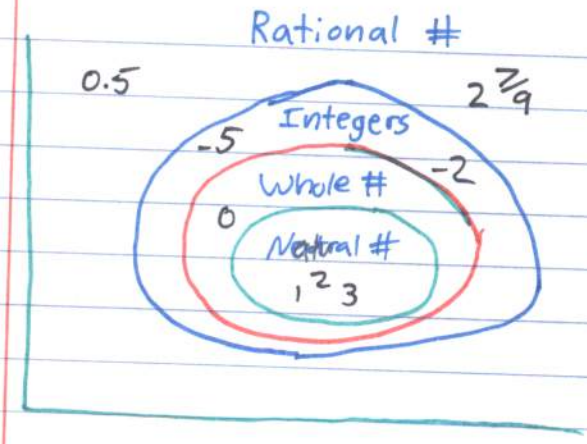
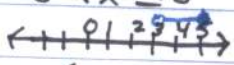


Rational numbers terminate or repeat

Real + Rational	Real + Irrational	Real + Rational
$-\frac{11}{2}$	$\sqrt{5}$	2.7652

Types of Notation:

In equality: $3 < x \leq 5$

Numberline: 

Roster Notation: {...3.....5}

Interval Notation: (open, #not included (3, 5]
[closed, # is included

Set Builder $\{x \mid 3 < x \leq 5\}$

Real
- Rat → Integer → Whole 0, 1, 2, 3, 4 ... → Natural 1, 2, 3, 4...
- IRR

Square Roots

Radical symbol

$\sqrt{36}$

Radicand

$\sqrt{25} = 5$ $\quad -\sqrt{25} = -5$ $\quad \pm\sqrt{25} = \pm 5$

Simplifying Square Roots

$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$ | $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

*Calculator $y = 48/x$

2nd + TABLE

If its not a perfect square, its an irrational number

$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$ $\quad \sqrt{\frac{500}{5}}$ $\quad 500/5 = 100$ $\quad \sqrt{100} = 10$

$-\sqrt{32} = -\sqrt{16 \cdot 2} = -4\sqrt{2}$

$\sqrt{3} \cdot \sqrt{12} = \sqrt{36} = 6$

$\frac{3\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{10}}{2}$

$6\sqrt{5} - \sqrt{20} = 6\sqrt{5} - \sqrt{4 \cdot 5}$

$6\sqrt{5} - 2\sqrt{5} = 4\sqrt{5}$

$9\sqrt{3} + 7\sqrt{3} = 16\sqrt{3}$

Simplifying Algebreic Expressions

ORDER OF OPERATIONS = PEMDAS

$7^2 + 4(7)(3) - 3^2$

$49 + 84 \quad -9$

124

$k^2 J = Jk^2$

$J(6k^2 + 7k) + 9Jk^2 - 7Jk$

$6Jk^2 + 7Jk + 9Jk^2 - 7Jk$

$15Jk^2$

Properties of Exponents

a^n ← Exponent

Base

Exponential = $-2x^3$ $\quad -(2x)^3$

Expanded = $-2(x \cdot x \cdot x)$ $\quad -(2x)(2x)(2x)$

* Anything raised to the zero power is 1 * $1^0 = 1$ $257^0 = 1$ $a^0 = 1$

* Never leave a negative exponent *

Negative Exponent Property

$7^{-2} = \left(\frac{1}{7}\right)^2 = 1/7^2$

$\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4$

To MULTIPLY powers with the same base, add the exponents

$4^3 \cdot 4^2 = 4^5$

$3+2=5$

To DIVIDE powers with the same base subtract the exponent.

$\frac{3^7}{2^2} = 3^5$

$7-2=5$

To raise one power to another, multiply the exponents

$(4^3)^2 = 4^6$ $\quad 3 \cdot 2 = 6$

SCIENTIFIC NOTATION

To move decimal left add one to the power.

To move decimal right subtract one to the power

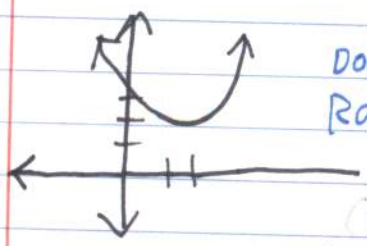
Function Notation

Output Value

$f(x) = 5x + 3$ When the set of ordered pairs

(y) Input Value

Range Domain



Domain = All real #

Range = $y \geq 2$

$f(x) = x^2 - 4x$

(0,0) $f(0) = 0$

$f(\frac{1}{2}) = (\frac{1}{2})^2 - 4(\frac{1}{2}) = -7/4$

$f(-2) = 12$

x = independant y = dependant

discrete function

NOT CONNECTED!!!!

Exploring Transformations

Transformation - change in position size or shape.
Translation - slide

HORIZONTAL STRECH OR COMPRESSION

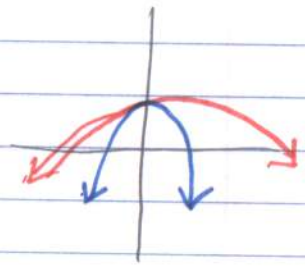
Each point is pulled away from y axis

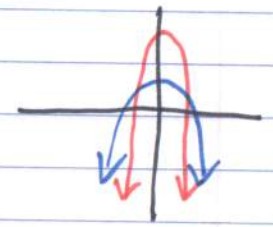
The x intercept changes Y DOES NOT !.

$(4,0) \rightarrow (2(4),0)$

$b=2 \quad (x,y) \rightarrow (bx, y)$

$|b| > |$





VERTICAL STRETCH OR COMPRESSION

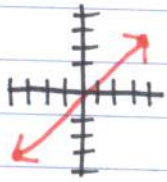
Each point is pulled away from y axis

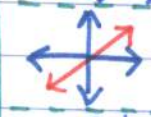
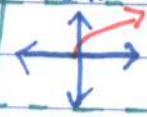
The Y intercept changes X DOES NOT!!

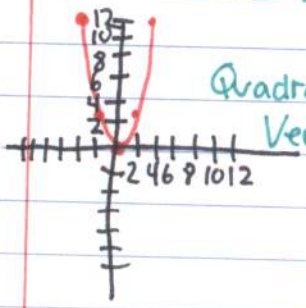
PIECE WISE FUNCTION =

Parent Functions

$y=x$



Family	Constant	Linear	Quadratic	Cubic	Square Root
Rule	$f(x)=c$	$f(x)=x$	$f(x)=x^2$	$f(x)=x^3$	$f(x)=\sqrt{x}$
Graph					
Domain	All real #	All real #	All real #	All real #	$x \geq 0$
Range	$y=c$	All real #	$y \geq 0$	All real #	$y \geq 0$
Intercepts y axis	$(0,c)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$



Quadratic

Vertical Stretch by 3

Solving Linear Equations and Enequalities

Equation - Mathematical statement that 2 expressions are equal.
Solution set of an equation - Values that make Equation true
Linear Equation in one variable - $ax=b$

TO BE A LINEAR EQUATION...
- Variable not under radical sign
- Variable not raised to a power
- Variable is not an exponent.
- Variable is not in a denominator

Properties of Equality

You can add, subtract, multiply, or divide both sides of the equation by the same quantity. Equation will still be true.

$3+2=3+2$ $\quad 3-2=3-2$ $\quad 3(2)=3(2)$ $\quad 3/2=3/2$

$5y-35=25$
$+35 \quad +35$
$\frac{5y}{5}=\frac{60}{5}$
$y=12$

$6y+28=9y-20$
$-6y \quad -6y$
$28=3y-20$
$48=3y$
$y=16$

Identity - An equation that is true for all vallues.

$3x+4x+5=7x+5$
$7x+5=7x+5$
$-7x \quad -7x$
$5=5$

ALL REAL NUMBERS

Contradiction - Equation that has no solution.

$8(y+7)=6y-8+2y$
$8y+56=8y-8$
$-8y \quad -8y$
$56=-8$

NO SOLUTION

Innequality
↳ Statement that compares 2 expressions.

If you multiply or divide both sides of an inequality you MUST reverse the sign in order to make the inequality true

Proportional Reasoning

Ratio - Comparison of 2 # by division.
Proportion - Equation stating 2 ratios are equal.

* The cross products of a proportion are equal *

$\frac{3}{5} = \frac{9}{15}$ $\quad 3(15) = 5(9)$

$45 = 45$

$\frac{22}{9} = \frac{x}{13.5}$

$\frac{297}{9} = \frac{9x}{9}$

$x = 33$

$\frac{512}{16} = \frac{64}{w}$

$512w = 1024$

$w = 2$

Rate - Ratio that involves two different units.

$\frac{400\text{ m}}{328\text{ strides}}$ $\quad \frac{x\text{ m}}{1\text{ stride}}$

$400 = 328x$

$x \approx 1.22$

Graphing Linear Functions

Linear Functions - Functions with a constant rate of change

Constant rate of change of a Linear function = it's slope

$\frac{\text{Rise}}{\text{Run}}$ $\frac{\text{change in } f(x)}{\text{change in } x}$

To find x intercept let $y = 0$

To find y intercept let $x = 0$

SLOPE INTERCEPT FORM = $y = mx + b$

slope Y intercept

$-4x + y = -1$ $y = 4x + 1$

slope y-intercept

$x + \frac{3}{4}y = 6$

$\frac{3}{4}y = -x + 6$

$y = -\frac{4}{3}x + 8$

$\frac{4}{3} \cdot \frac{6}{1}$ $\frac{24}{3} = \frac{8}{1} = 8$

$x = a$ = a vertical line

$y = b$ = a horizontal line

Writing Linear Functions

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

SLOPE FORMULA

Can only find slope if its a linear equation (constant change in y,x)

POINT SLOPE FORM

Slope

$$y - y_1 = m(x - x_1)$$

Parallel Lines have same slopes

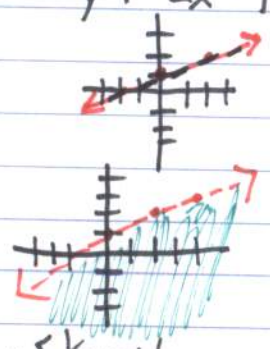
Perpendicular " have opposite recipricals

Linear Inequalities in 2 variables

Linear Inequality - Relates 2 values using innequality symbol.
Boundry Line - Divides the coordinate plane into 2 regions.

Example

$y < \frac{1}{2}x + 1$



$y < \frac{1}{2}x + 1$
$0 < \frac{1}{2}(0) + 1$
$0 < 1$ ✓

GRAPHING LINEAR INNEQUALITIES

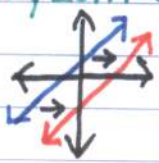
Step 1: Graph the boundry line
Dashed = not part of the solution ($<, >$)
Solid = Part of solution ($\leq, \geq$)
Step 2: Shade Accordingly
Above line = Points above line are included ($>, \geq$)
Below line = Points below ln are included ($<, \leq$)
Step 3: [CHECK]
Choose a point in solution region and test it on the innequality.

Linear Transformations

$y = f(x+2)$	Translate left 2
$y = f(x-2)$	Translate right 2
$y = f(x)+2$	Translate 2 up
$y = f(x)-2$	Translate 2 down
$y = f(2x)$	Horizontal Compression of $1/2$
$y = f(1/2x)$	Horizontal Strech of 2
$y = 2f(x)$	Vertical Strech by 2
$y = 1/2 f(x)$	Vertical Comp by $1/2$
$y = f(-x)$	Reflection over y axis
$y = -f(x)$	Reflection over x axis

Translations

Horizontal Shift of $|h|$ Units



Input value changes $f(x) \rightarrow f(x-h)$

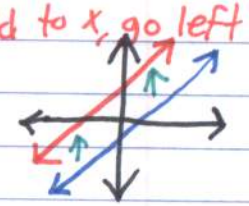
$h > 0$ moves right $h < 0$ moves left

* Add to y go high, add to x, go left *

Vertical Shift of $|k|$ units

output value changes $f(x) \rightarrow f(x) + k$

$k > 0$ moves up $k < 0$ moves down



Reflections

Reflection across y-axis

$f(x) \rightarrow f(-x)$

Lines symetric across y-axis

Reflection across x-axis

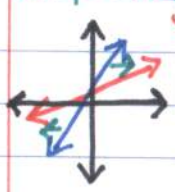
$f(x) \rightarrow -f(x)$

Lines symetric about y axis

Streches and Compressions

Horizontal streach/Compression

↳ only affects x

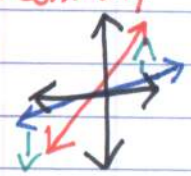


$f(x) \rightarrow f(\frac{1}{b}x)$

If your moving it horizontally

Vertical compression

$f(x) \rightarrow a \cdot f(x)$



IF THEY TELL IT TO YOU ON WORDS ITS OPPOSITE OF WHAT YOU SEE IN THE EQUATION

Let $g(x)$ be a horizontal shift of $f(x) = 3x$ left 6 units followed by a horizontal streach by a factor of 4. Write the rule for $g(x)$

$g(x) = f(x+6)$

$g(x) = 3(x+6)$ Translate

$g(x) = 3x + 18$ Simplify

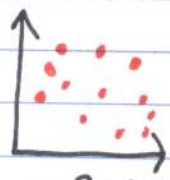
Opposite of what you think. $= 3(\frac{1}{4}x) + 18$

$g(x) = \frac{3}{4}x + 18$ Final

Curve Fitting with Linear Models

Correlation - streanth and direction on the linear relationship between 2 variables

$r \approx -0.95$



$r \approx .06$

1. Clear RAM 2nd + 7, 1, 2

   Turn diagnostics on 2nd + Catalog → Scroll down to diag on - Press enter twice

   Turn stat plot on 2nd + 1 → Enter → Highlight on → mark square

   Enter Values Stat → Edit → Enter x on L1 → Enter y on L2

   Stat → Calc → 4

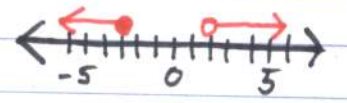
   Vars → Y vars → Enter, enter, enter

   Graph

Solving Absolute-Value Equations and Inequalities

Disjunction (OR) - Compound statement that uses word "or"

Compound statement - Made up of more than one equation or inequality



$x \leq -3$ OR $x > 2$

$\{x \mid x \leq -3 \cup x > 2\}$

Conjunction (AND) - Compound statement that uses the word "and"

$x \geq -3$ AND $x < 2$

$\{x \mid x \geq -3 \cap x < 2\}$

$\cup$ = OR $\qquad$ $\cap$ = AND

SOLVING COMPOUND INNEQUALITIES

$x+3 \geq 2$ or $3x < 9$ $\qquad$ Step 1. Solve both inequalities for x

$x \geq -1$ or $x < 3$

* Same process for conjunctions and disjunction *

$-2x < 8$ and $x - 3 \leq 2$

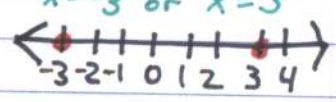
$x > -4$ $\qquad$ $x \leq 5$

Absolute value - $|x|$ - distance from x to 0 on number line

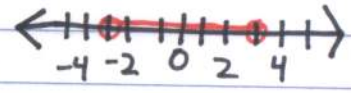
↳ Non negative

Solutions of $|x| = 3$ are the 2 points that are 3 units from zero

$x = -3$ or $x = 3$



Solutions of $|x| < 3$ are the points that are less than 3 units from zero.



Solutions of $|x| > 3$ are the points that are more than 3 units from 0.

Disjunction, Conjunction, Disjunction

ABSOLUTE-VALUE EQUATIONS AND INEQUALITIES

$\lvert x\rvert = a$	$\lvert x\rvert < a$	$\lvert x\rvert > a$
$x = -a$ OR $x = a$	$x > -a$ AND $x < a$	$x < -a$ OR $x > a$
	$-a < x < a$	

SOLVING ABSOLUTE VALUE EQUATIONS

$|-4q+2| \geq 10$ great OR

$-4q+2 \geq 10$ OR $-4q+2 \leq -10$

$-4q \geq 8$ $\quad$ $-4q \leq -12$

$q \leq -2$ OR $q \geq 3$

$|x-7|=5$

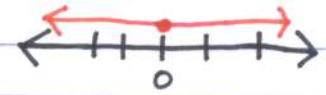
$x-7=5$ or $x-7=-5$

$x=12$ or $x=2$

$|0.5r|-3 \geq -3$

$|1.5r| \geq 0$ or $|1.5r| \leq 0$

$r \geq 0$ $\quad$ $r \leq 0$

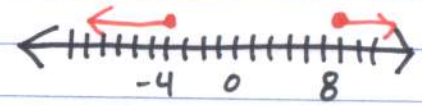


$-\frac{1}{2}|p-2| \geq 3$

$|p-2| \leq -6$

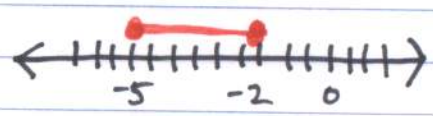
$p-2 \leq -6$ and $p-2 \geq 6$

$p \leq -4$ and $p \geq 8$



NO SOLUTION

$\frac{|2x+7|}{3} \leq 1$



$|2x+7| \leq 3$

$2x+7 \leq 3$ $\quad$ $2x+7 \geq -3$

$2x \leq -4$ $\quad$ $2x \geq -10$

$x \leq -2$ $\quad$ $x \geq -5$

STEP 1 - Isolate the absolute value expression

Step 2 - Rewrite the absolute value expression as a compound innequalities.

Step 3 - Solve each part of the compound inequality for x

2-7

Using Graphs and Tables to Solve Linear Systems

System of equations - Set of 2 or more equations containing 2 or more variables.
Linear System - System of equations containing only linear equations.

VERIFYING SOLUTIONS OF LINEAR SYSTEMS

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$(\overset{x}{2}, \overset{y}{4})$; $\begin{cases} x-2y=-6 \\ 2x+y=8 \end{cases}$

① Substitute 2 for x and 4 for y

$x-2y=-6$ $\quad 2-2(4)=-6$ $\quad -6=-6$ $\quad -6=-6$ ✓

$2x+y=8$ $\quad 2(2)+(4)=8$ $\quad 8=8$ ✓

SOLUTION OF SYSTEM

$(3,2)$; $\begin{cases} 2x+3y=12 \\ 8x-6y=24 \end{cases}$

$2x+3y=12$ $\quad 2(3)+3(2)=12$ $\quad 12=12$ ✓

$8x-6y=24$ $\quad 8(3)+6(2)=24$ $\quad 36 \neq 24$ X

NOT A SOLUTION OF THE SYSTEM because

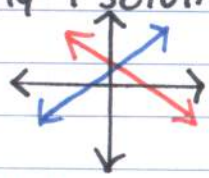
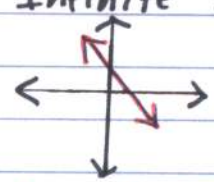
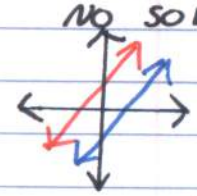
SOLVING LINEAR SYSTEMS WITH GRAPHS

$\begin{cases} x+y=4 \\ 2y+4=x \end{cases}$

$y=-x+4$
$y=\frac{1}{2}x-2$
$(4,0)$

① Solve each equation for y
② Find were lines intercept
SOLUTION TO SYSTEM $(4,0)$
③ CHECK

Consistent system - set of equations that has at least 1 solution
Inconsistent system - set of equations that has no solutions
Independent system - Has equations with different slopes
Dependent Systems - Equations with = slopes and = y-intercept

Exactly 1 solution	Infinite Solutions	NO SOLUTION
		
Consistent, independent	Consistent Dependent	Inconsistent

3.2

Solving Systems Using Algebra

$$\begin{cases} y = x-1 \\ x+y=7 \end{cases}$$

USING SUBSTITUTION

$x+x-1=7$

$2x=8 \quad x=4$

$y=4-1 \quad y=3$

1. Substitute y for x-1
2. Solve for x
3. Plug in x to equation and get y

USING ELIMINATION

$$\begin{cases} 3x+2y=4 \\ 4x-2y=-18 \end{cases}$$

$7x=-14$

$x=-2$

1. Eliminate y

$$\begin{cases} 5x-3=42 & \times 5 \\ 8x+5y=28 & \times 3 \end{cases}$$

$25x-\cancel{15y}=210$

$24x+\cancel{15y}=84$

$49x=294$

$x=6 \quad y=-4$

1. Eliminate y by making them cancel out by multiplying one equation by 5 and one by 3
2. Add the equations so y will cancel out
3. Plug in x into equation
4. Solve for y

Identity $0=0$

↳ = ALL REAL #

Contradiction $1=3$

↳ = no solution

Preview of ~~3.3~~ ↓

GHOST

$$\begin{cases} 5\left(\frac{1}{x+2}\right)+6\left(\frac{1}{y+1}\right)=-9 \\ 2\left(\frac{1}{x+2}\right)+\left(\frac{1}{y+1}\right)=2 \end{cases}$$

1. Let $u=\frac{1}{x+2}$ Let $v=\frac{1}{y+1}$
2. Solve for u and v

$5u+6v=-9$

$2u+v=2$

$v=-4 \qquad u=3$

3. Plug in u and v
4. Solve for x and y

$\frac{3}{1}=\frac{1}{x+2} \qquad \frac{-4}{1}=\frac{1}{y+1}$

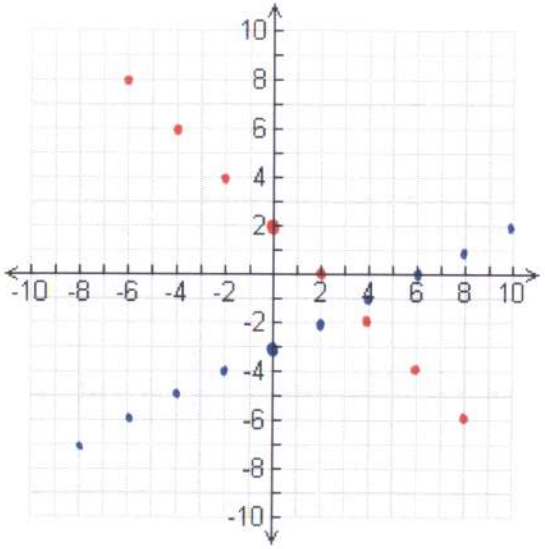
$x=-5/3 \qquad y=-5/4$

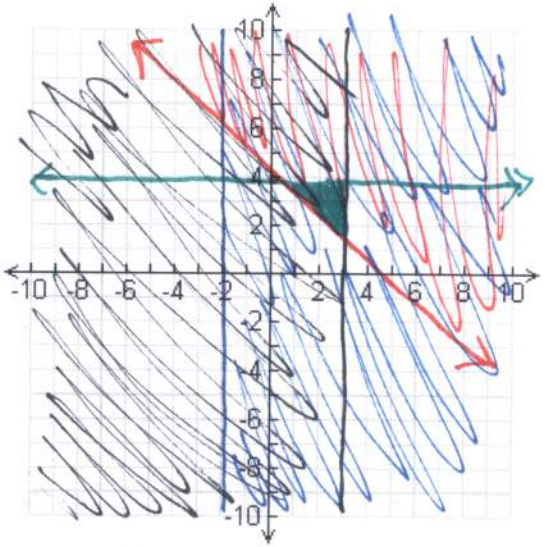
$(-5/3, -5/4)$

3-3

$$\begin{cases} y+3 < \frac{1}{2}x \\ y \geq -x+2 \end{cases}$$

$y < \frac{1}{2}x - 3$





dotted

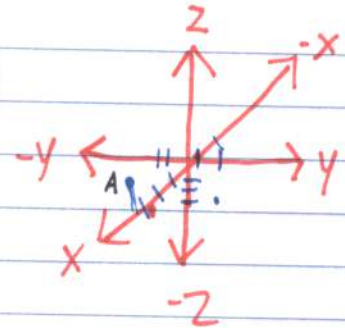
$x > -2$

$x \leq 3$

$y \geq -x + 4$

$y \leq 4$

A Graph $(\overset{x}{3}, \overset{y}{-2}, \overset{z}{1})$
B $(2, -1, -3)$

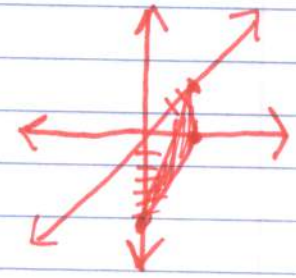


$2x - 3y + z = -6$

$2x = -6$ $\quad -3y = -6$

$x = -3$ $\quad y = 2$ $\quad z = -6$

$(-3, 0, 0)$ $\quad (0, 2, 0)$ $\quad (0, 0, -6)$



$$\begin{cases} ① \; 5x - 2y - 3z = -7 \\ 2\,③ \; 2x - 3y + z = -16 \\ ③ \; 3x + 4y - 2z = 7 \end{cases}$$

① $5x - 2y - 3z = -7$ $\quad$ ② $4x - 6y + 2z = -32$

② $6x - 9y + 3z = -48$ $\quad$ ③ $3x + 4y - 2z = 7$

Ⓐ $11x - 11y = -55$ $\quad$ Ⓑ $7x - 2y = -25$

2 Ⓐ $11x - 11y = -55$ $\quad$ Ⓐ $22x - 22y = -110$

-11 Ⓑ $7x - 2y = -25$ $\quad$ Ⓑ $-77x + 22y = 275$

$-55x = 165$

$x = -3$

Ⓑ $-21 - 2y = -25$

$y = 2$ $\quad$ ② $-6 - 6 + z = -16$

$z = -4$

$$\boxed{\overset{x \quad y \quad z}{(-3, 2, -4)}}$$

Element

(A) $\begin{bmatrix} 5 & -2 \\ 7 & 4 \end{bmatrix}$ 2x2

(B) $\begin{bmatrix} 4 & -3 \\ 2 & 6 \end{bmatrix}$ 2x2

(C) $\begin{bmatrix} 3 & -2 \\ 6 & 1 \\ 5 & 4 \end{bmatrix}$ 3x2

*Can add or subtract if dimensions are the same

① A+B

$\begin{bmatrix} 9 & -5 \\ 9 & 2 \end{bmatrix}$

2x2

(row) (clum)

② A-B

$\begin{bmatrix} 1 & 1 \\ 5 & -10 \end{bmatrix}$

2x2

③ A+C

NO SOLUTION

Scalar Mult

Row
Clum

$\begin{bmatrix} 4 & 6 & 9 \\ 7 & 8 & 1 \end{bmatrix}$ (column)

2·3

$\begin{bmatrix} 8 & 12 & 18 \\ 14 & 16 & 2 \end{bmatrix}$

Matrices can be mutiplied only if the # of column in A equals the # of rows in B

$A_{3\times4}\ B_{4\times2}$; AB

same

dimension of new matrix

3x2

$C_{1\times4}\ D_{3\times4}$; CD

undifined

$$W = \begin{array}{c} r1 \\ r2 \\ r3 \end{array} \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \qquad X = \begin{array}{c} \\ \end{array} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} \end{array}$$

3x2 same 2x3

3x3

$$\begin{array}{c} \\ \text{row1} \\ r^2 \\ r^3 \end{array} \begin{array}{c} \begin{array}{ccc} \text{column 1} & c2 & c3 \end{array} \\ \begin{bmatrix} 2 & 19 & -4 \\ 14 & 7 & -2 \\ 3 & 13 & -3 \end{bmatrix} \end{array}$$

12+7-4=15

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \qquad \begin{array}{c} r^1 \\ r^2 \end{array} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} \begin{array}{c} c_1 \quad c_2 \\ \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \end{array} \qquad \begin{bmatrix} 15 & -6 \\ 14 & -9 \end{bmatrix}$$

2x3 3x2

r1c1 r1c2 r2c1 r2c2

15+1-2=14

12+7=19-4=15

Square Matrix - matrix that ~~mall~~ has the same # of rows and columns

Main Diagonal of square matrix is the diagonal from upper left corner to lower right.

