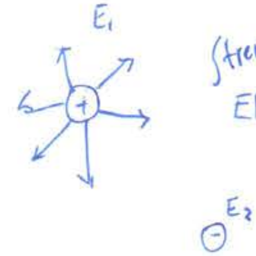
$\vec{F} = q\vec{E}$

$U = qV$

$\frac{\Delta U}{q} = -\int \frac{\vec{E}}{q} \cdot d\vec{x}$

$\Delta V = -\int \vec{E} \cdot dx$

$\vec{E} = \nabla V$



Strength of Electric Field = density of lines

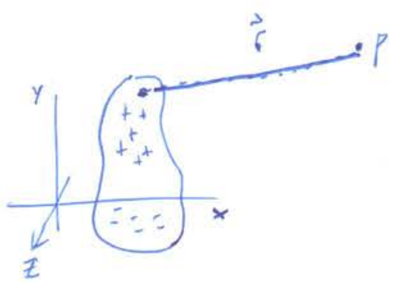
Superposition Principle

Total $E = E_1 + E_2$

Coulombs law $F = \frac{k q_1 q_2}{r^2}$

$E = \sum_i \frac{k q_1}{|r - r_1|^3}(r - r_1)$

Scalar $\rightarrow V = \sum_i \frac{k q_1}{|r - r_1|}$



Find Electric Field at P

$dE = \frac{k\,dq}{\vec{r}}\hat{r}$

① What is r?

$r = r(x, y, z, P)$

② Find dq

charge per volume

$dq = \rho\, dV_{ol} = \rho\, dx\,dy\,dz$

$= \lambda\, dx$ → line

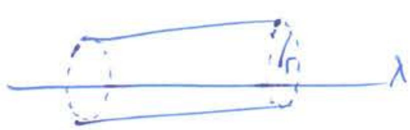
$\sigma\, dA$ → surface

③ Use symmetry

④ Integrate

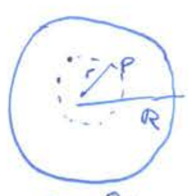
	$\lambda = \frac{Q}{L}$	$\lambda = \frac{Q}{2\pi R}$	
dq	λdx	λdl	$\sigma (2\pi R)\, dR$
r	$r = \sqrt{x^2+y^2}$	$\sqrt{R^2+h^2}$	
dE	$\frac{k\,dq}{r^2} = \frac{k\lambda dx}{x^2+y^2}$	$\frac{k\lambda dl}{R^2+h^2}$	
dE_y	$\lvert d\vec{E}\rvert \frac{h}{r} = \frac{k\lambda h\, dx}{(x^2+y^2)^{3/2}}$	$\lvert dE\rvert \frac{h}{\sqrt{R^2+h^2}} = \frac{R\lambda h}{(R^2+h^2)^{3/2}}$ $2\pi R$	$\sigma(2\pi R)\, dR$ $k\,dQ\,h$

$$\Phi = \int E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0}$$



$$\Phi = EA = E\, 2\pi r L = \frac{q_{enclosed}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\lambda \frac{L}{L}$$

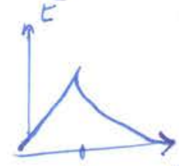
$$\boxed{= \frac{\lambda}{2\pi\varepsilon_0 r}}$$



$\rho = Q/_{Vol}$

$$\Phi = E 4\pi r^2 = \frac{1}{\varepsilon_0} \underbrace{\rho \frac{4}{3}\pi r^3}_{Q}$$

$$\boxed{E = \frac{\rho}{3\varepsilon_0} r}$$



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8.02 **Spring 2013**

Exam One Equation Sheet

$\vec{\mathbf{F}}_{12} = k_e \dfrac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$, force by 1 on 2

$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$

$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = k_e \dfrac{q}{r^2}\hat{\mathbf{r}} = k_e \dfrac{q}{r^3}\vec{\mathbf{r}}$

$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = k_e \sum_{i=1}^{N} \dfrac{q_i}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|^3}(\vec{\mathbf{r}} - \vec{\mathbf{r}}_i)$

$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = k_e \int_{\text{source}} \dfrac{dq}{r^2}\hat{\mathbf{r}} = k_e \int_{\text{source}} \dfrac{dq'(\vec{\mathbf{r}} - \vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3}$

$\underset{\text{closed surface}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{q_{enc}}{\varepsilon_o}$, $d\vec{\mathbf{A}} = \hat{n}_{out} dA$

$\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}}$

$V(r) - V(\infty) = k_e \dfrac{q}{r}$

$V(\vec{\mathbf{r}}) - V(\infty) = k_e \sum_{i=1}^{N} \dfrac{q_i}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|}$

$V(\vec{\mathbf{r}}) - V(\infty) = k_e \int_{\text{source}} \dfrac{dq'}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|}$

$\Delta U = q\Delta V$

$q\Delta V + \Delta K = 0$

$U_{\text{stored}} = k_e \sum_{\text{all pairs}} \dfrac{q_i q_j}{r_{ij}}$; $U(\infty) = 0$

Electric Dipole

$\vec{\mathbf{p}} = \sum_{i=1}^{N} q_i \vec{\mathbf{r}}_i$

$V(\vec{\mathbf{r}}) - V(\infty) = k_e \dfrac{\vec{\mathbf{p}} \cdot \hat{\mathbf{r}}}{r^2}$

$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$

Constants

$k_e = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \mathrm{N \cdot m^2 \cdot C^{-2}}$

Circumferences, Areas, Volumes

1) The area of a circle of radius r is πr^2. Its circumference is $2\pi r$.

2) The surface area of a sphere of radius r is $4\pi r^2$. Its volume is $(4/3)\pi r^3$.

3) The area of the sides of a cylinder of radius r and height h is $2\pi rh$. Its volume is $\pi r^2 h$.

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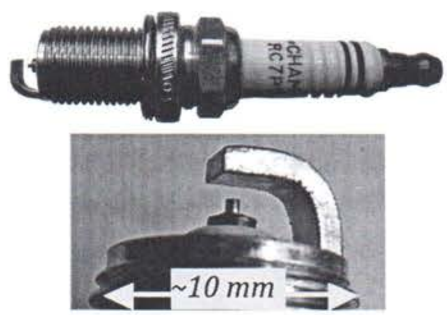
8.02 **Spring 2013**

Exam 1 Practice Problems

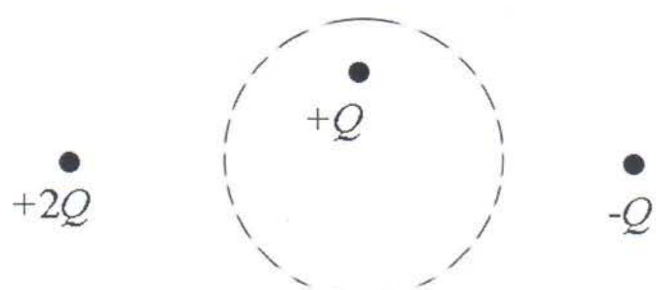
Part I: Short Questions and Concept Questions

Problem 1: Spark Plug Pictured at right is a typical spark plug (for scale, the thread diameter is about 10 mm).



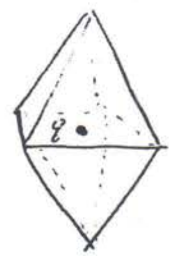
About what voltage does your car ignition system need to generate to make a spark, if the breakdown field in a gas/air mixture is about 10 times higher than in air? For those of you unfamiliar with spark plugs, the spark is generated in the gap at the left of the top picture (top of the bottom picture). The white on the right is a ceramic which acts as an insulator between the high voltage center and the grounded outer (threaded) part.

Problem 2 Consider the three charges $+Q$, $+2Q$, and $-Q$, and a mathematical spherical surface (it does not physically exist) as shown in the figure below.



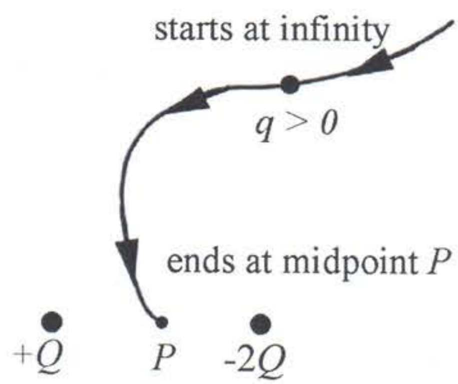
What is the net electric flux on the spherical surface between the charges? Briefly explain your answer.

Problem 3: A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the center of the base of the pyramid. What is the net flux of electric field emerging from one of the triangular faces of the pyramid?



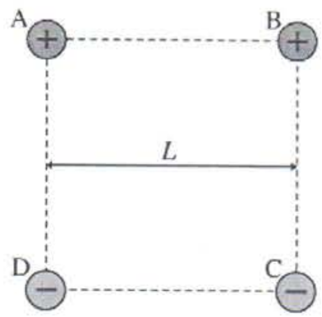
1. 0
2. $\dfrac{Q}{8\varepsilon_0}$
3. $\dfrac{Qa^2}{2\varepsilon_0}$
4. $\dfrac{Q}{2\varepsilon_0}$
5. Undetermined: we must know whether Q is infinitesimally above or below the plane?

Problem 4: The work done by an external agent in moving a positively charged object that starts from rest at infinity and ends at rest at the point P midway between two charges of magnitude $+Q$ and $-2Q$,



1. is positive.
2. is negative.
3. is zero.
4. cannot be determined – not enough info is given.

Problem 5



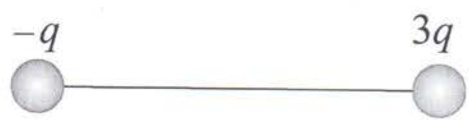
Four charges of equal magnitude (two positive and two negative) are placed on the corners of a square as pictured at left (positive charges at positions A & B, negative at positions C & D).

You decide that you want to swap charges B & C so that the charges of like sign will be on the same diagonal rather than on the same side of the square.

In order to do this, you must do…

1. … positive work
2. … negative work
3. … no net work
4. … an indeterminate amount of work, as it depends on exactly how you choose to move the charges.

Problem 6 Two opposite charges are placed on a line as shown in the figure below.



The charge on the right is three times the magnitude of the charge on the left. Besides infinite, where else can electric field possibly be zero?

1. between the two charges.

2. to the right of the charge on the right.

3. to the left of the charge on the left.

4. the electric field is nowhere zero.

Part II Analytic Problems

Problem 1 Non-uniformly charged sphere

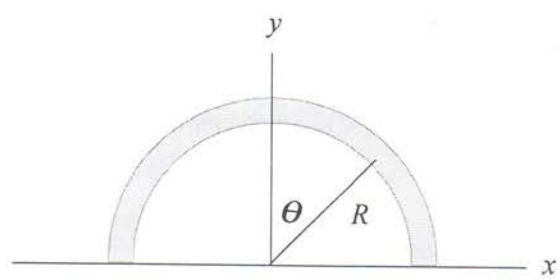
A sphere of radius R has a charge density $\rho = \rho_0(r / R)$ where ρ_0 is a constant and r is the distance from the center of the sphere.

a) What is the total charge inside the sphere?

b) Find the electric field everywhere (both inside and outside the sphere).

Problem 2 Electric field and force

A positively charged wire is bent into a semicircle of radius R, as shown in the figure below.



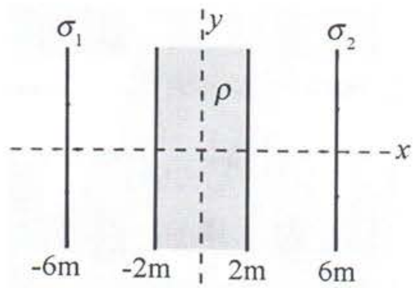
The total charge on the semicircle is Q. However, the charge per unit length along the semicircle is non-uniform and given by $\lambda = \lambda_0 \cos\theta$.

a) What is the relationship between λ_0, R and Q?

b) If a particle with a charge q is placed at the origin, what is the total force on the particle? Show all your work including setting up and integrating any necessary integrals.

Problem 3: Charged slab and sheets

An infinite slab of charge carrying a uniform volume charge density ρ has its boundaries located at x = -2 meters and x = +2 meters. It is infinite in the y direction and in the z direction (out of the page). Two similarly infinite charge sheets (zero thickness) are located at x = -6 meters and x = +6 meters, with uniform surface charge densities σ_1 and σ_2 respectively.



In the accessible regions you've measured the electric field to be:

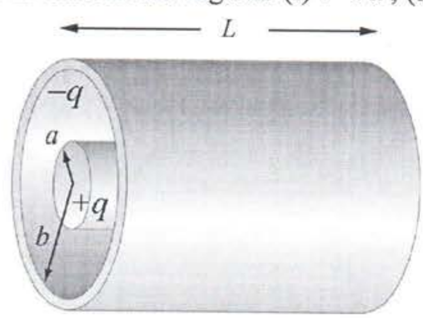
$$\vec{\mathbf{E}}(x) = \begin{cases} \vec{\mathbf{0}}\ ; & x < -6\ \mathrm{m} \\ (10\ \mathrm{N\cdot C^{-1}})\hat{\mathbf{i}}\ ; & -6\ \mathrm{m} < x < -2\ \mathrm{m} \\ (-10\ \mathrm{N\cdot C^{-1}})\hat{\mathbf{i}}\ ; & 2\ \mathrm{m} < x < 6\ \mathrm{m} \\ \vec{\mathbf{0}}\ ; & x > 6\ \mathrm{m} \end{cases}$$

(a) What is the charge density ρ of the slab?

(b) Find the two surface charge densities σ_1 and σ_2 of the left and right charged sheets.

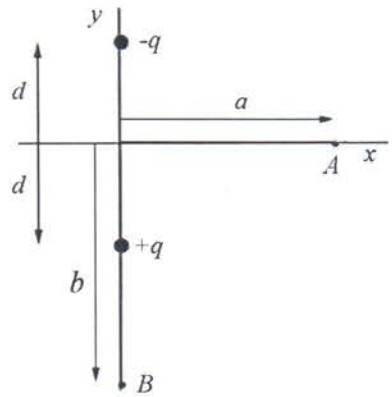
Problem 4 Concentric charged cylinders

A very long uniformly charged solid cylinder (length L and radius a) carrying a positive charge $+q$ is surrounded by a thin uniformly charged cylindrical shell (length L and radius b) with negative charge $-q$, as shown in the figure. You may ignore edge effects. Find a vector expression for the electric field in each of the regions (i) $r < a$, (ii) $a < r < b$, and (iii) $r > b$.



Problem 5 Electric field and potential of charged objects

Two charges $+q$ and $-q$ lie along the y-axis and are separated by a distance $2d$ as shown in the figure.



(a) Calculate the total electric field $\vec{\mathbf{E}}$ at position A, a distance a from the y-axis. Indicate its direction on the sketch (draw an arrow).

(b) Calculate the total electric field $\vec{\mathbf{E}}$ at position B, a distance b from the x-axis. Indicate its direction on the sketch (draw an arrow).

(c) Find the electric potential V at position A with the electric potential zero at infinity.

(d) Find the electric potential V at position B.

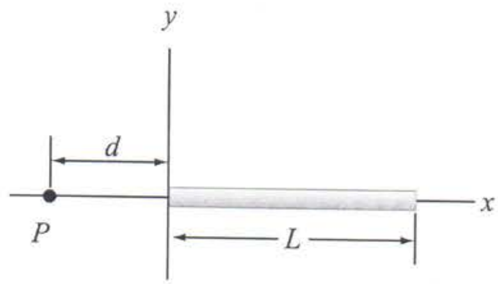
(e) A positively charged dust particle with mass m and charge $+q$ is released from rest at point B. In what direction will it accelerate (circle the correct answer)?

LEFT RIGHT UP DOWN It Won't Accelerate

(f) Again, assuming the positively charged dust particle was released from rest, what is its speed after it has traveled a distance s from its original position at point B? HINT: Don't try to simplify any fractions.

Problem 6: Electric field and electric potential of a non-uniformly charged rod

A rod of length L lies along the x-axis with its left end at the origin. The rod has a *non-uniform* charge density $\lambda = \alpha x$, where α is a positive constant.



(a) Express the total charge Q on the rod in terms of α and L.

(b) Calculate the electric field at point P, shown in the Figure. Take the limit $d \gg L$. What does the electric field look like in this limit? Is this what you expect? Explain. Hint: the following integral may be useful:

$$\int \frac{x\,dx}{(x+a)^2} = \frac{a}{x+a} + \ln(x+a)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \cdots \quad \text{(for small } x\text{)}$$

(c) Calculate the electric potential at P. Take the limit $d \gg L$. What does the electric potential look like in this limit? Is this what you expect? Explain. Hint: the following integral may be useful:

λ point
σ area
ρ volume

Problem 3

a)

$E_1 = 0$

$$E\,4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^R \rho_0\, 4\pi r^2\, dr$$

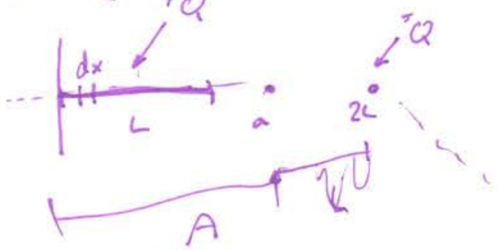
$$E_2\,4\pi r^2 = \frac{\rho_0\, 4\pi R^3}{3\varepsilon_0 r^2}\hat{r}$$

$(R+t/2)^2$

$$EA = \frac{AH\rho}{\varepsilon_0}$$

$$E = \frac{h\rho}{\varepsilon_0}$$

$\lambda = \frac{Q}{L}$



$r = (2L - A)$

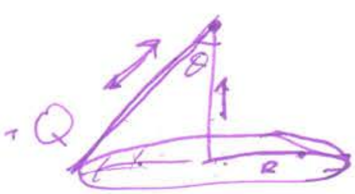
$$E = \frac{kQ}{r^2} \qquad E = \frac{kQ}{(2L-A)^2}$$

$$E = \frac{k\,dq}{(a-x)^2} = k\int_0^L \frac{\frac{Q}{L}dx}{(a-x)^2} \quad \frac{kQ}{L}\int_0^L \frac{dx}{(a-x)^2}$$

$dq = \frac{Q}{L}\cdot dx$

$$\frac{kQ}{L}\left(\frac{1}{a-x}\right)\Big]_0^L$$

$$\frac{kQ}{L}\left(\frac{1}{a-L} - \frac{1}{a}\right) = \frac{kQ}{(2L-A)^2}$$



dq

$2\pi r$

$$E = \frac{kQ}{r^2}$$

Problem 3

a) B

$E_1 = 0$

$\cancel{E_2}\ F$

$$4\pi r^2 E_2 = \frac{1}{\varepsilon_0}\int_0^R \rho_0 4\pi r^2\, dr$$

$$\cancel{4\pi r^2} E_2 = \frac{\rho_0 \cancel{4\pi} R^3}{\varepsilon_0 3 r^2}$$

$$\boxed{E = \frac{\rho_0 R^3}{\varepsilon_0 3\left(r+\frac{t}{2}\right)^2}}$$ ✓

b) C

$$E_2 = \frac{\rho_0 R^3}{\varepsilon_0 3 r^2} = \frac{\rho_0 R^3}{\varepsilon_0 3\left(R+\frac{3t}{4}\right)^2}$$

$$E_1 = E\cancel{A} = \frac{t\rho\cancel{A}}{4\varepsilon_0}$$

$$\boxed{E = E_2 + E_1}$$ ✓

c) ρ_0

A

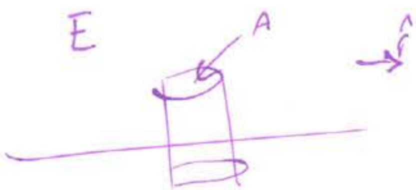
$$E_2 = \frac{\rho_0 R^{\cancel{3}}}{\varepsilon_0 3 \cancel{R^2}} \qquad \frac{\rho_0 R}{\varepsilon_0 3} = \frac{t\rho}{2\varepsilon_0}$$

$$E_1 = \frac{t\rho}{2\varepsilon_0} \qquad 3t\rho = 2\rho_0 R$$

$$\boxed{\rho_0 = \frac{3t\rho}{2R}}$$ ✓

Problem 3

a)



