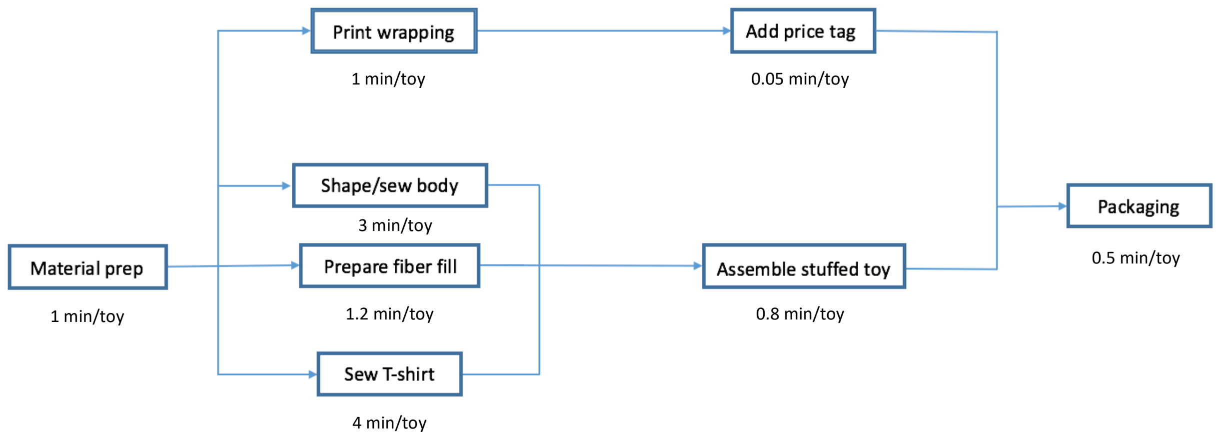
RC TOM SAMPLE QUIZ QUESTIONS FOR PRACTICE

(WITH SOLUTIONS)

Note: The midterm quiz will contain more questions and will cover additional concepts than the sample questions in this document. The quiz will cover materials up to and including the Toyota case.

Part A: Stuffed Animal Production

Yessica owns and operates a small toy factory in Peru. She has hired you as a consultant to assess a production process that produces stuffed animals. The production process is summarized in the process flow diagram below (with processing time below each step). Each of the eight workstations is staffed by a single, specialized worker dedicated to just that workstation (that is, there are eight workers in total, none of whom are cross-trained). The process begins each day without work-in-progress inventory, and there are no buffers in the process where work-in-process could accumulate.



1. How much time does it take to complete the first unit of the day?

a. 2.55 minutes
b. 4 minutes
c. 6.05 minutes
d. 6.3 minutes
e. 16 minutes

Answer: (d) 6.3 minutes

This question asks us to find the throughput time for the first unit of the day to go through the system. To do so, we trace the longest duration path through the process flow diagram to ensure that there is time for every operation to be completed. This longest path through the flow diagram is material prep → sew T-shirt → assemble stuffed toy → packaging, which has a total time of 1 minutes + 4 minutes + 0.8 minutes + 0.5 minutes = 6.3 minutes.

2. When the process is in steady state, what is the capacity of the entire process?

a. 10 toys/hour
b. 15 toys/hour
c. 20 toys/hour
d. 30 toys/hour
e. 60 toys/hour

Answer: (b) 15 toys/hour

The bottleneck step is the “sew T-shirt” workstation because it has the longest cycle time, and thus it sets the pace of the process. Its cycle time is 4 minutes per toy. Converting this output rate to “toys per hour” yields an output rate of: (1 toy/4 minutes) * (60 minutes/hour) = 15 toys/hour.

3. What is the labor content of a finished toy?

a. 4 minutes
b. 10.55 minutes
c. 6.3 minutes
d. 11.05 minutes
e. 11.55 minutes

Answer: (e) 11.55 minutes.

The direct labor content is the total time spent by all workers on a single product. If we sum up all workers’ time spent on a single toy, we have 1 minute + 1 minute + 0.05 minutes + 3 minutes + 1.2 minutes + 4 minutes + 0.8 minutes + 0.5 minutes = 11.55 minutes.

4. When the process is in steady state, what is the highest labor utilization of any single worker on the production process?

a. 75%
b. 80%
c. 90%
d. 100%
e. Not enough information to determine

Answer: (d) 100%

Recall that the bottleneck step determines the cycle time of the process, which in turn determines the labor time available for each worker. Here, the “sew T-shirt” workstation is the bottleneck, with a processing time of 4 minutes. Thus every worker has 4 minutes to complete his or her task.