Multiplicative Identity matrix - Square matrix that hass all of the entries along main diagonals equal to one and all others =0

Matrices to Solve systems

$$\begin{cases} 6x + 2y = 5 \\ 8x + 4y = 4 \end{cases}$$

2nd Matrix → edit A → type → 2nd Quit → edit B → type → 2nd Quit → 2nd Matrix → A-enter^{-1} • B = $\begin{bmatrix} 1.5 \\ -2 \end{bmatrix}$

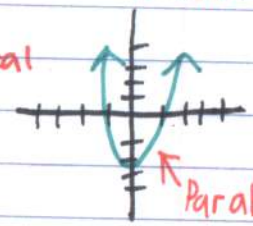
$$\overset{A}{\begin{bmatrix} 6 & 2 \\ 8 & 4 \end{bmatrix}}^{-1} \cdot \overset{B}{\begin{bmatrix} 5 \\ 4 \end{bmatrix}} = \begin{bmatrix} 1.5 \\ -2 \end{bmatrix}$$

Using Transformations to Graph Quadratic Functions

Quadratic function - $f(x) = a(x-h)^2 + k$ (horizontal comp; Horizontal; Vertical; vertical comp)

$a \neq 0$

Parabola - "U" shaped graph.



GRAPHING USING A TABLE

$f(x) = x^2 - 6x + 8$

x	$f(x) = x^2 - 6x + 8$	(x, y)
1	$f(1) = 1^2 - 6(1) + 8 = 3$	(1, 3)
2	$f(2) = 2^2 - 6(2) + 8 = 0$	(2, 0)
3	$f(3) = 3^2 - 6(3) + 8 = -1$	(3, -1)

H

HORIZONTAL SHIFT

$f(x) = x^2$

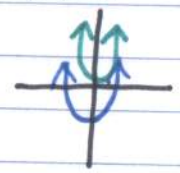
$f(x-h) = (x-h)^2$

$h < 0$ = left $h > 0$ = right

K = Vertical Shift

$f(x) = x^2$ $f(x) + k = x^2 + k$

$k < 0$ = down $k > 0$ = up



Examples: $g(x) = (x+3)^2 + 1$

$g(x) = (x - (-3))^2 + 1$

$h = -3$ H K

$k = 1$ Moves 3 units left and one up from parent $f(x) = x^2$

Identify H and K

REFLECTIONS

Y AXIS

Input value changes

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

Same thing.

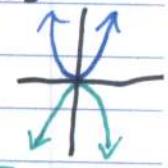
X AXIS

Output Value Changes

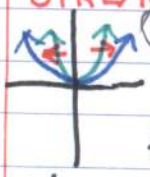
$f(x) = x^2$

$-f(x) = -(x^2) = -x^2$

Flipped across x axis



STREACHES + COMPRESSIONS



(x) Input Value changes

$f(x) = x^2$ HORIZONTAL

$f(\frac{1}{b}x) = (\frac{1}{b}x)^2$

$|b| > 1$ streaches away y axis

$0 < |b| < 1$ compresses toward y axis

VERTICAL

(y) Output value changes

$f(x) = x^2$ $a \cdot f(x) = ax^2$

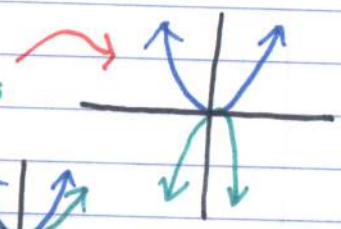
$|a| > 1$ streaches away x axis

$0 < |a| < 1$ compresses toward x axis

Examples: $g(x) = -4x^2$

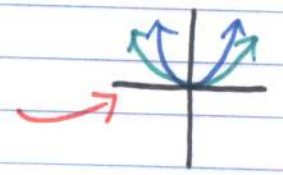
a is negative = Reflection across x axis

$|a| = 4$ = Vertical streach by 4



$g(x) = (\frac{1}{2}x)^2$ $\frac{1}{b}x$

$b = 2$ = Horizontal streach by 2



Vertex of a parabola = lowest or highest point (h, k)

Vertex form

$$f(x) = a(x-h)^2 + k$$

K = Vertical

H = Horizontal

indicates a reflection across x and/or Vertical Streach / Compression

indicates a horizontal translation

Indicates a vertical translation

VERTEX OF A PARABOLA IS AT (h, k)

WRITE TRANSFORMED QUADRATIC FUNCTIONS

$f(x) = x^2$ reflected across x-axis Vertical streach by 6 translated 3 units left

Reflection across x = a is negative

Vertical streach by 6 = $|a| = 6$

Translation 3 left = $h = -3$

$a = -6$

$h = -3$

$g(x) = a(x-h)^2 + k$ Plug in

$= -6(x-(-3))^2 + 0$

$= -6(x+3)^2$ Simplify

Properties of Quadratic Functions in Standard form

Vertexform = $f(x) = a(x-h)^2 + k$

STandard form = $f(x) = ax^2 + bx + c$

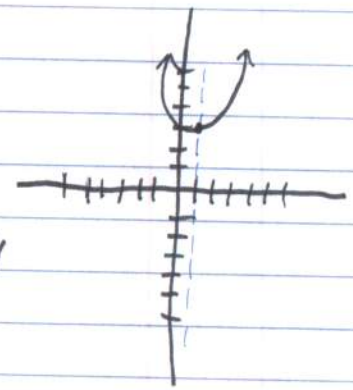
$a > 0$ = Opens upward

$a < 0$ = Opens downward

↖ y intercept

Axis of symmetry = $x = \frac{-b}{2a}$

Vertex = $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$



$2x^2 - 4x + 5$ → opens upward because $a > 0$

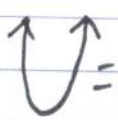
$x = \frac{-b}{2a}$ ~~with~~ $\frac{4}{4} = 1$ $x = 1$ → axis of symmetry

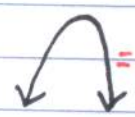
$(1, 3)$ Vertex

$f(x) = 3x^2 - 12x + 5$

A) UP B) $x = 2$ c) $(2, -7)$ d) $(0, 5)$

↳ y intercept

 = minimun value

 = maximun value

Doman = All real #

Range = ~~greater ≥ minimun~~

≥ minimun

Domain = All real #

range ≤ maximun

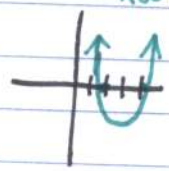
Solving Quadratic Equations

x^2-6x+8

$(x-4)(x-2)=0$

$x=4 \quad x=2$

Root = Solutions = Roots



$3x^2+18x$

$0=3x(x+6)$

$3x=0 \quad x+6=0$

$x=0 \quad x=-6$

$h(t)=-16t^2+V_0t+h_0$

always 16

Initial Vertical Velocity

Initial height in ft $t=0$

$h(t)=-16t^2+48t+0$

$0=-16t(t-3)$

$0=-16t \quad t-3=0$

$0=t \quad t=3$

DIFF OF 2 SQUARES

$a^2-b^2=(a+b)(a-b)$

$4x^2-25=0$

$4x^2-25=0$

$(2x+5)(2x-5)$

$x=-5/2 \quad x=5/2$

$18x^2=48x-32$

$\frac{18x^2-48x+32=0}{2}$

$9x^2-24x+16=0$

$(3x-4)^2=0$

$x=4/3$

Write the quadratic function with zeros 5 and -5

$x=5 \quad x=-5$

$(x-5) \quad (x+5)$

$x^2-25=f(x)$

Write the quadratic function

$x=1/2 \quad x=-1/3$

$2x=1 \quad 3x=-1$

$(2x-1) \quad (3x+1)$

$f(x)=6x^2-x-1$

5.4

5.4 Completing the Squares

$4x^2+11=59$
$\quad -11 \quad -11$
$4x^2=48$
$x^2=12$
$x=\pm\sqrt{12}$

$x^2+12x+36=28$
$\sqrt{(x+6)^2}=\sqrt{28}$
$x+6=\pm 2\sqrt{7}$
$x=-6\pm 2\sqrt{7}$

SOLVING QUADRATIC EQUATIONS $ax^2+bx+c=0$

1. Collect variable terms on one side and constants on other
2. Divide both sides to make x^2 by itself eg $\frac{2x^2}{2}=\frac{4}{2}$ $\quad x^2=2$
3. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides
4. Factor variable expression as perfect square
5. Take $\sqrt{\ }$'s of both sides
6. Solve

$x^2=12x-20$
$x^2-12x+36=-20+36$
$\quad -6^2=36$
$\sqrt{(x-6)^2}=\sqrt{16}$
$x-6=\pm 4$
$x=10 \quad x=2$

$18x+3x^2=45$
$\frac{3x^2+18x=45}{3}$
$x^2+6x+9=15+9$
$(x+3)^2=24$
$x=-3\pm\sqrt{24}$
$x=-3\pm 2\sqrt{6}$

"You can complete the square to rewrite ANY quadratic function in vertex form"

Write in vertex form $\quad f(x)=x^2+16x-12$
$x^2+16x+\underline{64}-12-\underline{64}$
$(x+8)^2-76 \qquad a=1$
$(-8,-76)$ vertex

$3x^2-18x+7=g(x)$
$3(x^2-6x+\underline{9})+7\underline{-27}$
27
$=3(x-3)^2-20$
$a=3$ – vertical stretch by 3
$(3,-20)$ = vertex

Imaginary Numbers

Imaginary Number - Square root of a negative #

When b is a rational #, it comes first

Memorize!!

$\boxed{i} = \sqrt{-1}$ $\quad i^3 = \boxed{-i}$

$i^2 = \boxed{-1}$ $\quad i^4 = \boxed{1}$

$\sqrt[4]{10} = i^2 = -1$

$10 = -1$ $\quad i^5 = i$

$\sqrt[4]{20}$ $\quad i^{20} = 1$ $\quad i^{30} = i^2 = -1$

$\sqrt[4]{85}$ $\quad i^{85} = i$

$\frac{4i^5}{2i^3} = 2i^2 = 2(-1) = -2$

$\sqrt{-1} = i$

$\sqrt{-2} = \sqrt{-1 \cdot 2} = i\sqrt{2}$

$\sqrt{-4} = \sqrt{-1 \cdot 4} = 2i$

$5\sqrt{-121} = \sqrt{} = 5 \cdot i \cdot 11 = 55i$

$-\sqrt{-96} = -\sqrt{-16 \cdot 6} = -4i\sqrt{6}$

$5x^2 + 90 = 0$

$5x^2 = -90$

$\sqrt{x^2} = \sqrt{-18}$

$x = \pm\sqrt{-1 \cdot 9 \cdot 2}$

$x = \pm 3i\sqrt{2}$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

real, imaginary

$a + bi$

real

$4x + 10i = 2 - (4y)i$

$4x = 2$ $\quad 10 = -4y$

$x = 1/2$ $\quad y = -5/2$

$-8 + (6y)i = 5x - i\sqrt{6}$

$-8 = 5x$ $\quad 6y = -\sqrt{6}$

$x = -8/5$ $\quad y = \frac{-\sqrt{6}}{6}$

$f(x) = x^2 + 10x + 26$

$-26 = x^2 + 10x + 25$

$-1 = (x+5)^2$

$\sqrt{-1} = x + 5$

$i = x + 5$

$x = -5 \pm i$

Memorize Quadratic Formula

completing the square

$f(x) = x^2 - 2x + 5$

$x^2 - 2x + 1 = -5 + 1$

$(x-1)^2 = -4$

$x - 1 = \pm\sqrt{-4}$

$\pm\sqrt{-1 \cdot 4}$

$x = \pm 2i$

Quadratic Formula

$x = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2}$

$x = \frac{2 \pm \sqrt{-16}}{2}$

$x = \frac{2 \pm 4i}{2}$

$x = \pm 2i$

Complex Conjugate

real $a + bi$, imaginary $a - bi$

$3i + 5 \rightarrow 5 + 3i$ $\quad (5 - 3i)$

imaginary, real

$25 - 9i^2 = 9$

$25 + 9 = 34$

$\sqrt[3]{8x^7y^6z^{10}}$

$\sqrt[3]{8x^6 x y^6 z^9 z}$

$2x^2y^2z^3\sqrt[3]{xz}$

$\sqrt[3]{54} \rightarrow \sqrt[3]{27 \cdot 2} \rightarrow 3\sqrt[3]{2}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2x^2 + x + 12 = 0$$

$a = 2 \quad b = 1 \quad c = 12$

$$x = \frac{-1 \pm \sqrt{1 - 4(2)(12)}}{4}$$

$$x = \frac{-1 \pm \sqrt{-95}}{4} = \frac{-1 \pm i\sqrt{95}}{4}$$

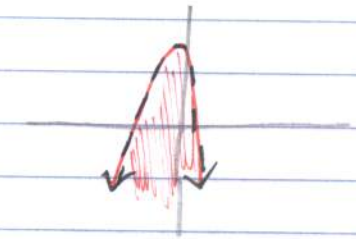
Solving Quadratic Inequalities

dotted line

$y < -2x^2 - 4x + 6$

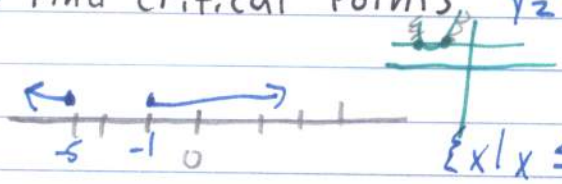
A) ↶↷ c) $(-1, 8)$

B) $x = -1$ D) $(0, 6)$



$x^2 + 8x + 20 \geq 5$ $y_1 = x^2 + 8x + 20$

① Find Critical Points $y_2 = 5$



$x = -5$

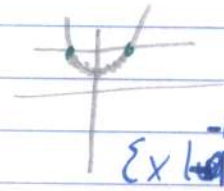
$x = -3$

$\{x \mid x \leq -5 \text{ or } x \geq -3\}$

$x^2 - x + 5 < 7$

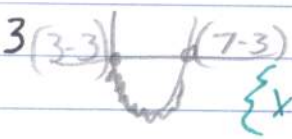
$(-1, 7)$ $x = -1$

$(2, 7)$ $x = 2$



$\{x \mid -1 < x < 2\}$

$x^2 - 10x + 18 \leq -3$



$\{x \mid 3 < x < 7\}$

Quadratic Regression

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9
	-5	-3	-1	1	3	5	
	2	2	2	2	2		

Calc → $f(x) = x^2$

2nd 0 → diagnosticOn

stat → edit

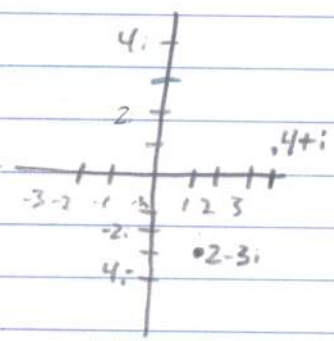
enter x and y

stat → calc → 5

enter

The x axis is the real one

y axis = imaginary



$$|a+bi| = \sqrt{a^2+2}$$

$$\frac{(2+8i)}{4-2i} \cdot \frac{(4+2i)}{4+2i} \quad \& \quad \frac{8+4i+32i+16i^2}{16-4i^2} = \frac{-8+36i}{20} \quad \frac{-2+9i}{5}$$

Polynomials

Monomial - A number with whole # exponents

Polynomial - a monomial or sum/sub of monolias

- No variables in denominator or exponent
- No Roots
- No absolute value
- Variables have whole # exponent

DEGREE OF MONOMIAL

sum of exponents of the variables

x^4	12	$4a^2b$	x^3y^4z
4	0	3	8

DEGREE OF POLYNOMIAL → Degree

Leading Coefficient $-5x^3 + 8x^2 + 3x - 17$

3 2 1 0

Binomial - Polynomial with 2 terms

Trinomial - Polynomial with 3 terms

NAME	DEGREE
Constant	0
Linear	1
Quadratic	2
Cubic	3
Quartic	4
Quintic	5

ADDING AND SUBTRACTING

$(3x+7+x)+(14x^3+2+x^2-x)$

$3x^2+x+7$ Write in standard form

$+14x^3+x^2-x+2$ alignn like terms

$14x^3+4x^2+9$ add

$(1-x^2)-(3x^2+2x-5)$

$(-x^2+1)+(-3x^2-2x+5)$

$(-x^2-3x^2)+(-2x)+1+5$

$-4x^2\ 2x+6$

Polynomial function

Function whose rule is a polynomial

Find the Product

$(a-3)(2-5a+a^2)$

$2a-5a^2+a^3-6+15a-3a^2$

$a^3-8a^2+17a-6$

$(y^2-7y+5)(y^2-y-3)$

$y^4-y^3-3y^2-7y^3+7y^2+21y$

$+5y^2-5y-15$

$y^4-8y^3+9y^2+16y-15$

PASCAL'S TRIANGLE

1	Row 0
1 1	Row 1
1 2 1	Row 2
1 3 3 1	Row 3
1 4 6 4 1	Row 4
1 5 10 10 5 1	Row 5

For a binomial expansion in the form of $(a+b)^n$

- $n+1$ terms
- Coeficients = # from n^{th} row of pascals triangle
- the exponents of a is n in the 1st term and the exponent decreases by 1 each time
- The exponent of b is 0 in first term and increases by 1 each time
- the sum of the exponents in any term is n

Expand $(2x-4)^5$

$+1\cdot(2x)^5 - 5\cdot(2x)^4\cdot 4^1 + 10\cdot(2x)^3\cdot 4^2 - 10\cdot(2x)^2\cdot 4^3 + 5\cdot(2x)^1\cdot 4^4 - 1\cdot 4^5$

$5\cdot 16\cdot 4$ $\quad 10\cdot 8\cdot 16$ $\quad 10\cdot 4\cdot 64$ $\quad 5\cdot 2\cdot 256$

$32x^5-320x^4+1280x^3-2560x^2+2560x-1024$

$$\begin{array}{r|l} & x+5+\frac{21}{x-3} \\ x-3 & x^2+2x+6 \\ & -\,x^2-3x \\ & \quad 5x+6 \\ & -\,5x-15 \\ & \quad 21 \end{array}$$

$$\begin{array}{r|l} & 38\tfrac{1}{2} \\ 2 & 77 \\ & -6 \\ & 17 \\ & -16 \\ & 1 \end{array}$$

Dividing Polynomials

$(2x^2 + 7x + 9) \div (x + 2)$

$x = -2$

-2	2	7	9
		-4	-6
	$2x$	$+3$	$r\,3$

$(x^2 - 3x - 18) \div (x - 6)$

$+6$	1	-3	-18
		6	18
	$1x$	$+3$	

$(3x^4 - x^3 + 5x - 1) \div (x + 2)$

-2	3	-1	0	5	-1
		-6	14	-28	46
	$3x^3$	$-7x^2$	$+14x$	-23	R $45/x+2$

$2x^3 + 5x^2 - x + 7 \quad x = 2$

2	2	5	-1	7
		4	18	34
	2	9	17	41

$= 4i$

Factoring Polynomials

$(x+1)$; (x^2-3x+1)

$$\begin{array}{r|rrrl} -1 & 1 & -3 & 1 & 5 \neq 0 \\ & \downarrow & -1 & 5 & \text{not a factor} \\ & & -4 & 5 & \end{array}$$

$(x+2)$; $(3x^4+6x^3-5x-10)$

$$\begin{array}{r|rrrrrl} -2 & 3 & 6 & 0 & -5 & -10 & \\ & \downarrow & 6 & 0 & 0 & 10 & 0=0 \\ & 3 & 0 & 0 & -5 & \underline{|0} & x+2 \end{array}$$

$(x^3-x^2)-(25x+25)$

$x^2(x-1)-25(x+1)$

$(x-1)(x^2-25)$

$(x-1)(x+5)(x-5)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$125d^3-8$

$(5d-2)(25d^2+10d+4)$

$(2x^3+x^2)(+8x+4)$

$x^2(2x+1)\ 4(2x+1)$

$(2x+1)(x^2+4)$

$(2x+1)(x+2i)(x+2i)$

6-5

FINDING REAL ROOTS OF POLYNOMIAL EQUATIONS

$4x^6 + 4x^5 - 24x^4$

$4x^4(x^2 + x - 6) = 0$

$4x^4(x+3)(x-2) = 0$

$x = 0 \quad x = -3 \quad x = 2$

$x^4 - 26x^2 + 25 = 0$

$(x^2 - 1)(x^2 - 25) = 0$

$x^2 = 1 \quad x^2 = 25$

$x = \pm 1 \quad x = \pm 5$

Even multiplicity = graph touches x axis but does not cross "bounces"

$x^3 + 6x^2 + 12x + 8 = 0$ Root = -2 → on Calc

-2	1	6	12	8
	↓	-2	-8	-8
	1	4	4	0

$x^2 + 4x + 4 = 0$ – factor / quadratic formula / solve

$(x+2)(x+2)$

$x = -2$

$\boxed{-2 \text{ mult } 3}$

$x^4 + 8x^3 + 18x^2 - 27 = 0$ Roots = 1, -3 → From calc → 2nd → Graph

1	1	8	18	0	-27
	↓	1	9	27	27
-3	1	9	27	27	0
	↓	-3	-18	-27	
	1	6	9	0	

$x^2 + 6x + 9 = 0$

$(x+3)^2$

$\boxed{1, -3 \text{ mult } 3}$

$\frac{p}{q}$ p = Factor of constant

q = Factor of leading coefisient.

$x^4 + 8x^3 + 18x^2 - 27$

$\pm \frac{p}{q} \quad \frac{(27)}{1} = \pm 1 \quad \pm 3 \quad \pm 9 \quad \pm 27$

± 1

$a + b\sqrt{c}$ and : : come in pairs

$2 + 3\sqrt{5}, \; 2 - 3\sqrt{5}$

$w = w$

$w + 4 =$ lenth

$w - 1 =$ height

$w(w+4)(w-1) = 12$

$(w^2 + 4w)(w-1)$

$w^3 + 3w^2 - 4w - 12 = 0$

$\pm \frac{p}{q}$

$\frac{\pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 12}{1}$

$w = 2$

$2x^3 - 9x^2 + 2 = 0$

.5	2	-9	0	2
	↓	1	-4	-2
	2	-8	-4	0

$2x^2 - 8x - 4 = 0 \quad x^2 - 4x - 2 = 0$

$x = \frac{4 \pm \sqrt{16 - 4(1)(-2)}}{2}$

$= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$

$\boxed{.5, 2 \pm \sqrt{6}}$

FUNDAMENTAL THEOREM OF ALGEBRA

$-1, \frac{2}{3}, 4$

$(x+1)(x-4)(3x-2) = F(x)$

$x^2 - 4x + x - 4$

$(x^2 - 3x - 4)(3x-2)$

$3x^3 - 11x^2 - 6x + 8$

$2+i, \sqrt{3},$

$2-i, -\sqrt{3},$

$(x-1)(x-\sqrt{3})(x+\sqrt{3})$ $(x-(2+i))(x-(2-i))$

$(x-1)(x^2-3)$ $(x-2-i)(x-2+i)$

$x^3 - 3x - x^2 + 3$

$(x^3 - x^2 - 3x + 3)(x^2 - 4x + 5)$

	x	-2	$-i$
x			
-2			
$+i$			

$x^4 - 3x^3 + 5x^2 - 27x - 36$ -1 4

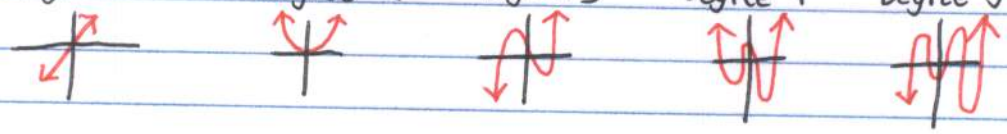
-1	1	-3	5	-27	-36
	↓	-1	4	-9	+36
4	1	-4	9	-36	0
	↓	4	0	36	
	1	0	9	0	

$x^2 + 9 = 0 \rightarrow x^2 = 9$

$x = \pm 3i$

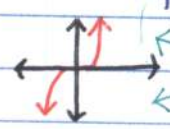
Investigating Graphs of Polynomial Functions

Linear Function	Quadratic	Cubic	Quartic	Quintic
Degree 1	Degree 2	Degree 3	Degree 4	Degree 5



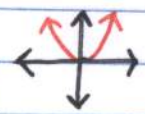
If leading coefficient is positive (+)

ODD DEGREE
↳ Split starts down ends up



← If right quadrant goes up. → Leading coefisient is positive

EVEN DEGREE
↳ same



pos-odd = As $x \to +\infty$, $y \to +\infty$; As $x \to -\infty$, $y \to -\infty$

$y = p(x)$

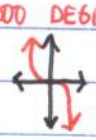
neg-odd = As $x \to -\infty$, $y \to +\infty$; As $x \to \infty$, $y \to -\infty$

$y = p(x)$

Possible Rational Roots $= \frac{p}{q}$

Leading coefficient negative (−)

ODD DEGREE → split
starts up ends down



EVEN DEGREE

pos-even = As $x \to -\infty$, $y \to +\infty$; As $x \to +\infty$, $y \to +\infty$

$y = p(x)$

neg even As $x \to -\infty$, $y \to -\infty$; As $x \to +\infty$, $y \to -\infty$

$y = p(x)$

Identify leading co., degree + end behavior

$-x^4 + 6x^3 - x + 9$

LC = −1 neg even

degree = 4

4 − 1 = 3 terms

$2x^5 + 6x^4 - x + 4$

LC 2 poss

deg = 5 odd

STEPS FOR GRAPING

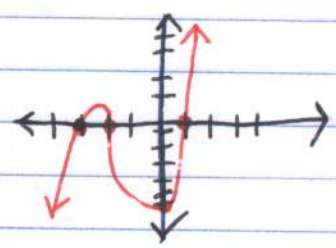
1. Find x and y intercepts
2. Plot the x and y intercept
3. Determine the end behavior of graph
4. sketh the graph

$x^3 + 4x^2 + x - 6$

yintercept = 6 zeros = −3, −2, 1

$(x+3)(x+2)(x-1)$

degree 3 odd poss



Radical Expressions and Rational Exponents

Index - The radical $\sqrt[n]{x}$ → represents the nth root of x

Numbers an Types of real roots

Case	Roots	Example
Odd index	1 real root	The 3rd real root of 8 is 2
Even index Pos radicand	2 real roots	The 4th real roots of 16 = ± 2
Even index Neg radicand	0 real roots	-16 has no 4th real roots
Radicand of 0	1 root of 0	The 3rd Root of 0 is 0

FIND REAL ROOTS

4th rt 81

even pos. radicand = 2 real roots $3^4 = 81$ $(-3)^4 = 81$

ROOTS ARE 3 and -3

6-5 → 6-7
Review

pg 476 #38-64

38 $x^3-5x^2+8x-4=0$ $x=1$

1 | 1 -5 +8 -4
↓ 1 -4 4
1 -4 4 | 0

x^2-4x+4

$(x-2)(x-2)$ $x=2$ mult 2, $x=1$

$x=2$ $x=2$
$x=1$

39 $x^3+6x^2+9x+2=0$ $x=-2$

-2 | 1 6 9 2
↓ -2 -8 -2
1 4 1 x

x^2+4x+1

$x=\frac{-4\pm\sqrt{16-4(1)(1)}}{2}$

$x=-4\pm\sqrt{3}$ $x=-2$

40 $x^3+3x^2+3x+1=0$ $x=-1$

-1 | 1 3 3 1
↓ -1 -2 -1
1 2 1 | 0

x^2+2x+1

$(x+1)(x+1)$ $x=-1$ mult 3
$x=-1$ $x=-1$ $x=-1$

41 $x^4-12x^2+27=0$ $x=3$ $x=-3$

3 | 1 0 -12 0 27
↓ 3 9 -9 -27
-3 | 1 3 -3 -9 0
↓ -3 0 +9
1 0 -3 0

(x^2-3) $x^2=3$

$x=\pm\sqrt{3}$ $x=3$ $x=-3$

42 x^3+x^2-2x-2 $x=-1$

-1 | 1 1 -2 -2
↓ -1 0 +2
1 0 -2 | 0

(x^2-2) $x^2=2$

$x=\pm\sqrt{2}$ $x=-1$
$x=1$

43 x^3-5x^2+4

1 | 1 -5 0 4
↓ 1 1 -4 -4
1 -4 -4 0

x^2-4x-4

$x=\frac{4\pm\sqrt{16-(4)(-4)}}{2}$
$\sqrt{32}$

$x=2\pm2\sqrt{2}$ $x=1$

44 length $=2w$ $2w^2(w+4)$
height $=(w+4)$ $2w^3+8w^2=48$
$w(2w)(w+4)=48$ $2w^3+8w^2-48=0$

2 | 2 8 0 -48
↓ 4 24 48
2 12 24 0

$2x^2+12x+24=0$ $\boxed{w=-3\pm i\sqrt{3}}$
$2(x^2+6x+12)$
$x=-6\pm\sqrt{36-4(12)}$
$-3\pm2i\sqrt{3}$

45 $(x+3)(x-2)(x-4)(x^2+x-6)$
$x^3-3x^2-10x+24$ $x^3+x^2-6x-4x^2-4x+24$

46 $(x+\frac{1}{2})(x+2)(x-3)(x^2+1.5x+1)$
$x^3-1.5x^2-3.5x-3$ $x^3+1.5x^2+x-3x^2-3$

47 $(x+\sqrt{2})(x-\sqrt{2})(x+1)(x^2-2)$
x^3+x^2-2x-2 x^3-2x+x^2-2

48 $(x+3)(x+i)(x-i)(x^2+1)(x+3)$
x^3+3x^2+x+3 x^3+x+3x^2+3

49 $(x+\sqrt{2})(x-\sqrt{2})(x+\sqrt{3})(x-\sqrt{3})$
$(x^2-2)(x^2-9)$ $x^4-9x^2-2x^2+18$
$x^4-11x^2-2x^2+18$

50 $(x-1-\sqrt{3})(x-1+\sqrt{3})(x-2i)(x+2i)$
$x^2-x+\sqrt{3}x-x+1-\sqrt{3}-\sqrt{3}x$ (x^2+4)
(x^2-2x-3) $+\sqrt{3}-3$ (x^2+4)
$x^4-2x^3-3x^2+4x^2-8x-12$
$x^4-2x^3+x^2-8x-12$

51 $x^3-x^2+4x-4=0$ $x=1$

1 | 1 -1 4 -4
↓ 1 0 4
1 0 4 0

$x=1$ $x=\pm2i$! (x^2+4) $(x+2i)(x-2i)$

52 $x^4-x^2-2=0$ $x=\sqrt{2}$ $x=-\sqrt{2}$

$\sqrt{2}$ | 1 0 -1 0 -2
↓ $\sqrt{2}$ 2 $\sqrt{2}$ 2
$-\sqrt{2}$ | 1 $\sqrt{2}$ 1 $\sqrt{2}$ 0
↓ $-\sqrt{2}$ 0 $-\sqrt{2}$
1 0 1 | 0
(x^2+1) $x\pm i$

$x^2+1=0$ $x=\pm i$
$x^2=-1$ $x=\sqrt{2}$
$x=\pm\sqrt{-1}$ $x=2$

53 $x^4+3x^2-5x-15=0$

54 $x^3+3x^2-5x-15=0$ $x=-3$

-3 | 1 3 -5 -15
↓ -3 0 +15
1 0 -5

$(x+\sqrt{5})(x-\sqrt{5})$ (x^2-5)

$x=-3$ $x=\sqrt{5}$ $x=\sqrt{5}$

55 $-2x^3+5x^2+3$ LC $=-2$ deg $=3$ odd neg — As $x\to\infty$ $y\to-\infty$; As $x\to-\infty$ $y\to\infty$

56 x^4+2x^3-3x+1 LC $=1$ deg $=4$ even pos — As $x\to-\infty$ $y\to\infty$; As $x\to\infty$ $y\to\infty$

57 LC $=-3$ deg $=6$ even neg — As $x\to-\infty$ $y\to-\infty$; As $x\to\infty$ $y\to-\infty$

58 $7x^5+x^4-2x^2+5$ LC $=7$ deg $=5$ poss odd — As $x\to\infty$ $y\to\infty$; As $x\to-\infty$ $y\to-\infty$

7-1

Exponential Functions growth and decay

Parent Function for exponential = $f(x) = b^x$

If $b > 1$ = growth As x increases y increases

$0 < b < 1$ = exponential decay As x increases y decreases

initial amount

$A(t) = a(1 \pm r)^t$ number of time periods.