$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} \qquad \sigma A$$

$$E_1 \cancel{A} = \frac{\cancel{\sigma} A}{2\varepsilon_0}$$

$$E_2 \cancel{A} = \frac{\rho \cancel{A} \cancel{k} d}{\varepsilon_0 2}$$

$$\boxed{E = \frac{\sigma + \rho d}{2\varepsilon_0} \hat{r}}$$

b)

$$\cancel{2} E_1 \cancel{A} = \frac{\sigma \cancel{A}}{2\varepsilon_0}$$

$$E_2 \cancel{A} = \frac{\rho d \cancel{A}}{\varepsilon_0 4}$$

$$\boxed{E = -\frac{\sigma}{2\varepsilon_0} + \frac{\rho d}{4\varepsilon_0} \hat{r}}$$

c)

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

$$E_2 = \frac{\rho d}{\varepsilon_0 2}$$

$$E_P = E_2 - E_1$$

$$\frac{\sigma}{\cancel{2\varepsilon_0}} = \frac{\rho d}{\cancel{2\varepsilon_0}}$$

$$\boxed{\rho = \sigma / d}$$

Part II

Problem 1

$\rho(r) = \rho_0 \left(\frac{r}{R}\right)$

$\rho(r) = \rho_0 \left(\frac{r^2}{a^2}\right)$

$r < a \quad E = 0$

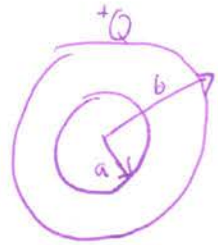
$\Phi = E 4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_0}$

$\rho_0 \frac{r^2}{a^2} = \frac{Q}{\frac{4}{3}\pi r^3}$

$\rho = \frac{Q}{Vol}$

Problem 1

$\rho(r) = \rho_0 \left(r^2/a^2 \right)$



a) $r < a$

Inside first sphere

$\rho = Q/_{SA}$

$$\int E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho_0 \left(\frac{r^2}{a^2}\right) 4\pi r^2 dr$$

$$= \frac{4\pi\rho_0}{\varepsilon_0 a^2}\int_0^r r^4 dr$$

$$= \frac{4\pi\rho_0}{\varepsilon_0 a^2} \frac{r^5}{5}\Big|_0^r$$

$$E \cancel{4\pi r^2} = \frac{\cancel{4\pi} r^{3} \rho_0}{5\varepsilon_0 a^2}$$

$$\boxed{E = \frac{r^3 \rho_0}{5\varepsilon_0 a^2} \hat{r}}$$

b) $a < r < b$

$$E 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^a \rho_0 \frac{r^2}{a^2} 4\pi r^2 dr$$

$$= \frac{\rho_0 4\pi}{a^2 \varepsilon_0}\int_0^a r^4 dr$$

$$E \cancel{4\pi} r^2 = \frac{\rho_0 \cancel{4\pi} a^{3}}{\cancel{a^2} \varepsilon_0 5}$$

$$\boxed{E = \frac{\rho_0 a^3}{5\varepsilon_0 r^2}}$$

c) $r > b$

$$E 4\pi r^2 = \frac{Q}{\varepsilon_0} + \frac{1}{\varepsilon_0}\int_0^a \rho_0 \frac{r^2}{a^2} 4\pi r^2 dr$$

$$= \frac{Q}{\varepsilon_0} + \frac{\rho_0 4\pi a^5}{a^2 \varepsilon_0 5}$$

Exam 1 Practice

Problem 1

$$\rho = \rho_0 (r/R)$$

a) $$Q = \int_0^R \rho_0 \frac{r}{R} 4\pi r^2 \, dr$$

$$= \rho_0 \frac{1}{R} 4\pi \int_0^R r^3 \, dr$$

$$\frac{\rho_0 R^{\cancel{4}3} 4\pi}{\cancel{4} R} = \rho_0 R^3 \pi$$

b)

1) $r < R$

$$\int E \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \cancel{\frac{\rho_0 R^3 \pi}{3_0}} \quad \frac{1}{\varepsilon_0} \int_0^r \rho_0 \frac{r}{R} 4\pi r^2 \, dr = \frac{1}{3_0} \rho_0 \frac{1}{R} 4\pi \int_0^r r^3 \, dr$$

$$\cancel{E = \frac{\rho_0 R^3 \pi}{4\pi r^2 \varepsilon_0}}$$

$$E\, 4\pi r^2 = \frac{\rho_0 4\pi}{3_0 R} \frac{r^4}{\cancel{4}}$$

$$E = \frac{\rho_0 \cancel{\pi} r^{4\,2}}{\varepsilon_0 R\, 4 \cancel{\pi} \cancel{r^2}}$$

$$\boxed{E = \frac{\rho_0 r^2}{4\varepsilon_0 R}}$$

b) $r > R$

$$\int E \cdot dA = \frac{Q_{total}}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{\rho_0 R^3 \pi}{3_0}$$

$$\boxed{E = \frac{\rho_0 R^3 \pi}{4\pi r^2 \varepsilon_0}}$$

Problem 2

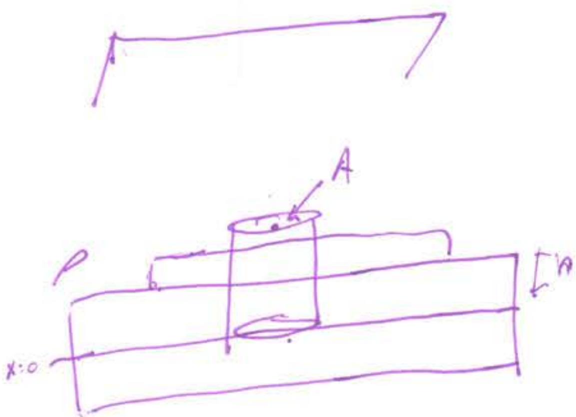
$$\lambda = \lambda_0 \cos\theta$$

$$\lambda = Q/_{length} = Q/_{R\theta}$$

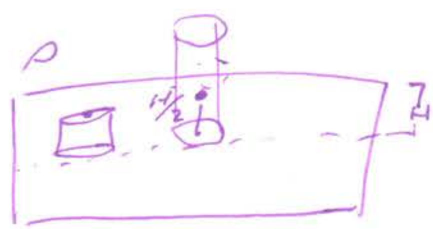
$$\cancel{\lambda = \frac{Q}{R\theta} = \lambda_0 \cos\theta}$$

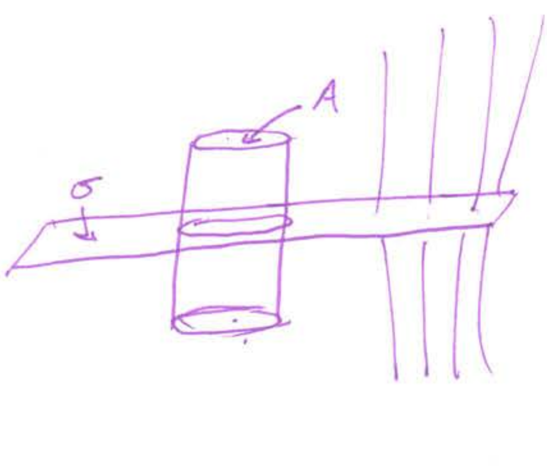
$$Q = \lambda R\theta$$

$$Q = \int \lambda_0 \cos\theta \, R d\theta$$



$$EA = \frac{\rho A h}{\varepsilon_0}$$

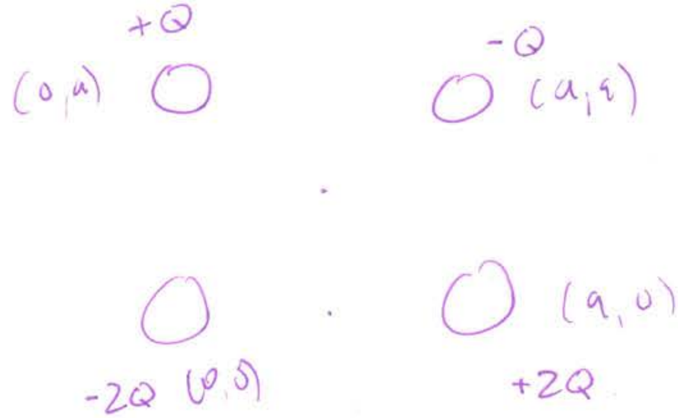




$$\int E \cdot da = \frac{\overbrace{Q_{enc}}^{\sigma A}}{\varepsilon_0}$$

$$E\,2A = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



$$\vec{p} = \Sigma q \vec{r}$$

$$Q\langle 0,a\rangle + (-Q)\langle a,a\rangle + 2Q\langle a,0\rangle$$

$$\cancel{Qa\hat{j}} - \cancel{Qa\hat{j}} - Qa\hat{i} + 2Qa\hat{i}$$

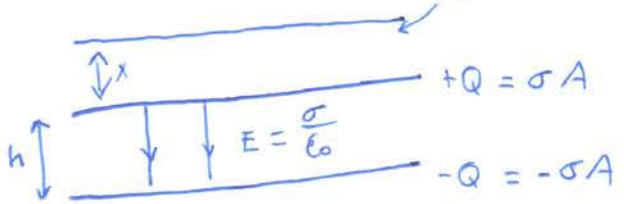
$$Qa\hat{i}$$

8.02 Lecture 7 OCW

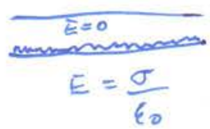
Capacitance and Field Energy

creates an electric field → need to do work

$W = Fx$



Inside one plate



average of the two E fields

$\rightarrow F = \frac{1}{2} Q E$

$$W = \frac{1}{2} Q E x$$

$$= \frac{1}{2} \sigma A E x \frac{\varepsilon_0}{\varepsilon_0}$$

$$= \frac{1}{2} \varepsilon_0 E^2 \underbrace{A x}_{\text{Volume}}$$

$$\frac{\text{work}}{\text{volume}} = \frac{1}{2} \varepsilon_0 E^2 \quad J/m^3$$

Field Energy Density

★

$$\boxed{\mathcal{U} = \int_{\text{all space}} \frac{1}{2} \varepsilon_0 E^2 \, dV} \text{ volume}$$

- By creating E field you do work

$$U = \int \frac{1}{2} \varepsilon_0 E^2 \, dV_{Ah} = \frac{1}{2} \varepsilon_0 \Big(\underbrace{\frac{\sigma}{\varepsilon_0}}_{E}\Big)^2 A h$$

$Q = A\sigma \quad V = Eh \qquad U = \frac{1}{2} Q V$ ← Potential Difference in plates

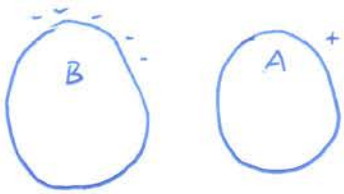
Capacitance $= \frac{Q}{V}$ Farads

$\boxed{C = Q/V}$

$V = \frac{Q}{4\pi\varepsilon_0 R}$

$C = 4\pi\varepsilon_0 R$ of sphere

Capacitance is the capability of holding charge for a given potential



$$C_B = \frac{Q_B}{V_B}$$

work per unit charge

V_B goes $\downarrow$ because of $B \rightarrow C_B \uparrow$

Two conductors. Same charge. Different Polarity

$$C = \frac{\text{charge}}{\text{potential difference}}$$

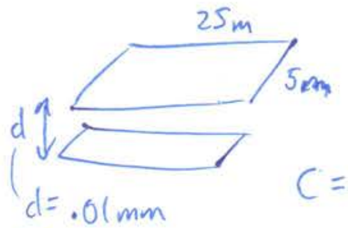
Parallel Plates

$$C = \frac{Q}{V} = \frac{\sigma A}{E d} = \frac{\sigma A \varepsilon_0}{\sigma d} = \frac{A \varepsilon_0}{d}$$

$E = \frac{\sigma}{\varepsilon_0}$ $d = h$

$$\boxed{C = \frac{A \varepsilon_0}{d}}$$

Not dependent on charge only geometry

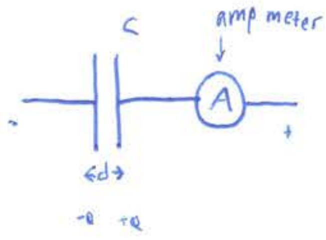


$d = .01\,mm$ $C = 1\mu F$

Parallel Plate Capacitors usually rolled up, to save space

How much energy can a capacitor store?

$$U = \frac{1}{2} Q V = \frac{1}{2} C V^2$$



$\Delta V = 1000\,V$

potential difference between the two plates

$\Delta V = Ed$

- If I charge the capacitor and remove power supply
  - Charge becomes trapped
  - does not change
- Electric field inside remains constant

$\uparrow \Delta V = E d \uparrow$

Must change

$\uparrow U = \frac{1}{2} Q V \uparrow$

Work has been done by moving plates apart

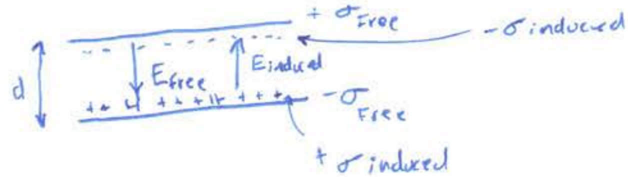
Used in photo flash

- Charge capacitor $\Rightarrow$ discharge over a light = bright flash (in short amount of time)

8.02 Lecture 8 ocw

Polarization and DiElectrics

- Dielectrics undergo polarization



$$E_{Free} = \frac{\sigma_{Free}}{\varepsilon_0}$$

$$E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}$$

$$\vec{E}_{Net} = \vec{E}_{Free} + \vec{E}_{induced}$$

opposite direction

$$E_{net} = E_{Free} - E_{ind}$$

$\sigma_i = b\sigma_F \quad b < 1$

$E_i = bE_F$

$$E_{net} = E_F\,(\underbrace{1-b}_{\frac{1}{K}})$$

$\frac{1}{K}$ ← Dielectric constant

dimentionless

$$\boxed{\vec{E} = \frac{E_F}{K}}$$

$$V = Ed$$

potential b/w plates

$$Q_{net} = \overset{+}{Q}_{Free} + \overset{-}{Q}_{ind}$$

Gauss's Law

$$\oint E \cdot d\vec{A} = \frac{1}{\varepsilon_0}\sum Q_{net}$$

$$= \frac{1}{\varepsilon_0}\frac{\sum Q_{Free}}{K}$$

I) $d = 1\,mm$ $V = 1500\,V$

dissconect Power S

-charge trapped

$d \to 7\,mm$

Q_{Free} no change

E no change

$V = Ed$ $V \uparrow 7$ ← up by a factor of 7

II) $V = 10\,KV$ $d = 7\,mm$

$K = 1$

insert K ← dielectric

Q_{Free} no change

$E \downarrow K$

$V \downarrow K$

remove K

- Everything resets

III) $\uparrow C = \frac{Q_{Free}}{V\downarrow}$

$$C = \frac{A\varepsilon_0}{d} K$$

$$E = \frac{\sigma_{Free}}{\varepsilon_0} K \quad (1)$$

$$V = Ed \quad (2)$$

$$V = \int E \cdot dl$$

$$C = \frac{Q_{Free}}{V} = \frac{A\varepsilon_0}{d} K \quad (3)$$

$V = 1500\,V$ $d = 1\,mm$

-Never disconnect PS

$d \to 7\,mm$

V_{Fixed} b/c

$E \downarrow 7$

$C \downarrow 7$

charge Flows

$Q_{Free} \downarrow 7$

IV) $V = 1500\ V$ $d = 7\mu m$

Insert K

V no change « connected to power supply

$Q_{free} \uparrow K$

$C \uparrow K$

E will NOT change

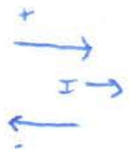
Because V, d constant

$V = Ed$

$$C = \frac{A\varepsilon_0}{d} K$$

8.02 Lecture 9 αw

Current, Resistivity and Ohm's Law

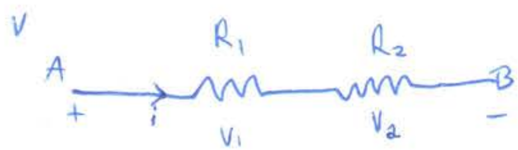


Ohm's Law

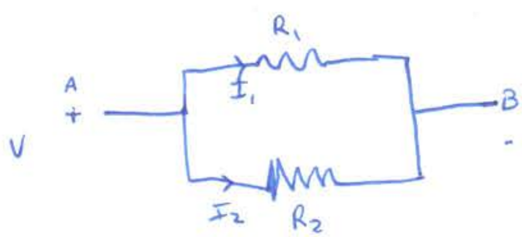
$$V = \frac{l}{\sigma A} I \qquad R = \frac{l}{\sigma A} = \frac{l\rho}{A}$$

$\rho = \frac{1}{\sigma}$ resistivity

$V = IR$



$$V = I(R_1 + R_2)$$
$$V_1 = IR_1$$
$$V_2 = IR_2$$



$$V = I_1 R_1$$
$$V = I_2 R_2$$

$$I = I_1 + I_2$$

8.02 Notes Capacitance and Dielectrics

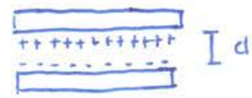
capacitor - stores electric charge
- two conductors carrying equal but opposite charges

Capacitance

$$C = Q/\Delta V$$

Parallel Plate Capacitor

$$C = \frac{\epsilon A}{d} =$$



Cylindrical Capacitor

$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(b/a) / 2\pi\epsilon_0} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

Spherical Capacitor

$$C = 4\pi\epsilon_0 \left(\frac{ab}{b-a}\right)$$

Electrical Potential Energy

$$U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Electric Energy Density

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{U_E}{Vol}$$

Energy stored in a Spherical Shell

$$u_E = \frac{1}{2}\epsilon_0 \underbrace{E^2}_{E = \frac{Q}{4\pi\epsilon_0 r^2}} = \frac{Q^2}{32\pi^2\epsilon_0 r^4}$$

$$U_E = \int_a^\infty \frac{Q^2}{32\pi^2\epsilon_0 r^4}\, \underbrace{dV}_{4\pi r^2 dr} = \frac{1}{2}QV$$

Dielectrics

Insulating material that increases capacitance by a factor of k_e

$C = k_e C_0$

Dielectric Constant

3/13/14 8.02 Lecture

Magnetic Force

Right hand Rule

$\underbrace{F_{elec} = qE}_{\text{Electric Force}}$ $\underbrace{F_{mag}}_{\text{F magnetic}} = q\vec{v} \times B$

$$F = q(\vec{E} + \vec{v} \times \vec{B})$$

Magnetic force on current carrying wire.

