6.002 Math Review

Monday, September 12, 2016 7:04 AM

Linear First Order Constant Coefficient Differential Eqn.

$Ax' + Bx = f(t)$ A, B const

(want to solve $x(t)$)

Superposition Principle

Suppose x_p is soln to ①

$\rightarrow Ax_p' + Bx_p = f(t)$

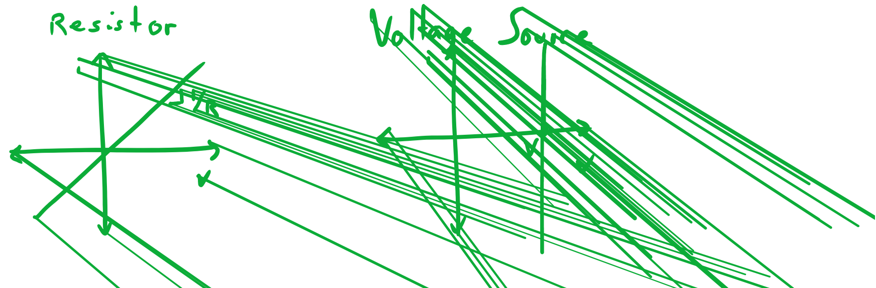
and x_h is homogeneus solution

$Ax_h' + Bx_h = 0$

then $x = x_p + x_h$ is solution to ①

W1: Circuit Elements

Tuesday, September 13, 2016 10:12 AM



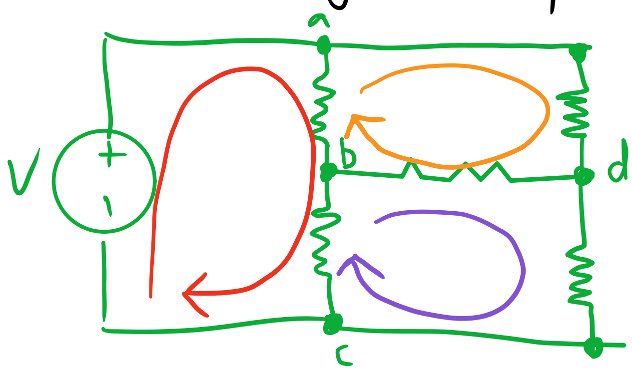
Formulas:

$$V = IR$$
$$P = IV$$

KVL (Kirchoffs Voltage Law)

- Sum of voltages in loop = 0

$$\sum_{j \text{ loop}} V_j = 0$$

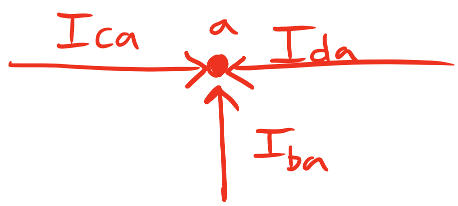


$$V_{ca} + V_{ab} + V_{bc} = 0$$

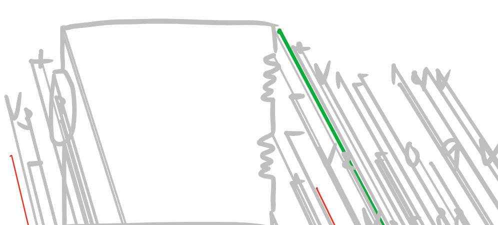
KCL

Sum of currents into node = 0

$$\sum_{i \text{ node}} i_i = 0$$

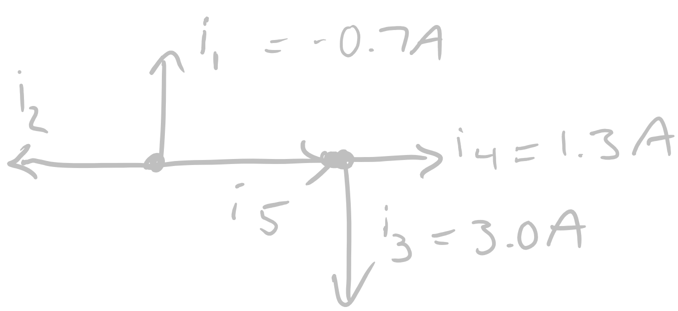


Sample Problem



By KVL $V_3 - V_1 + V_2 = 0$

$V_3 = V_1 - V_2$



$$i_5 = i_4 + i_3 = 4.3A$$

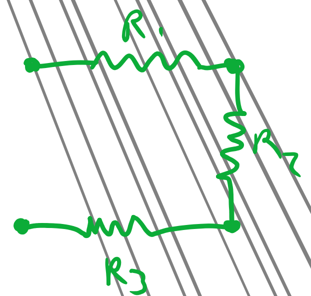
$$-4.3 + .7A - i_2 = 0$$

$V_B = V_A - V_B$

$-4.3 + .7A - i_2 = 0$

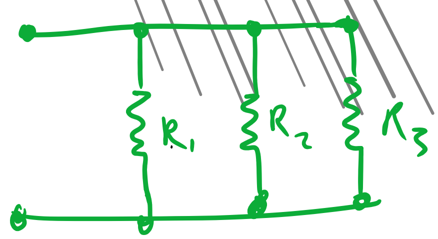
Resistors

Series:



$R_T = R_1 + R_2 + R_3$

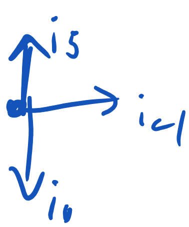
Parallel:

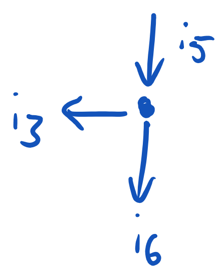


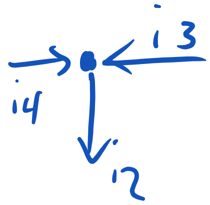
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

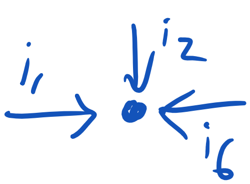
Scratch Space

Given i_1, i_2, i_3









$-i_4 + i_2 - i_3 = 0$

$i_4 = i_2 - i_3$

$i_1 + i_2 + i_6 = 0$

$\Rightarrow i_6 = -i_1 - i_2$

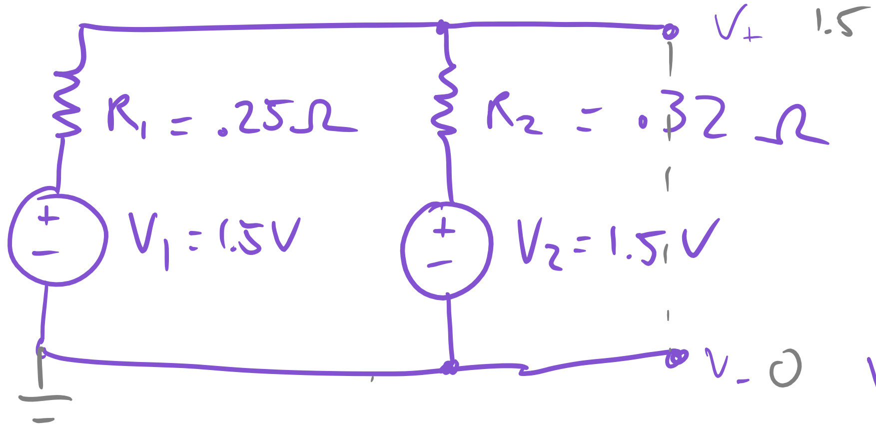
$\Rightarrow i_6 = -i_1 - i_2$

$-i_5 - i_4 - i_6 = 0$

$i_5 = i_3 + i_1$

$i_5 = -i_4 - i_6$

$-\cancel{i_2} + i_3 + i_1 + \cancel{i_2}$



$V_+ - V_- = ?$

$V = 1.5$

$V = IR$

$I = R/V$

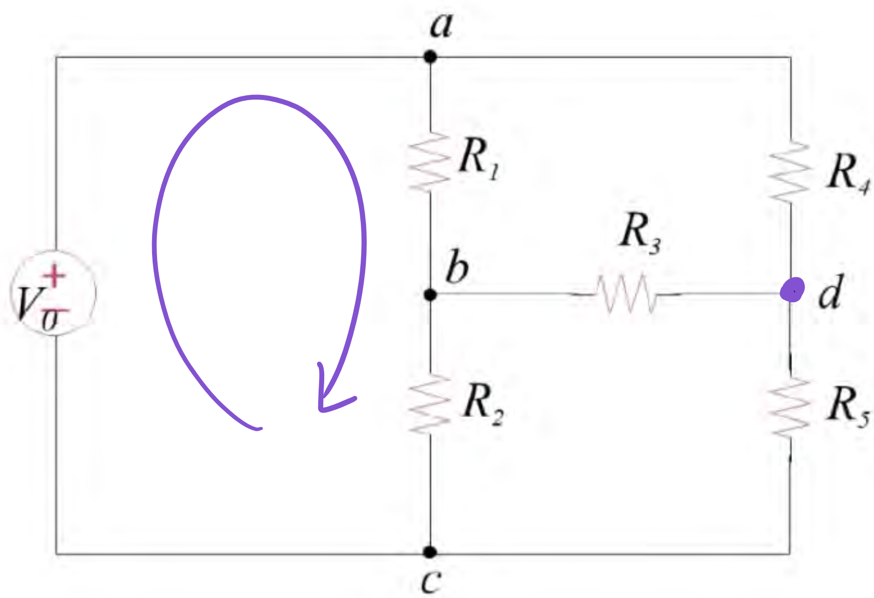
W1: Circuit Analysis Toolset

Wednesday, September 14, 2016 1:29 PM

Review

KVL: $\sum_j v_j = 0$ For all loops

KCL: $\sum_j i_j = 0$ For all nodes



KVL, KCL Method (Method 1)

Goal: Find all elements v's and i's

1) Write element v-i relationships
2) Write KCL for all nodes
3) Write KVL for all loops.
4) Solve eqns

3) Write KVL for all loops.

4) Solve eqns

Conventions

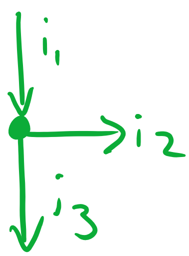
Assign currents flowing into positive terminal

$V = iR$

$v = V_0$

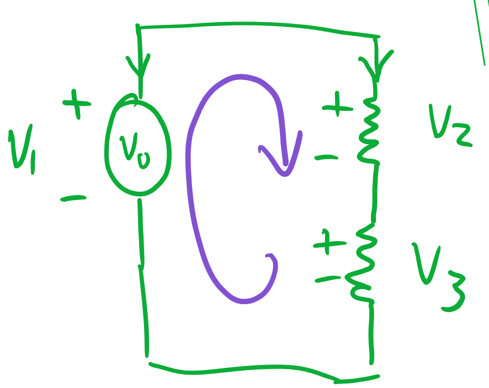
$i = I_0$

Currents leaving node are positive



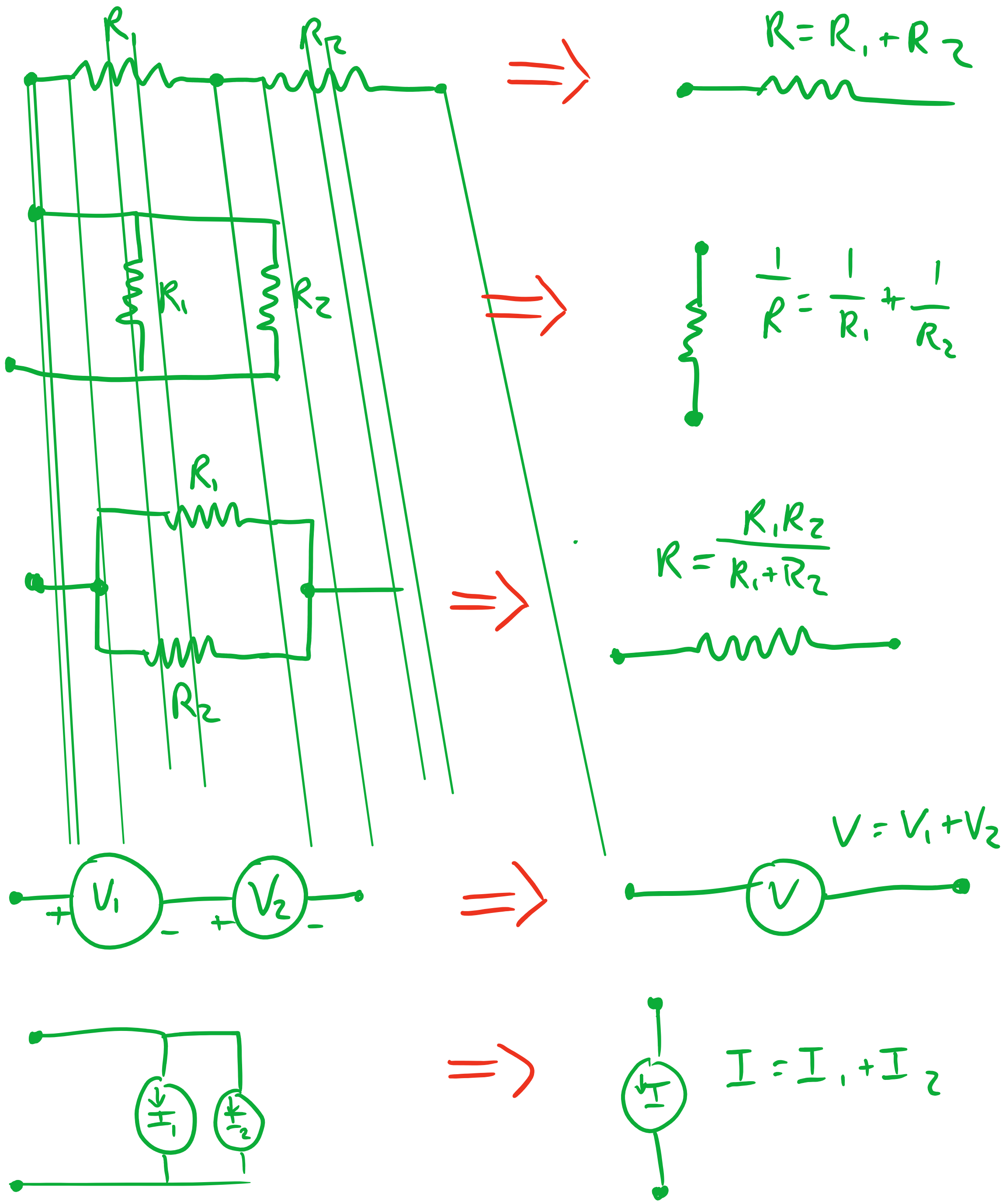
$i_2 + i_3 - i_1 = 0$

In KVL, assign voltage to first encountered sign



$-V_1 + V_2 + V_3 = 0$

Element Combinations (Method 2)



$R = R_1 + R_2$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$R = \frac{R_1 R_2}{R_1 + R_2}$

$V = V_1 + V_2$

$I = I_1 + I_2$

Node Analysis (Method 3)

1) Select reference node $\perp$ from which voltages are measured.

2) Label voltages of remaining nodes w/ respect to ground. ← primary unknowns

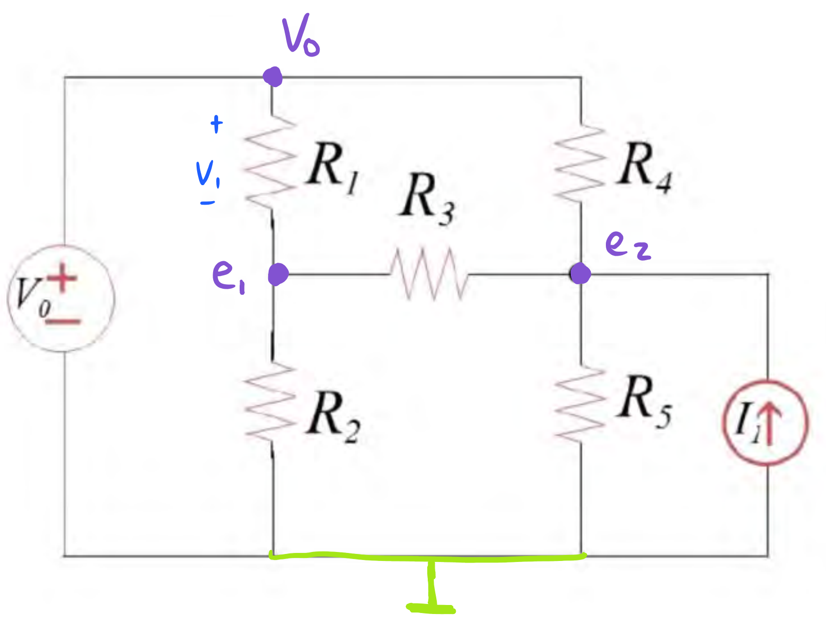
3) Write KCL for all but ground node (substituting device laws and KVL)

Zf5

4) Solve for node voltages

5) Back solve for branch voltages and currents

Example:



1) Ground node $\perp$

Tips: Pick with:

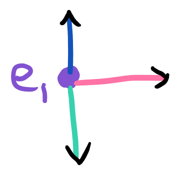
- Largest # of elements connected to it
- Largest # of sources connected to it

2) Label voltages

3) Write KCL

For convenience: $G_i = \frac{1}{R_i}$

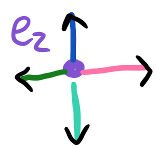
KCL e_1



(1) $$\underbrace{(e_1 - V_0)}_{V} \underbrace{G_1}_{1/R_1} + (e_1 - e_2) G_3 + (e_1 - 0) G_2 = 0$$

$I = V/R$

KCL e_2



(2) $$(e_2 - e_1) G_3 + (e_2 - V_0) G_4 + -I_1 + (e_2 - 0) G_5 = 0$$

4) Solve voltages

1) Move constants to RHS

(1) $$e_1(G_1 + G_2 + G_3) + e_2(-G_3) = V_0 G_1$$

(2) $$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0 G_4 + I_1$$

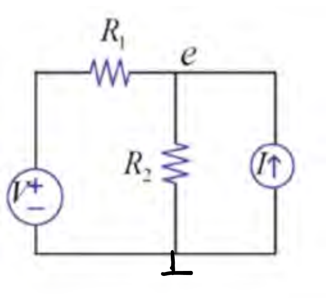
5) Branch voltages

$$V_1 = V_0 - e_1$$

$$i_1 = \frac{V_i}{R_1} = \frac{V_0 - e_i}{R_1}$$

W2: Linearity

Monday, September 19, 2016 6:45 PM



Node Method

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0$$

Linear!

$$\Rightarrow \underbrace{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}_{\text{Conductance Matrix } G} e = \underbrace{\frac{V}{R_1} + I}_{\text{sum of sources } S}$$

$Ge = S$

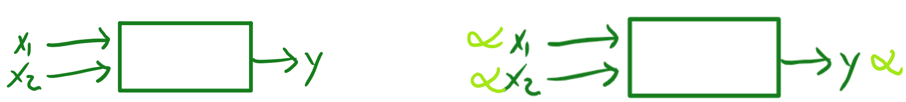
$[\because][:] = [:]$

$$e = \frac{R_2 V}{R_1 + R_2} + \frac{R_1 R_2 I}{R_1 + R_2}$$

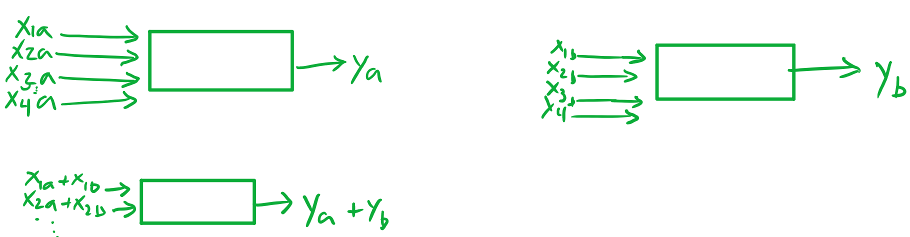
Linear

$e = a_1 V_1 + a_2 V_2 + \ldots$

Homogeneity



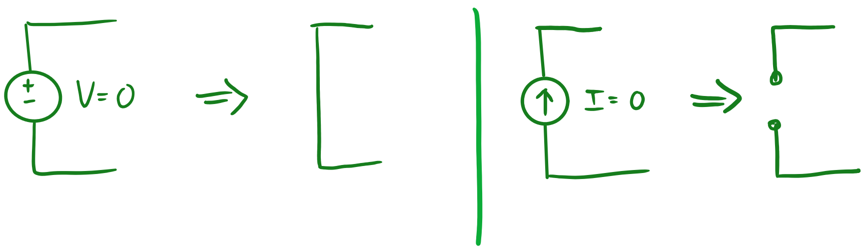
Superposition



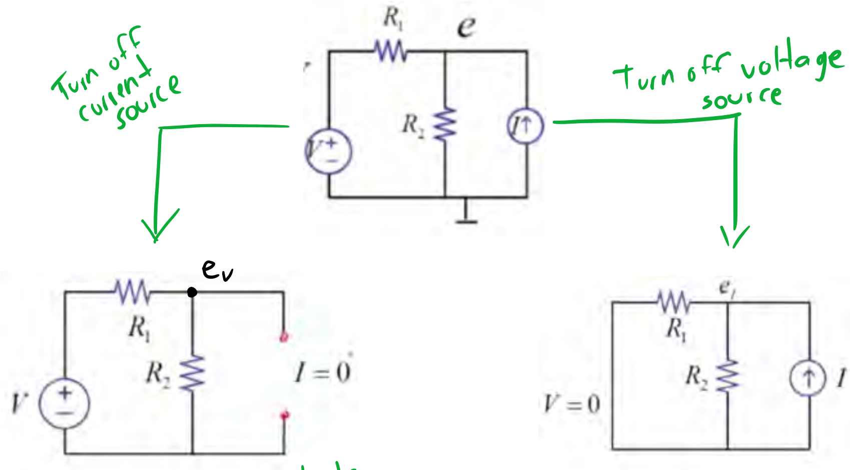
Superposition Method

1) Find responses to each source acting alone

2) Sum the individual responses



Example:

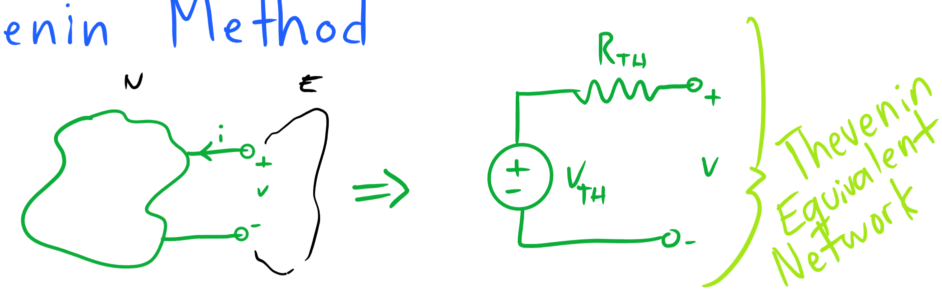


$$e_v = \frac{R_2}{R_1+R_2}V$$ shortcuts

$$e_I = \frac{R_1R_2}{R_1+R_2}I$$

$$e = e_v + e_I = \frac{R_2}{R_1+R_2}V + \frac{R_1R_2}{R_1+R_2}I$$

Thevenin Method



V_{TH}

- Don't connect anything and measure voltage

R_{TH}

Turn off sources and measure resistance

The easiest way to calculate the Thevenin resistance is to connect a short circuit to the output terminals and calculate the current through it. Rth is then Vth/Isc, where Isc is the short-circuit current.

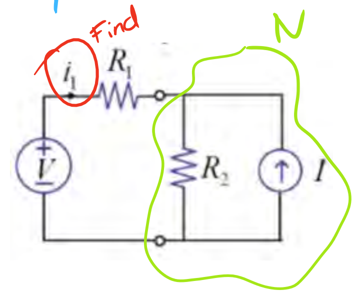
1) Replace network N with Thevenin Equivalent

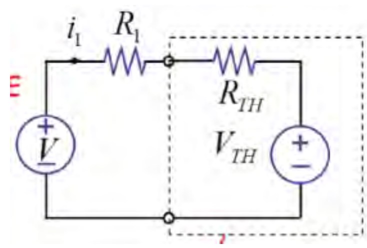
2) Solve external network E.

Example:

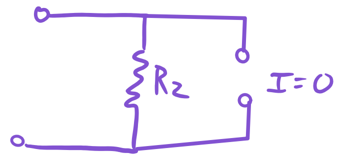
STEP 1

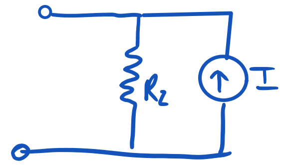
Example:





STEP 1

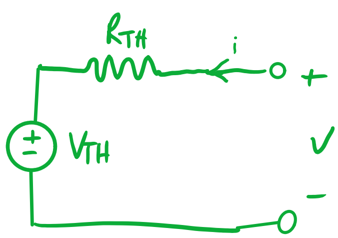


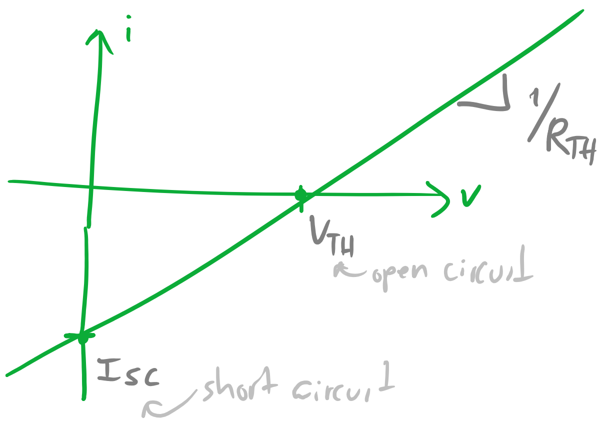


$V_{TH} = R_2 I$
$R_{TH} = R_2$

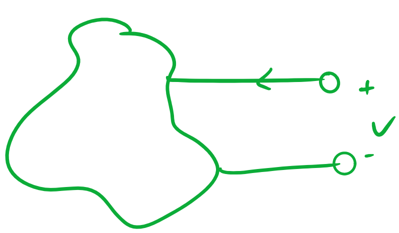
$$i_1 = \frac{V - V_{TH}}{R_1 + R_{TH}}$$

GRAPHICALLY

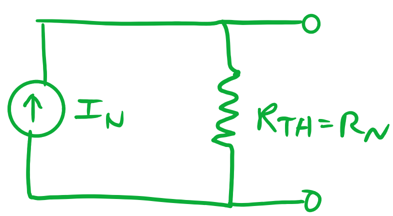




Norton Method



$\Longrightarrow$



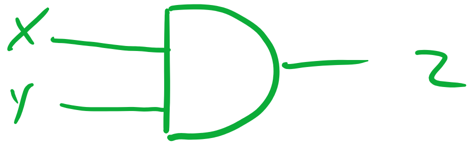
$I_N \rightarrow$ Short circuit current at port
$R_N \rightarrow$ Resistance seen from port with sources turned off

$R_N \rightarrow$ Resistance seen from port with sources turned off

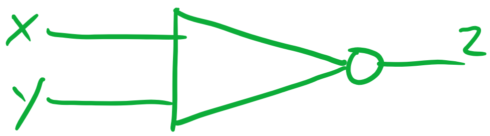
To calculate the Norton resistance, we first calculate the Thevenin voltage by leaving the output terminal as an open circuit. The circuit can be solved by the node method, but it is also possible to use the loop method. With the loop method, we get one equation for an assumed current "i" with its positive sign leaving the positive terminal of the independent source

W2: Static Discipline and Boolean logic

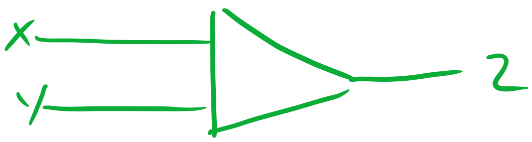
Tuesday, September 20, 2016 1:58 PM



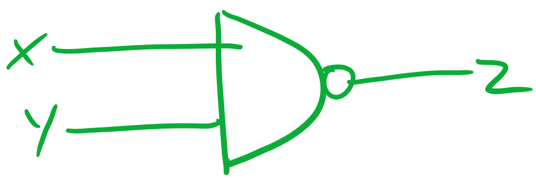
AND Gate



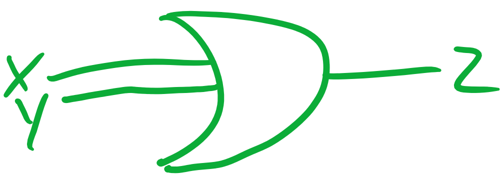
NOT Gate



BUFFER Gate



NAND



OR

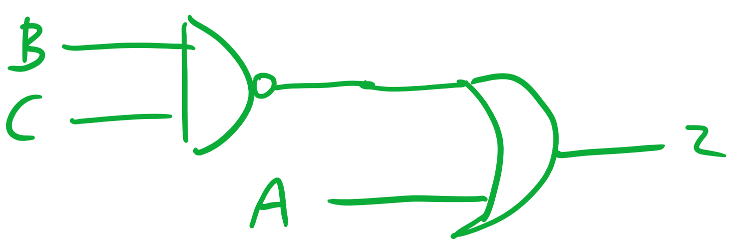
Example

Implement $A + \overline{B \cdot C} = Z$

NOT

OR

AND

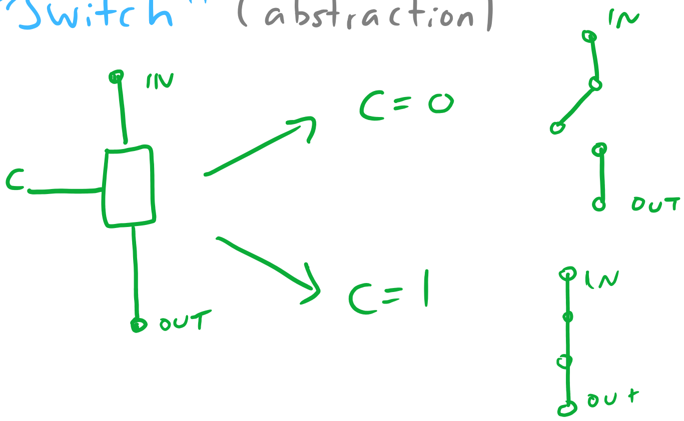


W3: Gates

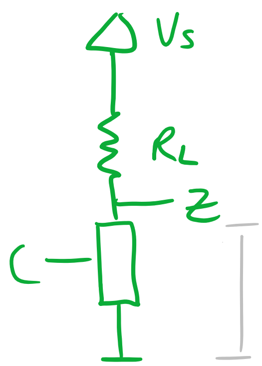
Wednesday, September 28, 2016 4:38 PM

How to build a digital gate

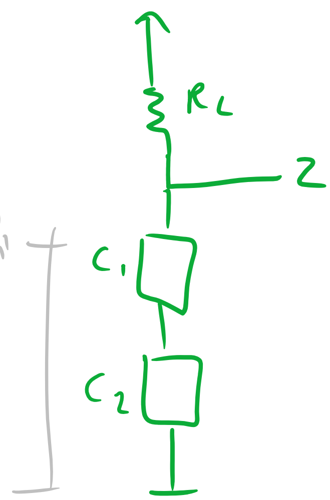
"Switch" (abstraction)



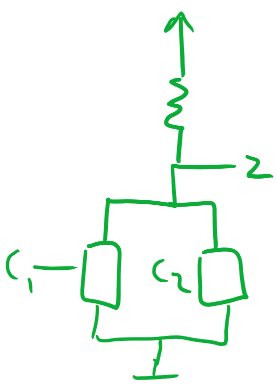
Inverter



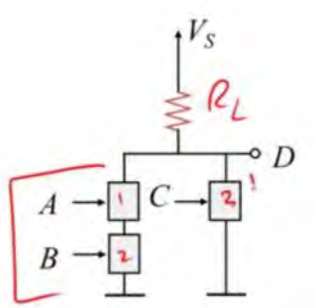
NAND



NOR



Compound Gate

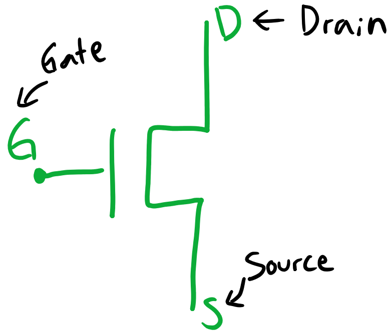


$$D = \overline{(A \cdot B) + C}$$

MOSFET

Transistor

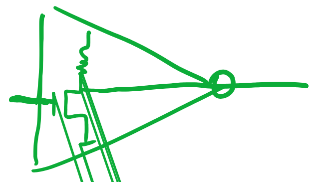
MOSFET ≈ Transistor



S-Model

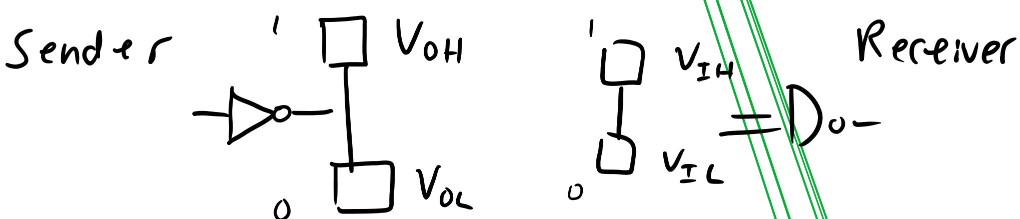
Can behave as a simple switch

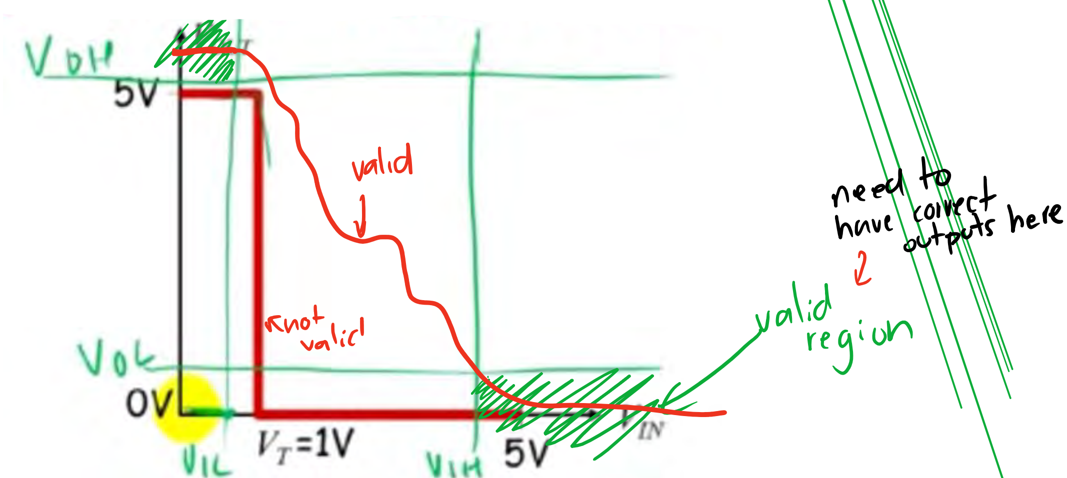
Mosfet Inverter



Static Disipline questions:

