Seat Racing 4+/4-

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Seat racing is a procedure used by rowing coaches to find the strongest athletes from a squad for a crew boat. While the general fitness of an athlete can be observed from land training, his or her ability to "move the boat" within a crew is more difficult to quantify. Seat racing is used to bring objectivity and transparency to the selection process of finding the best crew for a boat. This paper takes a fresh look at a method that works along the following principles:

1. Eight rowers are split repeatedly into two crews of four and race against each other in two boats under controlled conditions for a fixed distance (like 1000m).
2. After each race rowers between boats are swapped according to a fixed plan. The process continues over a total of 6 races.
3. Each rower is either rowing always on starboard (right) or port (left). This implies that the eight rowers are comprised of four for each side. The 4+/4- in the title refers to the class of boat being used: 4 rowers in a boat with or without a coxswain.
4. For each race the finishing time for each boat is recorded.

The goal of the method is to infer from the recorded finishing times a ranking of the athletes (per side) in terms of speed contribution.

1 Example

We will discuss the method based on the example in Table 1 from [1]. It is comprised of 6 races over 1000m. In each race two boats of 4 rowers race against each other. Rowers are identified as 1 to 4 (rowing on starboard) and A to D (rowing on port). Each crew has two rowers on each side.

What crews are racing each other is fixed and not random. We will see that this *swapping matrix* has important properties for this method to work.

crew	race	boat	time [s]
12AB	1	1	211.54
34CD	1	2	211.46
24AC	2	1	199.46
13BD	2	2	204.30
23BC	3	1	200.97
14AD	3	2	206.29
12CD	4	1	202.49
34AB	4	2	199.47
24BD	5	1	205.79
13AC	5	2	205.98
23AD	6	1	205.78
14BC	6	2	205.81

Table 1: Crews of 4 rowers race pairwise in 6 races over 1000m with the resulting times in seconds. Rowers are identified as 1 to 4 and A to D.

2 Problem Definition

Most descriptions of seat racing in the rowing literature talk about one rower being faster than another by a margin of time. While this is what it looks like, it is more the effect than the cause. The speed of a crew boat is the result of the effort of its crew and time cannot be attributed to individual crew members. What is happening is that each rower contributes power to the propulsion of the boat. It is helpful to think about individual power and the combined power of the crew. What seat racing really aims for is to uncover the power contribution of each rower.

When a boat with 4 crew members travels a distance d in time t, its average speed is $v = d/t$ and the average power P required depends on a drag factor k:

$$P = k \times v^3 \tag{1}$$

Drag factor k depends on the boat's shape, crew weight and other details. However, we are mostly interested in the relative power of rowers and can live with an approximation of $k = 2.8 \times 4$. This gives use for each race the average power of the crew. For example, a time of 211.54s corresponds to

$$P = 2.8 \times 4 \times (1000/211.54)^3 = 1183.15\,\text{watt}.$$

We make the following assumptions:

1. The power of a crew is the total of the power of its members and

2. each crew member m produces in each race the same amount of power x_m.

Power x is the number that we are looking for each rower. Power is an interesting measure because it is known for each rower from land training, too, and so it offers a comparison. Restating Table 1 in terms of power using Equation 1 leads to Table 2.

crew	race	boat	time [s]	power [W]	adj	adj. power [W]
12AB	1	1	211.54	1183.15	1.10	1303.02
34CD		2	211.46	1184.50		1304.49
24AC	2	1	199.46	1411.40	0.96	1350.62
13BD		2	204.30	1313.45		1256.89
23BC	3	1	200.97	1379.83	0.98	1354.82
14AD		2	206.29	1275.80		1252.69
12CD	4	1	202.49	1348.99	0.94	1274.37
34AB		2	199.47	1411.19		1333.14
24BD	5	1	205.79	1285.12	1.02	1305.56
13AC		2	205.98	1281.57		1301.95
23AD	6	1	205.78	1285.31	1.01	1304.04
14BC		2	205.81	1284.75		1303.47

Table 2: Race results restated in terms of implied crew power. Column *power* is the power based on equation 1. This leads to the total power of both crews in a race to vary across races. *Adjusted power* corrects this such that the total power in each race is equal to the avarage total power.

Race 4 is faster than race 1, maybe because of wind or stream or the distance was slightly shorter than 1000 meters. The total power of the rowers in race 4 therefore is seemingly higher than the total power of the same rowers in race 1 – which contradicts our assumptions. We do not know yet how significant this is, as it affects both crews in a race, but we can try to adjust the power in each race by a factor *adj* such that the total power in all races is constant and equals the average total power. This results in column *adj. power* in Table 2.