$$F_{mag} = \int_{wire} I\, d\vec{s} \times \vec{B}$$

- Uniform B

$$F_{mag} = \left(\int_{wire} I\, d\vec{s}\right) \times B$$

- Uniform + Straight

$$F_{mag} = I(\vec{L} \times \vec{B})$$

Single charge

$q \xrightarrow{\text{Feel}} \vec{F}_{elec} = q\vec{E}$

$q \xrightarrow{\text{create}} E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

moving charges $q\vec{v} \xrightarrow{\text{Feel}} F_{mag} = q\vec{v} \times B$

moving charges $q\vec{v} \xrightarrow{\text{create}} B = \frac{\mu_0}{4\pi}\frac{q\, v \times \hat{r}}{r^2}$

- What you feel is ~~[illegible]~~

Continious moving charge Distributions

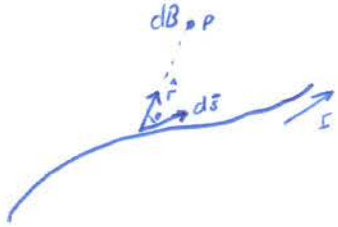
$$dB = \frac{\mu_0}{4\pi}\frac{dq\, v \times r}{r^2}$$

$\underbrace{dq}_{\text{little charge}}\, \underbrace{v}_{\text{moving}} = \overset{\text{current}}{I}\, \underbrace{ds}_{\text{little piece of wire}}$

$$dq \frac{ds}{dt} = \frac{dq}{dt} ds$$

Biot - Savart Law

$$dB = \frac{\mu_0}{4\pi}\frac{I\, d\vec{s} \times r}{r^2}$$



$$dB = \frac{\mu_0}{4\pi}\,\frac{I\,ds \times r}{r^2}$$

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Magnetic Field

Magnetic Monopoles do not exist

Magnetic Field lines are continious

Go from $S \to N$

$$\oiint B \cdot d\vec{A} = 0$$

Magnetic Gauss Law

Current carrying loop creates dipole field

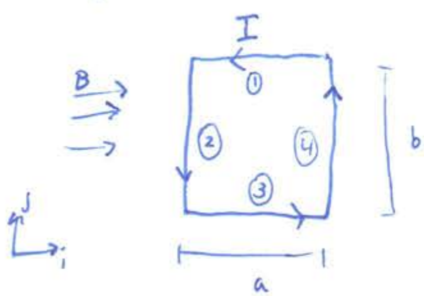
Electric Dipole

$$\vec{p} = \sum_i q_i \vec{r}_{p \to i} \quad (\text{assuming } \sum q = 0)$$

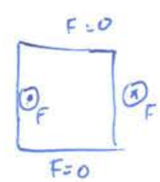
Magnetic Dipole

$$\mu = IA\hat{n}$$

Torque on Current carrying loop in a uniform magnetic field



1)



Total Force $= 0$

2) Find Torque

$\tau = \vec{\mu} \times B$

$\tau = \vec{r} \times \vec{F}$

$\tau = I\vec{A} \times B$

ab normal

curl Fingers on direction of current thumb points to normal

$\tau = I\vec{A}\hat{k} \times B\hat{i} = \hat{j} IAB$

Dipoles in non-uniform fields

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Problem Solving 4: Capacitance, Stored Energy, Capacitors in Parallel and Series, Dielectrics

Section ______ Table ______________________

Names ______________________________

____

____

____

Hand in one copy per group at the end of the Friday Problem Solving Session.

OBJECTIVES

1. To calculate the capacitance of a simple capacitor.

2. To calculate the energy stored in a capacitor in two ways.

REFERENCE: Course Notes: Sections 5.1-5.6, 5.8-5.9
http://web.mit.edu/8.02t/www/materials/StudyGuide/guide05.pdf

PROBLEM SOLVING STRATEGIES (see Section 5.8, 8.02 Course Notes)

(1) Using Gauss's Law, calculate the electric field everywhere.

(2) Compute the electric potential difference ΔV between the two conductors.

(3) Calculate the capacitance C using $C = Q / |\Delta V|$.

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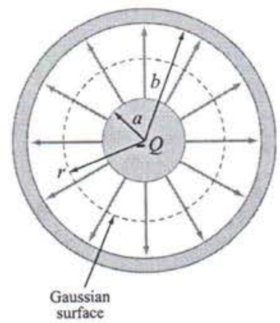
Problem 1: The Cylindrical Capacitor:



Two concentric conducting cylinders have radii a and b and height l, with $b > a$. The inner cylinder carries total charge -Q, and the outer cylinder carries total charge Q. ***We ignore end effects***. The goal of this problem is to calculate the capacitance.

Question 1: The Electric Field

Use Gauss's Law to find the direction and magnitude of the electric field in the between the inner and outer cylinders ($a < r < b$). NOTE: The inner cylinder has negative charge $-Q$.

Answer:

$$\oint E \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E 2\pi r h = \frac{-Q}{\varepsilon_0 l} h$$

$$E = \frac{-Q}{2\pi r \varepsilon_0 l} \hat{r}$$

Question 2: Electric Potential Difference (Voltage Difference)

The voltage difference between the cylinders, ΔV, is defined to be the work done per test charge against the electric field in moving a test charge q_t from the inner cylinder to the outer cylinder

$$\Delta V \equiv V(b) - V(a) = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}\ .$$

Find an expression for the voltage difference between the cylinders in terms of the charge Q, the radii a and b, the height l, and any other constants that you may find necessary.

Answer:

$$\Delta V = -\int_a^b \underbrace{\frac{-Q}{2\pi r \varepsilon_0 l}}_{E} dr = \frac{+Q}{2\pi\varepsilon_0 l}\int_a^b \frac{1}{r}\,dr$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

Question 3: Calculating Capacitance

Our two conducting cylinders form a capacitor. The magnitude of the charge, $|Q|$, on either cylinder is related to the magnitude of the voltage difference between the cylinders according to $|Q| = C|\Delta V|$ where ΔV is the voltage difference across the capacitor and C is the constant of proportionality called the 'capacitance'. The capacitance is determined by the geometrical properties of the two conductors and is therefore independent of the applied voltage difference across the cylinders.

What is the capacitance C of our system of two cylinders? Express your answer in terms, a, and b, the height l, and any other constants which you may find necessary.

Answer: $C = Q/\Delta V = \dfrac{\cancel{Q}\, 2\pi\varepsilon_0 l}{\cancel{Q}\,\ln\left(\frac{b}{a}\right)}$

Question 4: Stored Electrostatic Energy

The total electrostatic energy stored in the electric fields is given by the expression,

$$U = \frac{\varepsilon_0}{2}\int_{all\,space} \vec{\mathbf{E}}\cdot\vec{\mathbf{E}}\,dV_{vol}\ .$$

Starting from your expression for E in question (1), calculate this electrostatic energy and express your answer in terms of Q, a, b, and l (and any other constants which you may find necessary). If you use your expression for C from question 3 above, can you write you expression in terms of Q and C alone? What is that expression?

Answer:

$$U = \frac{\varepsilon_0}{2}\int E \cdot E \, dV_{ol}$$

$$dVol = 2\pi r l \, dr$$

$$= \frac{\varepsilon_0}{2}\int \left(\frac{Q}{2\pi \cancel{r} \varepsilon_0 \cancel{l}}\right)^2 2\pi \cancel{r} \cancel{l} \, dr$$

$$= \frac{\varepsilon_0}{2}\int \left(\frac{Q^2}{\varepsilon_0^2 \, 2\pi r l}\right) dr = \frac{\cancel{\varepsilon_0} Q^2}{2\varepsilon_0^{\cancel{2}} \, 2\pi l}\int_a^b \frac{1}{r}\, dr$$

$$U = \frac{Q^2}{4\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

$$U = \frac{Q^2}{2C}$$

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Question 5: Charging the Capacitor

Suppose instead of using a battery we charge the capacitor ourselves in the following way. We move charge from the inside of the cylinder at $r = b$ to the surface of the cylinder at $r = a$. Suppose we start off with zero charge on the conductors and we move charge for awhile until at time t we have built up a change $q(t)$ on the inner cylinder.

a. What is the voltage difference between the two cylinders at time t, in terms of C and $q(t)$?

Answer:

$C = Q/V$

$$V(t) = \frac{Q(t)}{C}$$

b. Now we move a very little additional charge dq from the outer to the inner cylinder. How much work dW do we have to do to move that dq from the outer to the inner cylinder, in the presence of the charge $q(t)$ already there, in terms of C, $q(t)$, and dq?

$V = \frac{W}{Q}$

Answer:

$$dW = dq\, V(t) = dq \frac{Q(t)}{C}$$

c. Using your result in (b), calculate the total work we have to do to bring a total charge Q from the outer to the inner cylinder, assuming the cylinders start out uncharged (Hint: integrate with respect to dq from 0 to Q).

Answer: $$\int_0^Q \frac{Q\,dq}{C} = \frac{Q^2}{2C}\Big|_0^Q = \frac{Q^2}{2C}$$

d. Is the work we did in charging the capacitor greater than, equal to, or less than the stored electrostatic energy in the capacitor that you calculated in question 4? Why?

Answer: Same

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Introduction: Capacitors in Parallel and in Series (Course Notes Section 5.3)

Parallel Connection

Suppose we have two capacitors C_1 with charge Q_1 and C_2 with charge Q_2 that are connected in parallel, as shown in Figure 4.1.

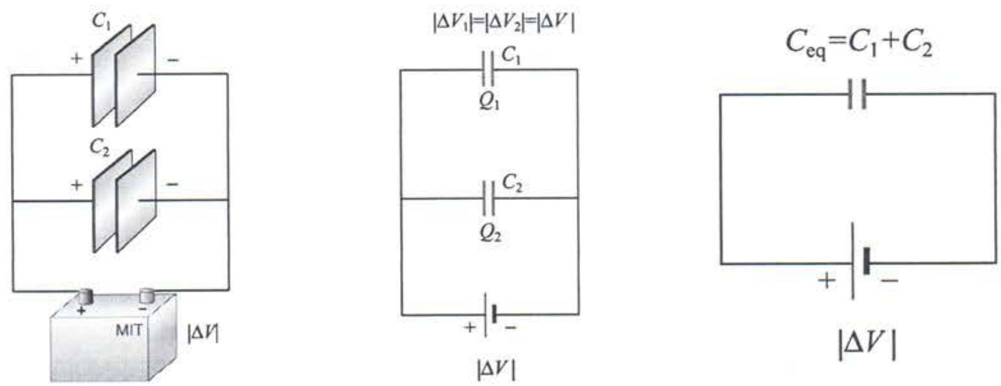

Figure 4.1 Capacitors in parallel and an equivalent capacitor.

The left plates of both capacitors C_1 and C_2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference $|\Delta V|$ is the same across each capacitor. This gives

$$C_1 = \frac{Q_1}{|\Delta V|}, \quad C_2 = \frac{Q_2}{|\Delta V|} \tag{1}$$

These two capacitors can be replaced by a single equivalent capacitor C_{eq} with a total charge Q supplied by the battery. However, since Q is shared by the two capacitors, we must have

$$Q = Q_1 + Q_2 = C_1|\Delta V| + C_2|\Delta V| = (C_1 + C_2)|\Delta V| \tag{2}$$

By definition the equivalent capacitance is given by

$$C_{eq} \equiv \frac{Q}{|\Delta V|} \tag{3}$$

We can bow substitute Eq. (2) into Eq. (3) and find that

$$C_{\mathrm{eq}} = \frac{Q}{|\Delta V|} = C_1 + C_2 \tag{4}$$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is

$$\boxed{C_{\mathrm{eq}} = C_1 + C_2 + C_3 + \cdots + C_N = \sum_{i=1}^{N} C_i \qquad \text{(parallel)}} \tag{5}$$

Series Connection

Suppose two initially uncharged capacitors C_1 and C_2 are connected in series, as shown in Figure 4.2. A potential difference $|\Delta V|$ is then applied across both capacitors. The left plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge $+Q$, while the right plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge $-Q$ as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge $-Q$ and the left plate of capacitor $+Q$.

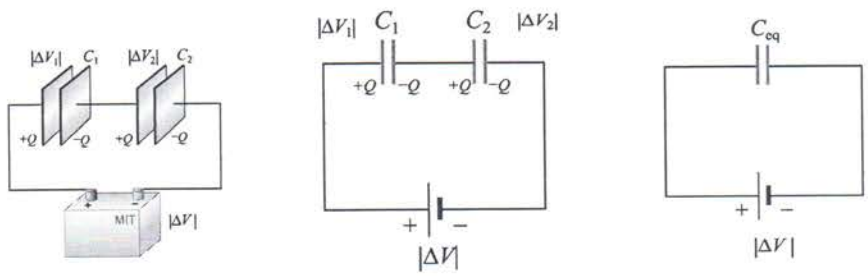

Figure 4.2 Capacitors in series and an equivalent capacitor

The potential differences across capacitors C_1 and C_2 are

$$|\Delta V_1| = \frac{Q}{C_1}, \quad |\Delta V_2| = \frac{Q}{C_2} \tag{6}$$

respectively. From Figure 4.2, we see that the total potential difference is simply the sum of the two individual potential differences:

$$|\Delta V| = |\Delta V_1| + |\Delta V_2| \tag{7}$$

In fact, the total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor $C_{\text{eq}} = Q/|\Delta V|$. Using the fact that the potentials add in series,

$$\frac{Q}{C_{\text{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

and so the equivalent capacitance for two capacitors in series becomes

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{8}$$

The generalization to any number of capacitors connected in series is

$$\boxed{\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i} \qquad (\text{series})} \tag{9}$$

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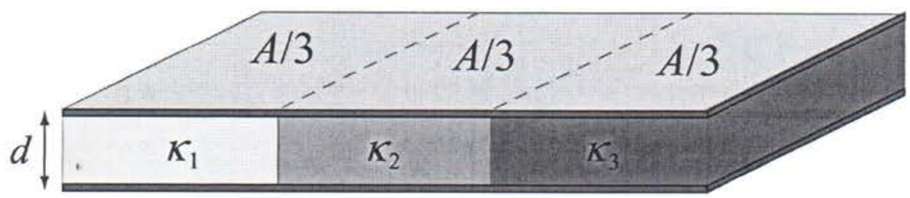
Problem 2: Capacitor Filled with Dielectric

a) A parallel-plate capacitor of area *A* and spacing *d* is filled with three dielectrics as shown in the figure below. Each occupies 1/3 of the volume. What is the capacitance of this system? [*Hint:* Consider an equivalent system to be three parallel capacitors, and justify this assumption.] Show that you obtain the proper limits as the dielectric constants approach unity, $\kappa_i \to 1$.]



Answer:

$C = Q/V$

$$C = \frac{k_1 \varepsilon_0 (A/3)}{d} + \frac{k_2 \varepsilon_0 (A/3)}{d} + \frac{k_3 \varepsilon_0 (A/3)}{d}$$

$$\frac{\varepsilon_0 A}{3d} (k_1 + k_2 + k_3)$$

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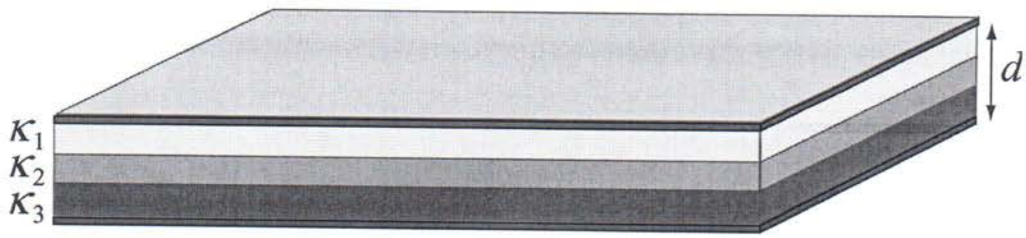
b) Suppose the capacitor is filled as shown in the figure below. What is its capacitance? Use Gauss's law to find the field in each dielectric, and then calculate ΔV across the entire capacitor. Again, check your answer as the dielectric constants approach unity, $\kappa_i \to 1$. Could you have assumed that this system is equivalent to three capacitors in series?



Answer:

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Problem Solving 5: Magnetic Fields

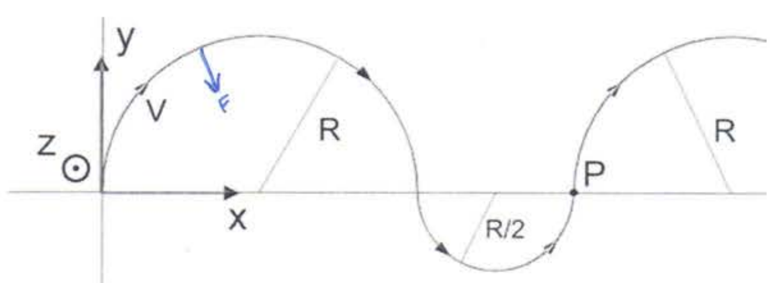
OBJECTIVES

1. To look at the behavior of a charged particle in a uniform magnetic field by studying the operation of a mass spectrometer
2. Understand how to apply the Biot-Savart Law

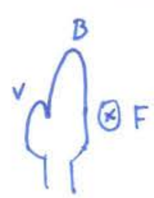
REFERENCE: 8.02 Course Notes Sections 8.9, 8.10, 9.10.1, 9.11.1-.3, 9.11.7-.8

http://web.mit.edu/8.02t/www/materials/StudyGuide/guide08.pdf

Problem 1 Circular Motion in a Magnetic Field A positively charged particle of mass m and charge q is at the origin at $t=0$ and moving upward with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{j}}$. Its subsequent trajectory is shown in the sketch. The magnitude of the velocity $v = |\vec{\mathbf{v}}|$ is always the same, although the direction of $\vec{\mathbf{v}}$ changes in time. For the region $y > 0$, the magnetic field is uniform with magnitude B_1. For $y < 0$, the magnetic field is also uniform but the magnitude is B_2.



(a) Is the direction of the magnetic field for the region $y > 0$ into or out of the page?



Out of the page

(b) Derive an expression for the magnitude B_1 of the magnetic field for the region $y > 0$ in terms of the given quantities, that is in term of q, m, R, and v.

$$F_{mag} = qvB_1$$

$$\frac{mv^2}{R} = qvB_1$$

$$B_1 = \frac{mv\cancel{^2}}{q\cancel{v}R}$$

(c) Is the magnetic field in the region $y < 0$ into or out of the page? What is the magnitude B_2 of that magnetic field in that region in terms of ~~in term~~ of q, m, R, and v?

Into the page

$$F_{mag} = qvB_2$$

$$\frac{mv^2}{R/2} = qvB_2$$

$$B_2 = \frac{2mv\cancel{^2}}{q\cancel{v}R}$$

(d) How long does it take the charged particle to move from the origin to point P located at $x = 3R$ (see figure above) along the x-axis? Give your answer in terms of the given quantities q, m, R, and v as needed.

$$v = d/t$$

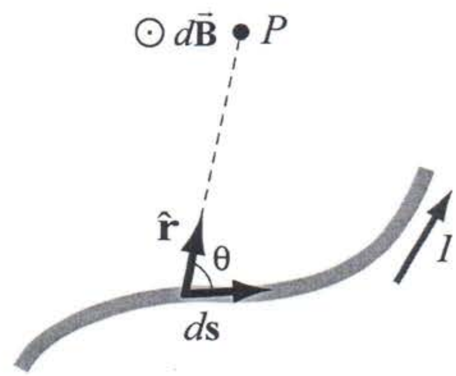
$$t = d/v$$

$$d = \pi R + \pi(R/2)$$

$$t = \frac{\pi R + \pi (R/2)}{v} = \frac{3\pi R}{2v}$$

Biot-Savart Law:

Moving charges (currents) are sources for magnetic fields. When the moving charges form a current I in a wire, the magnetic field at any point P due to the current can be calculated by adding up the magnetic field contributions, $d\vec{\mathbf{B}}$, from small segments of the wire $d\vec{\mathbf{s}}$, (see figure below).