Final amount rate of increase

$1+r$ growth factor

$1-r$ decay factor

$f(x) = 10(3/4)^x$ = decay

① $f(x) = a(1 \pm r)^x$

② $f(x) = 50(1+.0625)^x$

③ $100 = 50(1.0625)^x$

$2 = (1.0625)^x$ $y_1 = 1.0625^x$ $y_2 = 2$

$x = 11.43$

① graph on calculator

② Ajust window

③ Find Intersection.

① $f(x) = 15{,}500(1-.03)^x$

$8000 = 15500(.97)^x$

$.516 = (.97)^x$

An inverse is a reflection across $y = x$

$(x, y) \rightarrow (y, x)$

To write inverse flip variables and solve for y

Logarithic functions

A logarithm is the exponent to which a specified base is raised to obtain a given value.

$b^x = a$ Exponential

$\log_b a = x$ log form

$\log_b a = x$

$b^x = a$

$3^5 = 243$

$\log_3 243 = 5$

$25^{1/2} = 5$

$\log_{25} 5 = 1/2$

$\log_{10} 10 = 1$ $10^1 = 10$

$10^4 = 10{,}000$

$\log_{10} 10{,}000 = 4$

$\log_9 9 = 1$

$9^1 = 9$

$\log_8 2 = 1/3$

$8^{1/3} = 2$

Log with base 10 = common log *can do in calculator

If there is no base = 10

$\log_{10} 0.01 = x$

$10^x = .01$

$10^x = 1/100$

$10^x = 10^{-2}$

$x = -2$

$\log_5 125 = y$

$5^y = 125$

$5^y = 5^3$

$y = 3$

$\log_5 1/5 = z$

$5z = 1/5$

$5^z = 5^{-1}$

$z = -1$

The log is the inverse of an exponential fuction

7-4

Properties of Logs

Product property = $\log_b mn = \log_b m + \log_b n$

$\log_3 1000 = \log_3 (10 \cdot 100)$

$= \log_3 10 + \log_3 100$

$\log_6 4 + \log_6 9$

$\log_6 (4 \cdot 9)$

$\log_6 36 = x$

$6^x = 36$

$x = 2$

$\log_4 2 + \log_4 32$

$\log (64) = x$

$4^x = 64$

$x = 3$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log_5 (\frac{16}{2}) = \log_5 16 -$

$\log_5 100 - \log_5 4$

$\log_5 100/4$

$\log_5 25 = x$

$5^x = 25$

$x = 2$

$\log_7 49 - \log_7 7$

$\log_7 49/7$

$\log_7 7 = x$

$7^x = 7$

$x = 1$

Power Property $\log_b a^b = p \log_b 3$

$\log 10^3$

$\log (10 \cdot 10 \cdot 10)$

$\log 10 + \log 10 + \log$

$3 \log 10$

$\log_2 32^6$

$6 \log_2 32 = x$

$6 (2^x = 32)$

$6(5) = 30$

$4 \log_5 (1/5)^4$

$4 \log_5 (1/5) = x$

$5^x = 1/5$

$4 \; x = -1 \; = -4$

Inverse property $\log_b b^x = x$ $\quad b^{\log_b x} = x$

$\log_{10} 10^7 = 7$ $\quad 10^{\log_{10} 2} = 2$

$\log_3 3^{11} = 11$ $\quad \log_3 81 = x$

$3^x = 81$

$x = 4$

Change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

CALCULATOR

$\log_{32} 8$ $\quad \frac{\log 8}{\log 32} \xrightarrow{calc} .6$

$\log_{32} 8 = x$

$32^x = 8$

$(2^5)^x = 2^3$

$2^{5x} = 2^3$

$5x = 3$

$x = 3/5 = .6$

Exponential and logarithmic Equations and inequalities

Exponential Equation - variable as an exponent

If x is an exponent, take the log of both sides

$8^x = 2^{x+6}$

$(2^3)^x = 2^{x+6}$

$3x = x+6$

$x = 3$

$5^{x-2} = 200$

① Take log of both sides

$\log 5^{x-2} = \log 200$

② Move exponents to front

$(x-2)\frac{\log 5}{\log 5} = \frac{\log 200}{\log 5}$

③ Divide by log 5 * on calc *

④ solve

$x = 5.292$

$\log_3 (x-5) = 2$

$3^{\log_3 (x-5)} = 3^2$

① Use 3 as base for both sides

$x-5 = 9$

② ~~$3^{\log 3}$~~ remove

$x = 14$

③ Solve for x

$\log 45x - \log 3 = 1$

$\log_{(10)} \frac{45x}{3} = 1$

① Combine logs

$10^{\log 45x/3} = 10^1$

② Use 10 as base for both sides

$45x/3 = 10$

③ Inverse property to remove log

$45x = 30$

④ Solve

$x = 2/3$

$\log_4 x^2 = 7$

$\sqrt{4^7} = \sqrt{x^2}$

① Use ear method $4^7 = x^2$

$x = 128$

② Take $\sqrt{\ }$ of both sides

$\log x + \log (x+9) = 1$

$\log x^2 + 9x = 1$

① Write as single log

$10 = x^2 + 9x$

② Ear method

$x^2 + 9x - 10 = x$

③ Subtract 10 from both sides

$(x+10)(x-1)$

④ Factor

$x = -10 \quad x = 1$

⑤ Solve

Cannot have a negative log

The Natural Base e

e is like π. constant representing irrational number ≈ 2.72

$\log_e = \ln$ = natural logarithm

$f(x) = \ln^x$ is inverse of $f(x) = e^x$

SAME PROPERTIES APPLY

$\ln(e) = 1$

$\ln e^{-2t}$ $= -2t$

$e^{\ln(t-1)}$ $= t-1$

$e^{5\ln x^5}$ $= x^5$

NATURAL DECAY

$$\underbrace{N(t)}_{\text{amt remaining}} = \underbrace{N_0}_{\text{initial amt @ } t=0} e^{-kt}$$

k: decay constant

t: time

Sample Problems

(1) $0.04^{1/2}$

$\sqrt{.04}$

$.2$

(2) $\left(\frac{1}{36}\right)^{-1/2}$

6

(3) $27^{4/3} \cdot 4^{0/5}$

81

(4) $\frac{\sqrt{50x^8}}{\sqrt[3]{-x^6}}$ $\frac{5x^4\sqrt{2}}{-x^2}$

$-5x^2\sqrt{2}$

(5) $\log .01$

$x=-2$

(6) $\log_{36} 6 = x$

$36^x = 6$

$x = .5$

(7) $\log_5 1$

$x=0$

(8) $\log_2 16$

$x=4$

(9) $\ln e^{8.6}$

$x=8.6$

(10) $\log 12.5$

$x=1.096$

(11) $\log_7 49^3 = x$

$7^x = 49^3$

$7^x (7^2)^3$

$x=6$

(12) $\log_x 8 = -3$

$x^{-3} = 8$

$(x^{-3})^{-1/3} = 8^{1/3}$

$x = 1/2$

(13) $\log_4 (3x-1) = \log_4 (x+9)$

$3x-1 = x+9$

$x=5$

(14) $\ln 4 = 2x + \ln 12$

$\ln 4 - \ln 12 = 2x$

$\frac{\ln 4}{\ln 12} = 2x$

$1/\ln 3 = 2x$

$\ln e^{1/3} = 2x$

$e^{2x} = 1/3$

$2x = -1.098$

$x = -.0549$

(15) $\log_2 x^2 - \log_2 x - 2 = 0$

$\log_2 x^2/x = 2$

$\log_2 x = 2$

$x=4$

(16) $\log_{20} 3 = a$ $\log_2 5 = b$

$\log_2 (5 \times 4)$ $\log_2 5 + \log_2 4$

$b+2$

(17) $\ln_e (\ln x) = 0$

$e^0 = \ln x$

$e^1 = x$

$x = e$

(18) $\log_7 (\log_2 x) = 1$

$7 = \log_2 x$

$2^7 = x$

$x = 128$

(19) $8^{3x} = 16$

$\log 8^{3x} = \log 16$

$3x \log 8 = \log 16$

$3x = 4/3$

$x = 4/9$

(20) $4^x = 17^{x+1}$

$\log 4^x = \log 17^{x+1}$

$x \log 4 = (x+1) \log 17$

$x = (x+1)\ 2.043$

$\frac{x}{x+1} = \frac{2.044}{1}$

$x = 2.044 \cdot + 2.044$

$1.044x = 2.044$

$x = 1.958$

(21)

Sample Problems #2

$A(t) = a(1 \pm r)^t$

① 2,000,000 growing 5% 2025?

$2{,}000{,}000(1+.05)^{14}$

3959863

② \$4,000 3% int how long dbl?

$4000(1+0.03)^x$

$x = 23.4$ years

$f(x) = \ln x$

① $f(x) = \ln x + 3$ 3 up

② $f(x) \ln(x+3)$ 3 left

③ $f(x) = 3\ln(x)$ Vert strech 3

③ \$10,000 for 3 yrs compounded continously at 6% $A(t) = Pe^{(r \cdot t)}$

$A(t) = 10{,}000e^{(.06 \cdot 3)}$

$A(t) = \$11972.17$

④

1	3.5
2	5
3	6.5
4	7
5	8
6	8.5

2nd 0 clear on stat edit stat calc

a) exponential reg $f(x) =$

$r = .950$

b) power reg $f(x) =$

$r = .996$

Power reg is better b/c r is closest to 1

7-7

Transforming Exponential and Logarithmic Functions

EXPONENTIAL FUNCTIONS

$y = 2^x + 3$	3 up	Vertical translation
$y = 2^x - 6$	6 down	
$y = 2^{x-2}$	2 right	Horizontal translation
$y = 2^{x+1}$	1 left	
$y = 6(2x)$	Vertstrech by 6	Vertical Streach/comp
$y = 1/2(2x)$	Vert comp by 1/2	
$y = 2^{(1/5x)}$	Horiz strech 5	Horiz S/C
$y = 2^{(3x)}$	Horic comp by 1/3	
$y = -2^x$	Reflection Across x	Reflection
$y = 2^{-x}$	Reflection Across y	

LOGARITHMIC FUNCTIONS

$y = \log x + 3$	3up	Vertical translation
$y = \log x - 4$	4 down	
$y = \log(x-2)$	2 right	Horizontal translation
$y = \log(x+1)$	1 left	
$y = 6\log x$	V Strech by 6	Vertical S/C
$y = 1/2 \log x$	V comp by 1/3	
$y = \log(1/5\, x)$	H strech 5	Horiz S/C
$y = \log(3x)$	H strech by 1/3	
$y = -\log x$	Reflection x axis	Reflection
$y = \log(-x)$	" y axis	

8-1

Variation Function

$y = kx$

Direct variation - relationship b/w x and y that can be given $y = kx$ constant of variation

$y = mx + b$ $b = 0$ → Always goes through origin

$y = kx$ $y = 14$ $x = 3.5$

$14 = k(3.5)$ substitute x and y

$4 = k$ Solve for k

$y = 4x$ write equation using k

"if k is $+^{pos}$ then y increases as x increases"

$y = kx^2$

Joint variation - relationship b/w 2 variables

$A = 12m^2$ $b = 6m$ $h = 4m$ Find b when $A = 36m^2$ $h = 8m$

$A = kbh$

$12 = k(6)(4)$ Substitute

$1/2 = k$ Solve for k

$A = \frac{1}{2}bh$ Substitute k

$36 = 1/2 b(8)$ Substitute

$b = 9$ Solve for b

Inverse variation = $y = k/x$ y varies inversly as x

$f = k/s$ $k = 26.1954$ $3.5 = \frac{26.1954}{s}$

$2.97 = k/2.82$ $s = 7.4844$

Direct Variation

$y = kx \rightarrow k = \underbrace{y/x}_{\text{constant ratio}}$

Inverse Variation

$y = k/x \rightarrow k = \underbrace{xy}_{\text{Constant product}}$

x	6.5	13	104
y	8	4	.5

$\frac{y}{x}$ xy inverse

x	5	8	12
y	30	48	72

$\frac{y}{x}$ direct

8-1

pg 574 #17-36, 40, 41, 45-57

17 $y = kx$
$4 = k8$
$k = .5$
$y = \frac{1}{2}x$

18 $y = kx$
$12 = 2k$
$k = 6$
$y = 6x$

19 $y = kx$
$-15 = k5$
$k = -3$
$y = -3x$

20 $\frac{100}{55} = \frac{x}{70}$
$x = 127.27$

21 $\frac{25}{3.5} \quad \frac{x}{12.3.5}$
$x = 8.821$

24 $y = k/x$
$1 = k/.8$
$k = .8$
$y = .8/x$

25 $y = k/x$
$1.75 = k/6$
$k = 10.5$
$y = \frac{10.5}{x}$

26 $y = k/x$
$-2 = k/3$
$k = -6$
$y = -6/x$

27 $\frac{3}{20} = \frac{x}{12}$
$x = 1.8$

28 direct

29 neither

30 inverse

32 always

33 sometimes

34 never

35 always

36 never

40 $y = kx2$

41 $y = kx$
$y = k/x$

45 C

46 H

47 D

48 $\frac{250}{24} \quad \frac{200}{x}$
$x = 19.2$

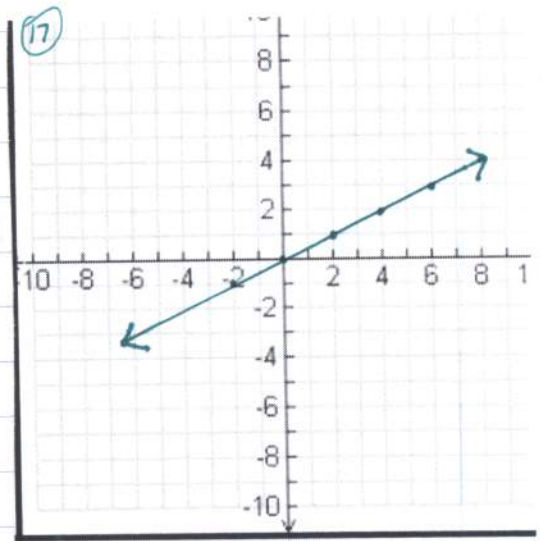
49 $y = kx\sqrt{2}$
$189 = k7.3$
$k = 9$
$y = 9x\sqrt{2}$

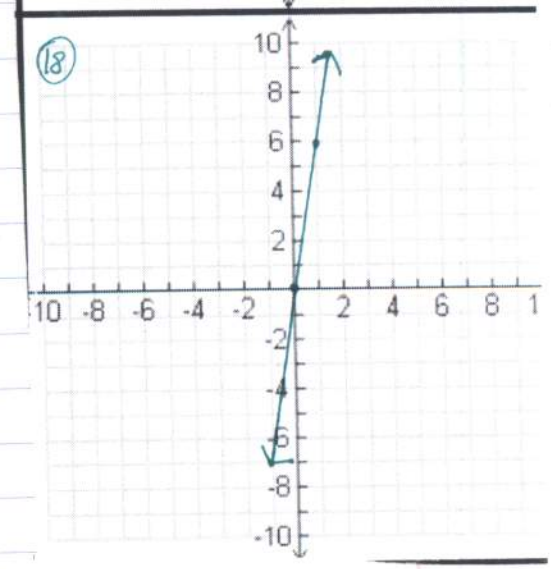
52 $\frac{80}{112} \quad \frac{11820}{x}$
$x = 1379$ ft

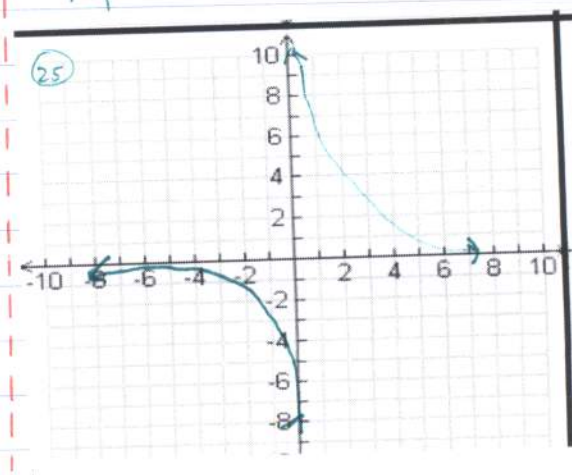
53 $y = 1.25x + .25$

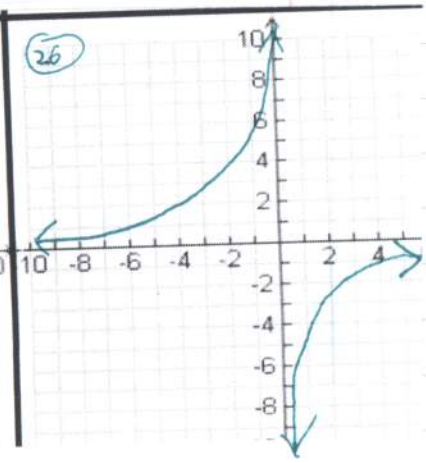
55 2 down

56 1 right 1 up
1 up









31 $V_1 = 20L$ $P_1 = 1atm$ $T_2 = 330K$ $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $\frac{1atm \mid 20L \mid 330K}{330K \mid 15L} = 1.375$ atm

8-2

Multiplying and Dividing Rational Expressions

$\frac{10x^8}{6x^4} = \frac{5x^4}{3}$ undefined $x=0$

$\frac{x^2+x-2}{x^2+2x-3} = \frac{(x+2)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \frac{x+2}{x+3}$

$x \neq -3 \quad x \neq 1$

x^2 must be positive

$\frac{4x-x^2}{x^2-2x-8} = \frac{-x(x-4)}{(x-4)(x+2)} \quad \frac{-x}{x+2}$

$x \neq 4 \quad x \neq -2$

1. Factor all numeratous and denominators
2. Divide out common Factors of num and dem
3. multiply numeretors. Then multiply dem
4. num and dem = no common Factors other than 1

$\frac{x-3}{4x+20} \cdot \frac{x+5}{x^2-9} \quad \frac{x-3 \cdot x+5}{\cdot \quad \cdot \quad \cdot \quad \cdot}$

$\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5} \quad \frac{\overset{x^2}{\cancel{5x^4}}}{\cancel{8x^2y^2}} \cdot \frac{\cancel{8}y^{\cancel{5}\,3}}{\underset{3}{\cancel{15}}} = \frac{x^2y^3}{3}$

$\frac{x^4-9x^2}{x^2-4x+3} \div \frac{x^4+2x^3-8x^2}{x^2-16}$

$\frac{\cancel{x^2}(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} \cdot \frac{(x+4)\cancel{(x-4)}}{\cancel{x^2}\cancel{(x+4)}(x-2)} = \frac{(x+3)(x+4)}{(x-1)(x-2)}$

$x^2(x^2+2x-8)$

$\frac{x^2-25}{x-5} = 14$

$\frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} = \frac{14}{1} \qquad x+5=14$

$x=9$

8.3

$$\frac{x-3}{x+4}+\frac{x-2}{x+4}=\frac{x-3+x-2}{x+4}=\frac{2x-5}{x+4}$$

$$\frac{x-3}{x+4}-\frac{x-2}{x+4}=\frac{x-3-(x-2)}{x+4}$$

$$\frac{x-3-x+2}{x+4}=\frac{-1}{x+4}$$

$$\frac{3x-4}{x^2+1}-\frac{6x+1}{x^2+1}=\frac{3x-4-(6x+1)}{x^2+1}=\frac{-3x-5}{x^2+1}$$ NO REAL #'S

$$\frac{x-3}{\cancel{x^2+3x-4}}+\frac{2x(x-1)}{(x+4)(x-1)}=\frac{x-3+2x(x-1)}{(x+4)(x-1)}=\frac{x-3+2x^2-2x}{(x+4)(x-1)}=\frac{2x^2-x-3}{(x+4)(x-1)}$$

$(x+4)(x-1)$

$$=\frac{(2x-3)(x+1)}{(x+4)(x-1)}$$

$$\frac{2x^2-30}{x^2-9}-\frac{x+5}{x+3}=\frac{2x^2-30}{(x+3)(x-3)}-\frac{x+5(x-3)}{x+3(x-3)}$$

$(x+3)(x-3)$

$$=\frac{2x^2-30-x^2-2x+15}{(x+3)(x-3)}=\frac{2x^2-30-(x^2+2x-15)}{(x+3)(x-3)}$$

$$\frac{x^2-2x-15}{(x+3)(x-3)}=\frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}=\frac{(x-5)}{(x-3)}$$

$$\frac{\frac{x+1}{x^2-1}}{\frac{x}{x-1}}\quad \frac{\cancel{x+1}}{\cancel{(x+1)}\cancel{(x-1)}}\cdot\frac{\cancel{(x-1)}}{x}=1/x$$

8-4

Radical Functions

$f(x) = \frac{\cancel{(x-1)}(x-2)}{(x+4)\cancel{(x-1)}} = \frac{(1-2)}{(1+4)} = -1/5$

$\frac{\cancel{(x-1)}(3x-2)}{(x+4)\cancel{(x-1)}}$

$y = 3$

Hiatuspoint $= (1, -1/5)$

Vertica asintope $x = -4$

kissing spliter

$y = 1$

y intercept $= (0, 1/2)$

Solving Rational Equations and inequalities

Rational Equation- contains one or more rational expressions

$x + 8/x = 6$

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4)$

$x = 2$ or $x = 4$

Extraneous Solution- solution of an equation derived from original equation that is not a solution of original equation

$$\frac{3x}{x-3} = \frac{2x+3}{x-3}$$

$$\frac{(3x)\cancel{(x-3)}}{\cancel{(x-3)}} = \frac{(2x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$3x = 2x + 3$

$x = 3$ - extraneous b/c 3 makes denominator $= 0$

NO SOLUTION

$$\frac{2x-9}{x-7} + \frac{x}{2} = \frac{5}{x-7}$$

$$\frac{(2x-9)(2\cancel{(x-7)})}{\cancel{(x-7)}} + \frac{x(\cancel{2}(x-7))}{\cancel{2}} = \frac{5 \cdot (2\cancel{(x-7)})}{\cancel{(x-7)}}$$

$2(2x-9) + x(x-7) = 10$

$4x - 18 + x^2 - 7x = 10$

$x^2 - 3x - 28 = 0$

$(x-7)(x+4) = 0$

$x = 7$ or $x = -4$

↳ x cannot be 7 (extraneous)

$x = -4$

8-5

$$\frac{x}{x-4} \leq 2$$

Calculator: $Y_1 = x/x-4$ $\quad Y_2 = 2$

2nd → table

x	Y_1	$1/2$
4	error	2
8	2	2

$x < 4$ or $x \geq 8$

$8/_{x+5}\,(x+5) \leq 4\,(x+5)$

$8 \leq 4x + 20$

$-12 \leq 4x$

$-3 \leq x$

$x + 5 > 0$

$x > -5$

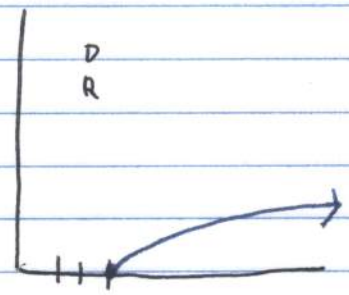
$x \geq -3$ and $x > -5$

$x \geq -3$

Radical Functions

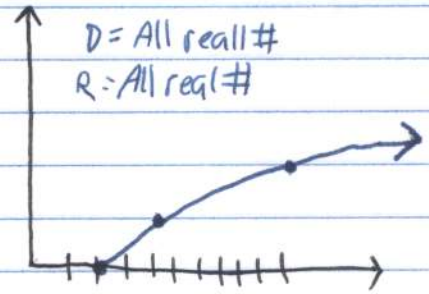
$f(x) = \sqrt{x-3}$

x	y
3	0
4	1
7	2
12	3



$f(x) = 2\sqrt[3]{x-2}$

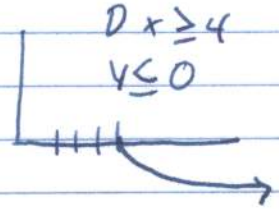
x	y
-6	-4
1	-2
2	0
3	2
10	4



$g(x) = -\sqrt{x-4}$

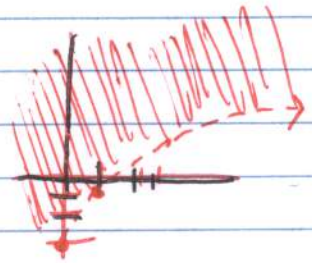
flip over x

right 4



$y > 2\sqrt{x} - 3$

0	-3
1	1
4	1
9	3



8-8

1. Isolate the radical $\sqrt[3]{x} - 2 = 0$
2. Raise both sides to power $(\sqrt[3]{x})^3 = (2)^3$
3. Simplify $x = 8$

$5 + \sqrt{x+1} = 16$

$-5 \qquad -5$

$\sqrt{x+1} = 11$

$x + 1 = 121$

$x = 120$

$\frac{7}{7}(\sqrt[3]{5x-7})^3 = (7)^3$

$(7)^3$

$5x - 7 = 1$

$x = 8/5$

$(\sqrt{-3x+33})^2 = (5-x)^2$

$-3x + 33 = 25 - 10x + x^2$

$0 = x^2 - 7x - 8$

$(x-8)(x+1)$

$x = 8 \quad x = -1$

Algebra II PreAP
Sec 8.8 Worksheet on Radicals

Name Fernando Trujano
Date_____ Period___

Solve each equation. Check to eliminate extraneous solutions.