Recall the definition of labor utilization:

Utilization for a worker = (the worker’s direct labor content)/(the total labor time available for that worker). For example, the utilization of the worker at the “material prep” station is = (1 minute)/(4 minutes) = 0.25, or 25%.

The worker at the bottleneck station is working 4 minutes out of 4 available minutes, that is (4 minutes)/(4 minutes) = 1 or 100% of the time.

5. When the process is in steady state, what is the average labor utilization of all workers on the production process?

a. Approximately 25%
b. Approximately 30%
c. Approximately 34%
d. Approximately 36%
e. Approximately 50%

Answer: (d) Approximately 36%

One way to calculate the average labor utilization across the entire process is to calculate the total direct labor content of *all* workers per cycle, and divide that by the total labor time available for *all* workers per cycle. We have already calculated the cycle time for the process as 4 minutes, so the labor time available for *each* worker is 4 minutes. Thus, the total labor time available for all workers per cycle is 8 workers * 4 minutes per worker. We know from previous calculation that the direct labor content of all workers per cycle is 11.55 minutes.

Avg. utilization = (11.55 minutes) / (8 workers * 4 minutes per worker)
= 11.55 minutes / 32 minutes
= 0.3609 or about 36%

An alternative approach is to calculate the utilization of each of the 8 workers (that is, their individual working times divided by the previously calculated available time for each of 4 min) and then calculate the average of the 8 workers. For example, the utilization of the material preparation worker is 1 minute of working time divided by 4 minutes of available time, or 25%. The individual worker utilization percentages of the various workers is 25% + 25% + 1.3% +75% + 30% + 100% + 20% + 12.5% which total 288.8%. Dividing this sum by 8 workers to calculate the average yields 36.1%.

6. For this question only, assume Yessica is considering sending as many as three different workers to a training program to improve their efficiency. Though expensive, this training would enable them to halve the processing time of their particular processing step. Assuming Yessica's goal in sending workers to the training is to maximize the capacity of the production process, which workers should she send to this training program? Note: A particular worker can be sent to this training only one time.

a. The three workers operating the sew T-shirt, assemble stuffed toy, and packaging tasks.
b. The three workers operating the sew T-shirt, shape/sew body, and prepare fiber fill tasks.
c. The two workers operating the shape/sew body and assemble stuffed toy tasks.
d. The two workers operating the sew T-shirt and shape/sew body tasks.
e. The two workers operating the material prep and packaging tasks.

Answer: (d) The two workers operating the sew T-shirt and shape/sew body tasks.

Sending these two workers to training will maximize the capacity of the production process. Paying to send a third worker for training beyond these two would be a waste because it would not further increase capacity.

Let us analyze this one worker at a time. We are looking to determine whether the overall process output rate would increase if a worker worked twice as quickly, considering that as many as two other workers might also be trained to work twice as quickly. The first worker to consider sending to training is the one staffing the bottleneck operation, the "sew T-shirt" step, which has the longest processing time of 4 minutes. After training, this worker would now have a processing time of 2 minutes and would no longer be the bottleneck. The new bottleneck would be the 3 minute station doing the "shape/sew body" operation. If this worker were trained, the processing time for "shape/sew body" would drop to 1.5 minutes.

If both of these workers were trained, the bottleneck would shift back to the "sew T-shirt" step, which, at its new 2 minutes processing time, would now be the slowest step and thus would determine the capacity of the entire process. Since we can't send the same worker to training twice, it would not make sense to send any other worker to training because reducing any of their processing times would not increase the overall capacity, which (after the "sew T-shirt" and "shape/sew body" workers were trained, would be set by the 2 minute bottleneck step ("sew T-shirt"). For example, while training the "prepare fiber fill" worker (the step with the next highest processing time among workers who hadn't been trained yet) would reduce the processing time at that station from 1.2 minutes to 0.6 minutes, it would not improve capacity of the overall process which will still be set by the 2 minute bottleneck step. Thus, Yessica should not pay to train a 3rd worker and option (d) is the correct answer.

