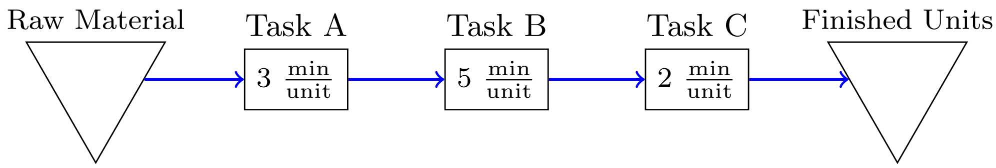
1. Consider the following three-step production process:



The numbers refer to the time necessary to process one unit of the product (run time) at that step. The process is staffed by three operators, with one operator assigned to each of the three steps. Assume no setups are required, that there is no variability in the process, that operators are capable of working at 100% all the time.

(a) What is the hourly capacity of the process?

A. 6 $\frac{\text{units}}{\text{hour}}$

B. 12 $\frac{\textbf{units}}{\textbf{hour}}$

C. 18 $\frac{\text{units}}{\text{hour}}$

D. 30 $\frac{\text{units}}{\text{hour}}$

In order to determine the hourly capacity of the process, the bottleneck of the process must be found. The bottleneck in the process is Step B. The capacity is calculated directly from the bottleneck; once we know the longest step, we need only calculate the capacity of that step and we have determined the capacity of the whole process. Thus, the hourly capacity of the process is:

$$60 \ \frac{\cancel{minutes}}{hour} \times \frac{1 \ unit}{5 \ \cancel{minutes}} = 12 \ \frac{units}{hour}$$

Notice that because there is no setup time, the bottleneck is the longest processing step in this case. Alternatively, as in Donner, we can compute the capacity of each processing step (20 $\frac{units}{hour}$, 12 $\frac{units}{hour}$, and 30 $\frac{units}{hour}$, respectively) and then identify the bottleneck (the one with the lowest capacity, i.e., Step B). The capacity of the process is the capacity of the bottleneck (i.e., 12 $\frac{units}{hour}$).

(b) What is the utilization rate of the operator at Task A?

A. 30%

B. 40%

C. 60%

D. 100%

The utilization rate of an operator is the portion of time that the operator is busy (not idle). From (a) we know that a unit is completed every 5 minutes. During each of these 5-minute cycles, with no buffer capacity between Tasks A and B, the worker at Task A is busy (i.e., actually working on the product) for 3 minutes. Thus, his or her utilization rate is:

$$\frac{3 \ \frac{minutes}{unit}}{5 \ \frac{minutes}{unit}} = 60\%$$

(c) What is the direct labor content of a unit produced by this process?

A. 3.3 $\frac{\text{minutes}}{\text{unit}}$

B. 5 $\frac{\text{minutes}}{\text{unit}}$

C. 10 $\frac{\textbf{minutes}}{\textbf{unit}}$

D. 12 $\frac{\text{minutes}}{\text{unit}}$

The direct labor content of a unit is the time actually spent working on each unit. Thus, the direct labor content per unit in this example is simply the sum of the direct labor content at each step:

$$3\ \frac{minutes}{unit} + 5\ \frac{minutes}{unit} + 2\ \frac{minutes}{unit} = 10\ \frac{minutes}{unit}$$

[Notice that if we instead wanted to compute direct labor cost, we would have to include not only the direct labor content, but also the idle time (since we pay for that, too). See part (d) below.]

The operators receive wages of \$12/hour for 8 hours/day, and \$18/hour for any time over 8 hours. Each operator can work no more than 12 hours per day. A finished unit sells for \$10 (with unlimited demand at that price) and total variable cost per unit is \$6 plus the cost of direct labor

(d) To maximize profits, how many hours a day would you have operators work?