$(x-1)^2$
x^2-2x+1

1. $(\sqrt{x+4})^2-(\sqrt{x-1})^2=1$

$x+4 = 1 + 2\sqrt{x-1} + x - 1$
$x+4 = x + 2\sqrt{x-1}$
$4 = 2\sqrt{x-1}$
$(2)^2 = (\sqrt{x-1})^2$
$4 = x-1$
$x = 5$

$\boxed{x=5}$

2. $3-\sqrt{y-6}=\sqrt{y}$

$(3-\sqrt{y-6})^2 = (\sqrt{y})^2$
$(3-\sqrt{y-6})(3-\sqrt{y-6}) = y$
$9-6\sqrt{y-6} = 0$
$-6\sqrt{y-6} = -3$
$(2\sqrt{y-6})^2 = (1)^2$
$4(y-6) = 1$
$4y-24 = 1$
$y = 25/4$

$\boxed{y = 25/4}$

3. $\sqrt{a+10}=5-\sqrt{a}$

$(\sqrt{a+10})^2 = (5-\sqrt{a})^2$
$a+10 = 25-10\sqrt{a}+a$
$-15 = -10\sqrt{a}$
$(-3)^2 = (2\sqrt{a})^2$
$9 = 4a$
$a = 9/4 = 2.25$

$\boxed{a = 2.25}$

4. $\sqrt{k}-\sqrt{k-8}=2$

$(\sqrt{k})^2 = (2+\sqrt{k-8})^2$
$k = 4+4\sqrt{k-8}+k-8$
$k = k-4+4\sqrt{k-8}$
$4 = 4\sqrt{k-8}$
$1^2 = (\sqrt{k-8})^2$
$1 = k-8$
$k = 9$

$\boxed{k=9}$

5. $\sqrt{r+10}+\sqrt{3-r}=5$

$(\sqrt{r+10})^2 = (5-\sqrt{3-r})^2$
$r+10 = 25-10\sqrt{3-r}+3-r$
$r+10 = 28-r-10\sqrt{3-r}$
$2r-18 = -10\sqrt{3-r}$
$(r-9)^2 = (-5\sqrt{3-r})^2$
$r^2-18r+81 = 25(3-r)$
$r^2-18r+81 = 75-25r$
r^2+7r+6
$(r+6)(r+1)=0$
$r=-6 \quad r=-1$

$\boxed{r=-6 \quad r=-1}$

6. $\sqrt{p+5}+\sqrt{p-2}=7$

$(\sqrt{p+5})^2 = (7-p-2)^2$
$p+5 = 49-14\sqrt{p-2}+p-2$
$p+5 = 49-14\sqrt{p-2}+p-2$
$5 = 47-14\sqrt{p-2}$
$-42 = -14\sqrt{p-2}$
$(-21)^2 = (-7\sqrt{p-2})^2$
$441 = 49(p-2)$
$441 = 49p-98$
$539 = 49p$
$p = 11$

$p=11$

7. $3\sqrt{c-1}-\sqrt{c-8}=9$

$3\sqrt{c-1} = 9+\sqrt{c-8}$
$(3\sqrt{c-1})^2 = (9+\sqrt{c-8})^2$
$9(c-1) = 81+18\sqrt{c-8}+c-8$
$9c-9 = 73+c+18\sqrt{c-8}$
$8c-82 = 18\sqrt{c-8}$
$4c-41 = 9\sqrt{c-8}$
$(4c-41)^2 = (9\sqrt{c-8})^2$
$16c^2-328c+1681 = 81(c-8)$
$16c^2-328c+1681 = 81c-648$
$16c^2-409c+2329$
$(16c-137)(c-17)=0$
$c = 137/16 \quad c=17$

$\boxed{c=17}$

8. $5\sqrt{n+4}-\sqrt{n-1}=11$

$(5\sqrt{n+4})^2 = (11+\sqrt{n-1})^2$
$25(n+4) = 121+22\sqrt{n-1}+n-1$
$25n+100 = 120+n+22\sqrt{n-1}$
$(24n-20)^2 = (22\sqrt{n-1})^2$
$(12n-10)^2 = (11\sqrt{n-1})^2$
$144n^2 = 243n+100 = 121(n-1)$
$144n^2-240n+100 = 121n-121$
$144n^2-361n+221$
$x = 17/16 \quad x = 13/9$

$\boxed{x = 17/16 \quad x = 13/9}$

9. $\sqrt{2x+1}-\sqrt{x-3}=2$

$(\sqrt{2x+1})^2 = (2+\sqrt{x-3})^2 \quad (2+\sqrt{x-3}$
$2x+1 = 4+4\sqrt{x-3}+x-3$
$2x+1 = 1+4\sqrt{x-3}+x$
$x+1 = 1+4\sqrt{x-3}+x$
$x = 4\sqrt{x-3}+x$
$0 = (4\sqrt{x-3})^2$
$0 = 16(x-3)$
$16x-48=0$
$16x=48$
$x=3$

10. $\sqrt{b+3}+\sqrt{3b+10}=3$

$(\sqrt{b+3})^2 = (3-\sqrt{3b+10})^2 \quad (3-\sqrt{3b+10})$
$b+3 = 9-6\sqrt{3b+10}+3b+10$
$b+3 = 19-6\sqrt{3b+10}+3b$
$-3b \quad -3b$
$-2b+3 = 19-6\sqrt{3b+10}$
$-19 \quad -19$
$-2b-16 = -6\sqrt{3b+10}$
$-b-8 = -3\sqrt{3b+10}$
$(-b-8)^2 = 9(3b+10)$
$b-11b-26=0$
$-166 \quad -166$
$b^2+16b+64 = 27b+90$
$(-b-8)(-b-8) = 27b+90$

11. $\sqrt{u-2}-\sqrt{2u+3}+2=0$

$u=11 \quad u=3$

12. $\sqrt{2z+8}-\sqrt{z+2}=2$

$z=14 \quad \cancel{z=-2}$

13. $\sqrt{3x+1}+\sqrt{x-1}=6$

$x=5$
$x=65$

14. $\sqrt{2w+3}-\sqrt{w+1}=1$

$(\sqrt{2w+3})^2 = (1+\sqrt{w+1})^2$
$2w+3 = 1+\sqrt{w+1}^2+1$
$(2w+1)(2w+1) = (\sqrt{w+1})^2$
$4w^2+4w+1 = w+1$
$4w^2+3w+0=0$

15. $\dfrac{2d-3}{\sqrt{3d-2}}=\sqrt{d}$

$\dfrac{4d^2-12d+9}{3d-2} = d$
$4d^2-12d+1 = 3d^2-2d$
$d^2-10d+1=0$
$(x-9)(x-1)$
$x=9 \quad x=1$

$\boxed{x=9 \quad \cancel{x=1}}$

16. $\sqrt[3]{x^3+7}-1=x$

$x=-2$
$x=1$

17. $\sqrt[3]{y^3-3y-1}=y-1$

$y^3-3y-1 = y^2-2y+1(y-1)$
$y^3-3y-1 = y^3-2y^2+y-y^2+2y-1$
$y^3-3y+1 = y^3-3y^2+3y-1$
$3y^2-3y+2$

18. $\sqrt{x+9}=\sqrt{x}+1$

$x+9 = x+1$
$x+8 = x$

$x+9 = x+2\sqrt{x}+1$
$8 = 2\sqrt{x}$
$\frac{64}{4} = \frac{4x}{4}$
$x=16$

8-8

(7) $3\sqrt{c-1} - \sqrt{c-8} = 9$

$3\sqrt{c-1} = 9 + \sqrt{c-8}$

$(3\sqrt{c-1})^2 = (9 + \sqrt{c-8})^2$

$9(c-1) = 81 + 18\sqrt{c-8} + c - 8$

$9c - 9 = 73 + c + 18\sqrt{c-8}$

$8c - 82 = 18\sqrt{c-8}$

$4c - 41 = 9\sqrt{c-8}$

$(4c-41) = (9\sqrt{c-8})^2$

$16c^2 - 328c + 1681 = (\sqrt{c-8})^2$

$16c^2 - 328c + 1681 = 81c - 648$

$16c^2 - 409c + 2329$

$(16c - 137)(c - 17) = 0$

$c = 137/16 \quad c = 17$

(3) $\sqrt{a+10} = 5 - \sqrt{a}$

$(\sqrt{a+10})^2 = (5 - \sqrt{a})^2$

$a + 10 = 25 - 10\sqrt{a} + a$

$-15 = -10\sqrt{a}$

$(-3)^2 = (2\sqrt{a})^2$

$9 = 4a$

$a = 9/4 = 2.25$

(16) $\sqrt[3]{x^3+7} - 1 = x$

$(\sqrt[3]{x^3+7})^3 = (x+1)^3 (x+1)$

$x^3 + 7 = (x^2 + 2x + 1)(x + 1)$

$x^3 + 7 = x^3 + 2x^2 + x + x^2 + 2x + 1$

$x^3 + 7 = x^3 + 3x^2 + 3x + 1$ $\quad 36 \quad 32$, $2 \quad 16$

$0 = 3x^2 + 3x - 6$

$0 = 3(x^2 + x - 2) = 3(x+2)(x-1)$

$3x^2 + 3x - 6$

$(x+2)(x-1) = 0$

$x = -2$

$x = 1$

(6) $\sqrt{p+5} + \sqrt{p-2} = 7$

$(\sqrt{p+5})^2 = (7 - \sqrt{p-2})^2 (7 - \sqrt{p-2})$

$p + 5 = 49 - 14\sqrt{p-2} + p - 2$

$p + 5 = 49 - 14\sqrt{p-2} + p - 2$

$5 = 47 - 14\sqrt{p-2}$

$-42 = -14\sqrt{p-2}$ $\quad (-21)^2 = (-7\sqrt{p-2})^2 (-7$

$p = 11$

$539 = 49p$

$441 = 49p - 98$

$441 = 49(p-2)$

$(2 + \sqrt{(x+2)})$

$(2 + \sqrt{x+2})$

(2) $3 - \sqrt{y-6} = \sqrt{y}$

$(3 - \sqrt{y-6})^2 = (\sqrt{y})^2$

$(3 - \sqrt{y-6})(3 - \sqrt{y-6}) = y$

$9 - 6\sqrt{y-6} + y - 6 = y$

$3 - 6\sqrt{y-6} = 0$

$-6\sqrt{y-6} = -3$

$(2\sqrt{y-6})^2 = (1)^2$

$4(y-6) = 1$

~~$y - 6 = 1/2$~~

~~$y = 6.5$~~

~~$2y - 12 = 1$~~

$4(y-6) = 1$

$4y - 24 = 1$

$y = 25/4$

(8) $5\sqrt{n+4} - \sqrt{n-1} = 11$

$(5\sqrt{n+4})^2 = (11 + n - 1)^2$

$25(n+4) = 121 + 22\sqrt{n-1} + n - 1$

$25n + 100 = 120 + n + 22\sqrt{n-1}$

$24n - 20 = 22\sqrt{n-1}$

$(12n - 10)^2 = (11\sqrt{n-1})^2$

$144n^2 - 240n + 100 = 121(n-1)$

$144n^2 - 240n + 100 = 121n - 121$

$144n^2 - 361n + 221 = 0$

$(16n - 17)(9n - 13) = 0$

$n = 17/16 \quad \cancel{n = 13/9}$

(12) $\sqrt{2x+8} - \sqrt{x+2} = 2$

$(\sqrt{2x+8})^2 = (\sqrt{x+2} + 2)^2$

$2x + 8 = 4 + 4\sqrt{x+2} + x + 2$

$(x+2) = (4\sqrt{x+2})^2$

$x^2 + 4x + 4 = 16(x+2)$

$x^2 + 4x + 4 = 16x + 32$

$x^2 - 12x - 28 = 0$

$(x-14)(x+2)$

$x = 14 \quad x = -2$

(13) $\sqrt{3x+1} + \sqrt{x-1} = 6$

$(\sqrt{3x+1})^2 = (6 + \sqrt{x-1})^2$

$3x + 1 = 36 + 12\sqrt{x-1} + x - 1$

$3x + 1 = 35 + 12\sqrt{x-1} + x$

$(2x - 34)^2 = (12\sqrt{x-1})^2$

$4x^2 - 136x + 1156 = 144(x-1)$

$4x^2 - 136x + 1156 = 144x - 144$

$4x^2 - 280x + 1012$

$x^2 - 34x + 289 = 36x - 36$

$x^2 - 70 + \cancel{253}\ 325 = 0$

$x^2 - 70 + 325 = 0$

$(x - 65)(x - 5) \quad x = 5$

(11) $\sqrt{v-2}\ \sqrt{2v+3} + 2 = 0$ $(\sqrt{v-2})$

$(\sqrt{v-2} + 2)^2 = (\sqrt{2v+3})^2$ $(\sqrt{v-2} + 2)$

$(v-2)' = 2v + 3$

$(\sqrt{v-2} + 2)(\sqrt{v-2} + 2)$

$v - 2 + 4\sqrt{v-2} + 4 = 2v + 3$

$4\sqrt{v-2} + 2 = v + 3$

$(4\sqrt{v-2})^2 = (v+1)^2 (v+1)$

$16(v-2) = v^2 + 2v + 1$

$16v - 32 = v^2 + 2v + 1$

$v^2 - 14v + 33 = 0$

$(v-11)(v-3)$

$v = 11 \quad v = 3$

Multiple Representations of Functions

Table - - Find differences or ratios to determine best parent functions.
- Graph points and look for a patterns.
- Match data to function and perform regression

Graph - Use key points from graph to write an equation
Locate points and write them in a table. Use slope

Equation - make a table of values
- Make a graph

Verbal description - make table of values.

Shintaro rappels down a 500 ft cliff at 6 ft per second. when will he reach the ground.

① Write an equation $h = 500 - 6f$

② Solve for $h=0$ $0 = 500 - 6f$

$f = 83\frac{1}{3}$

Stacy runs and records the time every 2 laps. How long will it take to run 20 laps.

Laps	Time
2	13
4	16
6	19
8	22
10	25

① Graph the points → linear

② Write linear equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ use any 2 points

③ Find the slope $\frac{16-13}{4-2} = 3/2$

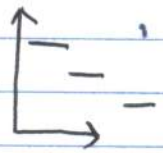
④ Write linear equation

⑤ Solve for $x = 20$ $y = 3/2\,x + 10$ $y = (3/2)20 + 10 = 40$ $y = 40$

can use calculator to graph (LineReg) (quadReg) and find minimums or maximums.

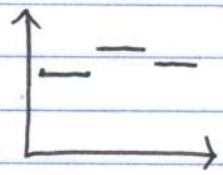
Piecewise Functions

Combination of one or more functions

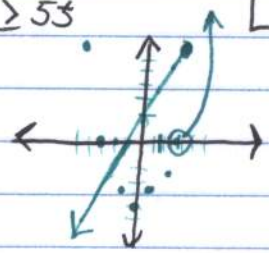


Step function: 1 - 1.99 = 3
2 - 3 = 2
3 - 5 = 1

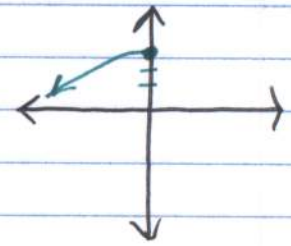
$$f(x) = \begin{cases} 5 & \text{if } 0 < x < 13 \\ 9 & \text{if } 13 \le x < 55 \\ 6.5 & \text{if } x \ge 55 \end{cases}$$



$$h(x) = \begin{cases} 2x+1 & \text{if } \le 2 \\ x^2-4 & \text{if } x > 2 \end{cases}$$



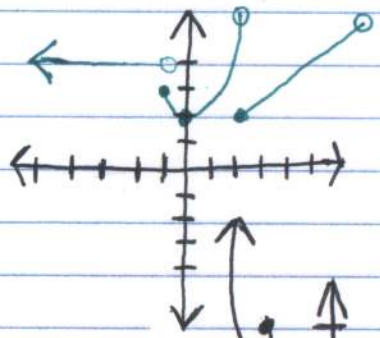
$$g(x) = \begin{cases} \frac{1}{4}x+3 & \text{if } x < 0 \\ -2x+3 & \text{if } x \ge 0 \end{cases}$$



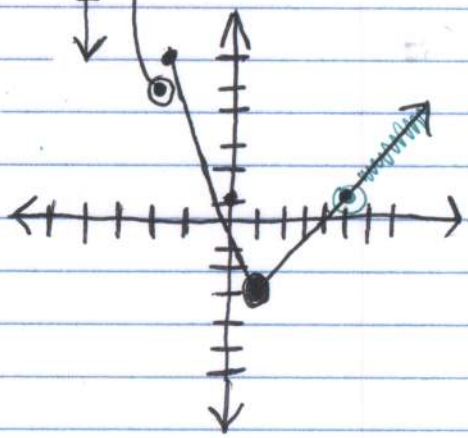
For linear put in beginning and end of interval and solve

$$h(x) = \begin{cases} 4 & \text{if } x < -1 \\ x^2+2 & \text{if } -1 \le x < 2 \\ x & \text{if } 2 \le x < 5 \end{cases}$$

plug in -1 in x^2+2
2



$$h(x) = \begin{cases} x^2+1 & \text{if } x < -2 \\ -3x & \text{if } -2 \le x < 1 \\ x-4 & \text{if } 1 \le x < 5 \\ -\frac{1}{2}x+\frac{7}{2} & \text{if } x \ge 5 \end{cases}$$



4-#5

TRANSFORMATIONS

Horizontal translations change the rule and the interval of the piecewise function

Vertical translations change only the rule of piecewise function

$$f(x)\begin{cases} -\frac{1}{2}x & \text{if } x<0 \\ \frac{1}{2}x^2 & \text{if } x \geq 0 \end{cases}$$ Interval

RULE

Vertical strech by 3

$$g(x)\begin{cases} -\frac{3}{2}x & \text{if } x<0 \\ \frac{3}{2}x^2 & \text{if } x \geq 0 \end{cases}$$

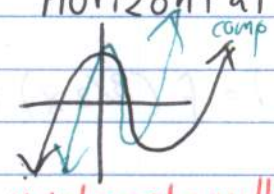
Horizontal 3 right

$$g(x)\begin{cases} -\frac{1}{2}(x-3) & \text{if } x-3<0 \\ \frac{1}{2}(x-3)^2 & \text{if } x-3>0 \end{cases}$$

$x<3$

$x \geq 3$

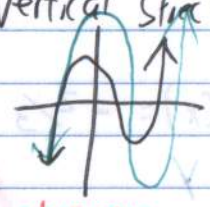
Horizontal



x intercepts multiplied by factor.

y intercept remains the same

Vertical Strech



y change

x intercepts remain same

Horizontal strech 2

$f(x) = -2x-4$ $\quad g(x) = f(\frac{1}{2})x$

x intercept: $(0,-4)$ $\quad$ x $(-4,0)$

y intercept: $(-2,0)$ $\quad$ y $(0,-4)$

In Reflection accross **y** axis - y intercept same

x intercepts negated signs change

Reflection across **X** x stay same

y negated

$f(x) = (x-2)^2-1$ $\quad g(x) = f(-x)$

x inter $(3,0)(1,0)$ $\quad (-3,0)(-1,0)$

y inter $(0,-1)$ $\quad (0,-1)$

$0=(x-2)^2-1$

$1=(x-2)^2$

$\pm\sqrt{1}\ (x-2$

$\pm 1 = x-2$

$2\pm 1 = x$

4-4

Operations with functions

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$(f/g)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$

$f(x) = 4x^2 + 3x - 1 \qquad g(x) = 6x + 2$

$(f+g)(x) = f(x) + g(x) \qquad \boxed{4x^2 + 9x + 1}$

$(f-g)(x) \quad 4x^2 + 3x - 1 - (6x+2)$

$4x^2 - 3x - 3$

$\left(\frac{f}{g}\right)x \qquad \frac{6x^2 - x - 12}{2x - 3} \qquad \frac{\cancel{(2x-3)}(3x+4)}{\cancel{(2x-3)}} = (3x+4)$

$(fg)x = (fx)(gx) \qquad (6x^2 - x - 12)(2x - 3)$

$12x^3 - 18x^2 - 2x^2 + 3x - 24x + 36$

$12x^2 - 20x^2 - 21x + 36$

$(f \circ g)(x) = f(g(x))$

NOT MULTIPLICATION

$f(g(4)) \qquad f(x) = 2^x \quad g(x) = 7 - x$

$f(3) = 2^3 \qquad g(4) = 7 - 4$

$\boxed{f(3) = 8} \qquad g(4) = 3$

$g(f(4) \qquad f(x) = 2^x \quad g(x) = 7 - x$

$g(16) = 7 - 16 \qquad f(4) = 2^4$

$g(16) = -9 \qquad f(4) = 16$

$f(x) = x^2 - 1 \qquad g(x) = x/1 - x$

$f(g(x))$

$f(g(x)) = \left(\frac{x}{1-x}\right)^2 - 1$

$\frac{x^2}{(1-x)^2} - \frac{1}{1} \quad \frac{(1-x)^2}{(1-x)^2} \quad \frac{x^2 - (1 - 2x + x^2)}{(1-x)^2}$

$f(g(x)) = \frac{-1 + 2x}{(1-x)^2} \qquad \frac{\cancel{x^2} - 1 + 2x - \cancel{x^2}}{(1-x)^2}$

$\boxed{\frac{-1 + 2x}{(1-x)^2}}$

$f(x) = \begin{cases} x+3 & \text{if } x>0 \\ 2x+3 & \text{if } x\leq 0 \end{cases}$

① Hor trans 4 rt $g(x-4)$

② Vert strech of 3

① $g(x) \begin{cases} x-1 \\ x-4+3 & \text{if } x>4 \\ 2(x-4+3) \\ 2x-8+3 \\ 2x-5 & \text{if } x-4\leq 0 \\ & x\leq 4 \end{cases}$

② $\begin{cases} 3(x+3) & \text{if } x\geq 0 \\ 3(2x+3) & \text{if } x\leq 0 \end{cases}$

$f(x) = \begin{cases} x^2 & \text{if } x\leq 0 \\ x-3 & \text{if } x> \end{cases}$

Hor str 2

$g(x) = \begin{cases} x^2/2 & \text{if } x\leq 0 \\ \cancel{(x-3)/2} \\ \frac{1}{2}x-3 & \text{if } \frac{1}{2}x>0 \\ & x>0 \end{cases}$

$f(x) = \begin{cases} x^2-5x+7 & \text{if } x<0 \\ 4-2x & \text{if } x\geq 0 \end{cases}$

① hor trans 5 left

$(x+5)^2-5(x+5)+7$ $\quad x<-5$

$4-2(x+5)$ $\quad x\geq -5$

$x^2+10x+25-5x-25+7$

$4-2x-10$

$g(x) = \begin{cases} x^2+5x+7 & \text{if } x<-5 \\ -2x-6 & \text{if } x\geq -5 \end{cases}$

② Vert strech 3

$g(x) \begin{cases} 3x^2-15x+21 & \text{if } x<0 \\ 12-6x & \text{if } x\geq 0 \end{cases}$

$f(x) = x^2-1$ $\quad 0=x^2-1$

x intercept $(1,0)(-1,0)$

y intercept $=(0,-1)$

$g(x) = -f(x)$

$(1,0)$ $(-1,0)$

$(0,1)$

$g(x) = f(-x)$

$(-1,0)(1,0)$

$(0,-1)$

$g(x) = 3f(x)$

$(1,0)(-1,0)$

$(0,-3)$

$g(x) = f(2x)$

$(\frac{1}{2},0)(-\frac{1}{2},0)$

$(0,-1)$

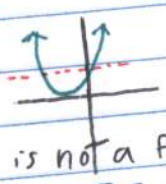
9-5

Functions and Their Inverses

The horizontal line test - determine wheather inverse is a function

To write the inverse of a function: exchange the x and y

Find Inverse of $(\frac{1}{2}x+2)^2$



① Graph the Function

Horizontal line test → inverse is not a function

② Swich x and y

$x=(\frac{1}{2}y+2)^2$

③ Solve for y

$\pm\sqrt{x}=\frac{1}{2}y+2$

$\pm\sqrt{x}-2=\frac{1}{2}y$

$y=\pm2\sqrt{x}-4$

CANNOT USE $F^{-1}(x)$ because INVERSE IS NOT A FUNCTION.

④ Check by graphing and → inverse should be a reflection across $y=x$

one to one function - when both a relation and it's inverse are function

each y value is paired with one x value

IDENTIFYING INVERSE FUNCTIONS

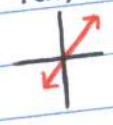
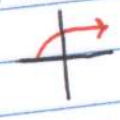
If $f(g(x))=g(f(x))=x$ then $f(x)$ and $g(x)$ inverse

$f(x)=3x$ $g(x)=\frac{1}{3}x$ $f(g(x))=3(\frac{1}{3}x)=x$ $g(f(x))=\frac{1}{3}(3x)=x$

for $x=1$ or 0 $f(x)=\frac{1}{x}-1$ and $g(x)=\frac{1}{x}+1$ $f(g(x))=\frac{1}{\frac{1}{x}}+1-1=x$ $g(f(x))=\frac{1}{x-1}+1=x$

9-6

Modeling Real World Data

Functions	Linear	Quadratic	Exponential	Square Roots
	$f(x) = x$	$f(x) = x^2$	$f(x) = b^x, b>0$	$f(x) = \sqrt{x}$
				
	Constant 1st Difference	Constant 2nd difference	Constant ratios	Constant 2nd difference b/w x
	y - x	y - x	y - x	x - y
			y/y	Example 2

11-1

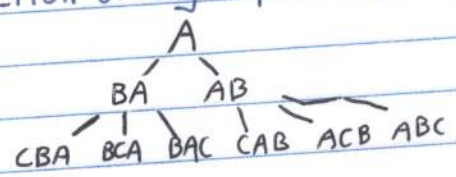
Permutations and Combinations

Fundamental Counting Principle - If there are n items and m_1 ways to choose a 1st item, m_2 ways to choose a second item and so on.

There are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items

Example. You have 4 pairs of shoes, 10 shirts and 5 shorts.
how many combinations? $4 \cdot 10 \cdot 5 = 200$

Permutation - selection of a group of objects in which order is important.



1 permutation
$2 \cdot 1$ permutation
$3 \cdot 2 \cdot 1$ permutation

n Factorial

The Factorial of a # is the product of the natural #'s ≤ to the #. $0! = 1$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$

Permutations

The # of permutations of 7 items taken 3 at a time is

$_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}$

The # of permutations of n items taken r at time is

$_nP_r = \frac{n!}{(n-r)!}$

Combinations - grouping of items in which order does not matter

# of combinations $= \frac{\text{ways to arrange items}}{(\text{selected items})(\text{not selected})}$

The # of combinations of 7 items taken 3 at a time

$_7C_3 = \frac{7!}{3!(7-3)!}$

$_nC_r = \frac{n!}{r!(n-r)!}$

11-2

Theoretical and Experimental Probability

Probability - How likely something is to happen.

