8.01T

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Reading Assignments due before each class **

3 Exams + Final

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PDF Print before class

Math Review after p-set is due - Tues @ 9

Newton's Laws of Motion

1| Object in motion/rest remains in motion/rest unless acted upon by an outside form.

$\nearrow$ change in momentum

2| $F = ma = \frac{dP}{dt}$

$\downarrow$ Force causes momentum to change

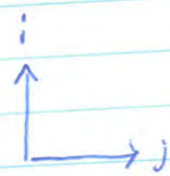
equal in magnitude and direction

Interaction Pair $\vec{F}_{2,1}$ = Force on ① due to interaction b/w ① and ②

$\hookrightarrow$ on different objects

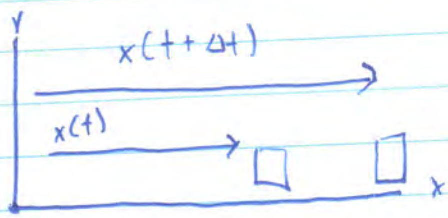
$\nearrow$ opposite in direction

3| $\vec{F}_{1,2} = -\vec{F}_{2,1}$



At rest - All forces = 0

$F_1 + F_2 + F_3 = 0$



$$r(t) = x(t)\,\hat{i}$$
$$r(t+\Delta t) = x(t+\Delta t)\,\hat{i}$$
$$\Delta r = r(t+\Delta t) - r(t)$$
$$= x(t+\Delta t) - x(t)\,\hat{i}$$
$$\vec{\Delta r} = \Delta x\,\hat{i}$$
$$\Delta x = x(t+\Delta t) - x(t) = \begin{cases} >0 \\ =0 \\ <0 \end{cases}$$

Displacement never bigger than distance traveled.

$V_{ave} = \frac{\text{displacement}}{\text{time}}$

4/6/13

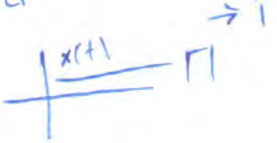
8.01
W1D3

Fernando Trujan

Derivative

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} \overset{not}{=} \frac{dr}{dt}$$

$$\vec{v}(t) = v_x(t)\,\hat{i}$$
$$\vec{r}(t) = x(t)\,\hat{i}$$



$$v_x(t) = \frac{dx}{dt}$$

Derivative - Slope of tangent lines

$$f = t^n$$
$$\frac{df}{dt} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 + t^n}{\Delta t}$$
$$= \lim_{\Delta t \to 0} n t^{n-1} \frac{\cancel{\Delta t}}{\cancel{\Delta t}} + \dots \Delta t$$
$$\frac{df}{dt} = n t^{n-1}$$

Mean Value Theorem  - same slope somewhere

Integral

└ area under curve

Fundamental Theorem

$$V_x(t_f) - V_x(t_i) = \int_{t_i}^{t_f} \frac{dV_x}{dt}\,dt = \int_{t_i}^{t_f} a_x(t)\,dt$$

$$\int t\,dt = \frac{t^{n+1}}{n+1} + C$$

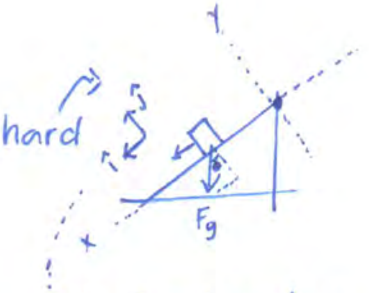
6/6/13

8.01
W02D1

Projectile Motion

Mathematical tools:

1) Coordinate System / can make problem easy or hard



Set of unit vectors
$|\hat{i}| = |\hat{j}| = 1$

$F^g = |\vec{F}| \sin\theta \, \hat{i} - |\vec{F}| \cos\theta \, \hat{j}$

2) Integration

$$V_x(t) - V_x(t=0) = \int_{t'=0}^{t'=t} a_x(t')\,dt'$$

Variable

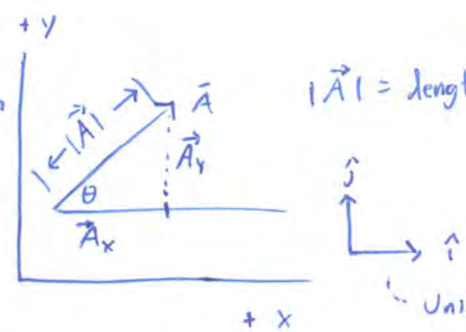
$$V_x(t) = V_x(t=0) + \int_0^t a_x(t')\,dt$$

same as +C

Integration of acceleration is the <u>change</u> of velocities

* 3) Vectors — Force, Velocity, Position, Acceleration

- Magnitude and direction



$|\vec{A}| = \text{length}$

Unit vectors same at every point

Vector Decomposition

$\vec{A} = |\vec{A}| = \vec{A}_x + \vec{A}_y$

$A_x = |\vec{A}| \cos\theta$

$A_y = |\vec{A}| \sin\theta$

$A_y = |A| \sin\theta$

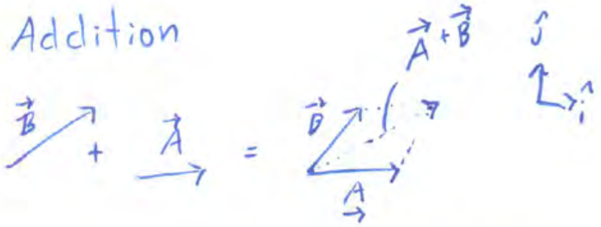
$\vec{F} = ma$

$F_x = ma_x$

$F_y = ma_y$

1/5

Vector Addition



Can also add individual components

$$\vec{A}+\vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$|\vec{A}| = +\left(A_x^2 + A_y^2\right)^{1/2}$$

$$\tan\theta = \frac{A_y}{A_x}$$

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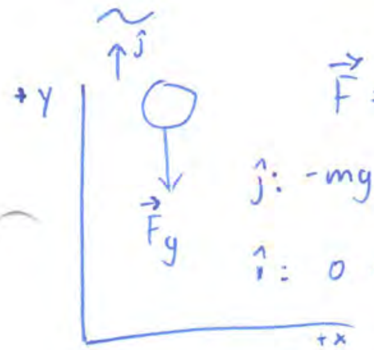

$$\vec{F} = m\vec{a}$$

$\hat{j}$: $-mg = ma_y$ $\quad a_y = -g$

$\hat{i}$: $0 = ma_x$ $\quad a_x = 0$

$$\vec{F}_g = -mg\hat{j}$$

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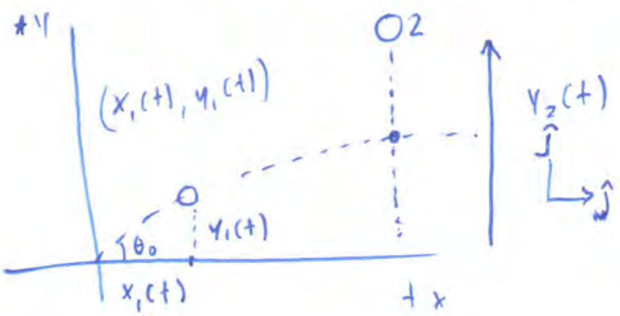
Monkey Drop

Q1: How many objects are moving?

Q2: How many steps of motion for each problem



$x_2 = d$

$a_{y_2} = a_{y_1} = -g$

Condition: $y_2(t_f) = y_1(t_f)$

$x_1(t_f) = d = x_2(t_f)$

Plan:

find θ_0

2/5

$$y_1(t_f) = V_{y_1}(0)\,t_f - \frac{1}{2} g t_f^2$$

$$x_1(t_f) = V_{x_1}(0)\,t_f$$

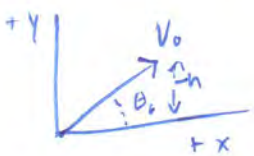
$$y_2(t_f) = y_2(0) - \frac{1}{2} g t_f^2$$

(a) = (c) $\Rightarrow$

$$V_{y_1}(0)\,t_f - \cancel{\frac{1}{2} g t_f^2} = y_2(0) - \cancel{\frac{1}{2} g t_f^2}$$

$$V_{x_1}(0)\,t_f = d$$

Initial Conditions

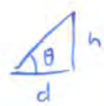


$$V_{y_1}(0) = V_0 \sin\theta_0$$

$$V_{x_1}(0) = V_0 \cos\theta_0$$

$$y_2(0) = h$$

$$\frac{V_{y_1}(0)\,\cancel{t_f}}{V_{x_1}(0)\,\cancel{t_f}} = \frac{y_2(0)}{d} = \frac{h}{d}$$



- Aim at the object

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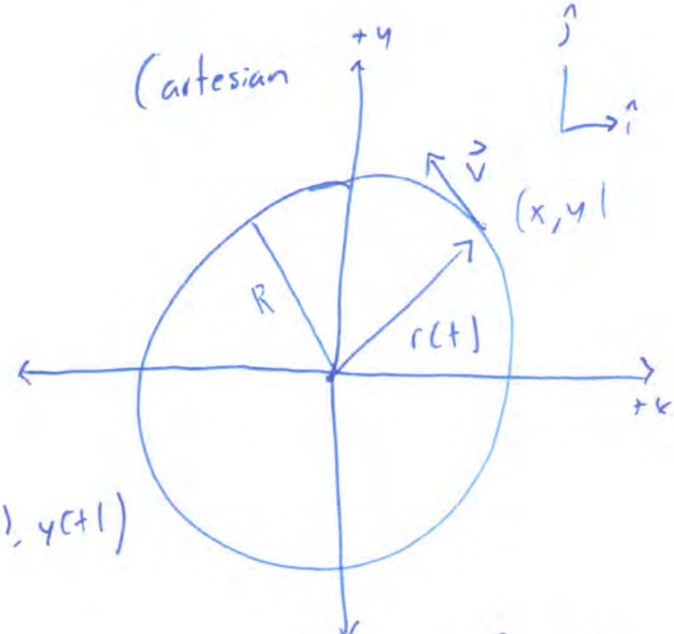
Rotational Motion

Polar Coordinates
4) TOOL

Coordinates (r, θ)
Unit vector $(\hat{r},$

Cartesian



$r = \sqrt{x^2 + y^2}$

$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\theta = \tan^{-1}(y/x)$

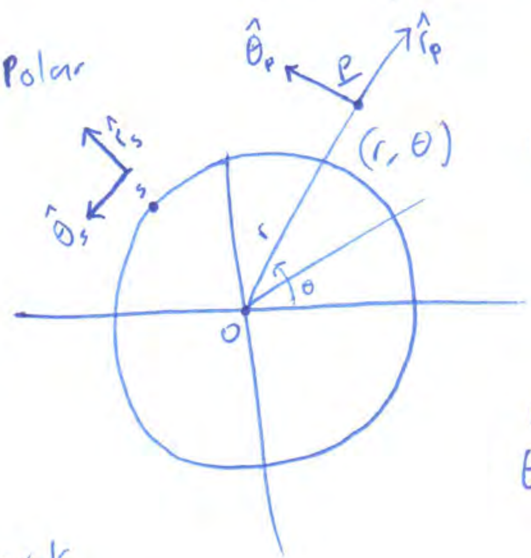
$\hat{r}$

$(x(t), y(t))$

$x(t)^2 + y(t)^2 = R^2$

Polar

Unit Vectors change



$0 \le \theta < 2\pi$
positive

counter-clock

$0 < r < \infty$

$\theta(t)$

$r =$ constant
$\theta =$ varies w/

Simple Summary

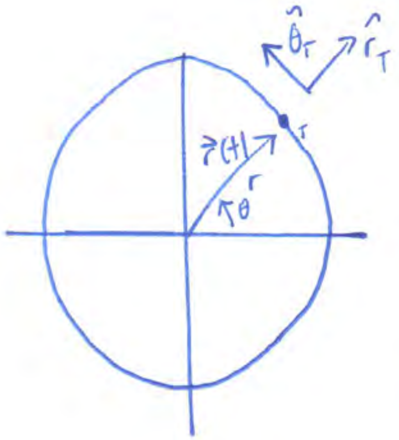
$\text{mag of velocity} = \frac{|ds|}{dt}$

$ds = r\,d\theta$
arc length

$v = r\omega$

$\text{mag of vel} = \frac{r\,d\theta}{dt} = r\left|\frac{d\theta}{dt}\right|$

omega $\omega = \left|\frac{d\theta}{dt}\right|$ Angular speed



$$\vec{r}(t) = r\,\hat{r}_p$$

$$\vec{v} = r\cancel{\alpha}\frac{d\theta}{dt}\,\hat{\theta}_p$$

Positive for ccw

$$\frac{d\theta}{dt} > 0$$

$$\frac{d\theta}{dt} < 0 \rightarrow \text{cc}$$

5/5

9/11/13

8.01 Lecture

Fernando Trujano

Newtons Laws

1) object in motion/rest remains in motion/rest until acted upon an external force
2) $F = ma$
2) Every action has an equal or opposite reactions

$\vec{F}$	$m\vec{a}$
experimental force laws catalogue	math
$\hat{i}$: $\hat{j}$: $\hat{k}$:	$F_x = ma_x$

$F_x = ma_x$ ← a_x not always a constant

$F_x = m\dfrac{d^2x}{dt^2}$

└ Find the equations of motion and solve for $x(t)$ and $v_x(t)$, $y(t)$, $v_y(t)$

Force Law: Universal Law of Gravity

3^{rd} law $F_{2,1} = F_{1,2}$

Interaction pairs must act on different objects

Force on 2 $\quad |F_{1,2}| = G\dfrac{m_1 m_2}{r_{12}^2}$

constants

$G = 6.7 \cdot 10^{-11} \dfrac{Nm^2}{Kg^2}$

Unit vectors - how we assign directions

① $\quad \overset{\longleftarrow}{F_{1,2}}$ ② $\longrightarrow \hat{r}_{1,2}$

$\vec{F}_{1,2} = -\dfrac{Gm_1m_2}{r^2}(\hat{r}_{1,2})$

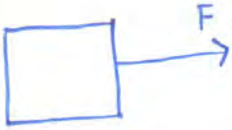
$F = \dfrac{Gm_E m}{R_E^2} = mg$

$g = \dfrac{GM_E}{R_E^2}$ ↱ Always positive

$= 9.81\ m/s^2$

Group 2
Table 10

Tension Force



F pulling with rope
↑
Tension

(1) massless
(2) inextensible, does not stretch

Free body force diagram

1) Object
2) Choice of Unit vectors
3) forces acting on the objects

Static

, one dimentional vector
$F = ma$

$\downarrow \hat{j}$: $mg - T = ma_y$

$a_y = 0$ static

$mg - T$

$T = mg$

motion

$\downarrow \hat{j}$ $\quad mg - T = ma_y$

$m(g - a_y) = T$

Tension decreases when cart released.

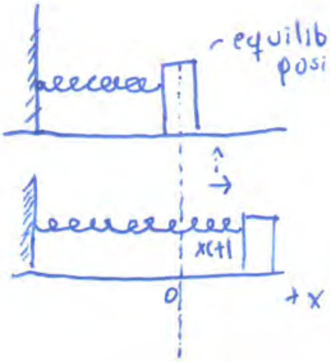
Hooke's Law

Mass attached to spring

Stretch or compress spring by Δx

$$|\vec{F}| = k|\Delta x|$$

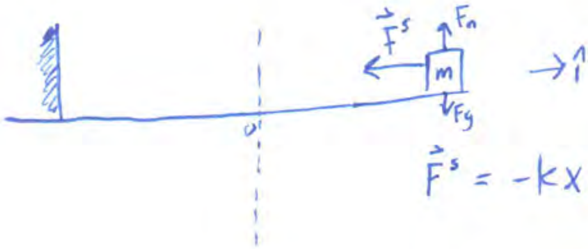
Restoring Force - always directed towards equilibrium



Choose origin at equilibrium point

$$\vec{r}(t) = x(t)\hat{i}$$

$$x(t) = \begin{cases} > 0 & \text{extended} \\ = & \text{equilibrium} \\ < 0 & \text{compressed} \end{cases}$$



$\vec{F}^s = -kx(t)\,\hat{\imath}$

$\vec{F} \quad m\vec{a}$

$\hat{\imath}: \quad F_x = ma_x$

$F_x = m\dfrac{d^2x}{dt^2}$

$-kx = m\dfrac{d^2x}{dt^2}$

Don't think of a as a constant but rather as second derivative of position

Find $x(t)$ such that

$\dfrac{d^2x}{dt^2} = \dfrac{-k}{m}x$

DON'T WORRY TOO MUCH.

$x(t) = A\cos(w_0 t) + B\sin(w_0 t)$

Constant

$\dfrac{dx}{dt} = -w_0 A\sin(w_0 t) + w_0 B\cos($

$\dfrac{d^2x}{dt} = -w_0^2 A\cos(w_0 t) - w_0^2 B\sin($

$= -w_0^2(A\cos(w_0 t) + B\sin(w_0 t$

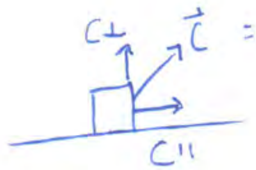
$\dfrac{d^2x}{dt^2} = -w_0^2 x$ Math

$\dfrac{d^2x}{dt^2} = \dfrac{-k}{m}x$ Newton

$w_0 = \sqrt{\dfrac{k}{m}}$

Contact Force

- Two components
  - Normal and f_s



$\vec{C} = C_\perp + C_\parallel$

$C_\perp \equiv \vec{N}$

$C_\parallel \equiv \vec{f}_s$

└ static friction

$0 \leq |\vec{f}_s| \leq |\vec{f}_{s,max}|$

direction depends on other forces

Force Law: $|\vec{f}_{s,max}| = \mu_s N$

dependent on mass

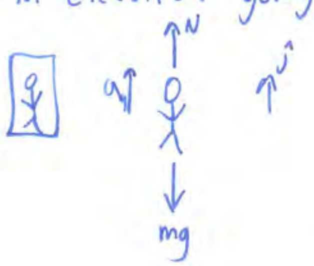
Kinetic Friction

└ once already moving

Force law $|\vec{F}_k| = \mu_k N$

direction opposes motion

Person in elevator going up



$F_{net} = ma$

$\hat{j}: \; N - mg = ma_y$

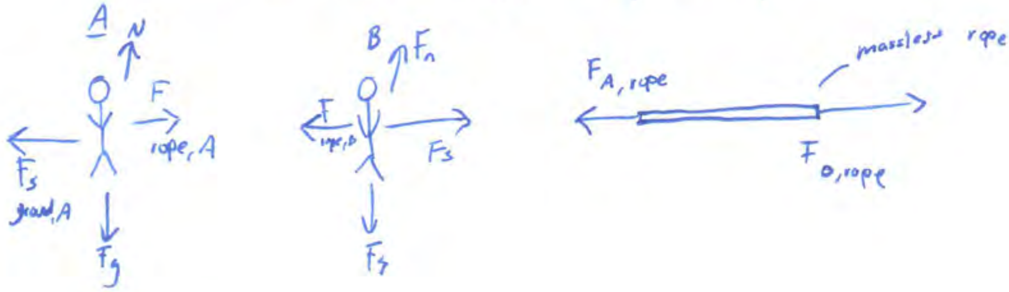
$N = mg + ma_y$

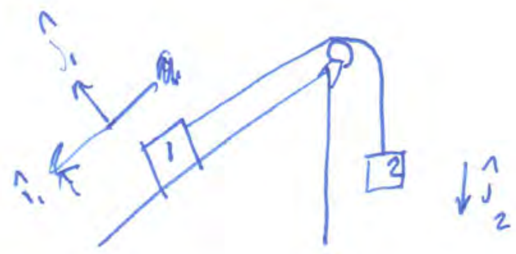
Normal forces depend on what is happening on the system. Not necessarily $N = mg$

Tug of War

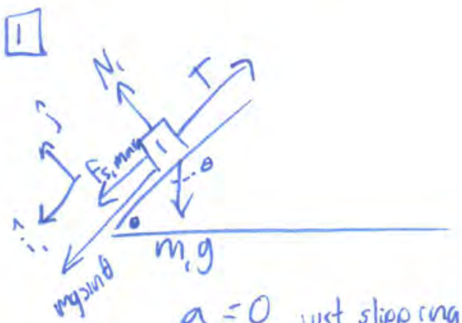
FBD

Treat each object separately





Can pick a separate coordinate system for each object



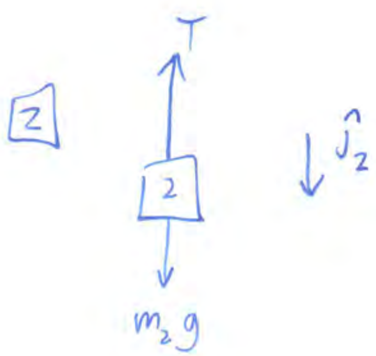
$a = 0$ just slipping

$$\vec{F} = m\vec{a}$$

$$\hat{i}:\ f_1^s\ max - T + m_1 g \sin\theta = 0$$

$$\hat{j}:\ N_1 - m_1 g \cos\theta = 0$$

$$F_{s1\,max} = \mu_s N_1$$



$$\vec{F}_2 = m_2 \vec{a}_2$$

$$\hat{j}: m_2 g - T \quad a_{y2} = 0$$

Substitute!