7. Yessica decided not to send any workers to the training mentioned above. Instead, she is considering introducing a radical new labor policy that would pay workers based only on the time they are working on products (that is, adding labor content to products) as specified in the process flow diagram, but not for idle time. To implement this new wage scheme, she would invest in technology that could detect when workers are working productively and when they are idle. If Yessica implemented this scheme by paying each of her workers 50 Peruvian Soles per hour of labor content they add to products, how much would she pay in total to the eight workers *per hour the factory operates*, assuming steady state?

a. About 50 Soles per hour the factory operates
b. About 121 Soles per hour the factory operates
c. About 144 Soles per hour the factory operates
d. About 189 Soles per hour the factory operates
e. About 280 Soles per hour the factory operates

Answer = (c) About 144 Soles per hour the factory operates

This question proposes a scenario where workers are paid according to their direct labor content. We know the labor content is 11.55 minutes per unit, and that 15 units are produced per hour. Thus, for each hour of production, Jessica would pay for (15 units) * (11.55 minutes of labor content/unit) = 173.25 minutes, or (173.25 minutes)/(60minutes/hour) = 2.8875 hours of labor content. Paying 50 Soles per hour of labor content for 2.8875 hours of labor content that occurs per hour the factory operates means she would pay 144.4 Soles per hour the factory operates (that is, 50 Soles/hour * 2.8875 labor content hours/hour = 144.4 Soles, or about 144).

Another way to approach this is to multiply the 50 Peruvian Soles per hour of labor content by the labor utilization ratio (calculated above to be 36%) and then by the number of workers, that is, (50 Soles/hour of labor content per worker) * 36% * (8 workers) = 144 Soles/hour. (If you consider utilization to be 0.3609, this yields 144.36 Soles/hour, or about 144 Soles/hour.)

Part B: Fidget Spinner Production

Sourobh reads an exposé in *Poets and Quants* about the new craze sweeping business schools across the world: fidget spinners. Looking for ways to supplement his doctoral stipend, Sourobh convinces a reluctant fellow doctoral student, Meitong, to enter into business with him to produce and sell fidget spinners to the MBA Class of 2019. Sourobh argues that the spinners are mechanically simple and easy to produce and assemble, yielding the promise of a potentially profitable venture.

Meitong locates an abandoned machine shop in Cambridge to produce the spinners using a three-step production process. First, an injection molding machine creates the spinner's plastic body. Second, high-precision ball bearings are inserted into the plastic body using a "press fit" machine. Third, a machine places stickers on the assembled body. The shop processes each order as a batch. Because the machine shop is small, there is no space for buffers between these three steps. Sourobh and Meitong seek to design the process to maximize its capacity.

The shop employs four full-time workers who set up machines; their training enables them to set up only their own assigned machine. Each machine has to be set up by its worker for each new order, during which time that machine cannot produce units. Once set up, the machines are automated and do not require labor to run.

The Create Body station has one injection molding machine with one worker. There are two identical Insert Bearing stations, each of which has one press fit machine and one worker. There is one Add Stickers station with one worker.

Units are passed between stations as a batch. Once the batch has been processed at the Create Body station, it is passed to the two Insert Bearing stations, where the batch is divided equally between the two stations. Once the batch has completed the Insert Bearing step, the batch is moved to the Add Stickers station, where the units are finished.

The setup and run times of each station are as follows:

	Setup time	Run time
Create Body	45 minutes/batch	0.1 minutes/unit
Insert Bearings (each station)	25 minutes/batch	0.5 minutes/unit
Add Stickers	10 minutes/batch	0.7 minutes/unit

8. If every batch is of the same size, 30 units, what is the bottleneck of the operation?

a. Create Body
b. Insert Bearings
c. Add Stickers
d. None of the above

Answer: (a) Create Body

Since the material handling for this process occurs at the batch level, we must calculate the bottleneck by analyzing the processing time per batch, which consists of the total setup time and total run time for each station. Let's look at the time it takes each operation to complete a batch of 30 units, as specified in the question.

Create Body

Setup time: 45 minutes/batch

Run time: (30 units/batch) * (0.1 min/unit) = 3 min/batch

Total time: 45 min/batch + 3 min/batch = 48 min/batch

Insert Bearings (*both* stations)

Setup time: 25 min to set up each of the two stations for each batch, which occurs in parallel because each machine has its own worker.

Run time: (30 units/batch) * (0.5 min/unit/station)/2 stations] = 7.5 minutes/batch (in other words, each machine processes in parallel half the batch, or 15 units, at 0.5 min/unit, which takes 7.5 minutes)

Total time: 25 min/batch + 7.5 min/batch = 32.5 min/batch

Add Stickers

Setup time: 10 minutes/batch

Run time: (30 units/batch) * (0.7 minutes/unit) = 21 min/batch

Total time: 10 min/batch + 21 min/batch = 31 min/batch

The Create Body operation takes the longest, so it is the bottleneck if all batches are 30 units.