A. 0 $\frac{\text{hours}}{\text{day}}$

B. 6 $\frac{\text{hours}}{\text{day}}$

C. 8 $\frac{\textbf{hours}}{\textbf{day}}$

D. 12 $\frac{\text{hours}}{\text{day}}$

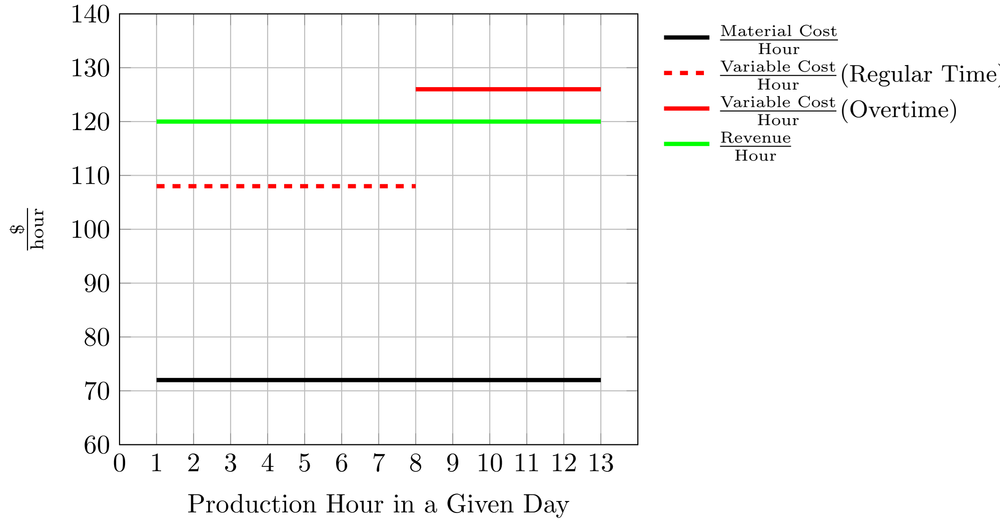
A third alternative approach is to compare the cost per unit to the revenue per unit. In part (a), we found that the hourly capacity was 12 $\frac{\text{units}}{\text{hour}}$. The hourly labor cost during regular production is 3 workers$\times$ $\frac{\$12}{\text{hour}} = \frac{\$36}{\text{hour}}$. The hourly labor cost during overtime production is 3 workers$\times$ $\frac{\$18}{\text{hour}} = \frac{\$54}{\text{hour}}$. Therefore, for units produced in the first 8 hours, the labor cost is $\frac{\$36}{\text{hour}} \times \frac{1\text{hour}}{12\text{units}} = \frac{\$3}{\text{unit}}$. For units produced in the last 8 hours, the labor cost is $\frac{\$54}{\text{hour}} \times \frac{1\text{hour}}{12\text{units}} = \frac{\$4.5}{\text{unit}}$. Given that the variable cost is \$6 per unit, this means the cost per unit during regular production is $\$3+\$6 = \$9$, whereas while the overtime cost per unit is $\$4.5 + \$6 = \$10.5$. Since the units sell for \$10, no overtime is cost effective. *The key to understanding this question is to think in terms of marginal costs and marginal revenues. The decision to incur overtime production depends on whether profitability is enhanced or eroded beyond the 8 hours of regular production. One approach is to calculate revenue and costs per production hour:*

- Revenue*:* $\frac{\$10}{unit} \times 12\ \frac{units}{hour} = \frac{\$120}{hour}$ *(using the capacity that was calculated in part a).*
- Material cost per production hour*:* $\frac{\$6}{unit} \times 12\ \frac{units}{hour} = \frac{\$72}{hour}$
- Labor cost per production hour at regular time*:* $3\ workers \times \frac{\$12}{hour} = \frac{\$36}{hour}$
- Total variable cost at regular time*:* $\frac{\$72}{hour} + \frac{\$36}{hour} = \frac{\$108}{hour}$

Therefore, profit per hour during normal production is $\frac{\$120}{hour} - \frac{\$108}{hour} = \frac{\$12}{hour}$