See slides for solution

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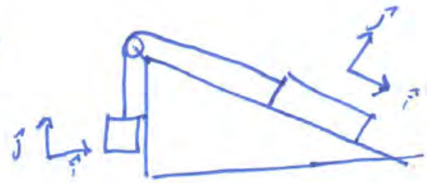
8.01 Exam Review

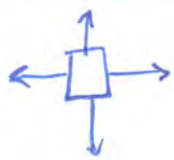
Fernando Trujano

Media Lab

1) Define Coordinates

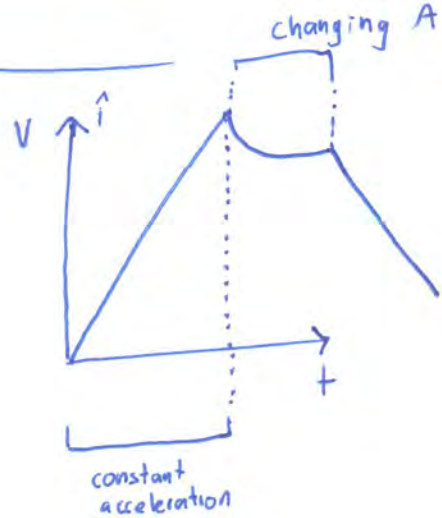


2) Draw Free Body Diagram for each object



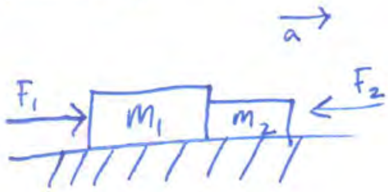
3) F = ma

4) Constraint



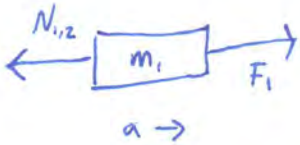
Concept Problems

Frictionless



$$a = \frac{F_1 - F_2}{m_1 + m_2}$$

$F_{net} = ma$



$|\vec{F}_1| > |\vec{F}_2|$

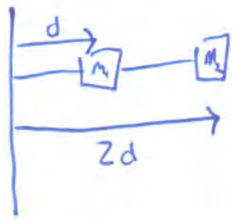
↑?

N

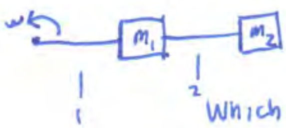
↑ bigger or smaller?

$N_{1,2} = -N_{2,1}$

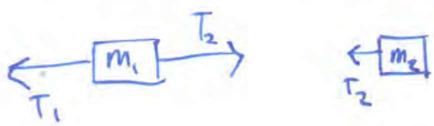
$|F_1| > |N| > F_2$



T max will break.



Which rope will break first.



$a = w^2 d$

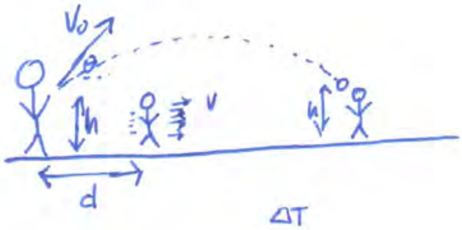
$m_1 a = T_1 - T_2$

$w^2 2d m_2 = T_2$

$w^2 d m_1 = T_1 - T_2$

$T_1 = T_2 + w^2 d m_1$

T_1 is bigger



Find V_0 and θ

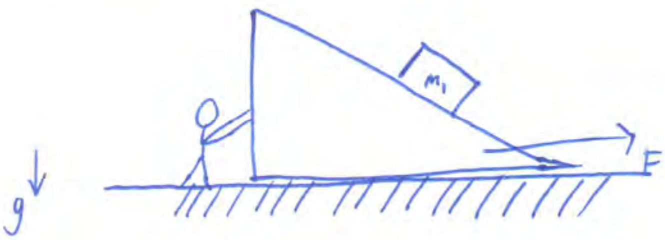
How far ball travels

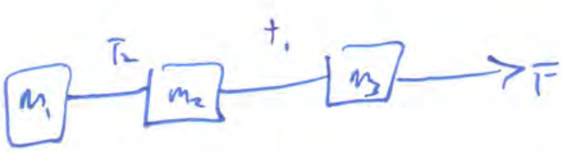
$$\Delta t \, v + d = V_0 \cos\theta$$

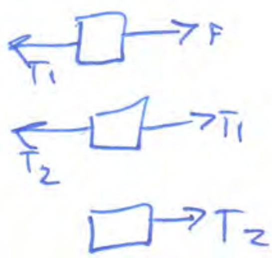
distance second guy traveled.

$$\cancel{V_0 \sin\theta \, \Delta t = \frac{1}{2} g \Delta t^2}$$

$$-V_0 \sin\theta = V_0 \sin\theta - g \Delta t$$







$$a = \frac{F}{m_1 + m_2 + m_3}$$

For all

8.01 Lecture

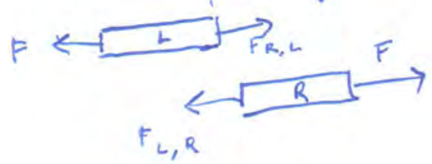
Fernando Trujano

Tension in a Rope

Action reaction pair

$T(x) = F_{left, right}(x) = F_{right, left}(x)$

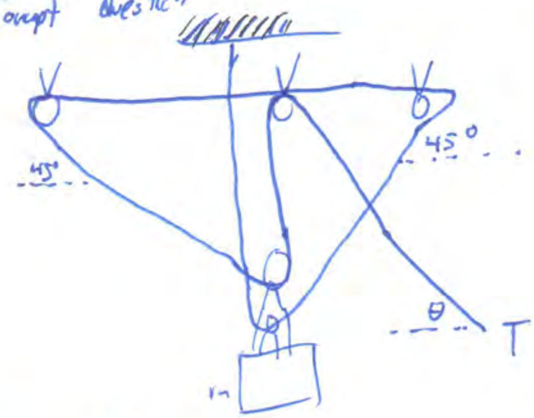
- imaginary cut at distance x, $T(x)$



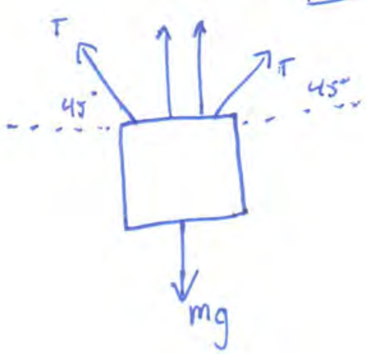
3rd Law $\rightarrow F_{LR} = -F_{RL}$ ... magnitude of the action reaction pair

$$T(x) = \left|\vec{F}_{L,R}(x)\right| = \left|\vec{F}_{R,L}(x)\right|$$

Concept Question



Find m



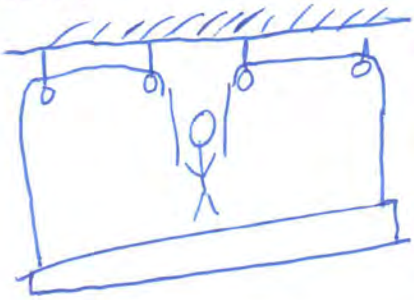
$\sin 45 = \frac{\sqrt{2}}{2}$

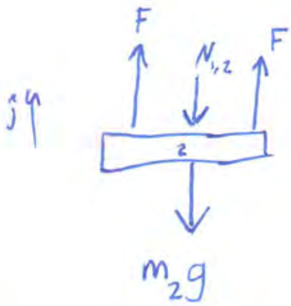
$$2T + 2T\sin 45^\circ - mg = 0$$

$$\frac{2T + 2T\frac{\sqrt{2}}{2}}{g} = m$$

$$m = \frac{2T\left(1 + \frac{\sqrt{2}}{2}\right)}{g}$$

Painter and Platform Problem





$$2F - N_{1,2} - m_2 g = m_2 a_2$$

$$a = a_1 = a_2$$

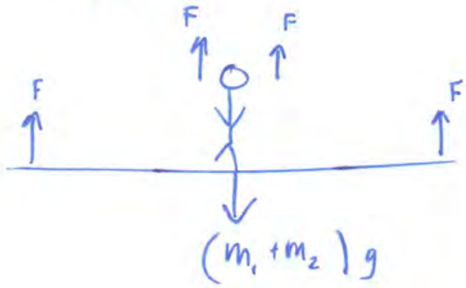
$$2F + N_{2,1} - m_1 g = m_1 a_1$$

$$2F + N_{2,1} - m_1 g = m_1 a$$

$$+ \quad 2F - N_{1,2} - m_2 g = m_2 a$$

$$4F - (m_1 + m_2) g = (m_1 + m_2) a$$

OR



$$4F - (m_1 + m_2) g = (m_1 + m_2) a$$

Tension in a rope

$\hat{j}$ $\hat{i}$

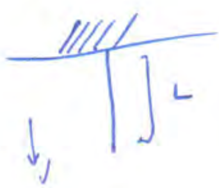
$$2T_e \cos\theta - mg = 0$$

$$T_m - T_{end}\sin\theta = 0$$

$$T_m = \frac{m}{2} g \tan\theta$$

$$j: \; T_{end}\cos\theta - \frac{m}{2}g = 0$$

Tension is suspended rope



Tension at top:

$$mg - T_{Top} = 0$$

Tension at distance y

$$m_u = \frac{m}{L} y$$

$$m_l = \frac{m}{L}(L-y)$$

$T_{bottom} = 0$

$$m_l g - T(y) = 0$$

$$T(y) = m_l g$$

$$T(y) = m\left(1 - \frac{y}{L}\right) g \checkmark$$

How tension varies as you go down.

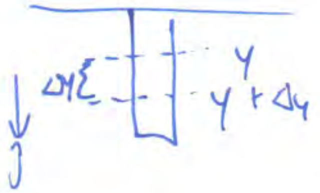
$$\frac{dT}{dy} = \frac{-mg}{L}$$

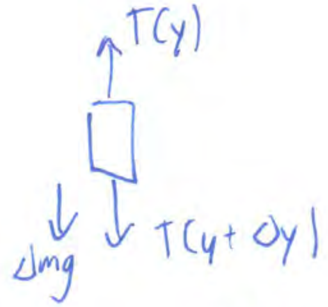
$$T(y) - T_{Top} + m_u g = 0$$

$$T(y) = T_{Top} - m_u g$$

$$T(y) = mg - \frac{m}{L}(y) g$$

$$= mg\left(1 - \frac{y}{L}\right) \checkmark$$





$$T(y+\Delta y) - T(y) + \Delta m g = 0$$

$$* \quad \Delta m = \frac{m}{L}\Delta y$$

$$T(y+\Delta y) - T(y) = -\Delta m g$$

$$T(y+\Delta y) - T(y) = \frac{-m}{L}\Delta y$$

$$\frac{T(y+\Delta y) - T(y)}{\Delta y} = \frac{-m}{L} g$$

$$\lim_{\Delta y \to 0} \frac{T(y+\Delta y) - T(y)}{\Delta y} \Rightarrow \frac{dT}{dy} = \frac{-m}{L} g$$

derivative

$$\frac{dT}{dy} = \frac{-m}{L} g$$

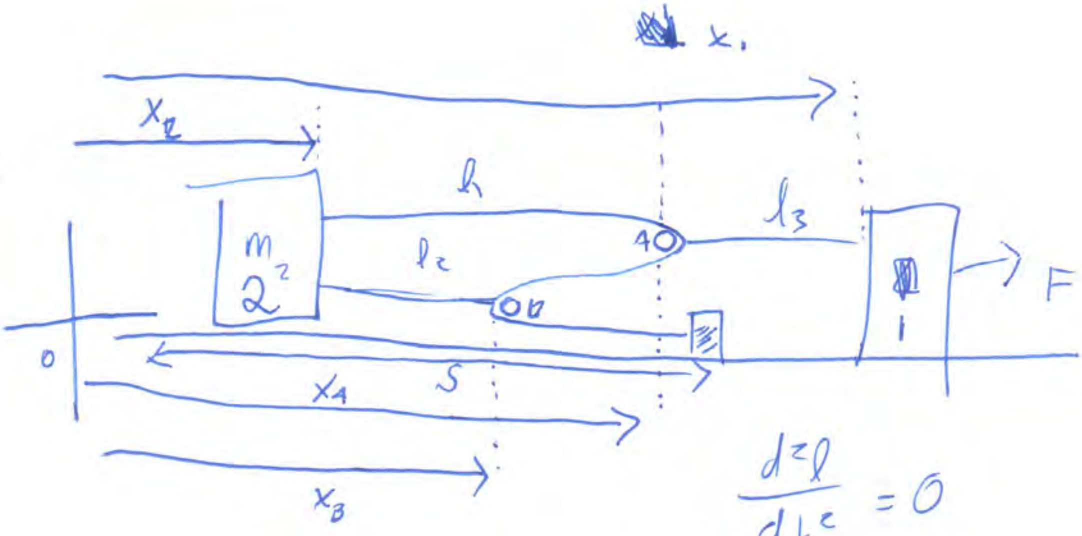
Separation of variables

$$\int_{T(0)=mg}^{T(y)} dT = \int_{y=0}^{y'=y} -\frac{m}{L} g \, dy'$$

match

$$T(y) - T(0) = -\frac{m}{L} g y \Rightarrow$$

$$T(y) = T_0 - \frac{m}{L} g y = mg - \frac{m}{L} g y$$



$$\frac{d^2 l}{dt^2} = 0$$

$l_1 = \cancel{x_B - x}\ (x_A - x_2) + (x_A - x_B) + (s - x_B)$

$l_2 = x_B - x_2$

$l_3 = x_1 - x_A$

$$0 = \frac{d^2 l_1}{dt^2} = 2a_A - a_2 - 2a_B$$

$$0 = \frac{d^2 l_2}{dt^2} = \frac{d^2 x_3}{dt^2} - \frac{d^2 x_2}{dt^2} = a_B - a_2$$

$a_B = a_2$

$$0 = \frac{d^2 l_3}{dt^2} = a_1 - a_A = 0$$

$a_1 = a_A$

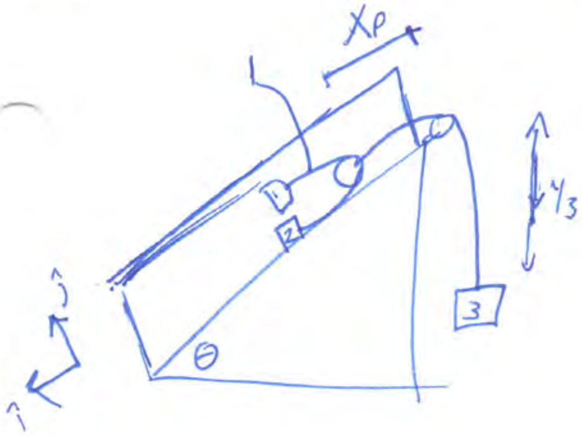
$$0 = 2a_1 - 3a_2$$

$$2a_1 = 3a_2$$

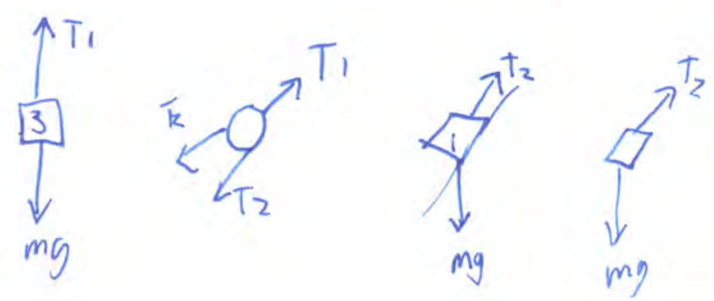
5/5

9/18/13

8.01 Lecture



1) Coordinate functions
2) Constraint b/w a's
3) Free Body Diagram for each movable
4) Apply $F = ma$ vectorially
5) Access



$$l_1 = (x_1 - x_p) + \pi R + (x_2 - x_p)$$

$$l_2 = x_p + c_1 + y_3$$

$$f(a_1, a_2, a_3)$$

$$\frac{d^2 l_1}{dt^2} = 0 = a_1 - a_p + 0 + a_2 - a_p$$

$$\Rightarrow 0 = a_1 + a_2 - 2a_p$$

$$l_2 = x_p + c_1 + y_3$$

$$\frac{d^2 l_2}{dt^2} = a_p + 0 + a_3 = 0$$

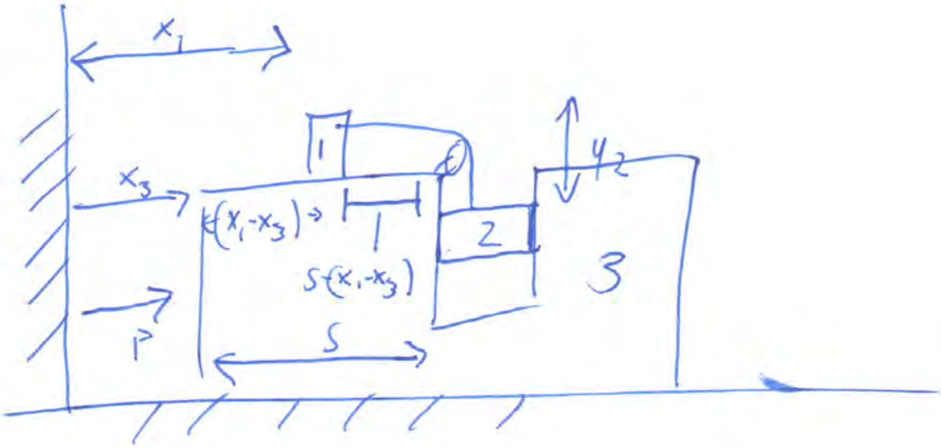
$$a_p = -a_3$$

$F = ma$

$\hat{i}$ $-T_1 + m_1 g \sin\theta = m_1 a_1$

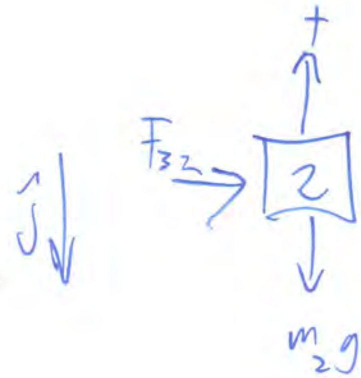
$\hat{j}$ $= N_1 - m_1 g \cos\theta = 0$

$2T_2 - T_1 = m_p a_1$
"0"



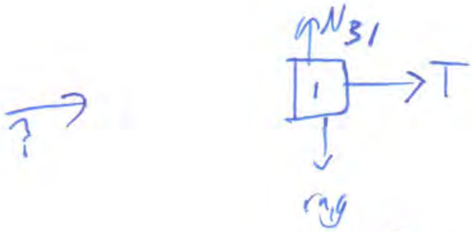
$$l = s-(x_1-x_3)+\frac{\pi}{2}R+y_2$$

$$\frac{d^2 l}{dt^2} = 0 - a_{x_1} + a_{x_3} + 0 + a_{y_2}$$

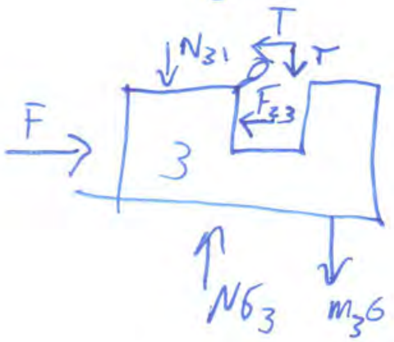


$$F_{32} = m_2 a_{2x} = m_2 a_{3x}$$

$$m_2 g - T = m_2 a_{2y}$$



$$T = m_1 a_{1x}$$



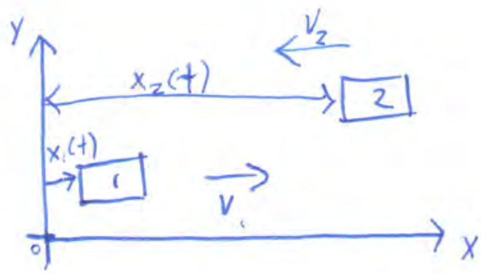
$\hat{\imath}$ $F - F_{32} - T = m_3 a_{3x}$

$$a_{x_2} = a_{x_3}$$

9/17/13

8.01 Exam 1 Review

Fernando Trujano



1

$V_0 = 0$

$a(t) = A - Bt$

$x(0) = 0$

$$v(t) - v(0) = \int_0^t a(t)\,dt$$

$$v(t) = A - \frac{Bt^2}{2}$$

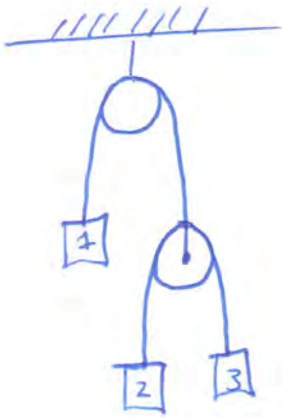
$$x_1(t_f) = x_2(t_f)$$

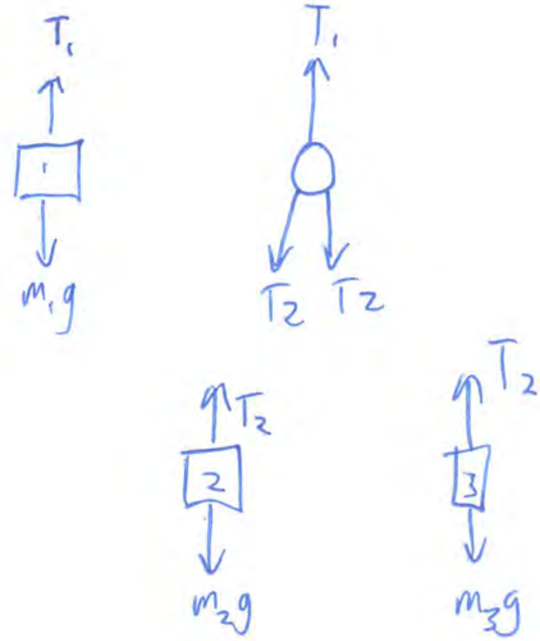
$$V_1(t_f) = 0$$

2

$V_{02} = -V_0$

$x_2(0) = S$





$$T_1 - m_1 g = m_1 a_1$$

$$T_1 - 2T_2 = m^0 a$$

$$T_1 = 2T_2$$

$$T_2 - m_2 g = m_2 a$$

$$T_2 - m_2 g = m_3 a$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.01 Physics **Fall Term 2013**

Exam 1: Equation Summary

One-Dimensional Kinematics:

$$\vec{\mathbf{v}} = \frac{d\,\vec{\mathbf{r}}}{dt}, \quad \vec{\mathbf{a}} = \frac{d\,\vec{\mathbf{v}}}{dt}$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')\,dt', \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')\,dt'$$

Initial conditions: x_0 and $v_{x,0}$ are the values at $t = 0$.