Outcome - Each possible result.

Sample Space - Set of all possible outcomes.

Event - Outcome - set of Outcomes

Example. Rolling a # cube. Sample s. $=\{1, 2, 3, 4, 5, 6\}$

Theoreical Probability - ratio of # of Favaroble outcomes to total # of outcomes.

Favorable outcomes - outcomes in a specified event

$$P(\text{event}) = \frac{\text{number of Favorable outcome}}{\text{\# of outcome in sample space}}$$

Complement - all outcomes in sample space that are not E

$P(\text{not } E) = 1 - P(E)$

Geometric probability - theoreical probability determined by ratio of lenths, area, volume.

Each repetition of an experiment is a trial.

Experimental probability - $\frac{\text{number of times the event occurs}}{\text{\# of Trials}}$

11-3

Independent and Dependent Events

Independent Events - occurance of one event does not affect the probalility of the other.

$P(A \text{ and } B) = P(A) \cdot P(B)$

Dependent Events - occurance of 1 event affects probability of the other

Conditional Probability - Probability that event B given that A occured $P(B|A)$

$P(A \text{ and } B) = P(A) \cdot P(B/A)$

11-4

Compound Events

Simple event - describes a single outcome

Compound event - 2 or more simple events

Mutually exclusive events - cannot both occur in same trial

$P(A \cup B) = P(A) + P(B)$ $P(1 \text{ or } 2)$ $P(1) + P(2)$ $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Binomial Distributions

$$\begin{array}{ccccccc} & & & {}_0C_0 & & & \\ & 1 & & {}_1C_0 & {}_1C_1 & & \\ & 1 & 1 & & & & \\ & 1 & 2 & 1 & {}_2C_0 & {}_2C_1 & {}_2C_2 \\ 1 & 3 & 3 & 1 & {}_3C_0 & {}_3C_1 & {}_3C_2 \quad {}_3C_3 \end{array}$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

Binomial Theorem

$$(x+y)^n = {}_nC_0 x^n y^0 + {}_nC_1 x^{n-1} y^1 + {}_nC_2 x^{n-2} y^2 + \dots + {}_nC_{n-1} x^1 y^{n-1} + {}_nC_n x^0 y^n$$

$$(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Binomial Probability - if a binomial experiment has n trials in which p is the probability of success and q is probability of failure

binomial probability $= P(r) = {}_nC_r p^r q^{n-r}$

10-2

Circles

Circle - set of points in a plane → Fixed distance (radius) from Fix point (center)

Center $(2,1)$ $r=5$

Distance formula. $(x_2,y_2)=(x,y)$ $(x_1,y_1)=(2,1)$ $d=5$

$$d=\sqrt{(x^2-x^1)^2+(y_2-y_1)^2}$$
$$5=\sqrt{(x-2)^2+(y-1)^2}$$
$$5^2=(x-2)^2+(y-1)^2$$
$$25=(x-2)^2+(y-1)^2$$

Equation of a circle

Center (h,k) Radius r

$$(x-h)^2+(y-k)^2=r^2$$

Center $(5,-2)$ $r=8$

$$(x-5)^2+(y+2)^2=64$$

Tangent - line in same plane as circle that intersects circle at 1 point.

↳ perpendicular to radius at point of tangency.

tangent to circle $25=x^2+y^2$ at point $(3,4)$

center $(0,0)$ $r=5$ — Find center and radius from equation

$m=\frac{y_2-y_1}{x_2-x_1}$ — Find slope at point of tangency

$m=4/3$

$y-y_1=m(x-x_1)$ — Slope-intercept equation by using $(3,4)$ and $m=4/3$

$y-4=-3/4(x-3)$

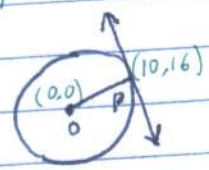
$y=-3/4x+25/4$ — Substitute

① Write the ⊥ Bisector $\overset{x_1}{(-5}, \overset{y_1}{12)}$ $\overset{x_2}{(8}, \overset{y_2}{-20)}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad \frac{-20-10}{8+5} = \frac{-30+2}{13} = \frac{13}{32}$

$y - y_1 = m(x - x_1)$ $\quad$ MP(1.5, -4)

$y + 4 = \frac{13}{32}(x - 1.5)$

② 

Find Equation of Tangent Line $\overset{x_1}{(0}, \overset{y_1}{0)}$ $\overset{x_2}{(10}, \overset{y_2}{16)}$

$\frac{16-0}{10-0} = 8/5 \quad -5/8$

$y - y_1 = m(x - x_1)$

$y - 16 = -5/8(x - 10)$

10-3

Ellipses

Ellipse - set of points $P(x,y)$ in a plane such that the sum of the distances from any point P on the ellipse to 2 fixed points F_1 and F_2 called the foci.

Foci - constant sum $d = PF_1 + PF_2$

① Find constant sum for an ellipse with foci $F_1(-3,0)$ and $F_2(3,0)$ point on ellipse $(0,4)$

$d = PF_1 + PF_2$

$d = \sqrt{(x_1-x_3)^2 + (y_1-y_3)^2} + \sqrt{(x_2-x_3)^2 + (y_2-y_3)^2}$ Distance Formula

$d = \sqrt{(-3-0)^2 + (0-4)^2} + \sqrt{25}$

$d = 10$

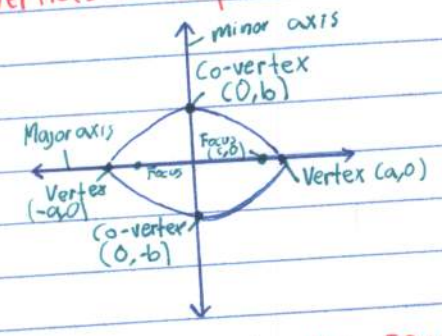
CONSTANT SUM IS 10

Major axis - Longer axis of an ellipse; passes through both Foci

Vertecis of the ellipse - Endpoints of mayor axi

Minor axis - Shorter axis of the ellipse

Co-Vertices of ellipse - Endpoints of the minor axis



Values a, b, c

$c^2 = a^2 - b^2$

Lenth of major axis $2a$

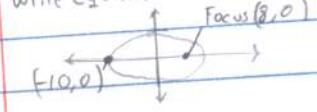
Lenth of minor axis $2a$

$a > b$

STANDARD FORM for the EQUATION of an ELLIPSE (center at $(0,0)$)

	Horizontal	Vertical
Major Axis	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Equation	$(a,0)$ $(-a,0)$	$(0,a)$ $(0,-a)$
Vertecies	$(c,0)$ $(-c,0)$	$(0,c)$ $(0,-c)$
Foci		
Co Verticies	$(0,b)$ $(0,-b)$	$(b,0)$, $(-b,0)$

Write equation in standard form.



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a = 10$
$c = 8$

Horizontal axis longer
Vertex gives a
Focus gives c

Substitute

$8^2 = 10^2 - b^2$ $b^2 = 36$

$\frac{x^2}{100} + \frac{y^2}{36} = 1$

Hyperbolas

Hyperbola - Set of points $P(x,y)$

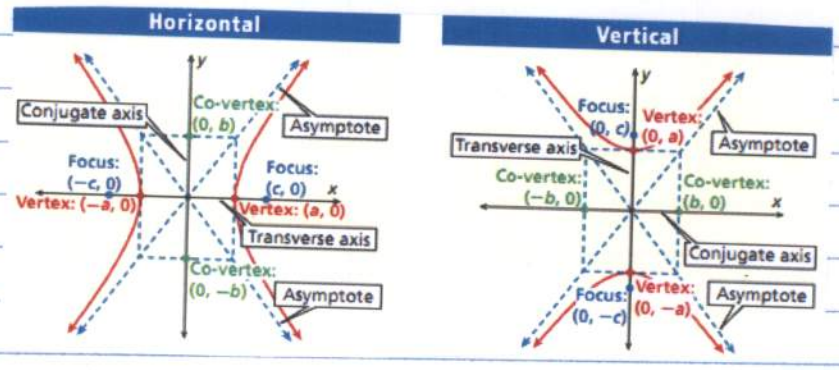
$$d = |PF_1 - PF_2|$$

$$= \left|\sqrt{(x_1-x_3)^2+(y_1-y_3)^2} - \sqrt{(x_2-x_3)^2+(y_2-y_3)^2}\right|$$

$$\left|\sqrt{1156} - \sqrt{900}\right|$$

$$|34-30|$$

$$=4$$



Transverse axis - Contains vertecis, foci and hyperbola $2a$

Conjugate axis - Separates 2 branches of parabola. $2b$

$$c^2 = a^2 + b^2$$

Standard Form for the Equation of a Hyperbola Center at (0, 0)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$
Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

Chapter 10 Review

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Mid Pt $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Circle $(x-h)^2+(y-k)^2=r^2$

Ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

1. $c^2=a^2-b^2$
2. a is always largest
3. a^2 under x = horizontal under y = vertical
4. a = distance from Center to Vertex
   b = distance from Center to Covertex
   c = distance from Center to Focus

Hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}$

1. a is always 1st denominator
2. $c^2=a^2+b^2$
3. a^2 under x = horizontal under y = vertical
4. a = d from center → Vertex
   b = d from center → co vertex
   c = d from center → Focus

5. Equation of asyntope
   ± slope and Center
   $y-y_1=\pm m(x-x_1)$

Parabola $(x-h)=\frac{1}{4p}(y-k)^2$ $\quad y-k=\frac{1}{4p}(x-h)^2$

$(x-h)=-\frac{1}{4p}(y-k)^2$ $\quad y-k=-\frac{1}{4p}(x-h)^2$

Ex $(x-3)=-\frac{1}{16}(y-2)^2$

$\frac{1}{4p}=-\frac{1}{16}$ $p=4$ $\quad V(h,k)$ $\quad p$ = d from Vertex to focus
p = d from Vertex to directrix

B^2-4AC - Discriminant
Bxy Ax^2 Cy^2

>0 Hyperbola
0 = Parabola
< 0 ellipse of circle

if b=0 then
if A or C = 0 → Parapola
$A=C\neq 0$ → Circle

A same sign C $A\neq C$ → Ellipse
A opposite sign C → Hyperbola

TEKS ★ TAKS Practice

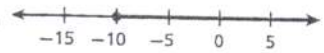
Chapter 1 Skills Practice

Lesson 1-1

Order the given numbers from least to greatest. Then classify each number by the subsets of the real numbers to which it belongs.

1. $2.3, \frac{5}{2}, \sqrt{10}, 2.\overline{4}, 2\sqrt{3}$
2. $-3, -\sqrt{12}, \frac{2}{5}, -2\pi, -\pi$
3. $\sqrt{9}, 3.0\overline{2}, 3\frac{1}{16}, 3\frac{2}{30}, \pi$

Use interval notation to represent each set of numbers.

4. $-40 \le x < -12$
5. $-1 < x < 5$ or $x \ge 13$
6. 

Rewrite each set in the indicated notation.

7. $x \le 0$ or $4 < x < 8$; set-builder notation
8. all odd natural numbers; roster notation

Lesson 1-2

Identify the property demonstrated by each equation.

9. $12 + a = a + 12$
10. $3 \cdot (9 \cdot 2) = (3 \cdot 9) \cdot 2$
11. $2(\sqrt{10} + 4) = 2(\sqrt{10}) + 2(4)$

Use mental math to find each value.

12. a 15% tip on a bill of \$34.60
13. a 30% discount on a \$67.80 item

Classify each statement as sometimes, always, or never true. Give examples or properties to support your answer.

14. $a + 4 = b + 4$
15. $12b = 6b + 6b$
16. $ab = ac$

Lesson 1-3

Estimate to the nearest tenth.

17. $\sqrt{90}$
18. $\sqrt{62}$
19. $-\sqrt{48}$
20. $\sqrt{23}$

Simplify each expression.

21. $\frac{\sqrt{242}}{\sqrt{2}}$
22. $\frac{\sqrt{20}}{\sqrt{120}}$
23. $2\sqrt{5} + 4\sqrt{20}$
24. $2\sqrt{72} - \sqrt{18}$

Lesson 1-4

Write an algebraic expression to represent each situation.

25. the area in square inches of a triangle with base b inches and height 12 inches
26. the amount in dollars remaining from \$55 after spending d dollars

Evaluate each expression for the given values of the variables.

27. $2a^2 + 5a - 3b$ for $a = 4$ and $b = -3$
28. $\frac{x + y}{2xy + 2}$ for $x = 3$ and $y = 5$

Simplify each expression.

29. $3n + 5n - 2(n + 2)$
30. $3(x - 7) + 4x^2$
31. $-4a + 2(12 - 4a)$

Lesson 1-5

Simplify each expression. Assume all variables are nonzero.

32. 4^0
33. $3^2 \cdot 3^{-5}$
34. $(3x^2y)^4$
35. $\frac{(5x)^2}{5y^{-4}}$

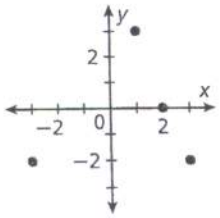
Simplify each expression. Write the answer in scientific notation.

36. $(1.4 \times 10^{12})(2.2 \times 10^3)$
37. $\frac{(9.9 \times 10^6)}{(2.2 \times 10^3)}$
38. $\frac{24 \times 10^{-5}}{6 \times 10^4}$

Lesson 1-6

Give the domain and range for each relation.

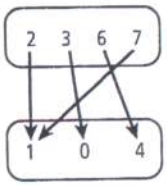
39.



40.

x	y
−5	−10
−2	−2
0	5
4	19

41.



Determine whether each relation is a function.

42.

Average Regular Gasoline Prices August 2005				
Date	8/8	8/15	8/22	8/29
Cost ($/gal)	2.37	2.55	2.61	2.61

43. from a person's age to his or her height

Lesson 1-7

For each function, evaluate $f(0)$, $f(3)$, and $f(-2)$.

44. $f(x) = -4x + 10$

45. $f(x) = \frac{1}{2}x^2$

46. $f(x) = x^2 - 2x + 5$

Graph each function.

47. $g(x) = \frac{1}{2}x - 4$

48.

x	1	2	3	4
y	1	3	5	7

49. $h(x) = -2x + 5$

Lesson 1-8

Perform the given translation on the point $(-3, 4)$. Give the coordinates of the translated point.

50. 3 units right

51. 5 units up

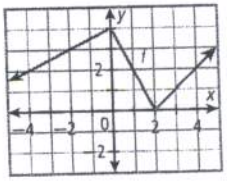
52. 2 units left, 2 units down

Use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.

53. reflection across the y-axis

54. translation 2 units up

55. vertical compression by a factor of $\frac{1}{2}$



Lesson 1-9

Identify the parent function for g from its function rule. Then graph g on your calculator and describe what transformation of the parent function it represents.

56. $g(x) = (x + 3)^3$

57. $g(x) = \sqrt{x - 4}$

58. $g(x) = x^2 + 3$

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

59.

x	−2	−1	0	1	2
y	−4	−0.5	0	0.5	4

60.

x	1	3	5	7	9
y	16	4	0	4	16

Lesson 2-1

Solve.

1. $4(x-3)=48$
2. $6x+10=-2x+26$
3. $\frac{1}{2}(10a+12)=a-6$
4. $3z+12=\frac{1}{2}(4z+4)$
5. $11w+4=58-7w$
6. $-5p+32=2(p-2)$

Solve and graph.

7. $3x+7<28$
8. $12y-3\le 57$
9. $2(4-x)<10$

Lesson 2-2

Solve each proportion.

10. $\frac{3x}{15}=\frac{3}{5}$
11. $\frac{8}{5x}=\frac{2}{11}$
12. $\frac{-4}{5}=\frac{14}{y}$
13. $\frac{2.2}{3}=\frac{n}{5}$
14. $\frac{9.5}{3}=\frac{6+m}{6}$
15. $\frac{-1}{9}=\frac{1.5}{3-x}$

Lesson 2-3

Determine whether each data set could represent a linear function.

16.

x	−2	1	4	7
$f(x)$	−14	−5	4	13

17.

x	−2	−1	0	1
$f(x)$	6	0	−2	0

Graph each line.

18. slope $\frac{2}{3}$; passes through $(3, 4)$
19. slope $-\frac{5}{3}$; passes through $(6, -1)$

Find the intercepts of each line, and graph the lines.

20. $-2y+x=8$
21. $3x+y=6$

Write each function in slope-intercept form. Then graph the function.

22. $3y-2x=3$
23. $4y+3x=20$

Lesson 2-4

Find the slope of each line.

24.

x	−2	1	4	7
$f(x)$	−14	−5	4	13

25. a line through $(-1, 20)$ and $(3, -4)$

Write the equation of each line in slope-intercept form.

26. a line with slope 3 and x-intercept $\frac{4}{3}$
27. a line with slope $-\frac{3}{2}$ passing through $(4, 1)$

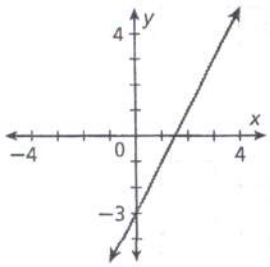
28.

x	−2	0	5	6
$f(x)$	14	15	17.5	18

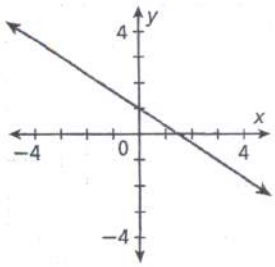
29.

x	7	10	13	16
$f(x)$	−6	−3	0	3

30.



31.



Lesson 2-5

Graph each inequality using intercepts.

32. $y - x > 4$ **33.** $2y - 8x \leq -4$ **34.** $4y + 3x \geq 12$

Solve each inequality for y. Graph the solution.

35. $6x + 6y < 18$ **36.** $12 > 2(x - 3y)$ **37.** $6y \leq 9x - 36$

Lesson 2-6

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

38. $f(x) = \frac{4}{7}x + 1$; vertical translation 3 units down

39. $f(x) = -4x + 9$; horizontal stretch by a factor of 4

40. linear function defined in the table; reflection across the y-axis

x	−1	0	2	5
y	−17	−11	1	19

Let $g(x)$ be the indicated transformation of $f(x) = x$. Write the rule for $g(x)$.

41. vertical stretch by a factor of 2 followed by a horizontal shift 2 units right

42. horizontal shift 5 units left followed by a reflection across the x-axis

43. horizontal compression by a factor of $\frac{3}{2}$ followed by a vertical shift 8 units down

Lesson 2-7

44. If the points in a scatter plot have a positive correlation, then the r-value is ___?___. If the points have no correlation, then the r-value is ___?___.

45. Make a scatter plot of the data shown in the table.

x	0	2	3	4	6	9
y	18	15	14	10	5	1

46. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.

Lesson 2-8

Solve each compound inequality. Then graph the solution set.

47. $3 - x > 4$ or $2x + 7 \geq 17$ **48.** $8x \leq 0$ and $4x + 6 \geq -10$

Solve each equation.

49. $|x - 9| = 1$ **50.** $|5x + 5| = 20$ **51.** $4|-2x| = 48$

Solve each inequality. Then graph the solution set.

52. $|x + 7| > 4$ **53.** $|2x - 5| < 21$ **54.** $3\left|\frac{1}{2}x + 1\right| \geq 12$

Lesson 2-9

Let $g(x)$ be the indicated transformation of $f(x) = |x|$. Write the rule for $g(x)$.

55. 7 units up **56.** 3 units right

Translate $f(x) = |x|$ so that the vertex is at the given point. Then graph.

57. $(1, 4)$ **58.** $(-3, 2)$ **59.** $(1.5, -2.5)$

Perform each transformation. Then graph.

60. Stretch $f(x) = |x - 1|$ vertically by a factor of 2.

61. Reflect $f(x) = |x + 4| - 1$ across the y-axis.

Lesson 3-1

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.

1. $(2, 8)$ $\begin{cases} y - 2x = 4 \\ 2y + x = -8 \end{cases}$

2. $(4, 13)$ $\begin{cases} y - 3x = 1 \\ 4x - y = 3 \end{cases}$

3. $(3, 2.5)$ $\begin{cases} 2y - x = 2 \\ 3x - 2y = 4 \end{cases}$

4. $(5, 4)$ $\begin{cases} x - y = 1 \\ x - 2y = 8 \end{cases}$

Use a graph and a table to solve each system. Check your answer.

5. $\begin{cases} 4y - x = 12 \\ 3x - 4y = -16 \end{cases}$

6. $\begin{cases} 3x + 3y = 6 \\ 2x - y = 4 \end{cases}$

7. $\begin{cases} 2y - x = 12 \\ 5x - 2y = -4 \end{cases}$

8. $\begin{cases} y - x = 4 \\ 2y + x = -4 \end{cases}$

Classify each system and determine the number of solutions.

9. $\begin{cases} y - 3x = 2 \\ 2y - 6x = 10 \end{cases}$

10. $\begin{cases} y - x = 3 \\ 3x - 4y = 10 \end{cases}$

11. $\begin{cases} 2y + 3x = 8 \\ 3x + 2y = 8 \end{cases}$

12. $\begin{cases} 4y - x = 6 \\ 2x - 8y = -12 \end{cases}$

Lesson 3-2

Use substitution to solve each system of equations.

13. $\begin{cases} x + y = 22 \\ y = x - 4 \end{cases}$

14. $\begin{cases} y = 2x + 2 \\ 3x + 2y = 18 \end{cases}$

15. $\begin{cases} 4x - y = 3 \\ 3y + 3x = 36 \end{cases}$

16. $\begin{cases} 9x - 3y = 3 \\ 2y - 4x = 16 \end{cases}$

Use elimination to solve each system of equations.

17. $\begin{cases} 3x + y = 5 \\ -2x - y = 1 \end{cases}$

18. $\begin{cases} 3x + y = 11 \\ 3y - 3x = -3 \end{cases}$

19. $\begin{cases} 2y + 5x = 7 \\ 2x - 4y = 10 \end{cases}$

20. $\begin{cases} \frac{1}{3}y + 2x = 11 \\ y - 3x = -12 \end{cases}$

Classify each system and determine the number of solutions.

21. $\begin{cases} 2y + x = 10 \\ -y + 4x = 4 \end{cases}$

22. $\begin{cases} 2y - x = 2 \\ 2x - 4y = 12 \end{cases}$

23. $\begin{cases} 4x - 8y = 16 \\ 12y - 6x = -24 \end{cases}$

24. $\begin{cases} y - \frac{2}{3}x = 3 \\ 2x - 3y = 15 \end{cases}$

Lesson 3-3

Graph each system of inequalities.

25. $\begin{cases} y \le 2x + 3 \\ y \ge x + 4 \end{cases}$

26. $\begin{cases} x + 2y \le 10 \\ -x + 2y > 12 \end{cases}$

27. $\begin{cases} y > 3x + 3 \\ 2y - 3x > 12 \end{cases}$

28. $\begin{cases} 3x + y < 4 \\ 2y - \frac{1}{2}x \ge 8 \end{cases}$

Graph each system of inequalities and classify the figure created by the solution region.

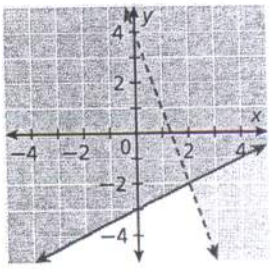
29. $\begin{cases} y \le 2x + 4 \\ y \ge 2x - 1 \\ y \le 4 \\ y \ge -1 \end{cases}$

30. $\begin{cases} y \le 3x \\ y \le -3x + 13 \\ y \ge 0 \end{cases}$

31. $\begin{cases} x \ge -1 \\ x \le 3 \\ y \ge 1 \\ y \le 8 \end{cases}$

32. $\begin{cases} y + x \le 8 \\ y - x \le 1 \\ y \le 3 \\ y \ge -1 \end{cases}$

33. Write a system of inequalities to describe the graph.



Lesson 3-4

Graph each feasible region.

34. $\begin{cases} y \geq 0 \\ x \geq 1 \\ y \leq -x + 8 \\ y \leq 3x \end{cases}$

35. $\begin{cases} y \geq -1 \\ x \geq -2 \\ y \leq -2x + 10 \\ y \leq \frac{1}{2}x + 5 \end{cases}$

36. $\begin{cases} y \geq -8 \\ x \geq -4 \\ y \leq -2x + 1 \\ y \leq -\frac{1}{4}x - 6 \end{cases}$

37. $\begin{cases} y \geq -3 \\ x \leq 1 \\ y \leq x + 8 \\ y \geq -x - 6 \end{cases}$

Maximize or minimize each objective function.

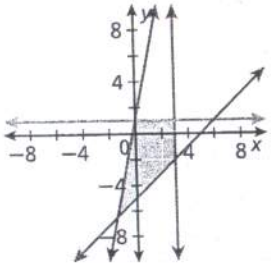
38. Maximize $P = 5x + 3y$ for the constraints from Exercise 34.

39. Maximize $P = 1.2x + 9.5y$ for the constraints from Exercise 35.

40. Minimize $P = 11x - 2.5y$ for the constraints from Exercise 36.

41. Minimize $P = 8x + 24y$ for the constraints from Exercise 37.

42. Maximize $P = 5.5x + 9y$ for the constraints shown on the coordinate grid below.



Lesson 3-5

Graph each point in three-dimensional space.

43. $(0, 4, -2)$ **44.** $(1, 3, 3)$ **45.** $(2, -3, -5)$ **46.** $(-3, -1, 4)$

Graph each linear equation in three-dimensional space.

47. $2x + 2y + z = 10$

48. $3x - 2y + 2z = 6$

49. $6x + 4y + 3z = 12$

50. $\frac{1}{2}x + 4y - z = 4$

Lesson 3-6

Use substitution or elimination to solve each system of equations.

51. $\begin{cases} 3x + y - z = 2 \\ 5x + 3y + 4z = -5 \\ -2x + y + 8z = -12 \end{cases}$

52. $\begin{cases} 2x + 2y - z = 16 \\ 4x - 2y + 2z = 0 \\ -3x - y + 3z = -19 \end{cases}$

53. $\begin{cases} 3x + 4y + 2z = 1 \\ -x + y - 4z = -17 \\ 2x + 8y + 4z = 14 \end{cases}$

54. $\begin{cases} -x + 3y + 3z = 11 \\ -3x + 5y - 7z = 1 \\ 4x - 2y + 3z = 11 \end{cases}$

Classify each system as consistent or inconsistent, and determine the number of solutions.

55. $\begin{cases} 3x + 3y - z = -3 \\ 5x + y + 2z = 14 \\ -4x + 2y + z = -9 \end{cases}$

56. $\begin{cases} 2x + 3y + z = 12 \\ 2x + 3y + z = -8 \\ 4x - y - 4z = 15 \end{cases}$

57. $\begin{cases} 8x - 4y - 16z = 12 \\ -2x + y + 4z = -3 \\ 3x - 2z + 9z = 18 \end{cases}$

Lesson 4-1

Use the following matrices for Exercises 1–4. Add or subtract, if possible.

$$A = \begin{bmatrix} 1 & 3 & 6 \\ 2 & -5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1.2 & 3.5 & 4 \\ 2.2 & 2.7 & -0.5 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & 3 & 9 \\ 4 & -5 & -2.2 \\ 2 & 1 & 12 \end{bmatrix}$$

1. $A + B$ **2.** $A + C$ **3.** $B - A$ **4.** $C - B$

Use the following matrices for Exercises 5–8. Evaluate, if possible.

$$A = \begin{bmatrix} 4 & 7 & 3 \\ 2 & 12 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 10 & -9 \\ 2 & 0 & -6 \end{bmatrix} \qquad C = \begin{bmatrix} 16 & 8 \\ -3 & 2 \\ 21 & 0 \end{bmatrix}$$

5. $4A$ **6.** $-2C$ **7.** $\frac{1}{2}A + B$ **8.** $2C - A$

Lesson 4-2

Tell whether each product is defined. If so, give its dimensions.