Make sure mosfet is "speaking the same language" so it can work with other components

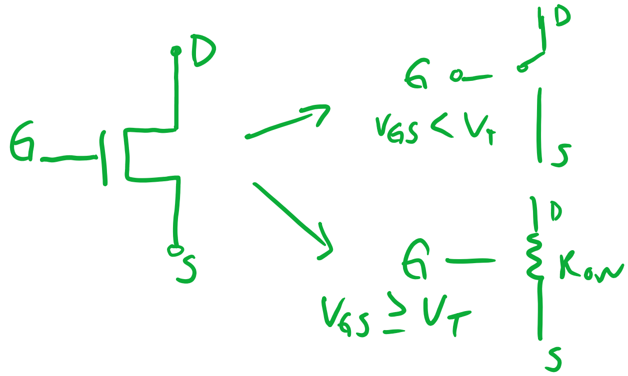


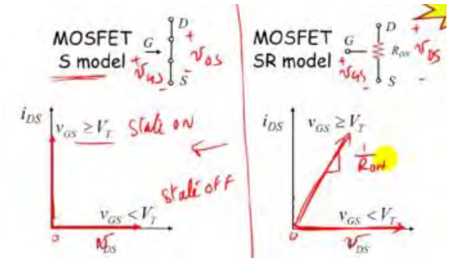


SR-Model for Mosfet

↳ More refined than S-Model

↳ More refined than S-Model



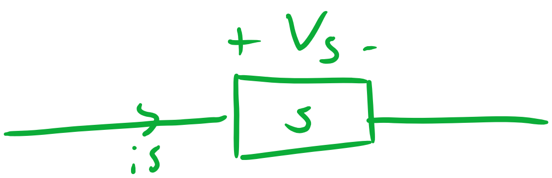


W3: Nonlinear Elements

Friday, September 30, 2016 7:17 PM

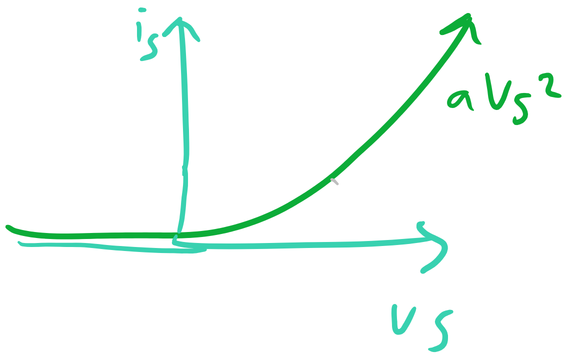
Recall: Cannot use substitution or Thevenin/Norton

Non linear Element

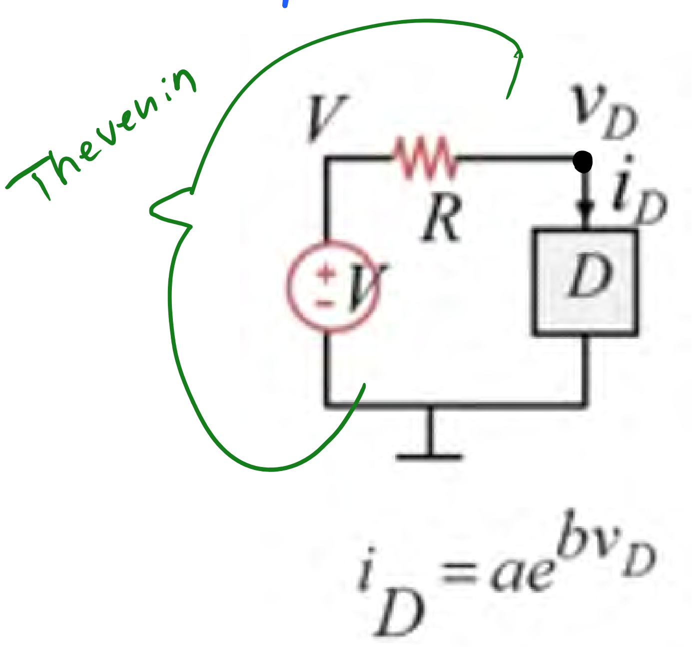


$$i_s = a v_s^2 \quad \text{for } v_s \geq 0$$
non linear

$$0 \quad \text{for } v_s < 0$$



Analytical Method (method 1)



Use Node Method

$$\frac{V_D - V}{R} + i_D \quad ①$$

$$i_D = a e^{bV_D} \quad ②$$

2) Solve by: Trial and error or numerical techniques

$$\Rightarrow V_D = 0.56V$$

Graphical Method (method 2)

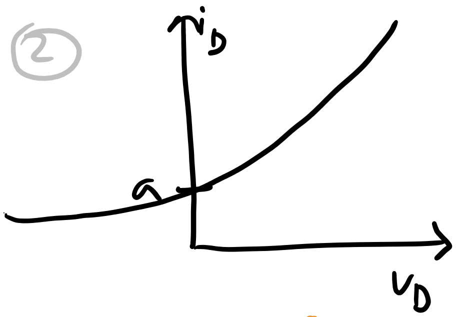
1) Use Node Method and Graph

$$\frac{V_D - V}{R} + i_D \quad (1)$$

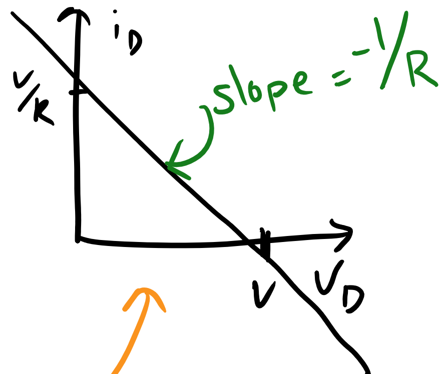
$$i_D = \frac{V}{R} - \frac{V_D}{R} \quad (1')$$

$$i_D = a e^{bV_D} \quad (2)$$

(2)

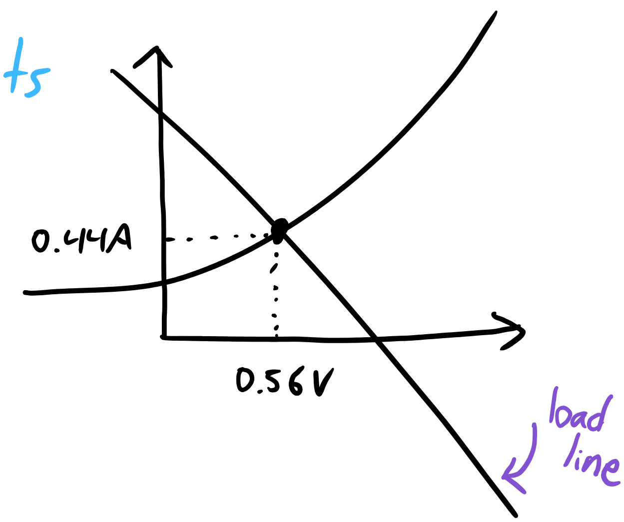


(1')



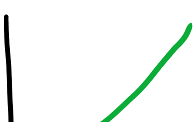
Constraint on i_D and V_D by device

2) Combine constraints



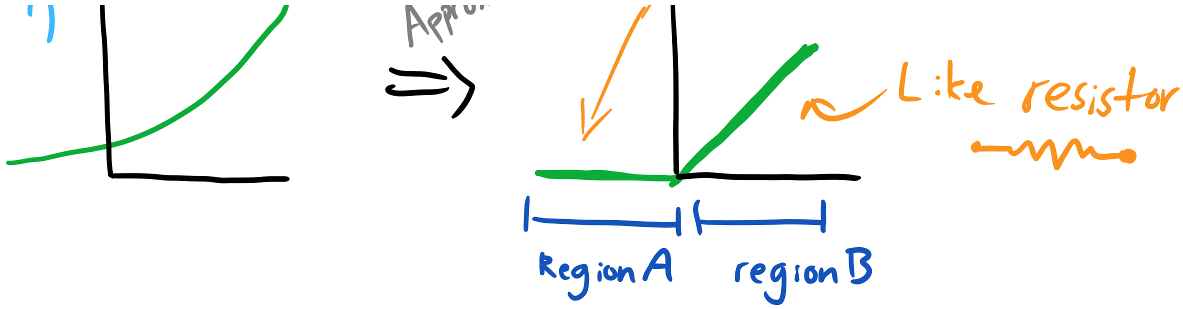
"not as important for course"

Piecewise Linear Method (Method 3)

1)  Approximate $\Rightarrow$ 

Like open circuit

Like resistor



2) Determine which of the linear region applies

3) Solve circuit assuming linear relation

Incremental Method (method 4)

"small signal method"

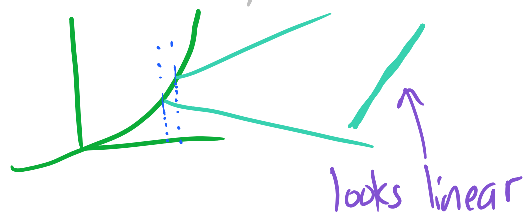
W4: Incremental Analysis

Saturday, October 1, 2016 8:10 PM

Small signal Method

- Get a linear response from something non-linear by focusing on small region of operation

  Boost + Shrink



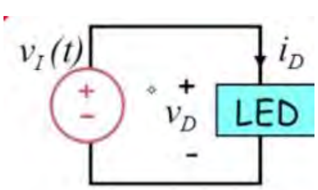
Incremental Method

1) Operate at some DC offset V_D, I_D
2) Superimpose small signal v_D on top of V_D
3) Notice that response i_d to small signal v_D is approx linear

NOTATION: $i_D = I_D + i_d$

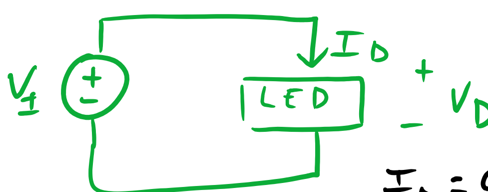
Total signal (i_D), DC offset (I_D), small superimposed signal (i_d)

Circuit View



$$v_D = f(v_D) = a\,e^{bV_D}$$
$$I_D = a\,e^{bV_D}$$
$$i_d = I_D \cdot b \cdot v_d$$

1) Write large signal circuit

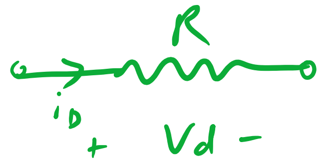


$I_D = ae^{bV_D}$

may need to solve a non-linear circuit to get I_D

2) Small Signal Response

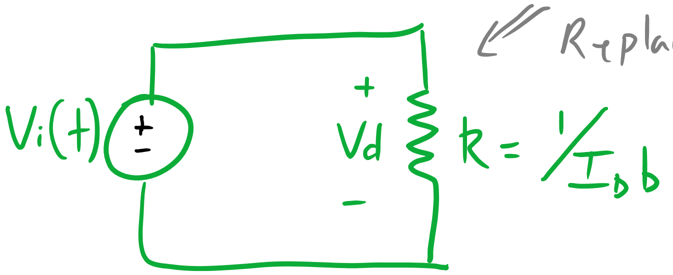
$$i_d = I_D \cdot b \cdot v_d$$



$\Rightarrow R = \frac{1}{I_D b}$

$\Rightarrow$ device was behaving like a resistor for small signals!

Replace by small signal equivalent



Small signal circuit is linear

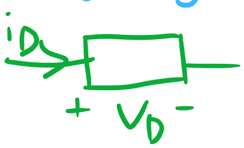
$$i_d = \frac{v_i}{R} = v_i \cdot I_D b$$

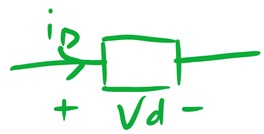
Small Signal Circuit Method

1) Find operating point using DC bias inputs from large signal circuit.
   Non linear analysis

2) Develop small signal (linearized) models for each element around the operating point.

Large Signal $\Rightarrow$ Small Signal





$$i_D = f(v_D) \qquad i_d = \frac{\partial f(v_D)}{\partial v_D}\bigg|_{v_D = V_D} \cdot v_d$$

$$\left.\frac{}{\partial v_D}\right|_{v_D = V_D}$$

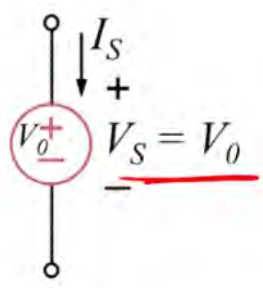
Derivative

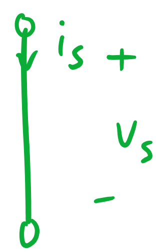
3) Analyze linearized circuit to get small signal response

Step 2 for different Elements

Voltage Source

Large Signal | Small signal



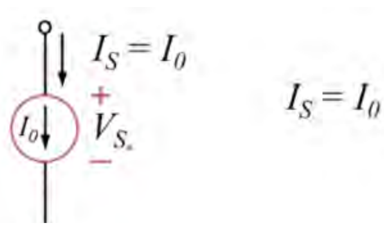


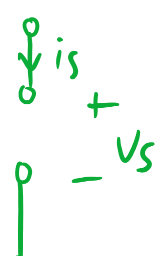
$$v_s = \left.\frac{\partial V_0}{\partial i_s}\right|_{i_s = I_s} \cdot i_s$$

$$= 0$$

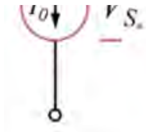
Current Source

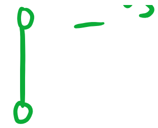
Large Signal | Small Signal





(no current)





(no current)

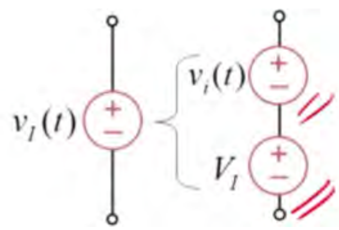
i_s does not change if v_s changes

$$i_s = \frac{\delta f(v_s)}{\delta V_s}\bigg|_{v_s = V_s} \cdot v_s$$

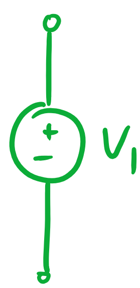
$$= \frac{\delta I_0}{\delta V_s}\bigg|_{V_s = V_s} \cdot v_s$$

$$= 0$$

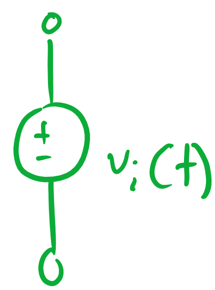
Voltage Source w DC and small signal



Large Signal



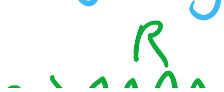
Small signal



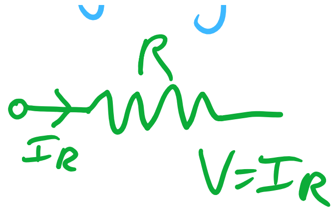
Resistance

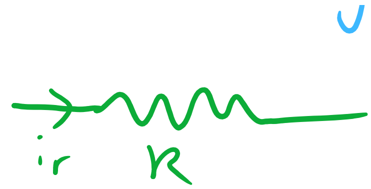
→ already linear element

Large Signal



Small Signal

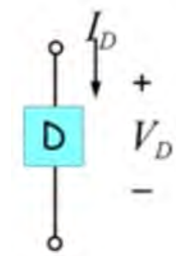




$V_r = i_r R$

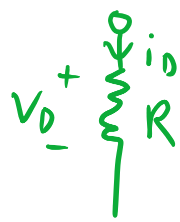
Non-linear device

Large Signal



$I_D = a e^{bV_D}$

Small Signal



$R = \frac{1}{I_D b}$

$$i_d = \left.\frac{\delta f(V_D)}{\delta(V_D)}\right|_{V_D = V_D} \cdot v_d$$

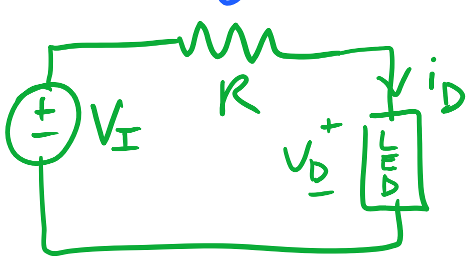
$$= \left. a e^{bV_D} b \right|_{V_D = V_D} \cdot v_d$$

$$= \underbrace{a e^{bV_D}} \cdot b v_d$$

$$i_d = \underbrace{I_D b}\, v_d$$

$$i_d = R v_d$$

Small Signal Example:



$i_D = a e^{bV_D}$

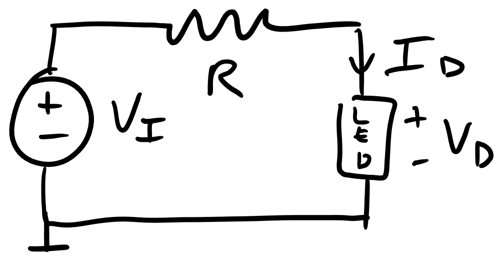
Find i_d for v_i

$R = 1\Omega \quad a = \frac{1}{4} A \quad b = 1V^{-1}$

Biaspoint set at $V_I = 1V$

1) Large Signal Analysis

1) Large Signal Analysis



Analytical Method

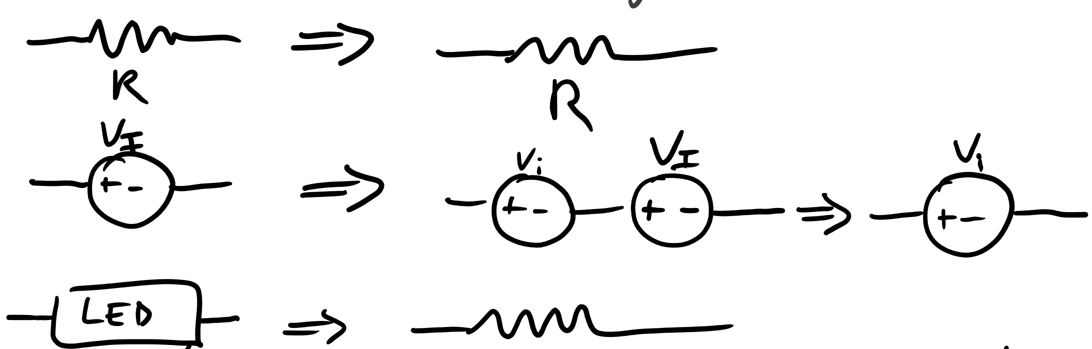
$$\frac{V_D - V_I}{R} + a e^{bV_D} = 0 \quad \text{(Node Method)}$$

$$V_D - 1 + \frac{1}{4} e^{V_D} = 0 \quad \text{(Plug in)}$$

$$V_D = 0.56\ V$$
$$I_D = 0.44\ A$$

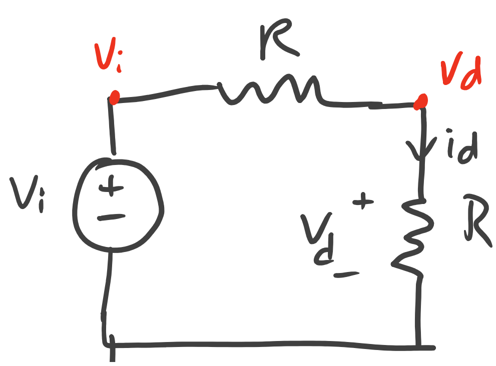
2) Create Linearized Models

small signal



$i_D = a e^{bV_D}$

$$R_D = 1 \Big/ \left.\frac{\delta f(v_D)}{\delta V_D}\right|_{V_D = V_{D_b}} = 1 \Big/ a e^{bV_{D_b}} = 1 \Big/ I_D b$$



$R_D = 1 / I_D b$

small signal circuit

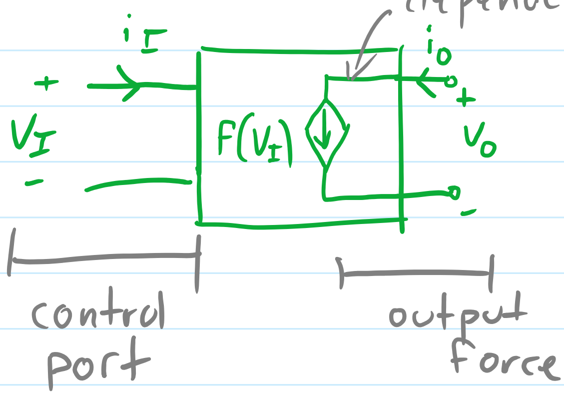
3) Solve

$$i_d = \frac{V_i}{R+R_d} = \boxed{\frac{V_i}{1+{}^{1}/_{I_D b}}}$$

W4: Dependent Sources and Amplifiers

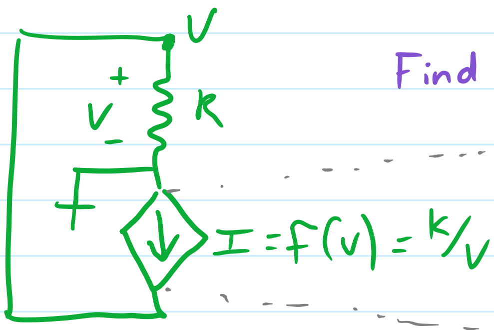
Wednesday, October 5, 2016 7:20 PM

Dependent Sources



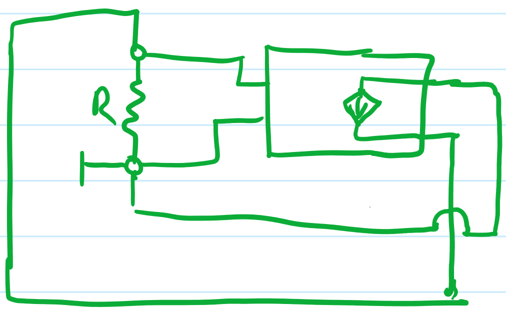
↳ Voltage Controlled Current Source

Example



Find V.

⟹



Node Method

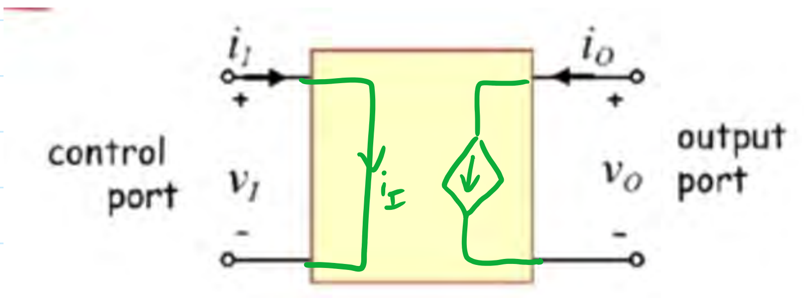
$$\frac{V}{R} - I = 0$$

$$V = IR$$

$$V = \frac{k}{V} R$$

$$V^2 = kR$$

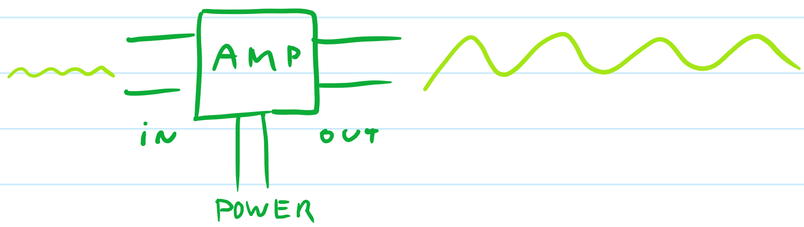
$$V = \sqrt{kR}$$



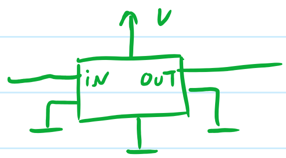
Current Controlled Current Source

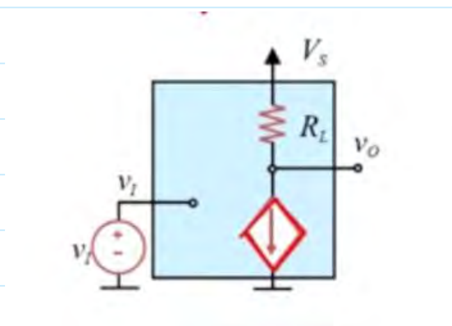
(CCCS)

Amplifiers



Used like





$i_D = \frac{K}{2}(v_{IN} - 1)^2$ for $v_{IN} \geq 1$

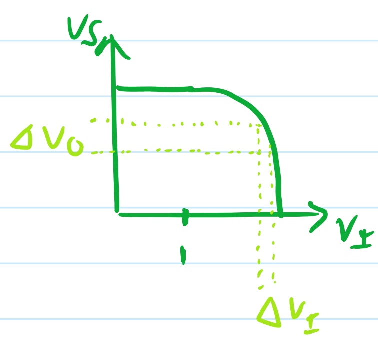
$i_D = 0$ otherwise

Node Method

$$\frac{V_O - V_S}{R_L} + i_D = 0$$

$$V_O = V_S - i_D R_L$$

$$\Rightarrow \quad V_O = \begin{cases} V_S - \frac{K}{2}(V_I - 1)^2 R_L & \text{for } v_I \geq 1 \\ V_S & \text{for } v_I < 1 \end{cases}$$



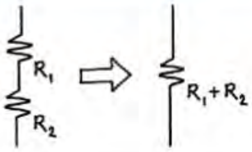
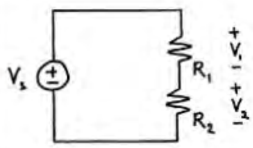
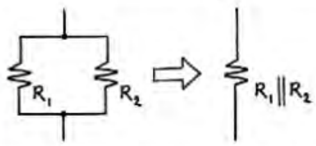
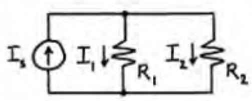
$$\frac{\Delta V_O}{\Delta V_I} > 1$$

$\Rightarrow$ Amplification

Exam I Review

Wednesday, October 12, 2016 10:17 PM

Common Shortcuts

Name	Diagram	Formulas
Series Resistors		Equivalent resistance $= R_1 + R_2$
Voltage Divider		$V_1 = \frac{R_1}{R_1 + R_2} V_s \qquad V_2 = \frac{R_2}{R_1 + R_2} V_s$
Parallel Resistors		Equivalent resistance $= R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$
Current Divider		$I_1 = \frac{R_2}{R_1 + R_2} I_s \qquad I_2 = \frac{R_1}{R_1 + R_2} I_s$

$$V = R * I = \frac{P}{I} = \sqrt{P * R}$$

$$R = \frac{V}{I} = \frac{V^2}{P} = \frac{P}{I^2}$$

$$I = \frac{V}{R} = \frac{P}{V} = \sqrt{\frac{P}{R}}$$

$$P = V * I = R * I^2 = \frac{V^2}{R}$$

Sample Problems

Q1

The power distribution network in a particular apparatus can be modeled by the circuit below.

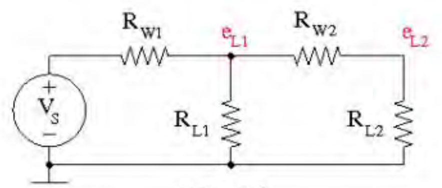


Figure 1-1

The power comes from a voltage source and is delivered to two loads. The loads are modeled by the resistors R_{L1} and R_{L2}. The resistances of the interconnects are modeled by the resistors R_{W1} and R_{W2}. The values of the element parameters are $V_S = 4.0$V, $R_{L1} = 15.0\Omega$, $R_{L2} = 20.0\Omega$, $R_{W1} = 1.0\Omega$, $R_{W2} = 2.0\Omega$.

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Node Method

$V = IR$

$I = V/R$

$$\frac{e_{L1} - V_S}{R_{W1}} + \frac{e_{L1} - e_{L2}}{R_{W2}} + \frac{e_{L1}}{R_{L1}} = 0 \qquad (1)$$

$$\frac{e_{L2} - e_{L1}}{R_{W2}} + \frac{e_{L2}}{R_{L2}} = 0 \qquad (2)$$

Plug in stuff

$$\frac{e_{L1} - 4}{1} + \frac{e_{L1} - e_{L2}}{2} + \frac{e_{L1}}{1} = 0$$

$$\frac{e_{L2} - e_{L1}}{2} + \frac{e_{L2}}{20} = 0$$

$$e_{L1} - 4 + \frac{1}{2}e_{L1} - \frac{1}{2}e_{L2} + e_{L1} = 0$$

$$2.5e_{L1} - \frac{1}{2}e_{L2} - 4 = 0 \qquad (1)$$

$$e_{L1} = \frac{4 + \frac{1}{2}e_{L2}}{2.5}$$

$$\frac{1}{2}e_{L2} - \frac{1}{2}e_{L1} + \frac{1}{20}e_{L2} = 0$$

$$\Rightarrow .55e_{L2} - \frac{1}{2}e_{L1} = 0$$

$$(2) \quad \frac{e_{L2} - \frac{4 + \frac{1}{2}e_{L2}}{2.5}}{2} + \frac{e_{L2}}{20} = 0$$

$$\begin{bmatrix} -1/2 & .55 \end{bmatrix}^{-1} \begin{bmatrix} 0 \end{bmatrix}$$

$$\frac{\quad\frac{\quad}{2.5}\quad}{2} + \frac{e_{L2}}{20} = 0 \qquad \begin{bmatrix} -\frac{1}{2} & .55 \\ 2.5 & -\frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 4 \end{bmatrix} =$$

$$\frac{20 e_{L2}}{2} - \frac{20(4 + \frac{1}{2} e_{L2})}{5} + \frac{e_{L2}}{\cancel{20}} = 0$$

$$\frac{1}{10} e_{L_2} - 16 + 2 e_{L2} + e_{L_2} = 0$$

$$3\tfrac{1}{10} e_{L_2} = 16$$

$$e_{L2} =$$

$2x + 3y - 2z = 8$
$x \quad\quad - 4z = 1$
$2x - y - 6z = 4$

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 0 & -4 \\ 2 & -1 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

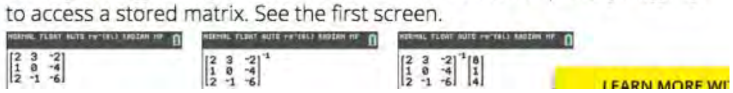
coefficient matrix A | variable matrix X | constant matrix B

Here's a short explanation of where this method comes from. Any system of equations can be written as the matrix equation, A * X = B. By pre-multiplying each side of the equation by A^{-1} and simplifying, you get the equation X = A^{-1} * B.

Using your calculator to find A^{-1} * B is a piece of cake. Just follow these steps:

1 Enter the coefficient matrix, A.

Press [ALPHA][ZOOM] to create a matrix from scratch or press [2nd][x^{-1}] to access a stored matrix. See the first screen.



2nd Try

$$\frac{e_{L1} - V_s}{R_{w_1}} + \frac{e_{L1} - e_{L2}}{R_{w_2}} + \frac{e_{L1}}{R_{L1}} = 0$$

$$\frac{e_{L1}-4}{1} + \frac{e_{L1}-e_{L2}}{2} + \frac{e_{L1}}{15} = 0$$

$$e_{L1} \cancel{-4} + \frac{1}{2}e_{L1} - \frac{1}{2}e_{L2} + \frac{e_{L1}}{15} = 4$$

$$\boxed{\tfrac{47}{30}e_{L1} - \tfrac{1}{2}e_{L2} = 4}$$

$$\frac{e_{L2}-e_{L1}}{R_{W2}} + \frac{e_{L2}}{R_{L2}} = 0$$

$$\frac{1}{2}e_{L2} - \frac{1}{2}e_{L1} + \frac{e_{L2}}{20} = 0$$

$$\boxed{-\tfrac{1}{2}e_{L1} + .55e_{L2} = 0}$$

$$\begin{bmatrix} 47/30 & -1/2 \\ -1/2 & .55 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 3.59 \\ 3.26 \end{bmatrix} \end{array} \begin{array}{l} e_{L1} \\ e_{L2} \end{array}$$

$V = IR \qquad I = V/R$

$P = IV$

$$P = \frac{V^2}{R} = \frac{(4-3.59)^2}{1} = .1681$$

$$= \frac{(3.59)^2}{} = .859$$

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$$= \frac{(3.59 - 3.26)^2}{2} =$$

$$= \frac{(3.26)^2}{20}$$

Power through voltage source?

=

Q2

0 points possible (ungraded)

A resistive load is attached to a two-terminal "black box".