Newton's Second Law: Force, Mass, Acceleration: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$

Newton's Third Law: $\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$

Then I saw her face
now I'm a believer

5/13 8.01 Lecture

Fernando Trujano

Summary

1) Cylindrical Coordinates with unit vectors $(\hat{\phi}, \hat{\theta}, \hat{k})$

2) Position: $\vec{r}(t) = R\hat{r}(t)$

3) Angular Velocity: $\vec{w} = w_z \hat{k} = \frac{d\theta}{dt}\hat{k}$

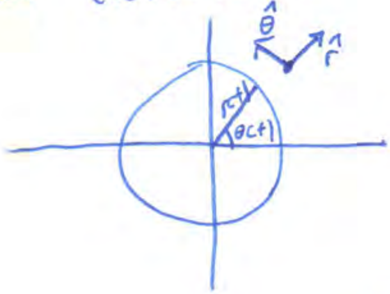
4) Velocity: $\vec{v}(t) = V_\theta \hat{\theta} = R\frac{d\theta}{dt}\hat{\theta}$

5) Acceleration: $\vec{a}(t) = a_r\hat{r} + a_\theta\hat{\theta} = -r\left(\frac{d\theta}{dt}\right)^2\hat{r} + r\frac{d^2\theta}{dt^2}\hat{\theta}$

Key Facts $\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\theta$ $\qquad \frac{d\hat{\theta}}{dt} = -\frac{d\theta}{dt}r$

6) Angular Acceleration: $\vec{\alpha} = \alpha_z\hat{k} = \frac{d^2\theta}{dt^2}\hat{k}$

Polar Coordinates



$r(t) = R$

└ For circular motion r constant

-- Direction changes with time
└ always outward

1:

$r(t) = R\,\hat{r}(t)$

component: constant

unit vector

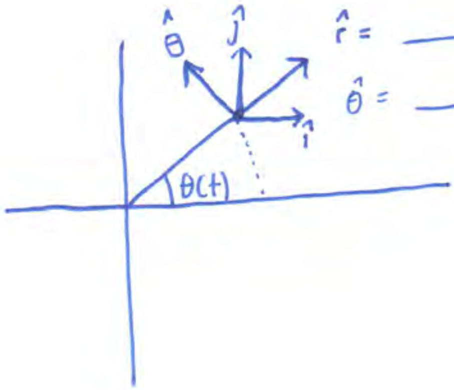
$\vec{v}(t) = \frac{d\vec{r}}{dt}$

$= R\frac{d\hat{r}(t)}{dt}$

constant

Proove $\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta}$

Vector Decomposition

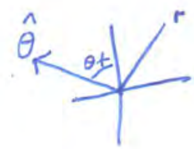


$\hat{r} = \_\_\_\ \hat{i} + \_\_\_\ \hat{j}$

$\hat{\theta} = \_\_\_\ \hat{i} + \_\_\_\ \hat{j}$

$\hat{r}(t) = \cos(\theta(t))\,\hat{i} + \sin(\theta(t))\,\hat{j}$

$\hat{\theta}(t) = -\sin(\theta(t))\,\hat{i} + \cos(\theta(t))\,\hat{j}$



Don't forget chain rule

$$\frac{df}{dt} = \frac{df}{d\theta}\frac{d\theta}{dt}$$

$$\hat{r}(t) = \cos(\theta(t))\,\hat{i} + \sin(\theta(t))\,\hat{j}$$

$$\frac{d\hat{r}}{dt} = -\sin(\theta(t))\frac{d\theta}{dt}\,\hat{i} + \cos(\theta(t))\frac{d\theta}{dt}\,\hat{j}$$

$$= \frac{d\theta}{dt}\underbrace{\left(-\sin(\theta(t))\,\hat{i} + \cos(\theta(t))\,\hat{j}\right)}_{\hat{\theta}}$$

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta}$$

$$\hat{\theta}(t) = -\sin(\theta(t))\,\hat{i} + \cos(\theta(t))\,\hat{j}$$

$$\frac{d\hat{\theta}}{dt} = -\cos(\theta(t))\frac{d\theta}{dt}\,\hat{i} + -\sin(\theta(t))\frac{d\theta}{dt}\,\hat{j}$$

$$\frac{d\theta}{dt}\underbrace{\left(-\cos(\theta(t))\,\hat{i} - \sin(\theta(t))\,\hat{j}\right)}_{-\hat{r}}$$

$$\frac{d\hat{\theta}}{dt} = \frac{-d\theta}{dt}(\hat{r})$$

2/7

$\vec{r}(t) = R\,\hat{r}(t)$

$\vec{v}(t) = \cancel{\frac{dR}{dt}\,\hat{r}(t)} + R\,\frac{d\hat{r}}{dt}(t)$ Chain Rule!

R = constant b/c circular motion

derivative of direction · Tangent to circle

$\vec{v}(t) = R\frac{d\hat{r}}{dt} = R\frac{d\theta}{dt}\,\hat{\theta} = V_\theta\,\hat{\theta}$

component

$V_\theta = \frac{Rd\theta}{dt}$

$V = \left|\frac{Rd\theta}{dt}\right|$

Angular Velocity

Normal to plane tells us orientation of plane

dir $\vec{\omega} \equiv \hat{n}_{RHR}$

Perpendicular to plane

$\vec{\omega}\,\odot$ out of board

$\hat{k}$ RHR w/ respect to ↑ θ(t)

$\vec{w} = w_z \hat{k}$ $\quad w_z = \frac{d\theta}{dt}$

$$\vec{w} = \frac{d\theta}{dt} \hat{k}$$

... independent of R

$w = \left| \frac{d\theta}{dt} \right|$ angular speed

Table Problem Angular Velocity

$$\theta(t) = At - Bt^3$$

a) Angular velocity vector

$w = \frac{d\theta}{dt} \hat{k}$

$\vec{w} = (A - 3Bt^2)\hat{k}$

b) Velocity vector

$v = R \frac{d\theta}{dt} \hat{\theta}$

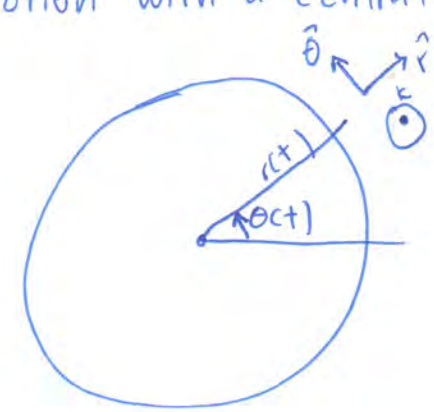
$v = R(A - 3Bt^2)\hat{\theta}$

c) when $w = 0$

$\vec{w} = (A - 3Bt_1^2)\hat{k}$

$t_1 = \sqrt{\frac{A}{3B}}$

Motion with a central point



Rotational motion in plane

$F = ma$

$\hat{r} = \quad F_r = ma_r$

$\hat{\theta} = \quad F_\theta = ma_\theta$

$\hat{k} = \quad F_z = ma_z$

4/7

$$\vec{v} = \underbrace{R\left(\frac{d\theta}{dt}\right)}_{\text{component}} \underbrace{\left(\hat{\theta}(t)\right)}_{\text{direction}}$$

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\underset{\text{constant}}{R}\frac{d\theta}{dt}\right)\hat{\theta} + R\frac{d\theta}{dt}\left(\frac{d\hat{\theta}}{dt}\right)$$

$$= R\frac{d^2\theta}{dt^2}\hat{\theta} + R\frac{d\theta}{dt}\left(-\frac{d\theta}{dt}\hat{r}\right)$$

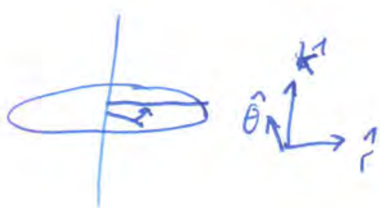
$$\vec{a} = \underbrace{a_r}_{\text{component in radial}}\hat{r} + \underbrace{a_\theta}_{\text{component in tangental}}\hat{\theta}$$

$$\vec{a} = -R\left(\frac{d\theta}{dt}\right)^2\hat{r} + R\frac{d^2\theta}{dt^2}\hat{\theta}$$

$$a_r = -R\left(\frac{d\theta}{dt}\right)^2 \qquad a_\theta = \frac{Rd^2\theta}{dt^2}$$

centripetal / inward

$$F_r = ma_r$$
$$= m\left(-R\left(\frac{d\theta}{dt}\right)^2\right)$$
$$F_\theta = ma_\theta$$
$$= m\left(R\frac{d^2\theta}{dt^2}\right.$$
$$F_z = m\frac{d^2z}{dt^2}$$



$$\vec{\alpha} = \frac{d^2\theta}{dt^2}\hat{k} = \alpha_z\hat{k}$$

$$\alpha = \left|\frac{d^2\theta}{dt^2}\right| \qquad \vec{a} = \frac{d^2x}{dt^2}\hat{i}$$

$$\vec{a} = a_x\hat{i}$$

circular object speeding up

two components to acceleration
a_θ a_r
centripetal

Concept Question: Cart in turn

Car in circular motion slows down
acceleration points

$$a_r = -r\left(\frac{d\theta}{dt}\right)^2$$

$$w = \left|\frac{d\theta}{dt}\right|$$

$$a_r = -rw^2 < 0$$

always negative b/c centripetal *

$$v = r\frac{d\theta}{dt}\hat{\theta}$$

$$v_\theta = r\frac{d\theta}{dt} \quad \begin{matrix} >0 \\ =0 \\ <0 \end{matrix}$$

speed: $v = |v_\theta| = rw$

$$\frac{v}{r} = |\ | = w$$

RQ

$$a_r = -rw^2 = -r\left(\frac{v}{r}\right)^2 = \frac{-v^2}{r}$$

$$\vec{F} = m\vec{a}$$

$$\frac{-mv^2}{R} = ma_r$$

$$2\pi R = vT = T = \frac{2\pi R}{v} = \frac{2\pi r}{rw} = \frac{2\pi}{w}$$

$$f = \frac{1}{T} = \frac{w}{2\pi} \qquad w = 2\pi f \ \frac{\text{radians}}{\text{second}}$$

$$a_r = -r\omega^2 = \frac{-r\,4\pi^2}{T^2} = \frac{-v^2}{r} = -r\,4\pi f^2$$

Concept Question

car is turning $R = 200m$

$a_c = a_r = 2 m/s^2$

what is the speed?

$$a = \frac{v^2}{r}$$

$$\sqrt{ar} = v$$

$$v = 20\ m/s$$

Table Problem

$$\theta(t) = At^3 - Bt$$

9/25/13

8.01 Lecture

Fernando Trujan

$\hat{r}$ $\quad F_r = mr\left(\frac{d\theta}{dt}\right)^2$

$\hat{\theta}$

Concept Question

Circular Motion and force

$\hat{r}$

$F = ma$

$mg - T = -\frac{mv^2}{r}$

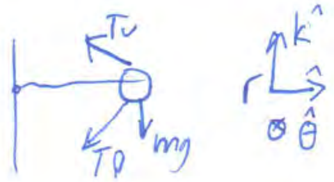
$mg + \frac{mv^2}{r} = T$

Strategy

- Always have a component of a inward
- May or may not have tangental component of a
- Draw Free Body Diagrams
- $\frac{mv^2}{r}$ is not on ~~forc~~ FBD
- Choose a sign convention for radial and tangental directions.

CQ

Tension and String Theory



F

9/29/13

8.01 Notes

Fernando Trujano

Momentum and Impulse

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = m\Delta\vec{v}$

change in momentum

$\vec{F}_{ave}\Delta t = \Delta\vec{p}$

Impulse

$\vec{I} = \vec{F}_{ave}\Delta t = \Delta\vec{p}$

Internal forces cancel

Center of Mass

Position of bodies

$$R_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

9/30/13 8.01 Lecture Fernando Trujano

Momentum Conservation

$\vec{p} = m\vec{v}$

Reference frame dependent

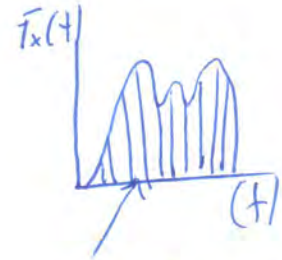
$F = \frac{dP}{dt}$ ... for a single object

$\vec{F}(t) = \lim_{\Delta T \to 0} \frac{\vec{P}(t+\Delta t) - \vec{P}(t)}{\Delta T}$

$\Delta\vec{P} = \int_{t'=0}^{t'=t} \vec{F}(t)' dt'$

$(\vec{P}(t) - \vec{P}(0))$

Change in momentum $\rightarrow \vec{I}$mpulse



$\vec{F}_{ave}$ over an inte~~gral~~rval ΔT

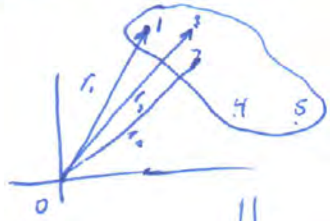
$\vec{I} = \vec{F}_{ave} \Delta t$

$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$

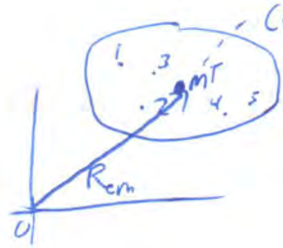
$\Delta P = m\Delta V$

Change in momentum $I = Ft$

Center of Mass



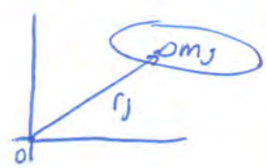
$\Downarrow$



$$\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 \ldots}{m_1 + m_2 + \ldots}$$ vector operation

$$= \frac{\sum_{j=1}^{N} m_j \vec{r}_j}{\sum_{j=1}^{N} m_j} = \sum_{j=1}^{N} \frac{m_j \vec{r}_j}{M^T}$$

continious system



$$\vec{R}_{cm} = \frac{\sum_j (\Delta m_j)(\vec{r}_j)}{\sum_j \Delta m_j} = \frac{\int_{object} dm\, \vec{r}\,'}{\int_{object} dm}$$

limit $\Delta m_j \to dm$

$\vec{r}_j = \vec{r}\,'$

$\sum_j = \int_{object}$

Analize motion of point at center of mass of system

$$\vec{V}_{cm} = \frac{dR_{cm}}{dt} = \frac{(m_1 v_1 + m_2 v_2 + m_3 v_3 + \ldots)}{m_T}$$

$$\vec{V}_{cm} = \frac{\vec{P}_{sys}}{m_T} \Rightarrow \vec{P}_{sys} = m_T V_{cm}$$

only need to think about external forces.
Because internal ones cancel out

$$\vec{F}^{ext}_{sys} = \sum_i F_i^{ext} = \sum_{i=1}^{N} F_i^{total}$$

$$= \sum_{i=1}^{N} \frac{d\vec{P}_i}{dt}$$

External Force => sum of changes in momentum

$$F_{ext} = \frac{dP_1}{dt} + \frac{dP_2}{dt} + \ldots = \frac{d}{dt}(P_1 + P_2 + P_3 \ldots)$$

$$\vec{F}_{ext} = \frac{dP_{sys}}{dt} = \frac{d}{dt}(m_T \bar{V}_{cm})$$

if M_T constant

$$F_{ext} = m_T A_{cm}$$

Center of Mass ~~u~~ undergoes projectile motion

$$R_{cm}(t_f) = 2X_0\,\hat{i}$$

$$\vec{R}_{cm}(t_f) = \frac{m_2\cancel{x_2(t_f)}\,\hat{i} + m_3 x_3(t_f)\,\hat{i}}{m_2 + m_3}$$

$$2x_0 = \frac{\frac{3m}{4}x_3(t_f)}{m}$$

$$\frac{8X_0}{3} = X_3(t_f)$$

Center of mass of system travels in parabolic trajectory

See solutions.

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internal forces cancel in pairs

$$\vec{F}^{ext} \Delta t_{int} = \vec{P}_{after} - \vec{P}_{before}$$

Forces ⇒ description of
physics motion
modeling momentum diagrams

$$\vec{F}^{ext} \Delta t_{int} = \vec{P}_{after} - \vec{P}_{before}$$

(1) $F_x^{ext} = 0$ vector eq: 3 eq

$$\vec{P}_{after} = \vec{P}_{before}$$

①

② (i) state system before interaction
iii) " after interaction $t_{after} \cdot \vec{p}^{sys}_{after}$

Model Interaction

$$\Delta t_{int} \approx 0$$

Laka instantaneous.

$$0 = \vec{p}^{sys}_{after} - \vec{p}^{sys}_{before}$$

Momentum Diagrams

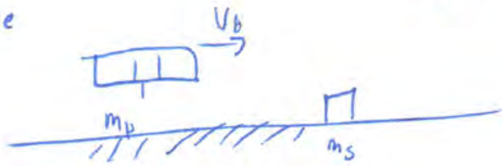
1) Identify all objects in system
2) Choose a reference frame
3) Draw Momentum Diagram for
   i) system before
   ii) system after

Show:
Unit Vectors
Velocity of each mass elem

Plane + Sandbag

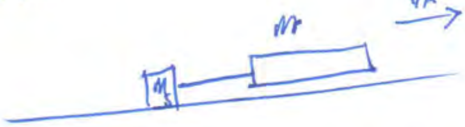
$\hat{\imath} \rightarrow$

t_{before}



$$P_{before} = m_p v_b \hat{\imath}$$

t after



$$P_{after} \; (m_s + m_p) v_f = \hat{\imath}$$

$$\vec{F}^{ext} \Delta t_{int} = \vec{P}_{after} - \vec{P}_{before}$$

$$\Delta t_{int} \approx 0$$

$$\hat{\imath}: \quad 0 = (m_s + m_p) V_f - m_p V$$

$$V_f = \frac{m_p v_b}{m_s + m_p}$$

If interaction time $\neq 0$

$$\mu_k m_s g \Delta t = (m_s + m_p)(v_f) - m_p$$

$$F_{ext} = \frac{dP_{syst}}{dt}$$

$$\bar{F}^{ext} \Delta t_{int} = P_{after} - P_{before}$$

Recoil

See Table Problem $\quad v_f = \frac{m_p v}{m_c + m_p}$

Continious Mass Transfer

$$\vec{F}_{ext}(t)\Delta t = \underbrace{\vec{P}(t+\Delta t)}_{\text{after}} - \underbrace{\vec{P}(t)}_{\text{before}}$$

Continious Recoil

speed of fuel ejected relative to rocket

Rocket Equation

Propulsion = series of recoils

10/7/13

8.01 Lecture

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Kinetic Energy and Work

$K = \frac{1}{2} m v^2 \qquad kg\, m^2/s^2 = J$

- Scalar quantity

$p = mv$

$K = \frac{1}{2} m \left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$

Concept Question

Two cars, masses m and 2m. Push both with force F. Frictionless surface.
KE of heavy car is ______ than smaller.