During overtime, labor costs increase to $3\ workers \times \frac{\$18}{hour} = \frac{\$54}{hour}$ *and total variable cost shifts to* $\frac{\$72}{hour} + \frac{\$54}{hour} = \frac{\$126}{hour}$*. Given revenue of* $\frac{\$120}{hour}$*, any production during overtime is unprofitable because the company incurs a loss of* $\frac{\$6}{hour}$ *during overtime. Thus, no overtime is profitable and to maximize profits we should employ the operators 8 hours per day.*

Figure 1: **Marginal Costs v. Marginal Revenue in Production**



A graphical analysis is another approach that can lead to this conclusion (Figure 1).
A third alternative approach is to compare the cost per unit to the revenue per unit. In part (a), we found that the hourly capacity was $12\ \frac{units}{hour}$. *The hourly labor cost during regular production is* 3 *workers*$\times\ \frac{\$12}{hour} = \frac{\$36}{hour}$. *The hourly labor cost during overtime production is* 3 *workers*$\times\ \frac{\$18}{hour} = \frac{\$54}{hour}$. *Therefore, for units produced in the first 8 hours, the labor cost is* $\frac{\$36}{hour} \times \frac{1hour}{12units} = \frac{\$3}{unit}$. *For units produced in the last 8 hours, the labor cost is* $\frac{\$54}{hour} \times \frac{1hour}{12units} = \frac{\$4.5}{unit}$. *Given that the variable cost is \$6 per unit, this means the cost per unit during regular production is* $\$3+\$6 = \$9$, *whereas while the overtime cost per unit is* $\$4.5 + \$6 = \$10.5$. *Since the units sell for \$10, no overtime is cost effective.*

(e) Would you hire a skilled operator whose wage rate is \$20/hour (regular time) but could perform Step B at a speed of 4 minutes/unit?

A. **Yes, because hourly profits increase.**

B. No, because hourly profits decrease.

C. Indifferent, because hourly profits remain the same.

D. None of the above.

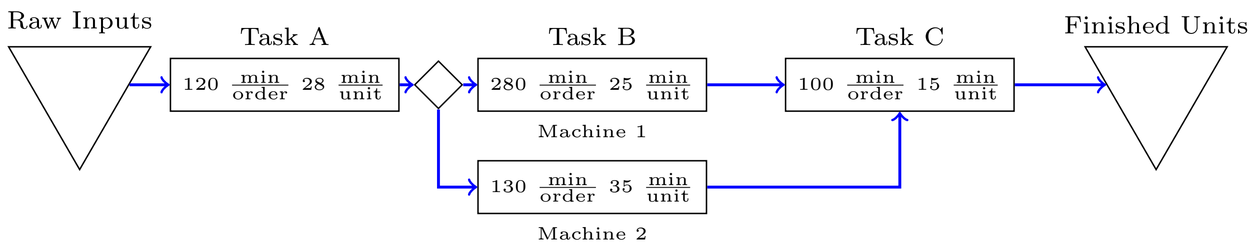
To determine if we would pay more for a skilled operator at Step B, we must compare the improvement in the cost per unit with and without the skilled worker. If costs per unit decrease, then hourly profits will increase because the price per unit doesn't change, and hence, revenues won't change either.

Without the skilled worker, we have already determined that the cost per unit is \$6 per unit for materials ($\frac{\$72}{hour} \times \frac{1hour}{12units} = \frac{\$6}{unit}$*)and \$3 per unit for labor (* $\frac{\$36}{hour} \times \frac{1hour}{12units} = \frac{\$3}{unit}$*) [see (d)]. As the new worker will not affect the material cost, we need only compare the direct labor cost. Thus, if the direct labor cost with the new worker is less than \$3, then it is cost effective to hire the new worker. With a skilled operator at Step B, we reduce the processing time to 4 minutes per* *unit. As Step B is still the longest step (i.e., it is still the bottleneck), the system will produce a unit every 4 minutes. The paid labor time per unit is $\frac{4 minutes}{unit}$ for each of the 3 workers, although now there are two pay scales. Thus, the total direct labor cost per unit is:*

$$\begin{aligned} Direct\ Labor\ Cost &= (2\ workers \times \frac{\$12/hour}{worker} \times \frac{1\ hour}{60\ minutes} \times \frac{4\ minutes}{unit}) \quad + \\ &\quad (1\ worker \times \frac{\$20/hour}{worker} \times \frac{1\ hour}{60\ minutes} \times \frac{4\ minutes}{unit}) \\ &= (\frac{\$1.60}{unit}) + (\frac{\$1.33\bar{3}}{unit}) \\ &\approx \frac{\$2.93}{unit} \end{aligned}$$

Since the direct labor cost with the additional skilled worker, $\frac{\$2.93}{unit}$, is less than the direct labor cost with the original workers, $\frac{\$3}{unit}$, you should hire the skilled worker.

