STAT395: Introduction to Bayesian Analysis

Introduction to Bayesian Analysis

What is Bayesian Analysis?
Bayes Theorem with examples.
Tutorial 1 - Questions
Bayesian vs Frequentist approach?
Some simple illustrations of the Bayesian approach.

… a Quick Tour

A

- We start with one hypothesis
- We collect some observations.
- We derive from them a test statistic.
- We calculate the probability of finding a test statistics atleast as extreme as the one we found.
- We compare the probability to some cut off mark.
- We either reject or fail to reject the null hypothesis

B

- We start with some pre-existing belief about the likelihood of each hypothesis.
- We collect some observations.
- We end up with a new (hopefully, refined) belief about the likelihood of each hypothesis.

… a Quick Tour – Frequentist approach

1. **Frequentist approach** – parameters (θ) are treated as **fixed** but unknown quantities.

2. Estimate these parameters - Use samples from a population.

3. Sampling distributions - and it quantifies the uncertainty of our estimate but the parameter itself is considered fixed.

… a Quick Tour – Bayesian approach

1. **Bayesian approach** – parameters (θ) are treated as **random variables**.

2. Prior distribution – Mathematical expression of our belief about the parameters.

3. Likelihood function – quantifies the results of our experiment/data.

4. Bayesian analysis allows us to update our beliefs about a parameter with the results of our experiment/data.

… a Quick Tour – Way forward

- In simple cases we can compute a **posterior** distribution, for inferential decisions, for the parameter (θ) by multiplying the **Likelihood** by the **Prior**:

$$Posterior \propto Likelihood \times Prior$$

$$P(\theta|y) \propto P(y|\theta) \times P(\theta)$$

- The effect of different priors (informative/noninformative priors) on the posterior distribution.
- Use posterior distribution to calculate different quantities of the posterior
    - Mean, mode, median and 95% credible intervals
- Today's posterior is tomorrow's prior.

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What is Bayesian Analysis?

Bayesian analysis

- is a field of statistics that is based on the notion of conditional probability
- can be viewed as the formalization of the process of incorporating scientific knowledge using probabilistic tools.
- provides uncertainty quantification of parameters by its conditional distribution in the light of available data.

Central Idea of Bayesian Approach

Combine data (likelihood) with Your prior knowledge (prior probability) to update information on the parameter to result in a revised probability associated with the parameter (posterior probability)

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Bayes Theorem

For two events A and B

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

- $P(A)$ is the prior probability of event A. It is called the *prior* because it does not take into account any information about event B.
- $P(B|A)$ is the conditional probability of event B given event A.
- $P(B)$ is the prior or marginal probability of event B.
- $P(A|B)$ is the conditional probability of event A given event B. It is called the posterior probability because it is derived from the specified value of event B.

Example 1 – Bayes Theorem

In a particular clinic, past data shows that 10% of patients have liver disease. Five percent of the clinic's patients are alcoholics. It is known that for patients diagnosed with liver disease, 7% are alcoholics. Find the probability that a patient in this particular clinic is having liver disease if he/she is an alcoholic.

$A = \ patient\ has\ liver\ disease$

$B = patient\ is\ an\ alcoholic$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Example 2 – Bayes Theorem

About 1% of people have a certain genetic defect. 90% of tests for the gene detect the defect (true positives) but 9.6% of the tests are false positives. If a person gets a positive test result, what is the probability he/she actually has the genetic defect?

A = Person having a genetic defect. (Note that A\`= a person not having genetic defect).

B = Positive test result detected.

Given: $P(A) = 0.01$ $P(A`) = 1 - 0.01 = 0.99$ $P(B/A) = 0.90$ $P(B/A`) = 0.096$

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Tutorial 1

Question 1 – Bayes Theorem

The probability of a certain medical test being positive is 90%, if a patient has disease D. 1% of the population have the disease, and the test records a false positive 5% of the time. If you receive a positive test, what is your probability of having disease D?

ANS = 0.1538

Tutorial 1

Question 2 – **Show**

$$
\begin{aligned}
P(B \mid A) &= \frac{P(A \mid B) \times P(B)}{P(A)} \\
&= \frac{P(A \mid B) \times P(B)}{P(A \mid B) \times P(B) + P(A \mid B^C) \times P(B^C)}
\end{aligned}
$$

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Statistical Inference

Statistical inference is an inductive process for making inferences about **parameters** of interest.