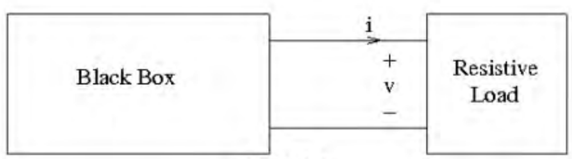


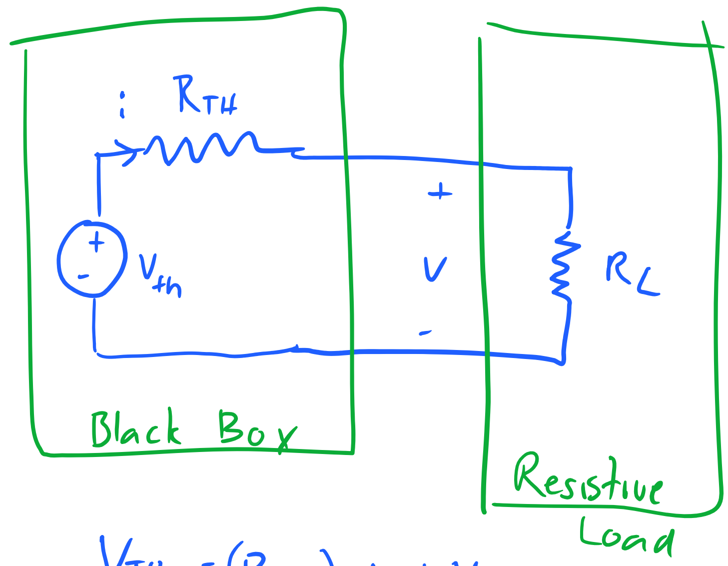
Figure 2-1

The "black box" contains only linear resistors and independent sources.
The voltage across the load is measured to be $v = 3.0V$ when the current through the load is measured to be $i = 22.0A$.

The load is then changed and the voltage across the new load is measured to be $v = 18.0V$ when the current through the load is measured to be $i = 17.0A$.

If the resistive load is replaced by a short circuit, what is the current, in Amperes, that flows through the short circuit?

If the resistive load is replaced by an open circuit, what is the voltage, in Volts, that appears across the open circuit?



$V_{TH} = (R_{TH}) \cdot i + V$

1st:

$V_{TH} = R_{TH}(22) + 3$

2nd:

$V_{TH} = R_{TH}(17) + 18$

1st - 2nd:

$0 = 5R_{TH} - 15$

$\Rightarrow R_{TH} = 3$

$\Rightarrow V_{TH} = 3(22) + 3 = \boxed{69}$ ✓

Short circuit $\Rightarrow R_L = 0$

$V_{TH} = iR_{TH}$

$i = V_{TH}/R_{TH} = 69/3 = \boxed{23}$ ✓

For many purposes of gate design, we can model a MOSFET used as a switch simply as as an ideal switch and an "on-state resistor" R_{ON}. This is the SR model.

Assuming this model for the MOSFET, consider the inverter in the figure. This inverter is intended to be used as an element in a logic family with NAND and NOR gates.

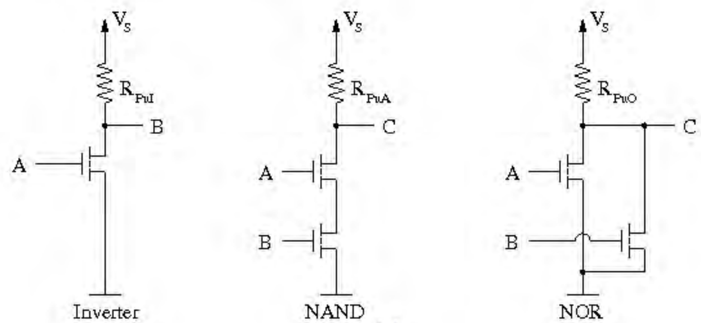


Figure 4-1

The static discipline required for this family is:

$$V_S = 5.0V, V_{OH} = 4.5V, V_{IH} = 4.0V, V_{IL} = 1.5V, V_{OL} = 1.0V.$$

Low noise margin

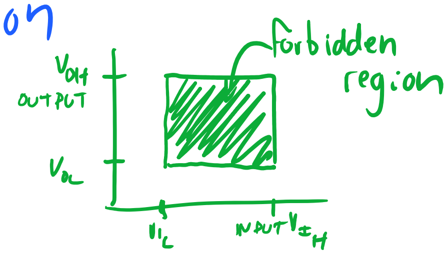
$$|V_{IL} - V_{OL}| = .5$$

High noise margin

$$|V_{IH} - V_{OH}| = .5$$

Width of forbidden region

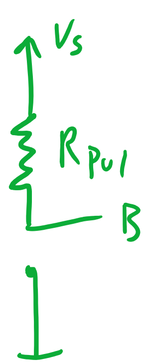
$$V_{IH} - V_{IL}$$



Suppose that the threshold voltage for the MOSFET is VT=2.0V and RON=7000.0Ω.

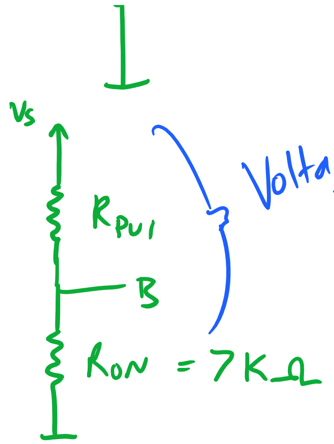
Min value of R_{PUL} to obey static discipline in Inverter

If $V_{GS} < V_T \Rightarrow$



$\Rightarrow$ "HIGH"

If $V_{GS} \geq V_T \Rightarrow$



Voltage divider!

$\Rightarrow$ "Low"

$$B = \frac{R_{ON}}{R_{pul} + R_{ON}} V_s$$

From static discipline B needs to be at least $= V_{OL} = 1V$

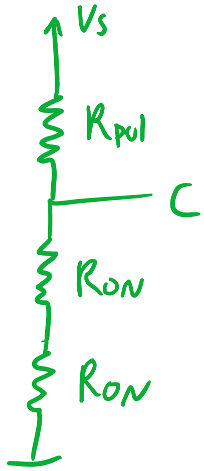
$$\Rightarrow \frac{1}{5} = \left(\frac{7000}{R_{pul} + 7000}\right) \cancel{5}$$

$$5(7000) = R_{pul} + 7000$$

$$R_{pul} = 4(7000) = \boxed{28K} \checkmark$$

Min value of R_{puA} to obay static discipline in NAND Gate

closed



$$C = \frac{2R_{ON}}{R_{pul} + 2R_{ON}} V_s = 1$$

$$\frac{1}{5} = \left(\frac{14K}{R_{pul} + 14K}\right) \cancel{5}$$

$$(14K)\cancel{5}^{4} = R_{pul} + \cancel{14K}$$

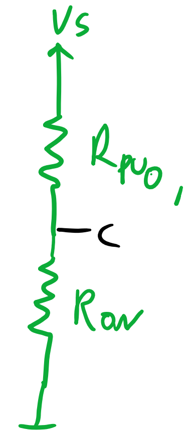
$$R_{pul} = 4(14K)$$

$$\boxed{= 56k} \checkmark$$

$$\boxed{= 56k}$$ ✓

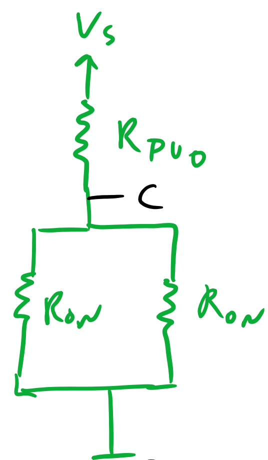
Min value of R_{puo} to obay static discipline in NOR Gate

Could be



same as inverter

OR



Try both and take min ?

$R_{puo_1} = 28K$

$R_{puo} = \max(R_{puo_1}, R_{puo_2})$

$\underline{R_{puo_2}}$

$$C = 1 = \frac{R_{on} \| R_{on}}{R_{pul} + R_{on} \| R_{on}} 5 \qquad R_{on} \| R_{on} = \frac{(R_{on})^2}{2 R_{on}} = \frac{7k^2}{14k} = 3.5K$$

$$\frac{1}{5} = \frac{3.5}{R_{puo_2} + 3.5} \cancel{5}$$

$$5(3.5) = R_{puo_2} + 3.\cancel{5}$$

$$R_{po_2} = 4(3.5) = 14K$$

$$\Rightarrow R_{pou} = 28K$$

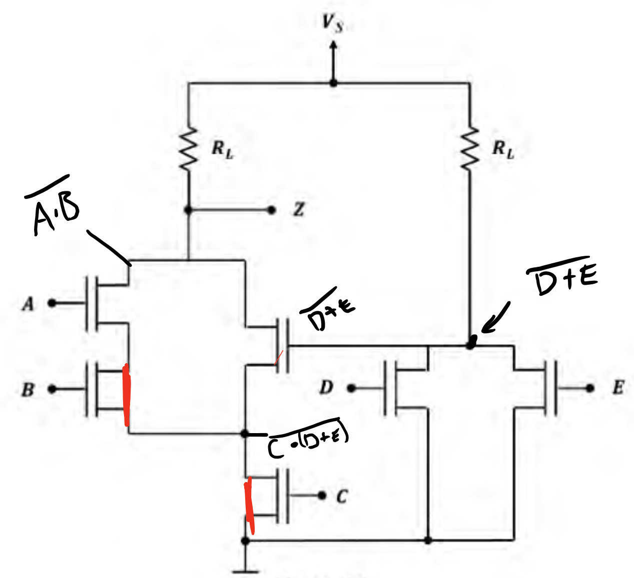


Figure 5-1

Write a boolean expression for Z in terms of A, B, C, D, and E. You need not simplify your expression. Use your expression to fill out the truth table below.

$$Z = \overline{(\overline{A\cdot B}) + (\overline{D+E})\cdot C}$$

How?

A	B	C	D	E	Z
1	0	1	0	1	1
1	1	1	1	1	0
0	0	1	1	0	1
0	1	1	0	0	0

A battery in combination with a dependent current source is connected to a lamp in the circuit shown below in Figure 6-1. The battery is modeled as a voltage source in series with a resistor, R_{bat}, and has an open circuit voltage of $4V$. The lamp turns on if the voltage across the lamp is greater than $2V$. When the lamp is on it has an internal resistance of 10Ω, and when it is off it acts like an open circuit.

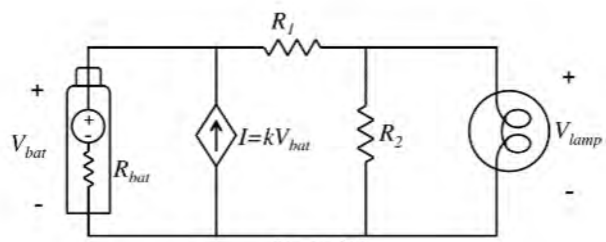


Figure 6-1

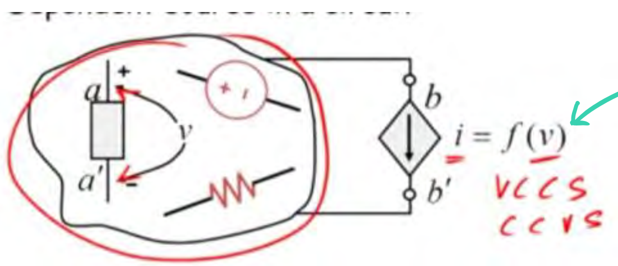
The elements in this circuit have the following values: $R_1 = 5\Omega$, $R_2 = 5\Omega$, and $k = 0.2$.

W1: MOSFET Large Signals

Friday, October 14, 2016 9:58 AM

Review

Dependent sources

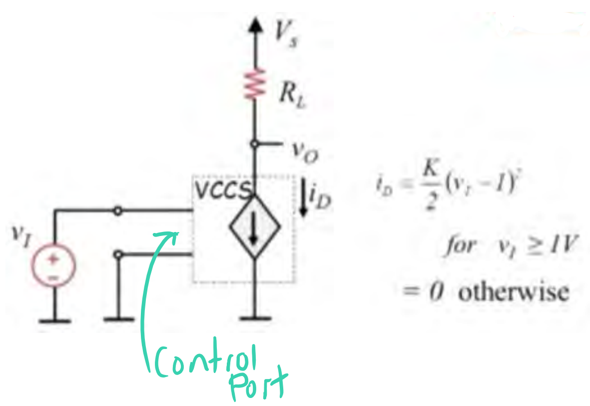


output depends on v somewhere else on the circuit

Superposition w/ dependent sources

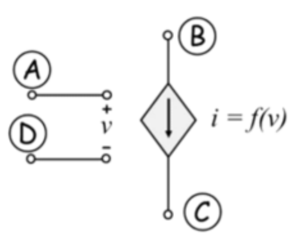
- leave all dependent sources in
- solve for independent source one at a time

Amplifier

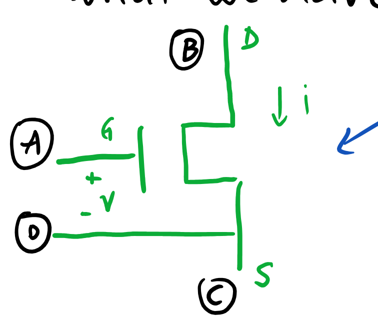


$v_o = V_S - i_D R_L$

What we need



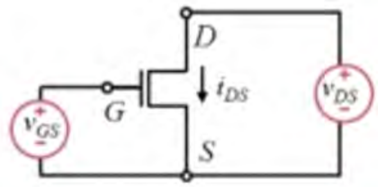
What we have

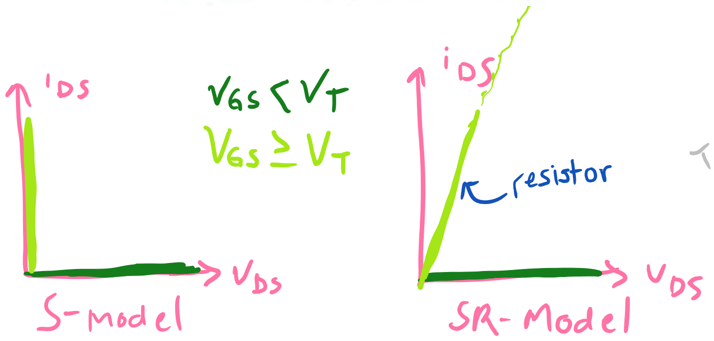


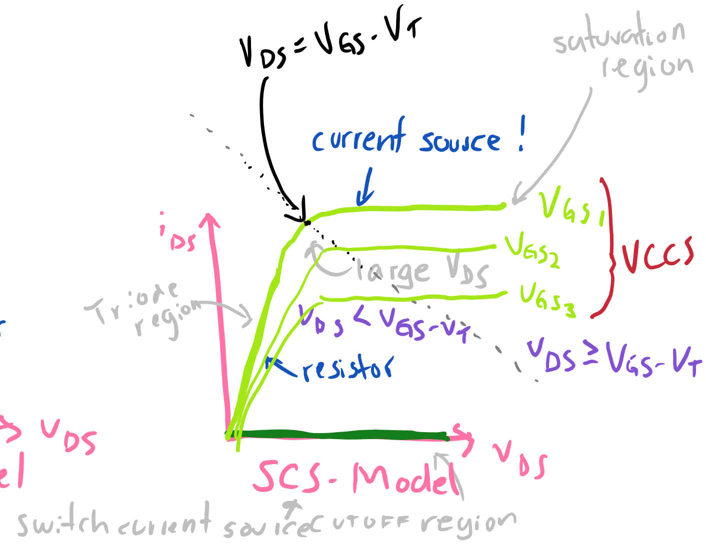
MOSFET

Mosfets

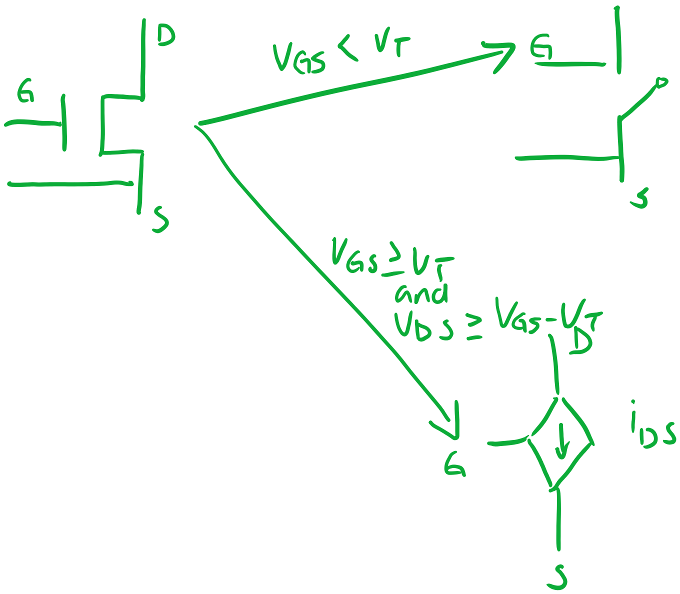
Mosfets







- On state behavior more complex than SR-Model



"OFF state"

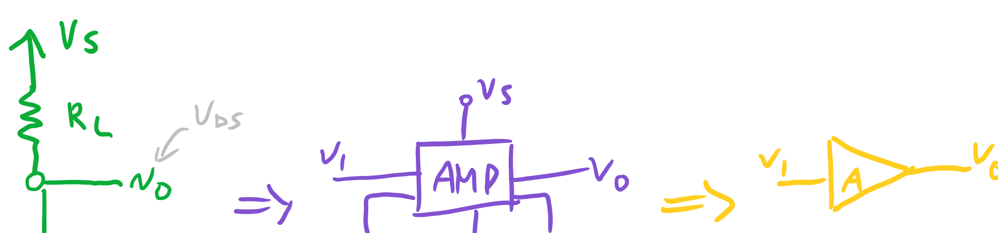
still behaves as open circuit

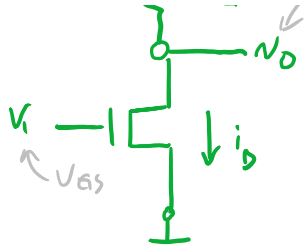
$i_{DS} = F(V_{GS})$ SATURATION REGION

$$i_{DS} = k/2 \, (V_{GS} - V_T)^2$$

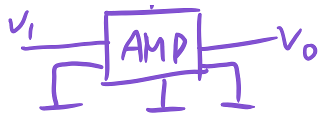
Use SR-Model for digital designs and SCS-Model for analog designs

Amplifier





$\Rightarrow$



$\Rightarrow$

V_{GS}

$$i_{DS} = \frac{K}{2}\left(v_I - V_T\right)^2$$

"In saturation region only!"

saturation discipline

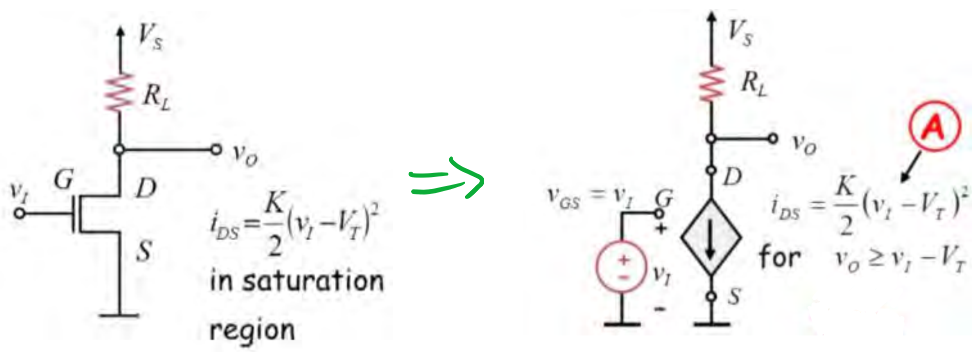
① $V_{GS} \geq V_T$

$v_I \geq V_T$

② $V_{DS} \geq V_{GS} - V_T$

$v_O \geq v_I - V_T$

Analysis



Analytical Method

Find v_O vs v_I

$$\frac{v_O - V_S}{R_L} + i_{DS} = 0 \qquad \text{(Node Method)}$$

$$v_O = V_S - i_{DS} R_L \quad \text{Ⓑ}$$

$$\Rightarrow v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

saturation region

$\leftarrow$ For $v_I \geq V_T$ and $v_O \geq v_I - V_T$

$v_O = 0 \leftarrow$ For $v_I < V_T$

Mosfet is off

Graphical Method

Ⓐ $i_{DS} = \frac{K}{2}\underbrace{(v_I - V_T)}^{2}$

Saturation begins: $v_O = \underbrace{v_I - V_T}$

$\geq$

$i_{DS} \leq \frac{K}{2} v_O^2$

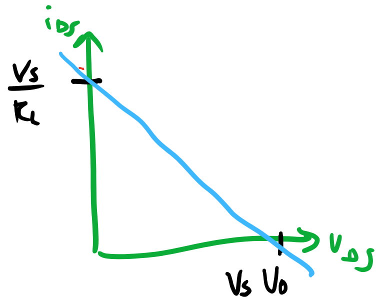
since v_O is greater

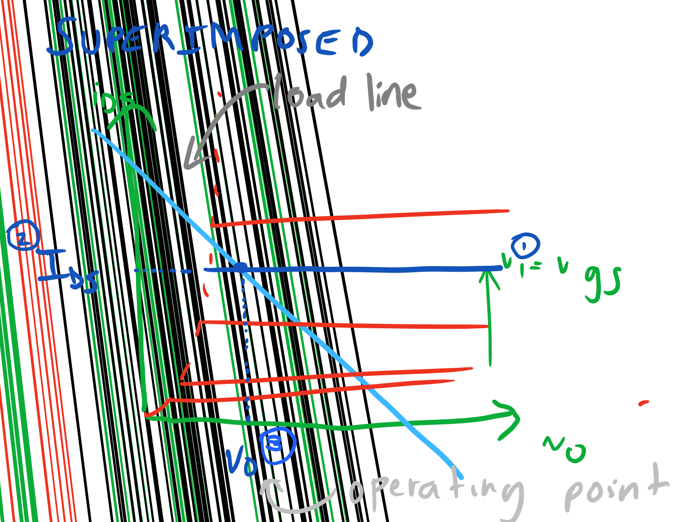
Ⓑ $v_O = V_S - i_{DS} R_L$

$\Rightarrow i_{DS} = \frac{V_S}{R_L} - \frac{v_O}{R_L}$

$i_{DS} = \frac{K}{2} v_O$

since v_O is greater



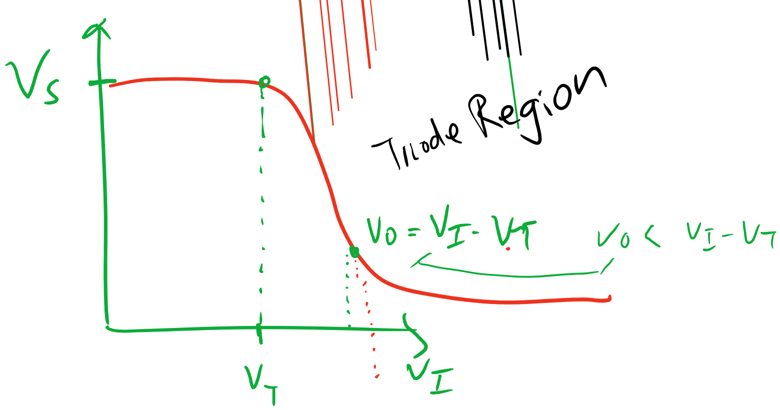


Large Signal Analysis

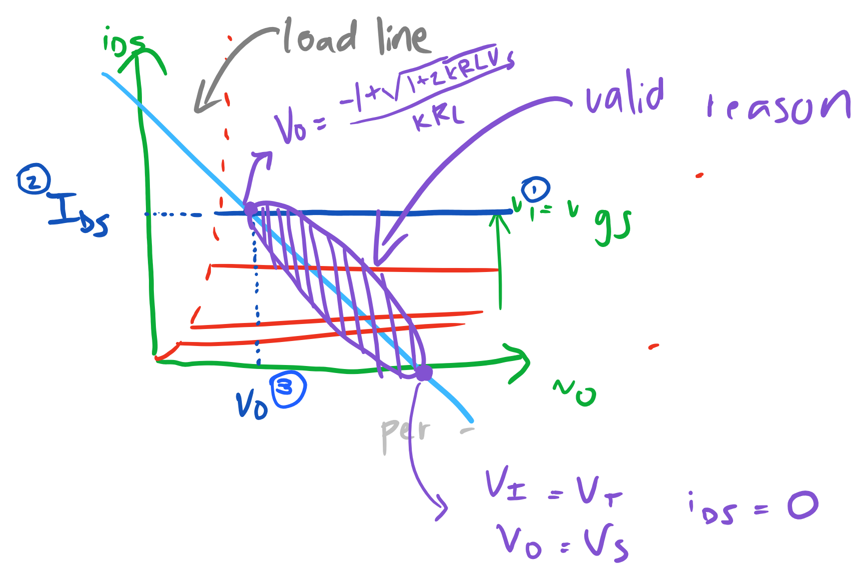
① V_O versus V_S

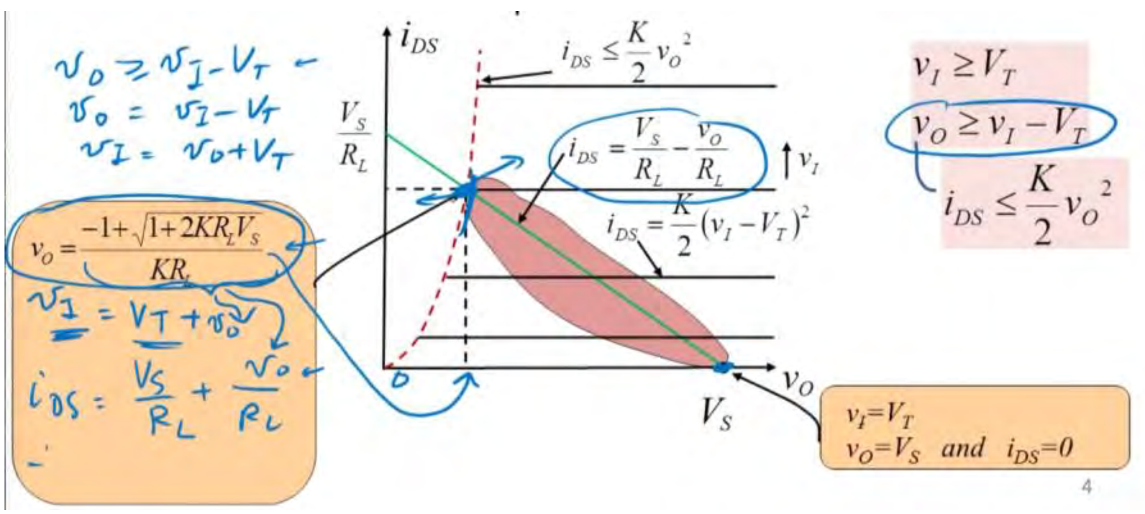
$$V_O = V_S - \frac{K}{2}(V_I - V_T)^2 R_L$$ for $V_I \geq V_T$, $V_O \geq V_I - V_T$

$$V_O = V_S$$ for $V_I < V_T$

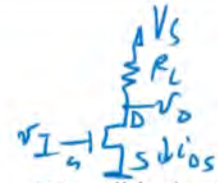


② Find valid operating ranges under static discipline





Large Signal Analysis Summary



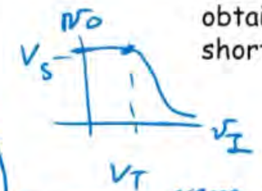
We will look at another way of obtaining (2) shortly

(1) v_O versus v_I

$$v_O = V_S - \frac{k}{2}(v_I - V_T)^2 \cdot R_L$$

(2) Valid operating ranges under the saturation discipline?

→ Valid input range:



$$v_I : V_T \longrightarrow V_T +$$

range for i_{DS}

→ Corresponding output range:

$$v_O : V_S \rightarrow \frac{-1 + \sqrt{1 + 2kR_LV_S}}{kR_L}$$

$\frac{V_S}{1}$

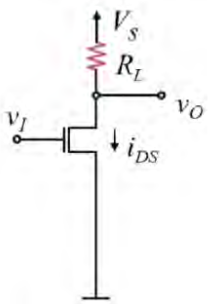
6

W1 : Amplifiers : small signal model

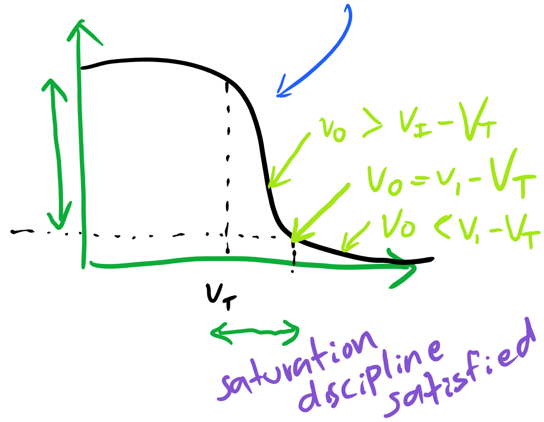
Monday, October 17, 2016 5:59 PM

Large Signal Analysis Review

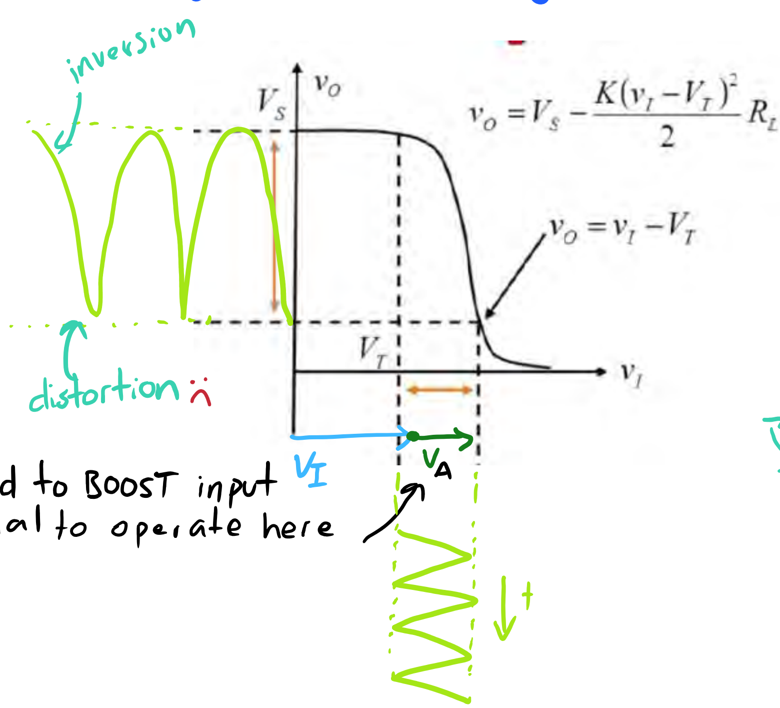
1. V_O vs V_I
2. Valid operating ranges



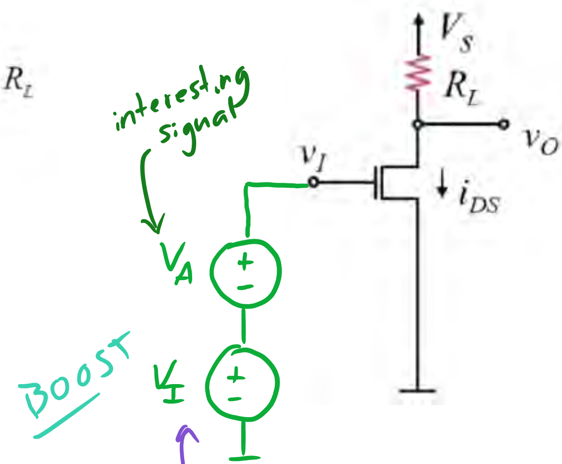
$$V_O = V_S - \frac{K(v_I - V_T)^2}{2} R_L$$



Operating in saturation region

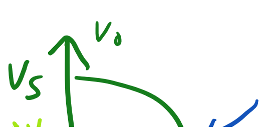


Need to BOOST input signal to operate here



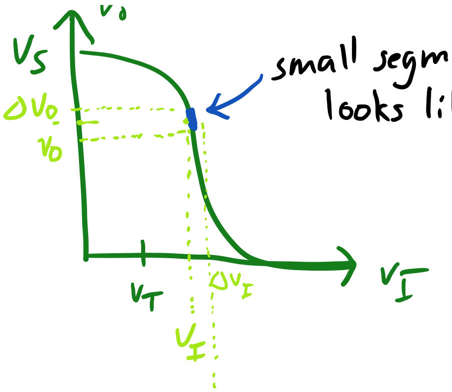
offset voltage
"DC bias"

Small Signal Trick



small segment

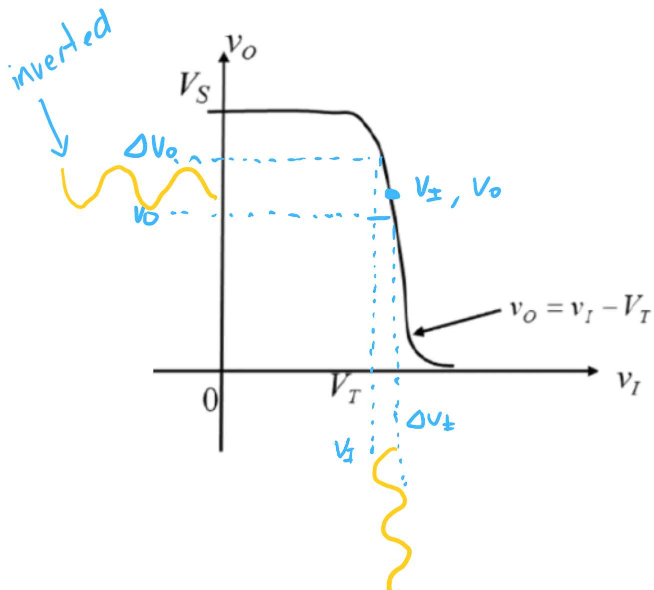
1. Operate amp at V_I V_O → "DC bias"

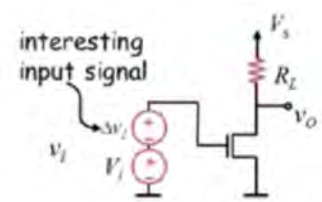


① Operate amp at V_I, V_O → "DC bias" ✓

② Superimpose small signal on top of V_I

Small Signal Method: Graphically





INPUT: $v_I = V_I + v_i$

DC bias (V_I), operating point voltage (V_I), small signal (v_i), Δv_i

OUTPUT $v_O = V_O + v_o$

II Mathematically

Bias Point Equation

$$v_O = V_S - \frac{R_L K}{2}(v_I - V_T)^2 \qquad V_O = V_S - \frac{R_L K}{2}(V_I - V_T)^2 \longrightarrow *$$

... watch my fingers!

substituting $v_I = V_I + v_i$, $\quad \overline{v_i << V_I}$

$$v_O = V_S - \frac{R_L K}{2}([V_I + v_i] - V_T)^2$$

$$= V_S - \frac{R_L K}{2}([V_I - V_T] + v_i)^2$$

neglect higher order terms in v_i

$$\cancel{V_O} + v_o = \cancel{V_S} - \frac{R_L K}{2}\left(\cancel{[V_I - V_T]^2} + \underbrace{2[V_I - V_T]v_i} + \cancel{v_i^2}\right)$$

From *

$$v_o = A\, v_i$$

$$v_o = -\frac{R_L K}{2}\left(2[V_I - V_T]v_i\right)$$

$$v_o = -\underbrace{R_L K (V_I - V_T)}\, v_i = -R_L \underbrace{g_m}_{K(V_I - V_T)} v_i$$

Transconductance related to MOSFET parameters and V_I

W2: Small signal circuit model

Monday, October 17, 2016 8:03 PM

Recall

$$v_o = \frac{-K(v_I - V_T)^2}{2} R_L$$

$$v_o = \frac{d}{dv_I}\left[V_S - \frac{K}{2}(v_I - V_T)^2 R_L\right]\Bigg|_{v_I = V_I} \cdot v_i$$

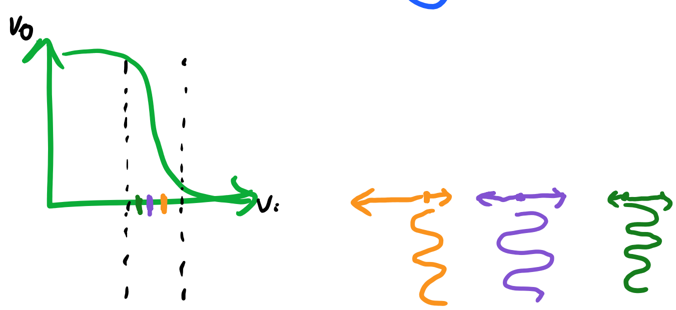
$$v_o = \underbrace{-K(V_I - V_T)}_{g_m} R_L \cdot v_i$$

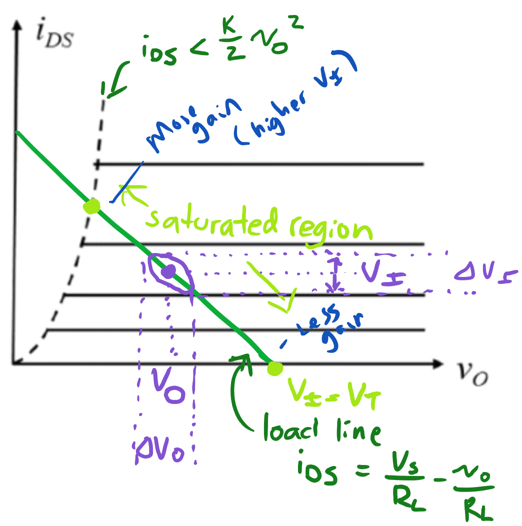
g_m ← transductance

Recall Notation

$$\underset{\text{Total}}{v_A} = \underset{\text{Operating Point}}{V_A} + v_a \leftarrow \text{Small Signal}$$

How to choose operating point?