The small segment can be thought of as a vector quantity with magnitude equal to the length of the segment and the direction of $d\vec{\mathbf{s}}$ is given by the direction of the tangent to the segment pointing in the direction of the current. The infinitesimal current source is then $Id\vec{\mathbf{s}}$. Let r denote the distance form the current source to the field point P. Let $\hat{\mathbf{r}}$ be a unit vector that points in the direction from the current source to the field point.

The Biot-Savart Law gives an expression for the magnetic field contributions, $d\vec{\mathbf{B}}$, from the current source, $Id\vec{\mathbf{s}}$,

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

where μ_0 is a constant called the **permeability of free space**:

$$\mu_0 = 4\pi\times 10^{-7}\ \mathrm{T\cdot m\cdot A^{-1}}.$$

Adding up these contributions to find the magnetic field at the point P requires integrating over the current source,

$$\vec{\mathbf{B}}(P) = \int_{wire} d\vec{\mathbf{B}} = \int_{wire} \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}.$$

The integral is a vector integral. That means that the expression for $\vec{\mathbf{B}}$ is really three integrals, one for each component of $\vec{\mathbf{B}}$. The vector nature of this integral appears in the cross product $Id\vec{\mathbf{s}}\times\hat{\mathbf{r}}$. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Biot-Savart Law.

When we introduce a coordinate system with origin we can rewrite the unit vector $\hat{\mathbf{r}}$ from the current element vector $Id\vec{\mathbf{s}}'$ to the field point P as follows. Define the vector $\vec{\mathbf{r}}'$ to be the vector from the origin to the current element $Id\vec{\mathbf{s}}'$. Define the vector $\vec{\mathbf{r}}$ to be the vector from the origin to the field point P. Then the distance from the current element to the field point is $r = \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|$ and the unit vector $\vec{\mathbf{r}}$ is

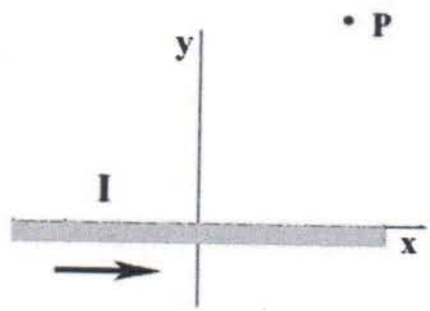
$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|}.$$

The Biot –Savart Law can then be rewritten as

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int_{wire}\frac{Id\vec{\mathbf{s}}'\times(\vec{\mathbf{r}} - \vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3}.$$

Problem 2: Straight Wire

Consider a straight wire of length L that has a current I flowing in the wire. (We will not worry about the return path of the current or the source for the current.) In this problem we will try to set up an integral vector expression for the magnetic field due to the straight wire at a point that does not lie on the perpendicular bisector of the wire.



Source Coordinates:

In order to apply the Biot-Savart, we choose a Cartesian coordinates system with the x-axis aligned along the wire, with the origin at the center of the wire.

Question 1: With this choice of coordinate system, choose an infinitesimal current element located anywhere along the x-axis except at the ends of the wire. What is the position vector for this infinitesimal current element, $\vec{\mathbf{r}}'$? What is your integration variable? What are the limits of your integration variable? Write down a vector expression for the infinitesimal current element $Id\vec{\mathbf{s}}'$.

Field Point Coordinates

The next step is to identify the location of your field point in terms of your coordinate system.

Question 2: Write down a vector expression for the position vector $\vec{\mathbf{r}}$ from the origin to the field point.

Question 3: What is the distance r from the source to the field point?

Question 4: The unit vector, $\hat{\mathbf{r}}$, located at the field point, has magnitude, $|\hat{\mathbf{r}}|=1$, and points from the source to the field point. Write down a vector expression for the unit vector $\hat{\mathbf{r}}$.

Biot-Savart Law:

We can now apply the Biot-Savart Law to calculate the contribution of the infinitesimal current source, $Id\vec{\mathbf{s}}'$, to the infinitesimal magnetic field, $d\vec{\mathbf{B}}$, at the field point P.

Question 5: Write down a vector expression in terms of your choice of source and field point coordinates for the contribution of the infinitesimal current source, $Id\vec{\mathbf{s}}$, to the infinitesimal magnetic field,

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{s}}' \times (\vec{\mathbf{r}} - \vec{\mathbf{r}}')}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|^3}.$$

Question 6: If you have not already done so, explicitly calculate the cross product that appears in the above equation and find a vector expression for $d\vec{\mathbf{B}}$.

The magnetic field at the point P is given by the integral over the wire

$$\vec{\mathbf{B}} = \int_{wire} d\vec{\mathbf{B}}.$$

The magnetic field at the point P is given by the integral over the wire

$$\vec{\mathbf{B}} = \int_{wire} d\vec{\mathbf{B}}.$$

Question 7: What are the endpoints of this integral?

Question 8: Set up the integral expression for the magnetic field at the field point based on your results up this point. (Can you integrate this?)

This integral can be evaluated using the fact that

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} \frac{Iy\,dx'}{((x-x')^2 + y^2)^{3/2}} \hat{\mathbf{k}} = -\frac{\mu_0 I}{4\pi y} \frac{(x-x')}{\sqrt{(x-x')^2 + y^2}} \Bigg|_{-L/2}^{L/2} \hat{\mathbf{k}}.$$

Question 9: Perform the integration, be careful with the limits and find an expression for the magnetic field as a function of the coordinates (x, y) of the field point P.

Parallel Plate Capacitor

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{A\sigma}{\varepsilon_0} \qquad C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d}$$

$$\Delta V = +ED$$

$$U = \frac{1}{2}CV^2$$

$$u_E = \frac{\varepsilon_0 E^2}{2}$$

Dielectric: - Reduces E

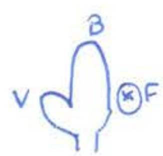
$$\frac{V}{K} \quad \frac{Ed}{k} \quad \frac{E}{K} \quad KQ$$

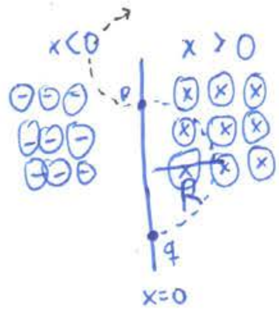
- Q can't change if disconnected
  V can't change if connected

. Electric Field = 0 inside a conductor

$$R = \frac{\overset{\text{resistivity}}{\rho_r} L}{A}$$

RHR





$\vec{B} =$ $\vec{B} = -B_0\hat{k}$

Particle moves in positive x direction w/ initial velocity

a) What is the sign of q?

Positive

b) Find R

$R = ?$

$F = q(v \times B) = qvB_0 = \frac{mv^2}{R}$

$R = \frac{mv}{qB_0}$

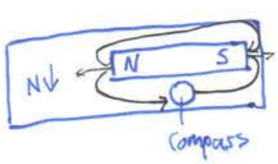
c) Time to get to P

$v = d/t$

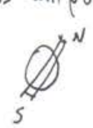
$t = d/v = \frac{\pi R}{v} = \frac{\pi m}{qB_0}$

Concept Question

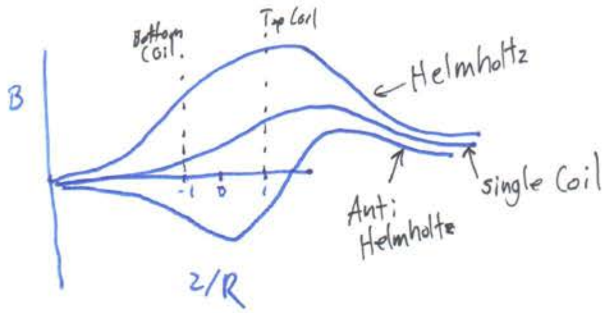
(1)



Compass will point



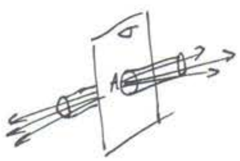
(2)



$\boxed{F}$

(4) Value of the electric Field and the potential along a line passing perpendicularly through and infinite planar charge distribution with uniform surface charge density

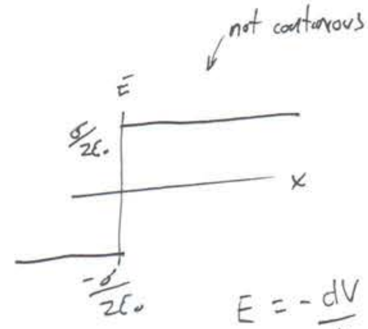
E? V?



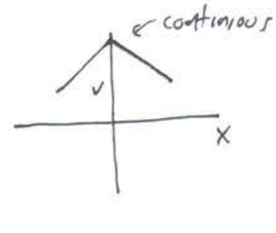
$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}\,\hat{\imath} \quad \text{for } x > 0$$

$$E = \frac{-\sigma}{2\varepsilon_0}\,\hat{\imath} \quad \text{for } x < 0$$



$$E = -\frac{dV}{dx}$$



Answer [C]

(5) non-conducting spheres

Charge uniformly distributed along:

A: Volume B: Surface C: $\rho = cr$

Rank V_A, V_B, V_C

$V_A > V_B$ $\quad V_B < V_C$ $\quad V_C < V_A$

(1) $E \to V$

(2) Be Smart

- Assume R is huge

$$dV = \int \frac{kqq}{r}$$

If charge closer → V↑

∴ $V_A > V_B$

$$\boxed{V_A > V_C > V_B}$$

8.02 Exam 2 Review

Electric Potential $V = -\int \vec{E} \cdot ds$

Electric Field $\vec{E} = -\nabla V$ In a dielectric $E \to \frac{E}{K}$

Electric Field $= 0$ inside a conductor
- Electric Potential is constant

$$\Phi = EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Capacitance

$C = Q/\Delta V$

Find E, integrate to get V

Biot-Savart Law

- Get directions right!!

Right hand rule!

Torque on a loop of current

$F = I ds \times \vec{B}$

along the current

$\vec{\tau} = \vec{\mu} \times \vec{B}$

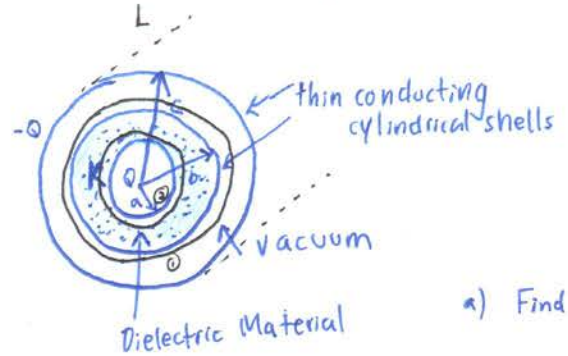
$\mu = I\vec{A}$

Area in normal direction

RHR curl Fingers in dir(i)
thumb = dir (A)

Problem 3

conflict exam 2 Sp2013



a) Find Capacitance per unit length C/L

$C = Q/\Delta V$

1) Add charges to diagram
2) Use Gaussian surfaces to find $\vec{E}$
3) Use $\vec{E}$ to find $\vec{V}$ $\quad V = -\int E \cdot ds$
4) $C = Q/\Delta V$

Gauss Law

$$\oint E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0} \qquad \oint \vec{E} \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0 K}$$

↓ Gaussian surface ↓

$$EA = \frac{q_{enclosed}}{\varepsilon_0}$$

$$E\, 2\pi r L = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\vec{E}_1 = \frac{Q}{\varepsilon_0 2\pi r L} \qquad E_2 = \frac{Q}{\varepsilon_0 2\pi r L K}$$

↑ outside dielectric ↑ inside Dielectric

$$V = -\int \vec{E} \cdot ds$$

Integrate from $a \to c$

$a \to b$ Inside dielectric $\qquad b \to c$ outside dielectric

$$V = -\int_b^c \frac{Q}{\varepsilon_0 2\pi r L}\, dr + -\int_a^b \frac{Q}{\varepsilon_0 2\pi r L K}$$

$$\Delta V = \frac{-Q}{\varepsilon_0 2\pi L} \ln\left(\frac{c}{b}\right) - \frac{Q}{\varepsilon_0 2\pi r L K} \ln\left(\frac{b}{a}\right)$$

$C = Q/\Delta V$ $\quad$ -Q's cancel out
-C/L → L's cancel

$$C/L = \frac{1}{+\frac{\ln\left(\frac{c}{b}\right)}{\varepsilon_0 2\pi} + \frac{\ln\left(\frac{b}{a}\right)}{\varepsilon_0 2\pi K}}$$

- Capacitance always positive
- change signs

2/4

b) When the conducting surfaces of the capacitor are connected to a battery with a potential difference V.

Find expression For stored energy per unit length U/L

$V \cdot q = U$

$$U = \int_0^Q V\,dq = \int_0^V V' C\,dV'$$

$C = Q/\Delta V$

$$U = \frac{1}{2} C V^2$$

$Q = C \Delta V$

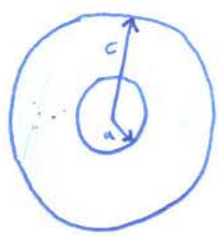
$dq = C\,dV$

$$U = \frac{1}{2} V^2 \frac{C}{L}$$

Plug in From last problem. Part a)

c) While the battery is still connected, the entire region b/w the cylindrical shells is Filled with dielectric material. By what amount did $\Delta(Q/L)$

V does not change



$Q_1 = C_1 \Delta V$
$Q_2 = C_2 \Delta V$

$$\frac{Q_2 - Q_1}{L} = \frac{\Delta V (C_2 - C_1)}{L}$$

$$\frac{Q_2 - Q_1}{L} = \frac{\Delta V (C_2 - C_1)}{L}$$

$$C_2 = \frac{Q_2}{\Delta V}$$ stays the same

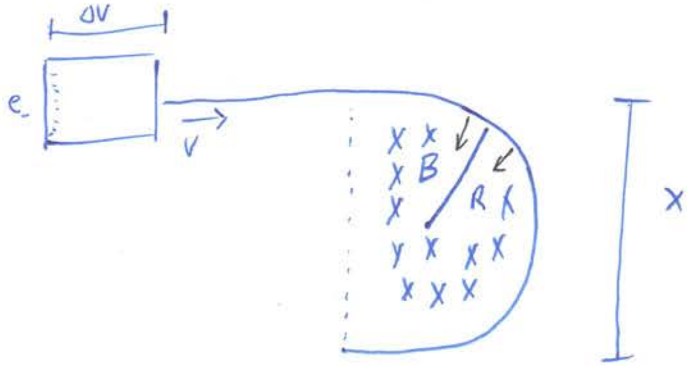
$$\oiint E \cdot dA = \frac{Q_{enc}}{\epsilon_0} \rightarrow Q_2$$

$$\Delta V = -\int_a^c \frac{Q}{K \epsilon_0 2\pi r L}$$

$$= \frac{Q}{K \epsilon_0 2\pi L} \ln\left(\frac{c}{a}\right)$$

$C = Q/\Delta V$

$$\frac{K \epsilon_0 2\pi}{\ln\left(\frac{c}{a}\right)} = C_2/L$$



$R = x/2$

$U = \frac{1}{2} m v^2$ - Work-Kinetic Energy Principle

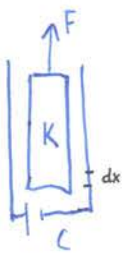
$V = U/Q$

$V(-e) = V = \frac{1}{2} m v_{el}^2$

$v_{el} = \sqrt{\frac{-2V(-e)}{m}}$

$$F_{mag} = \frac{mv^2}{\underset{x/2}{R}} = QVB$$

$$x = \frac{2Mv^2}{-eVR}$$



$F = \frac{-dU}{dx}$

$U_1 = \frac{1}{2} C_1 V^2$

$U_2 = \frac{1}{2} C_2 V^2$

$$\frac{dU}{dx} = \frac{1}{2}\frac{(C_2 - C_1)V^2}{dx}$$

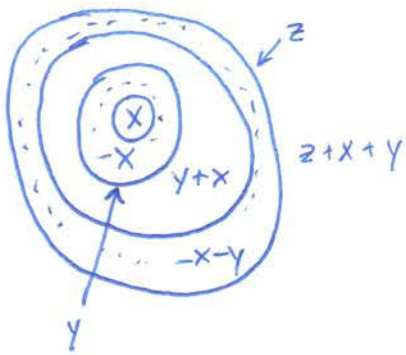
3/20/14 8.02 Lecture Exam II Review Fernando Trujano

Exam 2 Conflict s13

4A) A↓ → E↑ → U↑

4B)

Inside conductor $E = 0$ always



$\boxed{G}$

4C) Biot Savart

Total $B = B$ on semi-circular segment

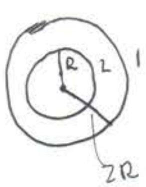
Right hand rule

Thumb- direction of current

Fingers point in dir B

-see if they add or subtract

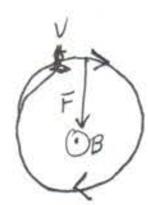
4D)



Lorentz Force Law

$F = qv \times B$

$|F|_{circ} = \frac{mv^2}{r}$



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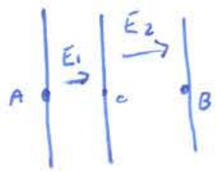


$V_A - V_B$

$V_A - V_C + V_C - V_B$

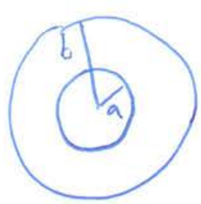
Particle Accelerators

- Generate a velocity
- Move in a ring (circ accelerator)

Increase velocity $\vec{E}$ or $\vec{B}$ ← make a ring

$$F = q(\vec{E} \times \vec{v} \times \vec{B})$$

Spherical Capacitor



8.02 Exam 2 Topics

- Determining Potential Difference From Electric Fields

- Conductors
  - Charges are free to move
  - Charge distributed on surface

$E_{inside} = 0$

$E_{surface} = \frac{\sigma}{\epsilon_0}$

$E \perp$ Surface

Induced surface charge distribution

- Capacitance

$C = Q/|V|$ - Always positive

Parallel Plate capacitor:

$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$ $\quad C = \frac{\epsilon_0 A}{d}$ only depends on A, d

$V = Ed$

- Stored Energy in Electric Fields

$U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

$V = U/Q$

- Dielectrics

$\frac{V}{K}$, $\frac{Ed}{K}$, $\frac{E}{K}$, KQ, CK

- Q can't change if battery disconnected
- V can't change if battery connected.
- Force pulls in dielectric into system

- Current, resistance, Ohm's Law

$I = \frac{dq}{dt}$

$V = IR$

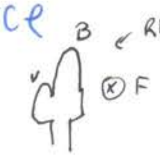
$R = \frac{\rho l}{A}$ resistivity

- Magnetic Field, Magnetic Force

Lorentz Force

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$F_B = q\,\vec{v} \times B$



on wire:

$F = \int_{wire} I\,d\vec{s} \times \vec{B}$

$F = I(\vec{L} \times \vec{B})$ if straight and uniform B

- Biot-Savart Conceptual

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{s} \times \hat{r}}{r^2}$

- RHR #2
  - Thumb in direction of I
  - Fingers curl in B

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2014**

Exam Two

Name Fernando Trujano

Table 5

Check Section

√ **L01 Tegmark** MW 10-12 am

___ **L02 Figueroa** MW 12 -2 pm

___ **L03 Soljačić** MW 2 -4 pm

√ **L04 Thaler** TR 9 -11 am

___ **L05 Checkelsky** TR 11-1 pm

___ **L06 Rajagopal** TR 1-3 pm

___ **L07 Dourmashkin** TR 3-5 pm

___ **L08 Bloomfield** MW 12 -2 pm

___ **L09 Belcher** MW 2-4 pm

	Score	Grader
Problem 1 (25 points)	23	RK
Problem 2 (25 points)	15	mk
Problem 3 (25 points)	14	YD
Problem 4 (25 points)	15	2d
TOTAL	67	

4/5

b) Determine a **vector expression** for the magnetic field $\vec{\mathbf{B}}_2$ in terms of q, m, R, and/or $|\Delta V|$, as needed. Be sure to specify the direction of the magnetic field.