The problem of inferring the power contribution of each rower from the race results can be stated as a system of linear equations in matrix form:

$$
\begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 0
\end{bmatrix}
\times
\begin{bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_A \\ x_B \\ x_C \\ x_D
\end{bmatrix}
=
\begin{bmatrix}
1303.02 \\ 1350.62 \\ 1354.82 \\ 1274.37 \\ 1305.56 \\ 1304.04 \\ 1304.49 \\ 1256.89 \\ 1252.69 \\ 1333.14 \\ 1301.95 \\ 1303.47
\end{bmatrix}
$$

or more succinctly as

$$Sx = P \tag{2}$$

where S is the swap matrix, x the power of each rower, and P the observed crew power (for which we used the adjusted power from Table 2). We are now looking for vector x, which assigns power to each rower, such that this matches the observed power.

3 Inferring Power

Solving $Sx = P$ for x is not straight forward: S is not square and therefore has no inverse. The left inverse S' with $S'S = I$ does not exist because $\operatorname{rank}(S) = 7 < 8$. But S has a generalised inverse S^+ which we can use to describe all solutions.

$$
S^+ =
\begin{bmatrix}
b & a & a & b & a & a & a & b & b & a & b & b \\
b & b & b & b & b & b & a & a & a & a & a & a \\
a & a & b & a & a & b & b & b & a & b & b & a \\
a & b & a & a & b & a & b & a & b & b & a & b \\
b & b & a & a & a & b & a & a & b & b & b & a \\
b & a & b & a & b & a & a & b & a & b & a & b \\
a & b & b & b & a & a & b & a & a & a & b & b \\
a & a & a & b & b & b & b & b & b & a & a & a
\end{bmatrix}
\tag{3}
$$

$$a = -0.1041667 \tag{4}$$

$$b = +0.1458333 \tag{5}$$

All solutions for rower power x given crew power P in Equation 2 are given by

$$x = S^+P + c\begin{bmatrix}+1 & +1 & +1 & +1 & -1 & -1 & -1 & -1\end{bmatrix}^T \tag{6}$$

for an arbitrary constant c. The fact that an infinite number of solutions exist may surprise you and call into the question the method. But it is not as bad as it sounds. First, you could verify that all solutions in Table 3 indeed respect Equation 2, i.e., the observed race times.

	Power x [W]		
rower	$c = 0$	$c = 10$	$c = 30$
1	293.40	303.40	323.40
2	343.42	353.42	373.42
3	334.14	344.14	364.14
4	332.80	342.80	362.80
A	331.67	321.67	301.67
B	334.53	324.53	304.53
C	342.74	332.74	312.74
D	294.82	284.82	264.82

Table 3: Power assignments that are consistent with race times. Observe that the difference in power between rowers of one side (1 to 4 and A to D) is constant across all solutions. Solutions differ only by shifting power between sides.

4 Results

Table 3 provides the answers we are looking for but we need to be a little careful with the interpretation:

1. The strongest rowers on starboard are 2, 3, 4, 1 in this order.
2. The strongest rowers on port side are C, B, A, D in this order.
3. We can't tell who is the strongest rower across sides because power shifts between sides are not detected by the method. Parameter c basically models this shift. Across all solutions, the difference in power between rowers of one side remains constant and leads to an order independent of c.
4. With $c = 0$ the combined power of rowers of 1 to 4 and A to D is equal. This makes it a somewhat more likely to be the true scenario than a solution with a large parameter c.
5. The swap matrix S and its generalised inverse S^+ are fixed and independent of the race results. They can be used to implement the method as a spreadsheet.

6. The drag factor $k = 2.8 \times 4$ chosen in Equation 1 could be adjusted without impacting the order of rowers but in order to match the observed power on the water to power observed in land training.

7. The swap matrix S used here is not the only possible but it needs to be chosen carefully for Equation 6 to be valid.

We have used the adjusted power in Table 2 to compensate for the fact that the total observed raw power between races varied. This might not be necessary as long as it affects both crews equally. Computing the power based on raw power and $c = 0$ yields an almost identical result with the largest difference being 1.21 W for rower D and the difference being smaller than 1 W for most rowers.

References

[1] Mike Purcer. Seat Racing Analysis. Excel Spreadsheet. Retrieved May 16. 2020. https://purcerverance.ca/files