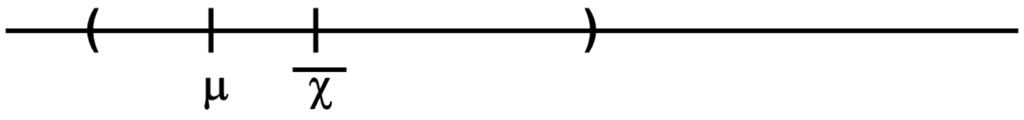
- **Frequentist Approach:** Classical Approach which considers the parameters to be fixed but unknown
- **Bayesian Approach:** Treat the unknown parameters as random variables.

Frequentist Approach to Inference

- Classical methods consider the parameters to be fixed but unknown.
- Based on probabilities that are only for observations given the unknown parameters.
- Judged by how they perform in an infinite number of hypothetical repetitions of the experiments.

Confidence Intervals – Classical Approach

95% Confidence Interval

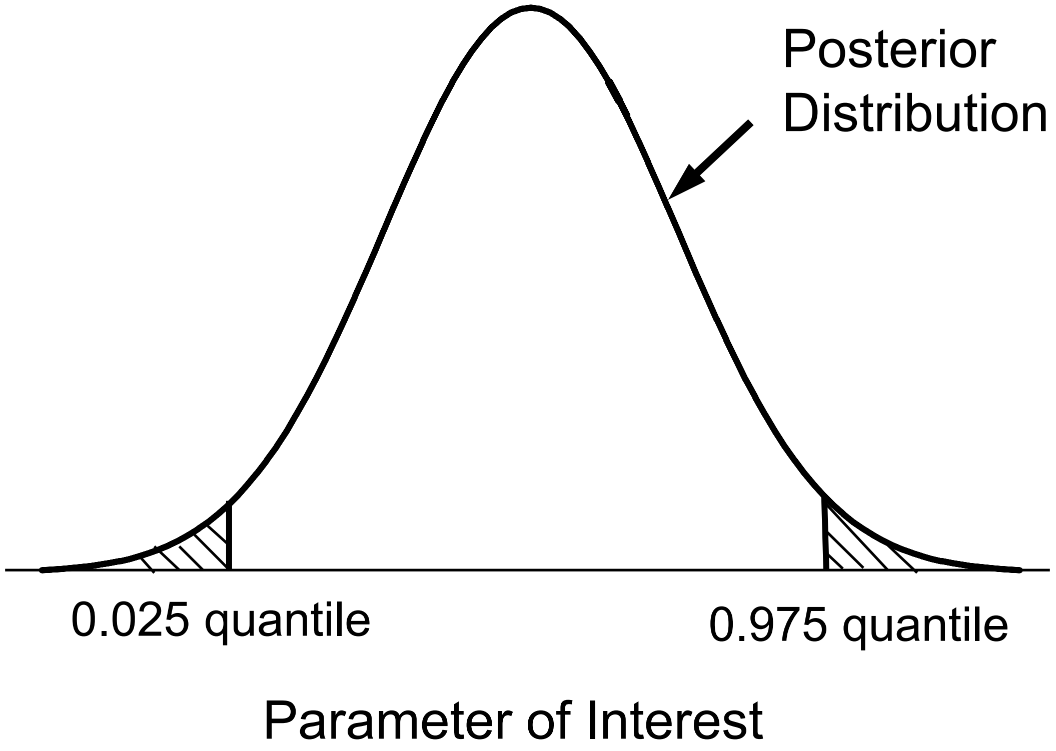


- A 95% confidence interval states that if 100 different samples were drawn from the same population and 100 intervals were calculated, approximately 95 of them would contain the population mean.

Bayesian Methods

- Treat the unknown parameters as random variables.
- Enables to make probability statements about parameters and observations.
- Interprets probabilities for parameters as “degree of belief” and can be subjective.
- Uses the rules of probability to revise “degree of beliefs” about the parameters given the observed data.
- Bases the inferences about the parameters on the probability distribution for the parameter.