9. $A_{2 \times 4}$ and $B_{4 \times 5}$; AB **10.** $C_{3\times3}$ and $D_{2\times3}$; CD **11.** $E_{4\times7}$ and $F_{7\times6}$; EF

Use the following matrices for Exercises 12–15. Find each product, if possible.

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 7 & 1 \\ 10 & 4 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 12 & 0 \\ -4 & 4 \\ 5 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 5 & 1 & -3 \end{bmatrix}$$

12. AD **13.** BC **14.** ED **15.** CB

Use the following matrices for Exercises 16–19. Evaluate, if possible.

$$A = \begin{bmatrix} 9 & 6 \\ 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & 10 \\ -3 & -1 & 6 \\ 2 & 3 & 6 \end{bmatrix} \qquad C = \begin{bmatrix} 12 & 0 & 5 \\ -5 & 7 & 8 \end{bmatrix}$$

16. A^2 **17.** A^3 **18.** B^2 **19.** C^2

Lesson 4-3

Translate the polygon with coordinates $M(3, 0)$, $N(2, 4)$, $O(-1, 3)$, and $P(-2, -1)$ as indicated. Find the coordinates of the vertices of the image, and graph.

20. 3 units right and 2 units down **21.** 1 unit left and 4 units up

Use a matrix to reduce or enlarge the polygon with coordinates $M(3, 0)$, $N(2, 4)$, $O(-1, 3)$, and $P(-2, -1)$ by the given factor. Find the coordinates of the vertices of the image, and graph.

22. Reduce polygon *MNOP* by a factor of 0.25.

23. Enlarge polygon *MNOP* by a factor of 3.

Reflect the figure with coordinates $A(-1, 1)$, $B(1, -3)$, $C(5, -1)$, and $D(2, 4)$ across the given line. Find the coordinates of the vertices of the image, and graph.

24. Reflect *ABCD* across the x-axis.

25. Reflect *ABCD* across the y-axis.

Use each matrix to rotate the figure with coordinates $E(2, 2)$, $F(4, 0)$, $G(-3, -3)$, and $H(-2, 3)$ about the origin. Graph and describe the image.

26. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

27. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Lesson 4-4

Find the determinant of each matrix.

28. $\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

29. $\begin{bmatrix} 0.75 & 3 \\ 1.5 & 4 \end{bmatrix}$

30. $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{2}{3} & 8 \end{bmatrix}$

31. $\begin{bmatrix} 10 & -5 \\ 12 & \frac{1}{2} \end{bmatrix}$

Use Cramer's rule to solve each system of equations.

32. $\begin{cases} 3x + 2y = 1 \\ -4x + 5y = -32 \end{cases}$

33. $\begin{cases} x + 4y = 15 \\ 3x - 10 = 2y \end{cases}$

34. $\begin{cases} 10x + 23 = 7y \\ 2y - 10 = 4x \end{cases}$

35. $\begin{cases} \frac{1}{2}x + \frac{3}{2}y = -1 \\ \frac{1}{4}x + 1 + y = 0 \end{cases}$

Find the determinant of each matrix.

36. $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & 4 \\ 5 & 0 & 9 \end{bmatrix}$

37. $\begin{bmatrix} 3 & -6 & -1 \\ 2 & 2 & 2 \\ 7 & 1 & -3 \end{bmatrix}$

38. $\begin{bmatrix} 9 & 3 & 0 \\ 5 & -5 & 1 \\ 2 & 3 & -2 \end{bmatrix}$

Lesson 4-5

Determine whether the given matrices are inverses.

39. $\begin{bmatrix} 1 & -8 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0.2 & 0.4 \\ -0.1 & 0.05 \end{bmatrix}$

40. $\begin{bmatrix} 9 & 3 \\ -6 & -6 \end{bmatrix} \begin{bmatrix} \frac{1}{6} & \frac{1}{12} \\ -\frac{1}{6} & -\frac{1}{4} \end{bmatrix}$

41. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$

42. $\begin{bmatrix} 14 & 7 \\ 20 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find the inverse of the matrix, if it is defined.

43. $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

44. $\begin{bmatrix} 3 & 6 \\ -9 & -6 \end{bmatrix}$

45. $\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$

46. $\begin{bmatrix} 1 & -4 \\ -\frac{1}{2} & 3 \end{bmatrix}$

Write the matrix equation for the system and solve.

47. $\begin{cases} 4x + 2y = 12 \\ 6x - y = -2 \end{cases}$

48. $\begin{cases} \frac{1}{3}x + 2y = -3 \\ y - 4 = -2x \end{cases}$

49. $\begin{cases} 3x + 3y = 12 \\ 2x + 9.5 = 5y \end{cases}$

Lesson 4-6

Write the augmented matrix for each system of equations.

50. $\begin{cases} 2x + 8 = 5y \\ 3y - 7 = 12x \end{cases}$

51. $\begin{cases} 9 - y = 2x \\ 3y = 18 \end{cases}$

52. $\begin{cases} 2x + 9y = 10 \\ 3x - z = 8 \\ 5z + 5 = 13y \end{cases}$

53. $\begin{cases} 4 - 5y = 8x \\ 13x + 12 = z \\ 4z - 2y = 0 \end{cases}$

Write the augmented matrix and use row reduction to solve.

54. $\begin{cases} 4x - 2y = 26 \\ x + 6y = -13 \end{cases}$

55. $\begin{cases} 8x - \frac{1}{2} = -3y \\ 4y - 8 = 4x \end{cases}$

56. $\begin{cases} 6x + \frac{1}{2}y = 6 \\ y + 14x = 12 \end{cases}$

57. $\begin{cases} 12x + y = -6 \\ 2y - 2x = 14 \end{cases}$

Lesson 5-1

Graph each function by using a table.

1. $f(x) = \frac{1}{2}x^2 - 4$
2. $f(x) = 2x^2 - x + 3$
3. $f(x) = -x^2 - 3x$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

4. $g(x) = (x + 2)^2 + 1$
5. $g(x) = -2x^2$
6. $g(x) = \frac{1}{4}x^2$

Use the description to write each quadratic function in vertex form.

7. The parent function $f(x) = x^2$ is vertically stretched by a factor of 3 and translated 6 units right to create g.
8. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 12 units down to create g.

Lesson 5-2

Identify the axis of symmetry for the graph of each function.

9. $f(x) = 2x^2 + 1$
10. $f(x) = (x + 3)^2 - 5$
11. $f(x) = 3(x - 2)^2$

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

12. $f(x) = 2x^2 - 4x + 5$
13. $f(x) = -\frac{1}{2}x^2 - 2x + 3$
14. $f(x) = -x^2 - 8x - 6$

Find the minimum or maximum value of each function. Then state the domain and range of the function.

15. $f(x) = 3x^2 + 60x + 294$
16. $f(x) = -2x^2 + 28x - 95$
17. $f(x) = 2x^2 + 14x + 30$

Lesson 5-3

Find the zeros of each function by using a graph and a table.

18. $f(x) = x^2 + 5x + 6$
19. $f(x) = x^2 - 3x - 28$
20. $f(x) = -x^2 + 12x - 20$

Find the zeros of each function by factoring.

21. $f(x) = x^2 + 2x - 35$
22. $f(x) = x^2 - 8x - 9$
23. $f(x) = 2x^2 - 9x$
24. $f(x) = x^2 + 10x + 25$
25. $f(x) = x^2 - 49$
26. $f(x) = x^2 - 12x + 36$

Write a quadratic function in standard form for each given set of zeros.

27. 5 and 8
28. −3 and 1
29. 6 and 6
30. 12 and 0

Lesson 5-4

Solve each equation.

31. $4x^2 - 10 = 90$
32. $x^2 + 8x + 16 = 10$
33. $x^2 + 4x + 4 = 8$

Complete the square for each expression. Write the resulting expression as a binomial squared.

34. $x^2 - 16x + \square$
35. $x^2 + 22x + \square$
36. $x^2 + 7x + \square$

Solve each equation by completing the square.

37. $x^2 + 8x = -10$
38. $x^2 - 12x = 13$
39. $x^2 + 20 = 10x$
40. $2x^2 + 12x = 14$
41. $3x^2 - 18 = 48x$
42. $x^2 - 5 = 2x$

Write each function in vertex form, and identify its vertex.

43. $f(x) = x^2 - 2x + 17$
44. $f(x) = x^2 + 4x - 8$
45. $f(x) = 4x^2 - 24x + 31$

Lesson 5-5

Express each number in terms of i.

46. $2\sqrt{-81}$ **47.** $-\sqrt{-144}$ **48.** $\sqrt{-128}$ **49.** $5\sqrt{-48}$

Solve each equation.

50. $169 + x^2 = 0$ **51.** $2x^2 = -200$ **52.** $x^2 = -90$

Find the zeros of each function.

53. $f(x) = x^2 + 8x + 20$ **54.** $f(x) = x^2 - 14x + 65$ **55.** $f(x) = x^2 - 2x + 46$

Find each complex conjugate.

56. $12i$ **57.** $3 - 6i$ **58.** $10i - 3$ **59.** $2\sqrt{7} - 10i$

Lesson 5-6

Find the zeros of each function by using the Quadratic Formula.

60. $f(x) = x^2 - 10x + 3$ **61.** $f(x) = 2x^2 + 5x + 1$ **62.** $f(x) = -x^2 + 8x - 3$

63. $f(x) = x^2 - 6x + 40$ **64.** $f(x) = x^2 + 7x + 13$ **65.** $f(x) = 2x^2 - 9x + 25$

Find the type and number of solutions for each equation.

66. $x^2 + 8x = -16$ **67.** $x^2 + 3 = 10x$ **68.** $5 + 2x^2 = 12x$ **69.** $4x^2 + 2x = -9$

Lesson 5-7

Graph each inequality.

70. $y \geq (x + 3)^2 + 2$ **71.** $y < 2x^2 - 4x - 1$ **72.** $y < -x^2 + 11x - 24$

Solve each inequality.

73. $x^2 + 13x + 20 < -2$ **74.** $x^2 - 11x \geq -10$ **75.** $x^2 + 6x + 3 > 10$

76. $x^2 - 2x - 20 > 28$ **77.** $2x^2 - 9x \leq 5$ **78.** $3x^2 + 1 \geq 4x$

Lesson 5-8

Determine whether each data set could represent a quadratic function. Explain.

79.

x	3	4	5	6	7
y	−2	−5	−6	−5	−2

80.

x	−2	−1	0	1	2
y	−5	2	3	4	11

81.

x	−6	−5	−4	−3	−2
y	19	10	7	10	19

Write a quadratic function that fits each set of points.

82. $(-2, 0)$, $(1, 6)$, and $(3, -10)$ **83.** $(-4, -25)$, $(0, -9)$, and $(2, 5)$

Lesson 5-9

Graph each complex number.

84. -3 **85.** $2i$ **86.** $2 + 4i$ **87.** $-3 - 3i$

Find each absolute value.

88. $|6 + 9i|$ **89.** $|-3 + 4i|$ **90.** $|-7i|$

Simplify. Write the result in the form $a + bi$.

91. $(3 + 7i) + (-2 + 3i)$ **92.** $(-9 - 4i) + (5 + i)$ **93.** $(10 + 6i) - (3i - 12)$

94. $-3i(9 - 2i)$ **95.** $(2 - i)(4 + 3i)$ **96.** $(6 + 4i)(4 - 5i)$

97. $\dfrac{11 + 3i}{2 + i}$ **98.** $\dfrac{-44 - 40i}{-8 + 2i}$ **99.** $\dfrac{5 + 12i}{3 + 2i}$

Lesson 6-1

Identify the degree of each monomial.

1. $7x^2$ 2. $-12x$ 3. $2x^3y^3$ 4. 8

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

5. $5x^2 + 6 + 9x - 10x^3$ 6. $3 - 12x^4 - 6x^2$ 7. $14x + 15x^5$

Add or subtract. Write your answer in standard form.

8. $(12x^2 + 4x - 9) + (3x^3 - 7x^2 - 1)$ 9. $(34 + 8x^3 - 9x^2) - (3x^3 + 10x^2 - 4x - 4)$

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros.

10. $f(x) = 5x^3 + 4x - 6$ 11. $g(x) = 2x^4 - 12x + 3$ 12. $h(x) = 3x^3 - 4x + 1$

Lesson 6-2

Find each product.

13. $3ab(2a^2 - 5ab + 9b)$ 14. $-5cd^3(8d + 3c - c^2d)$ 15. $(x + 3)(2x^2 - x + 6)$
16. $(2x - 1)(-x^2 + 5x + 5)$ 17. $(2x + 6)^3$ 18. $(y - 2)^4$

Expand each expression.

19. $(x - y)^5$ 20. $(y + 4)^4$ 21. $(2x + y)^5$ 22. $(x - 2y)^4$

Lesson 6-3

Divide by using long division.

23. $(6x^2 + 7x - 2) \div (x + 4)$ 24. $(2x^2 - 9x + 10) \div (2x - 1)$

Divide by using synthetic division.

25. $(3x^3 + 4x^2 - 8) \div (x - 2)$ 26. $(2x^3 + 3x^2 - 6x - 4) \div (x - 1)$

Use synthetic division to evaluate the polynomial for the given value.

27. $P(x) = -2x^3 + 7x^2 - 3x - 9$ for $x = -2$ 28. $P(x) = 6x^3 - 7x^2 + 10$ for $x = 0.5$

Lesson 6-4

Determine whether the given binomial is a factor of the polynomial $P(x)$.

29. $(x + 2)$; $P(x) = 3x^3 + 11x^2 + 2x - 16$ 30. $(x - 4)$; $P(x) = 12x^3 + 9x^2 - 2x + 8$
31. $(x + 1)$; $P(x) = x^4 - 3x^3 + 10x + 4$ 32. $(x - 3)$; $P(x) = x^3 - 3x^2 - 4x + 12$

Factor each expression.

33. $2x^3 + 12x^2 - 4x - 24$ 34. $2x^3 + 5x^2 - 18x - 45$ 35. $4x^3 + 12x^2 + 12x + 36$
36. $a^3 + 27$ 37. $128b - 2b^4$ 38. $4c^5 + 32c^2$

Lesson 6-5

Solve each polynomial equation by factoring.

39. $2x^3 + 3x^2 - 8x - 12 = 0$ 40. $-3x^3 + 30x^2 + 5x - 50 = 0$

Identify the roots of each equation. State the multiplicity of each root.

41. $x^3 + 15x^2 + 75x + 125 = 0$ 42. $x^3 - 2x^2 - 32x + 96 = 0$
43. $8x^3 - 12x^2 + 6x - 1 = 0$ 44. $4x^3 + 16x^2 - 25x - 100 = 0$

Identify all of the real roots of each equation.

45. $2x^4 - x^3 - 14x^2 - 5x + 6 = 0$ 46. $6x^3 - 11x^2 - 19x - 6 = 0$

Fernando Trujano

Review for Final

CHAPTER 1

1. 2.3, $2.\overline{4}$, $5/2$, $\sqrt{15}$, $2\sqrt{3}$
   Real Rational | Real Rational | Real Rational | Real Irrational | Real Irrational

2. -2π, $-\sqrt{12}$, π, -3, $2/5$
   Real Irrational | Real Irrational | Real Irrational | Real Rational | Real Rational

3. $\sqrt{9}$, $3.0\overline{2}$, $3\frac{1}{16}$, $3\frac{2}{30}$, π
   Real Rational | Real Rational | Real Rational | Real Rational | Real Irrational

4. $[-40, -12)$

5. $(-1, 5)$ or $[13, \infty)$

6. $[-10, \infty)$

7. $\{x \mid x \le 0 \text{ or } 4 < x < 8\}$

8. $\{1, 3, 5, 7, 9\}$

9. Additive Identity Property

10. Associative Property

11. Distributive Property

12. $34.6 \cdot .15 = \$5.19$

13. $67.8 \cdot .30 = 20.34$ $\quad 67.8 - 20.34 = \$47.46$

14. sometimes

15. always

16. sometimes

17. 9.5

18. 7.8

19. -6.9

20. 4.8

21. $\frac{\sqrt{242}}{\sqrt{2}} = \sqrt{121} = 11$

22. $\frac{\sqrt{20}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{24}}{12} = \frac{\sqrt{6}}{4}$

23. $2\sqrt{8} + 4\sqrt{32} = 2\sqrt{8} + 4\sqrt{4 \cdot 8} = 2\sqrt{8} + 8\sqrt{8} = 10\sqrt{8}$

24. $2\sqrt{18} + 4 - \sqrt{18} = 4\sqrt{18} - \sqrt{18} = 3\sqrt{18}$

25. $2^{12}b - 6b^2$

26. $x = 50 + b$

27 $32+20+9=43$

28 $8 \cdot 32 = 4 \cdot 2 \cdot 8 = 1/4$

29 $8x - 24 + 4 = 8x - 4$

30 $3x - 21 + 4y^2 = 4y^2 + 3x - 21$

31 $-4a + 24 - 8a = -12a + 24$

32 1

33 $3^{-3} = 1/3^3 = 1/27$

34 $81x^8y^4$

35 $25x^2 5y^4 = 125x^2y^4$

36 $3.08 \cdot 10^{15}$

37 $4.5 \cdot 10^3$

38 $4 \cdot 10^{-9}$

39 Domain $\{-2, 2, 3\}$ Range $\{-2, 2, 3\}$

40 Domain $\{-5, -2, 0, 4\}$ Range $\{-10, -2, 5, 19\}$

41 Domain $\{2, 3, 6, 7\}$ Range $\{1, 0, 4\}$

42 YES

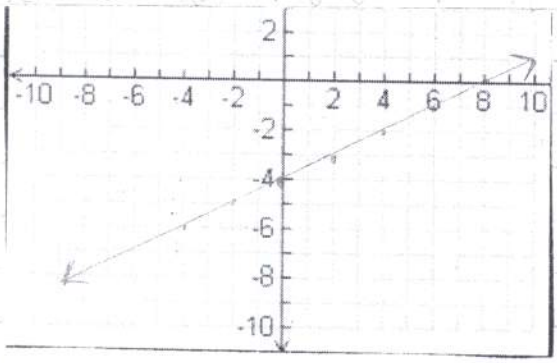
43 NO

44 $f(0) = 10$ $f(3) = -4(3) + 10 = -2$ $f(-2) = 8 + 10 = 18$

45 $f(0) = 0$ $f(3) = 4.5$ $f(-2) = 2$

46 $f(0) = 5$ $f(3) = 9 - 18 + 5 = -4$ $f(-2) = 4 + 4 + 5 = 13$

47



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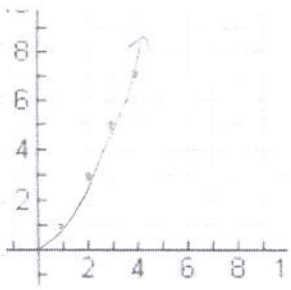

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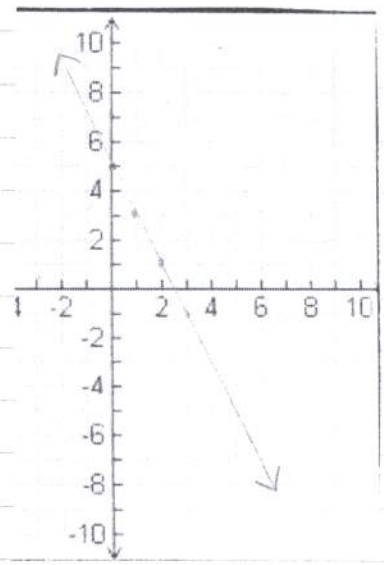


50 (0,4)

51 (-3,9)

52 (-1,2)

53 (0,4) (2,0) (4,2) (-4,2) ORIGINAL y=f(-x) (0,4) (-2,0) (-4,2) (4,2)

54 (0,6) (2,2) (4,4) (-4,4) y=f(x)+2

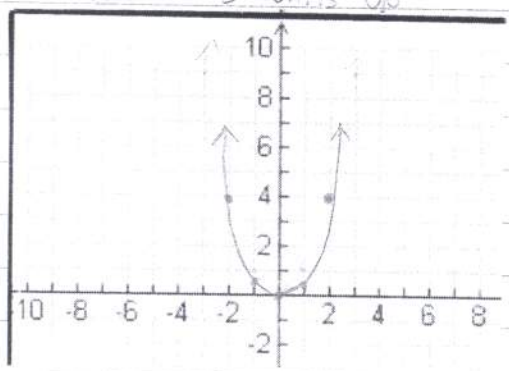
55 (0,2) (2,0) (4,1) (-4,1) y=½f(x) +y changes

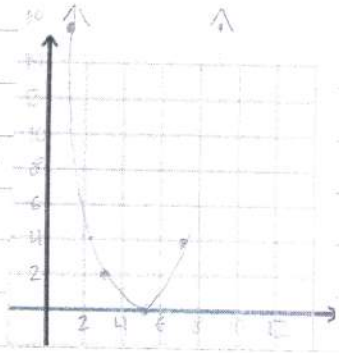
56 $y=x^3$ translated 3 units left

57 $y=\sqrt{x}$ translated 4 units right

58 $y=x^2$ translated 3 units up

59 $y=x^2$





60 $y=x^2$

translated 3 units right

CHAPTER 2

1 $4x-12=48$
$+12 \quad +12$
$4x=60$
$x=15$

2 $8x=16$
$x=2$

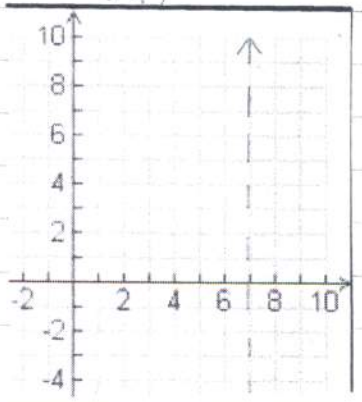
3 $5a+6=a-6$
$-a \quad -6 \quad -a \quad -6$
$4a=-12$
$a=-3$

4 $3z+12=2z+2$
$-2z \quad -12 \quad -2z \quad -12$
$z=-10$

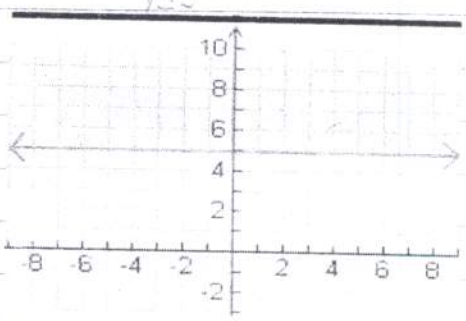
5 $18w=54$
$w=3$

6 $-5p+32=2p-4$
$+5p \quad -4 \quad +5p \quad +4$
$36=7p$
$p=36/7$

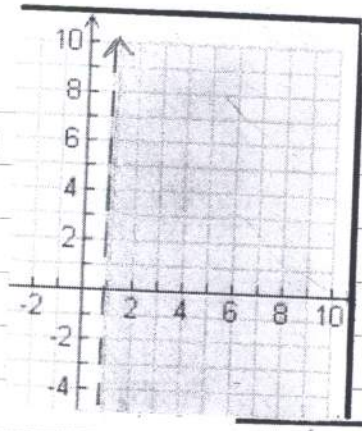
7 $3x+7<28$
$-7 \quad -7$
$3x<21$
$x<7$



8 $12y-3\leq 57$
$+3 \quad +3$
$12y\leq 60$
$y\leq 5$



9 $8 - 2x < 10$
$-8 \quad -8$
$-2x < 2$
$x > 1$



10 $15x = 45$
$x = 3$

11 $88 = 10x$
$x = 8.8$

12 $-4y = 70$
$y = -17.5$

13 $11 = 3n$
$n = 11/3$

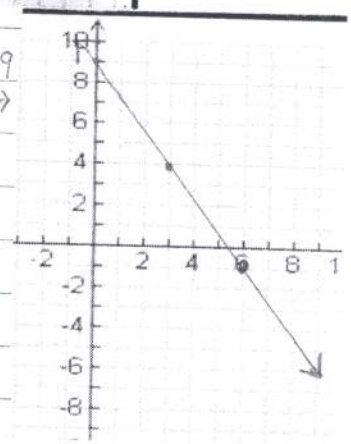
14 $57 = 18 + 3n$
$39 = 3n$
$n = 13$

15 $x - 3 = 13.5$
$x = 16.5$

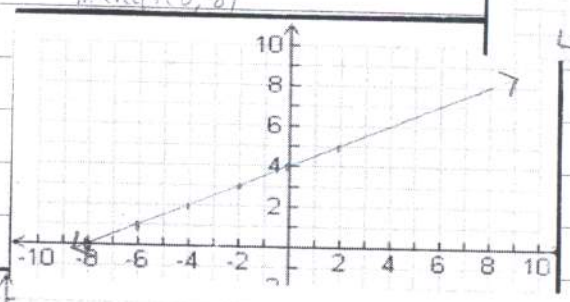
16 $+9 \quad +9 \quad +9 \quad +9$
YES

17 $-6 \quad -2 \quad +2$
NO

19



20 $-2y + x = 8$
$\frac{-2y}{-2} = -x + 8$
$y = \frac{1}{2}x - 8$
intercept $(0, -8)$



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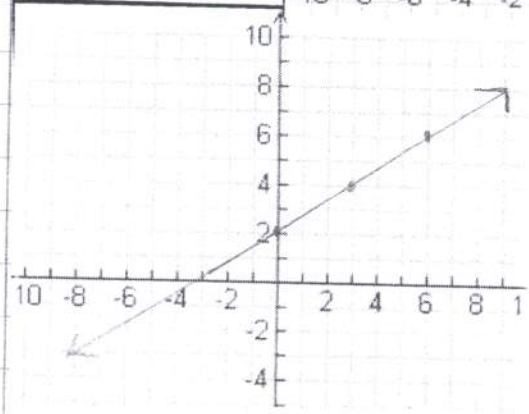
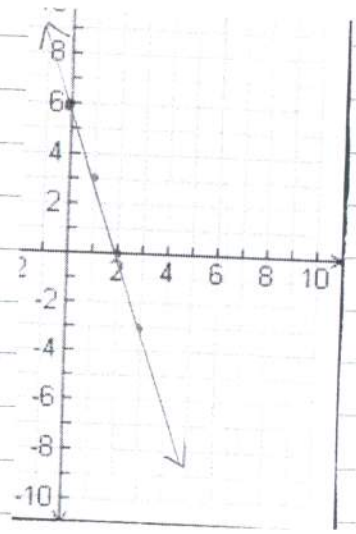
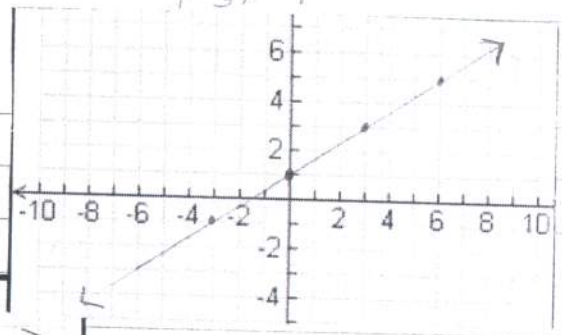


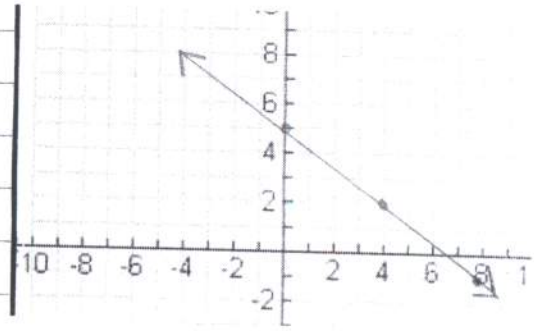
21 $y = \frac{-3}{1}x + 6$
Intercept $(0, 6)$