9. If every batch is of the same size, 30 units, what is the labor content of each batch?

a. 80 minutes
b. 94 minutes
c. 105 minutes
d. 144 minutes
e. 192 minutes

Answer: (c) 105 mins

Note the distinction between setup time and run time in how this process uses labor. As stated in the question, the setups require labor, but when the machine is running there is no labor involved. So the labor content for each batch only includes the setup times. The total labor content of each batch is calculated by summing the total time each worker spends to setup their own machine to process a batch. Thus the labor content for each batch is:

45 min + 25 min + 25 min + 10 min = 105 min.

10. Suppose Sourobh is considering increasing the output rate of the overall process by upgrading one of these steps to reduce its cycle time. For which of these steps is investing in an increase in output rate *never* worthwhile, regardless of batch size?

a. Add Stickers
b. Create Body
c. Create Body and Add Stickers
d. Insert Bearings
e. None; investing in upgrading any of these steps would increase the output rate of the overall process

Answer: (d) Insert Bearings

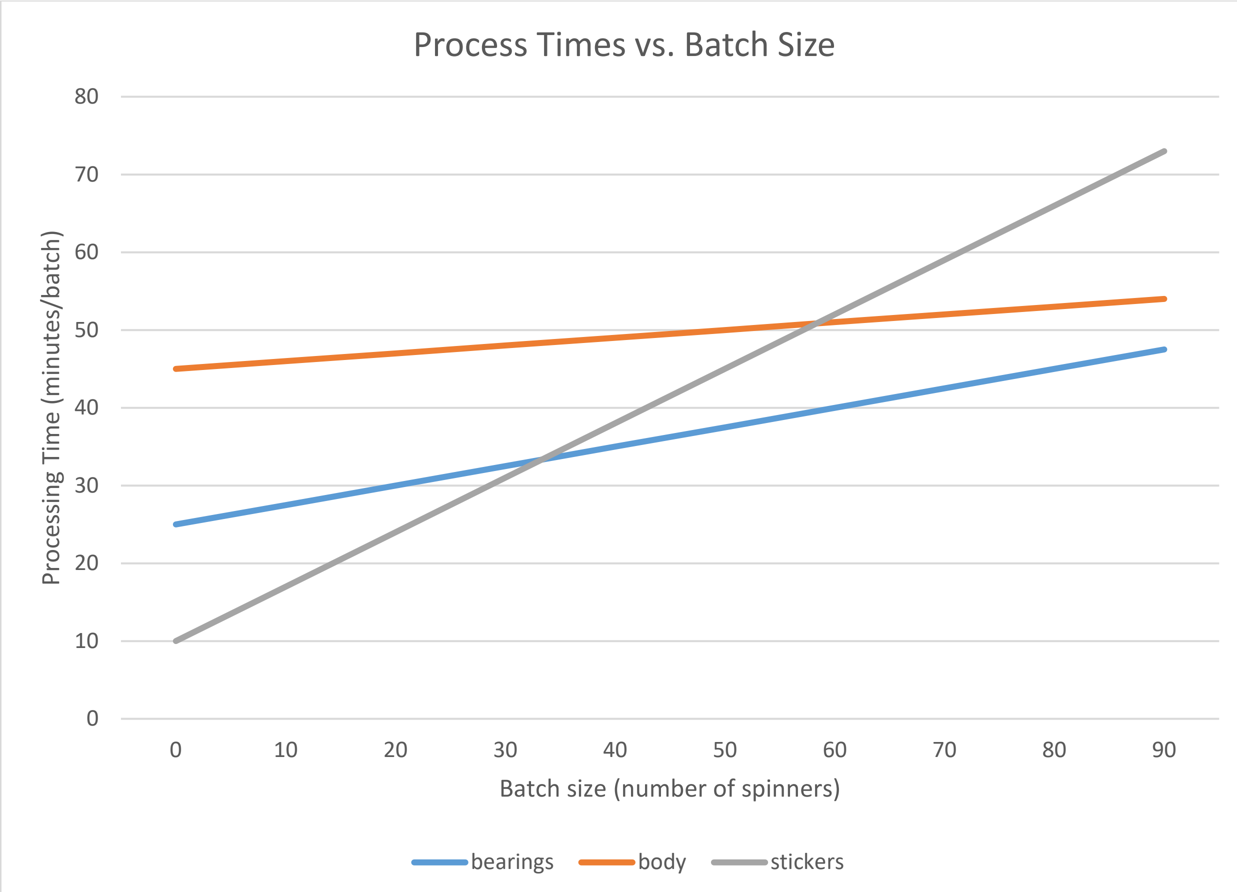
To improve the output rate of the overall process, we first need to identify the bottleneck(s), the step that limits the output rate of the process. Thus, this question is essentially asking us to identify which steps, if any, will never be the bottleneck, since improving a non-bottleneck operation's efficiency would be a wasted investment.