95% Credible Interval



There is a 95% chance that the parameter is in the credible interval.

Why the Bayesian Approach - Advantages?

1. Easier interpretation of parameters.
2. Computationally intensive.
3. More flexible.
4. Deals well with missing data.
5. Effectively models high-dimensional data.
6. Is a method of learning.
7. Does not require large samples.

Why the Bayesian Approach - Disadvantages?

1. Criticism of using subjectivity in the model.

2. Misunderstood.

3. Complicated.

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Examples of the Bayesian Approach Uses

- The Bayesian approach to inference is increasing in popularity.

- Applications can be found in:

  Clinical research, Pharmaceutical research, Genetics, Epidemiology, Psychology, Sociology, Forensics, Archeology, Cosmology, Ecology, Marketing, Physics, ...

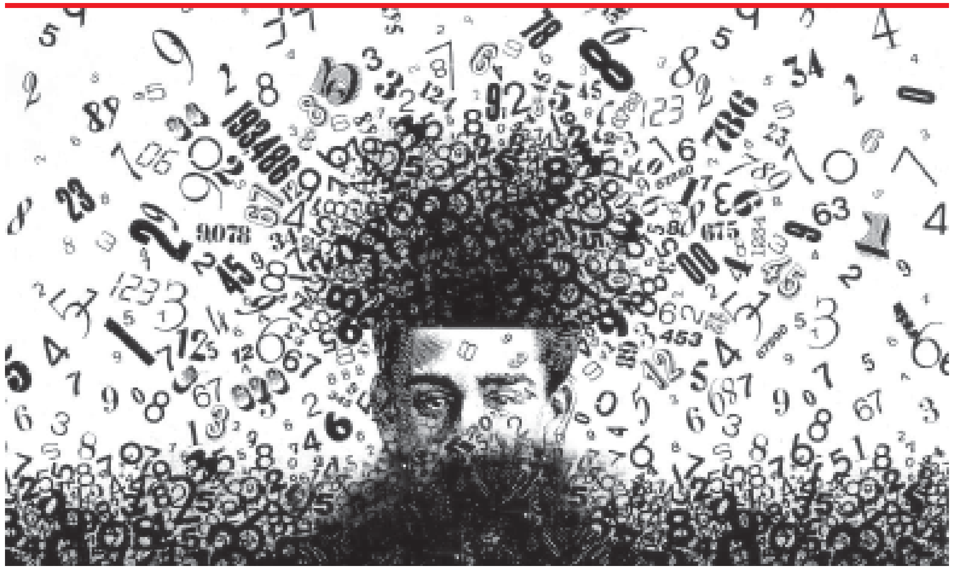
Pharmaceutical research

Bayesian statistics in

- Product development and manufacturing
- Process development and validation
- Analytical methods and assays
- Stability studies
- Sampling, release and content uniformity
- Dissolution testing
- Manufacturing
- Medical devices
- Program and portfolio decision-making

BAYESIAN HIERARCHICAL PK/PD MODEL TO CHARACTERIZE THE EXPOSURE-QT EFFECT RELATIONSHIP IN EARLY DRUG DEVELOPMENT
Muriel Boulton | May 20, 2015
Vincent Dubois, Roberta Bursi
Development – Data Sciences – Pharmacometrics
MANCHESTER 1824
NOVARTIS
A Bayesian population physiologically-based model to predict the impact of age and drug-drug interaction on mavoglurant pharmacokinetics
Thierry Wendling
Manchester Pharmacy School, The University of Manchester, Manchester, UK
Drug Metabolism and Pharmacokinetics, Novartis Institutes for Biomedical Research, Basel, CH
Robust meta-analytic-predictive priors in clinical trials with historical control information
Heinz Schmidli
Statistical Methodology, Novartis, Basel, Switzerland
BAYES2015, 20 May 2015, Basel
NOVARTIS
heinz.schmidli@novartis.com
Pfizer
Advantages of a wholly Bayesian approach to assessing efficacy in early drug development: a case study
Phil Woodward, Ros Walley, Claire Birch, Jem Gale