KEY PARTS

1) Gain

$g_m R_L$ ← since $v_o = -g_m R_L v_i$

2) Input swing

Symmetric swing:

closer to midpoint

= larger input swing

Small Signal circuit view

- Can derive small signal equivalent and conduct small signal analysis there small signal analysis there

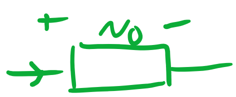
Small Signal

1. Find operating point using DC bias inputs using large signal model $v_I \to V_O$, $\to I_{ds}$
2. Develop small signal models of elements
3. Replace original elements w/ small signal models

$\Rightarrow$ can use superposition to analize!

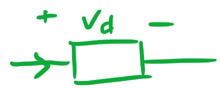
2 Linearized Model

LARGE SIGNAL



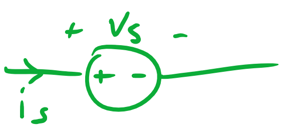
$i_D = F(v_D)$

SMALL SIGNAL

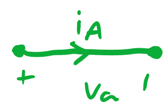


$$i_d = \underbrace{\left.\frac{\delta F(v_d)}{\delta v_d}\right|_{v_D = V_D}}_{\text{constant}} \cdot v_d$$

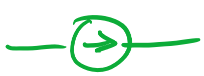
$$R_d = 1 \Big/ \left.\frac{\delta F(v_d)}{\delta v_d}\right|_{v_D = V_D}$$



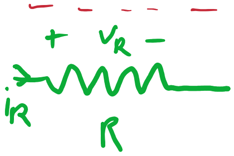
$\Rightarrow$



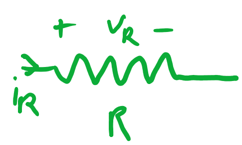
$v_A = 0$



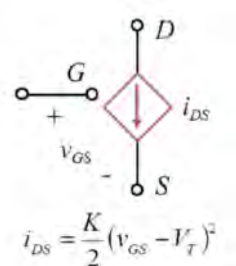
$\Rightarrow$



$\Rightarrow$



MOSFET



$\Rightarrow$

$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

$$i_{ds} = \left.\frac{\delta}{\delta V_{GS}}\left[\frac{K}{2}(V_{GS} - V_T)^2\right]\right|_{V_{GS} = V_{GS}} \cdot v_g$$

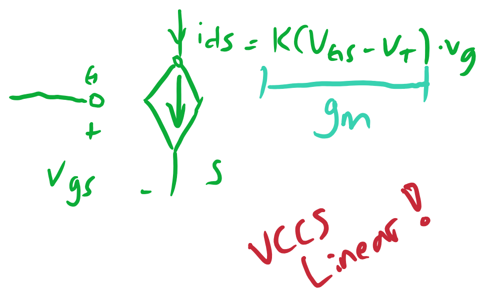
$$= \cancel{K/2 \cdot 2}\,(V_{GS} - V_T)\Big|_{V_{GS} = V_{GS}}$$

$i_{DS} = \frac{k}{2}(V_{GS} - V_T)$

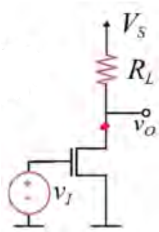
$= \frac{k}{2} \cdot \cancel{2} (V_{GS} - V_T) \big|_{V_{GS} = V_{GS}}$

$= K(V_{GS} - V_T) \cdot v_g$

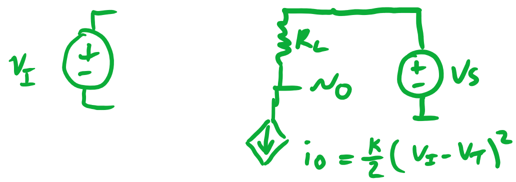
ids is linear in vgs



EX: AMPIFIER



Large Signal



① Find operating point

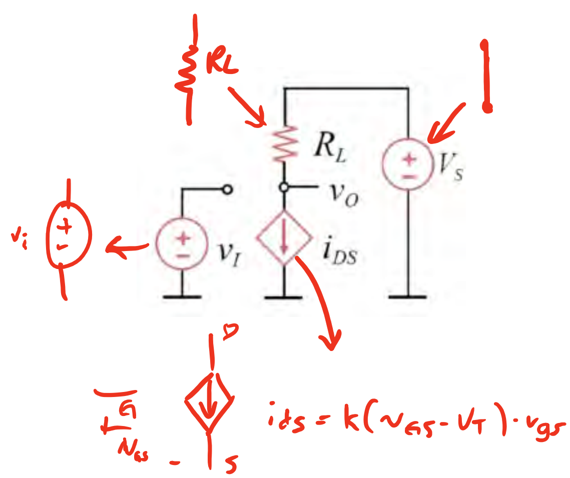
Large signal Analysis

$v_O = V_S - \frac{k}{2}(v_I - V_T)^2 R_L$ (node method)

V_O $\quad$ V_I

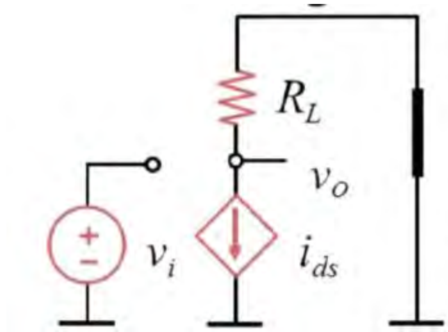
$i_{DS} = \frac{k}{2}(V_I - V_T)^2$

② Small signal model



③ Replace elements w/ small signal element.

③ Replace elements w/ small signal element and analyze



$$i_{ds} = K(\overset{V_{GS}}{V_I} - V_T)\cdot \overset{v_{gs}}{v_i}$$

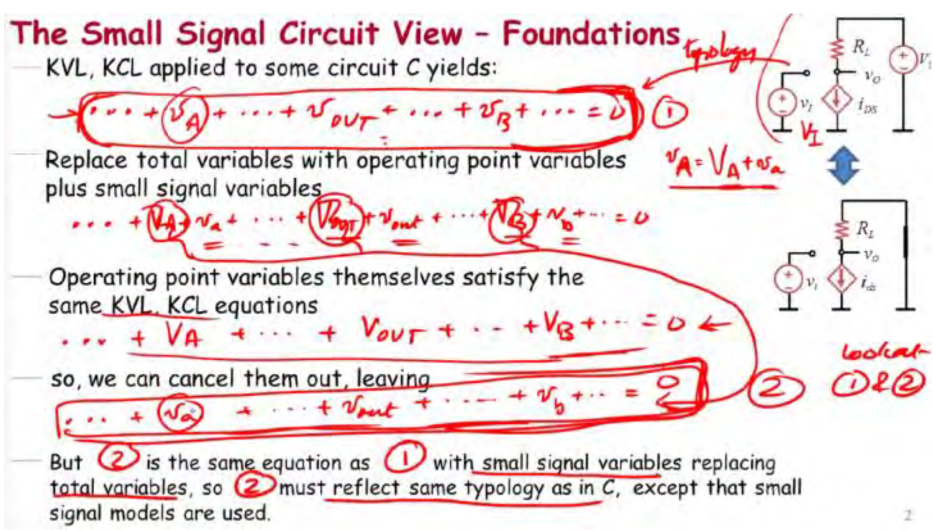
Node Method

$$\frac{v_O}{R_L} + i_{ds} = 0$$

$$v_O = -i_{ds} R_L$$
$$= -K(V_I - V_T)\cdot R_L v_i$$
$$= -g_m R_L \, v_i$$

need to do large signal analysis to get

Amplification ✓



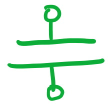
W2: Capacitors and First order circuits

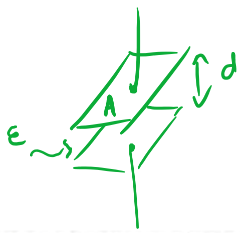
Friday, October 21, 2016 2:24 PM

one energy storage element

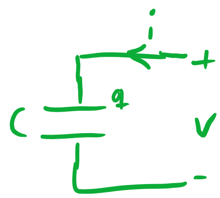
MOSFET structure looks like a capacitor 

Capacitor





$c = \frac{\varepsilon A}{d}$



Farads, charge, Coulombs, capacitance, volts

$q = Cv$

We have just learned the v-i characteristic for a linear capacitor:

$$i = C\frac{dv}{dt}$$

So the power entering the capacitor (as in any two-terminal device) is:

$$P = vi = Cv\frac{dv}{dt}$$

If we integrate this between two times we get the energy change in the capacitor over the interval:

$$\Delta E = \int_{t_0}^{t_1} P(t)dt = \int_{t_0}^{t_1} Cv(t)\frac{dv(t)}{dt}dt = \frac{1}{2}C(v(t_1))^2 - \frac{1}{2}C(v(t_0))^2$$

So the energy stored in a capacitor, as a function of the voltage across it, is:

$$E = \frac{1}{2}Cv^2$$

$$i = \frac{dq}{dt} = \frac{dCv}{dt} = C\frac{dv}{dt}$$

Assume c linear time invariant

$$\rightarrow v = \frac{1}{C}\int_{-\infty}^{t} i\, dt$$

$$P = iv = vC\frac{dv}{dt}$$

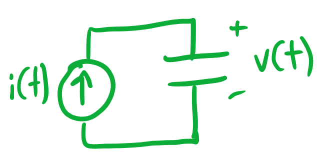
Energy

$$= \frac{d}{dt}\left(\frac{1}{2}cv^2\right)$$

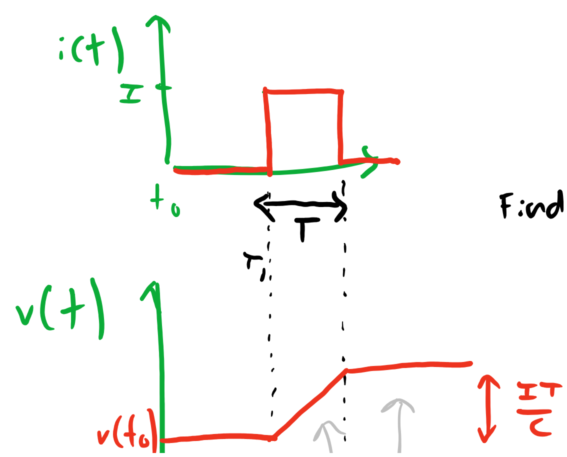
$$\boxed{E = \left(\frac{1}{2}\right)cv^2}$$

Capacitor stores energy → memory

EXAMPLE



Given



Find $v(t_0)$

$T: \Delta q = IT$

$$v = \frac{1}{C}\int_{-\infty}^{t} i\, dt$$

$$v = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{t_0} i\,dt + \frac{1}{C}\int_{t_0}^{t} i\,dt$$

given

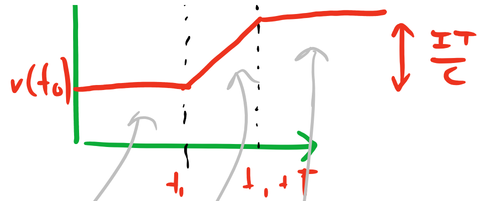
$$= v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\,dt$$

$\hookrightarrow t_0 \to T_1$

split integral

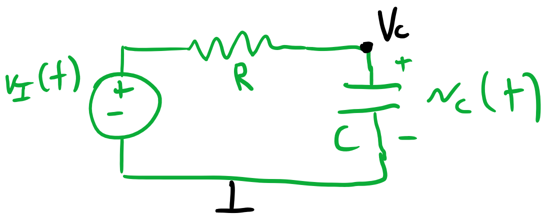
$t_1 \to T_1 + T$

$t_1 + T \to \infty$



$\Delta q = IT$

RC Circuit



Node method

$$\frac{V_c - V_I}{R} + C\frac{dV_c}{dt} = 0$$

$$RC\frac{dv_c}{dt} + V_c = v_I \quad (1)$$

first order differential equation

Given $V_c(0) = V_0$

$V_I(t) = V_I$ Solve.

(use homogeneous and particular solutions method)

1. Find particular soln
2. Find homogeneous soln
3. total soln = particular + homogenus

$$V_c(t) = V_{cH}(t) + V_{cP}(t)$$

Homogenus Particular

(1) Particular Solution

↳ Any solution that satisfies original equation

$$RC\frac{dV_{cP}}{dt} + V_{cP} = V_I$$

Guess: $V_{cP} = V_I$

Check: $RC\cancel{\frac{dV_I}{dt}} + V_I = V_I$

0 since V_I constant ✓

0 since V_I constant ✓

② Homogeneous solution

↳ solution to homogenous equation

set drive to 0

$$RC\frac{dv_{CH}}{dt} + V_{CH} = 0$$

Guess: $V_{CH} = Ae^{st}$ need to find A and s

Substitute:

$$RC\frac{dAe^{st}}{dt} + Ae^{st} = 0$$

$$RC\cancel{A}se^{\cancel{st}} + \cancel{A}e^{\cancel{st}} = 0$$

characteristic equation

$$RCs + 1 = 0$$

$$s = -1/RC \Rightarrow V_{CH} = Ae^{-1/RC\,t}$$

③ Total solution

$V_C = V_{CP} + V_{CH}$

$$V_C = V_I + Ae^{-1/RC\,t}$$

Find A using initial condition

Given $V_C = V_0$ at $t=0$

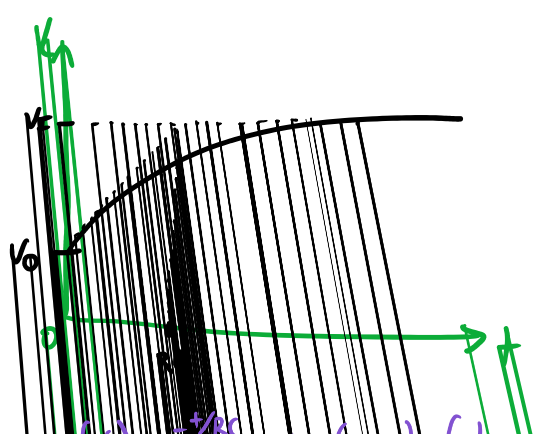
$$\Rightarrow V_0 = V_I + A$$

$$A = V_0 - V_I$$

$$\Rightarrow V_C = V_I + (V_0 - V_I)e^{-1/RC\,t}$$

$\square$

$$i_C = C\frac{dv_C}{dt} = -\frac{1}{R}(V_0 - V_I)e^{-1/RC\,t}$$



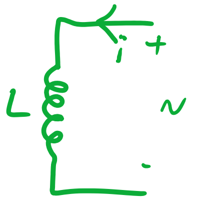
★ $v_C(t) = v_C(0)\, e^{-t/RC} + v_C(\infty)\left(1 - e^{-t/RC}\right)$

general form

W3: Inductors and First Order Circuits

Sunday, October 23, 2016 8:16 PM

Inductors



magnetic Flux, inductance, current

$\lambda = Li$

Webers, Henrys, Amps

$v = \frac{d\lambda}{dt}$

$= \frac{dLi}{dt}$ Assume L time invariant

$v = L\frac{di}{dt}$

$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$

Inductors and capacitors do not dissipate energy – they store it (or supply it)

$P = Vi = i\frac{Ldi}{dt} = \frac{d}{dt}\frac{Li^2}{2}$

$E = \frac{1}{2}Li^2$ stored in electric field

for capacitor: $E = \frac{1}{2}Cv^2$ stored in magnetic field

We have just learned the v-i characteristic for a linear inductor:

$$v = L\frac{di}{dt}$$

So the power entering the inductor (as in any two-terminal device) is:

$$P = vi = Li\frac{di}{dt}$$

If we integrate this between two times we get the energy change in the inductor over the interval:

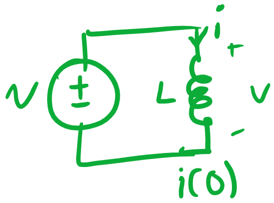
$$\Delta E = \int_{t_0}^{t_1} P(t)dt = \int_{t_0}^{t_1} Li(t)\frac{di(t)}{dt}dt = \frac{1}{2}L(i(t_1))^2 - \frac{1}{2}L(i(t_0))^2$$

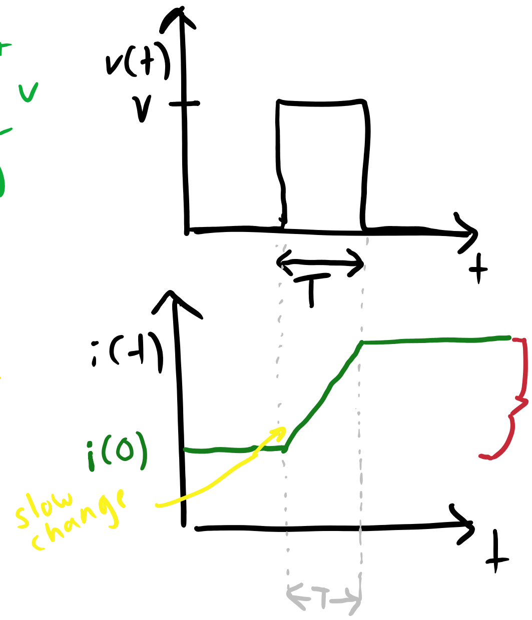
So the energy stored in an inductor, as a function of the current through it, is:

$$E = \frac{1}{2}L \cdot i^2$$

Inductor at voltage source

Inductor at voltage source





Inductors like to hold current the same!

$\Delta\lambda = vT$

$\frac{\Delta\lambda}{L} = \frac{vT}{L}$

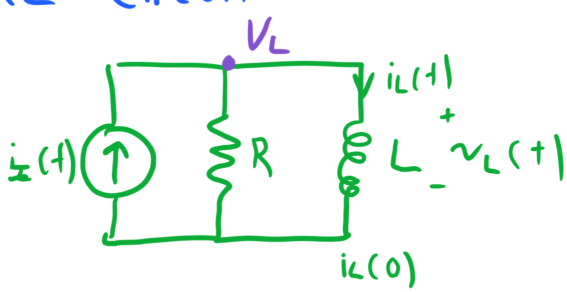
Inductor appears as open to external instantaneous current change

⇒ Voltage spike

Capacitors appears as short to external instant voltage change

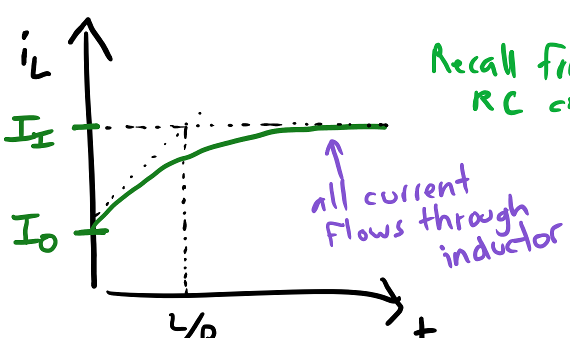
⇒ current spike

RL Circuit



Pick:- $i_I(t) = I_I$

$i_L(0) = I_0$ given



Recall from RC circuit

Node Method

$$-i_I + \frac{v_L}{R} + i_L = 0$$

$$= -i_I + \frac{L}{R}\frac{di_L}{dt} + i_L = 0$$

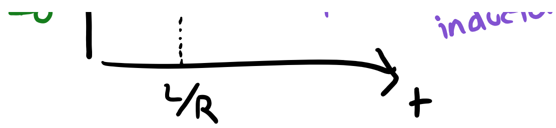
$$= \frac{L}{R}\frac{di_L}{dt} + i_L = 0$$

→ Time constant

$$= RC\frac{dv_C}{dt} + v_C = V_I$$

$$v_C = V_I + (V_0 - V_I)e^{-t/RC}$$

$$i_L = I_I + (I_0 - I_I)e^{-t/\frac{L}{R}}$$

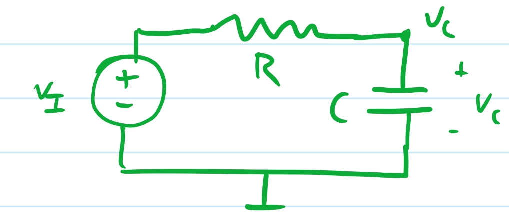


$$i_L = I_I + (I_0 - I_I)\, e^{-t/\frac{L}{R}}$$

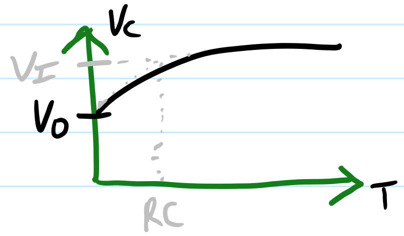
W3: Speed of Digital Circuits

Monday, October 24, 2016 2:31 PM

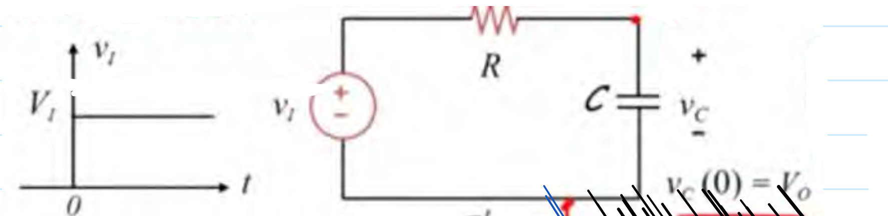
Review



$$v_C = V_I + (V_O - V_I)\, e^{-t/RC}$$



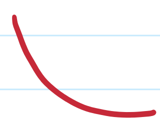
Intuitive Approach



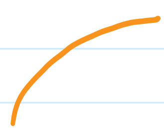
$$v_C = V_I + (V_O - V_I)\, e^{\frac{-t}{RC}}$$

① Initial Point
② Final Point
③ Draw line

Two Forms

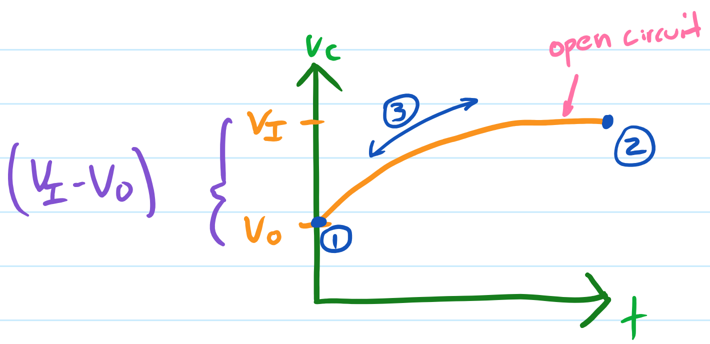


$e^{-t/RC}$,



$1 - e^{-t/RC}$

$V_I > V_O$



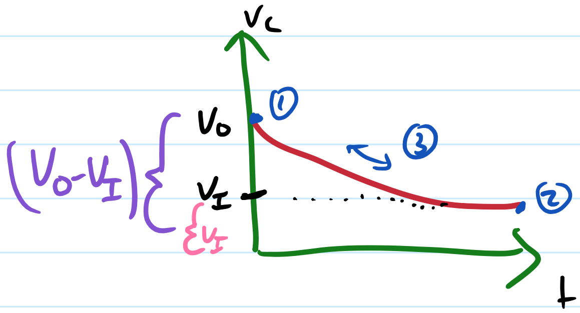
$$V_c = V_O + (V_I - V_O)\left(1 - e^{-t/RC}\right)$$

$V_I < V_O$

① V_O ② V_I ← capacitor discharges until reaches V_I

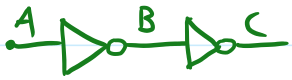
$V_I \leftarrow V_0$

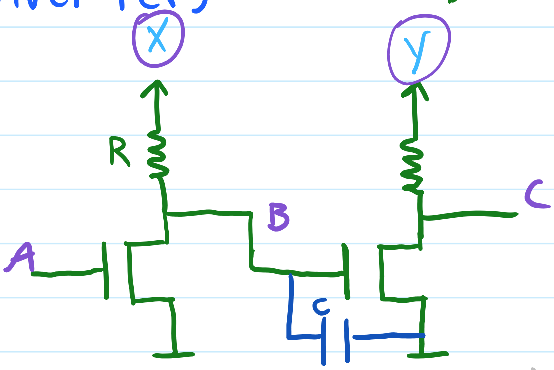
① V_0 ② V_I ← capacitor discharges until reaches V_I



$$V_c = V_I + (V_0 - V_I)e^{-t/RC}$$

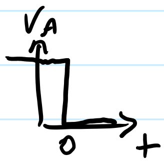
Inverters



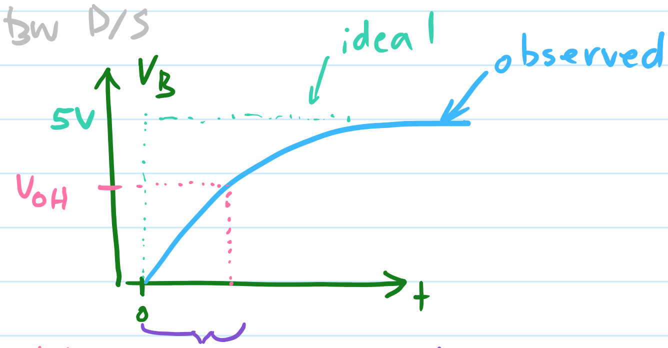


capacitance of mosfet b/w D/S

Given:



$1 \rightarrow 0$

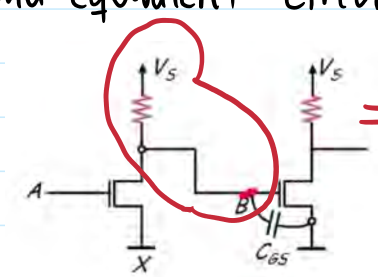


Find delay ← time until valid logical value

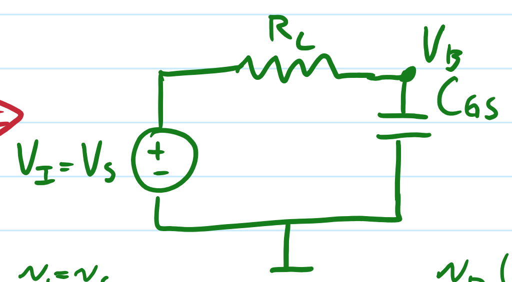
rising delay t_r of inverter X

Rising Delay

① Find equivalent circuit



$\Rightarrow$



$v_I = v_S$ $v_B(0) = 0$

$v_1 = v_s$ $\perp$ $v_B(0)=0$

Since $v_B(0)=0$ capacitor voltage will begin to charge $\Rightarrow$

$1-e^{-t/R_L C_{GS}}$

$$v_B = \underline{\ } \ \underline{V_S}\left(1 - e^{-t/R_L C_{GS}}\right)$$

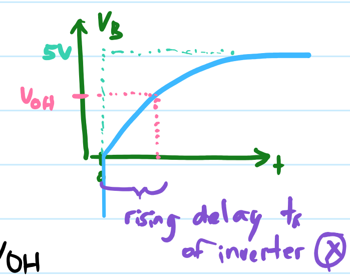
initial value (Final - Initial)

Sanity check:

$t=0 \Rightarrow V_B = 0$ ✓

$t \to \infty \Rightarrow V_B = V_S$ ✓ (open circuit)

② Find rising delay t_r



$$V_B = V_S - V_S e^{-t/R_L C_{GS}}$$

t_r: value of t for which V_B reaches V_{OH}

$$V_{OH} = V_S - V_S e^{-t_r/R_L C_{GS}}$$

$$V_S e^{-t_r/R_L C_{GS}} = \frac{V_S - V_{OH}}{V_S}$$

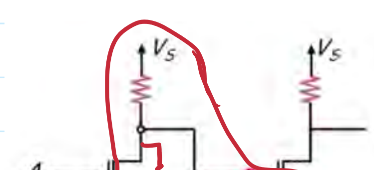
$$\frac{-t_r}{R_L C_{GS}} = \ln\left(\frac{V_S - V_{OH}}{V_S}\right)$$

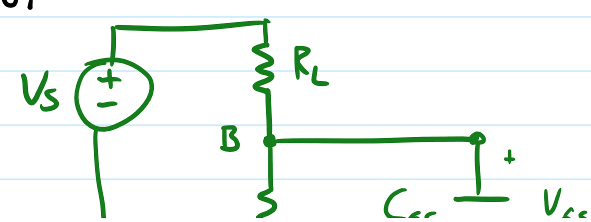
$$\Rightarrow t_r = -R_L C_{GS} \ln\left(\frac{V_S - V_{OH}}{V_S}\right)$$

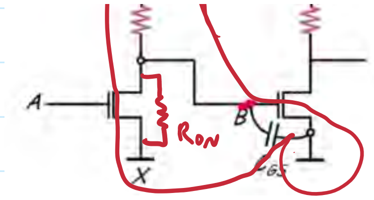
Falling Delay t_F

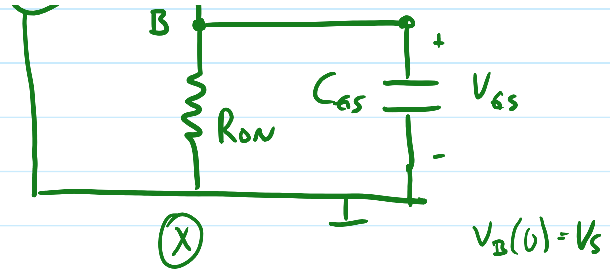
t_F: time for V_B to fall from V_S to V_{OL}