2. Consider the following high-setup-time production process below. Note that for the second step in sequence, an order can either be processed with Machine 1 or Machine 2 (but not both). In the process flow diagram below, a diamond indicates you must choose one of the two paths rather than having both occurring in parallel. For each step, the time on the left above the step is the setup time required for each new order; the time on the right is the run time. Assume that each order must be completed in one step before the order is moved to the next step. Each task has one operator who performs the setup and must be present during the run time as well; that is, all the setup and run times require labor content.



(a) Consider the second step in sequence (which features a choice between Machine 1 or Machine 2 for that task). Assuming our objective is to minimize labor content, for what order sizes should we use Machine 1? For what orders sizes should we use Machine 2?

A. Use Machine 1 for all order sizes that are less than or equal to 15 units.

B. **Use Machine 1 for all order sizes that are greater than or equal to 15 units.**

C. Use Machine 1 for all order sizes that are less than or equal to 10 units.

D. Use Machine 1 for all order sizes that are greater than or equal to 10 units.

E. Indifferent between Machine 1 or Machine 2 for all order sizes.

Since orders aren't split, any given order must be sent to either Machine 1 or Machine 2. Intuition tells us that big orders will be sent to Machine 1 for which the larger setup time can be amortized over more units.

If we want to minimize labor content, then the best policy is "use Machine 2 whenever the labor content for that machine is less than or equal to the labor content for Machine 1." If x represents *the size of the order (number of units), the policy above can be written as:*

$$\textit{Machine 2 Labor Content} \leq \textit{Machine 1 Labor Content}$$
$$130 + 35x \leq 280 + 25x$$
$$10x \leq 150$$
$$x \leq 15$$

That is, we'd want to use Machine B whenever the order size is less than 15 units. When the order size is 15 units, we are indifferent to using Machine 2 or Machine 1, since they have the same labor content. When the order size is greater than 15 units, Machine 2 yields a higher labor content than Machine 1 (i.e. $130\frac{min}{order} + 35\frac{min}{unit} \times x\frac{units}{order} \leq 280\frac{min}{order} + 25\frac{min}{unit} \times x\frac{units}{order}$*), and so for* $x\frac{units}{order} > 15\frac{units}{order}$*, we would prefer to use Machine 1.*

(b) Suppose now that all orders are for 10 units. What is the daily 10-unit order capacity of this process, assuming an 8-hour day? Continue to assume you want to minimize the labor content.

A. $4.2\frac{\text{units}}{\text{day}}$

B. $9\frac{\text{units}}{\text{day}}$

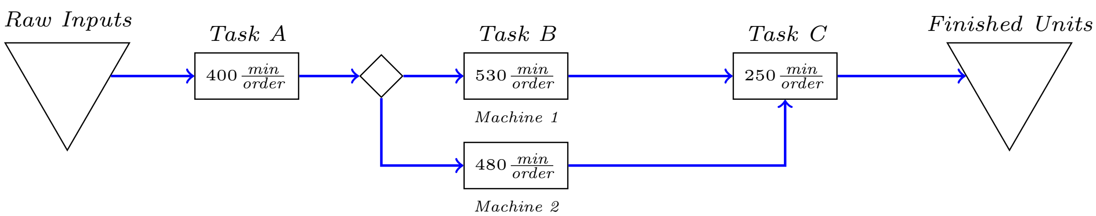
C. $10\frac{\textbf{units}}{\textbf{day}}$