B_2 = out of Page

$$F_{mag} = q v B_2 = \frac{m v^2}{R/3}$$

$$v = \sqrt{\frac{2Q\Delta V}{m}}$$

$$q v B_2 = \frac{3 m v^2}{R}$$

$$\boxed{B_2 = \frac{3mv}{qR}\,\hat{k}}$$

$q = Q$

$$\boxed{\vec{B}_2 = \frac{3m\sqrt{\frac{2q|\Delta V|}{m}}}{qR}\,\hat{k}}$$

-1 ✓

b) Determine an expression for the magnitude of the electric field E_2 in dielectric slab 2. Express your answer in terms of Q, κ_1, κ_2, d_1, d_2, w, L, ε_0, and/or d, as needed. Describe the direction of the electric field.

$$E_2 = \frac{Q}{\varepsilon_0 k_2 L w} \checkmark$$

$$C_2 = \frac{\varepsilon_0 L w k_2}{d_2} = \frac{Q}{|\Delta V|}$$

$$|\Delta V| = \frac{Q d_2}{\varepsilon_0 L w k_2} \hat{i}$$

$$\boxed{E_2 = \frac{Q}{\varepsilon_0 l w k_2} \hat{j}}$$

c) Using your expressions for the magnitudes of the electric fields from parts a) and b), determine the magnitude of potential difference, $|\Delta V|$, between the top and bottom plates of the capacitor. Express your answer in terms of Q, κ_1, κ_2, d_1, d_2, w, L, ε_0, and/or d, as needed.

superposition

$$V = -\int E \cdot ds$$

$$V = -\int_0^{d_1} E_1 \, dr + -\int_{d_1}^{d_2 + d_1} E_2 \, dr$$

$$\cancel{= -\int_0^1 \frac{Q}{\varepsilon d w}} \ ?$$

-2

$$\boxed{V = \frac{Q d_2}{\varepsilon_0 L w k_2} + \frac{Q d_1}{\varepsilon_0 L w k_1}}$$

a) Determine a **vector expression** for the electric field $\vec{\mathbf{E}}(x)$ for this problem in the following regions:

(i) For $x \le -1\,\text{m}$, (ii) $-1\,\text{m} \le x \le 1\,\text{m}$, and (iii) $x \ge 1\,\text{m}$. Be sure to include the correct units in your answer.

4

i) $x \le -1$

$V(x) = \frac{V}{m}x \; \cancel{+} \; V$

$E(x) = -\frac{dV}{dx}$

$\boxed{\hat{E}(x) = -\frac{V}{m}}$

ii) $-1m \le x \le 1$

$V(x) = \frac{-V}{m^2}x^2 + \frac{V}{m}x$

$E(x) = -\left(\frac{-2Vx}{m^2} + \frac{V}{m}\right)$

$\boxed{E(x) = \frac{2Vx}{m^2} - \frac{V}{m}}$

iii) $x \ge 1m$

$1 - \frac{V}{m}x$

$E(x) = -\left(-\frac{V}{m}\right)$

$\boxed{E(x) = \frac{V}{m}}$

-2

b) Determine the points(s) where the electric field vanishes.

1

$E = -$ slope dV

E vanishes when $\frac{dV}{dx} = 0$

$\frac{2Vx}{m^2} - \frac{V}{m} = 0$

-1

$\frac{2\cancel{V}x}{m^2} = \frac{\cancel{V}}{m}$

$2x = 1$

$\boxed{x = .5}$ m

e) Determine the volume charge density ρ. You may express your answer in terms of ε_0. [Hint: Using Gauss' Law]. Be sure to include the correct units in your answer.

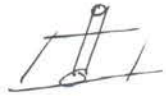
3

Gauss Law

$$\oiint E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0}$$

Pillbox

$$EA = \frac{\sigma A}{\varepsilon_0}$$

slab  $E = \frac{Q}{2\varepsilon_0}$

ρ units Q/V

3

$$\cancel{EA = \frac{\rho A}{\varepsilon_0}}$$

$$\cancel{\frac{Q\cancel{A}}{2\varepsilon_0} = \frac{\rho \cancel{A}}{\cancel{\varepsilon_0}}}$$

$$\frac{1}{\varepsilon_0}\int_0^r \rho$$

$$\cancel{\boxed{\rho = Q/2}}$$

$$\frac{QA}{2\varepsilon_0} = \frac{Q\,\cancel{A}}{V\,\cancel{\varepsilon_0}}$$

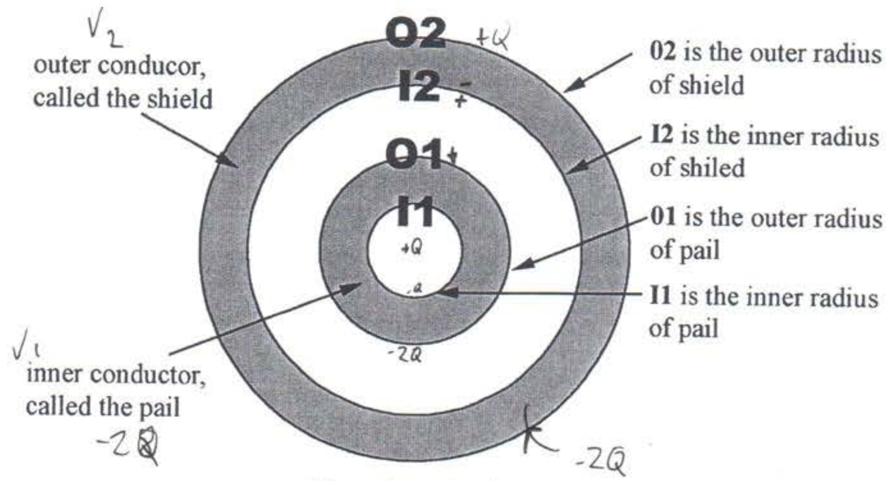
$$\boxed{\rho = \frac{Q}{2}}$$

- Units don't seem quite right

:(

Part B (5 points):

Consider two hollow concentric conducting shells. The outer conductor carries a total charge $-2Q$, and the inner conductor also carries a total charge $-2Q$. Place a point-like object with positive charge $+Q$ at the center of the shells. Let V_1 be the potential at the inner conductor and V_2 be the potential at the outer conductor.



Overhead view

$O2 = -1Q$

$V_2 = -1Q$

$V_1 = -2$

$V_1 < V_2$

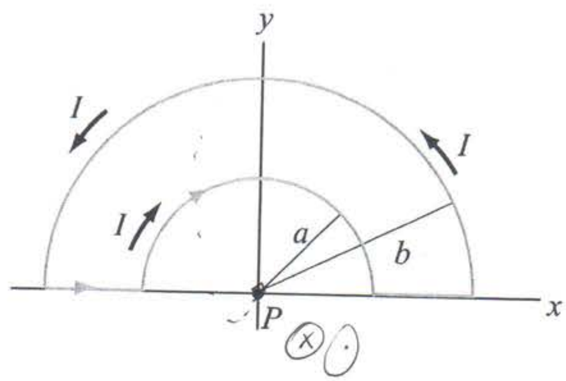
Which of the following statements is true?

1. The surface charge at O2 is positive and $V_1 > V_2$.
2. The surface charge at O2 is positive and $V_1 < V_2$.
3. The surface charge at O2 is positive and $V_1 = V_2$.
4. ~~4~~. The surface charge at O2 is negative and $V_1 > V_2$.
5. ~~5~~. The surface charge at O2 is negative and $V_1 < V_2$.
6. ~~6~~. The surface charge at O2 is negative and $V_1 = V_2$

Answer 2 ✗ ______________________.

Part D (5 points)

Consider the following loop of wire carrying a current I in the counterclockwise direction as shown in the figure below.



Which of the following statements is true for the magnetic field at the point P located at the origin?

1. The magnetic field at the point P is zero.
2. The magnetic field at point P is directed in positive y-direction.
3. The magnetic field at point P is directed in negative y-direction.
4. The magnetic field at point P is directed in positive x-direction.
5. The magnetic field at point P is directed in negative x-direction.
6. The magnetic field at point P is directed into the plane of the figure.
7. The magnetic field at point P is directed out of the plane of the figure.

Answer 1 ×.

11/14

Coulomb $\qquad$ Gauss

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \longrightarrow \int_{closed} \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\frac{Q}{4\pi\varepsilon_0}\hat{r}$$

Biot Savart $\qquad$ Amperes Law

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{wire}\frac{I d\vec{s} \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \longrightarrow \oint \vec{B}\cdot d\vec{s} = \mu_0 \iint \vec{J}\cdot dA$$

- current creates magnetic field

Line Integral of Magnetic Field

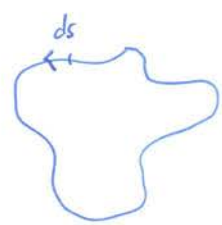
$$\oint \vec{B}\cdot ds$$

$$\int_0^{2\pi} \frac{\mu_0 I}{2\pi r}\hat{\theta}\cdot r\, d\theta\,\hat{\theta}$$

$$= \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\theta = \mu_0 I$$

Ampere's Law

$$\oint_{closed} \vec{B}\cdot d\vec{s} = \mu_0 \iint_{\substack{open\\surface}} \vec{J}\cdot\hat{n}\, da$$



① - Identify regions in which to calculate B field

② - Choose Amperian closed path so symmetry B is zero or constant

③ Calculate $\int B\cdot ds = \begin{cases} B \ast l \\ 0 \end{cases}$

④ $I_{enc} = \mu_0 \iint_{open} J\cdot\hat{n}\, da$

⑤ Amperes $\oint_{close} \vec{B}\cdot d\vec{s} = \mu_0 \iint J\cdot\hat{n}\, da$

A cylindrical conductor has radius R and a uniform current density

Find B for

(1) outside wire $r \geq R$

(2) inside wire $r < R$

(1)

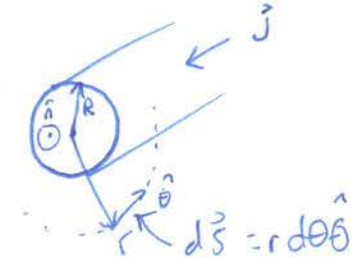
Cylindrical Symmetry $\Rightarrow$ Amperian Circle

B-field CCW

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r)$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}}$$

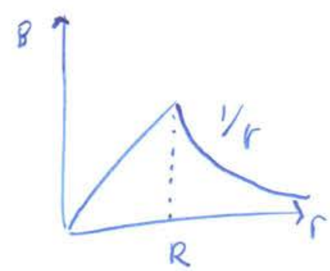


(2)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$|B| L_{AP} = \mu_0 \left(\frac{\pi r^2}{\pi R^2}\right.$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi R^2} r$$



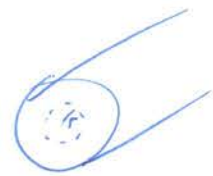
$$B_{in} = \frac{\mu_0 I r}{2\pi R^2}$$

$$B_{out} = \frac{\mu_0 I}{2\pi r}$$

A cylindrical conductor has radius R and a non-uniform current density with total current

$$\vec{J} = J_0 \frac{R}{r} \hat{n}$$

Find B everywhere



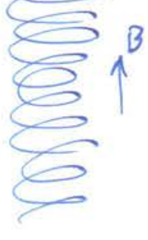
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

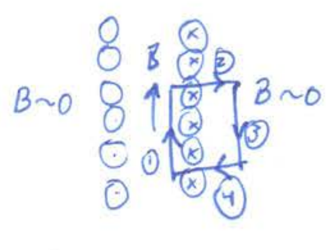
$$B \cdot 2\pi r = \mu_0 \iint \vec{J} \cdot d\vec{A}$$

$$= \mu_0 \int J_0 \frac{R}{\cancel{r}} \cancel{r}\, dr\, d\theta$$

$$B = \frac{\mu_0 J_0 R \cancel{2\pi}}{\cancel{2\pi} r}$$

Magnetic Field of Ideal Solenoid





$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

Side

1. $B_{want}\, L$
2. 0
3. 0
4. 0

— Perpendicular

windings

$$\oint B \cdot dS = \mu_0 N_{inc} I$$

$B_{want} L = \mu_0 N_{inc} I$ $\quad B_{wand} = \mu_0 \underbrace{\frac{N}{L}}_{n} I$

$$\boxed{\vec{B} = \mu_0 n I \hat{k}}$$

Current Sheet

A sheet of current carries uniform current density:

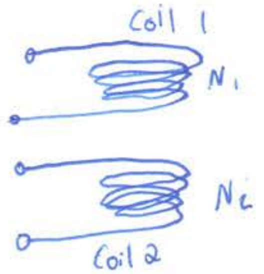
$$\vec{J} = J\hat{k}$$

Find direction and magnitude of B as a function of x

Faraday's Law of Induction

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{A}$$

Mutual Inductance



$$\Phi_{12} = M_{12} I_2$$

$$\mathcal{E}_{12} = -M_{12}\frac{dI_2}{dt}$$

Self Inductance

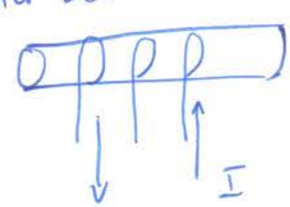
$$\Phi_B = LI$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

Energy in inductors

$$\mathcal{E} = -L\frac{dI}{dt}$$

Inductor Behavior



Inductors inductance

$U = \frac{1}{2}LI^2$

$L = \frac{\phi_{self}}{I}$

$u = U_M = \frac{N_0 B^2}{2}$ ⟵

Energy stored in B field

Capacitors

$U = \frac{1}{2}\frac{Q^2}{C}$ ←

$C = Q/\Delta V$

energy density → $u = U_E = \frac{1}{\varepsilon_0 2} E^2$

Energy stored in E field

8.02 Notes

Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \iint J \cdot \hat{n}\, da$$

tangental magnetic field

I positive $\otimes$ RHR
I negative $\odot$

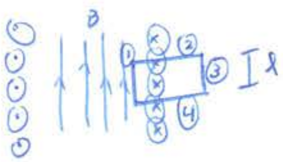
Biot Savart

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{s} \times \hat{r}}{r^2}$$

Ampere's Law - (symmetric current Source)

$$\oint \underbrace{\vec{B} \cdot d\vec{s}}_{B\ell \text{ or } 0} = \mu_0 \iint \vec{J}\cdot\hat{n}\,da = \mu_0 I_{enc}$$

Solenoid ~ 2 current sheets
- Multiple Wire loops
- B uniform inside, 0 outside (ideal)



②,③,④ $\rightarrow B = 0$

① $\int B \cdot d\vec{s} = B\ell = \mu_0 \underbrace{I_{enc}}_{n\ell I}$

n ← turns / unit length

$$B = \frac{\mu_0 n \ell I}{\ell}$$

$$\boxed{B = \mu_0 n I}$$

Faraday's Law

$$\underbrace{\mathcal{E} = -\frac{d\Phi_B}{dt}}_{\text{Lenz's Law}} \qquad \oint_c \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\iint_s \vec{B}\cdot d\vec{A}$$

Lenz's Law
- Induced EMF opposes change in flux

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8.02 **Spring 2012**

Exam 3 Equation Sheet

Force Law: $\vec{\mathbf{F}}_q = q(\vec{\mathbf{E}}_{ext} + \vec{\mathbf{v}}_q \times \vec{\mathbf{B}}_{ext})$

Force on Current Carrying Wire:

$$\vec{\mathbf{F}} = \int_{wire} I d\vec{\mathbf{s}}' \times \vec{\mathbf{B}}_{ext}$$

Magnetic Dipole

$\vec{\mu} = IA\hat{\mathbf{n}}_{RHR}$

Torque on a Magnetic Dipole

$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}_{ext}$

Force on a Magnetic Dipole

$\vec{\mathbf{F}} = \vec{\nabla}(\vec{\mu} \cdot \vec{\mathbf{B}})$

$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

Source Equations:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = k_e \int_{\text{source}} \frac{dq}{r^2}\hat{\mathbf{r}} = k_e \int_{\text{source}} \frac{dq'(\vec{\mathbf{r}} - \vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_o}{4\pi} \int_{source} \frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_o}{4\pi} \int_{source} \frac{I d\vec{\mathbf{s}}' \times (\vec{\mathbf{r}} - \vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3}$$

$\hat{\mathbf{r}}$ points from source to field point

Ampere's Law

$$\oint_{\substack{closed\\path}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$$

Faraday's law

$$\oint_{\substack{closed\\fixed\ path}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt} \iint_{\substack{open\\surface\ S}} \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, da$$

$$\varepsilon = I_{ind} R$$

Gauss's Law for Magnetism:

$$\oiint_{closed\ surface} \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}_{out}\, da = 0$$

Gauss's Law:

$$\oiint_{\text{closed surface}} \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}_{out}\, da = \frac{q_{enc}}{\varepsilon_o}$$

Electric Potential Difference Electrostatics:

$$\Delta V = V_b - V_a \equiv -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = -\vec{\nabla} V$$

Potential Energy:

$\Delta U = q\Delta V$

Capacitance:

$$C = \frac{Q}{\Delta V} \qquad U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C\Delta V^2$$

Inductance: $L = \dfrac{\Phi_{\text{B,Total}}}{I} = \dfrac{N\Phi_{\text{B}}}{I}$

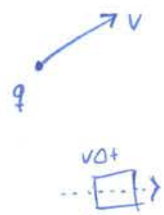
$$\mathcal{E}_{\text{back}} = -L dI / dt \; U_M = \tfrac{1}{2} LI^2$$

Energy Density Stored in Fields:

$u_E = \frac{1}{2}\varepsilon_0 E^2 \; u_B = \frac{1}{2} B^2 / \mu_0$

8.02 Review

Moving electric charges → Magnetic Field



$\lambda = \frac{q}{L}$

$K = I/\text{length}$

$J = I/\text{Area}$

Find B

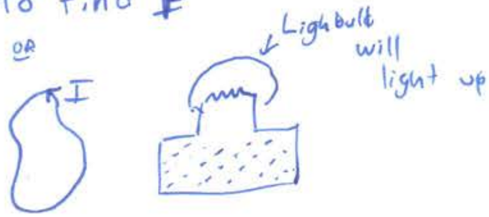
Ampare's

- B is constant For some part, 0 elsewere

If that doesn't work. Use Biot Savart

Force

Use B to find F



Faraday's Law

$\int \vec{B} \cdot n \, dA$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

voltage across lightbulb

↓

$V = IR$

$P = IV$

$= \frac{V^2}{R}$

* Magnetic Field Lines are <u>always</u> loops

Right hand Rule

$A \times B = C$ ← Thumb

↑ Fingers ↑ curl in direction

Dipole μ tries to line up with magnetic Field.