To identify the bottleneck(s) of the process, we first find each station's cycle time as a function of batch size (in terms of minutes/unit). Note that each batch is split evenly between two Insert Bearings stations. Each of those machines require 25-minute setup times that can occur in parallel, and their run times can also occur in parallel. After being set up, each machine's run time is 0.5 minutes/unit, so the two stations could together process a unit every 0.25 minute (that is, (0.5 min/unit/station)/2 stations), so their collective run time is 0.25 seconds/unit.

	Setup time	Run time
Create Body	45 min	0.1 minutes/unit
Insert Bearings (*both* stations)	25 min	0.25 minutes/unit
Add Stickers	10 min	0.7 minutes/unit

Solution Method 1, Graphical Approach:

Next, for each step we can plot three equations for the processing time per batch for batches of different size. The time, Y, for a step to complete a batch of X units is setup time + (run time * *X)*. We plot Y = 45 + (0.1 * *X)* (for the Create Body step), Y = 25 + (0.25 * *X)* (for the Insert Bearings step), and Y = 10 + (0.7 * *X)* (for the Add Stickers step):



Note that the line representing the Insert Bearings step never has the maximum processing time per batch, regardless of batch size. Thus, it is never the bottleneck at any batch size, and investing in an upgrade to this step (whether one or both machines, and whether setup time and/or run time) would never reduce the cycle time of the line.

Solution Method 2, Analytical Approach:

Instead of a graphical approach, we can solve for intersection points to see where the bottleneck is and where it may move. First, we find the intersection points for the three equations representing the three steps in our process. We have Y = 45 + (0.1 * *X)* (for the Create Body step), Y = 25 + (0.25 * *X)* (for the Insert Bearings step), and Y = 10 + (0.7 * *X)* (for the Add Stickers step).

Setting Create Body = Insert Bearings:
$45 + 0.1X = 25 + 0.25X$
$20 = 0.15X$
$X = 133.3$ units

Setting Create Body = Add Stickers:
$45 + 0.1X = 10 + 0.7X$
$35 = 0.6X$
$X = 58.3$ units

Finally, Insert Bearings = Add Stickers:
$25 + 0.25X = 10 + 0.7X$
$15 = 0.45X$
$X = 33.3$ units

Having calculated each of the intersection points, we can find the processing times for batch sizes at the corresponding intersection points to see if/how the bottleneck may move in the table below:

Intersection Point/ Batch Size	**Create Body Processing Time**	**Insert Bearings Processing Time**	**Add Stickers Processing Time**	**Bottleneck?**
X = 33.3 units	48.3 min	33.3 min	33.3 min	Create Body
X = 58.3 units	50.8 min	39.6 min	50.8 min	Create Body & Add Stickers
X = 133.3 units	58.3 min	58.3 min	103.3 min	Add Stickers

The first column shows the intersection point in consideration, while the Create Body, Insert Bearings, and Add Stickers columns show the processing times for each of those respective steps at that given batch size. Finally, the rightmost column shows the bottleneck at that point, which is the step with the highest processing time.

The table shows that at X = 33.3 units, there is only one bottleneck, the Create Body step, which has the highest processing time (48.3 min). After that, the point X = 58.3 units has two steps with the highest processing time (of 50.8 min), Create Body and Add Stickers, demonstrating that they are both the bottleneck at that step. The fact that there are two bottlenecks at this step signifies a switching of the bottleneck, from Create Body to Add Stickers. This is confirmed at X = 133.3 units, where the highest processing time (103.3 min) is at the Add Stickers step, which is clearly the bottleneck for that batch size.

In summary, the table above demonstrates that from batch sizes of X = 0 to X = 58.3 units, the bottleneck is the Create Body step. For batch sizes of X = 58.3 and greater, the bottleneck is at the Add Stickers step. Thus, we find that the Insert Bearings step is never the bottleneck. Therefore, Sourobh should never invest in an upgrade to that step.