$I = \Delta \vec{p}$

$\hat{i}: F\Delta t = \Delta p$

same for each object

$p_{f_1} = p_{f_2}$

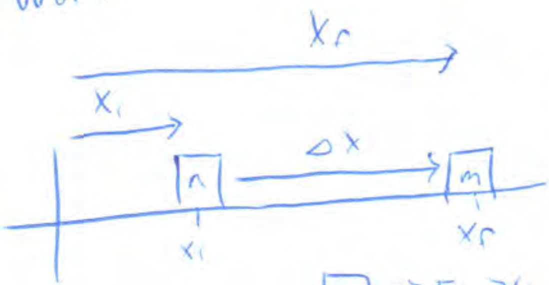
$K_{f_1} = \frac{p_{f_1}^2}{2m_1}$

$K_{f_2} = \frac{p_{f_2}}{2m_2}$

$K_{f_1} > K_{f_2}$

A) Smaller

Work



$\hat{i}$

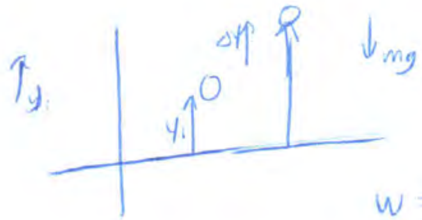
$F_x > 0$

constant, uniform

$Work \equiv F_x \Delta x$

Point of action of the force

Positive or negative



$F_y = -mg$

$W = f_y \Delta y$

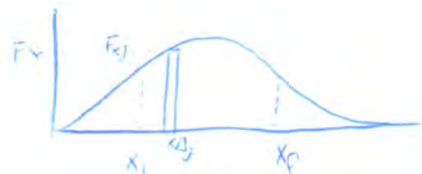
$W = (-mg)(y_f - y_i)$

Force opposes motion

			Work
a)	$y_f > y_i$	Rising	Negative
b)	$y_f = y_i$	back to start	~~Positive~~ zero
c)	$y_f < y_i$	falling	Positive

Force in direction of motion

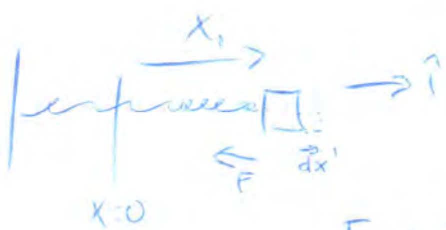
If F_x is non uniform



$\Delta W_j = F_{x_j} \Delta x_j$

$$W = \lim_{\substack{n \to \infty \\ \Delta x_j \to 0}} \sum F_{x_j} \Delta x_j$$

$$W = \int_{x_i}^{x_f} F_x \, dx$$



$F_x = -kx'$

$dW = F_x dx'$
$= -kx'dx'$ intermediate work from □ → □

$$w(x) = \int_{x'=0}^{x'=x} -kx'dx' = -\frac{1}{2}kx^2$$

$x > 0$ stretch
$w < 0$
$x < 0$ compr
$w < 0$

2/7

Review

$a_x(t)$

$$\int_{t'=t_i}^{t'=t_f} a_x(t')\,dt' = \int_{t_i}^{t_f} \frac{dv_x}{\cancel{dt'}}(t')\,\cancel{dt'}$$

$$\int_{i}^{t_f} dv_x = v_x(t_f) - v_x(t_i)$$

Function of time

For work you integrate with respect to space

$$\int_{x_i}^{x_f} (dv_x') \frac{dx'}{dt} = \int_i^f dv_x' \, v_x'$$

$$= \frac{1}{2}\left(v_{x_f}^2 - v_{x_i}^2\right)$$

$$\int_i^f a_x\,dx = \frac{1}{2}\left(v_{x_f}^2 - v_{x_i}^2\right)$$

$$\underbrace{m\int_i^f a_x\,dx}_{F=ma} = \frac{1}{2}m\left(v_{x_f}^2 - v_{x_i}^2\right) = K_f - K_i$$

$$= \Delta KE$$

Newton's second Law

$$\underbrace{\int_i^f F_x\,dx}_{} = \Delta K$$

$$W = \Delta K$$

Work $\equiv$ the change in Kinetic Energy

Work Kinetic Energy Principle

Concept Question

Two objects are pushed on a frictionless surface from a starting line to a finish line with equal constant force. One object is 4 times as massive as the other. Both are initially at rest.

$$W = \Delta K$$

$\int F_x \, dx$ | ΔK

uniform $\rightarrow F_x \Delta x$

Both objects $\rightarrow = \Delta K$

$= K_f - K_i$

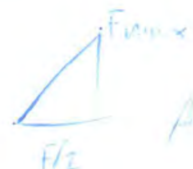
Both started at rest

$K_i = 0$

Same

Concept Question

A particle starts from rest at $x = 0$ and moves to $x = L$ under $F(x)$. What is the KE at $x = L/2$ and $x = L$



Most

$Area = \frac{1}{2} F_{max} \frac{L}{2} = \frac{F_{max} L}{4}$

Area $F_{max} \frac{L}{2}$

8.01 Lecture

Dot Product aka Scalar Product

$$0 \le \theta \le \frac{\pi}{2}$$

$$\cos\theta > 0$$

$$A \cdot B = \underbrace{|A||B|\cos\theta}_{\text{Scalar}}$$

$$\theta = \frac{\pi}{2}$$

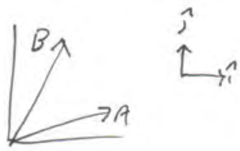
$$\cos\theta = 0$$

$$\frac{\pi}{2} \le \theta < \frac{3\pi}{2}$$

$$\cos\theta < 0$$

How much of B ~~projects~~ (B is) in the direction of A

OR vise versa



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j}$$

$$\vec{A} \cdot (c\vec{B}) = c(\vec{A} \cdot \vec{B})$$

#

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$(A_x \hat{i} + A_y \hat{j}) \cdot (B_x \hat{i} + B_y \hat{j})$$

distribute

$$A_x \hat{i} \cdot B_x \hat{i} + A_x \hat{i} \cdot B_y \hat{j} + A_y \hat{j} \cdot B_x \hat{i} + A_y \hat{j} \cdot B_y \hat{j}$$

$$\vec{A} \cdot \vec{B} = A_x B_x \underbrace{(\hat{i} \cdot \hat{i})} + A_x B_y \underbrace{(\hat{i} \cdot \hat{j})}_{0} + A_y B_x \underbrace{(\hat{j} \cdot \hat{i})}_{0} + A_y B_y \underbrace{(\hat{j} \cdot \hat{j})}_{1}$$

$$\hat{i} \cdot \hat{i} = |\hat{i}||\hat{i}|\cos\theta = 1$$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y}$$

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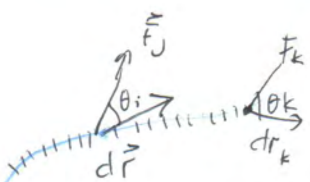
Power

Rate of doing work

$$P = \frac{\Delta W}{\Delta t} = \frac{F \Delta x}{\Delta t} = F v_x$$

1 watt = 1 J/s

$$W = F_{\parallel} \Delta r$$



$$= \int \vec{F} \cdot d\vec{r}$$

line integral

$$\Delta W_j = \vec{F}_j \cdot d\vec{r}_j$$

$$W = \lim_{\substack{n \to 0 \\ d\vec{r} \to 0}} \sum_{1}^{N} \vec{F}_j \cdot d\vec{r}_j$$

6/7

$m = m$

$$\vec{F} = F_x \hat{\imath} + F_y \hat{\jmath}$$

$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$

$$\vec{F} \cdot d\vec{r} = F_x dx + F_y dy$$

$$W = \int F_x\,dx + \int F_y\,dy$$

$$= \int m a_x dx + m a_y dy$$

$$= \frac{1}{2}m(V_{x_f}^2 - V_{x_i}^2) + \frac{1}{2}m(V_{y_f}^2 - V_{y_i}^2)$$

$$W = \frac{1}{2}m(V_{x_f}^2 + V_{y_f}^2) - \frac{1}{2}m(V_{x_i}^2 + V_{y_i}^2)$$

$$W = \Delta K$$

Work = Change in Kinetic Energy

Friction is not a conservative Force

↗ └ depends on path

non-conservative

└ path dependent

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Work Kinetic Energy Principle

$$W_{A\to B} = K(B) - K(A)$$

$$\vec{F}\cdot\hat{\Delta} s = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

uniform force

$$\begin{cases} F\ |\Delta\vec{s}| \\ |F|\ \Delta s_n \end{cases}$$

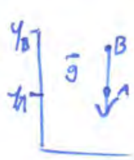
$$\int_a^b \vec{F}\cdot ds$$

Calculate on force diagram.

Potential Energy

$$W_{A\to B} = \int_A^B \vec{F}\cdot ds \quad \leftarrow \text{hard to do}$$

path

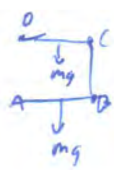
Example 

$\hat{j}$

$$W_{A\to B} = \vec{F}\cdot\Delta s$$

$$(-mg)(y_B - y_A)$$

$$W_{A\to B} = -mg(y_B - y_A)$$



gravity

Perpendicular force = 0 work

$$\int_{③} \vec{F}\, d\vec{s}$$

$$d\vec{s} = dx\,\hat{i} + dy\,\hat{j}$$

$$F = -mg\,\hat{j}$$

$$d\vec{s} = dx\hat{i} + dy\,\hat{j}$$

$$F\cdot d\vec{s} = (-mg\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$

$$= -mg\,dx\,\hat{j}\cdot\hat{i} - mg\,dy\,\hat{j}\cdot\hat{j}$$

← only displacement in direction of force matters

$$F\cdot ds = -mg\,dy$$

$$\int \vec{F} \cdot d\vec{s} = \int_{y'=y_A}^{y'=y_B} -mg\,dy'$$

$$W_{A \to B} = -mg(y_B - y_A)$$

Potential Energy Difference

work independent of path
F is conservative $\vec{F}^c$

Friction → non conservative force
depends on the path taken

-- Defined only in conservative force

$\vec{F}^c$ acting on object going from A to B

$$U(B) - U(A) \stackrel{def}{=} -W^c_{A \to B}$$

Example

$$\vec{F}^c = -mg\,\hat{j}$$

$$U(y_B) - U(y_A) = +mg(y_B - y_A)$$

Potential Energy Difference b/w floor and height $h = mgh$

reference pt P

$$U(P) = 0$$

$$y_B = y \leftarrow \text{variable}$$

$$y_A = y_P \leftarrow \text{reference pt}$$

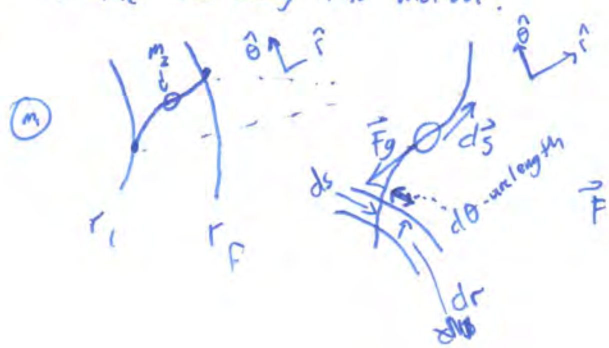
Gravity $y_P = 0$ $U(0) = 0$

$$U(y) - U(0) = mg(y - 0)$$

$U(y) = mgy$ with respect $U(0) = 0$

Change of Potential Energy for Inverse Square Gravitational Force

- Consider an object of mass m_1 moving towards m_2 (sun). Initially the object is r_0 from center of sun. The object moves to r_f from center. What is the PE during this motion.



$$\vec{F}^G = \frac{-Gm_1m_2\hat{r}}{r^2}$$

$$d\vec{s} = dr\,\hat{r} + \underbrace{r\,d}_{\text{arclength}}\hat{\theta}$$

$$\vec{F}^G \cdot ds$$

$$\left(\frac{-Gm_1m_2\hat{r}}{r^2}\right) \cdot dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

How much of one vector is in the direction of the other.

$\hat{r}\cdot\hat{r} = 1$

$\hat{r}\cdot\hat{\theta} = 0$

Perpendicular

PE = -Work $\quad F_1 ds = \dfrac{-Gm_1m_2}{r^2}dr$

Reminder

$\int \frac{dr}{r^2} = \frac{-1}{r}$

$$W^G_{1\to f} = -\int_{r_1}^{r_f} \frac{Gm_1m_2}{r^2}dr = Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_1}\right)$$

$\ominus\ominus\ominus$

$$U(r_f) - U(r_i) = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Concept Question:

A comet is speeding along hyperbolic orbit towards the Sun.
While the comet is moving away from the Sun, the change in PE of the Sun-comet system.

(1) positive
(2) zero
(3) negative

moving away → more PE → positive

OR

Displacement away from earth
Force toward earth. > Work is negative

$PE = -W$

PE is positive

Table Problem: Change in PE → spring Force

$$W_{AB} = -\int_{x'=x_A}^{x'=x_B} k x' dx = -\frac{1}{2} k (x_B^2 - x_A)^2$$

$$U(x_B) - U(x_A) = \frac{1}{2} k (x_B^2 - x_A^2)$$

First case: $F_{nc} = 0$

only forces are conservative.

$W^c_{AB} = \Delta K$

$-\Delta U^{total} = \Delta K$

$0 = \Delta U^{total} + \Delta K$ if $\boxed{F_{nc}=0}$

$E^{mech} = U + K$

choosing $U=0$ for each conservative force

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$$\Delta E^{mech} = \Delta U + \Delta K$$

$$\Delta E^{mech} = 0$$

Energy Diagram

1) System
2) Choose $U = 0$ for each PE
3) Draw initial state diagram

$$E_i^{mech} = K_i + U_i^{total}$$

4) Draw a final state diagram

$$E_f^{mech} = K_f + U_f^{total}$$

$F_{nc} = 0$	Energy Diagrams
0	$\frac{\text{energy}}{\text{principle}}$

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Table Problem: Loop - the - Loop

$$W_{A \to B} = \Delta K$$

$$W_{nc} + W_c = \Delta K$$

$$W_{nc} - \Delta U^{total} = \Delta K$$

$$W_{nc} = \Delta K + \Delta U^{total}$$

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Today's Problems

1) Excellent Practice for P-set

2) Combination of ideas

$\vec{F} = m\vec{a} \iff F_r = -\frac{mv^2}{r}$

$W_{nc} = E_f^{mech} - E^{mech}$

$\vec{F}^{ext} \Delta t = \vec{p}_f^{sys} - \vec{p}_i^{sys}$

3) Techniques

(1) Free body force diagrams
(2) Momentum diagrams (Two): Initial + Final states
(3) Energy Diagram (Choose $U=0$). Initial + Final stage

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10/16/18

8.01 Lecture

Eq

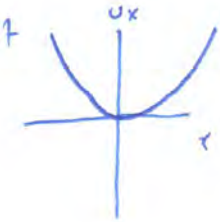
$x=0$

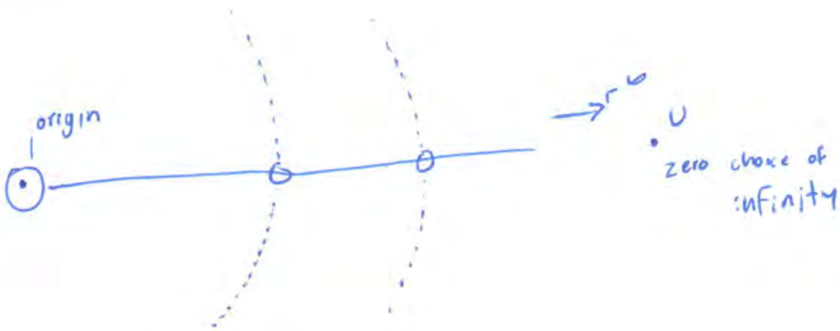
$U(0)=0$

$$U(x)-U(0)=\frac{1}{2}kx^2$$

$$F_x(x)=-kx$$

Plot



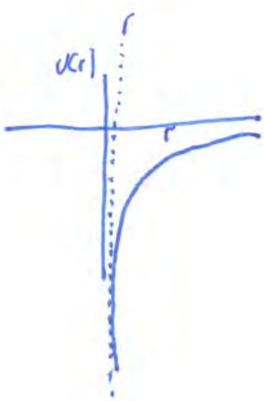


$$U(r)-U(\infty)=\frac{-Gm_1m_2}{r}$$

U = - work done by conservative force

$$F_r(r)=-\frac{dU}{dr}=-\frac{-Gm_1m_2}{-r^2}=\frac{-Gm_1m_2}{r^2}$$ Gravitational Law

Plot



* See and review slides online

Energy Diagram

$U(x) = \frac{1}{2}kx^2$

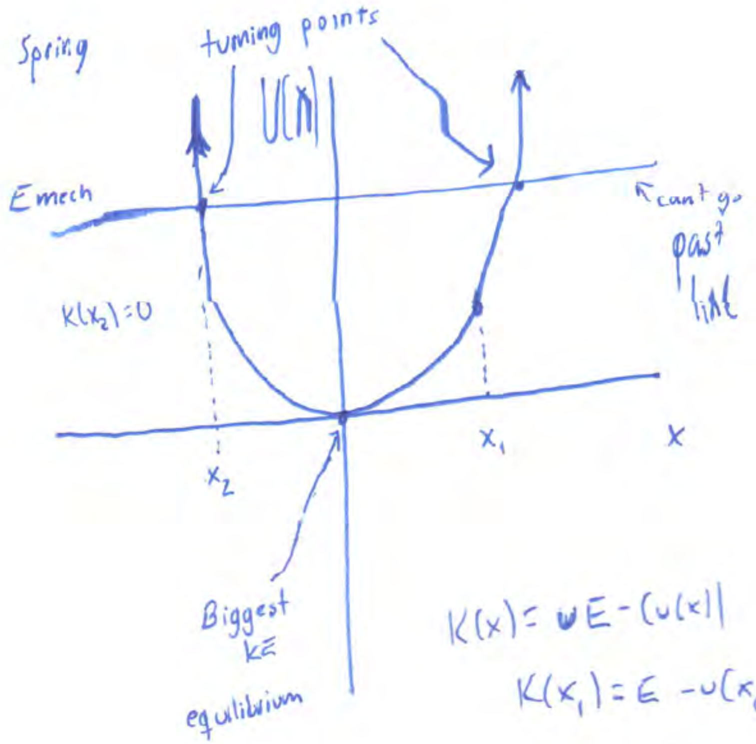
$W_{NC} = 0$

$E^{mech} = U + KE$

$\phantom{E^{mech}} = U(x) + k(x)$

constant

Spring

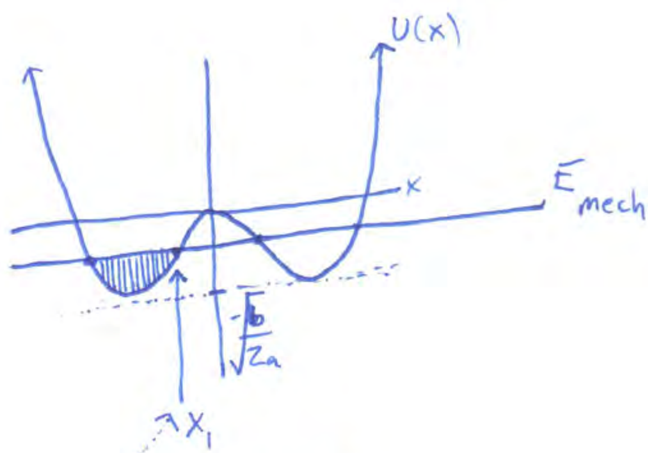


$K(x) = E - (U(x))$

$K(x_1) = E - U(x_1)$

Reading Question

$U(x) = ax^4 - bx^2$



Where released from?