D. $12\frac{\text{units}}{\text{day}}$

If all orders are for 10 units, then the individual task processing times per order are:

- Task A*:* $120\frac{min}{order} + 28\frac{min}{unit} \times 10\frac{units}{order} = 400\frac{min}{order}$
- Task B, Machine 1*:* $280\frac{min}{order} + 25\frac{min}{unit} \times 10\frac{units}{order} = 530\frac{min}{order}$
- Task B, Machine 2*:* $130\frac{min}{order} + 35\frac{min}{unit} \times 10\frac{units}{order} = 480\frac{min}{order}$
- Task C*:* $100\frac{min}{order} + 15\frac{min}{unit} \times 10\frac{units}{order} = 250\frac{min}{order}$

We can re-write the production process with these order times as:



Note that as per part (a) of this question, we opt for Machine 2 for order sizes of under 15 units. As predicted by the policy, Machine 2 yields a lower labor content of $480\frac{min}{order}$*. Thus, we use Machine 2 here.*

Now, we compare the individual task processing times per order to determine the bottleneck step, or the step with the longest processing times per order. This step will in turn set the capacity of the line. Comparing the individual task processing times per order of the first step ($400\frac{min}{order}$)*, the second step* ($480\frac{min}{order}$)*, and the third step* ($250\frac{min}{order}$)*, we see the second step is the bottleneck. Thus, the daily capacity is:*

$$8\frac{hours}{day} \times 60\frac{min}{hour} \times \frac{1\ order}{480\ min} = 1\frac{order}{day}$$

With a capacity of $1\frac{order}{day}$*, the capacity in units is* $1\frac{order}{day} \times 10\frac{units}{order} = 10\frac{units}{day}$.

(c) Suppose the production line was going to continue processing orders of a single size (number of units), and we wanted to know what the capacity would be if we processed orders of only 1 unit, only 2 units, etc., through 16 units. Management has suggested that graphing processing times vs. order size may help with this analysis. Which of the options below depicts a graph where the vertical axis is the task processing time, order size is the horizontal axis (for order sizes of up to 20 units), and one line is drawn depicting this relationship for each task (that is 4 lines in total, including one for Machine 1 and one for Machine 2)?

A. **Graph A**

B. Graph B

C. Graph C

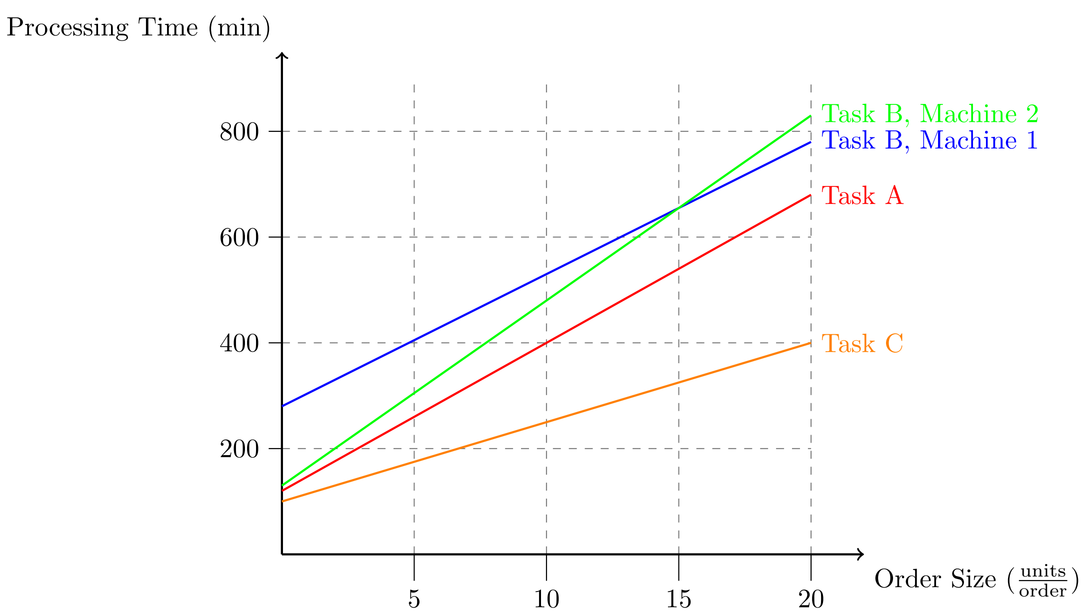
D. Graph D

As before, we find the individual task processing times per order. We can express these as equations, where x *is the size of the order in number of units:*