11. Compared to the bottleneck associated with a batch of 30, for which of the following batch sizes will the bottleneck be a different step?

a. 52 units
b. 71 units
c. 82 units
d. 71 units and 82 units
e. None of the above

Answer: (d) 71 units and 82 units

Solution Method 1, Graphical Approach:

Visually inspecting the graph from the prior solution, the Create Body step is the bottleneck for smaller batches, and the Adding Stickers step is the bottleneck for larger batches. To calculate the point at which the bottleneck switches, we consider the two equations reflecting their processing times in terms of batch sizes, and set them equal:

$45 + 0.1X = 10 + 0.7X$
$0.6X = 35$
$X = 58.333....$

Thus, for batches of 58 or fewer units—including the batch of 30 stated in the question, and of 52 units in (a) —the bottleneck is the Create Body step. For batches of 59 or more units—including batches of 71 and 82 units as indicated in (d) —the bottleneck is the Adding Stickers step.

Solution Method 2, Analytical Approach:

We have already found the intersection points and how/where the bottleneck shifts in the analytical solution approach to the prior problem. For batches of less than 58.3 units—including the batch of 30 stated in the question, and of 52 units in (a) —the bottleneck is the Create Body step. For batches of 58.3 or more units—including batches of 71 and 82 units as indicated in (d) —the bottleneck is the Adding Stickers step. Thus, the solution is option (d).

For the next three questions, please consider the following: Meitong convinced Sourobh that they should trade in their injection molding machine and pay $1,000 to get a 3D printer to perform the Create Body step. This enabled them to produce a fancier spinner they called a whirligig. After conducting some market research, they expected the whirligig to yield an additional $1 in gross profit per unit. The 3D printer has a setup time of four minutes per batch and a run time of 0.8 minutes/unit. Like the other machines, the 3D printer has to be setup by its worker for each new order, during which time the 3D printer cannot produce units. Once set up, the 3D printer is automated and does not require labor to run. To simplify operations, they produce whirligig orders in batches of 20 units of the same design.

12. What is the hourly capacity of the production line with the new 3D printer performing the Create Body step?

a. 30 units per hour
b. 40 units per hour
c. 50 units per hour
d. 60 units per hour
e. 70 units per hour

Answer: (b) 40 units per hour

We first consider how (if at all) the addition of the 3D printer influences which step is now the bottleneck. Recall from the problem setup that each order is produced as a batch, thus an order size of 20 is produced as a batch of 20. With batch sizes of 20, the processing time for the new 3D printer is 4 min + 0.8 min/unit * (20 units) = 20 minutes. The next potential bottleneck is the Insert Bearings step, with a processing time of 25 + 0.25 min/unit*(20 units) = 30 minutes/batch. Thus, the Insert Bearings step is now the bottleneck.

At 30 minutes/batch, the line now produces two batches/hour, or 2 batches/hour * 20 units/batch = 40 units/hour.

13. After fulfilling the aforementioned whirligig orders, Meitong and Sourobh begin to consider whether they should reduce the fixed production order size, but they disagree on the operational benefits of the smaller batches that would result. They ask you to help resolve their argument: which of the following would occur if the fixed production order size (and thus batch size) were cut in half from 20 units/batch to 10 units/batch?

a. The production process's capacity in terms of units per hour would increase
b. The direct labor content *per unit* would decrease
c. The labor utilization would decrease
d. The throughput time *per batch* would decrease
e. None of the above

Answer: (d) The throughput time per batch would decrease

Let's consider each possibility in turn:

a) For this process that has setup and run times, cutting the batch size in half would double the number of setups. While this could result in more batches being produced per hour, the line's capacity in terms of *units* per hour would not increase because these additional setups would decrease time available for run time when units are effectively being produced.

b) In this process, setups are performed with direct labor but run time is automated and does not require any labor. As explained for option a, smaller batches result in more time being spent on setups, which *increases* direct labor content *per unit*.

c) Recall that direct labor utilization is the percentage of time that workers are actually working on the products, and is calculated as direct labor content divided by total available labor time. We can calculate direct labor utilization here on a *per batch* basis.