Forensics

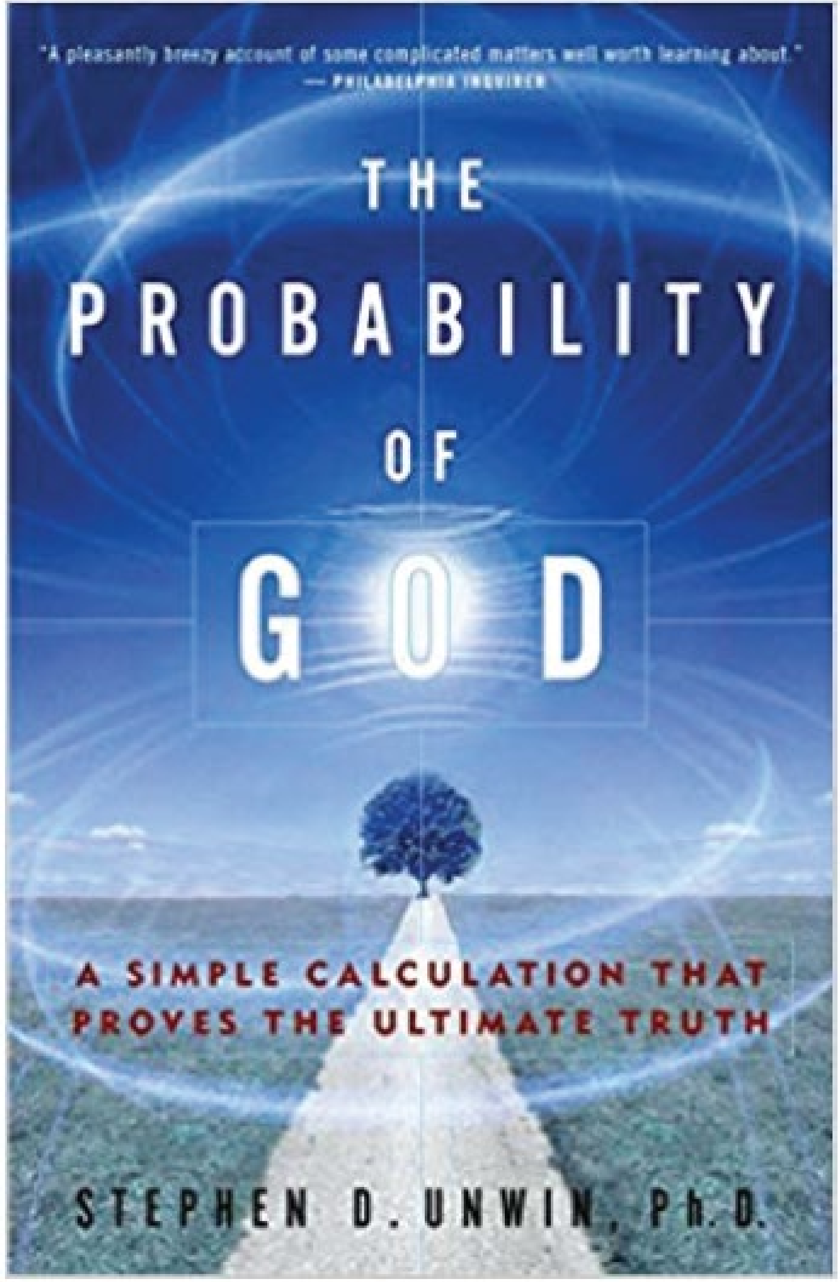


CONVICTION BY NUMBERS

In March 2003, when nurse Lucia de Berk faced trial in a Dutch court for charges of murder and attempted murder, the statistical evidence against her seemed compelling. Investigators had identified a number of 'suspicious' deaths and near deaths in hospital wards in which de Berk had worked from 1999 to 2001, and records showed that she had been present when many of those events took place. The statistical expert testifying in the case, Henk Elffers, reported that the chance that her presence was mere coincidence was only 1 in 342 million.

Lucia de Berk is still waiting to hear whether a judicial review of her conviction for murder will recommend that her case be reopened.

Even in theology…



FOR THE NEXT LECTURE - Go recap on:

- Differentiation
- Integration
- Exponential distribution
- Bernoulli distribution
- Poisson distribution
- Gamma distribution
- Beta distribution