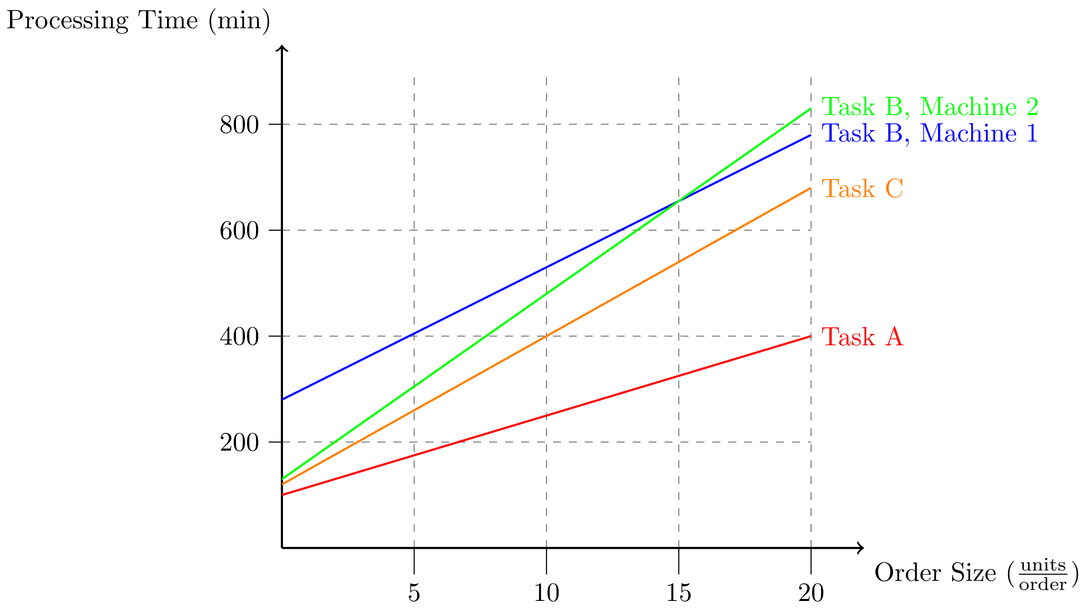
- Task A: $120\frac{min}{order} + 28\frac{min}{unit}x$
- Task B, Machine 1: $280\frac{min}{order} + 25\frac{min}{unit}x$
- Task B, Machine 2: $130\frac{min}{order} + 35\frac{min}{unit}x$
- Task C: $100\frac{min}{order} + 15\frac{min}{unit}x$

These equations correspond to Graph A.