$U(x_1) = E$

$K(x_1) = 0$

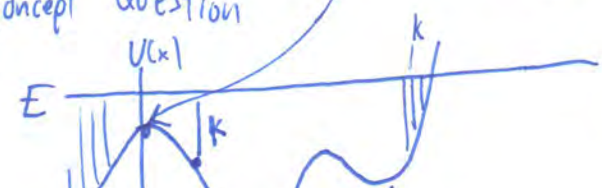
$au^2 - bu - E = 0$

quadratic

find $u \Rightarrow$ Find x

Concept Question

still has some KE and is going left



Escapes to ∞ in $-x$ direction

Table problem

$U_1 = mg(d+x)\sin\theta$

$K_1 = 0$

$E_1 = mg(d+x)\sin\theta$

$U_f = \frac{1}{2}kx^2$

$K_f = 0$

$E_f = \frac{1}{2}kx^2$

$$W_{nc} = E_f - E_1$$

$$-f_k(d+x) = \frac{1}{2}kx^2 - mg(d+x)\sin\theta$$

$$-\mu_k N(d+x)$$

$$-\mu_k mg(d+x)\cos\theta = \frac{1}{2}kx^2 - mg(d+x)\sin\theta$$

10/23/13 # 8.01 Midterm Review Notes Fernando Trujano

Polar Coordinates

Angular velocity: $\vec{w} = \frac{d\theta}{dt}\hat{k}$

Tangental Velocity: $\vec{v} = rw = r\frac{d\theta}{dt}\hat{\theta}$

Acceleration: $\vec{a} = a_c\hat{r} + a_\theta\hat{\theta}$

$\uparrow$ Centripetal $\quad$ $\uparrow$ tangental

$$\frac{dr}{dt} = \frac{d\theta}{dt}\hat{\theta}$$

$$F_r = ma_r \qquad F_\theta = ma_\theta$$

$$= \frac{mv^2}{r} = mw^2r \qquad = mR\frac{d^2\theta}{dt^2}$$

Centripetal Force not a separate force

Another Force, ie Tension or normal, acts as the centripetal force

Momentum

$$\vec{p} = m\vec{v}$$

$$I = F\Delta t$$

$$I = \Delta P$$

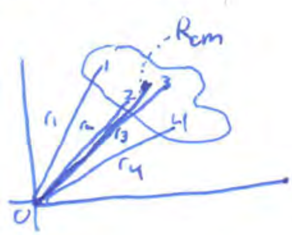
$$F = \frac{dp}{dt}$$

Internal Forces cancel out

$$F^{ext}\Delta T = P_{after} - P_{intial}$$

* No external forces means momentum is conserved! $P_{after} = P_{intial}$

Center of Mass



$$R_{cm} = \frac{m_1r_1 + m_2r_2 + \ldots}{m_1 + m_2}$$

* Center of mass of system travels in parabolic trajectory

* See and Understand Rocket Equation

Kinetic Energy and Work

$KE = \frac{1}{2} m v^2$

Negative work $\Rightarrow$ Force opposes motion

$W = F \cdot d$

$W = \int_{x_i}^{x_f} F_x \, dx$

Spring: $F_x = -kx$

$W = \cancel{} -\frac{1}{2} k x^2$

$W = \Delta KE$

Power

$P = \frac{\Delta W}{\Delta T} = \frac{F \Delta x}{\Delta T} = F V_x$

$W_{A \to B} = KE(B) - KE(A)$

$F \cdot \Delta s = \frac{1}{2} m V_B^2 - \frac{1}{2} m V_A^2$

Potential Energy

$U = mg \underset{h}{\Delta Y}$

$F_G = \frac{-G m_1 m_2}{r^2} \hat{r}$

$W = -U$

$W_{1 \to F} = -\int_1^F \frac{G m_1 m_2}{r^2} = G m_1 m_2 \left(\frac{1}{r_F} - \frac{1}{r_1} \right)$

$U(r_F) - U(r_1) = G m_1 m_2 \left(\frac{1}{r_F} - \frac{1}{r_1} \right)$

$U = -\frac{G m_1 m_2}{r}$

$W = -U \qquad U_s = \frac{1}{2} k x^2$

$W_{nc} = E_f^{mech} - E_i^{mech}$

* If no work by non conservative force

$E_f^{mech} = E_i^{mech} \Rightarrow$ Energy is conserved

* If non-conservative force like friction

$E_f^{mech} = E_i^{mech} + W_{Friction}^{c}$ negative work

$$W^{TOTAL} = \Delta KE$$

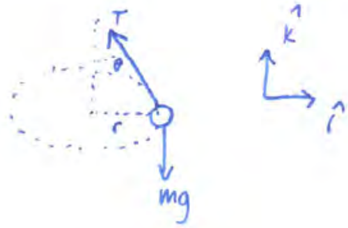
$$W^{C} + W^{nc} = \Delta KE$$

$$W^{nc} = \Delta KE - \underline{W^{C}}$$

$$W^{nc} = \Delta KE + \Delta V$$

$$W^{nc} = E_f^{mec} - E_i^{mech}$$

Tension in Ropes



$T = \frac{2\pi}{w}$

$F = ma$

$-mrw^2$

$-\frac{mv^2}{r}$

$\frac{-4\pi^2 r}{T_{per}^2}$

$r = L\sin\theta$

$K: \; T\cos\theta - mg = 0$

$T = \frac{mg}{\cos\theta}$

$$T\sin\theta = \frac{m 4\pi^2 L\sin\theta}{T_{per}^2}$$

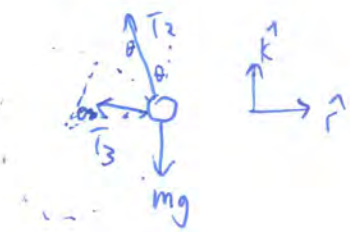
$$T_{perod}^2 = \frac{\cancel{m} 4\pi^2 L\cos\theta}{\cancel{m} g}$$

$$T_{period} = \sqrt{\frac{4\pi L\cos\theta}{g}}$$

- unit check
  $L =$ meters
  $g = m/s$

$\frac{\cancel{m}}{\cancel{m}}$ seconds $= T$

b) 



$-T\sin\theta = \frac{-mv^2}{r}$

$-T_3 - T_2\sin\theta = \frac{-mv^2}{r}$

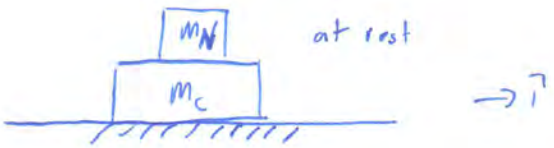
$\vec{a} = a_r\hat{r} + a_{\theta_2}\hat{\theta}$

radial $\quad a_r = -rw^2$

tandental $\quad a_\theta = \frac{r\,dw}{dt} = \frac{r\,d^2\theta_2}{dt}$

Jumping of a Flatcar

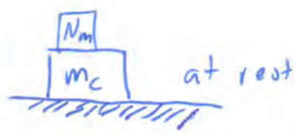
External Force $=0$ $\longrightarrow$ Momentum conserved

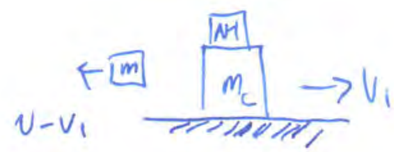


$$P_f = m_c V_f - (v - V_f) Nm$$
$$P_i = 0$$

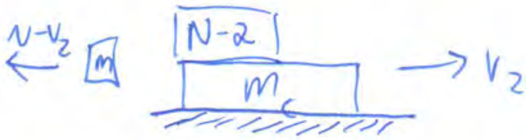
$$I = F^{ext}\Delta t = \overbrace{P_f - P_i}^{\Delta P}$$
$$0 = m_c V_f - Nm(v - V_f)$$
$$m_c V_f = Nm(v - V_f)$$

1step initial state





2 step



Algebra $\longrightarrow$ $V_1 = \dfrac{mv}{Nm + m_c}$

$$V_2 = V_1 + \frac{mv}{(N-1)m + m_c}$$

$$V_3 = V_2 + \frac{mv}{(N-2)m + \ldots}$$

Bigger than if they all jumped off at once

Collision

Elastic

$$\frac{1}{2}m_1 v_{1,0}^2 + \frac{1}{2}m_2 v_{2,0} + \dots = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f} \dots$$

Inelastic

$$K_0^{sys} > K_f^{sys}$$

Completely Inelastic:
only one body emerges

Super elastic

$$K_0^{sys} < K_f^{sys}$$

Inelastic Collision

$$m_1 v_1 + \underbrace{\cancel{m_2 v_2}}_{\text{at rest}} = (m_1 + m_2) v'$$

$$KE_1 = \frac{1}{2} m_1 v_1^2 \qquad v' = \frac{m_1 v_1}{m_1 + m_2}$$

$$KE_f = \frac{1}{2} \cancel{(m_1 + m_2)} \frac{(m_1 v_1)^2}{\cancel{(m_1 + m_2)}}$$

$$\Delta K / K_1 = \frac{\dfrac{m_1^2 v_1^2}{2(m_1 + m_2} - \dfrac{(m_1 v_1)^2}{2}}{\dfrac{m_1 v_1^2}{2}} = \frac{m_1}{m_1 + m_2} - 1$$

independent of v

Elastic

Energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 (V_1^2 - V_{1f}^2) = m_2 (V_{2f}^2 - V_{2}^2)$$

$$m_1 (V_1 + V_{1f})(V_1 - V_f) = m_2 (V_2 + V_{2f})(V_2 - V_{2f}) \quad (2)$$

Momentum

$$m_1 V_1 + m_2 V_2 = m_1 V_{1f} + m_2 V_{2f}$$

$$m_1 (V_1 - V_{1f}) = m_2 (V_{2f} - V_2) \qquad (1)$$

$$\frac{(2)}{(1)} \Rightarrow V_1 + V_{1f} = V_2 + V_{2f}$$

$$V_1 - V_2 = V_{2f} - V_{1f} \qquad (3)$$

* Relative Speed does not change.

Momentum is a vector

Rigid Body - Distance between two points never changes in time.

Two dimentional Rotational motion

- From reference frame following object, only rotational motion.
- Translational from fixed frame

$$F_{ext}^{total} = \frac{d\vec{p}_{sys}}{dt} = \frac{m_{sys}\, d\vec{V}_{cm}}{dt} = m_{sys}\vec{A}_{cm}$$

Torque produces angular acceleration about center of mass

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}_{cm}$$

moment of inertia

Angular acceleration about center of mass

... measure of mass distribution

$$\vec{w} = \frac{d\theta}{dt}\hat{k}$$

$\uparrow \hat{k}$

$\frac{d\theta}{dt} > 0$ $\hat{k}$ ccw

$\frac{d\theta}{dt} < 0$ $-\hat{k}$ cw

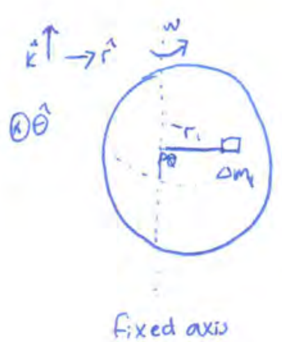
$\perp$ angular velocity perpendicular to plane.

$$w_z(t) - w_z(t_1) = \int_{t_1}^{t} \alpha_z\, t\, dt$$

var

$$\theta(t) - \theta(t_1) = \int_{t_1}^{t} w_z(t)\, dt$$

var, ← var



$\vec{V}_1 = r_1 w_z \hat{\theta}$

$K_1 = \frac{1}{2}\Delta m_1 v_1^2$

$= \frac{1}{2}\Delta m_1 (r_1 w_z)^2$

Moment of inertia about the z axis

$$KE = \sum_{i=1}^{N} \Delta m_1 r_1^2 w_z^2 = \frac{1}{2}\sum_{i=1}^{N} \frac{1}{2} m_1 r_1^2 (W_z)^2$$

$$I_z = \frac{1}{2}\sum_{i=1}^{N} \frac{1}{2} \Delta m_i r_i^2$$

$\Delta m_i \rightarrow dm$

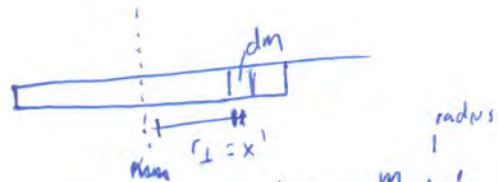
$r_i \rightarrow r_\perp$

$\sum_{i=1}^{N} \rightarrow \int_{body}$

$$I = \int_{body} dm \, r_\perp^2 \Rightarrow K = \frac{1}{2} I_z w_z^2$$

how a mass is dotributed about an ~~object~~ the axis of rotation

Table Problem: Moment of inertia of Rod



$$dm = \frac{M}{L} dx'$$
$$= \lambda dx'$$

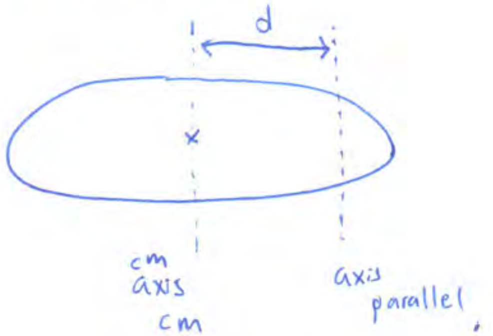
always a positive quantity

$$I_z = \int_{body} dm \, r_\perp^2$$

$$= \int_{-L/2}^{L/2} \frac{M}{L} dx' \, x'^2 = \frac{M}{L} \frac{x^3}{3} \Big|_{-L/2}^{L/2}$$

$$\frac{M}{3L}\left((L/2)^3 - \left(\frac{-L}{2}\right)^3 \right) = \frac{1}{12} ML^2$$

Parallel Axis Theorem



$$K_{cm} = \frac{1}{2} I_{cm} w^2$$
$$K_1 = \frac{1}{2} I_1 w^2$$

Same w

$$I_1 = I_{cm} + md^2$$

8.01

1) Rotational Coordinates

$\otimes \hat{n}_{RHR}$

$$\vec{\alpha} = \frac{d^2 t}{dt^2}\hat{n}_{RHR}$$

$$= \underbrace{\alpha}_{\text{component}} \hat{n}_{RHR}$$

2) Torque Diagram

a) choose points S

b) show were force acts at (Point P)

c) Draw $\vec{r}_{SP}$ vector from S to P

$$\tau_S = r_{SP} \times F_P$$

Torque causes angular acceleration
Force causes linear acceleration

$$\vec{T}_{cm} = I_{cm} \vec{\alpha}_{cm}$$

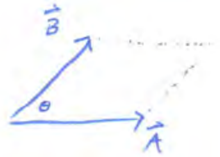
Point that we are calculating torque about

Point where Force is applied

$$\vec{r}_{s,p} \equiv \text{vector from } S \text{ to } P$$

$$T_s = \vec{r}_{s,p} \times \vec{F}_p$$

Cross Product



$$\vec{A} \times \vec{B} \equiv \vec{C}$$

new vector

need

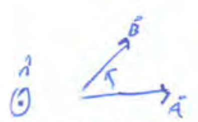
(1) Magnitude = area = $B_\perp |\vec{A}| = A_\perp |\vec{B}|$

(2) Direction

$B_\perp = |B| \sin\theta$

$A_\perp = |A| \sin\theta$

Direction



Right hand rule

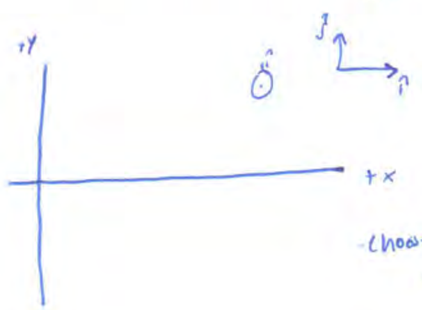
$$\text{Area} = \underbrace{|\vec{B}|\sin\theta}_{B_\perp}\ \underbrace{|\vec{A}|}_{A_\perp}$$

$$dir(\vec{A} \times \vec{B}) \equiv \hat{n}$$

$$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$$

start fingers at A

Curl to B



choose RHR coordinate system

$$\hat{i} \times \hat{j} \equiv \hat{k} = \hat{n} \text{ RHR}$$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\vec{A} = \overset{\swarrow \text{component}}{A_x} \hat{i} + A_y \hat{j} + \cancel{A_z \hat{k}}$

$\vec{B} = B_z \hat{k}$

$\vec{A} \times B = (A_x \hat{i} + A_y \hat{j}) \times B_z \hat{k}$

$(A_x \hat{i} \times B_z \hat{k}) + (A_y \hat{j} \times B_z \hat{k})$

$= A_x B_z (\hat{i} \times \hat{k}) + A_y B_z (\hat{j} \times \hat{k})$

$A_x B_z (-\hat{j}) + A_y B_z (\hat{i})$

Example

$\vec{r} = x \hat{i}$

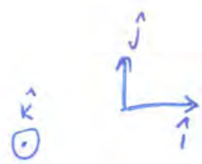
$\vec{F} = F_x \hat{i} + F_z \hat{k}$

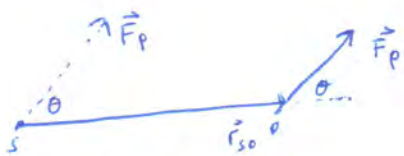
$\vec{r} \times \vec{F} = (x \hat{i}) \times (F_x \hat{i} + F_z \hat{k})$

$(x \hat{i} \times F_x \hat{i}) + (x \hat{i} \times F_z \hat{k})$

$x F_x (\hat{i} \times \hat{i}) + x F_z (\hat{j} \times k)$

$\hat{i} \times \hat{k}$





$r_{s,p} \times \vec{F}_p = \text{dir } \hat{k}$

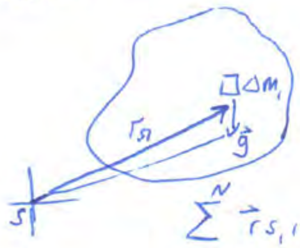
$\text{mag} = r F_\perp = |r_{so}| F_\perp$

$= |r_{so}||F| \sin \theta$

Torque Due to Uniform Gravitational Force

- Gravity acts at the center of mass

Calculation



$$\sum_{i=1}^{N} \vec{r}_{s,i} \times \Delta m_i \vec{g} = \tau_s$$

$$= \sum_{i=1} \Delta m_i \, \vec{r}_{s,i} \times \vec{g}_{\text{uniform } g}$$

$$= \sum_i (\Delta m_i r_{s,i}) \times \vec{g}$$

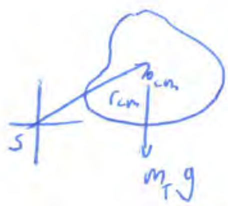
Recall

$$\vec{R}_{cm} = \frac{\sum \Delta m_i r_i}{m_T}$$

$$m_T \vec{R}_{cm} = \sum m_i r_i$$

$$m_T R_{cm} \times g$$

$$= \vec{R}_{cm} \times m_T \vec{g}$$



$$\vec{R}_{cm} = \vec{r}_{s,cm}$$

$\therefore$

Gravity acts at the center of mass

Rotation about a fixed axis

The direction of increasing θ changes $\hat{k}$ $\to$ RHR

Parallel Axis Theorem

$$I_s = Md^2 + I_{cm}$$

moment of inertia

Choose fixed axis in z-direction

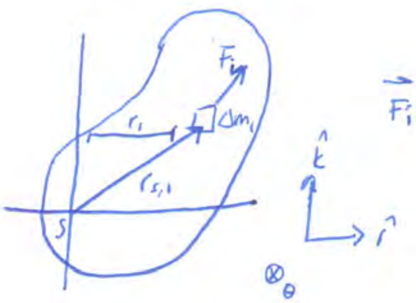
fixed axis

$$\tau_S = \tau_{S,z}\hat{k} + \sim$$

$$\tau_{S,z} = I_S \alpha_z$$

$$\vec{\alpha} = \alpha_z \hat{k}$$

Calculation



$$\vec{F}_i = F_{r,i}\hat{r} + F_{\theta,i}\hat{\theta} + F_{z,i}\hat{k}$$

$$r_{S,i} = r_i\hat{r} + z_i\hat{k}$$

$$\tau_{S,i} = (r_i\hat{r} + \cancel{z_i\hat{k}}) \times (\cancel{F_{r,i}}\hat{r} + F_{\theta,i}\theta + \cancel{F_{z,i}\hat{k}})$$

only want z component because $\hat{r}\times\hat{\theta} = \hat{k}$

$$(\tau_{S,i})_z = r_i\hat{r} \times F_{\theta,i}\theta$$

$$= r_i F_\theta \hat{k}$$

Newton's 2nd

$$F_{\theta,i} = \Delta m_i a_{\theta i}$$

$$F_{\theta,i} = \Delta m_r r_i \alpha_z$$

$$(\tau_{S,i})_z = r_i F_{\theta i} = \Delta m_i r_i^2 \alpha_z$$

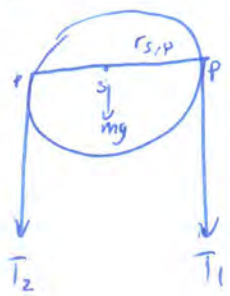
$$(\Sigma \Delta m_i r_i^2)\alpha_z = = I\alpha_z$$

Newton's 3^{rd} law

Internal ~~T~~ Torques Cancel in Pair

if Forces are radial along line connecting the 2 vectors

Mass Pulley



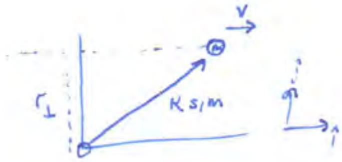
$$RT_1 - RT_2 = I_S \alpha_z$$

11/4/13 8.01 Lecture Fernando Trujano

Angular Momentum

$\hookrightarrow \vec{L}_s = \vec{r} \times \vec{p}$

$|L_s| = |r_\perp||\vec{p}|$



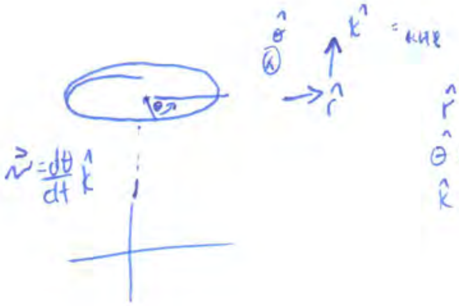
$\vec{p} = m\vec{v}$ $\hat{i} \times \hat{j} = \hat{k}$

$\vec{r}_{0,m} = x\hat{i} + y\hat{j}$

$\vec{p} = mv\hat{i}$

$\vec{L}_0 = (x\hat{i} + y\hat{j}) \times mv\hat{i} = -myv\hat{k}$

$|\vec{r}_\perp||\vec{p}|(-\hat{k})$



$\hat{r} \times \hat{\theta} = k$

$\hat{\theta} \times \hat{k} = \hat{r}$

$\hat{k} \times \hat{r} = \hat{\theta}$

Circular Motion

$\vec{L}_0 = \vec{L}_{0,m} \times \vec{p}$

$\vec{L}_0 = \vec{r}_{0,m} \times \vec{p} = R\hat{r} \times mv_\theta \hat{\theta}$

$= Rmv(\hat{r} \times \theta) = Rmv_\theta \hat{k}$

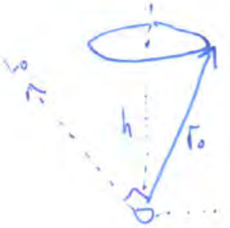
$V_\theta = R\frac{d\theta}{dt} = R\omega_z$

$\vec{L}_0 = RmR\omega_z \hat{k}$

$\vec{L}_0 = mR^2\omega_z \hat{k}$

$\vec{L}_0 = I\omega_z \hat{k}$

Angular Momentum along axis of rotation in circular motion
↳ Table Problem



$\hat{k}$
$\hat{\theta} \otimes \hat{r}$

$\vec{v} = r w_z$

$\vec{r}_0 \times m\vec{v}$

$(r\hat{r} + h\hat{k}) \times (mrw_z)\theta$

$mr^2 w_z(\hat{r} \times \hat{\theta}) + mrw_z(\hat{k} \times \hat{\theta})$

$mr^2 w_z \hat{k} - hmrw_z(+\hat{r})$

r_0 and L must be $\perp$

$K_{Trans} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$K_{rot} = \frac{1}{2} I_{cm} w^2 = \frac{L^2}{2I_{cm}}$
Symetric body

$L = I_{cm} w$

$w = \frac{L}{I_{cm}}$

$\vec{P}_{sys} = mv_1 + mv_2 = 0$

If $P_{sys} == 0$ then $\vec{L}_s$ is independent of the point s!