① Equivalent circuit





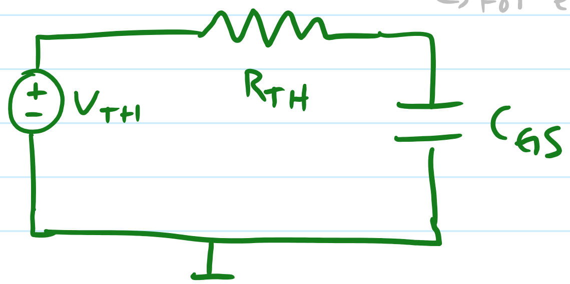




$V_B(0) = V_S$

1.5 Get Thevenin equivalent

↳ For easy analysis



Recall

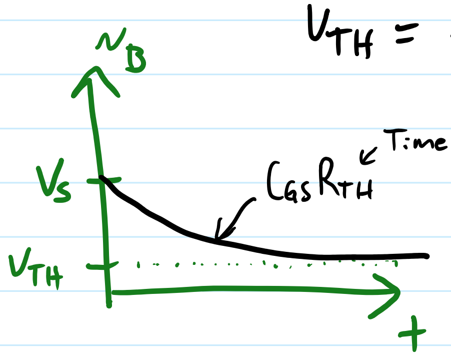
R_{TH}: Turn off souuces and find R

$$R_{TH} = R_L \parallel R_{ON} = R_{ON}R_L / R_{ON}+R_L$$

V_{TH}: open circuit voltage

in this case ⇒ voltage divider

$$V_{TH} = \frac{R_{ON}}{R_{ON}+R_L}$$



$$v_B = V_{TH} + \underbrace{(V_S - V_{TH})}_{\text{initial value}} e^{-t/R_{TH}C_{GS}}$$

2 Find Falling delay t_F

$$V_{OL} = V_{TH} + (V_S - V_{TH})\, e^{-t_F/R_{TH}C_{GS}}$$

$$e^{-t_F/R_{TH}C_{GS}} = \frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$

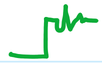
$$t_F = -R_{TH}\, C_{GS} \cdot \ln \frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$

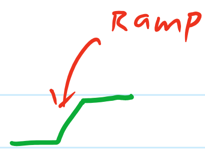
$$t_f = -R_{TH} C_{GS} \cdot \ln \frac{V_{oc} - V_{TH}}{V_S - V_{TH}}$$

much smalling than rising delay

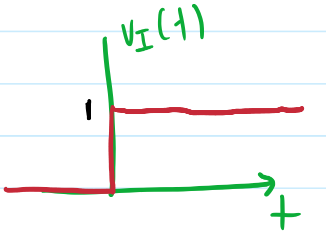
W4: Ramps, Steps, Impulses

Friday, November 4, 2016 1:06 AM

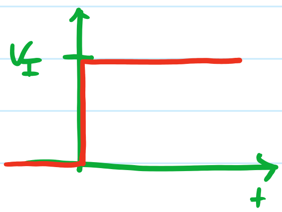
Recall: Trying to achieve  led to 

instead do 

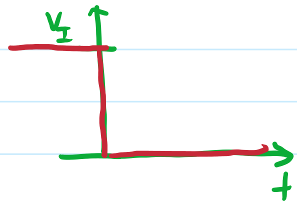
Step Inputs



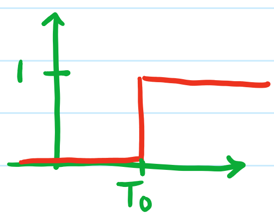
$V_I(t) = u(t)$ (notation)



$V_I u(t)$

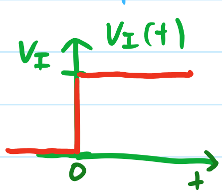


$V_I - V_I u(t)$



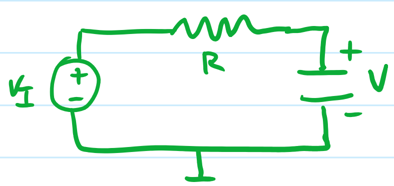
$u(t-T_0)$

Responses:



$V_I u(t)$

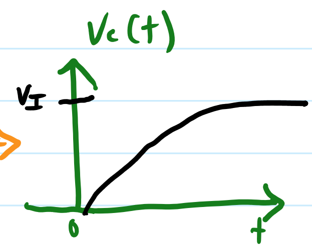
$\Rightarrow$



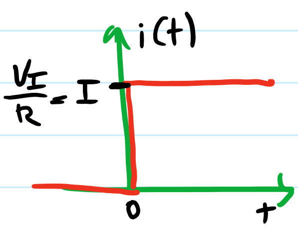
just before $t=0$

$V_c(0^-) = 0$

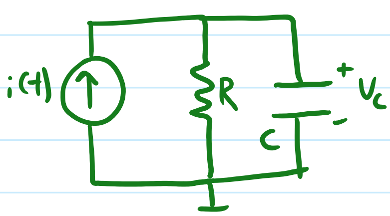
$\Rightarrow$



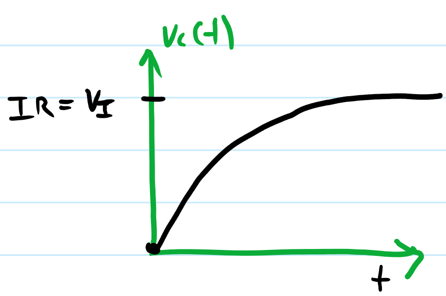
$V_I(1-e^{-t/RC})$

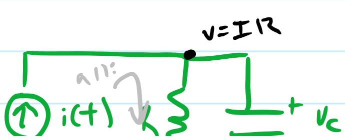


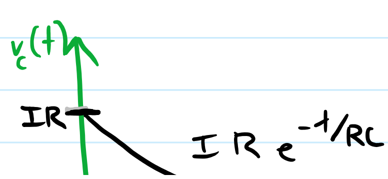
$I u(t)$

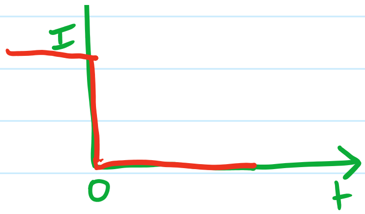


$V_c(0^-) = 0$

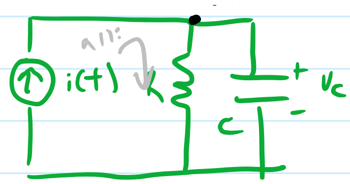






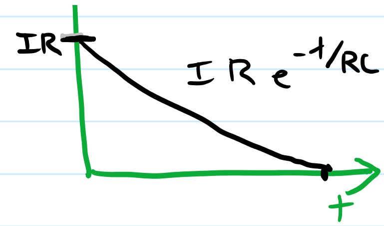


$I - I u(t)$

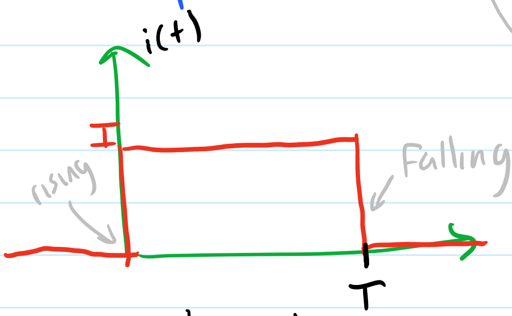


$V_c(0^-) = IR$

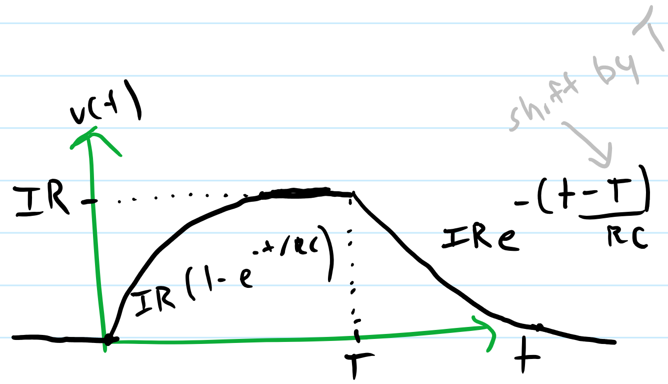
capacitor acts as open circuit

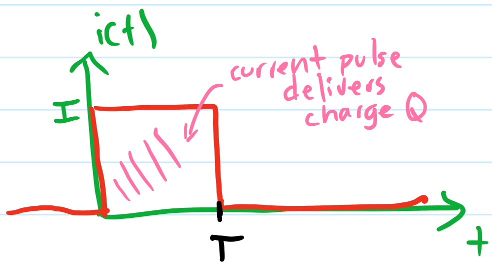


Pulse input

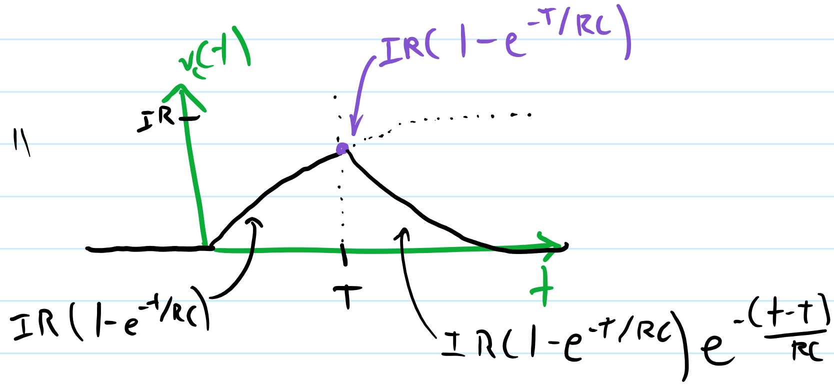


← large →

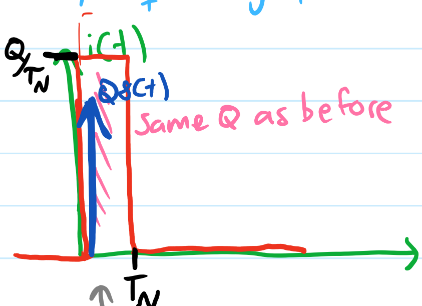




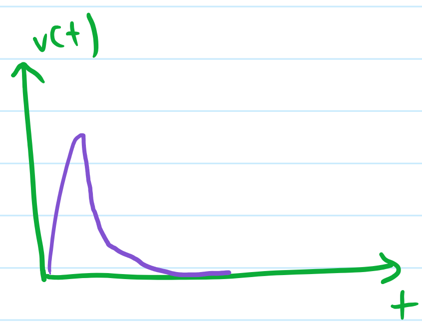
Area = IT = Q



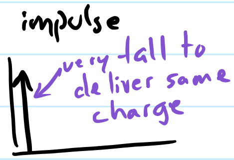
Try "squeezing" pulse

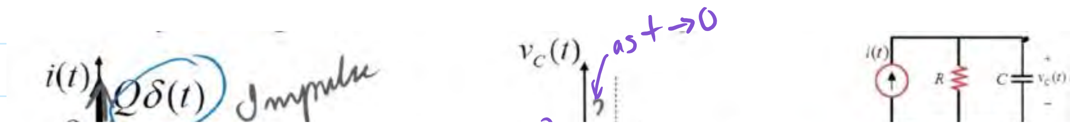


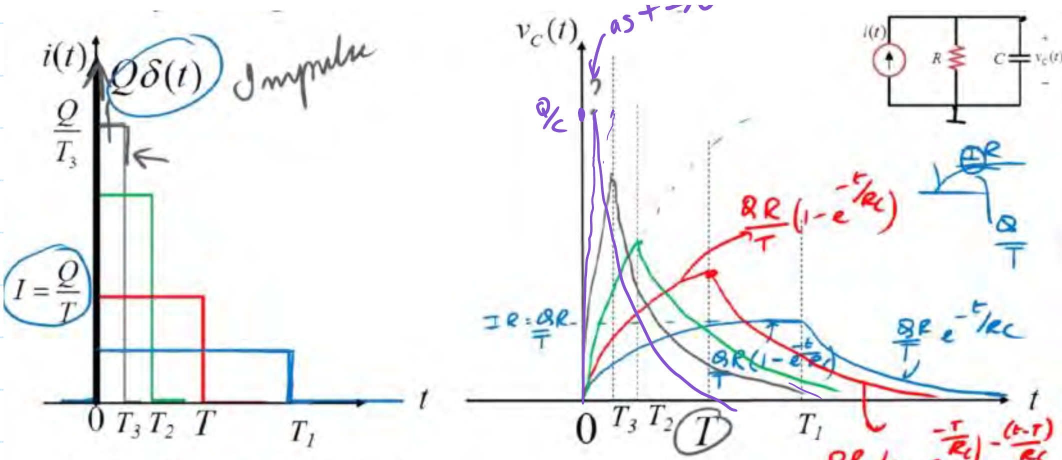
very narrow
⟹ impulse



IT=Q becomes impulse
as $T \to 0$





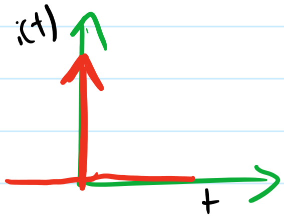


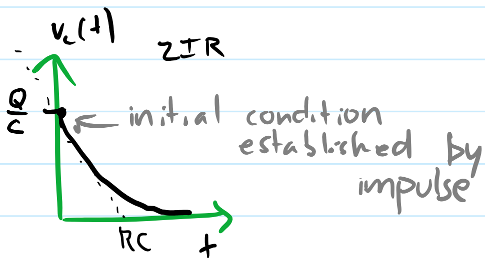
Becomes "impulse" as Lim T → 0
Denoted by $\delta(t)$
Impulse is an important signal type

As $T \to 0$ | all charge deposited into capacitor instantaneously

⇒ capacitor looks like short

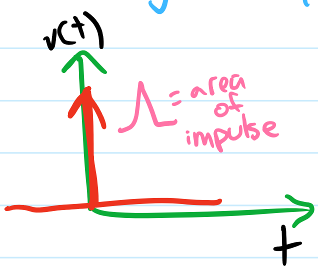
=> Impulse established an initial condition on the capacitor = Q/C

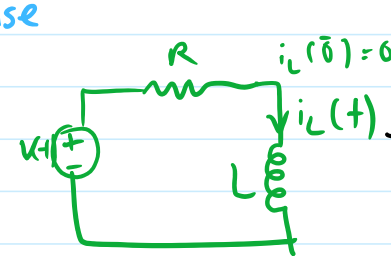


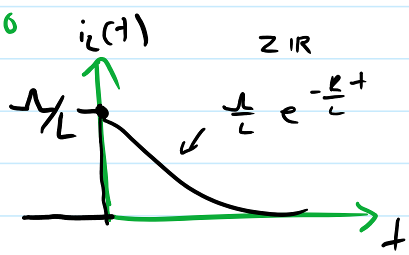


- $v_c(t)$ independent of T. what matters is Q.
- capacitor behaves like short circuit for fast changes
- all charge Q goes through capacitor

Voltage Impulse







- Inductor behaves like instantaneous open circuit

⇒ all voltage spike accross inductor

open circuit

$\Rightarrow$ all voltage spike accross inductor

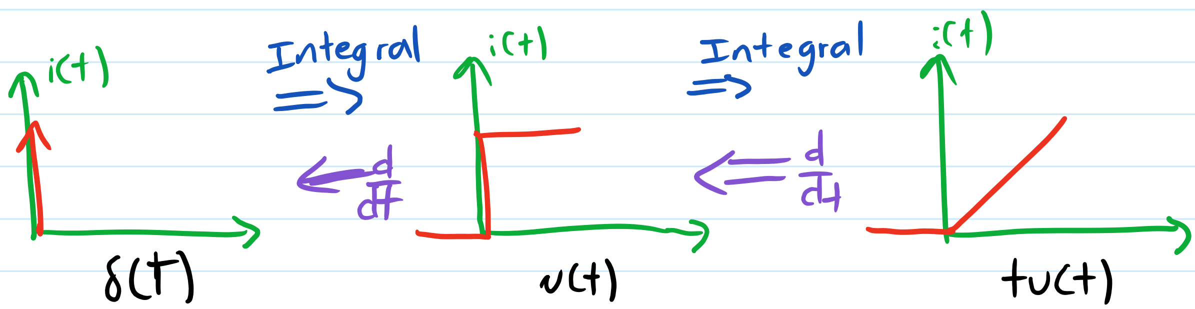
Superposition

- Superposition does not apply when initial conditions exists

To solve:

- Treat initial (independent) conditions like sources
  1. Turn off sources and set initial cndts to zero
  2. Find response to each source (and initial condition) acting alone
  3. Sum up partial responses

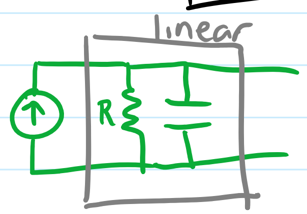
Impulses, Steps and Ramps



$x(t) \rightarrow \boxed{\text{linear}} \rightarrow y(t)$

$\frac{d}{dt}x(t) \rightarrow \boxed{\text{linear}} \rightarrow \frac{d}{dt}y(t)$

$\int x(t) \rightarrow \boxed{\text{linear}} \rightarrow \int y(t)$



$1\cdot\delta(t)$ $\qquad$ $\frac{1}{C}e^{-t/RC}$

$\Rightarrow \int_0^t \delta(t)\,dt = u(t) \longrightarrow \int_0^t \frac{1}{C}e^{-t/RC}\,dt = \frac{1}{C}\left(-\frac{1}{1/RC}\right)e^{-t/RC}\Big|_0^t$

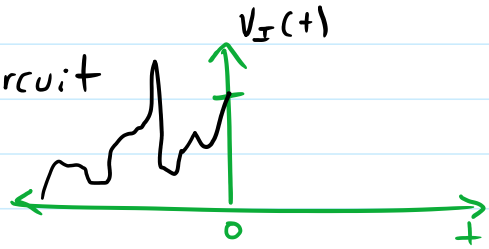
$= RC(1-e^{-t/RC})$

can find output of an input using output for another input!

W4: State and Memory

Tuesday, November 15, 2016 10:45 PM

Recall for RC circuit



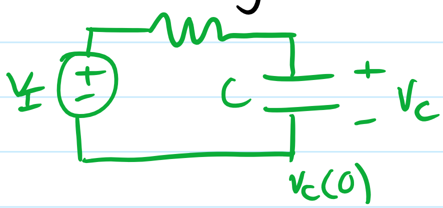
$V_I(0)$ captures entire history of V_I before $t=0$

State

↳ summary of past inputs relevant to predicting the future

state variable

$q = CV$ (for linear capacitors)



$$V_C = f\left(V_C(0),\ V_I(t)\right)$$

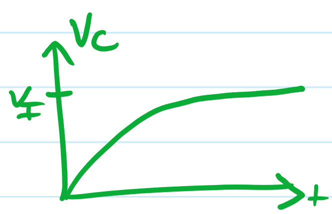
$$= V_C(0) + \left(V_I - V_C(0)\right)\left(1 - e^{-t/RC}\right)$$

ZIR and ZSR

↳ Zero Input Response ↳ Zero State Response

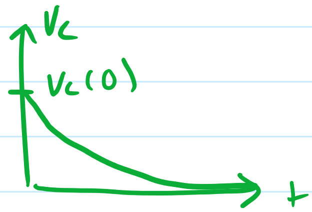
- ZS $\quad V_C(0) = 0 \quad$ state of capacitor is zero
- ZI $\quad V_I(t) = 0 \quad$ input is zero

- ZSR



$$V_I\left(1 - e^{-t/RC}\right) \quad ②$$

- ZIR



$$V_C = V_C(0)\, e^{-t/RC} \quad ③$$

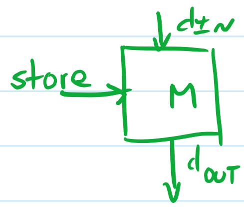
$$V_C = V_I + \left(V_C(0) - V_I\right) e^{-t/RC} \quad ①$$

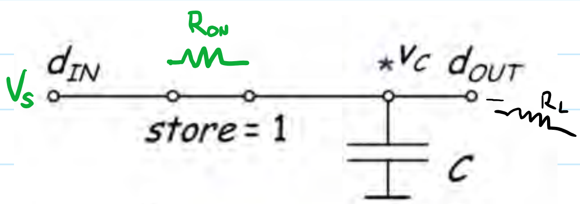
Note that ① = ② + ③

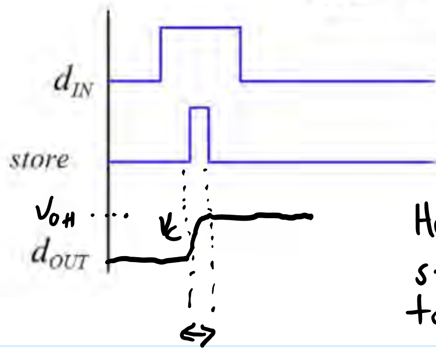
$\Rightarrow$ Total Response = ZSR + ZIR

Digital Memory

1 bit memory element







If store goes high change d_{out} to whatever d_{IN} is

minimum pulse width

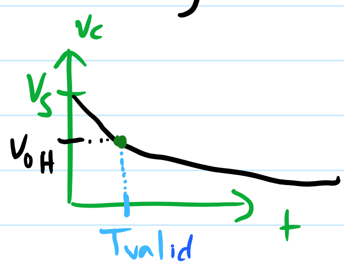
How long does store need to be on?

Recall: $V_C = V_s(1-e^{-t/R_{ON}C})$

we need V_C to be at least V_{OH} so

$$V_{OH} = V_s(1-e^{-T_{min}/R_{ON}C})$$

For how long will the capacitor store a "1" for?



ZIR: $V_C = V_s e^{-t/R_LC}$

$$V_{OH} = V_s e^{-T_{valid}/R_LC}$$

$$\ln\left(\frac{V_{OH}}{V_s}\right) = -T_{valid}/R_LC$$

$$\Rightarrow T_{valid} = -R_LC\ln\left(\frac{V_{OH}}{V_s}\right)$$

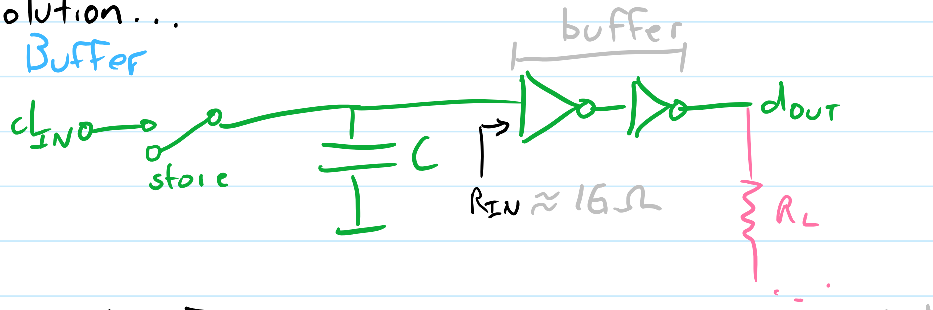
Problem: storage node directly connected to output so memory does not hold for long...

Solution...

memory does not hold for long...

Solution...

Add Buffer



What is T_{valid} now?

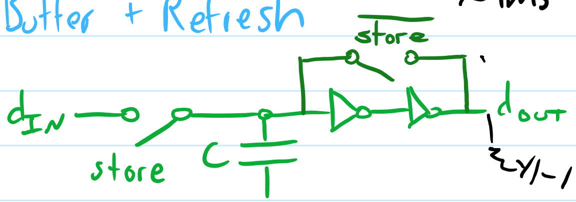
$$T_{valid} = -R_{IN}C \ln \frac{V_{OH}}{V_S}$$

$\approx 1ms$

can control — cannot control

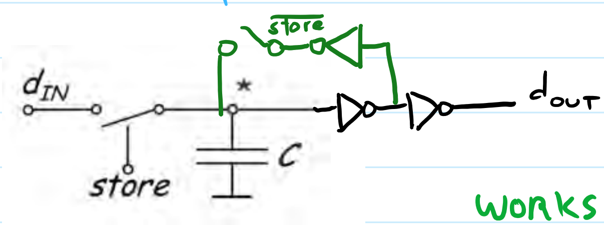
$$R_{IN} \gg R_{ON}$$

Add Buffer + Refresh



does not work since external value can influence storage node

Buffer + decoupled Refresh

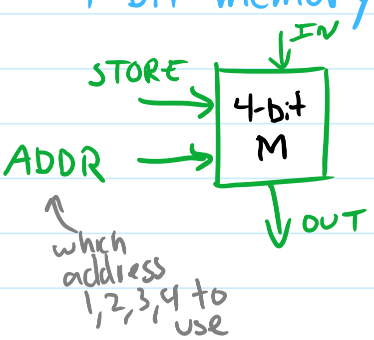


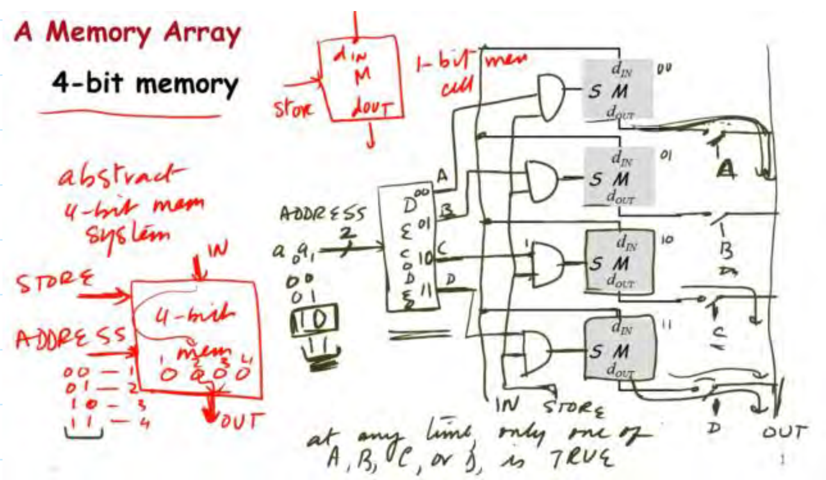
works!

no way for output to affect storage node

Memory Array

4-bit memory





This is how larger systems are built !

Quiz 2 Review

Friday, November 4, 2016 2:45 AM

Saturation Region Operation of the MOSFET A MOSFET operates in the saturation region when the following two conditions are met:

$$v_{GS} \geq V_T \qquad (7.6)$$

and

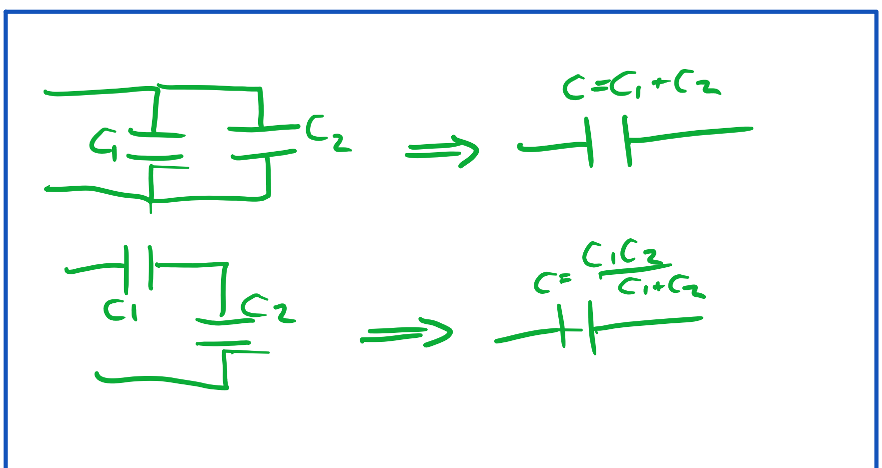
$$v_{DS} \geq v_{GS} - V_T. \qquad (7.7)$$

Metric prefixes in everyday use			
Text	**Symbol**	**Factor**	**Power**
exa	E	1 000 000 000 000 000 000	10^{18}
peta	P	1 000 000 000 000 000	10^{15}
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^{9}
mega	M	1 000 000	10^{6}
kilo	k	1 000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
(none)	(none)	1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}
femto	f	0.000 000 000 000 001	10^{-15}
atto	a	0.000 000 000 000 000 001	10^{-18}

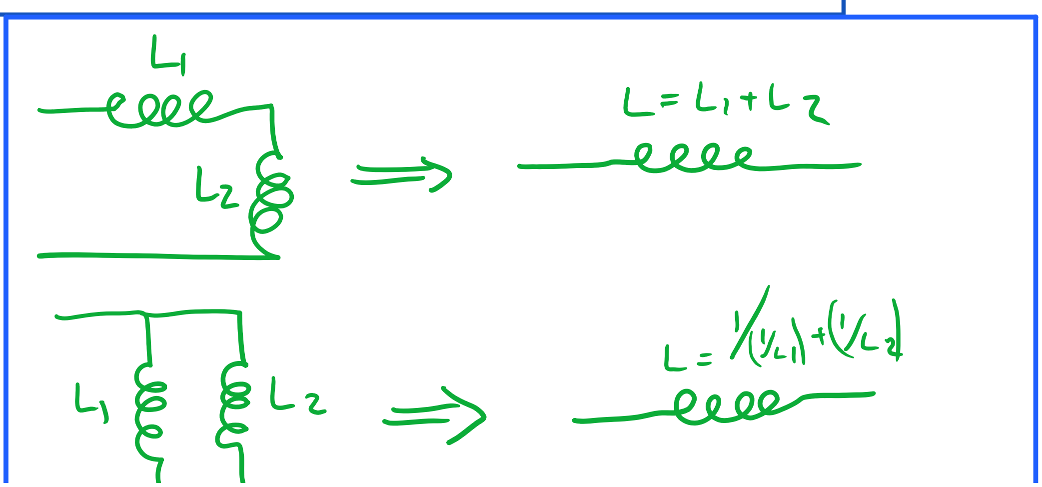
Transconductance

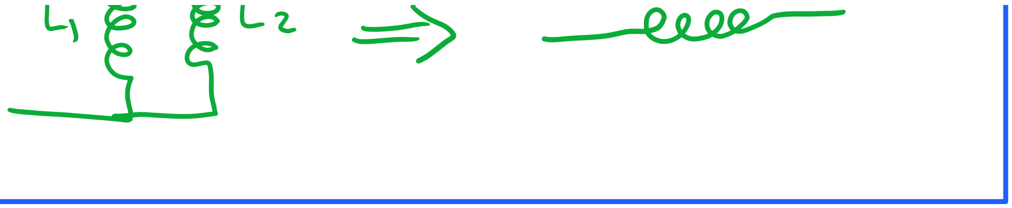
$$g_m = \frac{\delta i_D}{\partial V_{GS}}$$

- Decay time constant $= C_{total} R_{TOTAL}$



Current Impulse:
  Capacitor --> Short
Voltage Impulse
  Inductor --> Open Circuit





Q1

0 points possible (ungraded)

The circuit shown below is called a "common gate amplifier": the input is applied to the source rather than the gate of the MOSFET. As usual, however, the MOSFET is biased so that it is operated in saturation.

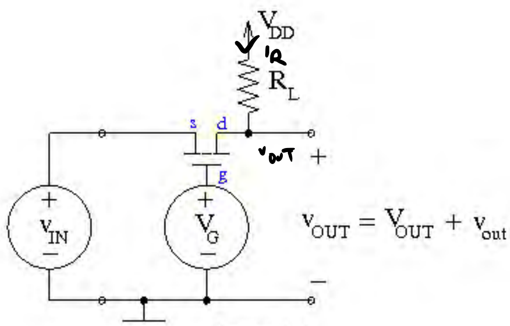


Figure 1-1

The MOSFET's parameters are: $V_T = 0.9$V and $K = 0.018$A/V^2. The resistance of the load resistor $R_L = 220.0\Omega$.

If in the application of this circuit the input voltage v_{IN} may swing between 1.0V and −2.0V, what is the minimum value of the bias voltage V_G, in Volts, needed to keep the MOSFET out of cutoff? $V_{GS} = V_G - V_{IN}$

1 -2

1.9V ✓

$V_{GS} \geq V_T$

$V_G - V_{IN} \geq V_T$

$V_G - 2 \geq .9$

OR

$V_G - 1 \geq .9$ so $V_G = 2.9$

$V_G = 1.9 \leftarrow$ min

b) Assume $V_G = 3.0$V, and the input swing is as specified.

What is the minimum value of the power-supply voltage V_{DD}, in Volts, needed to keep the MOSFET in saturation? Saturation when: $V_{DS} \geq V_{GS} - V_T$

35.3938 ✓

c) Choose the operating point where the input bias voltage $V_{IN} = 0$.

Now we go to a small-signal model: what is the transconductance g_m, in Amperes/Volt, of the MOSFET at the chosen operating point?