- For this process, the direct labor content per batch is the sum of the setup times per batch; the run time has no labor content. As such, the direct labor content per batch for this process does not vary by batch size.
- For this process, total available labor time per batch consists of both direct labor content per batch and idle time per batch. Because workers are idle during this process's run times, the idle time per batch is the sum of the run times per batch. As the batch size decreases from 20 units to 10 units, the sum of the run times per batch decreases, and thus idle time per batch decreases. As a result, the decrease in batch size reduces total available labor time.

Thus, when considering how a decrease in batch size affects direct labor utilization, note that direct labor utilization's numerator (direct labor content per batch) is unchanged, but its denominator (total available labor time) becomes smaller. As a result, direct labor utilization increases as the batch size decreases.

d) Producing in batches means the entire batch is processed in a single step before advancing to the next step. Reducing batch size reduces the processing time at each step, and thus the throughput time for the entire line. A common reason to reduce batch size is to decrease its throughput time, which enables operations to be more responsive to changing customer demands.

Part C: Edison Motors Assembly

The final assembly process of the Edison Motors Model Y electric car is conducted using a conveyor-paced assembly line that consists of five stations, each of which is five meters long. Each station has one worker. In this process, cars move along the conveyor at equally spaced increments.

As shown in the table below,

- the worker at Station 1 performs Tasks 1–3,
- the worker at Station 2 performs Tasks 4–7,
- the worker at Station 3 performs Tasks 8–11,
- the worker at Station 4 performs Tasks 12–15, and
- the worker at Station 5 performs Tasks 16–19.

Each worker performs his or her tasks sequentially, and an individual worker cannot perform a task in parallel with any of their other tasks. Management sets the speed of the conveyor to maximize the output rate of assembly process while maintaining the task allocations and task times (in seconds) depicted in the table below.

Station (Worker)	Task	Time (seconds)
1	1	80
	2	25
	3	10
2	4	70
	5	75
	6	80
	7	15
3	8	50
	9	15
	10	45
	11	45
4	12	35
	13	30
	14	35
	15	30
5	16	15
	17	5
	18	65
	19	115

14. What is the direct labor content of a car during the final assembly process?

a. 720 seconds
b. 840 seconds

c. 1,080 seconds
d. 1,200 seconds
e. 1,440 seconds

Answer: (b) 840 seconds

Recall that direct labor content refers to the amount of time workers actually spend producing a unit. In this case, the total direct labor content per car is the sum of the total task times for the five stations. We calculate the total task time for each worker on each station, as follows:

Station 1: 80 + 25 + 10 = 115 seconds

Station 2: 70 + 75 + 80 + 15 = 240 seconds

Station 3: 50 + 15 + 45 + 45 = 155 seconds

Station 4: 35 + 30 + 35 + 30 = 130 seconds

Station 5: 15 + 5 + 65 + 115 = 200 seconds

Total direct labor content per car = 115 + 240 + 155 + 130 + 200 = 840 seconds.

15. What is the throughput time of a car going through the final assembly process?

a. 720 seconds
b. 840 seconds
c. 1,080 seconds
d. 1,200 seconds
e. 1,440 seconds

Answer: (d) 1,200 seconds

The throughput time for the final assembly process is the amount of time required for a car to pass through all of the five stations – start to finish. Since this is a conveyor paced line, the conveyor can operate *no faster* than the amount of time the bottleneck station worker requires to complete all assigned tasks.

Here, because management sets the speed of the conveyor to maximize its output rate, the conveyor speed will *equal* the amount of time the bottleneck station worker requires to complete all assigned tasks. From our analysis of the prior question, we know that Station 2 takes the most time (240 seconds), and hence it is the bottleneck for the assembly process. We can thus note that each car spends 240 seconds at each station, and thus the throughput time of the final assembly process is (5 stations) * (240 seconds/stations) = 1,200 seconds.

An alternative approach is to recognize that since Station 2 sets the pace of the assembly process, the cycle time of the assembly process will be 240 seconds per car. We also know that WIP is 5 cars (one car per station). We can therefore use Little's Law to calculate the throughput time for a car to pass through the assembly process:

TPT = WIP * CT = (5 cars) * (240 seconds/car) = 1,200 seconds.

16. What is the hourly capacity of the final assembly process?

a. 15 cars/hour
b. 18 cars/ hour
c. 20 cars/ hour
d. 24 cars/ hour
e. 30 cars/ hour

Answer: (a) 15 cars/hour

The capacity of the assembly process is determined by the cycle time of the bottleneck station. Recall that average cycle time is the average time between the completion of successive units. The prior question identified cycle time as 240 seconds, or 4 minutes. Thus, in one hour (60 minutes) the capacity, or maximum output rate, is (60 min)/(4 min/car) = 15 cars.