Uniform magnetic Field → no $force_{net}$ on a dipole

Lenz's Law

- Wire wants to oppose change in flux

Solenoid

Inductance →

$\Phi_{12} = L_{12} I_2$

Taking away $\frac{1}{2}$ turns

Decrease area 2

Decrease B by 2

Total Inductance ↓ 4

$N_0 = \frac{B^2}{2N_0}$

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Problem Solving 6: Ampere's Law and Faraday's Law

Section ______ Table ______________________________

Names Fernando Trujano

____

____

Hand in one copy per group at the end of the Friday Problem Solving Session.

Part One: Ampere's Law

OBJECTIVES

1. To learn how to use Ampere's Law for calculating magnetic fields from symmetric current distributions
2. To find an expression for the magnetic field of a cylindrical current-carrying shell of inner radius a and outer radius b using Ampere's Law.
3. To find an expression for the magnetic field of a slab of current using Ampere's Law.

REFERENCE: http://web.mit.edu/8.02t/www/materials/StudyGuide/guide09.pdf
Section 9.3-9.4, 9.7, 9.10.2, 9.11, 8.02 Course Notes

Summary: Strategy for Applying Ampere's Law (Section 9.10.2, 8.02 Course Notes)

Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed loop is proportional to the total steady current passing through any surface that is bounded by the closed loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint \vec{\mathbf{J}} \cdot \hat{\mathbf{n}} \, dA$$

To apply Ampere's law to calculate the magnetic field, we use the following procedure:

Step 1: Identify the 'symmetry' properties of the current distribution.

Step 2: Determine the direction of the magnetic field

Step 3: Decide how many different spatial regions the current distribution determines

For each region of space…

Step 4: Choose an Amperian loop along each part of which the magnetic field is either constant or zero

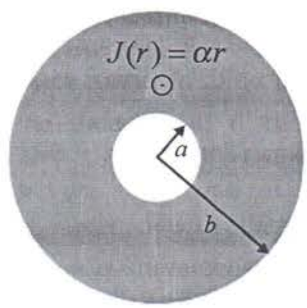
Step 5: Calculate the enclosed current through the Amperian Loop, $I_{enc} = \iint \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA$

Step 6: Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around the closed loop.

Step 7: Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

Problem 1: Magnetic Field of a Cylindrical Shell

A long cylindrical cable consists of a conducting cylindrical shell of inner radius a and outer radius b. The current density $\vec{\mathbf{J}}$ in the shell is out of the page (see sketch) and varies with radius as $J(r) = \alpha r$ for $a < r < b$ and is zero outside of that range, where α is a positive constant with units $\text{A} \cdot \text{m}^{-3}$.



Question 1: Problem Solving Strategy

Step 1: <u>**Identify Symmetry of Current Distribution**</u>
Either circular or rectangular which is it?

circular

Step 2: <u>**Determine Direction of magnetic field**</u>
Clockwise or counterclockwise?

counter clockwise

Step 3: <u>**How many regions?**</u>

3

$r < a$

$a < r < b$

$r > b$

For each region, you will repeat the following four steps:

Step 4: Draw Amperian Loop, choose an integration direction.

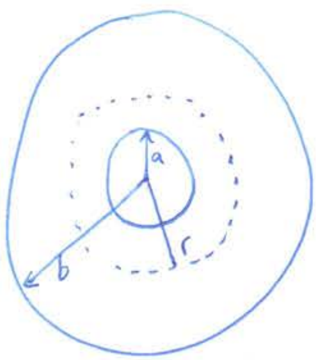
Step 5: Calculate current enclosed by Amperian Loop. Be careful about signs!

Step 6: Apply Ampere's Law to calculate magnitude of the magnetic field.

Step 7: Determine direction of magnetic field based on results of step 6.

The next step is to calculate the current enclosed by this Amperian loop. Because the current is non-uniform, you will need to set up and solve an integral relation for the current enclosed in each of your Amperian loops for the different regions.

Question 2: Draw a new figure showing the cross section of the cable and draw Amperian Loop on your figure for the region $a < r < b$, indicating your choice of integration direction.



$\int_a^r J \cdot d\vec{A}$

$2\pi r\,dr$

Question 3: Calculate current enclosed by Amperian Loop. Be careful about signs! Remember that $\vec{\mathbf{J}}$ is non-uniform so you will need to compute the integral $I_{enc} = \iint \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA$.

$$dA = 2\pi r\, dr$$

$$I_{enc} = \iint J \cdot d\vec{A} = \int_a^r (\alpha r)\, 2\pi r\, dr$$

$$= 2\pi\alpha \int_a^r r^2\, dr$$

$$= \frac{2\pi\alpha}{3}\left(r^3 - a^3\right)$$

Question 4: Your answer above should be zero when $r = a$ and when $r = b$, the total current is I in the wire. Find an expression for I.

$$r = b \Rightarrow I_{enc} = I$$

$$I = \int_a^b \alpha r\, 2\pi r\, dr = \frac{2\pi\alpha}{3}\left(b^3 - a^3\right)$$

Question 5: What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$? (That is, evaluate the integral, the left hand side of Ampere's law)

$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r)$$

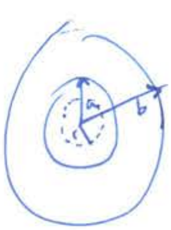
Question 6: Now apply Ampere's Law. What do you get for the magnitude of the magnetic field in the region $a < r < b$?

Ampere's Law:

$$\oint B \cdot d\vec{s} = \mu_0 I_{enc}$$

$$B(2\pi r) = \frac{\mu_0 2\pi\alpha}{3}\left(r^3 - a^3\right)$$

Question 7: Repeat the steps above to find the magnitude of the magnetic field in the region $r < a$.

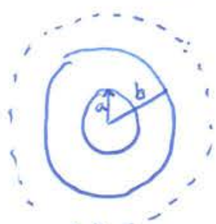


$$\int_0^a$$

Given

$$I_{enc} = 0 \Rightarrow B = 0$$

Question 8: Repeat the steps above to find the magnitude of the magnetic field in the region $r > b$.



$I_{enc} = I$

$$I_{enc} = \int_a^b \alpha r \, 2\pi r \, dr = \alpha 2\pi \int_a^b r^2 \, dr$$

$$= \frac{\alpha 2\pi}{3}\left(b^3 - a^3\right)$$

$$\oint B \cdot ds = B(2\pi r)$$

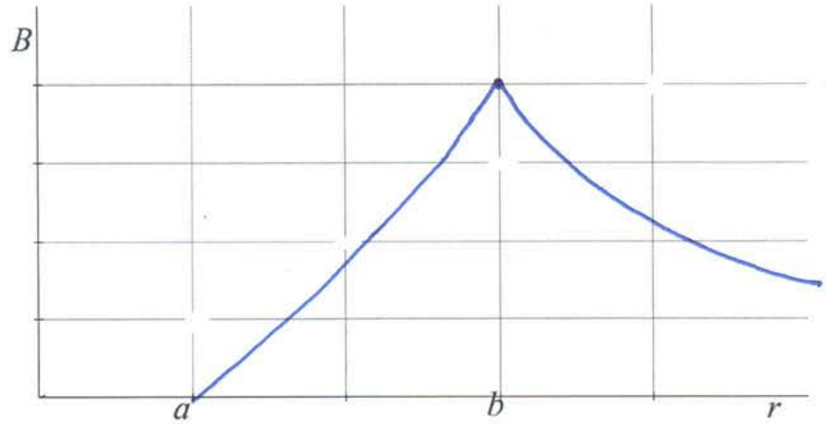
Ampare's Law:

$$\oint B \cdot ds = \mu_0 I_{enc}$$

$$B 2\pi r = \frac{\mu_0 \alpha 2\pi}{3}\left(b^3 - a^3\right)$$

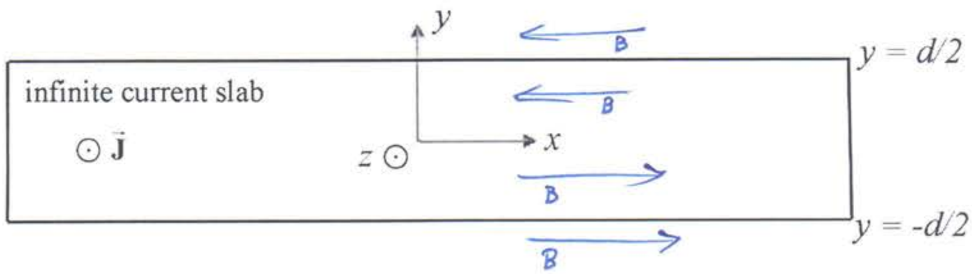
$$B = \frac{\mu_0 \alpha 2\pi}{2\pi r 3}\left(b^3 - a^3\right)$$

Question 9: Plot B vs. r on the graph below.



Problem 2: Magnetic Field of a Slab of Current

We want to find the magnetic field $\vec{\mathbf{B}}$ due to an infinite slab of current, using Ampere's Law. The figure shows a slab of current with current density $\vec{\mathbf{J}} = (2J_e|y|/d)\hat{\mathbf{z}}$, where J_e is positive constant with units of amps per square meter. The slab of current is infinite in the x and z directions, and has thickness d in the y-direction.



Question 1: What is the magnetic field at $y = 0$, where $y = 0$ is the exact center of the slab? Explain your reasoning.

0

Problem Solving Strategy

(1) <u>Identify Symmetry</u>
Either circular or rectangular. Which is it?

Rectangular

(2) <u>How many regions?</u>

3 Inside Slab, Outside Slap Top Bottom

(3) <u>Determine Direction</u>
Make sure you determine the direction of the magnetic field in all regions. Sketch on figure above.

For each region, you will repeat the following four steps.

Step 4: Draw Amperian Loop, choose a integration direction.

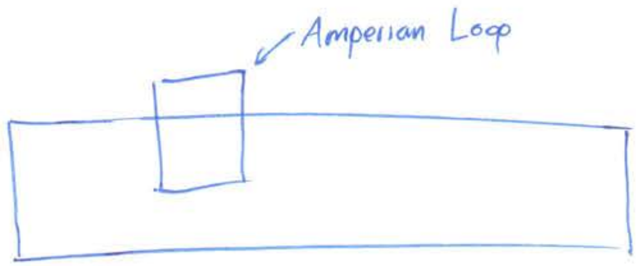
Step 5: Calculate current enclosed by Amperian Loop. Be careful about signs!

Step 6: Apply Ampere's Law to calculate magnitude of the magnetic field.

Step 7: Determine direction of magnetic field based on results of step 6.

We want to find the magnetic field for $y > d/2$, and we have from the answer to Question 1 for the magnetic field at $y = 0$. Therefore....

Question 2: What Amperian loop do you take to find the magnetic field for $y > d/2$? Make a new figure below and draw the Amperian loop on that figure indicating any variables that you may need.



The next step is to calculate the current enclosed by this Amperian loop. Hint: the current enclosed is the integral of the current density over the enclosed area.

Question 3: What is the total current enclosed by your Amperian loop?

$$J = \frac{2J_e y}{d}$$

$$I_{enc} = \iint J \cdot \underbrace{dA}_{\ell dy} = \frac{2J_e \ell}{d}\int_0^{d/2} y\,dy = \frac{J_e \ell d}{4}$$

Question 4: What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$?

$$\oint B \cdot d\vec{s} = B\ell + 0 + 0 + 0$$

Question 5: Now apply Ampere's Law to find the magnitude of the magnetic field in the region $y > d/2$?

$$\oint B \cdot ds = \mu_0 I_{enc}$$

$$B\cancel{\ell} = \frac{\mu_0 J_e \cancel{\ell} d}{4} \cdot \hat{\imath}$$

We now want to find the magnetic field in the region $0 < y < d/2$.

(4) <u>Draw Amperian Loop</u>:

We want to find the magnetic field for $0 < y < d/2$, and we have from the answer to Question 1 for the magnetic field at $y = 0$. Therefore…

(5) <u>Current enclosed by Amperian Loop</u>:

Question 6: What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Make a new figure below and draw the Amperian loop on that figure indicating any variables that you may need.

The next step is to calculate the current enclosed by this Amperian loop.

Question 7: What is the total current enclosed by your Amperian loop?

$$I_{enc} = \iint J \cdot \overbrace{dA}^{\ell\, dy} = \frac{2 J_e \ell}{d} \int_0^y y\, dy = \frac{J_e \ell y^2}{d}$$

Question 8: What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$?

$$\oint \vec{B} \cdot d\vec{s} = 0 + B\ell + 0 + 0 = B\ell$$

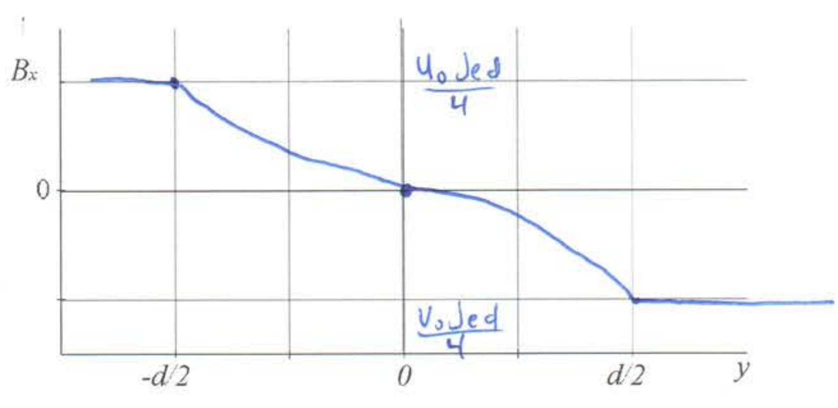
(7) <u>**Solve for B:**</u>

Question 9: Now apply Ampere's Law to find the magnitude of the magnetic field in the region $0 < y < d/2$?

$$\oint B \cdot dS = \mu_0 I_{enc}$$

$$B\cancel{\ell} = \mu_0 \frac{J_e \ell y^2}{d} - \hat{i}$$

Question 10 Plot B_x vs. y on the graph below. Use symmetry to determine B for $y < 0$. Label the y-axis as appropriate.



Part Two: Problem Solving Faraday's Law

OBJECTIVES

1. To explore a particular situation that can lead to a changing magnetic flux through the open surface bounded by an electric circuit.
2. To calculate the rate of change of magnetic flux through the open surface bounded by that circuit in this situation.
3. To determine the sense of the induced current in the circuit from Lenz's Law.
4. To look at the forces on the current carrying wires in our circuit and determine the effects of these forces on the dynamics of the circuit.

REFERENCE: http://web.mit.edu/8.02t/www/materials/StudyGuide/guide10.pdf
Section 10.1-10.4, 8.02 Course Notes

Problem-Solving Strategy for Faraday's Law

In Chapter 10 of the 8.02 Course Notes, we have seen that a changing magnetic flux induces an emf:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

according to Faraday's law of induction. For a conductor that forms a closed loop, the emf sets up an induced current $I = |\varepsilon| / R$, where R is the resistance of the loop. To compute the induced current and its direction, we follow the procedure below:

(1) For the closed loop of area A, define an area vector $\vec{\mathbf{A}}$ and choose a positive direction. For convenience of applying the right-hand rule, let $\vec{\mathbf{A}}$ point in the direction of your thumb. Compute the magnetic flux through the loop using

$$\Phi_B = \begin{cases} \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} & (\vec{\mathbf{B}} \text{ is uniform}) \\ \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} & (\vec{\mathbf{B}} \text{ is non-uniform}) \end{cases}$$

(2) Evaluate the rate of change of magnetic flux $d\Phi_B / dt$. Keep in mind that the change could be caused by

(i) changing the magnetic field $dB/dt \neq 0$,

(ii) changing the loop area if the conductor is moving ($dA/dt \neq 0$), or

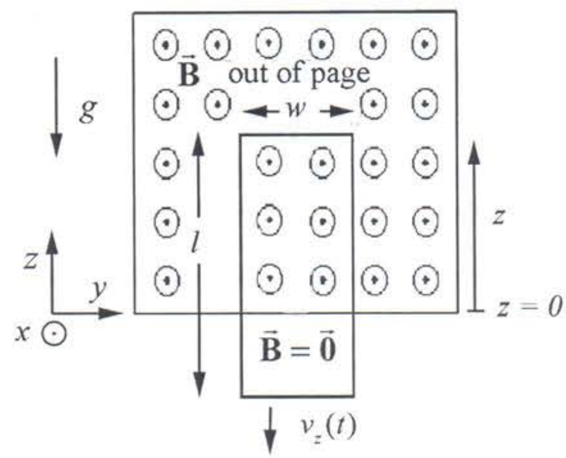
(iii) changing the orientation of the loop with respect to the magnetic field ($d\theta/dt \neq 0$).

Determine the sign of $d\Phi_B/dt$ (is the flux increasing or decreasing)

(3) The sign of the induced emf is opposite the sign of $d\Phi_B/dt$. The direction of the induced current can be found by using the Lenz's law discussed in Section 10.4.

Problem 1 Falling Loop

A rectangular loop of wire with mass m, width w, vertical length l, and resistance R falls out of a magnetic field under the influence of gravity. The magnetic field is uniform and out of the paper ($\vec{\mathbf{B}} = B\hat{\mathbf{i}}$, $B > 0$ within the area shown (see sketch) and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{\mathbf{v}}(t) = v_z(t)\hat{\mathbf{k}}$, where $v_z(t) < 0$ (meaning the loop is moving downward, not upward). Suppose at time t the distance from the top of the loop to the point where the magnetic field goes to zero is $z(t)$ (see sketch).



Question 1 What is the relationship between $v_z(t)$ and $z(t)$? Be careful of you signs here, remember that $z(t)$ is positive and decreasing with time, so $dz(t)/dt < 0$.

$$v_z(t) = \frac{dz(t)}{dt}$$

Problem Solving Strategy Step (1): Define $\vec{\mathbf{A}}$ and Compute Φ_B

Question 2: If we define the area vector $\vec{\mathbf{A}}$ to be out of the page, what is the magnetic flux Φ_B through our circuit at time t (in terms of $z(t)$, not $v_z(t)$).

Uniform magnetic Field $\Rightarrow$ $\Phi_B = B \cdot A$

$= B w\, z(t)$

Problem Solving Strategy Step (2): Compute $d\Phi_B / dt$

Question 3: What is $d\Phi_B / dt$? Is this positive or negative at time t? Be careful here, your answer should involve $v_z(t)$ (not $z(t)$), and remember that $v_z(t) < 0$.