.0378 ✓

b) Saturation when: $V_{DS} \geq V_{GS} - V_T$

$V_{DS} \geq V_G - V_{IN} - V_T$

$V_{IN} = 1$ or -2

$V_G = 3$

$V_T = .9$

$V_{DS} \geq V_G - V_{IN} - V_T$ $\quad V_G = 3$, $V_T = .9$

$V_{DS} \geq 3 - 1 - .9$ or $V_{DS} \geq 3 + 2 - .9$

$V_{DS} \geq 1.1$ $\quad V_{IN} = 1$

OR

$V_{DS} \geq 4.1$ $\quad V_{IN} = -2$

$V_{DS} = V_{OUT} - V_{IN}$

$V = IR$, $I = V/R$

$\cancel{i_{DS} = \frac{V_{DD}}{R_L} = \frac{K}{2}(V_{GS} - V_T)^2}$ $\quad \cancel{V_{GS} = V_G - V_{IN}}$

1 or $-.9$

$\cancel{\frac{V_{DD}}{220} = \frac{.018}{2}(3 - V_{IN} - .9)^2}$

$\cancel{V_{DD} = \cancel{2.178}}$, if $V_{IN} = 1$

OR $\cancel{V_{DD} = \cancel{8.9298}}$, if $V_{IN} = -2$

$V_{OUT} - 1 \geq 1.1 \Rightarrow V_{OUT} \geq 2.1$

$V_{OUT} + 2 \geq 4.1 \Rightarrow V_{OUT} \geq 2.1$

$V_{DS} = V_{OUT} - V_{IN}$

$V_{OUT} = V_{DD} - i_R R_L$

$V_{DD} - \frac{K}{2}(V_{GS} - V_T)^2 R_L \geq 2.1$

$i_R = i_{DS} = \frac{K}{2}(V_{GS} - V_T)^2$

$V_{DD} - \frac{.018}{2}(3 - V_{IN} - .9)^2 R_L \geq 2.1$ $\quad V_{GS} = V_G - V_{IN}$

IF $V_{IN} = 1$ $\quad$ IF $V_{IN} = -2$

$V_{DD} \geq 4.958$

$\boxed{V_{DD} \geq 35.3838}$ IF $V_{IN} = -2$

$\Rightarrow V_{DD}$ must supply at least $35.38\,V$ to allow MOSFET to stay in saturation when $V_{IN} = -2$

c) Recall: $g_m = \frac{\partial i_D}{\partial V_{GS}}$ $\quad (V_{GS} - V_T)(V_{GS} - V_T)$

$i_D = \frac{K}{2}(V_{GS} - V_T)^2 = \frac{K}{2}(\cancel{V_{GS}^2} - 2V_{GS}\cancel{V_T} - V_T^2)^2$

$$v \quad d\,v_{GS}$$

$$\cancel{i_{DS} = \frac{K}{2}(V_{GS}-V_T)^2 = \frac{K}{2}(V_{GS}^2 - 2V_{GS}V_T - V_T^2)^2}$$

$$\cancel{\frac{d}{V_{GS}} = \frac{K}{2}(V_G - V_{IN} - V_T)^2 = \frac{K}{2}(2V_{GS} - 2V_T)^2} \leftarrow \text{mistake!}$$

$$\cancel{= \frac{K}{2}(2(V_G - \overset{0}{V_{IN}}) - 2V_T)^2}$$

$$\cancel{= \frac{.018}{2}(6 - 2(.9))^2 =}$$

$$i_{DS} = \frac{K}{2}(V_{GS}-V_T)(V_{GS}-V_T)$$

$$= \frac{K}{2}(V_{GS}^2 - 2V_{GS}V_T - V_T^2)$$

$$\frac{d\,i_{DS}}{\delta V_{GS}} = \frac{K}{2}(2V_{GS} - 2V_T)$$

$$= \frac{.018}{2}(2(\overset{3}{V_G} - \overset{0}{V_{IN}}) - 2(.9))$$

$$= \frac{.018}{2}(6 - 2(.9)) = \boxed{.0378}$$

What is the incremental gain $\frac{v_{out}}{v_{in}}$ of this circuit at the chosen operating point?
Be careful with signs here.

✖

What is the equivalent incremental input resistance, in Ohms, of this small-signal model?
Note that the incremental input resistance is $\frac{v_{in}}{i_{in}}$, where i_{in} is the current taken from the input voltage source for the voltage v_{in}.

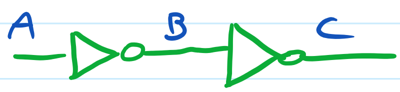
✖

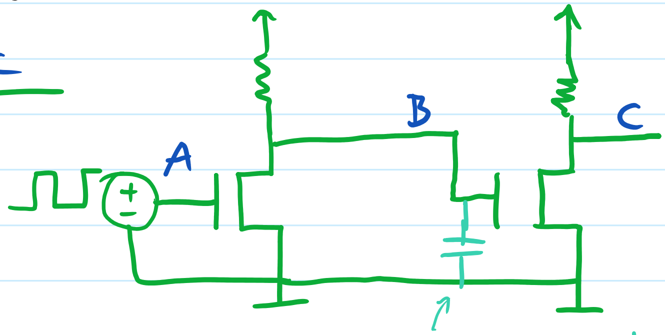
W1: Undamped Second Order Systems

Sunday, November 20, 2016 10:35 AM

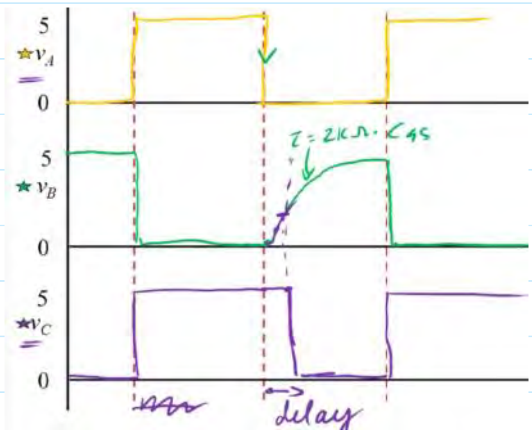
Review Complex Algebra
↳ Appendix C

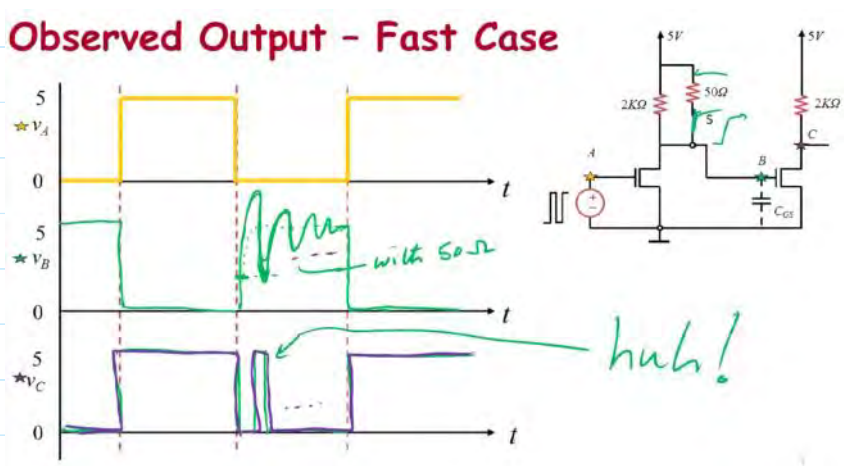
Recall Slow case





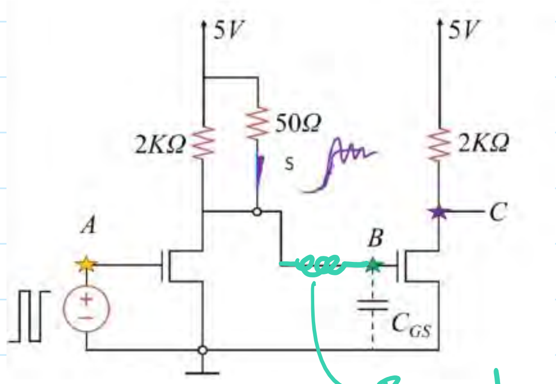
Parasitic Capacitance



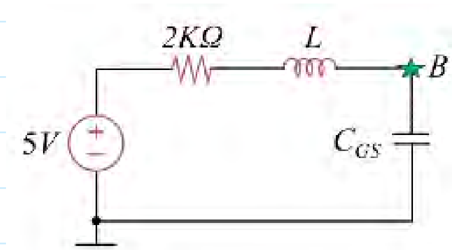


What is happening?

Relevant circuit

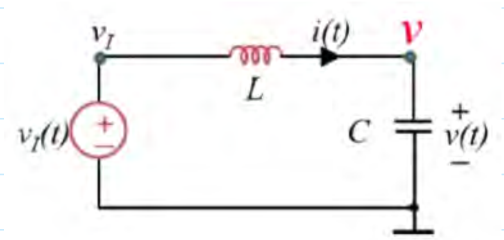


Paracitic inductance
because of large loops



RLC Circuit

LC network (simpler than RLC)



Node Method

$$i(t) = C\frac{dv}{dt}$$

$$V_I - V = L\frac{di}{dt}$$

$$\frac{1}{L}\int_{-\infty}^{t}(V_I - V)\,dt = i(t)$$

$$\rightarrow \frac{1}{L}\int_{-\infty}^{t}(V_I - v)\,dt = C\frac{dv}{dt}$$

$$\frac{1}{L}(V_I - v) = C\frac{d^2v}{dt^2}$$

$$LC\frac{d^2V}{dt^2} + V = V_I$$ ← second order equation

Solving second order differential equation

1. Find particular solutions
2. Find homo genus solution (4 steps)
3. Total solution = sum of particular and homogenus solution.
   use initial conditions to solve for constants

1. Any solution

$$LC\frac{d^2V}{dt^2} + v_p = V_I$$

Guess

$$V_p = V_I$$

Guess
$v_p = V_I$

(2) Solution to $LC \frac{d^2 V_H}{dt^2} + V_H = 0$ ← set drive to 0

(2a) Guess functions whose second derivative is same as function.

Guess: $V_H = Ae^{st}$

Plug in ⇒ $LC\cancel{A}s^2\cancel{e^{st}} + \cancel{A}\cancel{e^{st}} = 0$ A, s = ??

$\boxed{LCs^2 + 1 = 0}$ characteristic equation

(2b) $s^2 = -1/LC$ characteristic eqtn

(2c) Roots of chrcstic equations $j = \sqrt{-1}$

$s = \pm j\sqrt{1/LC}$ use $\boxed{\omega_0 = \sqrt{1/LC}}$ ← frequency

$\boxed{s = \pm j\omega_0}$

(2d) General solution of homogenous equation

$V_H = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$

(3) Total Solution Given $v(0) = 0$, $i(0) = 0$

$v(t) = v_p(t) + v_h(t)$

$= V_I + A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$

Find A_1, A_2 using initial conditions

$v(0) = 0 \rightarrow 0 = V_I + A_1 + A_2$

$i(0) = 0$ $i(t) = C\frac{dv}{dt} = CA_1 j\omega_0 e^{j\omega_0 t} - CA_2 j\omega e^{-j\omega_0 t}$

$0 = \cancel{C}A_1 \cancel{j\omega_0} - \cancel{C}A_2 \cancel{j\omega_0}$

$\Rightarrow A_1 = A_2$

$A_1 = \frac{-V_I}{2} = A_2$

$$A_1 = \frac{-V_I}{2} = A_2$$

$$v(t) = V_I - \frac{V_I}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$e^{jx} = \cos x + j\sin x$

Euler relation:

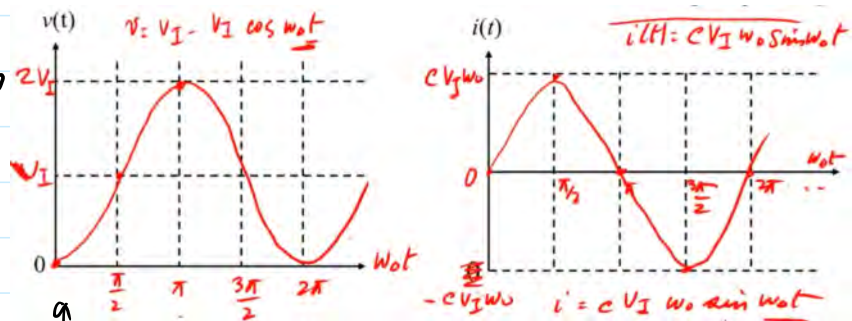
Algebra shortcut

$$\frac{e^{jx} + e^{-jx}}{2} = \cos(x)$$

For $t \geq 0$

$$\boxed{\begin{aligned} v(t) &= V_I - V_I\cos(\omega_0 t) \\ i(t) &= C\frac{dv}{dt} = C\,V_I\,\omega_0 \sin(\omega_0 t) \end{aligned}}$$

can output more than input!



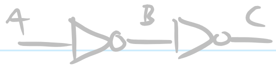
sinusoidal output given a step input

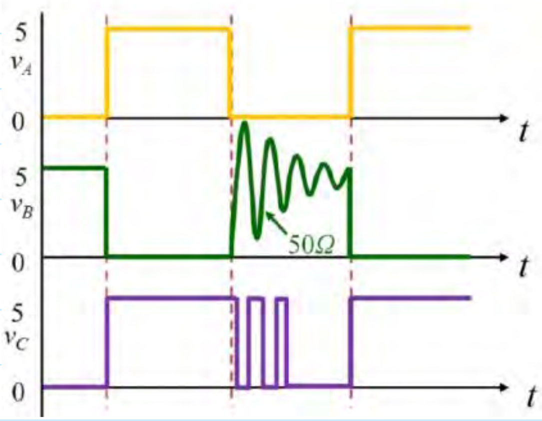
Summary of Method

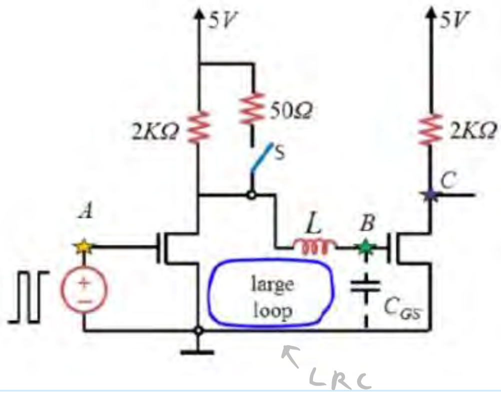
1. Write DE for circuit by applying node method.
2. Find particular solution v_P by guessing and trial & error.
3. Find homogeneous solution v_H
   - (A) Assume solution of the form Ae^{st}.
   - (B) Obtain characteristic equation. $LCs^2 + 1 = 0$
   - (C) Solve characteristic equation for roots s_i.
   - (D) Form v_H by summing $A_i e^{s_i t}$ terms.
4. Total solution is $v_P + v_H$, then solve for remaining constants using initial conditions.

W1 : Damped Second Order Systems

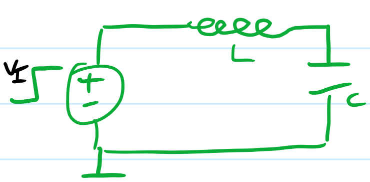
Tuesday, November 22, 2016 8:25 AM

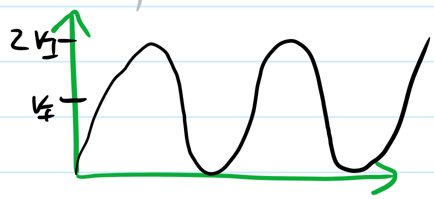




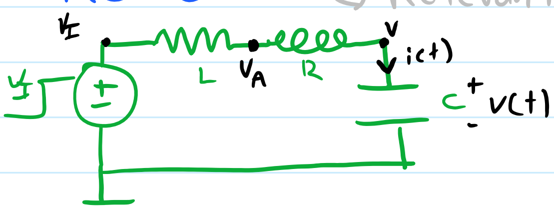


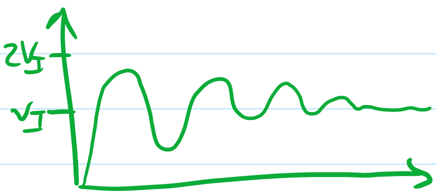
LC Circuit (From last lecture)





LRC Circuit ↳ Relevant circuit





"Decaying" synusoid
"Damped"

A&L 13.6

Analysis

Node Method :

v: $\frac{V_A - v}{R} = C\frac{dv}{dt}$

V_A: $\frac{1}{L}\int_{-\infty}^{t}(V_I - V_A)\,dt = \frac{V_A - v}{R}$

want to get rid of V_A (to solve for v)

v: $V_A = RC\frac{dv}{dt} + v$

$\frac{1}{L}\int^{t}(V_I - V_A)\,dt = C\frac{dv}{dt}$.

Review :

L : $V_L = \frac{L\,di_L}{dt}$

$\frac{1}{L}\int_{-\infty}^{t} V_L\,dt = i_L$

C : $i_C = C\frac{dV_C}{dt}$

$\frac{1}{C}\int_{-\infty}^{t} i_C\,dt = V_C$

$$\frac{1}{L}\int_{-\infty}^{t}(v_I - v_A)\,dt = C\frac{dv}{dt}$$

derivative of both sides

$$\frac{1}{L}(V_I - V_A) = C\frac{d^2v}{dt^2}$$

$$\frac{1}{LC}(V_I - V_A) = \frac{d^2v}{dt^2}$$

$$\frac{1}{LC}\left(V_I - RC\frac{dv}{dt} - v\right) = \frac{d^2v}{dt^2}$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{1}{LC}V_I$$

Solving differential equation

1. Find particular solution $v_p(t)$
2. Find homogenus solution $v_H(t)$
3. Total solution $= v_p(t) + v_H(t)$ and solve for constants using initial conditions

For input $V_I u(t)$ ZSR $v(0)=0$ $i(0)=0$

1. $$\cancel{\frac{d^2v_p}{dt^2}}^{0} + \cancel{\frac{R}{L}\frac{dv}{dt}}^{0} + \frac{1}{LC}v_p = \frac{1}{LC}V_I$$

Guess $V_p = V_I$ ✓

2. Solution $$\frac{d^2v_H}{dt^2} + \frac{R}{L}\frac{dv_H}{dt} + \frac{1}{LC}v_p = 0$$] homogenus equation

2.a) Guess $V_H = Ae^{-st}$ A, s unknown

Plug-in:

$$A\cdot s^2\cancel{e^{st}} + \frac{R}{L}A s\cancel{e^{st}} + \frac{1}{LC}\cancel{Ae^{st}} = 0$$

$$s^2 + R/L + 1/LC = 0$$

Characteristic Equation !!

2b) $s^2 + 2\alpha s + \omega_0^2 = 0$ - put ↑ in canonical form

$$\omega_0 = 1/\sqrt{LC}$$

$$\alpha = R/2L$$

2c) Find Roots

$$s_{1,2} = \frac{-2\alpha \pm \sqrt{(2\alpha)^2 - (4)(1)(1)}}{2}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

2d) General Solution

$$v = A_1 e^{s_1 t} + A_2 e^{st}$$

2d) General Solution

$v_A = A_1 e^{s_1 t} + A_2 e^{s t}$

$v_H = A_1 e^{-\alpha + \sqrt{\alpha^2 + \omega_0^2}} + A_2 e^{-\alpha - \sqrt{\alpha^2 + \omega_0^2}}$

3) Total solution: $v(t) = v_p(t) + v_H(t)$

$v(t) = V_I + A_1 e^{(-\alpha + \sqrt{\alpha^2 + \omega_0^2})t} + A_2 e^{(-\alpha - \sqrt{\alpha^2 + \omega_0^2})}$ for $t \geq 0$

Find A_1, A_2. We know $v(0) = 0 \quad i(0) = 0$

(1) $0 = V_I + A_1 + A_2$ $\quad i(t) = C \frac{dv}{dt}$

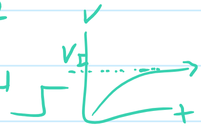
$i(t) = C A_1 (-\alpha + \sqrt{\alpha^2 - \omega_0^2}) e^{-\alpha + \sqrt{\alpha^2 - \omega_0^2} t}$
$+ C A_2 (-\alpha - \sqrt{\alpha^2 - \omega_0^2}) e^{-\alpha - \sqrt{\alpha^2 - \omega_0^2} t} = 0$ when $t = 0$

(2) $= C A_1 (-\alpha + \sqrt{\alpha^2 - \omega_0^2}) + C A_2 (-\alpha - \sqrt{\alpha^2 - \omega_0^2})$

Solve for A_1, A_2 using (1) and (2)

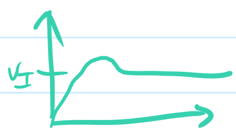
3 cases:

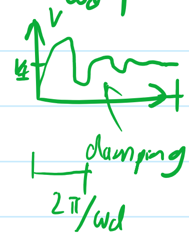
$\alpha > \omega_0$: overdamped : $v(t) = V_I + A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$ constant



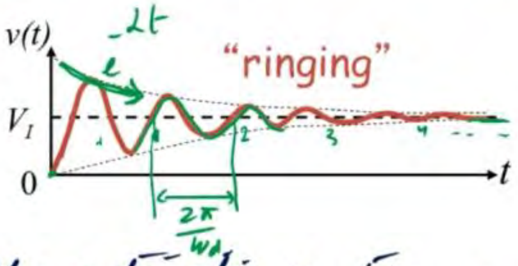
$\alpha < \omega_0$: underdamped : $v(t) = V_I \frac{\omega_0}{\omega_d} e^{-\alpha t} \cos(\omega_d t - \tan^{-1} \frac{\alpha}{\omega_d})$

$\alpha = \omega_0$: critically damped





Easy way: Characteristic equation tells the whole story!



Characteristic equation tells the story (for series RLC ckt)

$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$

$s^2 + 2\alpha s + \omega_0^2 = 0$

canonic form

from char. eq

$\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
↳ oscillation freq

α = governs the rate of decay

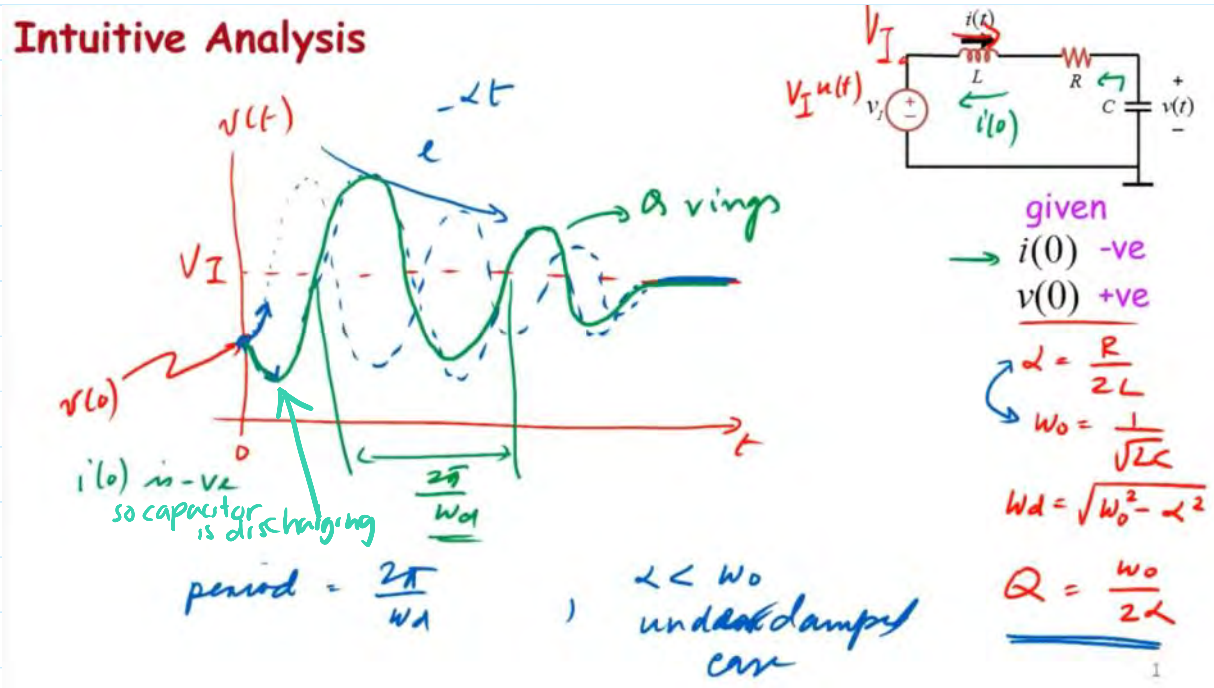
V_I = final val

$v(0)$ = initial val

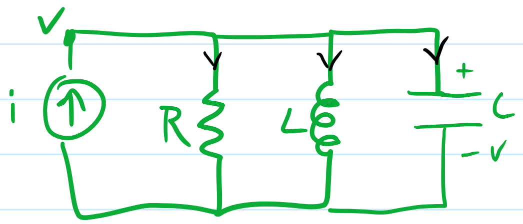
$Q = \frac{\omega_0}{2\alpha}$: Quality factor (approx. the number of cycles of ringing)

$\boxed{\text{NO DIFF EQs!}}$

See Sec. 12.7 of A&L textbook



Parallel RLC



$$C\frac{dv}{dt} + \frac{v}{R} - i + \frac{1}{L}\int_{-\infty}^{t} v\,dt = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = i$$

Set drive to 0 to get homogenus equation

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$ characteristic equation

$$\alpha = 1/2RC$$
$$w_o = 1/\sqrt{LC}$$
$$w_d = \sqrt{w_o^2 - \alpha^2}$$
$$Q = w_o/2\alpha$$

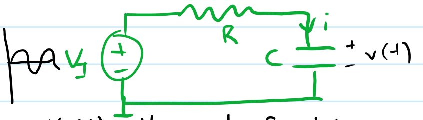
W2: Sinusoidal Steady State

Tuesday, November 22, 2016 9:01 PM

13.1

- Any signal can be represented as a sum of sinusoids

Sinusoidal Response of RC Network



$v(0)=0$

$V_I(t) = V_i \cos\omega t$ for $t>0$

Solution:

Could write out diffeq but it gets really hard instead...

- Find Particular solution to another input

$$RC\frac{V_{ps}}{dt} + V_{ps} = V_{Is} = V_i e^{st}$$

Related "sneaky path"

real part of $V_i\cos\omega t$ ← input

Try: $V_{ps} = V_p e^{st}$ Find V_p.

$$RC\frac{dV_{p_s}e^{st}}{dt} + V_{p_s}e^{st} = V_i e^{st}$$

easy to find

$$RC\,V_{p_s} s\cancel{e^{st}} + V_{p_s}\cancel{e^{st}} = V_i\cancel{e^{st}}$$

$$V_{p_s}(RC_s+1) = V_i$$

$$V_{p_s} = \frac{V_i}{(RCs+1)}$$

$V_p e^{st} = \frac{V}{RCs+1}e^{st}$

so now we know:

$$V_p = \frac{V_i}{j\omega RC+1}\cdot e^{j\omega t} \longrightarrow \text{particular solution for } V_i e^{j\omega t}$$

$\boxed{V_p}$ $s=j\omega$

Inverse superposition: Real input → Real output

so to find V_p from V_{ps} take the real part of V_{ps}.

$$v_p = \text{Re}[v_{ps}] = \text{Re}\left[V_p e^{j\omega t}\right] = \text{Re}\left[\frac{V_i}{1+j\omega RC}\cdot e^{j\omega t}\right]$$

complex conjugate

Polar form

$$= \text{Re}\left[\frac{V_i(1-j\omega RC)}{1+\omega^2R^2C^2}\cdot e^{j\omega t}\right] = \text{Re}\left[\frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cdot e^{j\phi}e^{j\omega t}\right]$$

$\phi = \tan\phi = -\omega RC$

$$\Rightarrow V_p = \frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cos(\omega t+\phi)$$

(3) Find V_H

$$V_H = Ae^{-t/RC}$$

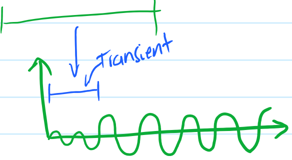
(4) Total Solution:

$\tan^{-1}-\omega RC$

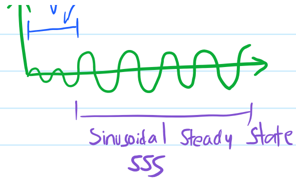
$$V = \frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cos(\omega t+\phi) + \underbrace{Ae^{-t/RC}}_{}$$

Given $v(0)=0$

$$A = \frac{-V_I\cos(\phi)}{\sqrt{1+\omega^2R^2C^2}}$$



$$A = \frac{-V_I \cos(\phi)}{\sqrt{1 + w^2 R^2 C^2}}$$



All info about SSS is contained in V_p

ie: step 3 and 4 are unecessary

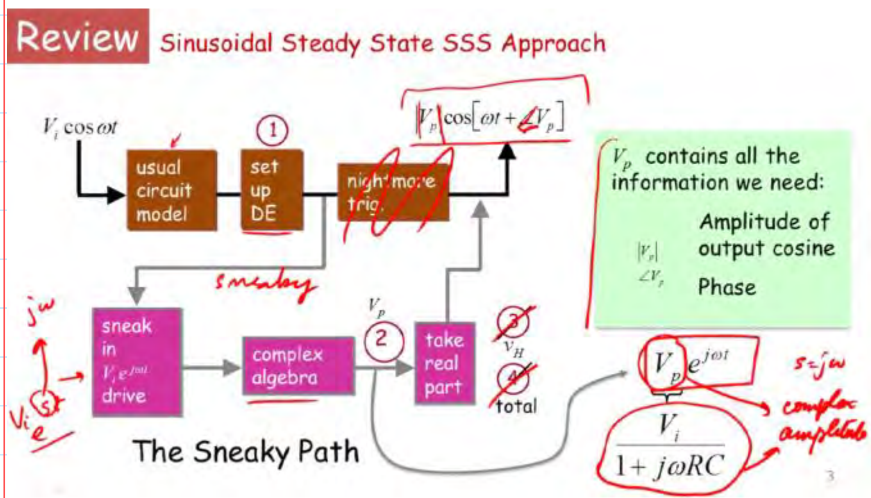
$$\frac{V_p}{V_i} = \frac{1}{1 + jwRC} = \frac{1}{\sqrt{1 + w^2 R^2 C^2}} e^{j\phi}$$

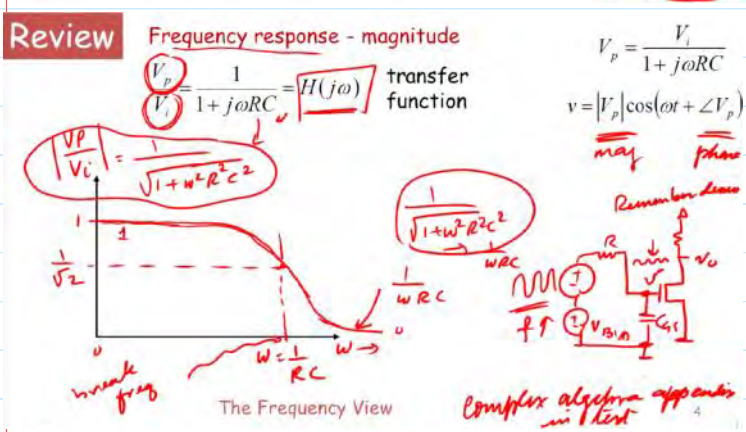
$\phi = \tan^{-1} -wRC$

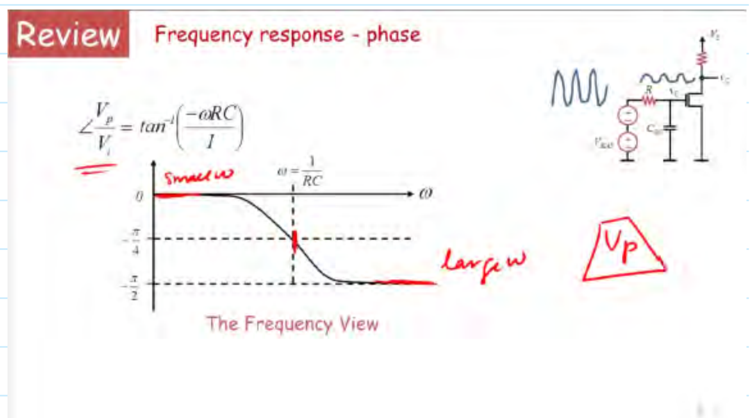
phase magnitude form

W2: Impedance Model

Monday, November 28, 2016 6:32 PM







Finding Vp (more easilly)

s=jw

$$V_p = \frac{V_i}{1+j\omega RC}$$

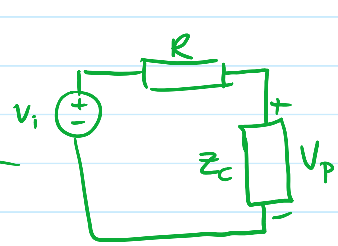
$j\omega \to s$

$$= \frac{V_i \frac{1}{sC}}{\frac{1}{sC} + R}$$

divide by $1/sC$

↳ looks like voltage divider

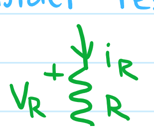
$Z_c = 1/sC$



The Impedence Model

complex amplitudes s=jw

Consider resistor



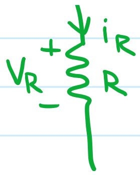
given

$$i_R = I_r e^{st}$$

$$\Rightarrow V_R = V_r e^{st}$$

$$V_R = R i_R$$

$$V_r \cancel{e^{st}} = R I_r \cancel{e^{st}}$$



$i_R = I_r e^{st}$

$\Rightarrow v_R = \underbrace{V_r}_{} e^{st}$

$z_R = R$

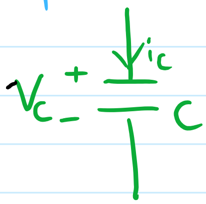
$v_R = R i_R$

$V_r \cancel{e^{st}} = R I_r \cancel{e^{st}}$

$V_r = R I_r$

complex amplitudes related according to ohms law !!

Capacitor



$i_C = I_C e^{st}$

$v_C = V_c e^{st}$

$i_C = C \frac{dv_c}{dt}$

$I_c \cancel{e^{st}} = C V_c s \cancel{e^{st}}$

$I_c = C V_c s$

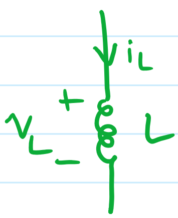
$Z_C = \frac{1}{sC}$

$V_c = \underbrace{\frac{1}{sC}}_{z_c} \cdot I_c$ ← Impedance

$V_c = z_c I_c$

complex amplitudes are related by generalization of ohms law.

Inductor



$i_L = I_L e^{st}$

$v_L = V_L e^{st}$

$v_L = L \frac{di_L}{dt}$

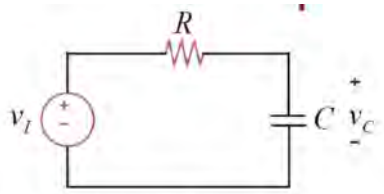
$V_\ell \cancel{e^{st}} = L \cdot I_\ell \, s \, \cancel{e^{st}}$

$V_\ell = \underbrace{L s}_{z_L} I_\ell$

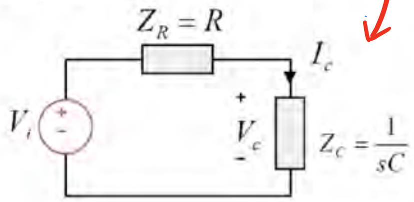
$Z_L = Ls$

$V_\ell = z_L I_\ell$

⇒ For a drive of the form $V_c e^{st}$, the complex amplitude V_c is related to the complex amplitude I_c by a generalization of ohms law.