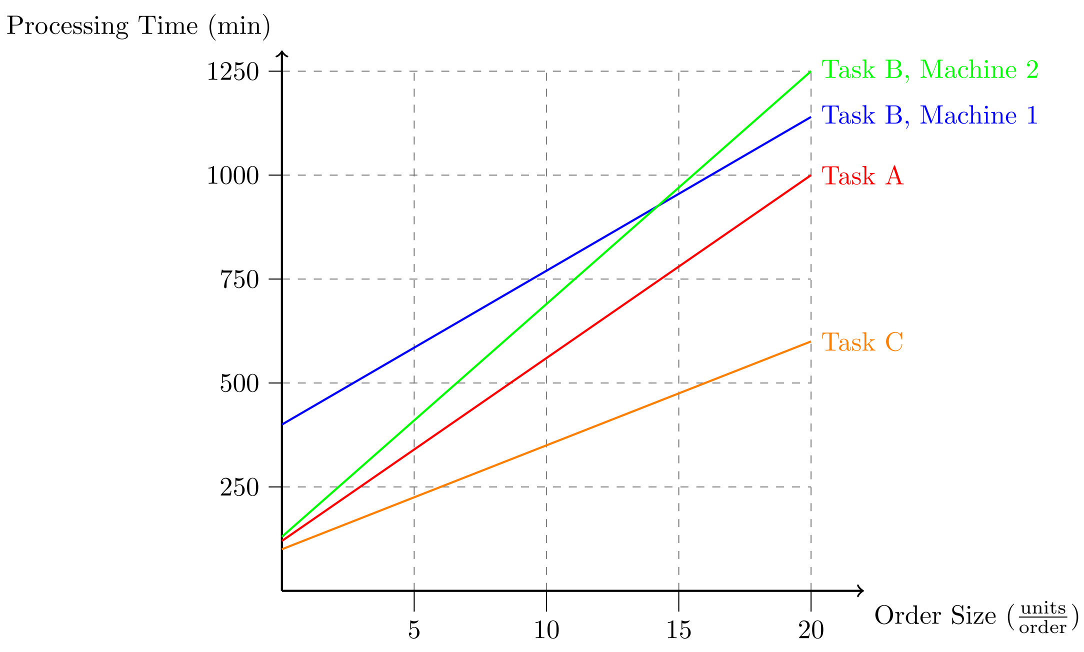
Graph A



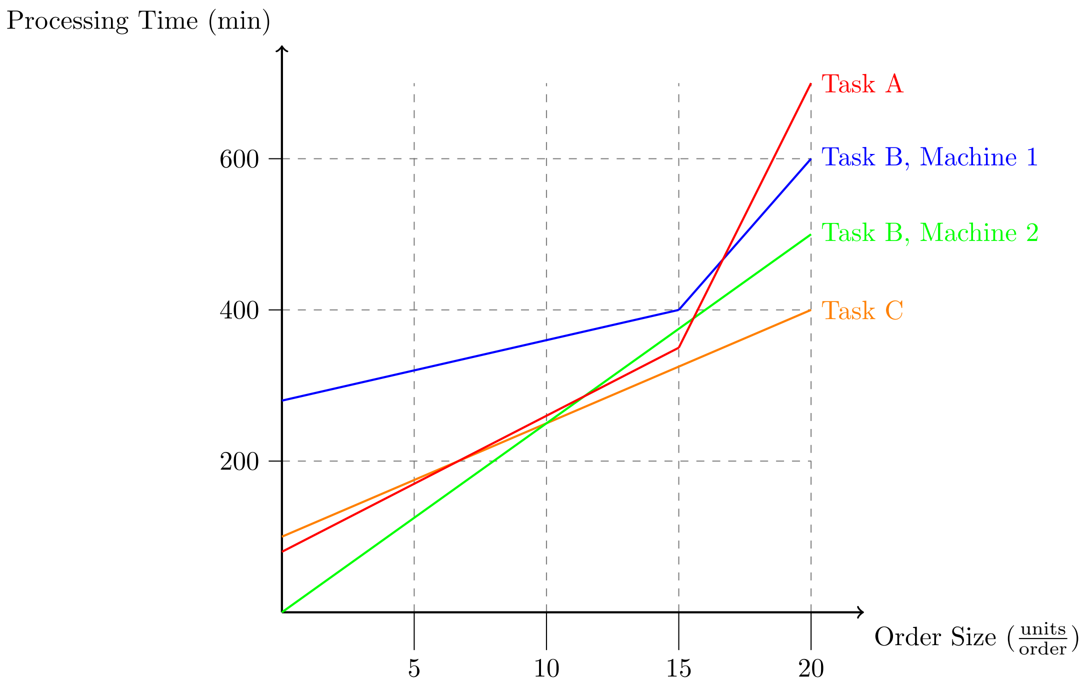
Graph B



Graph C



Graph D


Processing Time (min)
600
400
200
5
10
15
20
Order Size ($\frac{\text{units}}{\text{order}}$)
Task A
Task B, Machine 1
Task B, Machine 2
Task C