$\frac{d}{dt}\Phi_B = \frac{d}{dt} B w\, z(t)$

$\frac{d\Phi_B}{dt} = B w\, v_z(t)$

$\frac{d\Phi_B}{dt}$ is negative

Problem Solving Strategy Step (3): Determine the sign of the induced emf (the same as the direction of the induced current)

Question 4: If $d\Phi_B / dt < 0$ then your induced emf (and current) will be right-handed with respect to $\vec{\mathbf{A}}$, and vice versa. What is the direction of your induced current given your answer to Question 3, clockwise or counterclockwise?

Counterclockwise

Question 5: Lenz's Law says that the induced current should be such as to create a self-magnetic field (due to the induced current alone) which tries to keep things from changing. What is the direction of the self-magnetic field due to your induced current inside the circuit loop, into the page or out of the page? Is it in a direction so as to keep the flux through the loop from changing?

Positive magnetic flux opposes decrease in external flux

Question 6: What is the magnitude of the current flowing in the circuit at the time shown (use $I = |\varepsilon| / R$)?

$$\varepsilon = -\frac{d\Phi_B}{dt} = B w v_z(t)$$

$$I = \left| \frac{B w v_z(t)}{R} \right|$$

Question 7: Besides gravity, what other force acts on the loop in the $\pm\hat{\mathbf{k}}$-direction? Give the magnitude and direction of this force in terms of the quantities given (hint: $d\vec{\mathbf{F}}_{mag} = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$).

$$F_B = \int I\, d\vec{s} \times B$$

$$= I_{ind} \vec{L} \times B = \frac{B w v_z(t)}{R} w (-\hat{j}) \times B \hat{i} = \frac{B^2 w^2 v_z(t)}{R} \hat{k}$$

- no force on bottom leg ∴ ↳ outside of field

Question 8: Assume that the loop has reached "terminal velocity"--that is, that it is no longer accelerating. What is the magnitude of that terminal velocity in terms of given quantities?

$F_B = F_G$

$$\frac{B^2 w^2}{R} V_f = mg$$

$$|V_f| = \frac{Rmg}{B^2 w^2}$$

Question 9: Show that at terminal velocity, the rate at which gravity is doing work on the loop is equal to the rate at which energy is being dissipated in the loop through Joule heating (the rate at which a force does work is $\vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$)?

8.02: Electricity and Magnetism

Exam 3 Review Problems

Spring 2014

By: Jason Choi

Directions: Solve the following problems. Use the 8.02 Test 3 formula sheet. Calculators should not be needed. This review set is slightly longer than what a test would be, so aim to take around 2.5 hours. Good luck! Solutions will be discussed at the review session.

1. **Magnetic Fields.** An infinitely long wire of radius a carries a current density J_0 which is uniform and constant. The current points out of the page, as shown in the figure. Additionally, there is a hole in the wire with radius b as shown (where $2b < a$). There are four labeled points: O, L, M, and N. The point O is the center of the wire and the point M is the center of the hole. The points L and N lie on the circumference of the hole.

   (a) First consider a wire with *no* hole with the same uniform current density J_0. Compute the magnetic field of a filled wire for:
   (i) $r < a$ and (ii) $r > a$.

   (b) Now, consider again the wire with the hole as shown in the figure. Use the principle of superposition to compute the magnetic field at:
   (i) point M, (ii) point L, and (iii) point N.

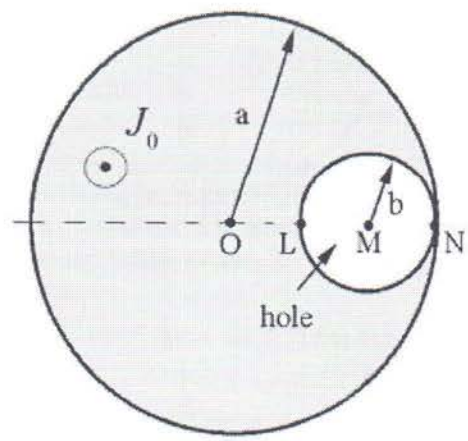


Figure to Problem 1.

3. **Fun with Ampère's Law.** A circular conducting loop is connected to two, long straight wires across its diameter, as shown in the figure. The top half of the loop ($y > 0$) has resistance R and the bottom half ($y < 0$) has resistance $2R$. A current I flows from one wire to the other, as shown.

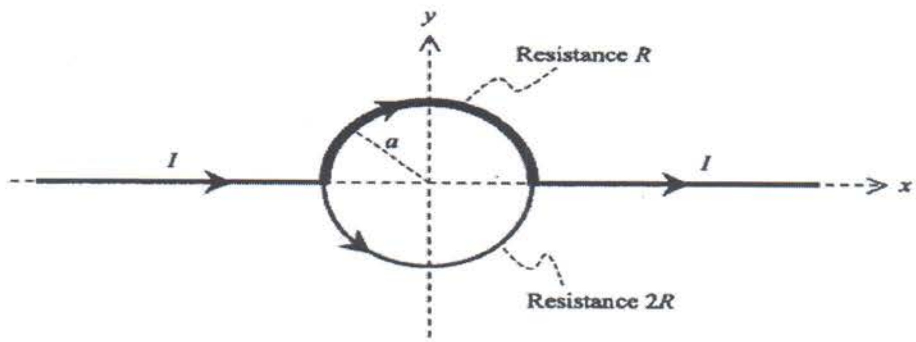
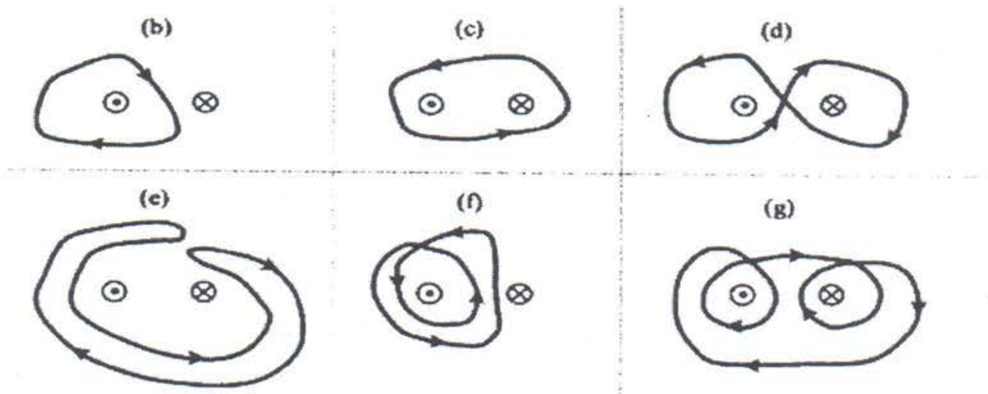


Figure to Problem 3(a).

(a) Calculate the magnitude and direction of the magnetic field **B** at the center of the loop.

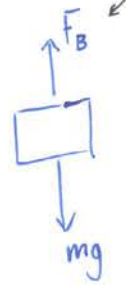
(b) through (g): Each of the following diagrams shows a pair of infinitely long parallel wires, and a particular closed curve. The wires carry equal currents I, with one current directed into the page and one directed out of the page. The arrows on the curve indicate the sense of rotation we consider positive. Calculate the line integral of the magnetic field around the curve in each case using Ampère's Law.



Figures to Problem 3(b)-(g).

Problem 4

b)



b/c cw I and $\odot$ B. RHR

$$F_{net} = mg - F_B = mg - \Delta I(\vec{l} \times \vec{B})$$

$$= mg - I(aB(z+a) - aB(z))$$

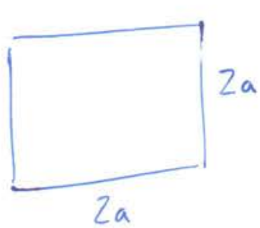
$$= mg - I(ak(z+a) - akz)$$

$$= mg - \underbrace{I}_{\text{From a)}} a^2 k \Rightarrow = \boxed{mg - \frac{k^2 a^4 v}{R}}$$

c) Terminal velocity → no more a → $F = 0$ ⇒ $mg = \frac{k^2 a^4 v_f}{R}$

$$\boxed{v_f = \frac{mgR}{k^2 a^4}}$$

d)



$a \to 2a$
$m \to 2m \Rightarrow v_f' = \frac{(2m)g(2R)}{k^2(2a)^4} = \frac{4}{16} v_f = \frac{1}{4} v_f$
$R \to 2R$

5. **A DC Circuit.** Consider the following DC circuit with four resistors ($R_1 = R$, $R_2 = 2R, R_3 = 3R, R_4 = 4R$) and a battery with emf ε.

(a) Rank the resistors according to the potential difference across them, from largest to smallest.

(b) Determine the potential difference across each resistor only in terms of ε.

(c) Rank the resistors according to the current in them, from largest to smallest.

(d) Determine the current in each resistor in terms of I only.

(e) If R_3 is increased, what happens to the current in each of the resistors? If we take the limit $R_3 \to \infty$, compute the new values of current in each resistor in terms of I.

(f) Determine the heat dissipated in 1 second in the circuit. Assume the battery has no internal resistance.

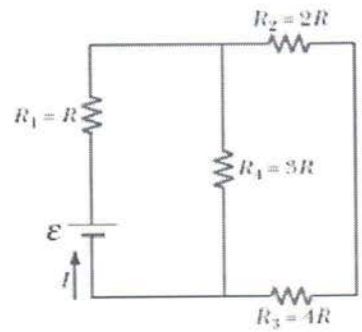


Figure to Problem 5.

a) $R_{23} = 6R$

$\frac{1}{R_{234}} = \frac{1}{3R} + \frac{1}{6R} \Rightarrow R_{234} = 2R$

$R_{1234} = 3R \Longrightarrow I = \frac{\varepsilon}{3R}$

$\rightarrow \Delta V_1 = \varepsilon/3$

$\Delta V_4 = \frac{2\varepsilon}{9R}(3R) = \frac{2\varepsilon}{3}$

$\Delta V_2 = \frac{\varepsilon}{9R}(2R) = \frac{2\varepsilon}{9}$

$\Delta V_3 = \frac{4\varepsilon}{9}$

$\boxed{4 > 3 > 1 > 2}$

b) See a)

c) $\boxed{1 > 4 > 2 = 3}$

d) $I_1 = I \qquad I_2 = I_3 = \frac{I}{3}$

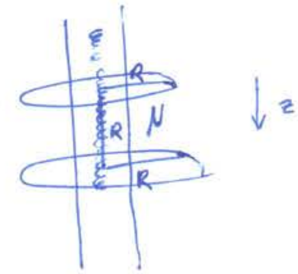
$I_4 = \frac{2I}{3}$

e) $I_4 \uparrow,\ I_1 \downarrow,\ I_2 I_3 \downarrow \qquad I_2 = I_3 = 0 \qquad I_1 = I_4 = \frac{\varepsilon}{3R}$

f) Heat $= Pt = P = IV = \boxed{\frac{\varepsilon^2}{3R}}$

$\uparrow$ 1 sec

Problem 6



a) $F_{dp} = \mu \nabla B$

$= \mu \frac{dB}{dz} = \boxed{\frac{\mu N N_0 I R^2 (-3z)}{2(z^2+R^2)^{5/2}}}$

b) Current ON: $F_B + mg - k\Delta z_1 = 0$

Current OFF: $F_B - k\Delta z_2 = 0$ $\Rightarrow mg = k\Delta z_2$

$F_B + k\Delta z_2 - k\Delta z_1 = F_B + k(\Delta z_2 - \Delta z_1) = F_B + k\Delta z = 0$

$F_B(z=z_0) = \frac{\mu N N_0 I R^2 (-3z_0)}{2(z_0^2+R^2)^{5/2}}$ ← From A

$\boxed{\Delta z = -F_B/k}$

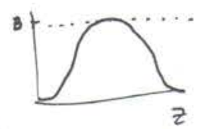
From class experiment

c) $\frac{dB}{dz} = 0 \Rightarrow z = \pm R/2$

-Force is largest at the ends R $R/2$ 0 $-R/2$

d) $F = 0$

← When the field is the largest



Probably not on exam!

e) $T = \vec{\mu} \times \vec{B}$ $\boxed{= \mu B \sin\theta}$

f) $T = I\alpha = \boxed{I\ddot{\theta} = \mu B \sin\theta}$

g) Small angle approximation $\boxed{T = 2\pi\sqrt{\frac{I}{\mu B}}}$

$\sin\theta = \theta$

$I\theta = \mu B\theta$

h) NOT VALID because of small angle approx

7. **Concept Questions.** Circle the correct answer.

1. Consider a triangular loop of wire with sides a and b. The loop carries current I in the direction shown and is placed in a uniform magnetic field with magnitude B, pointing in the direction shown. At this instant, the torque on the current loop:

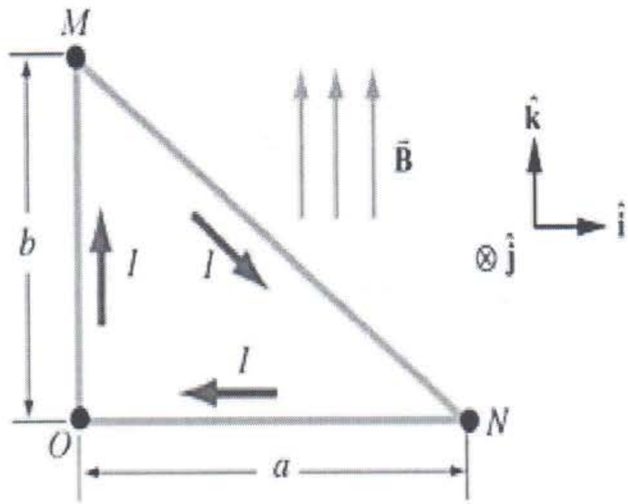


Figure to Question 1.

(a) points in the $-\hat{\imath}$ direction with magnitude $IabB/2$
(b) points in the $+\hat{\imath}$ direction with magnitude $IabB/2$
(c) points in the $-\hat{\jmath}$ direction with magnitude $IabB/2$
(d) points in the $+\hat{\jmath}$ direction with magnitude $IabB/2$
(e) points in the $-\hat{\imath}$ direction with magnitude $IabB$
(f) points in the $+\hat{\imath}$ direction with magnitude $IabB$
(g) points in the $-\hat{\jmath}$ direction with magnitude $IabB$
(h) points in the $+\hat{\jmath}$ direction with magnitude $IabB$
(i) None of the above. The correct answer is: ______

current   Area Vector

$\vec{T} = I\vec{A} = IA\hat{\jmath}$

$= \frac{IabB}{2}\hat{\jmath} = \mu \times B$

3. A particle with charge q and velocity v enters through the hole in screen 1 and passes through a region with non-zero electric and magnetic fields (see figure). If $q < 0$ and we have that $|E| > |vB|$, then the force on the particle:

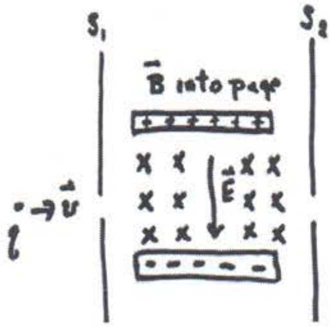


Figure to Question 3.

(a) is zero and the particle will move in a straight line passing through the hole in screen 2

(b) is constant and the particle will move in a parabolic path hitting screen 2 above the hole

(c) is constant and the particle will move in a parabolic path hitting screen 2 below the hole

(d) is constant in magnitude but changes direction and the particle will follow a circular path hitting screen 2 above the hole

(e) is constant in magnitude but changes direction and the particle will follow a circular path hitting screen 2 below the hole

(f) changes magnitude and direction and the particle will follow a curved path hitting screen 2 above the hole

(g) changes magnitude and direction and the particle will follow a curved path hitting screen 2 below the hole

(h) None of the above. The correct answer is: ______________________

$q < 0$

$|E| > |vB|$

5. Consider the following circuit, with 3 resistors and 3 batteries ($\varepsilon_1, \varepsilon_2, \varepsilon_3$ from left to right). Which of the following is the correct ranking of the power delivered by each battery?

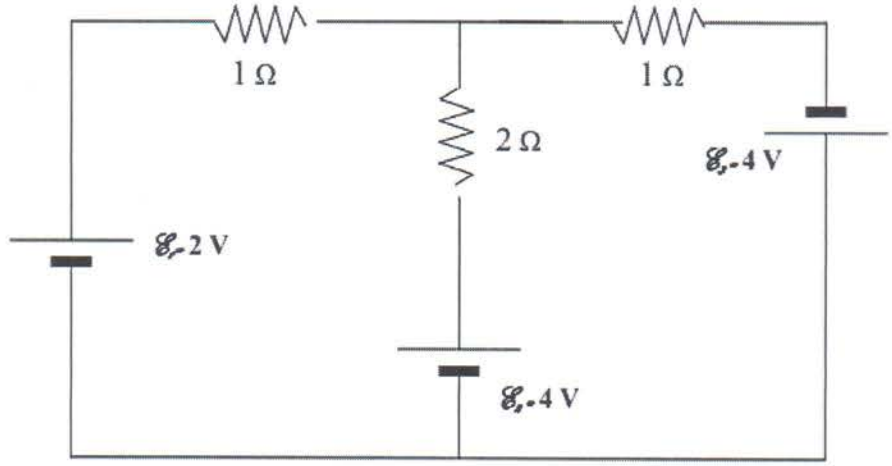
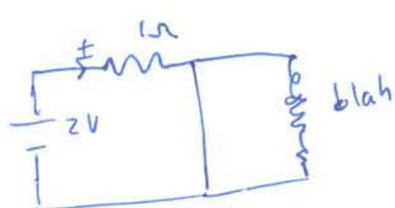


Figure to Question 5.

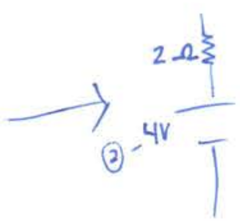
(a) $P_1 = P_2 > P_3$
(b) $P_1 = P_3 > P_2$
(c) $P_2 > P_1 = P_3$
(d) $P_2 = P_3 > P_1$
(e) $P_3 > P_1 = P_2$
(f) $P_3 > P_2 > P_1$
(g) $P_1 = P_2 = P_3$
(h) None of the above. The correct answer is: ______________________

The end!



$I = 2A$
$P_1 = 4W$

3 > 2 > 1



$P_2 = 2A(4V) = 8W$

$P_3 = 4A(4V) = 16W$

①

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2013

Exam Three

Name Fernando Trujano

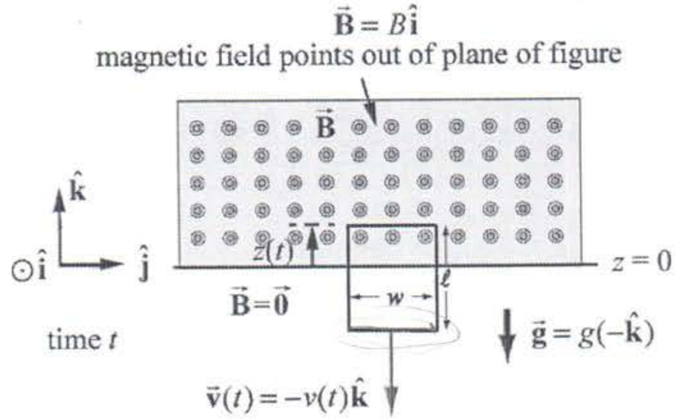
Table ____________

Check Section

____L01 Conrad	MW 10-12 am	____L06 Dourmashkin	TR 1-3 pm
____L02 Paus	MW 12 -2 pm	____L07 Dourmashkin	TR 3-5 pm
____L03 Paus	MW 2 -4 pm	____L08 Figueroa	MW 12 -2 pm
____L04 Tegmark	TR 9 -11 am	____L09 Gore	MW 2-4 pm
____L05 Belcher	TR 11-1 pm		

	Score	Grader
Problem 1 (25 points)		
Problem 2 (25 points)		
Problem 3 (25 points)		
Problem 4 (25 points)		
TOTAL		

Problem 1 (25 points): answers without work shown will not be given any credit.