Impedance model:



① Replace components with boxes and impedance.

Treat impedance like resistors

② Can solve algebreically !

(no diff eq)

↳ KVL, KCL

↳ superposition

$$V_c = \frac{\frac{1}{sC}}{\frac{1}{sC}+R} V_i$$

voltage divider

$$= \frac{1}{\underbrace{1+sCR}_{\text{Characteristic equation!}}} \quad \square$$

Signal notation

Pg 716

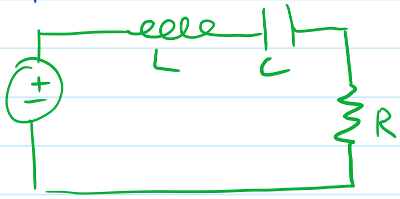
- V_A - DC or operating point variables
- v_A - Total instantaneous variables
- v_a - Small signal instantaneous variables
- V_a - complex amplitudes

Impedance Method Summary

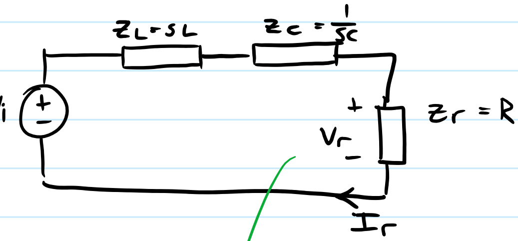
1. First, replace the (sinusoidal) sources by their complex (or real) amplitudes. For example, the input voltage $v_A = V_a \cos(\omega t)$ is replaced by its real amplitude V_a.
2. Replace circuit elements by their impedances, namely, resistors by R boxes, inductors by Ls boxes, and capacitors by $1/Cs$ boxes. Here $s = j\omega$. The resulting diagram is called the *impedance model* of the network.
3. Now, determine the complex amplitudes of the voltages and currents in the circuit by any standard linear circuit analysis technique — Node method, Thévenin method, intuitive method based on series and parallel simplifications, etc.
4. Although this step is not usually not necessary, we can then obtain the time variables from the complex amplitudes. For example, the time variable corresponding to node variable V_o is given by

$$v_O(t) = |V_o| \cos(\omega t + \angle V_o). \qquad (13.41)$$

Example
LCR Circuit



$\Longrightarrow$



Recall $|V_r| \cos(\omega t + \angle V_r)$

voltage divider

$$V_r = \frac{V_i \cdot Z_R}{Z_L + Z_C + Z_R}$$

$$= \frac{V_i R}{sL + \frac{1}{sC} + R} = \frac{V_i R \, s/L}{\boxed{s^2 + 1/LC + \frac{R}{L} s}}$$

characteristic equation

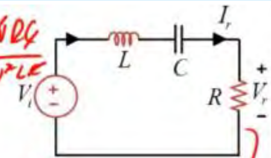
Transfer function:

$$\frac{V_r}{V_i} = \frac{\frac{j\omega R}{L}}{\frac{1}{LC} + \frac{j\omega R}{L} - \omega^2}$$

Graphically

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

by ω $\rightarrow$ $\frac{wRC}{\sqrt{w^4 L^2 C^2}} \rightarrow \frac{wRC}{w^2 LC}$



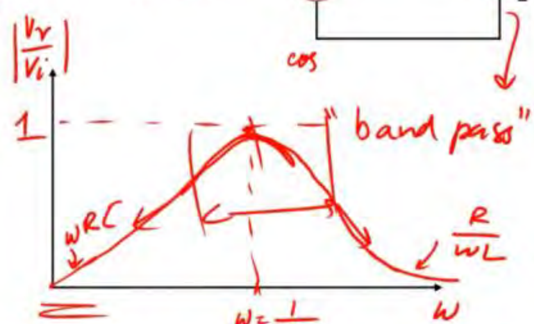
Observe $\left|\frac{V_r}{V_i}\right|$

Low ω: $\approx wRC$

High ω: $\approx \frac{R}{wL}$

$\omega = \frac{1}{\sqrt{LC}}$: ≈ 1

"alternate"

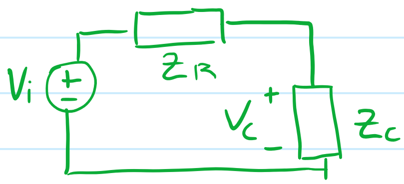


passes frequencies in a middle band

1

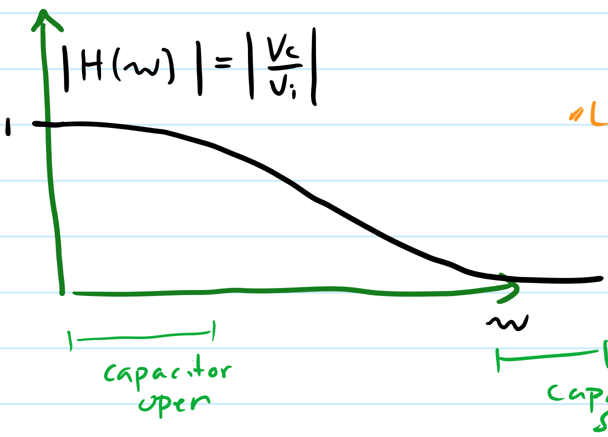
W3: Filters

Wednesday, November 30, 2016 3:51 PM



Transfer function

$$\frac{V_c}{V_i} = \frac{Z_c}{Z_c + Z_R} = \frac{1}{1 + j\omega RC}$$

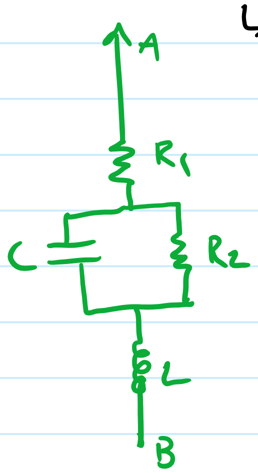


"Low Pass Filter"

- kills high frequency

Recall Impedance Model

↳ can treat impedance like resistor



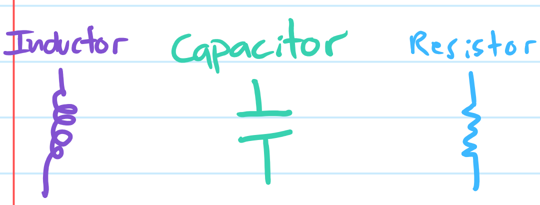
$$Z_{AB} = Z_{R1} + Z_c \| Z_{R2} + Z_L$$

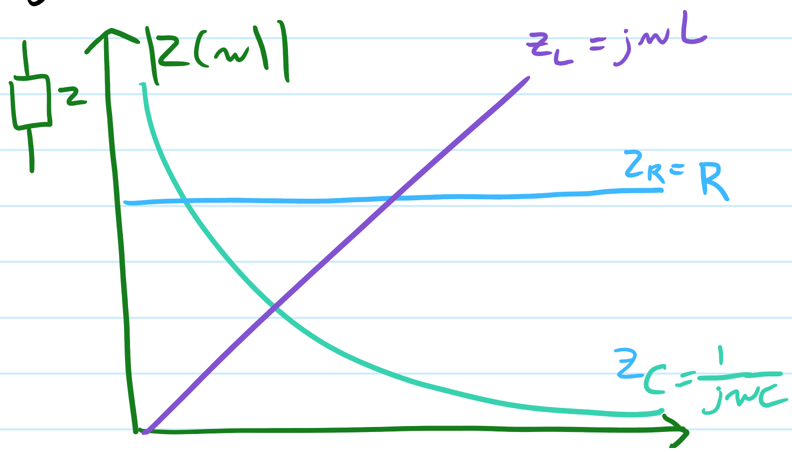
$$= R_1 + \frac{1}{j\omega C} \| R_2 + j\omega L$$

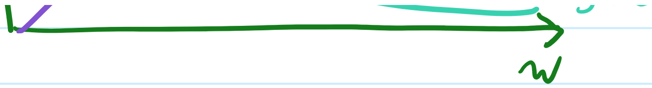
$$= R_1 + \frac{R_2}{1 + j\omega R_2 C} + j\omega L$$

Building Filters

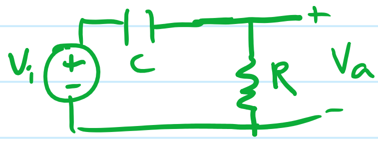
↳ By combining impedances



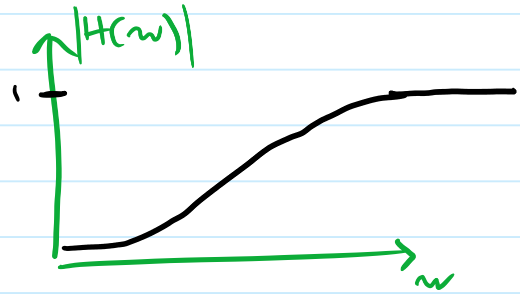




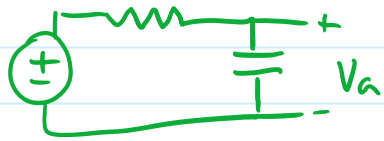
High Pass Filter

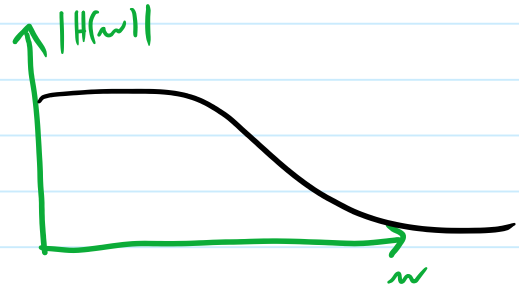


$$H(\omega) = \frac{V_a}{V_i}$$

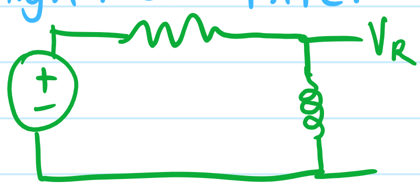


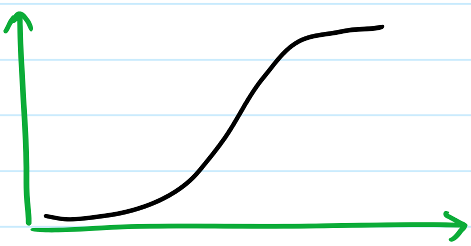
Low Pass Filter





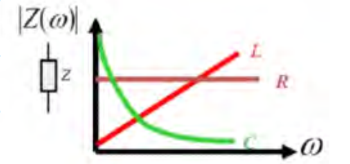
Another High Pass Filter

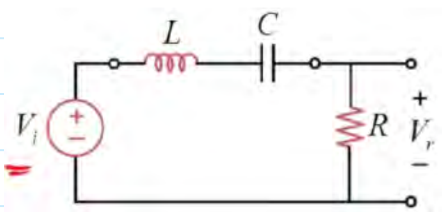




Example

Find $\frac{V_r}{V_i}$

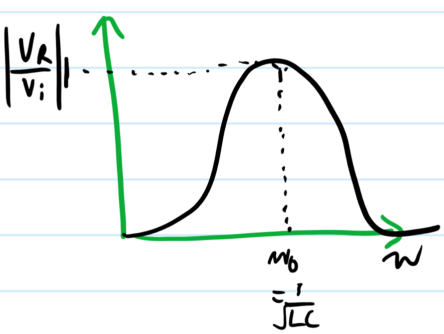


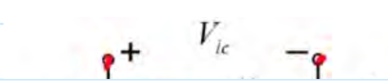


$$\frac{V_r}{V_i} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$$

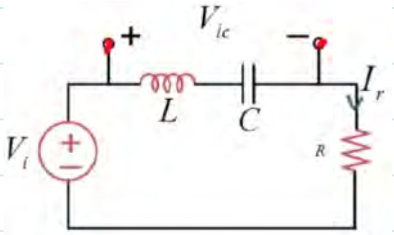
$$= \frac{j\omega C R}{1 - \omega^2 LC + j\omega CR}$$

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\underbrace{(1-\omega^2 LC)^2}_{\text{real part}} + \underbrace{(\omega RC)^2}_{\text{imaginary part}}}}$$





$$z_{LC} = j\omega L + \frac{1}{j\omega C} = j\omega L - j/\omega C$$



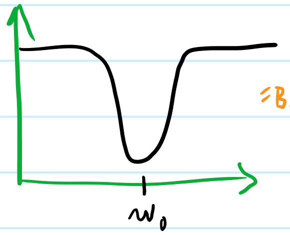
$$z_{LC} = j\omega L + \frac{1}{j\omega C} = j\omega L - j/\omega C$$

Set $\omega = \omega_0 = 1/\sqrt{LC}$

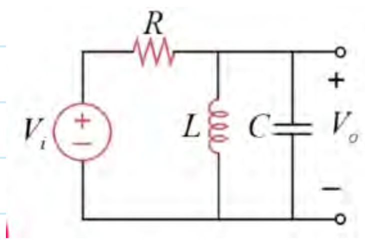
$$Z_{LC} = j\omega_0 L - j/\omega_0 C = j\sqrt{\tfrac{L}{C}} - j\sqrt{\tfrac{L}{C}}$$
$$= 0$$

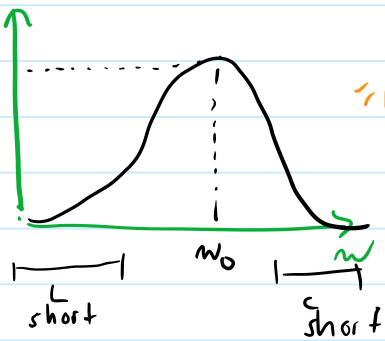
$$V_{LC} = V_L + V_C = I_r j\sqrt{\tfrac{L}{C}} - I_r j\sqrt{\tfrac{L}{C}} = 0$$

Drop accross inductor = drop accross capacitor



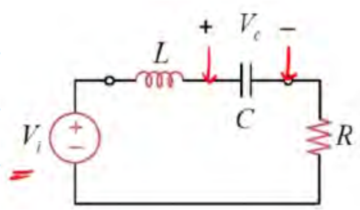
"Band stop Filter"
"notch"





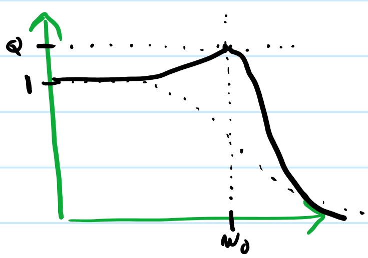
"Band Pass Filter"

$V_i(0)\omega_0 t$



$$\frac{V_c}{V_i} = \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C} + R} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

$$\left|\frac{V_c}{V_i}\right| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$



$\omega_0 = 1/\sqrt{LC}$

$2\alpha = R/L$

$s = j\omega$

$$\frac{V_c}{V_i} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC} + R} = \frac{1}{s^2 LC + 1 + sRC}$$

$\frac{V_c}{V_i}$ at $s = j\omega_0$

$$\frac{V_c}{V_i} = \frac{\omega_0^2}{\cancel{(j\omega_0)^2} + \cancel{\omega_0^2} + 2\alpha(j\omega_0)} = \frac{1/LC}{s^2 + 1/LC + sR/L} = \frac{\omega_0^2}{s^2 + sR/L + \omega_0^2}$$

$$= \omega_0^2 / 2\alpha(j\omega_0) \qquad = \frac{\omega_0^2}{\underline{s^2 + 2\alpha s + \omega_0^2}}$$

characteristic Equation

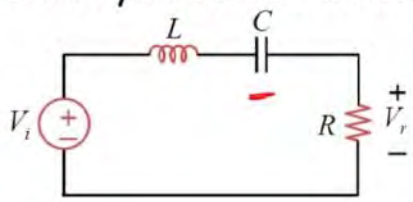
$$\left|\frac{V_c}{V_i}\right| = \frac{\omega_0^2}{2\alpha\omega_0} = \omega_0/2\alpha = Q$$

Magnitude of response $\left|\frac{V_c}{V_i}\right|$ when $s = j\omega_0$ is Q

Magnitude of response $\left|\frac{V_c}{V_i}\right|$ when $s = j\omega_0$ is Q

Selectivity: Look at series RLC in more detail

Show you some fun stuff regarding Q!



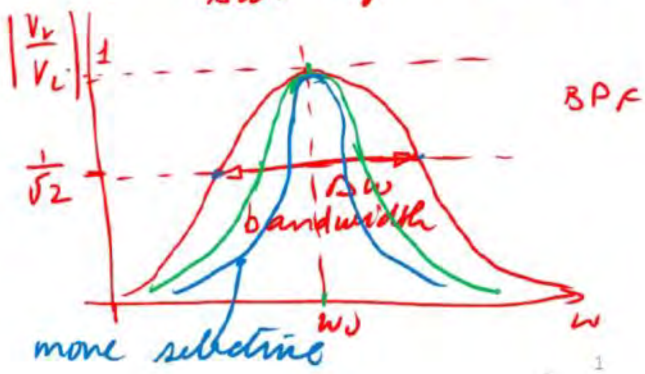
Selectivity look at $\frac{\omega_0}{\Delta\omega} = \frac{\text{resonant freq}}{\text{bandwidth}}$

higher $\frac{\omega_0}{\Delta\omega}$, higher the selectivity

Recall $\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$

$\omega = \omega_0$

$j\omega_0 L + \frac{1}{j\omega_0 C} = 0$

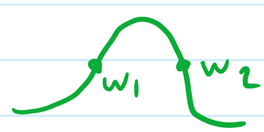


Selectivity $= \omega_0/\Delta\omega \qquad \omega_0 = 1/\sqrt{LC}$

$\Delta\omega = ?$

where transfer function $\frac{V_i}{V_c}$ reaches $1/\sqrt{2}$

$$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + 1/j\omega C}$$



$$= \frac{1}{1 + j\omega\frac{L}{R} - j/\omega CR} = 1/1 + j\left(\omega\frac{L}{R} - \frac{1}{\omega CR}\right)$$

Note that magnitude $= 1/\sqrt{2}$ when $\frac{V_r}{V_i} = \frac{1}{1 + j\underbrace{\left(\omega\frac{L}{R} - \frac{1}{\omega CR}\right)}_{\pm 1}}$

$$= \frac{1}{1 \pm j1}$$

$$\Rightarrow \frac{\omega L}{R} - \frac{1}{\omega CR} = \pm 1$$

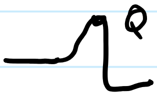
$$\Rightarrow \omega_1 = R/2L - \tfrac{1}{2}\sqrt{\tfrac{R^2}{2L} + \tfrac{4}{LC}}$$

$$\omega_2 = R/2L + \text{"} \quad \text{"}$$

$$\Delta\omega = \omega_2 - \omega_1 = R/L$$

$$\boxed{\text{selectivity} = \omega_0/RC = \omega_0/R_L = \omega_0/2\alpha = Q}$$

High Q → high selectivity
→ more ringing
→ "peakyness" of voltage



We have now seen three definitions for the quality factor Q of a resonant circuit. The first, encountered in Chapter 12, was based on the ratio of the undamped resonant frequency to the damping factor for transient excitation (Equation 12.65):

$$Q = \frac{\omega_o}{2\alpha}. \tag{14.95}$$

The second, derived in this chapter (Equation 14.47), was based on the width of the resonant peak in the frequency response for sinusoidal excitation:

$$Q = \frac{\omega_o}{\omega_2 - \omega_1} \tag{14.96}$$

where $\omega_2 - \omega_1$ is the bandwidth, and ω_1 and ω_2 are the frequencies where the response magnitude is down to 0.707 of its peak value.

The third is the relation in terms of stored and dissipated energy at resonance, Equation 14.91:

$$Q = \frac{\text{Stored energy}}{\text{Average energy dissipated per radian}}. \tag{14.97}$$

These definitions all reduce to the same value for second-order circuits, but they yield slightly different values in higher order circuits.

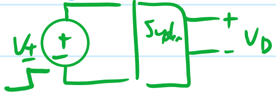
W3: Time Domain vs Frequency Domain Analysis

Thursday, December 1, 2016 12:43 AM

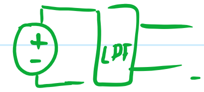
13.63

Review

Time domain

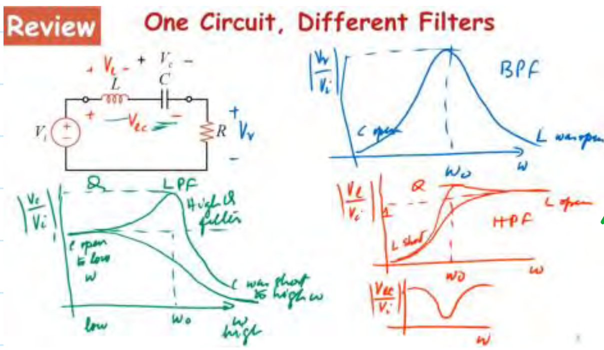


Filter view

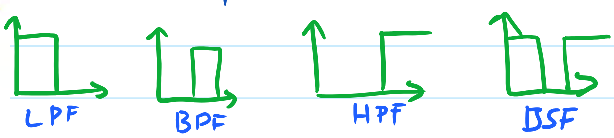


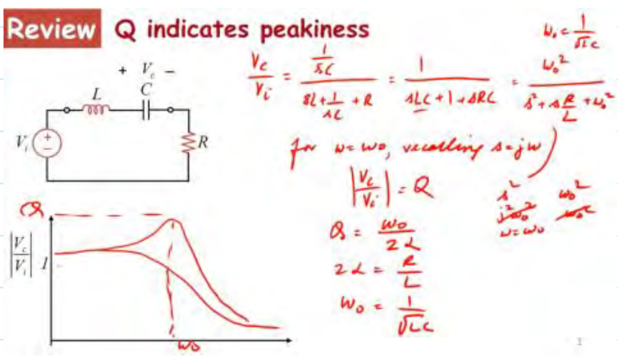
V_{in} $V_{out} = |V_{OUT}| \cos(\omega t + \angle V_{out})$

complex amplitude

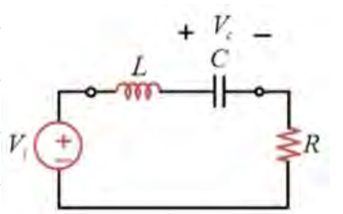


Process input according to frequencies





Impedance $\Longrightarrow$ Time domain



$$\frac{V_c}{V_i} = \frac{\omega_o^2}{s^2 + 2\alpha s + \omega_o^2}$$

① Replace s's with $\frac{d}{dt}$

$$V_c(s^2 + 2\alpha s + \omega_0^2) = V_i \omega_0^2$$

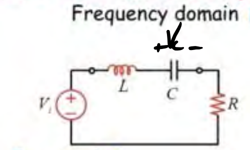
for example step input

$$\frac{d^2 V_c}{dt^2} + 2\alpha \frac{dV_c}{dt} + \omega_0^2 V_c = \omega_0^2 u(t)$$

$$Q = \frac{\omega_0}{2\alpha} = \frac{\omega_0}{R/L}$$

Let's use Q to compare time domain and frequency domain

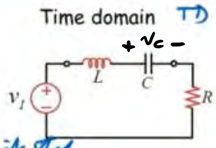
Frequency domain FD



Sinusoid

$v_I = V_i \cos \omega t$

Time domain TD



Unit step

$v_I = u(t)$

$Q = \frac{\omega_0}{2\alpha}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$2\alpha = \frac{R}{L}$

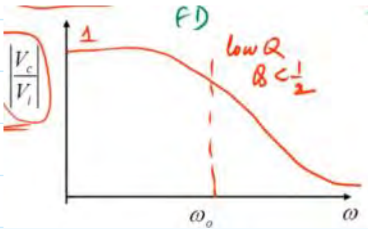
3 cases:

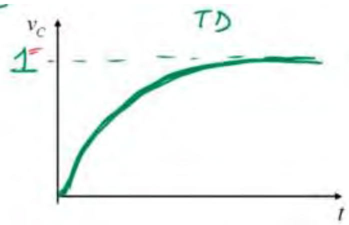
$\alpha > \omega_0$, or $Q = \frac{\omega_0}{2\alpha} < \frac{1}{2}$ → overdamped (low Q)

$\alpha < \omega_0$, or $Q = \frac{\omega_0}{2\alpha} > \frac{1}{2}$ → underdamped (high Q)

$\alpha = \omega_0$, $Q = \frac{1}{2}$ → critically damped

Over damped case

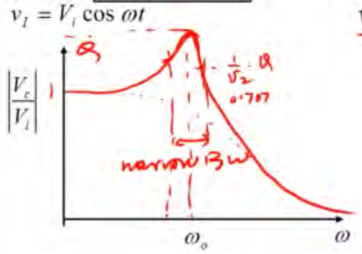




Underdamped Case

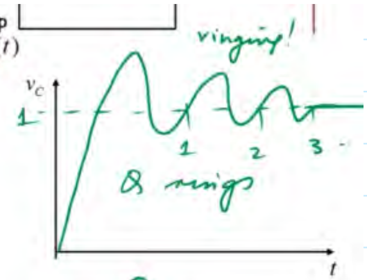
Sinusoid

$v_I = V_i \cos \omega t$



Unit step

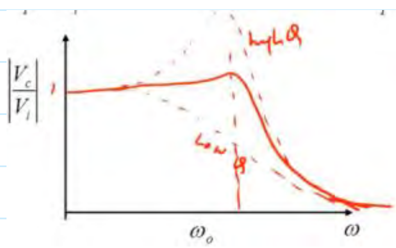
$v_I = u(t)$

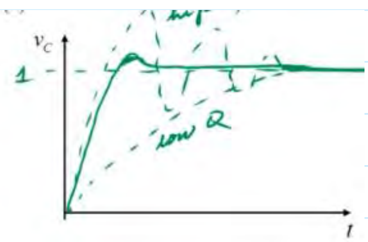


Q: Peakiness

Q: ringiness

Critically Damped Case





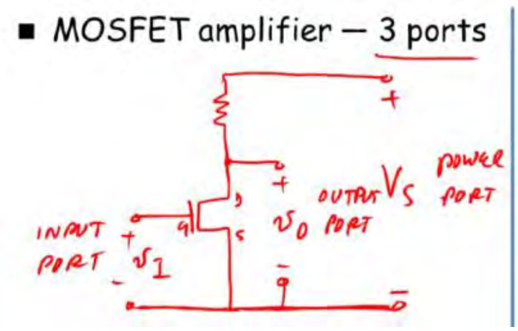
W4: Operational Amplifier

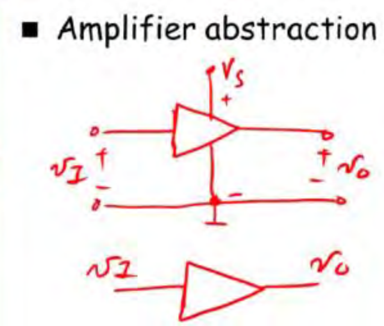
Monday, December 12, 2016 4:31 PM

"Op-amp"

Chapter 15

Review

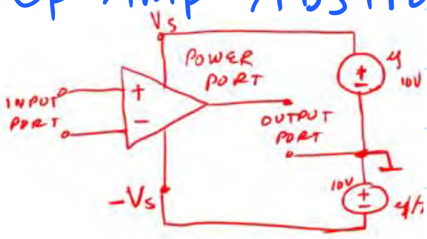




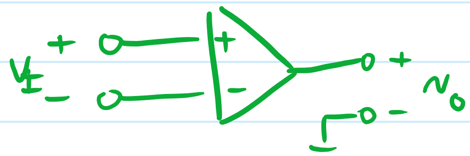
Basic Building block for analog circuits

Op Amp Abstraction

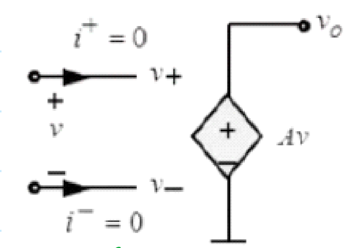
- ∞ input resistance
- 0 output resistance
- Gain → ∞



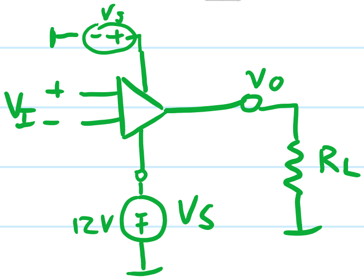
$\Rightarrow$

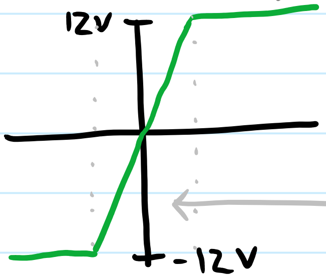


Equivalent circuit



$A \sim \infty$





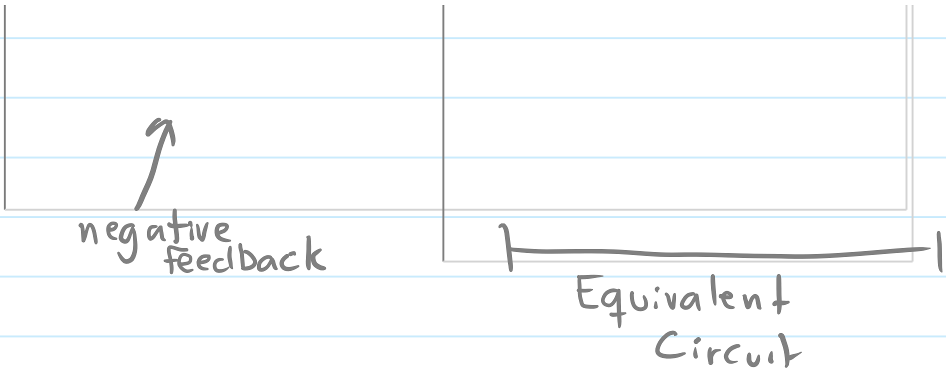
saturation

maxes out since power supply is 12 V

Active Region

in reality, A is unstable aka not reliable

Non-inverting Amplifier



Analysis

1. Find v_o in terms of v_I

$$v_o = A(\underbrace{v^+}_{v_I} - \underbrace{v^-}_{\frac{V_o R_2}{R_1+R_2}})$$

$\frac{V_o R_2}{R_1+R_2}$ — Voltage divider

$$= A\left(v_I - \frac{V_o R_2}{R_1+R_2}\right)$$

$$v_o\left(1 - \frac{A R_2}{R_1+R_2}\right) = A v_I$$

$$v_o = \frac{A V_I}{1 - \frac{A R_2}{R_1+R_2}}$$

when A is large

$$v_o \approx V_I \cdot \underbrace{\frac{R_1+R_2}{R_2}}_{\text{Gain}}$$

$\frac{R_1+R_2}{R_2}$ — external components

- not dependent on A

Negative Feedback

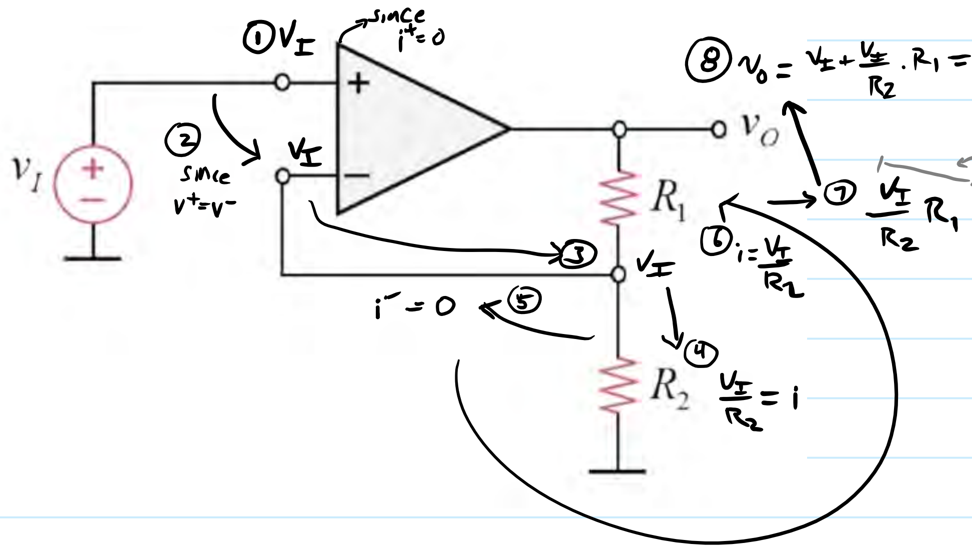
- If the output is perturbed the circuit will quickly pull up/down to "Fix"

$\Rightarrow$ Output does not change

$\Rightarrow \quad v^+ - v^- = 0 \Rightarrow v^+ = v^-$ — only under negative feedback

Virtual short method

Example (8 steps)



⑧ $v_o = V_I + \frac{V_I}{R_2}\cdot R_1 = V_1\frac{R_1+R_2}{R_2}$

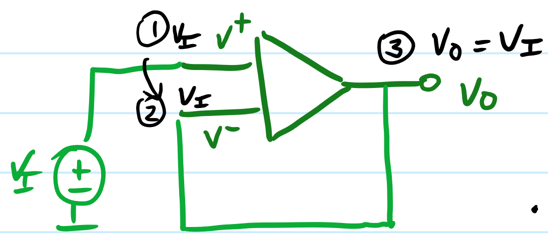
"Givens"

$i^+=0$

$i^-=0$

$v^+=v^-$

Buffer Circuit

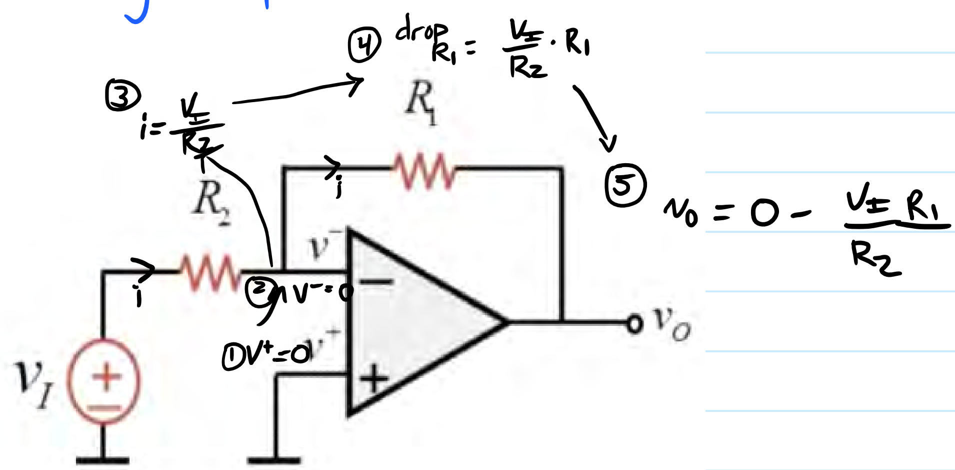


③ $V_O = V_I$

"source follower"

- used to isolate sources from the rest of the system

Inverting Amplifier



⑤ $v_o = 0 - \frac{V_I R_1}{R_2}$

Input Resistance

① Apply a V_I at input → ② measure current i_I

⇒ Impedance $= V_I / i_I = R_{IN}$

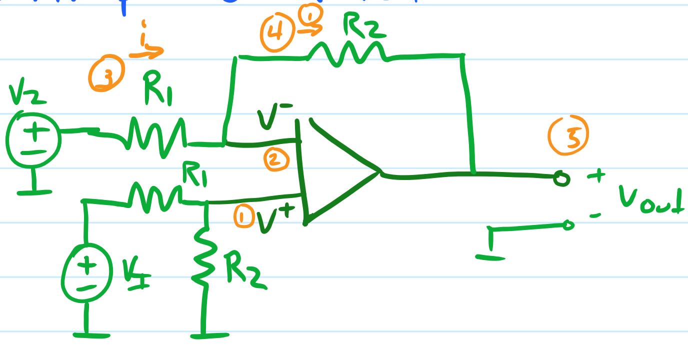
$$R_{\pm N} = \frac{V_{\pm}}{I} = \frac{V_{\pm}}{V_{\pm}/R_2} = R_2$$

* if need as a function of A need to use equivalent circuit method

W4: Operational Amplifier Circuits   15.5, 15.6

Tuesday, December 13, 2016   12:08 AM

Op Amp Subtractor



① $v^+ = V_1 \dfrac{R_2}{R_1 + R_2}$

② $v^+ = v^-$

③ $i = \dfrac{V_1 - V^-}{R_1}$

④ $i_2 = i$

⑤ $v_{OUT} = v^- - iR_2$

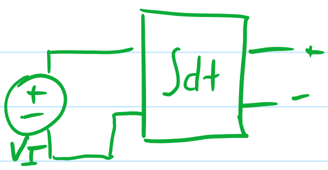
$= v^- - \dfrac{V_2 - V^-}{R_1} \cdot R_2$

$= V_1\left(\dfrac{R_2}{\cancel{R_1+R_2}}\right)\left(\dfrac{\cancel{R_1+R_2}}{R_1}\right) - \dfrac{V_2 R_2}{R_1}$

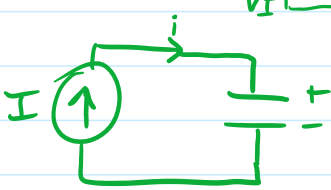
subtracts two voltages!