Conservation of Angular Momentum

$$\vec{L}_s = r_{s,m} \times m\vec{v}$$

$$\frac{d\vec{L}_s}{dt} = \frac{dr_{s,m}}{dt} \times m\vec{v} + r_{s,m} \times \frac{d(m\vec{v})}{dt}$$

$$= (\underbrace{v \times m\vec{v}}_{0}) + r_{sm} \times \vec{F}$$

$$= r_{s,m} \times F$$

$$\frac{d\vec{L}_s}{dt} = T_s$$

A

→ Torque causes angular momentum to change about point s

$$T_s \Delta t_{collision} = \Delta L_s$$

Decompose the motion

Translational motion of center of mass

(frame fixed to ground)

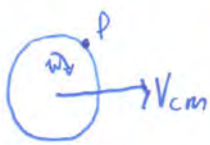
pure rotational about cm

(reference frame w/ center of mass)

★

	Translational motion of cm		Rotation about cm (in cm reference frame)	
Kinematix	$v(P) = \vec{V}_{cm}$	+	$\vec{V}_{rot}$	
	$\vec{L}_{s,cm} = \vec{r}_{s,cm} \times m_T \vec{V}_{cm}$	+	$\vec{L}_{cm} = I_{cm}\vec{w}$	
	$K_{trans} = \frac{1}{2} m_T V_{cm}^2$	+	$K_{rot} = \frac{1}{2} I_{cm} w^2$	
Dynamics	$\vec{F}^{ext} = m_T a_{cm}$		$\tau_{cm} = I_{cm}\alpha$	Fixed axis

Rolling Bicycle Wheel

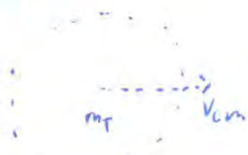


reference frame to ground.

$\vec{v}(P)$ - Think of wheel as point mass w/ translational motion

$$\vec{v}(P) = \vec{V}_{cm} + V_{rot}(P)$$

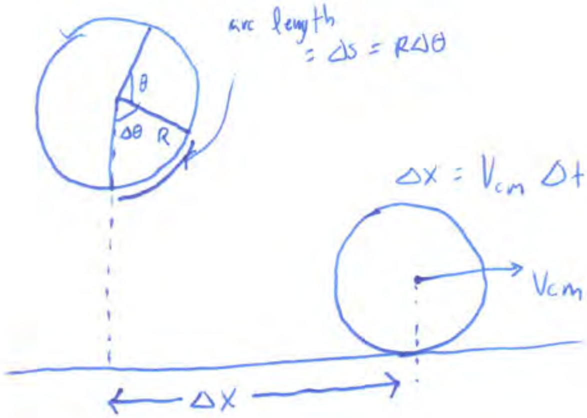
Translational Rotational



+

$\vec{v}_{rot} = RW\hat{\theta}$

Skidding
└ No rotation
Contact pt velocity Forward
Friction points back



If no skidding or slipping arc length should equal Δx!

If
~~Δx~~ Arc $> \Delta x$ = slipping
Arc $= \Delta x$ = Roll w/o slip
Arc $< \Delta x$ = skidding

Rolling w/o slipping

$$\Delta s = \Delta x$$

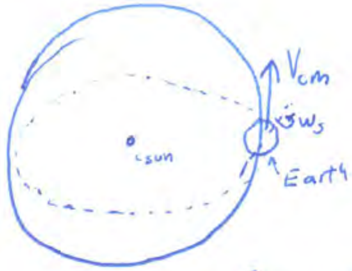
$$R\Delta\theta = V_{cm}\Delta t$$

$$R\frac{\Delta\theta}{\Delta t} = V_{cm}$$

$$R\omega = V_{cm}$$

$V_{rot} = V_{cm} \Rightarrow$ Rolling w/o slipping

+ Contact point w/ the ground is instantaneously at rest!
$V_{cm} + V_{rot} = 0$ at contact pt
↳ Static Friction



Decomposition

+

$$\underbrace{L_{s,cm} = r_{s,cm} \times M_E V_{cm}}_{\text{Orbital Angular Momentum}} \qquad \vec{L}_{cm} = \underbrace{I_{cm}}_{\text{Spin Angular Momentum}} \vec{\omega}$$

$$\vec{L}_{s\,total} = r_{s,cm} \times m_E V_{cm} + I_{cm} \vec{\omega}_s$$

$$L_{s,cm} = r_{ES} m_E V_{cm}$$

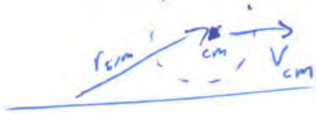
$$V_{cm} = r_{ES}\, \omega_{orbital}$$

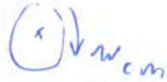
$$L_{s,cm} = m_E r_{ES}^2\, \omega_{orbital}$$

$$\omega_{orbital} = \frac{1}{365}\, \omega_{spin}$$

$$\vec{L}_{spin} = \frac{2}{5} m_E r_E^2\, \omega_{spin}$$

$$\vec{L}_s = \vec{r}_{s,cm} \times m V_{cm} + I_{cm} \vec{\omega} = \left(R m V_{cm} + I_{cm} \omega \right) \hat{k}$$





Static Friction does no work

Rolling w/o slipping → energy is conserved

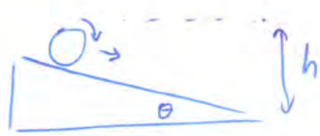
Case (1)

$I_1 = b_1 m R^2$

$I_2 = b_2 m R^2$ $\quad b_1 \neq b_2$

↳ not the same mass distribution

$V_{cm} = \sqrt{}$



$$mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$\omega = \frac{V_{cm}}{R}$$

$$mgh = \frac{V_{cm}^2}{2}\left(m + \frac{I_{cm}}{R^2}\right)$$

$$V_{cm} = \sqrt{\frac{2mgh}{m + I_{cm}/R^2}}$$

Angular Momentum and Torque

$$\vec{L}_s = \vec{L}_s^{\,orbital} + L_{cm}^{spin} = r_{s,cm} \times m_T V_{cm} + L_{cm}^{spin}$$

$$T_s = \sum_i T_{s,i}^{ext} = \frac{dL_s}{dt}$$

Decomposition

$$T_{cm}^{ext} = \frac{dL_{cm}^{spin}}{dt}$$

$$T_{s,cm} = \frac{dL_s^{orbital}}{dt}$$

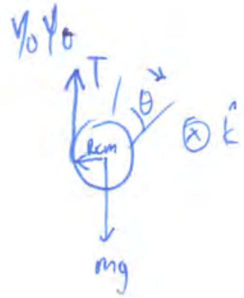
4/15

About center of mass Fixed Axis

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}$$

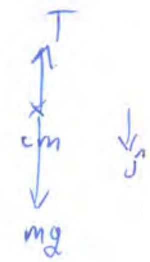
Translational Motion

$$\vec{F}^{ext} = m_T \vec{a}_{cm}$$



$$\vec{\tau}_{cm} = I_{cm}\alpha$$

$$\hat{k}: RT = I_{cm}\alpha$$



$$\vec{F}^{ext} = m\vec{a}_{cm}$$

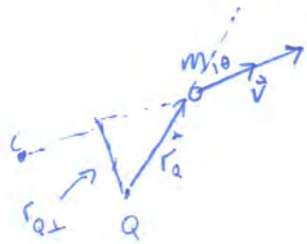
$$\hat{j}: mg - T = ma_y$$

See Slides

5/15

14/11/13 8.01 ocw Fernando Trejan

Angular Momentum
Torques
Conservation of Angular momentum



$\vec{p} = m\vec{v}$

$\vec{L}_Q = \vec{r}_Q \times \vec{p} = (r_Q \times \vec{v})m$

$|L_Q| = r_Q m v \sin\theta = mv \underbrace{r_Q \sin\theta}_{r_{\perp Q}}$

$\vec{L}_c = 0$ because $\sin\theta = 0$

Angular Momentum depends on choice of origin

point is at C

$t = 0 \quad \vec{L}_c = 0$

$t = t \quad \vec{L}_c \neq 0$

Angular Momentum changing throughout path

$\frac{p}{m} \to L$

$F \to T$

$\frac{dP}{dt} = F \quad \frac{dL}{dt} \to \tau$

$\vec{L}_Q = \vec{r}_Q \times \vec{p}$

$\frac{dL}{dt}_Q = \frac{d\vec{r}_Q}{dt} \times \vec{p} + \left[r_Q \times \left(\frac{d\vec{P}}{dt}\right) \right]$

$\frac{d\vec{r}_Q}{dt}$ = velocity => same direction as P => 0

$\frac{d\vec{P}}{dt}$ => F

$\frac{dL}{dt}_Q = \underbrace{r_Q \times F}_{\text{Torque} = \tau_Q}$

☆

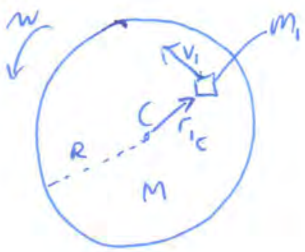
1/2

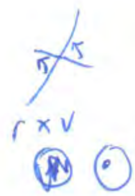
Torque changes angular momentum.

if $T = 0$

L is conserved

$T_c = 0$





$$L_{c_i} = m_i \, r_{i_c} \, v_i$$

$v = R w$

$$L_{c_1} = m_1 r_1^2 w$$

$v_1 = w r_i$

$$L_{disk} = w \underbrace{\sum m_1 r_1^2}_{I \text{ = moment of inertia}}$$

★

$$\boxed{L_{disk} = I_c w}$$

Holds regardless of point

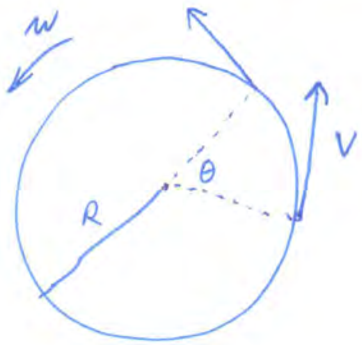
If more mass is concentrated on middle

$I \rightarrow \downarrow$ $\qquad L = I w$

so $w \uparrow$

2/12

Rotating Rigid Bodies
Moment of Inertia
Parallel and Perpendicular Axis Theorem
Rotational KE



$v = r\omega$

$a_{tan} = \frac{d\omega}{dt} R = \alpha R$

angular acceleration

rad/sec^2

→ makes speed change along circumference

$x \rightarrow \theta$
$v \rightarrow \omega$
$a \rightarrow \alpha$
$m \rightarrow I$

$\Delta s = V_0 t + \frac{1}{2} a t^2$ ← Linear

$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$ ← rotational

$V = V_0 + at$

$\omega = \omega_0 + \alpha t$

m, R

$KE_i = \frac{1}{2} m_i v_i^2$

$= \frac{1}{2} m_1 \omega^2 r_1^2$ ← same at all points

$KE_{disk} = \frac{\omega^2}{2} \underbrace{\sum m_1 r_1^2}$

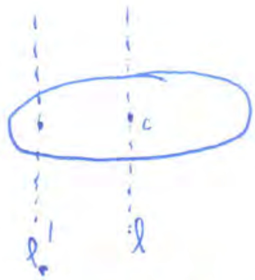
I = Moment of Inertia $= \sum m_i r_i^2$

$KE_{disk} = \frac{1}{2} I_c \omega^2$

disk $I_c = \frac{1}{2} m R^2$ rod $I = \frac{1}{12} m l^2$

sphere $I = \frac{2}{5} m R^2$

☆

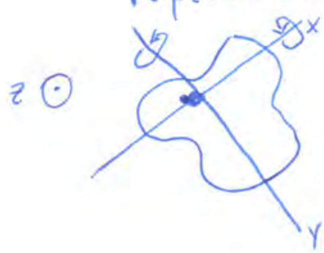


l', parallel to l

Parallel Axis Theorem

$$I_{l'} = I_l + md^2$$

Perpendicular Axis Theorem



$$I_z = I_x + I_y$$

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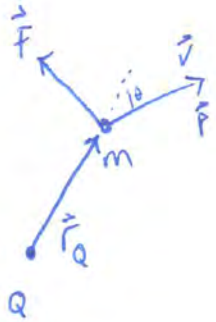
Torques, Oscillating Bodies

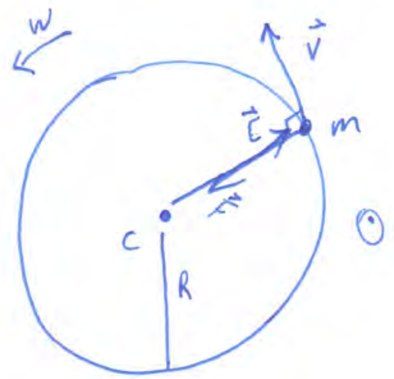


$\vec{L}_Q = \vec{r}_Q \times \vec{p}$ ⊗

$\vec{\tau}_Q = \vec{r}_Q \times \vec{F}$ ⊙

Angular Momentum ← depends on point Q you choose

Torque → Leads to change in



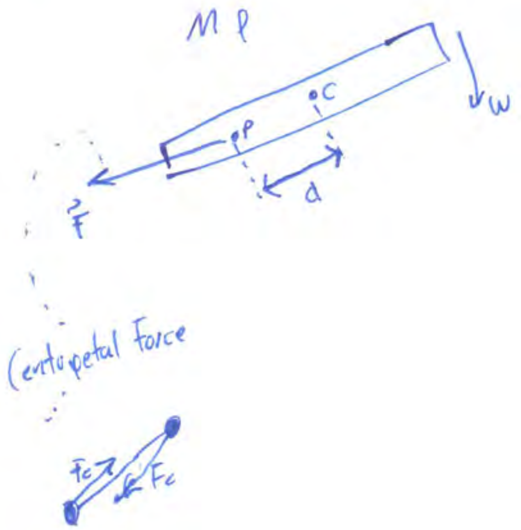
$$|L_c| = r_c m v = m r_c^2 w = mR^2 w$$

or

$$L_c = I w = \underbrace{mR^2}_{I} w$$

$\tau_c = 0$ b/c $\vec{F}$ and $\hat{R}$ are always 180°

=> Angular Momentum is conserved



$$|L_p| = I_p w = \left(\frac{1}{12} M\ell^2 + Md^2\right) w$$

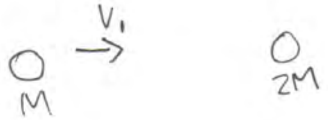
Parallel Axis Theorem

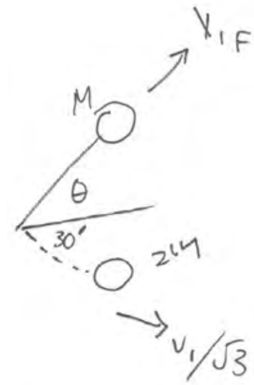
$$|\tau_p| = 0$$

=> Angular Momentum is conserved relative to P

- Two Dimentional Collision

Problem 1





X

$$MV_1 = MV_{1f}\cos\theta + \frac{2MV_1}{\sqrt{3}}\cos 30$$

$$\cancel{M}V_1 = \cancel{M}V_{1f}\cos\theta + \cancel{M}V_1$$

$$V_1 = V_{1f}\cos\theta + V_1$$

$$V_{1f}\cos\theta = 0$$

a)

$$\theta = 90^\circ$$

Y

$$0 = MV_{1f}\sin\theta - \frac{2MV_1}{\sqrt{3}}\sin 30$$

$$0 = MV_{1f}\sin\theta - \frac{MV_1}{\sqrt{3}}$$

$$V_{1f}\sin\theta = \frac{V_1}{\sqrt{3}}$$

$$V_{1f}^2\cos^2\theta + V_{1f}^2\sin^2\theta = \frac{V_1^2}{3}$$

$$V_{1f}^2\underline{(\cos^2\theta + \sin^2\theta)} = \frac{V_1^2}{3}$$

$$V_{1f}^2 = \frac{V_1^2}{3}$$

b)

$$V_{1f} = \frac{V_1}{\sqrt{3}}$$

c) $KE_1 = \frac{1}{2}MV_1^2$

$$KE_f = \frac{1}{\cancel{2}}\cancel{2}M\left(\frac{V_1}{\sqrt{3}}\right)^2 + \frac{1}{2}M(V_{1F})^2$$

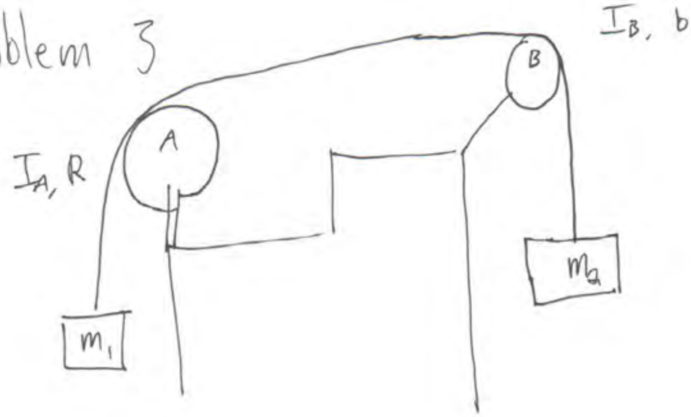
$$\left(\frac{V_1}{\sqrt{3}}\right)^2 M + \frac{1}{2}M\left(\frac{V_1}{\sqrt{3}}\right)^2 = \frac{1}{2}M(V_1^2)$$

$$\frac{V_1^2}{3}M + \frac{1}{2}M\frac{V_1^2}{3} = \frac{2V_1^2 M}{3} =$$

Problem 2

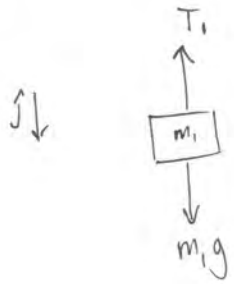
$$KE_i = KE_f$$

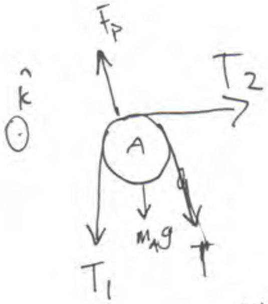
Problem 3

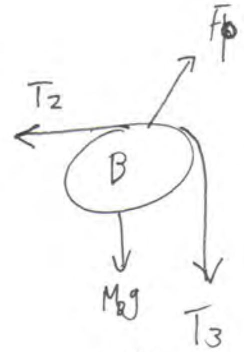


$I_A = \frac{1}{2} M_A R^2$

$I_B = \frac{1}{2} M_B b^2$







T RHR

$RT_1 - RT_2 = I_A \alpha_A$

$= \frac{1}{2} M_A R^2 \alpha$

RHR

$bT_2 - bT_3 = I_B \alpha_B$

$= \frac{1}{2} M_B b^2 \alpha_B$

$m_2 g - T_3 = m_2 a_B$

$m_1 g - T_1 = m_1 a_A$

$s = R\theta$

$v = R\omega$

$a = R\alpha$

$\alpha = \frac{a}{R}$

$T_1 = m_1 g - m_1 a$

$m_1 g - T_1 = m_1 \overset{a}{R\alpha} \rightarrow m_1 g - T_1 = m_1 a$

$RT_1 - RT_2 = \frac{1}{2} M_A R^2 \cancel{\alpha}\, a \rightarrow \cancel{R}(T_1 - T_2) = \frac{1}{2} M_A \cancel{R}\, a$