A rectangular loop of wire with mass m, width w, vertical length l, and resistance R falls out of a magnetic field under the influence of gravity, as shown in the figure above. The magnetic field is uniform and out of the paper ($\vec{B} = B\hat{i}$) within the area shown and zero outside of that area. The loop of wire is released from rest. At the time t shown in the figure, the loop is exiting the magnetic field with velocity $\vec{v}(t) = -v(t)\hat{k}$. The gravitational field is $\vec{g} = g(-\hat{k})$.

a) Is the direction of the induced current in the rectangular loop of wire at time t, clockwise or counterclockwise? Briefly justify your answer.

Flux is decreasing
Current induced in wire wants to oppose change.
↳ Lentz's Law

⇒ CCW Current

b) Determine an expression for the magnitude of the induced current in the rectangular loop of wire at time t? Express your answer in terms of the quantities m, w, l, R, B, t, $v(t)$, and g as needed.

Lentz Law:

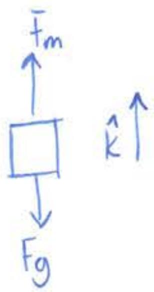
$$\mathcal{E} = \frac{-d\Phi}{dt}$$

$$\mathcal{E} = I_{ind} R$$

$$\frac{d\Phi}{dt} = B w v(t)$$

$$I_{ind} = \frac{+Bw(v(t))}{R}$$

c) Determine the direction and magnitude of the induced force on the rectangular loop of wire at time t. Express your answer in terms of the quantities m, w, l, R, B, t, $v(t)$, g, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as needed.



$$\bar{F}_m = \int I\,ds \times B$$

$$= IL \times B$$

$$= IwB\hat{k}$$

$$\left(I = +\frac{Bw\,v(t)}{R}\right.$$

$$\boxed{\bar{F}_m = \frac{B^2 w^2}{R} v(t)\,\hat{k}}$$

d) What is the maximum possible speed that the rectangular loop of wire can achieve before it exits the region of non-zero magnetic field? Express your answer in terms of the quantities m, w, l, R, B, and g as needed.

Terminal velocity $\rightarrow$ no acceleration $\rightarrow$ $F_{net} = 0$

$$\Rightarrow F_m = F_g$$

$$\frac{B^2 w^2}{R} v_f(t) = mg$$

$$v_f = \boxed{\frac{Rmg}{B^2 w^2}}$$

e) What is the maximum possible dissipated power in the rectangular loop of wire before it exits the region of non-zero magnetic field? Express your answer in terms of the quantities m, w, l, R, B, and g as needed.

$P = IV$, IR

$P = I^2 R$

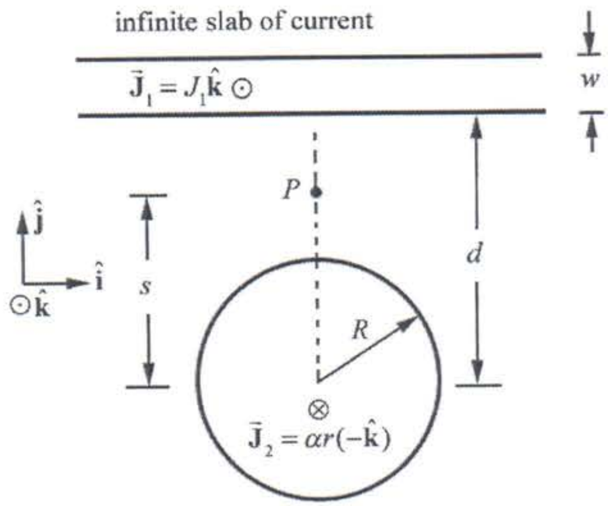
$I \to I_{max}$ when $v \to V_{max}$

$$P = \left(\frac{BwV_F}{R}\right)^2 R$$

$$= \left(\frac{\cancel{B}\cancel{w}\cancel{R}mg}{B^{\cancel{2}}w^{\cancel{2}}\cancel{R}}\right)^2 R$$

$$= \boxed{\left(\frac{mg}{Bw}\right) R}$$

Problem 2 (25 points): answers without work shown will not be given any credit.

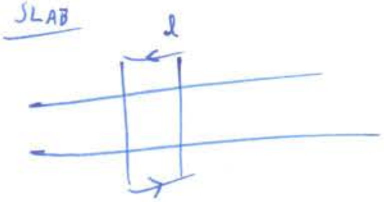


infinite wire with non-uniform current density

Consider the following two current distributions. An infinite slab of width w carries a uniform volume current density given by $\vec{\mathbf{J}}_1 = J_1\,\hat{\mathbf{k}}$. An infinite wire of radius R carries a non-uniform volume current density $\vec{\mathbf{J}}_2 = \alpha r(-\hat{\mathbf{k}})$ where r is the distance form the central symmetry axis of the wire and α is a positive constant with SI units $[\mathrm{A}\cdot\mathrm{m}^{-3}]$. The center of the wire is a distance d from the bottom surface of the slab.

a) Determine an expression for the direction and magnitude of the magnetic field at the point P located a distance s directly above the center of the wire. Express your answer in terms of μ_0, J_1, w, R, α, d, s, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as needed.

SLAB



$$\oint B\cdot ds = \mu_0 I_{enc}$$

$$2B\cancel{l} = \mu_0 J_1 w \cancel{l}$$

$$B = \frac{\mu_0 J_1 w}{2}\,\hat{i}$$

$$\oint B \cdot ds = \mu_0 I_{enc}$$

$$B 2\pi r = \mu_0 \int \alpha r 2\pi \, dr$$

$$= \mu_0 2\pi\alpha \int_0^R r^2 \, dr$$

$$B \cancel{2\pi} r = \frac{\mu_0 \cancel{2\pi} \alpha R^3}{3}$$

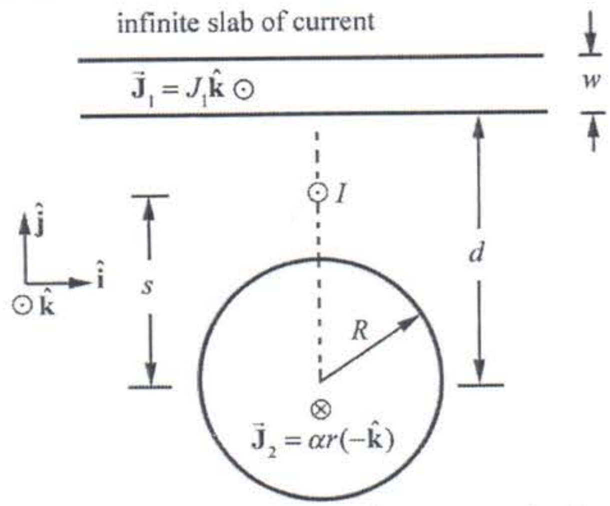
$$B = \frac{\mu_0 2 R^3}{3r} \hat{i}$$

Super Position

$$B_{TOTAL} = \overleftrightarrow{B_1 + B_2}$$

$$= \left(\frac{\mu_0 \alpha R^3}{3r} + \frac{\mu_0 J_1 w}{2} \right) \hat{i}$$

Now suppose that a very long straight wire carrying a current I is placed at the point P with the current directed out of the plane of the figure below (positive $\hat{\mathbf{k}}$ -direction).



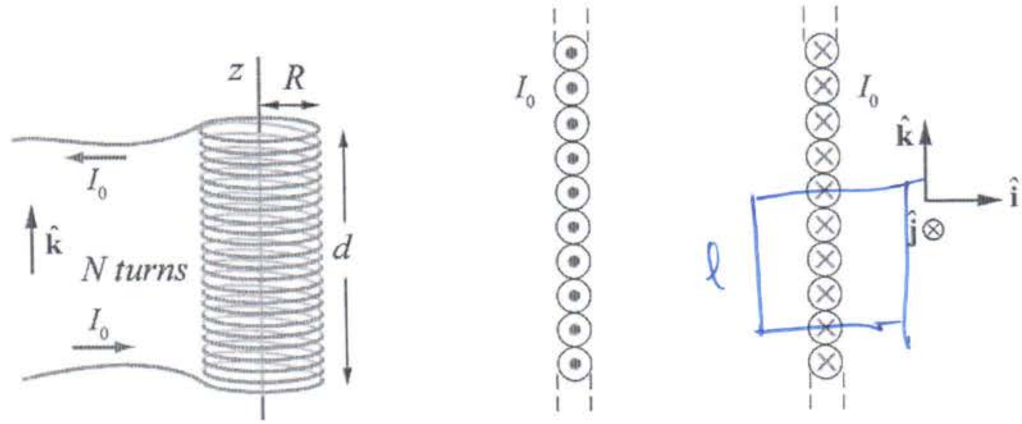
infinite wire with non-uniform current density

b) Determine an expression for the direction and magnitude of the force per unit length, $d\vec{\mathbf{F}}/ds$, on the current carrying wire. Ignore any effects of gravitation. Express your answer in terms of μ_0, I, J_1, w, R, α, d, s, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as needed.

$$dF = I \overset{\hat{k}}{ds} \times \overset{\hat{i}}{B}$$

$$\frac{dF}{ds} = IB\hat{j}$$

Problem 3 (25 points): answers without work shown will not be given any credit.



A very long solenoid of length d has N turns of wire. The wire carries a current I_0 directed as shown in the figures above. The solenoid has radius R.

a) Determine the direction and magnitude of the magnetic field within the solenoid. (Ignore edge effects). Express your answer in terms of μ_0, I_0, R, d, N, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as needed. **Clearly show your work including any Amperian loops that you use.**

$$\int B \cdot ds = \mu_0 I_{enc}$$

$$B\ell = \mu_0 \frac{N}{d} \ell I$$

$$B = \mu_0 \frac{N}{d} I \hat{k}$$ RHR

b) Determine an expression for the self-inductance L of the solenoid. Express your answer in terms of μ_0, I_0, R, d, and N as needed.

$$L = \frac{\Phi}{I} \,/\, \text{turn}$$

BA because constant B

N

$$L = \frac{\mu_0 \frac{N}{d} I \pi R^2}{I} N$$

$$L = \frac{\mu_0 N^2 \pi R^2}{d}$$

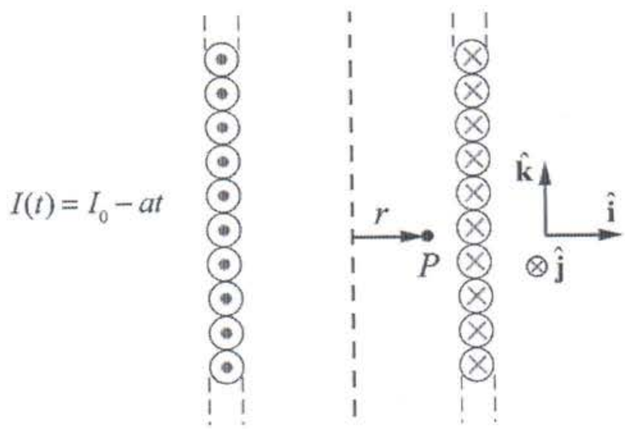
c) Determine an expression for the magnetic energy U_M stored in the solenoid. Express your answer in terms of μ_0, I_0, R, d, and N as needed.

$$U_m = \frac{1}{2} L I^2$$

Now suppose that the current through the wire decreases according to

$$I(t) = I_0 - at, \qquad 0 < t < I_0 / a.$$

d) During the interval, $0 < t < I_0 / a$, determine an expression for direction and magnitude of the electric field at the point P shown in the figure above that is a distance r from the central axis of the solenoid. Express your answer in terms of μ_0, I_0, a, R, r, d, t, N, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as needed.



Top View

$$\oint E\cdot ds = -\frac{d}{dt}\Phi$$

$$E\,2\pi r\,\cancel{ds} = -\frac{d}{dt}\,\pi r^2 N_0 \frac{N}{d}(I_0 - at)$$

$$2\cancel{\pi}\cancel{r}E = \cancel{\pi}r^2 N_0 \frac{N}{d}a$$

$$E = \frac{rN_0 Na}{2d}$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2014

Exam Three

Name Fernando Trujano

Table 5

Check Section

____L01 Tegmark MW 10-12 am

____L02 Figueroa MW 12 -2 pm

____L03 Soljačić MW 2 -4 pm

√ L04 Thaler TR 9 -11 am

____L05 Checkelsky TR 11-1 pm

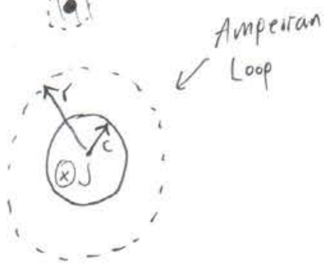
____L06 Rajagopal TR 1-3 pm

____L07 Dourmashkin TR 3-5 pm

____L08 Bloomfield MW 12 -2 pm

____L09 Belcher MW 2-4 pm

	Score	Grader
Problem 1 (25 points)	24	AN
Problem 2 (25 points)	20	YZ
Problem 3 (25 points)	15	KR
Problem 4 (25 points)	0	A.L.
TOTAL	59	



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \int_0^c -J\, 2\pi r\, dr$$

$$B\, 2\pi r = -\mu_0 J\, 2\pi \int_0^c r\, dr$$

$$B \cancel{2\pi} r = \frac{-\mu_0 J \cancel{2\pi} c^2}{2}$$

$$B_2 = \frac{+\mu_0 J c^2}{2r}\,\hat{j}$$

$$B_2(P) = \frac{\mu_0 J c^2}{2d}\,\hat{j}$$

$$B(P) = B_1(P) + B_2(P)$$

$$\boxed{B(P) = \frac{\mu_0 J}{2}\left(\frac{c^2}{d} + \frac{(b^2-a^2)}{(s-d)}\right)\hat{j}}$$

~~Loop 1 +2~~

~~$2\varepsilon - \varepsilon$~~

1

$$2\varepsilon - i_3R - i_3R - i_1R = 0$$

$$2\varepsilon - 2i_3R - i_1R = 0$$

$$2\varepsilon = 2i_3R - i_1R$$

$$\cancel{2\varepsilon = 2\varepsilon - 4\varepsilon}$$

~~Loo~~ $\cancel{= -2\varepsilon - 4\varepsilon}$

$$2\varepsilon = -2\varepsilon + 4\varepsilon$$

$$i_1 = -4q/R$$

KVL

KCL

Loop 1

$$i_2 + i_3 = i_1$$

$$-2\varepsilon + \varepsilon - i_2R - i_1R = 0$$

$$-3\varepsilon \cancel{+ i_2R} = R(i_2 + i_1)$$

Loop 2

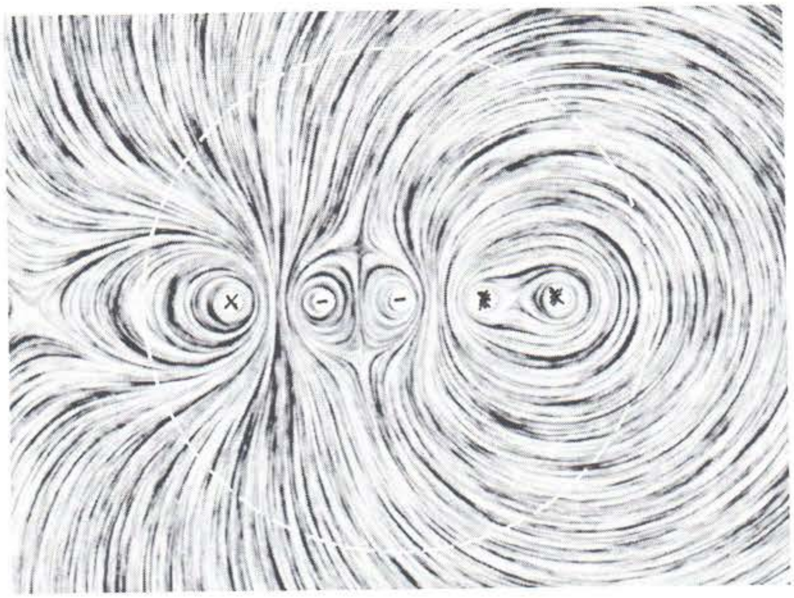
$$-\varepsilon - i_3R - i_3R + i_2R = 0$$

$$-\varepsilon = R(2i_3 - i_2)$$

$$-\varepsilon + Ri_2 = R2i_3$$

Loop 3 $-2\varepsilon - 2i_3R - i_1R = 0$

Part B (5 points):



Five lines of current each carry a current of magnitude I_o, directed either into or out of the plane of the paper. The leftmost line of current carries a current I_o into the ~~plain~~ plane of the paper. The iron filings plot above shows the magnetic field around these five line currents. If we calculate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ moving counterclockwise around the Amperean loop shown ~~about~~ above (dashed circle), what value do we find?

$\oint B \cdot ds = \mu_0 I$ enclosed

1) $3\mu_o I_o$.

2) $2\mu_o I_o$.

CCW = Positive direction

3) $\mu_o I_o$.

2+
3−

4) $-\mu_o I_o$.

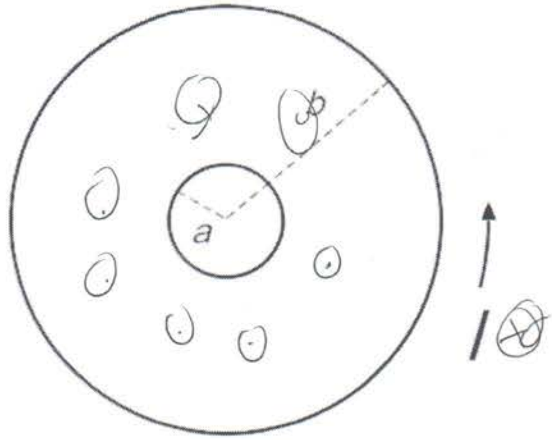
$-\mu_0 I_0$

5) $-2\mu_o I_o$.

6) $-3\mu_o I_o$.

Answer 4 ✗.

Part D (5 points)



Two concentric circular loops of wire are arranged as shown in the figure, with $a << b$. Initially neither loops carries current. The inner loop is not connected to a battery, and has resistance R. A power source (not shown) causes the current in the outer loop to start increasing in a counterclockwise direction at $t = 0$, such that

$$I = \begin{cases} 0, & t < 0 \\ I_o \dfrac{t}{T}, & t \geq 0 \end{cases}.$$

You may neglect the variation in the magnetic field of the larger loop at the center of the smaller loop. Which of the following statements is true just after $t = 0$?

~~1)~~ The current in the inner loop is clockwise and has magnitude $\dfrac{\pi a^2 \mu_o I_o}{2bTR}$.

~~2)~~ The current in the inner loop is clockwise and has magnitude $\dfrac{\pi b^2 \mu_o I_o}{2aTR}$.

3) The current in the inner loop is counterclockwise and has magnitude $\dfrac{\pi a^2 \mu_o I_o}{2bTR}$.

~~4)~~ The current in the inner loop is counterclockwise and has magnitude $\dfrac{\pi b^2 \mu_o I_o}{2aTR}$.

Answer 3 x.