$= \dfrac{R_2}{R_1}(V_1 - V_2)$

Integrator



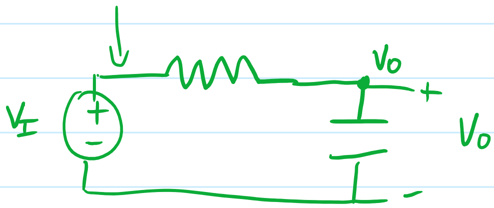
$+ \; v_o \; - = A \int v_I \, dt$



$+ \; V_o \; - = \frac{1}{C} \int i \, dt$

i not v_i.

would need to convert voltage v_i to current i



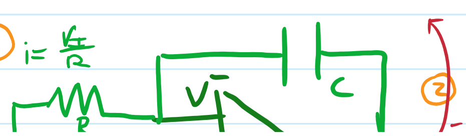
- V_o has to be very small!
  or $i \neq \frac{V_I}{R}$

OpAmp Integrator

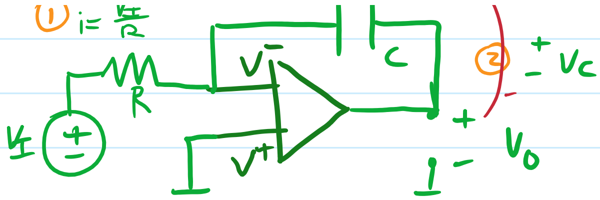
① $V_I \to i$

② i through capacitor

① $i = \frac{V_I}{R}$



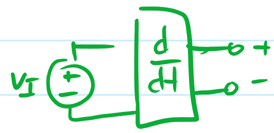
$V_C = \frac{1}{C} \int^t i \, dt$



$$V_c = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$
$$= \frac{1}{C}\int_{-\infty}^{t} \frac{V_I}{R}\,dt$$

$$v_o = 0 - V_c$$
$$= -V_c$$

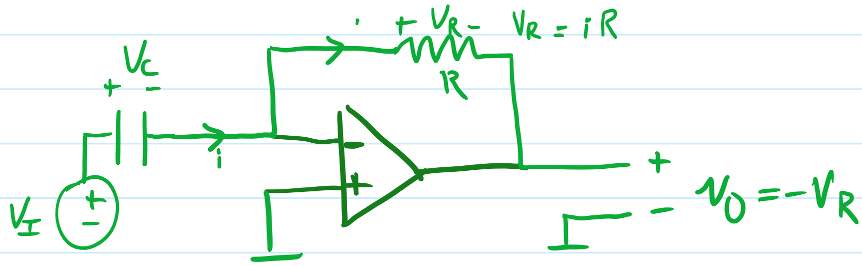
Differentiator



$$v_o = A\frac{d}{dt}V_I$$

① V_I across capacitor $\quad i \sim \frac{dV_I}{dt}$

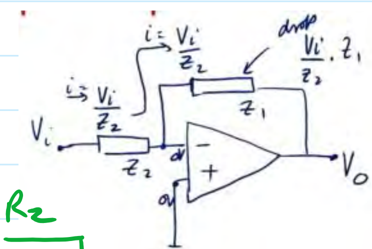
② convert $\quad i \to v_o$

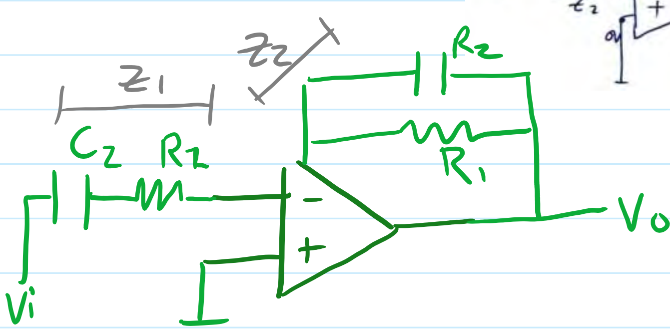


$$i = C\frac{dV_I}{dt}$$

$$V_O = iR = -C\frac{dV_I}{dt}\cdot R$$

Op Amp Filters





$$V_o = \frac{-Z_1}{Z_2}V_i$$

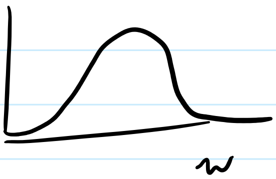
$$Z_1 = \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + 1/c_1} \qquad Z_2 = R_2 + \frac{1}{sC_2}$$

$$Z_1 = \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + 1/sC_1} \qquad Z_2 = R_2 + \frac{1}{sC_2}$$

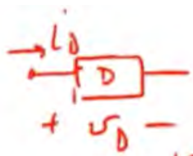
$$V_0 = -\frac{\frac{R \cdot \frac{1}{sC_1}}{R + 1/sC_1}}{R_2 + 1/sC_1} \cdot V_1 = \frac{sR_1C_2}{(1+sR_2C_2)(1+sR_1C_1)}$$

$|H(j\omega)|$

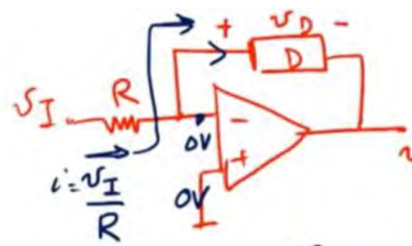
"Band Pass Filter"



Non-linear OpAmp Circuit



$i_D = a e^{b v_D}$



$v_o = -v_D$

$$i = \frac{v_I}{R} = a e^{b v_D}$$

$$\frac{v_I}{aR} = e^{b v_D}$$

$$\frac{1}{b} \ln \frac{v_I}{aR} = v_D$$

"log Amp"

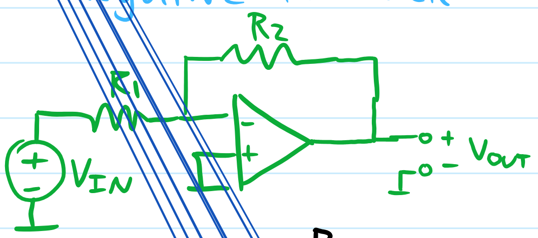
$$v_0 = -\frac{1}{b} \ln \frac{v_I}{aR}$$

W5: OpAmp Positive Feedback

Tuesday, December 13, 2016 2:07 AM

Negative vs Positive Feedback

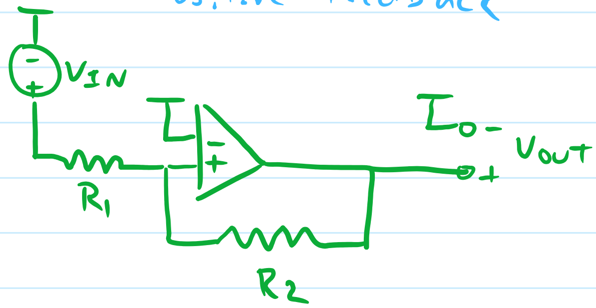
Negative Feedback



$$V_{OUT} = -\frac{R_2}{R_1} V_{IN}$$

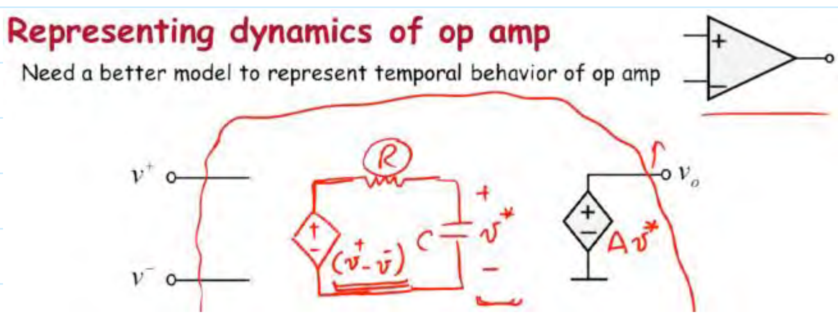
Change in input

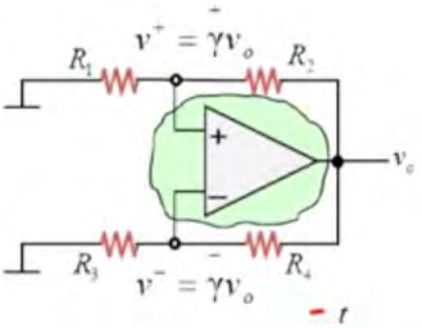
Positive Feedback



$$V_{OUT} = -\frac{R_2}{R_1} V_{IN}$$

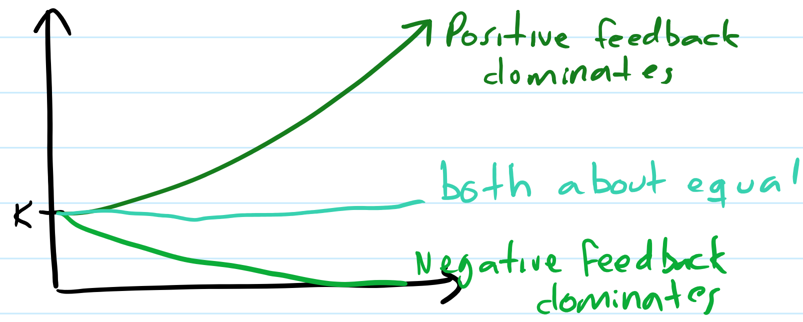
· drives opamp into saturation



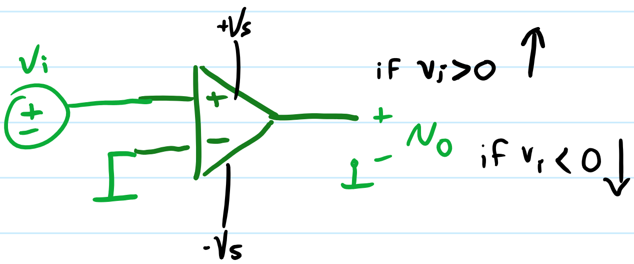


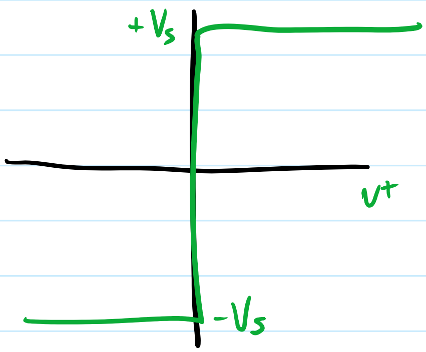
$$v_o = K e^{-\frac{t}{T}}$$

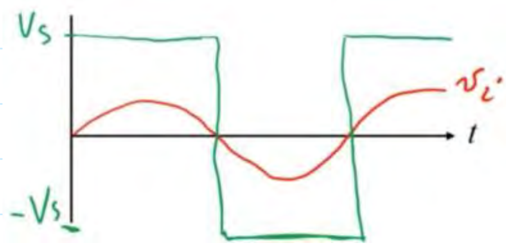
$$T = \frac{RC}{A(\gamma^- - \gamma^+)}$$

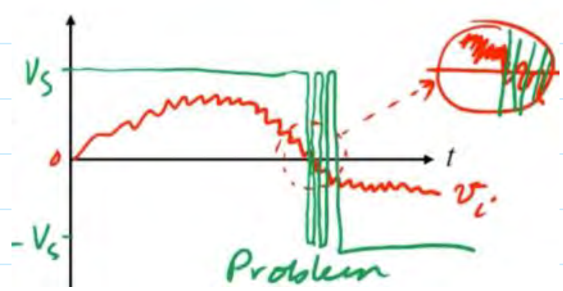


OpAmp Comparator



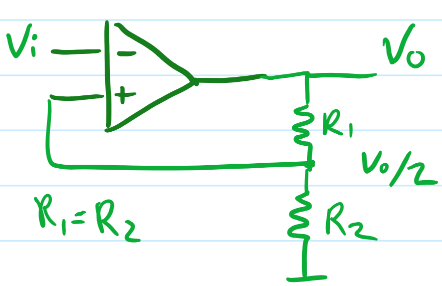


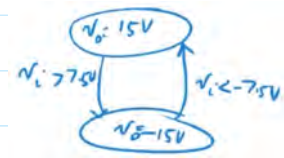




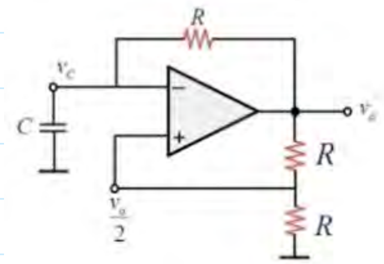
Better version - Hysterisis

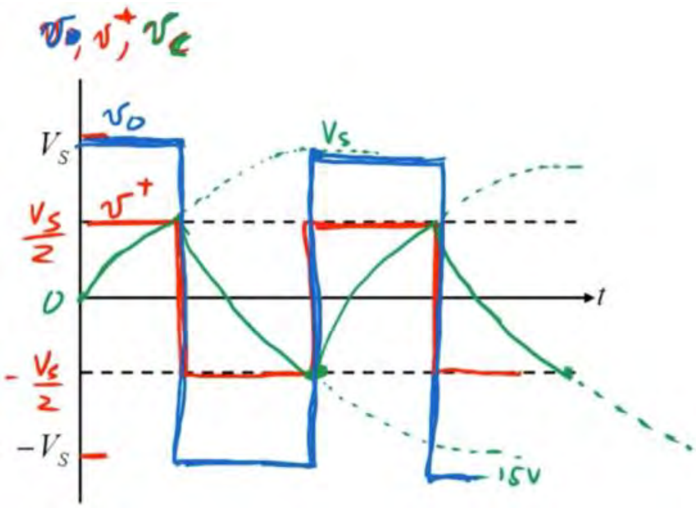
↳ circuit remembers the past





OpAmp Oscillator

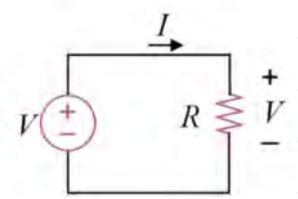




· Find rising and falling time
(see lecture)

W5: Energy and Power

Tuesday, December 13, 2016 1:25 PM



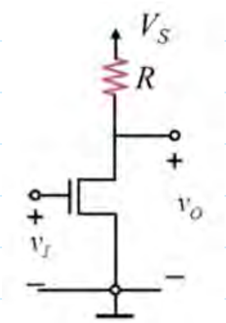
Power $P = IV = \frac{V^2}{R}$

Energy dissipated in time T

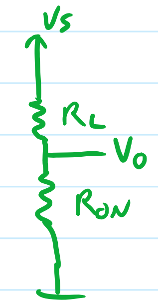
$E = PT = IVT$

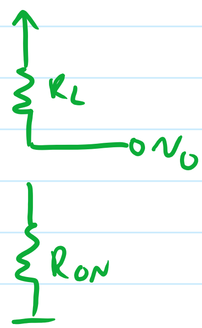
standby (not applying input)

Static Power for gate



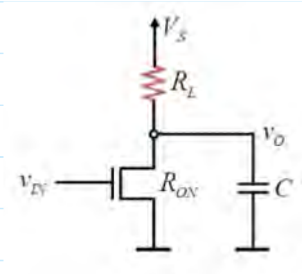
①

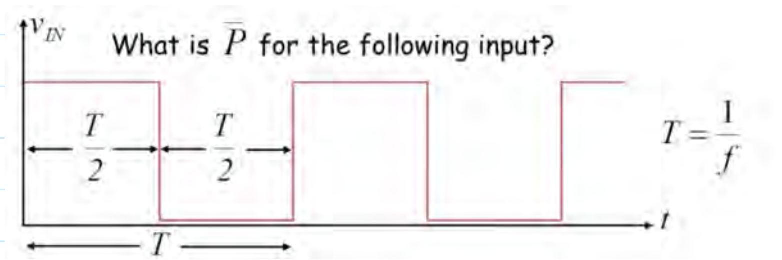




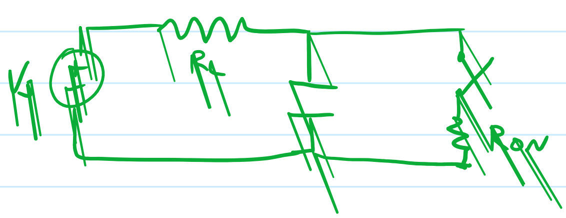
$$P = \frac{V_s^2}{R_L + R_{ON}}$$

$$P = 0$$





Equivalent Circuit



from section 11.3

$$\overline{P} = \frac{V_s^2}{2(R_L + R_{ON})} + CV_s^2 f \frac{R_L^2}{(R_L + R_{ON})^2}$$

if $R_L \gg R_{ON}$

$$\overline{P} = \underbrace{\frac{V_s^2}{2R_L}}_{\overline{P}_{static}} + \underbrace{CV_s^2 f}_{\overline{P}_{DYNAMIC}}$$

1/2 since MOSFET is ON for 1/2 of the time 'static 'DYNAMIC

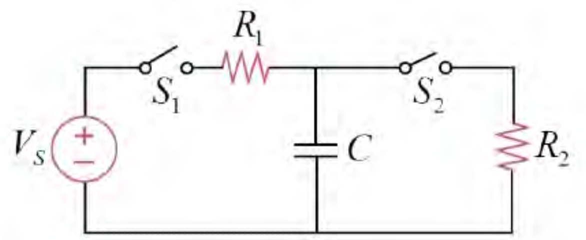
Vs matters a lot !

W6: Energy and CMOS Design

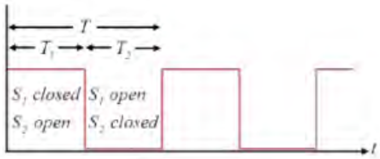
Tuesday, December 13, 2016 9:21 PM

11.4
11.5

NOT ON FINAL

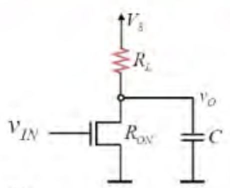


$$\overline{P} = C V_S^{\,2} f$$

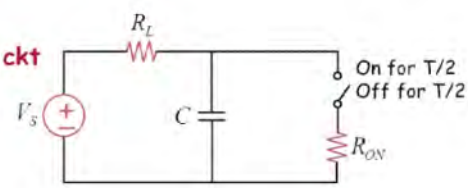


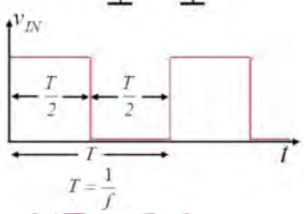
$$T = T_1 + T_2 = \frac{1}{f}$$

Review **Inverter**



Eqvt. ckt





$$\overline{P} = \frac{V_S^{\,2}}{2R_L} + C V_S^{\,2} f$$

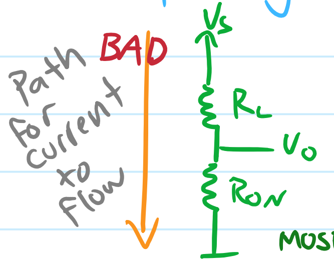
$\overline{P}_{STATIC}$ $\overline{P}_{DYNAMIC}$

↑

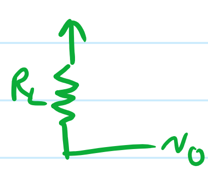
Because there is a direct path through $V_S \to R_S \to R_{ON}$

Get rid of static power

Input High



Input Low



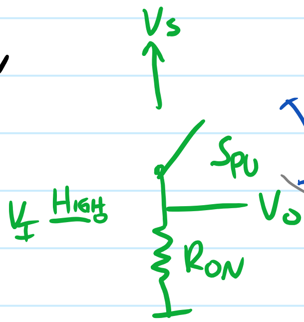
MOSFET OFF

$$\overline{P} = \frac{V_S^{\,2}}{R_L + R_{ON}}$$

$\overline{P} = 0$

$$P = \frac{V_S^2}{R_L + R_{ON}}$$

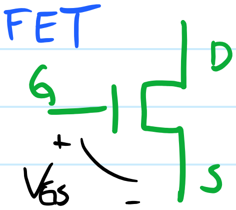
↳ Solution



How?

switch off when V_I is high

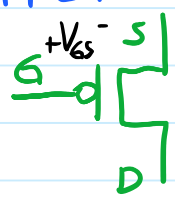
NFET



MOSFET we've seen before

$V_{GS} \geq V_{TN} \Rightarrow$ ON
$V_{GS} < V_{TN} \Rightarrow$ OFF

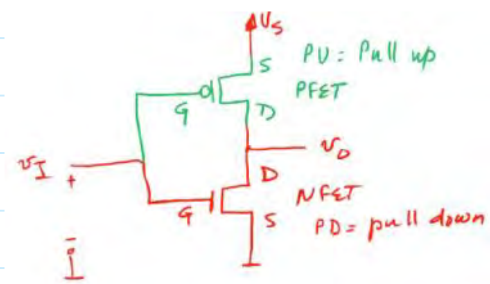
PFET *New MOSFET!!*



Complementary to NFET

$V_{GS} \leq V_{TP} \Rightarrow$ ON
$V_{GS} > V_{TP} \Rightarrow$ OFF

New Inverter



consumes much less power since never a path from $V_S \rightarrow$ GND

"Complementary MOS" — CMOS logic

$$\bar{P}_{DYNAMIC} = C V_S^2 F$$

comes from switching capacitors

can still draw alot of power for faster speeds

Reduce dynamic Power

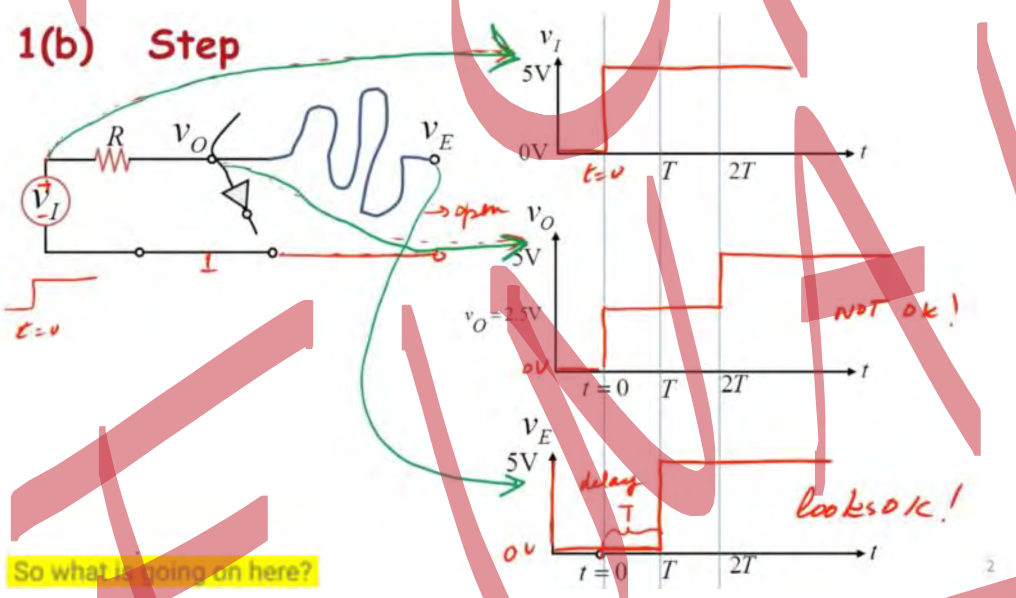
- Reduce V_S
- Shrink capacitors
- Clock Gating
  - Turn off clock for systems

- Clock Gating
  - Turn off clock for systems not in use

W6: Breaking The Abstraction Barrier

Tuesday, December 13, 2016 10:45 PM

NOT ON FINAL!



Final Review

Thursday, December 15, 2016 4:35 PM

Impedance:

ZR, ZC, ZL

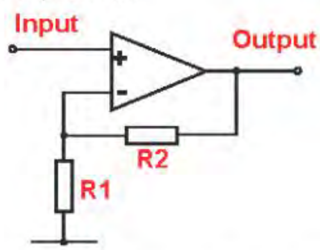
$Z_R = R \qquad Z_C = \frac{1}{sC} \qquad Z_L = Ls$

Low frequency --> Capacitor open circuit

High Frequency ---> Capacitor short curcuit

Basic non-inverting amplifier circuit

The basic non-inverting amplifier circuit using an op-amp is shown below. In this circuit the signal is applied to the non-inverting input of the amplifier. However the feedback is taken from the output via a resistor to the inverting input of the operational amplifier where another resistor is taken to ground. It is the value of these two resistors that govern the gain of the operational amplifier circuit.



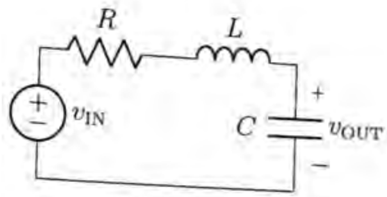
Basic non-inverting operational amplifier circuit

The gain of the non-inverting amplifier circuit for the operational amplifier is easy to determine. The calculation hinges around the fact that the voltage at both inputs is the same. This arises from the fact that the gain of the amplifier is exceedingly high. If the output of the circuit remains within the supply rails of the amplifier, then the output voltage divided by the gain means that there is virtually no difference between the two inputs.

As the input to the op-amp draws no current this means that the current flowing in the resistors R1 and R2 is the same. The voltage at the inverting input is formed from a potential divider consisting of R1 and R2, and as the voltage at both inputs is the same, the voltage at the inverting input must be the same as that at the non-inverting input. This means that Vin = Vout x R1 / (R1 + R2). Hence the voltage gain of the circuit Av can be taken as:

$$\boxed{\frac{V_{out}}{V_{in}} = A_v = 1 + \frac{R_2}{R_1}}$$

As an example, an amplifier requiring a gain of eleven could be built by making R2 47 k ohms and R1 4.7 k ohms.



(a) (20%) Find V_{out} and ϕ as functions of R, L, C, V_{in}, and ω.

$$V_{out} = V_{in}\left|\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L + R}\right| = \frac{V_{in}}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

In order to find ϕ, we focus on the ratio of imaginary to real in the transfer function and take the inverse tangent

$$\phi = \tan^{-1}\left(\frac{-\omega RC}{1-\omega^2 LC}\right)$$

FINAL

Friday, December 16, 2016 2:11 PM

$$i_D = \frac{K}{2} V_{GS}^2$$

$$V_T = 0$$

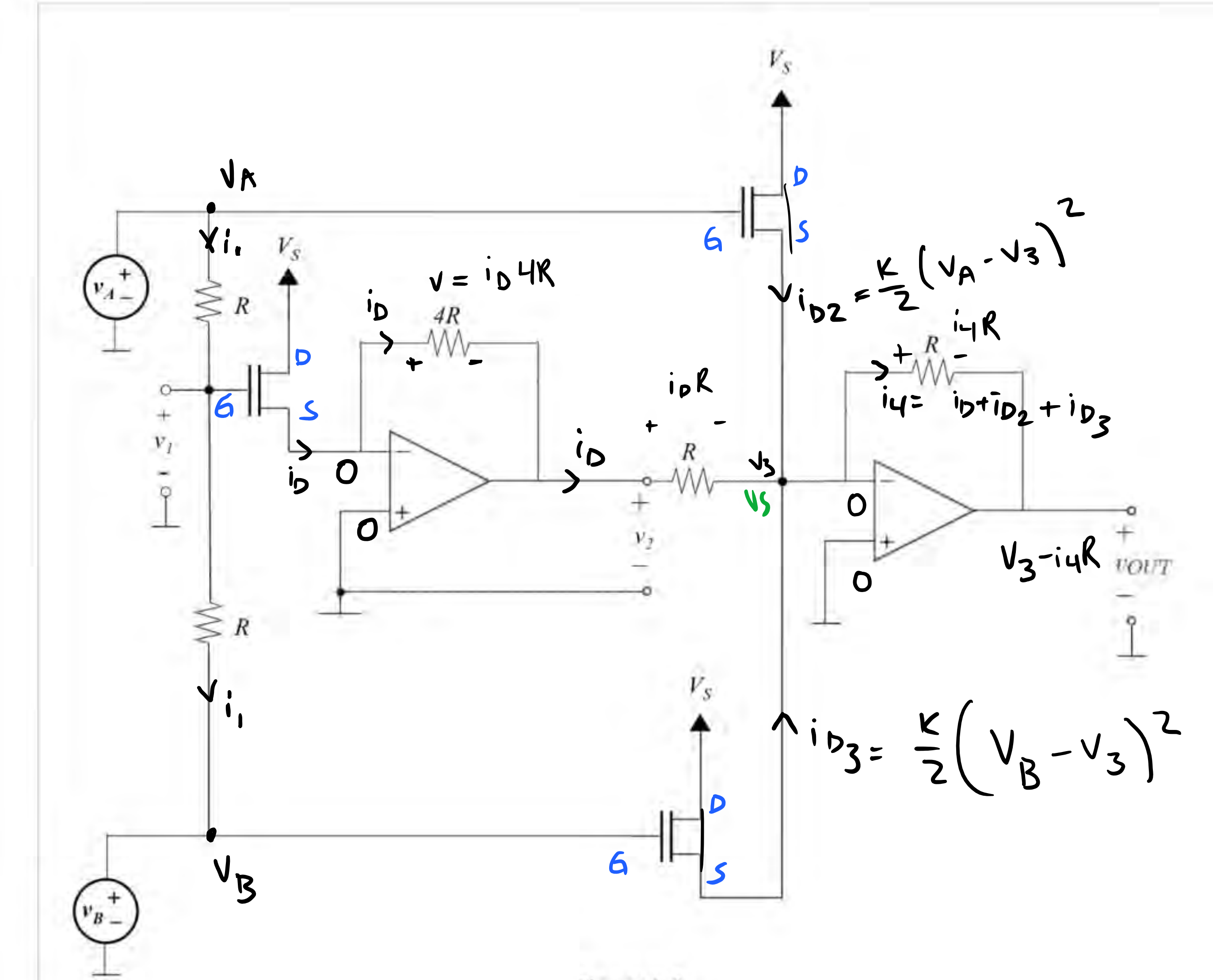


Figure 6-1

$$v_1 = v_A - i_1 R \Rightarrow i_1 R = v_A - v_1$$

$$v_1 = v_B + i_1 R$$

$$v_1 = v_B + v_A - v_1$$

$$V_B + V_A$$

$$\boxed{v_1 = \frac{V_B + V_A}{2}}$$

$$i_D = \frac{K}{2} v_{GS}^2$$

$$i_D = \frac{K}{2} v_1^2$$

$$v_2 = -i_D 4R$$

$$\boxed{v_2 = -\frac{K}{2} v_1^2 (4R)}$$