22 $3y = 2x + 3$
$y = \frac{2}{3}x + 1$



23 $4y = -3x + 20$
$y = -3/4x + 5$



24 $\frac{y_2-y_1}{x_2-x_1} = m = \frac{4+5}{4-1} = \frac{9}{3} = 3$

25 $\frac{-4-20}{3+1} = \frac{-24}{4} = -6$

26 $y = 3x + 4/3$

27 $y = -3/2x + 7$

28 intercept = 15 $\frac{15-14}{0+2} = \frac{1}{2}$ $y = \frac{1}{2}x + 15$

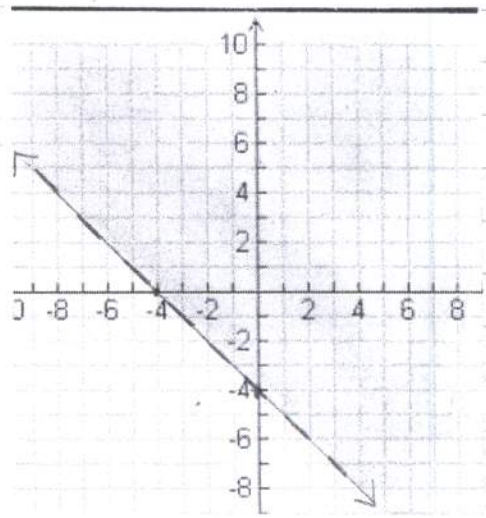
29 $\frac{0+3}{13-10} = \frac{3}{3} = 1$ $y = x - 7$

30 $y = 2x - 3$

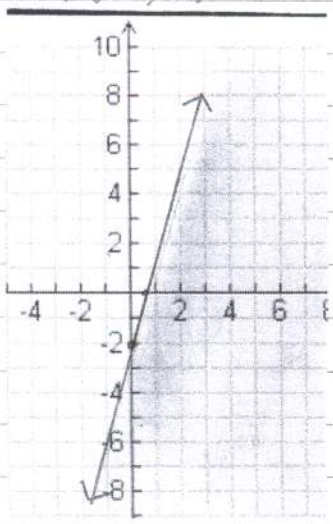
31 $y = -\frac{2}{3}x + 1$

32 $y = 4$ $x = -4$ $(-4,0)$ $(0,-4)$

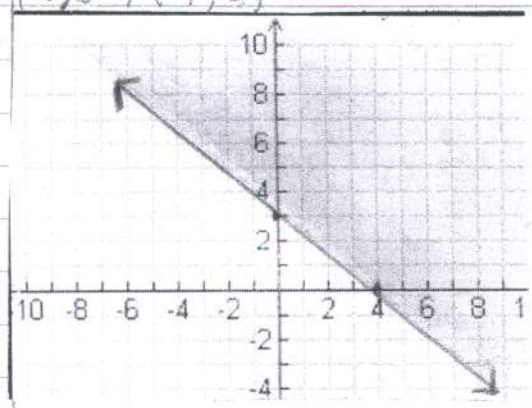
#32



33 $(0,-2)$ $(.5,0)$

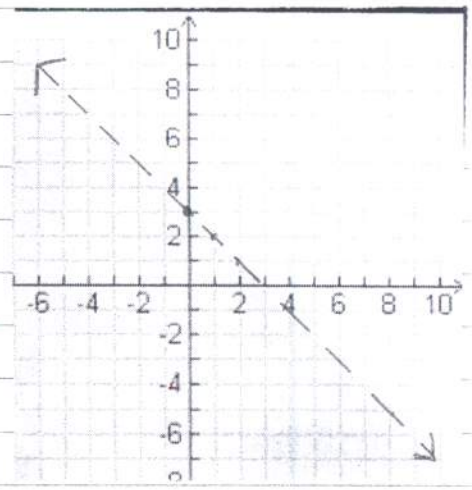


34 $(0,3)$ $(4,0)$



35 $6y < -6x + 18$

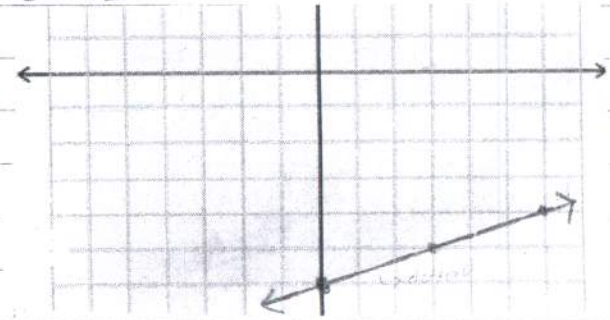
$y < -x + 3$



36 $12 > 2x - 6y$

$6y > -12 - 2x$

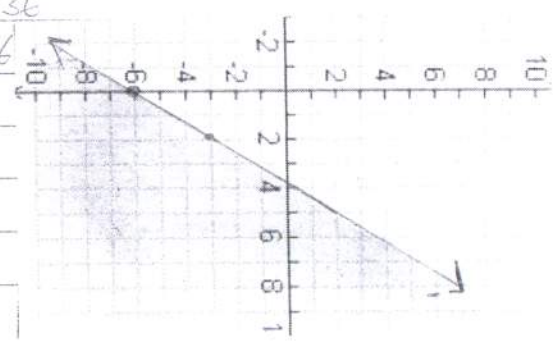
$y > \frac{1}{3}x - 12$



37

$6y \leq 9x - 36$

$y \leq \frac{3}{2}x - 6$



38 $g(x) = 11/7x - 2$

39 $g(x) = -x + 9$ 1/4 doesn't affect x

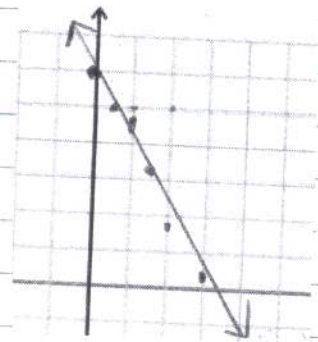
40 (1,17) (0,11) (2,-1) (5,-19)

41 $g(x) = 2(x-2)$

42 $g(x) = -(x+5)$

43 $g(x) = (2/3x) - 8$

44 1, 0

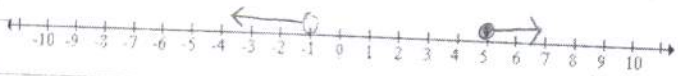


45 $y = -2.02x + 18.58$

46 $r = .9568$

47 $-x > 1$ $2x \geq 10$

$x < -1$ or $x \geq 5$



48 $x \leq 0$ and $4x \geq -16$ $x \geq -4$

49 $x - 9 = 1$ $x - 9 = 1$ $x - 9 = -1$ $x = 10$ $x = 8$

50 $5x + 5 = 20$ $5x + 5 = -20$ $x = 3$

-5 -5 -5 -5

$5x = 15$ $5x = -25$ $x = -5$

51 $-2x = 12$ $-2x = -12$

$x = -6$ $x = 6$

52 $x + 7 > 4$ $x + 7 > 4$ or $x + 7 < -4$

$x > 11$ $x < 3$

53 $2x - 5 < 21$ and $2x - 5 > -21$

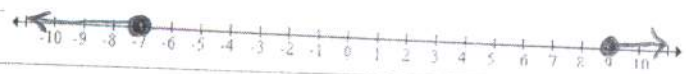
$+5$ $+5$

$2x < 26$ $2x > -16$

$x < 13$ $x > -13$

54 $1/2x + 1 \geq 4$ $1/2x + 1 \geq 4$, $1/2x + 1 \leq -4$

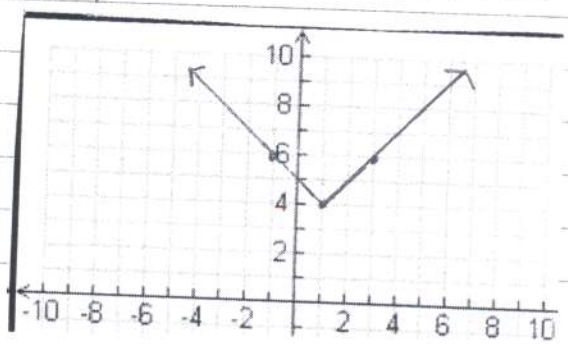
$x \geq 9$ or $x \leq -7$



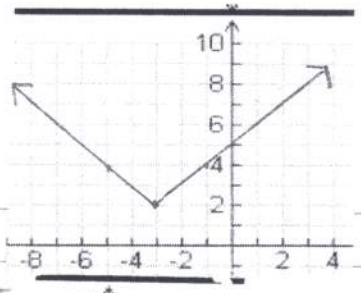
55 $g(x) = |x| + 7$

56 $g(x) = |x - 3|$

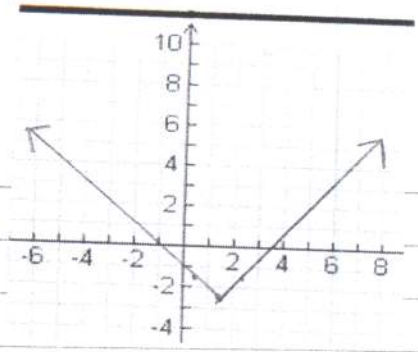
57 $g(x) = |x - 1| + 4$



58 $g(x) = |x+3| + 2$

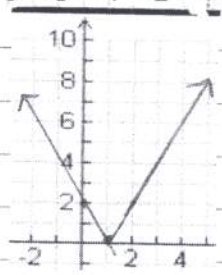


#59

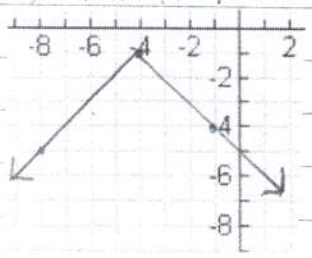


59 $g(x) = |x-1.5| - 2.5$

60 $7|x-1| = g(x)$



61 $g(x) = |-x+4| - 1$



CHAPTER 3

1 $8-4=4$ NO

$16 \div 2 = 8$ X

2 $13-12=1$ YES

$16-13=3$

3 $5-3=2$ YES

$9-5=4$

4 NO

5 $(-2, 2.5)$

6 $(2, 0)$

7 $(2, 7)$

8 $(-4, 0)$

9 $y=3x+2$ $2y=6x+10$

$y=3x+5$ Inconsistent No Solution

10 $y=x+3$ 1 solution

Consistent independent

11 Consistent dependent

Infinite Solutions

12 $y = \frac{1}{4}x + \frac{6}{4}$ Consistent dependent

infinite solutions

13 $2x=26$

$x=13$ $y=9$

14 $3x+4x+4=18$ $y=6$

$x=2$ $7x=14$ $x=2$

15 $12x-9+3x=36$ $x=3$

$x=3$ $15x=45$ $y=9$

16 $2y=4x+16$

$y=2x+8$ $9x-6x-24=3$ $x=9$

$x=9$ $3x=27$ $y=12$ $y=12$

$2y-36=-12$ $2y=24$

17 $x=6$ $y=-13$

18 $4y=8$ $y=2$ $3x=9$ $x=3$

19 $4y+10x=14$ $2y=5$

$-4y+2x=10$ $y=-2.5$

$10x=24$ $x=2.4$

20 $y+6x=33$ $-3 \cdot 3x=-12$

$2y-6x=-24$ $-3x=-15$

$3y=-9$ $y=-3$ $x=5$

21 $y=4x-4$ Consistent independent

One solution

22 Consistent independent one solution

23 $-8y=-4x+16$ $12y=6x-24$

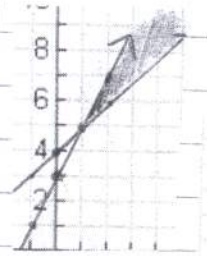
$y=\frac{1}{2}x-2$ $y=\frac{1}{2}x-2$

Consistent dependent infinite solution

24 $y = \frac{2}{3}x + 3$ $\quad -3y = -2x + 15$ $\quad$ Consistent independent one solution

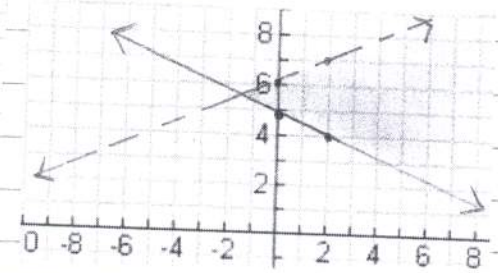
$\frac{2}{3}$ -3

25



26 $y \leq \frac{1}{2}x + 5$

$y > \frac{1}{2}x + 6$



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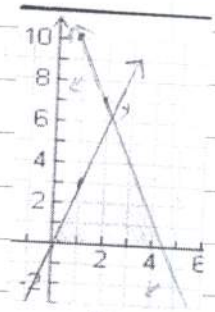
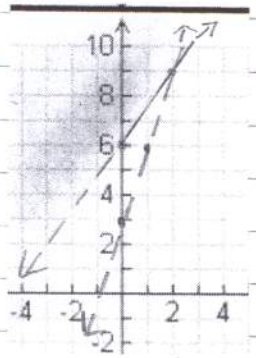


Isosceles Triangle

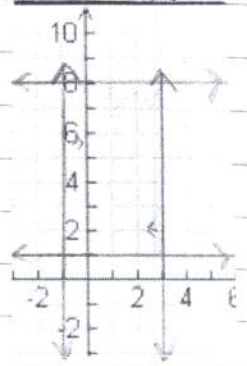
27 $y > 3x + 3$

$2y > 3x + 12$

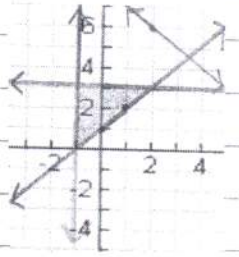
$y > \frac{3}{2}x + 6$



31 Rectangle



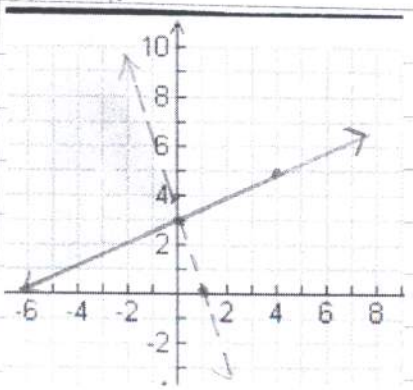
32



Right triangle

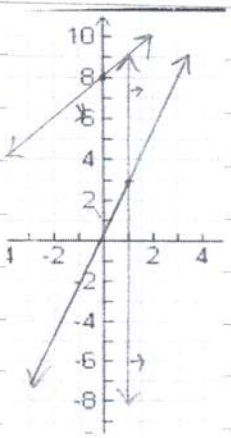
28 $y < -3x + 4$

$y \geq \frac{1}{4}x + 4$



33 $\begin{cases} y \geq \frac{1}{2}x - 3 \\ y > -3x + 4 \end{cases}$

34



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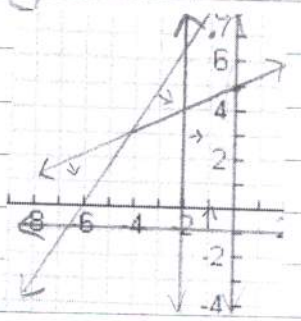
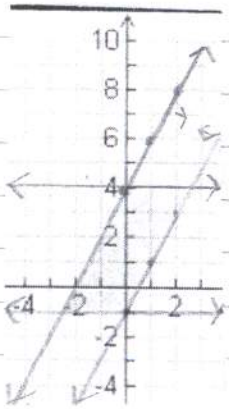


29 Parallelogram



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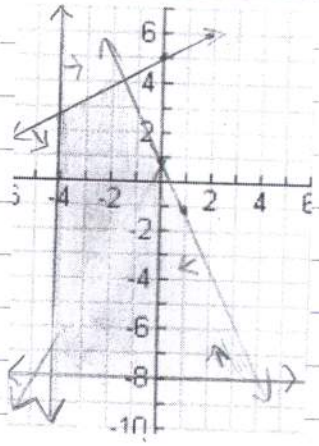

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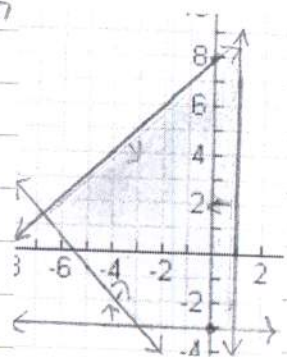

38

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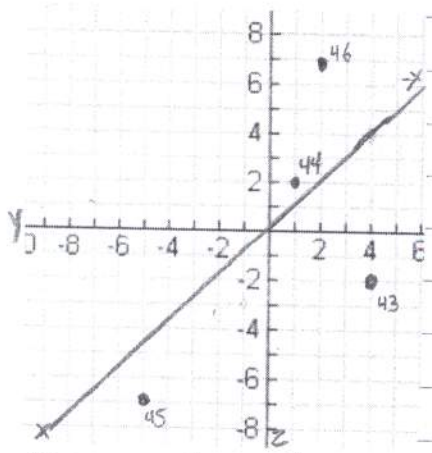
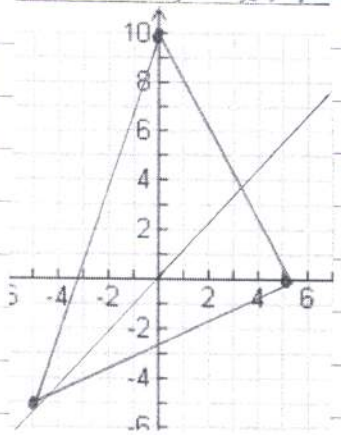
42

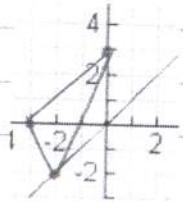
43-46



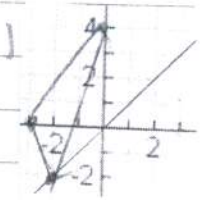
47 (5,0,0)(0,5,0)(0,0,10)



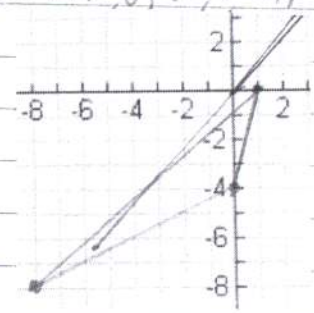
48 (2,0,0)(0,-3,0)(0,0,3)



49 (2,0,0)(0,-3,0)(0,0,4)



50 (8,0,0)(0,1,0)(0,0,-4)



51 (1,-2,-1) x=1 y=-2 z=-1

52 (3,4,-2) x=3 y=4 z=-2

53 (-3,4.5,5) x=-3 y=4 z=5

54 (4,4,1) x=4 y=4 z=1

55 consistent 1 solution (2 -2, 3)

56 inconsistent no solution

57 inconsistent no solution

Chapter 4

1 $\begin{bmatrix} 2.2 & 6.5 & 10 \\ 4.2 & -2.3 & -.5 \end{bmatrix}$

2 NO SOLUTION

3 $\begin{bmatrix} .2 & .5 & -2 \\ .2 & 7.7 & -.5 \end{bmatrix}$

4 NO SOLUTION

5 $\begin{bmatrix} 16 & 28 & 12 \\ 8 & 48 & -16 \end{bmatrix}$

6 $\begin{bmatrix} -32 & -16 \\ 6 & -4 \\ -42 & 0 \end{bmatrix}$

7 $\begin{bmatrix} 1 & 13.5 & -7.5 \\ 3 & 6 & -8 \end{bmatrix}$

8 NO SOLUTION

9 YES 2×5

10 NO SOLUTION

11 YES 4×6

12 $\begin{bmatrix} 5 & 13 \\ 40 & -16 \end{bmatrix}$

13 $\begin{bmatrix} 56 & 34 & -8 \\ 99 & 57 & -15 \end{bmatrix}$

14 $[41 \quad 1]$

15 NO SOLUTION

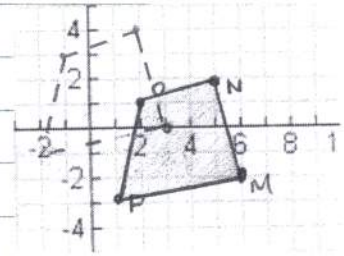
16 $\begin{bmatrix} 81 & 42 \\ 0 & 4 \end{bmatrix}$

17 $\begin{bmatrix} 729 & 407 \\ 0 & -8 \end{bmatrix}$

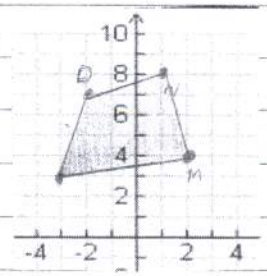
18 $\begin{bmatrix} 14 & 40 & 120 \\ 6 & 4 & 0 \\ 9 & 25 & 74 \end{bmatrix}$

19 NO SOLUTION

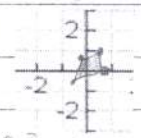
20



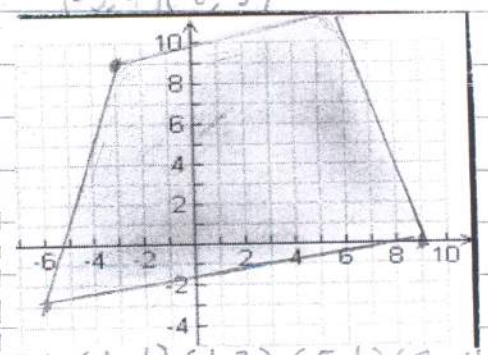
21



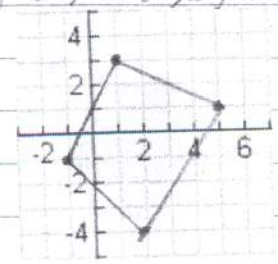
22 (.75,0) (.5,1) (-.25,.75) (-.5,-.25)



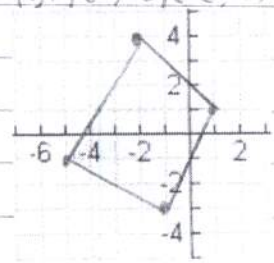
23 (9,0) (6,12) (-3,9) (-6,-3)



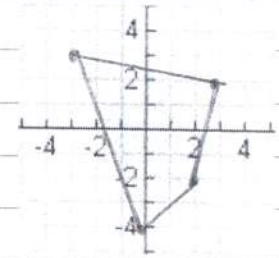
24 (-1,-1) (1,3) (5,1) (2,-4)



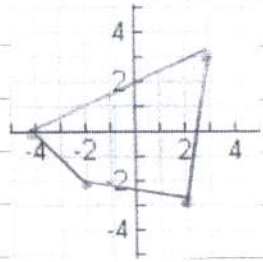
25 (-2,4) (1,1) (-1,-3) (-5,-1)



26 (2,-2) (0,-4) (-3,3) (3,2)



27 (-2,-2) (-4,0) (3,3) (2,-3)



28 6-20 = -12

29 3-4.5 = -1.5

30 2÷.33 = -1.67

31 5+60 = 65

32 (3,4) x=3 y=4

33 (5,2.3) x=5 y=2.3

34 (.5,4) x=.5 y=4

35 (4,-2) x=4 y=-2

36 59 2nd → matrix → math → det [A]

37 -132

38 99

39 NO

40 [A]x[B] =

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ YES

41 NO

42 NO

43 $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

44 $\begin{bmatrix} -1/6 & -1/6 \\ 1/4 & 1/12 \end{bmatrix}$

45 No Solution

46 $\begin{bmatrix} 3. & 4 \\ .5 & 1 \end{bmatrix}$

47 (.5, 5) x=.5 y=5

48 (15/11, 14/11)

49 (1.5, 2.5)

50 $\left[\begin{array}{cc|c} 2 & 5 & -8 \\ -12 & 3 & 7 \end{array}\right]$

51 $\left[\begin{array}{cc|c} -2 & -1 & -9 \\ 0 & 3 & 18 \end{array}\right]$

52 $\left[\begin{array}{ccc|c} 2 & 9 & 0 & 10 \\ 3 & 0 & -1 & 8 \\ 0 & -13 & 5 & -5 \end{array}\right]$

53 $\left[\begin{array}{ccc|c} -8 & -5 & 0 & -4 \\ 13 & 0 & -1 & -12 \\ 0 & -2 & 4 & 0 \end{array}\right]$

54 $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \end{bmatrix}$ = rref[A]

55 $\begin{bmatrix} 1 & 0 & 13/22 \\ 0 & 1 & -31/22 \end{bmatrix}$

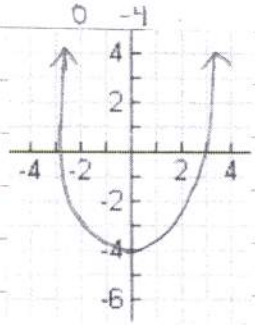
56 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 12 \end{bmatrix}$

57 $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 6 \end{bmatrix}$

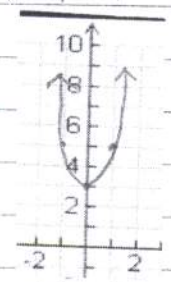
Chapter 5

1

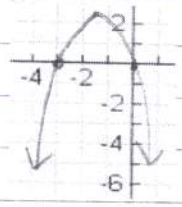
x	y
1	-3.5
2	-2
-1	-3.5
0	-4



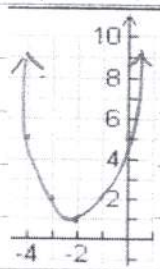
2 x=1/4 y=2.9



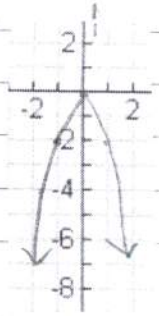
3 (0,-3) (0,0) (-1.5, 2.25)



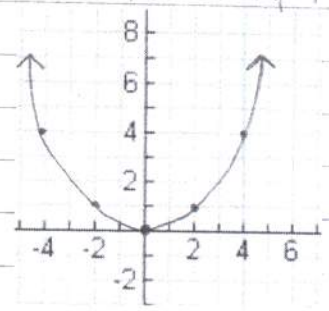
4 translated 2 units left
up



5 Horizontal comp by 1/2



6 Horizontal strech by 4

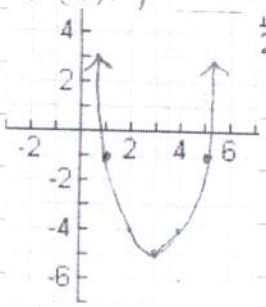


7 $3(x+6)^2$

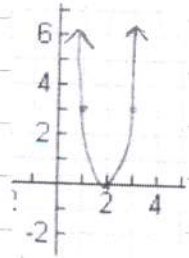
8 $-(x^2)-12$

9 x=0 y=1 (0,1)

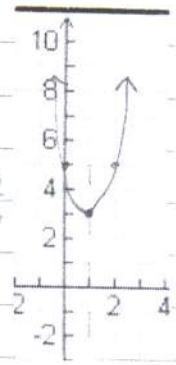
10 (-3,5)



11 (±2,0)

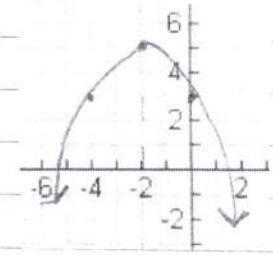


12 upward
x=1
(1,3)
(0,5)



#12

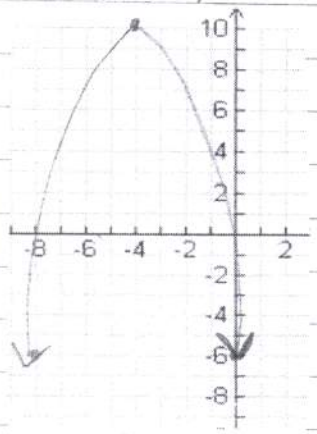
13 down
x=-2
(-2,5)
(0,3)