$bT_2 - bT_3 = \frac{1}{2} M_B b^2 \cancel{\alpha}\, a \rightarrow \cancel{R}(T_2 - T_3) = \frac{1}{2} M_B \cancel{b}\, a$

$m_2 g - T_3 = m_2 \overset{a}{R\alpha} \rightarrow m_2 g - T_3 = m_2 a$

$T_3 = m_2 g - m_2 a$

+

$T_1 - \cancel{T_2} + \cancel{T_2} - T_3 = \frac{1}{2} a (M_B + M_A)$

$T_1 - T_3 = \frac{1}{2} a (M_B + M_A)$

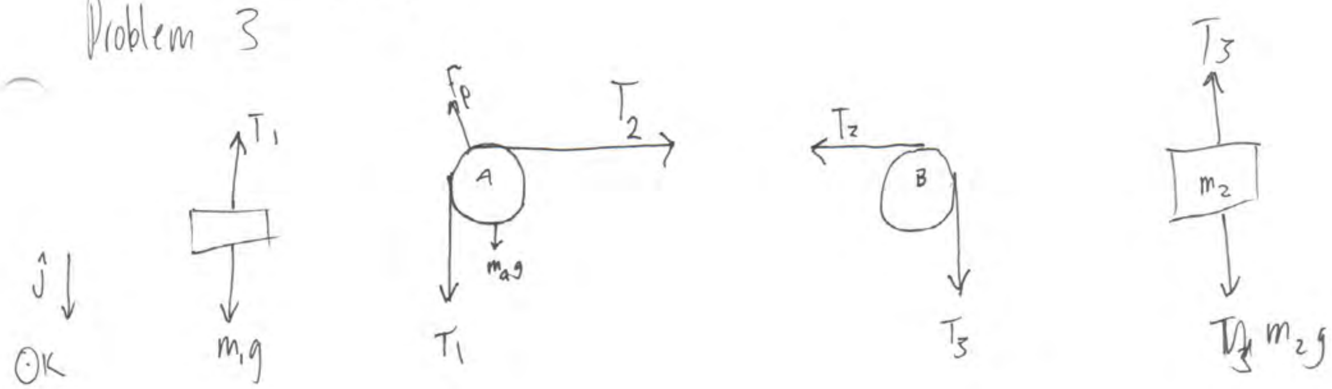
~~$m_1 g - m_1 a - m_2 g +$~~

$m_1 g - m_1 a - m_2 g + m_2 a =$

$\rightarrow$

By Myself

Problem 3



$T_2 b - T_3 b = I_B A$

$m_1 g - T_1 = m_1 a$ $\quad$ $T_1 R - T_2 R = I_A \alpha$ $\quad$ $m_2 g - T_3 = m_2 a$

$v = r\omega$

$a = r\alpha$

$\alpha = \frac{a}{r}$

$m_1 g - T_1 = m_1 a$

$T_1 \cancel{R} - T_2 \cancel{R} = \frac{1}{2} M_a \cancel{R^2} \frac{a}{\cancel{R}}$) +

$T_2 \cancel{b} - T_3 \cancel{b} = \frac{1}{2} M_B \cancel{b} a$

$m_2 g - T_2 = m_2 a$

$T_1 - T_2 + T_2 - T_3 = \frac{1}{2} M_a a + \frac{1}{2} M_b a$

$T_1 - T_3 = \frac{1}{2} a (M_a + M_b)$

$\cancel{a = \frac{2(T_1 - T_3)}{M_a + M_b}}$

$T_1 = m_1 g - m_1 a$

$T_3 = m_2 g - m_2 a$

$m_1 g - m_1 a - m_2 g + m_2 a = \frac{1}{2} a (M_a + M_b)$

$g(m_1 - m_2) + a(m_2 - m_1) = \frac{1}{2} a (M_a + M_b)$

$g(m_1 - m_2) = \frac{1}{2} a [(M_a + M_b) - (M_2 - m_1)]$

$a = \frac{2g(m_1 - m_2)}{(M_a + M_b) + (M_1 - M_2)}$ ✓

Problem 4

$$L_{m} = I_{cm} w_1$$

$$L_s = I_s w_1 \ \hat{k}$$

$$I_s = I_{cm} + m_1 R^2 = 2 m_1 R^2$$

$$L_s = 2 m R^2 w$$

Problem 5

a) ~~ma~~ $\frac{mv^2}{r_0} = \frac{G m_c m_s}{r_0^2}$

$$v = \sqrt{\frac{G m_c \cancel{m_s} \cancel{r}}{\cancel{r} \cancel{m} r_0}}$$

b) $E^{Mech} = U + KE$

$$\frac{G m_c m_s}{r_0} + \frac{1}{2} m_s \frac{G m_c}{R_0}$$

$\rightarrow$ c)

Problem 6

$v = r\omega$

$L_{s_i} = r \times p = r \times mv$

$r \times m_s r \omega$

$= a^2 m_s \omega$

also rotates

$L_{s_f} = b^2 (m_s + m_b) \omega_f$

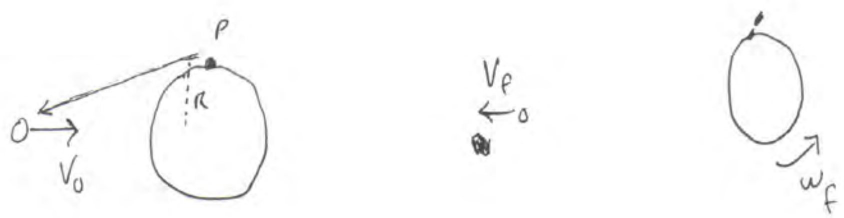
$a^2 m_s \omega_0 = b^2 (m_s + m_b) \omega_f$

$\omega_{f_{ater}} = \dfrac{a^2 m_s \omega_0}{b^2 (m_s + m_0)}$

$m_s = \lambda t$

$\omega_{f_{ater}} = \dfrac{a^2 \lambda t \, \omega_0}{b^2 (\lambda t + m_b)}$

Problem 7



$\cancel{P_i = P_f}$ $L = r \times p$

$\cancel{mV_0 = mV_f} +$ $r_p \times mV_0 = RmV_0 = -RmV_f + I\omega_f$

PAT

$RmV_0 = 2MR^2\omega_f - RmV_f$

$\frac{1}{2} m v_0^2 = \frac{1}{2} m (v_f)^2 + \frac{1}{2} 2mR^2\omega_f^2$

$\cancel{\sqrt{\frac{1}{2}\frac{mv_0^2 - mv_f^2}{mR^2}} = \omega_f}$

$\cancel{RmV_0 = mR^2}$

$Rm(V_0 + V_f) = 2mR^2\omega_f$

$\frac{1}{2} mV_0^2 = \frac{1}{2} m V_f^2 + \frac{1}{2} 2mR^2\omega_f^2$

$v_0^2 = v_f^2 + 2\omega_f^2 R^2$

$(V_0 + V_f)(V_0 - V_f) = 2R^2\omega_f^2$

$\cancel{V_0 + V_f = 2}$

$2R\omega_f (V_0 - V_f) = 2R^2\omega_f^2$

$\omega_f = \frac{V_0 - V_f}{R}$

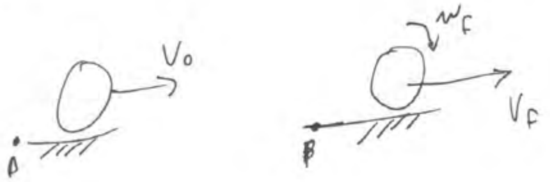
$V_0 + V_f = 2R\frac{V_0 - V_f}{R}$

$V_0 + V_f = 2(V_0 - V_f)$

$3V_f = V_0$

$\boxed{V_f = \frac{1}{3} V_0}$

Problem 8



$T_p = 0 \quad L_{p1} = L_{pf}$

$$RmV_0 = RmV_F + I\omega_F$$

$$RmV_0 = RmV_F + \frac{2}{5}mR^2\omega_F$$

Rolling w/o slipping

$$V_F = R\omega$$

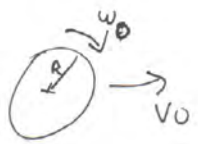
$$\omega = \frac{V_F}{R}$$

$$\cancel{Rm}V_0 = \cancel{Rm}V_F + \frac{2}{5}\cancel{Rm}V_F$$

$$V_0 = \cancel{V_F +}\ \frac{7}{5}V_F$$

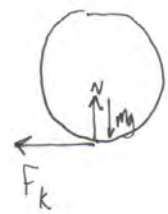
$$\boxed{V_f = \frac{5}{7}V_0}$$

Problem 9

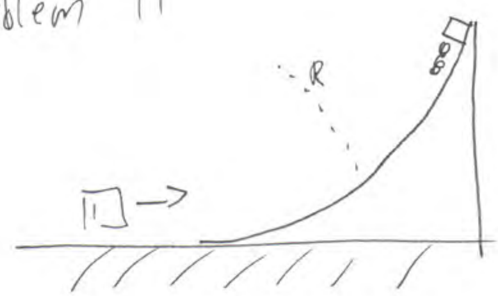


$I_{cm} = mR^2$

a)



Problem 11



$$m_b v_{c_0} = (m_c + m_b) v'$$

$$v' = \frac{m_b v_{c_0}}{m_c + m_b}$$

$$\frac{1}{2} m_c v_{c_0}^2 = \frac{1}{2} m_b v'^2 + mgR + \frac{1}{2} m_c v_1^2 + \frac{1}{2} kx^2$$

$$\frac{1}{2} m_c v_{c_0}^2 = \frac{1}{2} (m_b + m_c)(v')^2 + m_c g R + \frac{1}{2} kx^2$$

$$\frac{1}{2} m_c v_{c_0}^2 = \frac{1}{2} (m_b + m_c)\left(\frac{m_b v_{c_0}}{m_c + m_b}\right)^2 + m_c g R + \frac{1}{2} kx^2$$

$$\sqrt{\frac{\frac{1}{2} m_c (v_{c_0})^2 - \frac{1}{2}(m_b + m_c)\left(\frac{m v_{c_0}}{m_c + m_b}\right)^2 - m_c g R}{\frac{1}{2} k}} = x$$

b)

$$\cancel{(m_c + m_p) v' = m_p - m_c v_f}$$

$$m_c v_{c0} = -m_c v_{cf} + m_p v_f$$

$$m_c (v_{c0} + v_{cf}) = m_p v_f$$

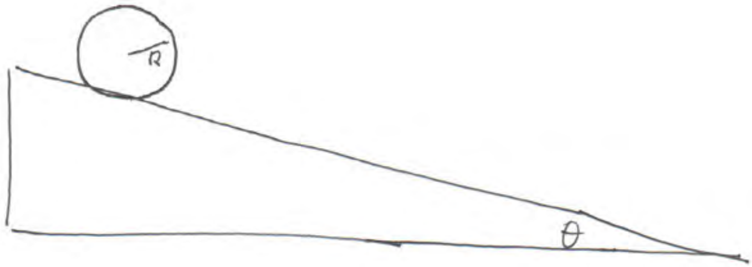
$$\cancel{m_c^2 (v_{c0} + v_{cf})^2 =}$$

$$\frac{1}{2} m_c (v_{c0})^2 = \frac{1}{2} m_c (v_{cf})^2 + \frac{1}{2} m_p (v_f)^2$$

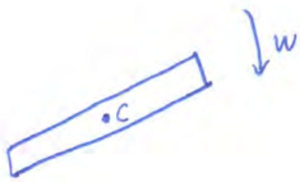
$$m_c (v_{c0}^2 - v_{cf}^2) = m_p v_f^2$$

$$m_c (v_{c0} - v_{cf})(v_{c0} + v_{cf}) = m_p v_f^2$$

Problem 10



$Mgh -$



$T_Q = 0$

Any Point
b/c no force

$$|L_{CM}| = I_c w = \frac{1}{12} M l^2 w$$

11/25/13

8.01 Lecture

Exam 3 Review

Fernando Trujano

Initial relative velocity = negative Final relative velocity

Derived from momentum and energy equations

Collision Theory

(1) Is momentum constant? $\vec{p}_1 \overset{?}{=} \vec{p}_f$ $\leftrightarrow$ $\vec{F}^{ext} = 0$ (sum)

(2) Is energy constant? $E_1 \overset{?}{=} E_f$ $W^{nc} = 0$ elastic $E_1 = E_f$, inelastic $E_1 \neq E_f$

(3) Is there a point such that $L_{p1} \overset{?}{=} L_{pf}$ $\tau_p = 0$

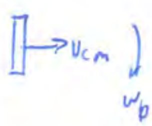
Pivot Force = external force

No Torque at point where force is acting

$\tau_p = r_{pF} \times \vec{F}$

Two ways to think about rotational energy

(1) $K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} w_f^2$



(2) Pivoted about P pure fixed axis rotation about P

$K = \frac{1}{2} I_p w_f^2$

$\frac{1}{2}\left(m\left(\frac{d}{2}\right)^2 w_f + I_{cm}\right) w_f^2$

Parallel Axis Theorem

$= \frac{1}{2} I_p w_f^2$ ✓

Concept Question 4

Static Equilibrium

$\hat{F}^{ext} = 0$

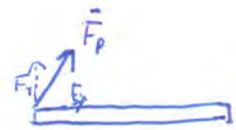
$\tau_P = 0$ about any point

$\alpha = 0$

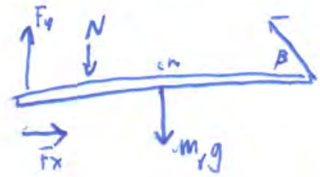
$a_{cm} = 0$

1) Free body diagram

pivot forces



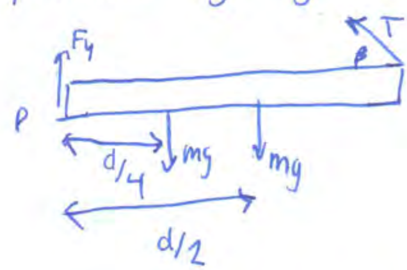
2) Choose P to get $\tau_P = 0$



$N - m_b g = 0$

$N = m_b g$

$\hat{j}$: $F_y + T\sin 30 - mg - mg = 0$

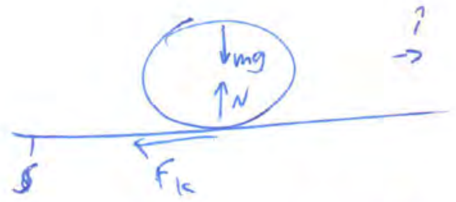


$\tau_P = -\frac{d}{4} mg\hat{k} - \frac{d}{2} mg\hat{k} + d\,T\sin\theta =$

$-\frac{3d}{4} mg + \frac{dT}{2} = 0$

$T = 3/2\, mg$

Concept Question 1



$P_1 \neq P_f$ b/c f_k

$E_1 \neq E_f$ b/c f_k

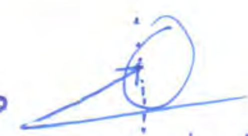
$\vec{L}_{S,i} = \vec{L}_{S,f}$

$$\tau_S = \overbrace{r_{S,f_k}}^{0} \times f_k + r_{S,N} \times N + r_{S,mg} \times mg$$

$0 + xN\hat{k} - x\,mg\,\hat{k}$

$N - mg = 0$

$= 0$



only $\perp$ distance counts

Angular Momentum about S is constant

treat as point object

pure rotation about cm independent of S

$$\vec{L}_S = r_{S,cm} \times m\vec{V}_{cm} + I_{cm}\vec{w}$$

11/25/13 # 8.01 Exam Review #3 Fernando Trujano

Collisions

Elastic - KE is conserved → $v_1 - v_2 = v_{2f} - v_{1f}$ ⇒ Relative speed does not change

Inelastic - KE is lost

Momentum constant if $F^{ext} = 0$

Energy constant if $W^{nc} = 0$

Angular Momentum constant if $\tau_p = 0$

Moment of inertia

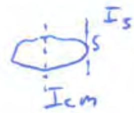
- measure of mass distribution

$v = r\omega$

$KE = \frac{1}{2}mv^2 \Rightarrow KE = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}\underbrace{mr^2}\omega^2$

Moment of Inertia $= I_z$ ⇒ $KE = \frac{1}{2}I\omega^2$

Parallel Axis theorem: $I_s = Md^2 + I_{cm}$



Torque

$\tau_{cm} = I_{cm}\alpha_{cm}$ $\quad$ $\tau_s = r_{sp} \times F_p$

$\hat{i} \times \hat{j} = \hat{k}$
$\hat{k} \times \hat{i} = \hat{j}$
$\hat{j} \times \hat{k} = \hat{i}$

- Torque causes angular acceleration
- Gravity acts at the center of mass

Angular Momentum

$\vec{L}_s = \vec{r}_s \times \vec{p}$

$p = mv$

$F\Delta T = \Delta P$

$\tau_s \Delta t = \Delta L_s$

$\vec{L}_o = I\omega_z$

if $\tau = 0$ then Angular Momentum is conserved

$v = r\omega \Rightarrow \omega = \frac{v}{r}$

$U = KE_{trans} + KE_{rot} = mgh = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{V_{cm}^2}{2}\left(m + \frac{I_{cm}}{R^2}\right)$

$I = \frac{1}{2}mr^2$ so if more mass is concentrated on middle $r\downarrow \Rightarrow I\downarrow$. $L_o = I\omega \Rightarrow \omega\uparrow$

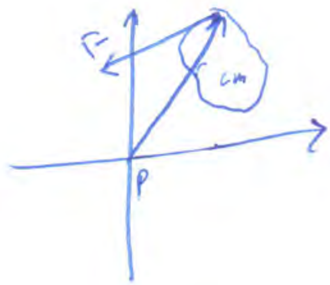
11/25/13 — 8.01 MAS Review — Fernando Trejo

① Concepts

$$\vec{L}_p = r_{p,cm} \times \hat{P}_{cm} + I_{cm}\hat{w}$$

$$T_p = \vec{r}_{p,s} \times \hat{F}_s$$

$$\hat{i} \times \hat{j} = \hat{k} \qquad \vec{T}_p = I_p \vec{\alpha}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$



$$I_p = \sum_{i=1}^{N} \Delta m_i \, r_{p,i}^2$$

$$I_p = I_{cm} + Md^2$$

one-dimensional elastic $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$

② "Before" and "After"

i) $\vec{P}_i = \vec{P}_f$ if $F_{ext}\Delta t = 0$

ii) $E_i = E_f$ if $W_{nc} = 0$ ... ie. Elastic

iii) $L_f = L_i$ if $\underbrace{T_p \Delta T}_{\text{Angular impulse}} = 0$

③ "During"

- Find acceleration "during"

i) $F_{ext} = M a_{cm}$

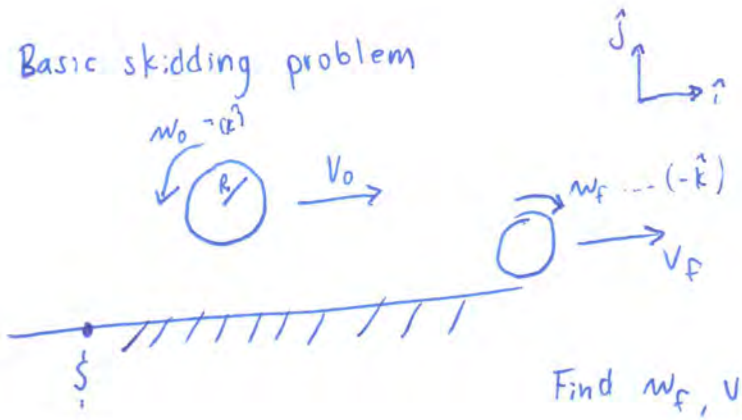
ii) $\vec{\tau}_{cm} = I_{cm} \vec{\alpha}$

$s = r\theta$

$v = r\omega$

$a = r\alpha$

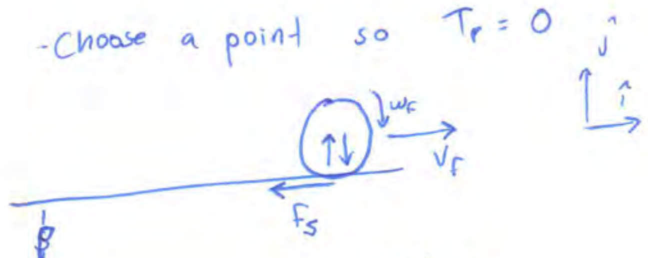
Basic skidding problem



Find ω_f, V_f

- "Before" and "after" problem

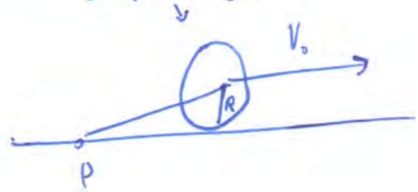
- Choose a point so $T_P = 0$



$F^{ext} \neq 0$

$W_{nc} \neq 0$

$$L_i = \underbrace{r_{P,cm} \times M V_0 \hat{i}}_{= RMV_0 (-\hat{k})} + I \omega_0 \hat{k}$$



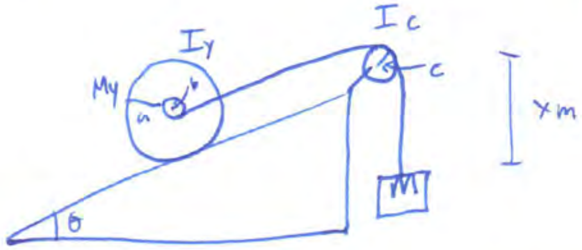
$$L_f = RMV_f (-\hat{k}) - I_{cm} \omega_f \hat{k}$$

Rolling w/o slipping at end so $V_f = R\omega_f$

Yo-Yo

l_0

Rolling w/o slipping



"During" Problem

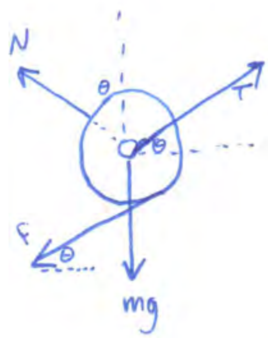
$F_{ext} = M\vec{a}$

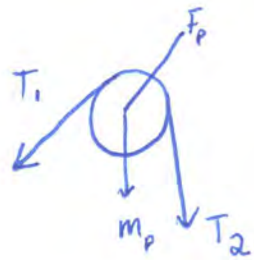
$\tau_{cm} = I_{cm}\alpha_{cm}$

$s = r\theta$

$v = r\omega$

$a = r\alpha$





$\tau = T_1 r_p \hat{k} - T_2 r_p \hat{k} = I_b \alpha_p \hat{k}$

r×r

$\tau_y = T_1 b\,\hat{k} - fa\,\hat{k}$

$\tau_y = I_y \alpha_y \hat{k}$

$T_1 b - fa = I_y \alpha_y$

$F = ma$

$-f + T_1 - m_y g \sin\theta = m_y a_y$

$r_y \alpha_y = a_y$

$\alpha_y b$

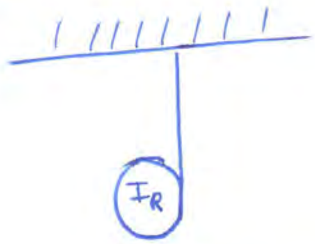
$a_b = r_p \alpha_y$

wrapping around

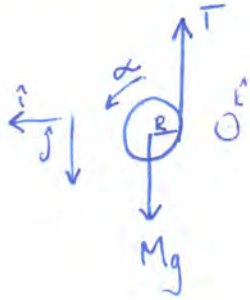
$l = l_0 + x_b + x_y + b\theta$

$0 = -a_b + a_y + b\alpha$

During Problem



- unrolls and falls down.



$$Mg - T = ma_y$$

$$T = RT = I\alpha$$

check directions
a and α in $\hat{j}$ dir

$$a_y = R\alpha_z$$

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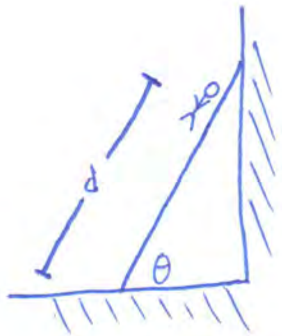
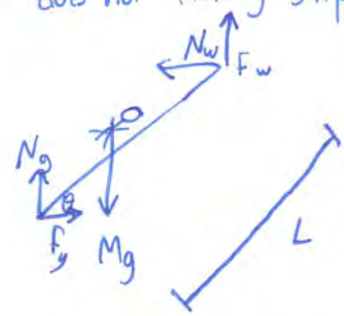


Find θ so ladder does not fucking slip.

$F_{ext} = 0$

$T_{ext} = 0$



$$F_{ext} = 0 \quad \begin{cases} N_g - M_g + \overbrace{\mu_{wall} N_{wall}}^{f_w} = 0 \\ \overbrace{\mu_g N_g}^{f_g} - N_w = 0 \end{cases}$$

$$T_{ext} = 0 \quad -Mgd\cos\theta + N_w L \sin\theta + \mu_s N_w \cos\theta = 0$$

Random Fact

- Static Friction doesn't do work

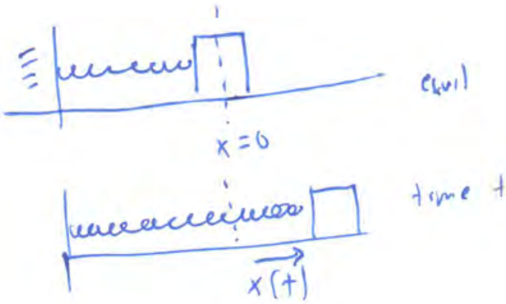
11/18/13

8.01 Lecture

Fernando Troja

Simple Harmonic motion

Hooke's Law



$$F = ma$$

$$-kx = \frac{m\, d^2x}{\cancel{m}\, dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}$$

Try it yourself

★ $x(t) = A\cos\left(\frac{2\pi}{T}\right)t$

$$\frac{dx}{dt} = -\frac{2\pi}{T} A \sin\left(\frac{2\pi}{T}\right)t$$

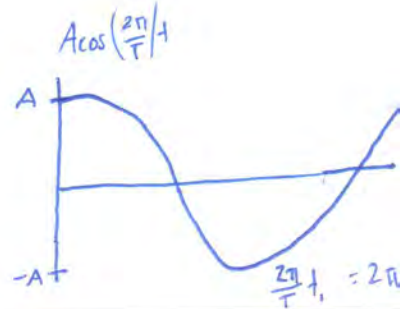
$$\frac{d^2x}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \underbrace{A\cos\left(\frac{2\pi}{T}t\right)}_{x}$$

$$\frac{d^2x}{dt} = -\left(\frac{2\pi}{T}\right)^2 x$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$$

$$\frac{2\pi}{T} = \sqrt{k/m}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$



$= 2\pi\sqrt{\frac{m}{k}}$

$$\omega_0 \overset{\text{def}}{\equiv} 2\pi \underset{\text{frequency}}{F} = \frac{2\pi}{T}$$

angular frequency

↳ not speed

rad/sec

$$\boxed{x(t) = A\cos(\omega_0 t) \qquad \omega_0 = \sqrt{\frac{k}{m}}}$$

Solution to Simple Harmonic Oscillator

(1) $x(t) = C\cos(\omega_0 t) + D\sin(\omega_0 t)$

$= x_1(t) + x_2(t)$

$\frac{d^2 x_i}{dt^2} = \frac{-k}{m} x_i \quad i = 1,2$

Super Position Principle

(2) $V_x(t) = \frac{dx}{dt} = -\omega_0 C\sin(\omega_0 t) + \omega_0 D\cos(\omega_0 t)$

C, D are constants

Initial conditions

$x(t=0) = X_0$

$V_x(t=0) = V_x(0)$

Set $t=0$ in (1) and (2)

$x(0) = C + 0 \quad C = X_0$

$V_x(0) = \omega_0 D$

$D = \frac{V_x(0)}{\omega_0} = \frac{V_x(0)}{\sqrt{k/m}}$

General Solution

$x(t) = X_0\cos(\omega_0 t) + \frac{V_x(0)}{\sqrt{k/m}}\sin(\omega_0 t)$

can do it:

$V_x(t) = \ldots$

$x(t) = A\cos(w_0 t + \phi)$

≥0

A = amplidude
min and max displacement

ϕ = phase constant

$w_0 t + \phi$ = phase

$x(t) = A\cos w_0 t \cos\phi - A\sin(w_0 t)\sin\phi$

Compare to
General Solution

$x(t) = \cancel{x_0 \cos w_0 t}\ x_0\cos(w_0 t) + \frac{V_x(0)}{w_0}\sin(w_0 t)$

★

$\Rightarrow A\cos\phi = x_0$ (1)

$-A\sin\phi = \frac{V_x(0)}{w_0}$ (2)

$\tan\phi = \frac{-V_x(0)}{(w_0)(x_0)}$

$(1)^2 + (2)^2$

$\Rightarrow A = \sqrt{x_0^2 + \left(\frac{V_{x0}}{w_0}\right)^2}$

$x_0 > 0$ $V_x(0) \neq 0$

$\phi \neq 0$ $A = x_0$

$A = \sqrt{x_0^2 + \frac{V_x(0)}{V_0}}$

constant

$x(t) = A\cos(w_0 t + \phi)$ $w_0^2 = \frac{k}{m}$

$E(t) = \frac{1}{2}k(A^2\cos^2(w_0 t) + \frac{1}{2}m w_0^2 A \sin^2(w_0 t + \phi)$

$= \frac{1}{2}kA^2(\cos^2(w_0 t) + \sin^2(w_0 t)$ $= \frac{1}{2}kA^2$

ϕ

3/4

Energy Method

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \text{constant}$$

$$\frac{dE}{dt} = 0$$

x, v are functions of time

$$\frac{dE}{dt} = \frac{1}{2} m v \frac{dv}{dt} + \frac{1}{2} k x \frac{dx}{dt}$$

$$0 = v \left(m \frac{d^2x}{dt^2} + kx \right)$$

$$\frac{dv}{dt} = \frac{d^2x}{dt^2}$$

if $v = 0$

Trivial

Non trivial

$$\frac{m\,d^2x}{dt^2} + kx = 0$$

same answer

$$-kx = m\frac{d^2x}{dt^2}$$

for small angle

$\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} = 0$$

$$\omega_0 = \sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

$\theta_0, \frac{d\theta}{dt}$

$$\theta(t) = \theta_0 \cos(\omega_0 t) + \frac{\omega_0(0)}{\omega_0} \sin \omega_0 t$$

12/2/13 8.01 Lecture Fernando Trujano

$$\tau_s = \frac{d\hat{L}_s}{dt} \Rightarrow \text{dir } T_{cm} = \text{dir } \Delta\vec{L}_{cm}$$

direction of L changes, in time direction.

$$L_S = L^{orbital}_{S,cm} + L^{spin}_{cm} = r_{S,cm} \times m_T V_{cm} + L^{spin}_{cm}$$

Time Derivative of a Vector w/ constant Magnitude that changes directions

↳ Example: Circular Motion

$\vec{v} = \frac{d\vec{r}}{dt}$

dir $\vec{v}$ = dir $\Delta\vec{r}$

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta}$$

$|\vec{r}| = R$

dir $\vec{r}$ is changing!

$|\vec{v}|$ = constant = V

$\vec{a} = \frac{dV}{dt}$

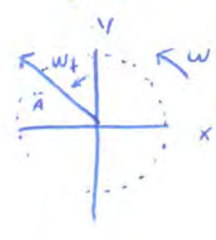
$= \underset{mag}{|v|}\ \underset{rate}{w}\ \underset{dir}{(-\hat{r})}$

a and v change in directions!

$V = \underset{magnitude}{|\vec{r}|}\ \underset{rate}{|w|}\ \underset{\text{Perp to } \hat{r}}{\hat{\theta}}$

* Concept Question: Rotating Vector

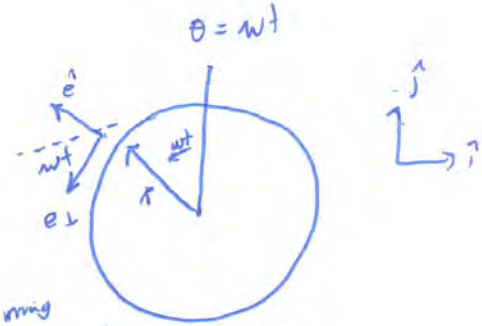
A vector $\vec{A}(t)$ of fixed length A is rotating about the z-axis at an angular speed w. At t=0 it is pointing in the positive y direction. $\vec{A}(t)$ is given by



$$A(t) = -A\sin(wt)\hat{i} + A\cos(wt)\hat{j}$$

$A(t) = -A\sin(wt)\,\hat{i} + A\cos(wt)\,\hat{j}$

$\frac{dA}{dt} = -A\cos(wt)w + -Aw\sin(wt)$

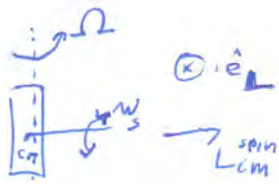


$$\frac{dA}{dt} = \underbrace{|A|}_{mag}\,\underbrace{w}_{rate}\,\underbrace{(e_\perp)}_{perpendicular}$$

$$= |A|w(-\cos wt\,\hat{i} - \sin wt\,\hat{j})\ \checkmark$$

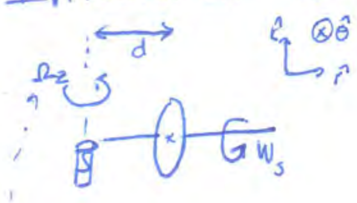
$$\left|L_{cm}^{spin}\right| = I_{cm} w_s$$

$$\frac{dL_{cm}^{spin}}{dt} = \underbrace{|L|}_{mag}\,\underbrace{\Omega}_{rate}\,\underbrace{\hat{e}_\perp}_{dir}$$



$$= (I_{cm}W_s)\,\Omega\,\hat{e}_\perp$$

Introduction to Gyroscopic Motion



$w = w_s$

Precessional angular velocity

$\Omega = \Omega_z \hat{k}$

Torque causes angular momentum to change

$$T_s^{ext} = \frac{dL_s}{dt} \quad \text{- Gyroscopic approx}$$

$$T_s^{ext} = \underbrace{|L_{s,\perp}|}_{mag}\,\underbrace{\left|\frac{d\theta}{dt}\right|}_{rate}$$

$\frac{d\theta}{dt} = \Omega$

$\vec{F} = ma$ uniform circular motion

analysis

$m\frac{dv}{dt}$

$m|\vec{v}|w(-\hat{r})$ $\qquad \vec{v} = rw$

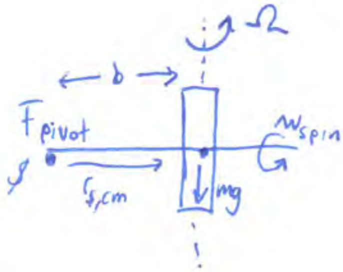
$-\frac{mv^2}{R}\hat{r}$ $\qquad a = \frac{dv}{dt}$

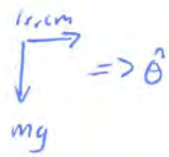
$-mRw^2\hat{r}$

$$\tau_s = \frac{d\vec{L}_s}{dt}$$

analysis

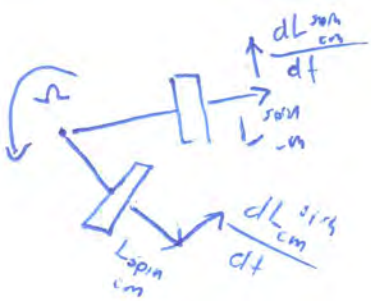
$\underbrace{|L_{s,\perp}|}_{\text{more to this}}\,\Omega\,\hat{e}_\perp$



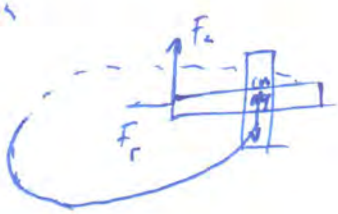


τ_s =	
$\vec{r}_{s,cm} \times m\vec{g}$	
$b\,mg\,\hat{\theta}$	$= (I_{cm} w_s)(\Omega_z \hat{e})$

Torque about S changes direction of angular momentum leaving magnitude unchanged. Flywheel processes, does not fall.



The radial component of the pivot force makes center of mass travel in rotational motion



$$\tau_{cm} = \frac{dL_{cm}^{spin}}{dt}$$

$$\bar{F}_r = \frac{m v_{cm}^2}{d} \quad \text{radial}$$

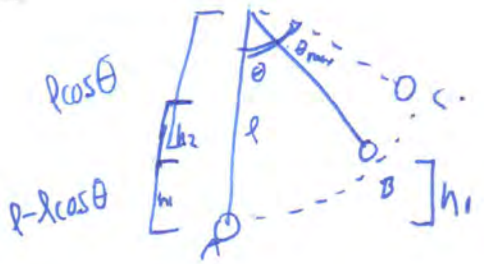
$$F_z - mg = 0 \quad \text{vertical}$$

$$F_z = mg$$

$$mgd = I_{cm}\, \omega_s\, \Omega_z$$

If you calculate about cm, you have to deal w/ pivot force.
↳ not about S

- See Slides



$\theta = \theta_{max} \cos(\omega t + \varphi)$

$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

$h_1 + h_2 = l - l\cos\theta$

Conservation of energy

$K_A + U_A = K_B + U_B = K_C + U_C$

$\frac{1}{2} m V_A^2 = \frac{1}{2} m V_B^2 + mgh_1 = mg(h_1 + h_2)$

$\frac{1}{2} m v_B^2 = mgh_2$

$h_2 = \frac{v_B^2}{2g}$

Same thing

Work Energy Theorem

$W_{AB} = K_B - K_A$

$= \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -mgh_1$

Simple Harmonic Oscillator

$x = x_{max} \cos(\omega t + \varphi)$

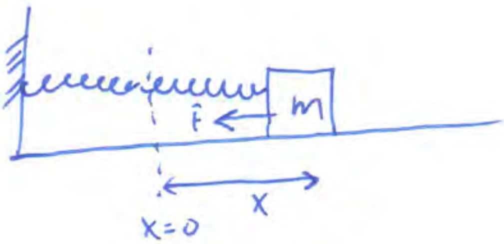
$\omega = \sqrt{k/m}$

$T = \frac{2\pi}{\omega}$

8.01

Simple Harmonic Motion

ocw



$$T = 2\pi\sqrt{\frac{m}{k}}$$

not dependent on x

Proof:

$$F = -kx$$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

differential equation

$$\star \quad \boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

$$\Downarrow$$

phase angle

$$x = A\cos(\omega t + \varphi)$$

amplitude

angular frequency

$$T = \frac{2\pi}{\omega} \Rightarrow \text{Period}$$

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \varphi)$$

$$= -\omega^2 x$$

→

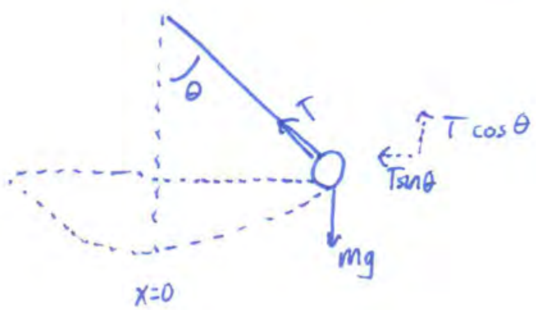
$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$-\omega^2 x + \frac{k}{m}x = 0$$

$$\omega^2 = \frac{k}{m}$$

$$\boxed{\begin{aligned} \omega &= \sqrt{\frac{k}{m}} \\ T &= 2\pi\sqrt{m/k} \end{aligned}}$$



$F = ma$

x: $-T(\theta)\sin\theta = -T(\theta)\frac{x}{l}$

$$m\frac{d^2x}{dt^2} = -T(\theta)\frac{x}{l} \qquad (1)$$

y: $$m\frac{d^2y}{dt^2} = T(\theta)\cos\theta - mg \qquad (2)$$

Small Angle Approximations $\theta << 1$

$\therefore \cos\theta = 1$

$\therefore \frac{d^2y}{dt^2} \approx 0$ no acceleration in y

(2) $0 = T(\theta) - mg$

$T = mg$

(1) $m\dfrac{d^2x}{dt^2} + mg\dfrac{x}{\ell} = 0$

$$\boxed{\frac{d^2x}{dt^2} + \frac{g}{\ell}x = 0}$$

Simple Harmonic Oscsallation

$$\boxed{\begin{gathered} x = A\cos(\omega t + \varphi) \\ \omega = \sqrt{\tfrac{g}{\ell}} \\ T = \tfrac{2\pi}{\omega} \therefore T = 2\pi\sqrt{\tfrac{\ell}{g}} \end{gathered}}$$