#9

14 down $x=-4$

$(-4,10)(0,-6)$



15 $(-10,-6)$

16 $(7,3)$

17 $(-3.5,-21.5)$

18 $x=-3$ $x=-2$ 2nd > Calc > zero

19 $x=7$ $x=-4$

20 $x=2$ $x=10$

21 $x^2+2x-35$

$(x+7)(x-5)$

$x=-7$ $x=5$

22 x^2-8x-9

$(x+1)(x-9)$

$x=-1$ $x=9$

23 $x=0$ $x=4.5$

24 $x=-5$ $(x+5)(x+5)$

25 $(x+7)(x-7)$

$x=-7$ $x=7$

26 $(-x+6)(x+6)$

$-x+6=$ $x=6$

27 $(x-3)(x-8)$

$x^2-8x-3x+24$

$x^2-11x+24$

28 $(x+3)(x-1)$

x^2+2x-3

29 $(x-6)(x-6)$

x^2+36 $x^2-12x+36$

30 $(x-12)(x+10)$

x^2-12x

31 $4x^2=100$

$x=\pm 5$

32 $(x+4)^2=10$

$x=-4\pm\sqrt{10}$

33 $(x+2)=\sqrt{8}$

$x=-2\pm 2\sqrt{2}$

34 $x^2-16x+64$

$(x+8)(x-8)$

35 $x^2+22x+121$

$(x+11)^2$

36 12.25

$(x+3.5)^2$

37 $x^2+8x+16=6$

$(x+4)^2=\pm\sqrt{6}$

$x=-4\pm\sqrt{6}$

38 $x^2-12x+36$

$(x+6)(x-6)=49$

39 $x^2-10x+25=20+25$

$(x+5)(x-5)=45$ $(x-5)^2=5$

$x=5\pm\sqrt{5}$

40 $x^2+6x+9=7$ $+9$

$(x+3)=\sqrt{16}$

$x=-3\pm 4$

41 $x-6=16x$

$x-16x+64=6$

$(x-8)^2=70$

$x=8\pm\sqrt{70}$

42 $x^2-2x+1=5$

$(x-1)^2=6$

$x=1\pm\sqrt{6}$

43 $x^2-2x+1+16$

$(x-1)^2+16$

$(1,16)$

44 $x^2+4x+4-4$

$(x+2)^2-4$

$(-2,-4)$

45 $x^2-6x+7.75$

x^2-6x+9

$(x-9)^2-1.25$

$(9,-1.25)$

46 $18i$

47 $-12i$

48 $-4i\sqrt{8}$

49 $2i$

$10i\sqrt{12}$

50 $x^2=-169$

$x=\sqrt{169}$

$x=\pm 13i$

51 $2x^2=-200$

$x^2=-100$

$x=\pm 10i$

52 $x^2=\pm 3i\sqrt{10}$

$x=\pm 3i\sqrt{10}$

53 $x^2+8x+20$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$-8\pm\sqrt{64-(4)(20)(1)}$

$64-80$

$-8\pm\sqrt{16}$

$\dfrac{-8\pm 4i}{2}$ $\boxed{-4\pm 2i}$

54 $x = 14 \pm \sqrt{196 - (4)(65)(1)}$

$x = 14 \pm \sqrt{-64}$

$\frac{14 \pm 8i}{2}$

$x = 7 \pm 4i$

55 $2 \pm \sqrt{4 - (4)(1)(46)}$

$2 \sqrt{-180}$

$1 \pm \sqrt{-90}$

$1 \pm 3i\sqrt{10}$

56 $0 + 12i$

$-12i$

57 $3 + 6i$

58 $-3 - 10i$

59 $2\sqrt{7} + 10i$

60 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{10 \pm \sqrt{100 - 4(3)(1)}}{2}$

$5 \pm \sqrt{88}$

$x = 5 \pm 2\sqrt{22}$

61 $\frac{-5 \pm \sqrt{25 - (4)(2)(1)}}{4}$

$\frac{-5 \pm \sqrt{17}}{4}$

62 $\frac{-8 \pm \sqrt{64 - 4(-1)(-3)}}{-2}$

$\frac{-8 \pm \sqrt{52}}{-2}$

63 $\frac{6 \pm \sqrt{36 - 4(40)(1)}}{2}$

$\frac{6 \pm \sqrt{-124}}{2}$

64 $x = -7 \pm \sqrt{49 - (4)(13)}$

$49 - 52$

$\frac{-2\sqrt{-3}}{2}$

65 $\frac{9 \pm \sqrt{81 - (4)(2)(25)}}{4}$

$9 \pm \sqrt{-119}$

$\frac{9 \pm i\sqrt{119}}{4}$

66 $b^2 - 4ac$

$64 - (4)(1)(16)$

0

one distinct real solution

67 $100 - (4)(3)$

2 distinct real solutions

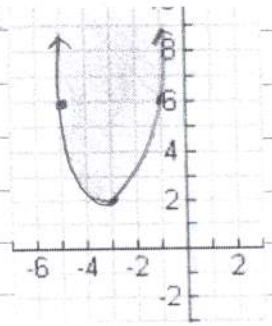
68 $144 - (4)(5)(2)$

2 distinct real

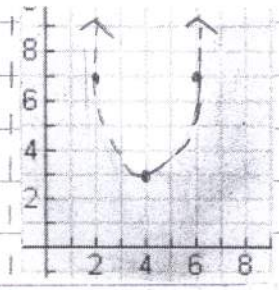
69 $4 - (4)(4)(-9)$

2 distinct real solutions

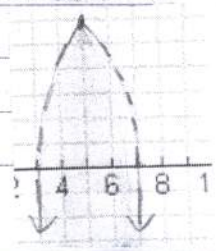
70



71 $(x-4)^2 + 3$



72



73 $-11 < x < -2$

74 $x < 1$ or $x > 10$

75 $x > 1$ or $x < -7$

76 $x < -6$ or $x > 8$

77 5 .5

$.5 < x < 5$

78 1 .333

$x \leq .33$ or $x \geq 1$

79 YES second differences =

80 NO 2nd dif ≠

81 YES 2nd dif =

82 $y = -2x + 8$

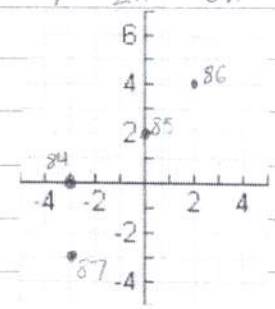
83 $y = \frac{1}{2}x^2 + 6x - 9$

84

85

86

87



88 $\sqrt{36 + 81} = \sqrt{117}$

89 $\sqrt{9 + 16} = \sqrt{27} = 3\sqrt{3}$

90 $= 7$

91 $1 + 10i$

92 $-4 - 3i$

93 $22 + 3i$

94 $-27i + 6i^2 = -6 - 27i$

95 $8 + 6, 4i, -4i^2$

$8 + 2i + 4$ $12 + 2i$

96 $24 - 30i + 16i + 20$

$44 - 14i$

97 $\frac{11 + 3i}{2 + i} \frac{(2 - i)}{(2 - i)}$ $\frac{22 - 11i + 6i + 3}{4 - 2i + 2i - i^2}$ $\frac{25 - 5i}{5} = 5$

98 $\frac{-44-46i}{-8+2i}$ $\quad \frac{-8-2i}{-8-2i}$ $\quad 352-88i+320i-80$ $\quad 64+4=68$ $\quad \frac{272+232i}{68}$ $\quad 5.5+5i$

99 $\frac{(5+12i)}{3+2i}$ $\quad \frac{(3-2i)}{3-2i}$ $\quad 15-10i+36i+24i^2$ $\quad 9+4=13$ $\quad \frac{39+26i}{13}$ $\quad 3+2i$

Chapter 6

1 2

2 1

3 6

4 0

5 $-10x^3+5x^2+9x+6$
LC = -10
Cubic Polynomial

6 $-12x^4-6x^2+3$
LC = -12
Quartic Trinomial

7 $15x^5+14x$
LC = 15
Quintic Bynomial

8 $3x^3+5x^2+4x-10$

9 $5x^3-19x^2+4x+38$

10 From ← to → increases sharply, then increases then increases sharply
Appears to be 1 zero

11 From ← to → increases then decreases.
Appears to have 2 real zeros

12 From left to → increases then decreases then increases.
Appears to have 3 real zeroes.

13 $6a^3b-15a^2b^2+27ab^2$

14 $-40cd^4-15c^2d^3+5c^3d^4$

15 $2x^3-x^2+6x+6x^2-3x+18$
$2x^3+5x^2+3x-18$

16 $-2x^3+10x^2+10x+x^2-5x-5$
$-2x^3+11x+5x-5$

17 $1(2x)^3+3(2x)^2(6)+3(2x)(6)^2+1(6)^3$
$8x^3+72x^2+216x+216$

	1			0	1
	1	1		1	1
	1	2	1	2	1
	1	3	3 1	3	1
	1	4	6 4 1	4	1
	1	5	10 10 5 1	5	1

18 $1y^4 \quad 4y^3 2 \quad 6y^2(2)^2 \quad 4y(2)^3 \quad 1(2)^4$
$y^4-8y^3+24y^2-16y+16$

19 $1x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-1y^5$

20 $1y^4+4y^3 4+6y^2(4)^2+4y(4)^3+1(4)^4$
$y^4+16y^3+96y^2+256y+256$

21 $1(2x)^5+5(2x)^4y+10(2x)^3y^2+10(2x)^2y^3+5(2x)y^4+1y^5$
$32x^5+80x^4y+80x^3y^2+40x^2y^3+10xy^4+y^5$

22 $1x^4-4x^3 2y+6x^2(2y)^2-4x(2y)^3+1(2y)^4$
$x^4-8x^3y+24x^2y^2-32xy^3+16y$

23 $6x-17+66/x+4$
$x+4 \overline{\big)\, 6x^2+7x-2}$
$-6x^2-24x$
$-17x-2$
$+17x+68$
66

24 $x-4x+6/2x-1$
$2x-1 \overline{\big)\, 2x^2-9x+10}$
$2x^2+x$
$-8x+10$
$-8x+4$
6

25 $2\,\underline{)}$

3	4	0	-8
↓	6	20	40
3	10	20	32

$3x^2+10x+20 \quad R=32$

26 $1\,\underline{)}$

2	3	-6	-4
↓	2	5	-1
2	5	-1	-5

$2x^2+5x-1 \quad R=-5$

27 $-2\,\underline{)}$

-2	7	-3	-9
↓	4	-22	50
-2	11	-25	42

28 $.5\,\underline{)}$

6	-7	0	10
↓	3	-2	-1
6	-4	-2	9

29 $-2 \mid$ 3 11 2 -16
↓ -6 -10 16
YES 3 5 -8 0

30 $4 \mid$ 12 9 -2 8
↓ 24 132 520
NO 12 33 130 528

31 $-1 \mid$ 1 -3 0 10 4
↓ -1 4 -4 -6
NO 1 -4 4 6 -2

32 $3 \mid$ 1 -3 -4 12
↓ 3 0 -12
YES 1 0 -4 0

33 $2x^3+12x^2-4x-24$
$2x^2(x+6)-4(x+6)$
$(x+6)(2x^2-4)$
$(x+6)(2x^2-4)$

34 $2x^3+5x^2-18x-45$
$x^2(2x+5)-9(2x+5)$
$(2x+5)(x^2-9)$
$(2x+5)(x+3)(x-3)$

35 $4x^3+12x^2+12x+36$
$4x^2(x+3)+12(x+3)$
$(x+3)(4x^2+12)$
$(x+3)(4x^2+12)$

36 a^3+27
$(a+3)(a^2-3a+27)$
$(a+3)(a+3)(a-9)$

37 $(b^2+8)(b^2+8)$

38 $c^2(4c^3+32)$
$c^2(c^3+8)$
$4c^2(c+2)(c^2-2c+4)$

39 $2x^3+3x^2-8x-12$
$x^2(2x+3)-4(2x+3)$
$(2x^2+3)(x-4)$
$(x-4)$
$(2x^2+3)$ $x=4$

y

40 $-3x^3+30x^2+5x-50$
$-3x^2(x-10x)+5(x-10)$
$(x-10)(-3x^2+5)$
$x=10$ $(-3x^2+5)$

45 $2x^4-x^3-14x^2-5x+6$
$x=-2$ $x=-1$ $x=3$ $x=.5$

46 $x=3$ $6x^3-11x^2-19x-6$

41 $x=-5$ $x^3+15x^2+75x+125$

42 $x=4$ $x^3-2x^2-32x+96$

43 $x=0$ $8x^3-12x^2+6x-1$

44 $x=-4$ $4x^3+16x^2-25x-100$

45 $2x^4-x^3-14x^2-5x+6$
$x=-2$ $x=-1$ $x=3$ $x=.5$
46 $x=3$ $6x^3-11x^2-19x-6$

Solving Quadratic Equations by Using Square Roots

$x^2 = a$

If $x^2 = a$ and "a" is a positive real number then $x = \pm\sqrt{a}$

$\sqrt{x}^2 = \sqrt{144}$

$x = \pm 12$

Solve using square roots.

$x^2 = 169$

$\sqrt{x}^2 = \sqrt{169}$

$x = \pm 13$

$x^2 = -49$

$\sqrt{x^2} = \sqrt{-49}$

NO REAL SOLUTION

$16x^2 - 49 = 0$

$+49 \quad +49$

$\frac{16x^2}{16} = \frac{49}{16}$

$x^2 = \frac{49}{16}$

$\sqrt{x^2} = \sqrt{\frac{49}{16}}$

$x = \pm \frac{7}{4}$

$x^2 = 15$

$x \approx \pm 3.87$

Solving Quadratic equations by graphing.

p.

Graph $y = x^2 + 4x + 3$

① Vertex $x = -\frac{b}{2a}$ $-\frac{4}{2}$ $x = -2$

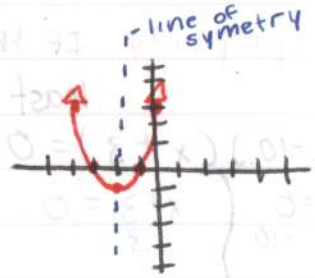
$(\overset{x}{-2}, \overset{y}{-1})$ $4 - 8 + 3 = -1$

② Intercept

$(0, 3)$

③ Choose ①

$(-1, 0)$



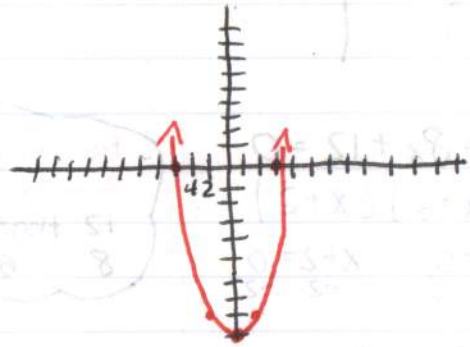
Graph $2x^2 - 18 = 0$ $\boxed{0 = y}$

$y = 2x^2 - 18$

① $(0, -18)$

② $(3, 0)$

③ $(1, -16)$



Geometric Sequences

Geometric Sequence - A sequence of numbers with a common ratio.

Common Ratio - Ratio of any term and the previus term

Step 1. Find the common ratio

1, 4, 16, 64 4

Step 2 - Multiply each term by r (4)

64·4 256·4 1024·4

EXAMPLE

$-9, 3, -1, \frac{1}{3}, -\frac{1}{9}$

[redacted] $\frac{3}{-9} = -1/3$

$\frac{1}{3} \cdot -1 = -1/3$

$\frac{-\frac{1}{9}}{\frac{1}{3}} = -1/3$

$-\frac{1}{9} \cdot -\frac{1}{3} = 1/27$

$1/27 \cdot -\frac{1}{3} = -1/81$

$1/81 \cdot -1/3 \quad 1/243$

Common Ratio

$a_n = a_1 r^{n-1}$

n^{th} term 1st Term n^{th} term

$a_n = a_1 r^{n-1}$

$a_7 = 500(.2)^{7-1}$

$a_7 = 500(.2)^6$

$a_7 = 500(.000064)$

$a_7 = .032$

$a_6 = 5\ 2^5$

$a_6 = 160$

Exponential growth occurs when a quantity increases by the same rate r in each period t

$$y = a(1+r)^t$$

Final Amount

Rate of growth as a decimal

time

$$y = a(1+r)^t \quad \text{where } a > 0$$

Original Amount

$y = 1200(1+.08)^6$

$y = 1200(1.08)^6$

$y = 1904.249188$

Compound interest is interest earned or paid on both the principal and previously earned interest.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Balance after t years

Anual interest rate expressed as a decimal

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

time in years

Principal Original Amount

Number of times interest is compounded per year.

Annualy - Once per year $(n=1)$

Quarterly - 4 times per year $(n=4)$

monthly - 12 times per year. $(n=12)$

$A = 1200\left(1+\frac{.02}{4}\right)^{4\cdot 3}$

$A = 1200(1.005)^{12}$

$\$1274.01$

Linear, Quadratic and Exponential models

$y = mx + b$

LINEAR

x	y
	3
	5
	7
	9

+2, +2, +2

CONSTANT FIRT DIFFERENCE

(+,-)

$y = ax^2 + bx + c$

QUADRATIC

x	y
	300
	260
	231
	213

-40, -29, -18

+11, +11

CONSTANT SECOND DIFERENSES

(+,-)

$y = ab^x$

EXPONENTIAL

x	y
	10
	20
	40
	80

x2, x2, x2

MULTIPLY BY A COMMON RATIO

(*)

IN ALL EXAMPLES X MUST INCREASE AT A CONSTANT RATE

EX 1,3,5,7,9,11,13

Square Root Functions

Perfect Squares - 1 4 9 16 25 36 49 64 81 100 121 144 169 196 225
256 289 324 361 400

Radical expression - an expresion that has a radical sign $\sqrt{}$

EX

$\sqrt{14}$ $\sqrt{l^2+w^2}$ $\sqrt{2gd}$ $\frac{\sqrt{d}}{4}$ $5\sqrt{2}$ $\sqrt{18}$

Radicand - The part that is under the radical symbol

SIMPEST FORM

- The radican has no perfect squares
- The radican has no fractions
- No square roots in any denominator.

EX

$\sqrt{\frac{256}{4}}$
$\sqrt{16}$
$=4$

$\sqrt{40+9}$
$\sqrt{49}$
$=7$

$\sqrt{5^2+12^2}$
$\sqrt{25+144}$
$\sqrt{169}$
$=13$

$\sqrt{(3-x)^2}$
$=(3-x)$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ where $a \geq 0$ and $b \geq 0$

$\sqrt{x^3y^4}$

$\sqrt{x^3y^4} = \sqrt{x^3}\sqrt{y^4}$
$= \sqrt{y^4}\sqrt{x^2}\sqrt{x^1}$
$= y^2x\sqrt{x}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $a \geq 0$ $b \geq 0$

$\sqrt{\frac{12}{27}} = \frac{\sqrt{12}}{\sqrt{27}} = \sqrt{3} \cdot \sqrt{4} = \sqrt{3}\,2$

$\frac{\sqrt{4}}{\sqrt{9}}$ $\boxed{\frac{2}{3}}$

Multiplying & dividing Radicals

① $\sqrt{3}\sqrt{6} = \sqrt{18}$
$= \sqrt{9 \cdot 2}$
$= 3\sqrt{2}$

② $(5\sqrt{3})^2 = (5\sqrt{3})(5\sqrt{3})$
$= 25\sqrt{9}$
$= 25 \cdot 3$
$= 75$

③ $2\sqrt{8x}\sqrt{4x}$
$2\sqrt{32x^2}$
$2\sqrt{16 \cdot 2x^2}$
$8x\sqrt{2}$

Distributative Property

$\sqrt{2}(5+\sqrt{12})$
$5\sqrt{2}+\sqrt{24}$
$5\sqrt{2}+2\sqrt{6}$

$\sqrt{3}(\sqrt{3}-\sqrt{5})$
$3-\sqrt{15}$

$(4+\sqrt{5})(3-\sqrt{5})$
$12-\sqrt{5}-5$
$7-\sqrt{5}$

F O I L F O I L

$(\sqrt{7}-5)^2$
$(\sqrt{7}-5)(\sqrt{7}-5)$
$7 - 5\sqrt{7}$
$-5\sqrt{7} + 25$
$32 - 10\sqrt{7}$

Rationalizing the denominator.

$\frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$

④ $\frac{\sqrt{7}}{\sqrt{8n}} = \frac{\sqrt{7}}{2\sqrt{2n}} \cdot \frac{\sqrt{2n}}{\sqrt{2n}}$ $\frac{\sqrt{14n}}{8n}$

$\frac{\sqrt{7a}}{\sqrt{12}} = \frac{\sqrt{7a}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{21a}{3 \cdot 2} = \frac{\sqrt{21a}}{6}$

Exponential decay occurs when a quantity decreases by the same rate r in each time period t

$\# \; y = a(1-r)^t \; \#$

$y = a(1-r)^t$

$y = 1700(1-.03)^{12}$

$y = 1700(.97)^{12}$

1179.532014

Half-life

$A = P(.05)^t$

Final Amount

Original Amount

# of halflifes in a given time period

$t = \frac{\text{timeperiod}}{\text{half life}}$

$A = 500(0.5)^5$

15.625

Exponential Functions

Form $f(x)=ab^x$ where $a \neq 0$ $b \neq 1$ and $b > 0$

Identify:

In order to be an exponential function:

① the x values increase by a constant

② the y values are multiplied by a constant.

x	y
0	4
1	12
2	36
3	108

+1, +1, +1 ; ·3, ·3, ·3

YES

{(0,4) (1,12) (2,36) (3,108)}

This function is exponential!

{(-1,1) (0,0) (1,1) (2,4)}

NO! This Function is not exponential

x	y
-1	1
0	0
1	1
2	4

+1, +1, +1 ; ·0, ·NO

NO

Evaluate:

$f(x) = 500(1.035)^x$ $f(6) = 500(1.035)^6 = 6 = 614.6276632$

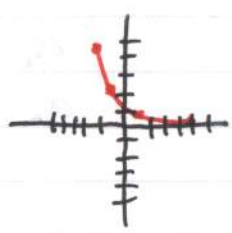
Plug in x into the exponential function $x=6$ $\$614.63$

Graph:

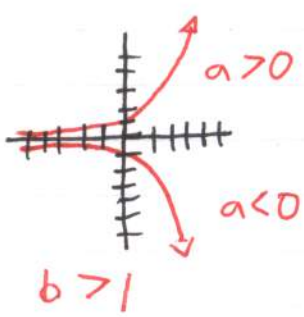
$y = 4(0.6)^x$

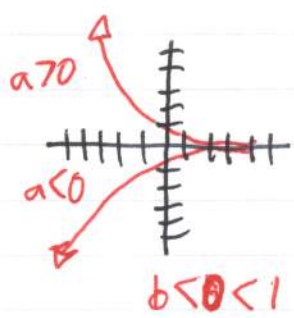
Pick 5 points for "x" and plug them in the equation to get "y"

x	$4(0.6)^x$	y
-2	$4(0.6)^{-2}$	11.11
-1	$4(0.6)^{-1}$	6.66
0	$4(0.6)^0$	4
1	$4(0.6)^1$	2.4
2	$4(0.6)^2$	1.44



X WHAT DO GRAPHS LOOK LIKE X





Fernando Trujano

Solving Quadratic Functions by Factoring

Zero product property - If the product of two Quantities = 0, at least one of the quantities = 0

Example: $(5x-10)(x+3)=0$

et equation equal to 0

olve for x

x=2

$5x-10=0$
$+10 \quad +10$

$\frac{5x}{5}=\frac{10}{5}$

$x=2$

$x+3=0$
-3

$x=-3$

ZEROS

$x=2$

$x=-3$

Graph should look like this.

or

$x^2+8x+12=0$

Factor Polynomial $(x+6)(x+2)$

Factoring Polynomials
Remember. Product of 12 that adds to 8 6, 2

zeros

$x=2$

$x=-3$

et equation equal to zero

$x+6=0$
$-6 \quad -6$

$x+2=0$
$-2 \quad -2$

Solve

$x=-6$ $\quad x=-2$

Set the equation to x^2+bx+c

$-3x^2=6x+3$
$+3x^2 \quad +3x^2$

x^2+bx+c

Factor-Simplify $0=3x^2+6x+3$

Factor Polynomial $0=3(x^2+2x+1)$

Set each equation to zero $0=3(x+1)(x+1)$

$x+1=0$ $\quad x+1=0$

Solve

$x=-1$ $\quad x=-1$

ZEROS

$x=-1$

One Solution

The Quadratic Formula and the Discriminant

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad ax^2+bx+c=0$$

Example

$$-3x^2+5x+2=0$$

$a=-3$
$b=5$
$c=2$

$$x = \frac{-5 \pm \sqrt{25+24}}{-6}$$

$$x = \frac{-5 \pm \sqrt{49}}{-6}$$

$$\frac{-5+7}{-6} \qquad \frac{-5-7}{-6}$$

$x=-1/3$ $\qquad$ $x=2$

The discriminant → b^2-4ac

- Is positive → 2 real solutions → $\sqrt{\frac{1}{36}} = \pm 1/6$
- Is 0 → 1 real solution $=0$ → $\sqrt{0}=0$
- Is negative → No real Solution → $\sqrt{-36}=i$

EXAMPLE WRONG $9=2x^2+3x$ WRONG

$$2x^2+3x-9=0$$

$$x = -3 \pm \sqrt{6-72}$$

b^2

$$\sqrt{64}$$

$$x = -3 \pm 8$$

WRONG

$\underline{-3}$

$x=-3+8$ $\qquad$ $x=-3-8$

$x=5$ $\qquad$ $x=-11$

$\frac{5}{-6}$ $\quad$ $\frac{11}{-6}$

3
18
×4
72

$$x = \frac{-3 \pm 8}{4}$$

Solving Radical Equations

Power Property of Equality -

If a ~~and b~~ and b are real # and $a = b$, then $a^2 = b^2$

$\sqrt{x} = 8$

$(\sqrt{x})^2 = 8^2$

$x = 64$

Isolate
remove radical
solve

① $\sqrt{x-5} = 4$

$(\sqrt{x-5})^2 = 4^2$

$x - 5 = 16$

$+5$

$x = 21$

② $\sqrt{2x-1} + 4 = 7$

$-4 \quad -4$

$(\sqrt{2x-1})^2 = (3)^2$

$2x - 1 = 9$

$+1 \quad +1$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

③ $\frac{3\sqrt{x}}{3} = \frac{21}{3}$

$\sqrt{x} = 7$

$x = 49$

$x = 7$

④ $\sqrt{x+1} = \sqrt{3}$

$x + 1 = 3$

$-1 \quad -1$

$x = 2$

⑤ $\sqrt{x+8} - \sqrt{3x} = 0$

$+\sqrt{3x}$

$x + 8 = 3x$

$-x \quad -x$

$\frac{8}{2} = \frac{2x}{2}$

$x = 4$ ~~x=10000~~

⑥ $\sqrt{6-x} = x$

$(\sqrt{6-x})^2 = x^2$

$6 - x = x^2$

$-6-x \quad -6-x$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3 \quad x = 2$

Adding and Subtracting Radical Expressions

$6\sqrt{5}+2\sqrt{5}$

Like terms

$\sqrt{5x}-7\sqrt{3x}-4\sqrt{5x}$

$-3\sqrt{5x}-7\sqrt{3x}$

$$\begin{aligned}\sqrt{8}+\sqrt{72} &= \sqrt{4\cdot 2}+\sqrt{36\cdot 2}\\ &= \sqrt{4}\sqrt{2}+\sqrt{36}\sqrt{2}\\ &\quad 2\sqrt{2}+6\sqrt{2}\\ &\quad 8\sqrt{2}\end{aligned}$$

Square Root Functions

Square-root function - Function whose rule contains a variable under a square root sign

EXAMPLES	NON EXAMPLES
$y = \sqrt{x}$	$y = x^2$
$y = \sqrt{2x+1}$	$y = \frac{2}{x+1}$
$y = 3\sqrt{\frac{x}{2}} - 6$	$y = \sqrt{3}\,x$

$y = \sqrt{4(x+3)}$

$4(x+3) \geq 0$

$-3 \quad -3$

$\frac{4x}{4} \geq -3$

$x \geq -\frac{3}{4}$

$y = \sqrt{2x-1}$

$2x - 1 \geq 0$

$+1 \quad +1$

$\frac{2x}{2} \geq \frac{1}{2}$

$x \geq \frac{1}{2}$

TRASLATIONS

$y = \sqrt{x} + c$

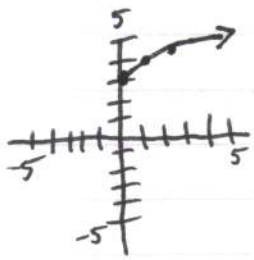
will make it move up or down

$y = \sqrt{x-a}$

will make it move lef or right

OPPOSITE

$y = 2\sqrt{x} + 3$

$x \geq 0$

x	y
0	3.46
1	4
2	4.47