Alternatively, we can use Little's Law, which indicates that WIP = TPT*OR, and thus OR = WIP/TPT. We know from the prior question that TPT = 1,200 seconds or 1/3 hour, and WIP is 5 cars. Thus OR = (5 cars)/(1/3 hour) = 15 cars/hour.

17. What is the labor utilization of the final assembly process?

a. 68%
b. 70%
c. 72%
d. 85%
e. Approximately 100%

Answer: (b) 70%

Labor utilization is the total direct labor content divided by the total labor time available. One way to calculate this is to consider the time it takes for one car to pass through the five-step assembly line.

Labor time available per car is (240 seconds/worker) × (5 workers) = 1,200 seconds per car, that is, the labor time available per car is the process throughput time since there is one worker per station and there are no parallel processes.

In the answer to question 21, we computed the total direct labor content per car above as the sum of the station task times:

115 + 240 + 155 + 130 + 200 = 840 seconds/car.

Thus, the labor utilization is (840 seconds/car)/(1,200 seconds/car) = 0.7 or 70%, which is choice (b).

18. For this question only, you can reallocate one task from one worker to another (adjacent) worker—without changing the sequence of the tasks or the workers—in order to maximize the capacity of the final assembly process. What would the capacity of the final assembly process be after you reallocated that one task?

a. Approximately 4 cars/hour
b. Approximately 16 cars/hour
c. Approximately 18 cars/hour
d. Approximately 19 cars/hour
e. Approximately 21 cars/hour

Answer: (c) 18 cars/hour

Capacity can be increased only if you can shorten processing time of the bottleneck station (Station 2) to less than 240 seconds (4 minutes), since that determines the cycle time of the entire five-step line. Given the constraints of the problem that prohibit changing the sequence of the tasks, you have two options: A) you could shift Task 4 to Station 1, or B) shift Task 7 to Station 3.

Option A: If you shift Task 4 from Station 2 to Station 1, the resulting processing time for Station 1 becomes 80 + 25 + 10 + 70 = 185 seconds and for Station 2 becomes 75 + 80 + 15 = 170 seconds. After calculating the total task time for each station, we observe that Station 5 has the largest processing time of 200 seconds (3.33 minutes) and so it would become the bottleneck.

Option B: If you shift Task 7 from Station 2 to Station 3, the resulting processing time for Station 2 becomes 70 + 75 + 80 = 225 seconds and for Station 3 becomes 15 + 50 + 15 + 45 + 45 = 170. After calculating the total task time for each station, we observe that Station 2 would still have the largest processing time of 225 seconds (3.75 minutes) and it would remain the bottleneck.

Option A yields a shorter processing time of the bottleneck station, which results in a higher capacity for the final assembly process. Thus, we should choose to shift Task 4 to Station 1. The capacity of the line is its maximum output rate, which we can compute as (60 minutes/hour)/(3.33 minutes/car) = 18 cars/hour.

Alternatively, we can compute the capacity as the inverse of the final assembly process' cycle time, thus 1 / (200 seconds per car) = 1/200 cars per second. We convert this capacity to "cars per hour" as 1/200 car per second * 60 seconds/minute * 60 minutes/hour = 18 cars/hour.

19. For this question only: Suppose that the plant manager wanted to maximize labor utilization of the final assembly process and discovered that the activities completed in the 19 tasks could be divided into any number of small subtasks, where the total processing time of these subtasks is the same as the total processing time of the original 19 tasks. For example, task 1 (80 seconds) could be divided into a task 1a that required 30 seconds, to be followed by a task 1b that required 50 seconds. Note that the processing times of the subtasks must exactly add up equal the original task's processing time (here, 30 seconds + 50 seconds = 80 seconds).

These subtasks created by dividing up the original tasks would be assigned to five stations, each of which would be staffed by one worker. Each of the 5 workers would continue to work at just one station, and the overall sequence of the activities would be required to remain the same as the sequence in the original configuration.

What would the final assembly process's capacity be if these adjustments were made to maximize labor utilization?

a. Approximately 20 cars/hour
b. Approximately 21 cars/hour
c. Approximately 30 cars/hour
d. Approximately 33 cars/hour
e. Approximately 60 cars/hour

Answer: (b) Approximately 21 cars/hour

To maximize labor utilization, we would minimize the total idle time across the five workers. Since we can split tasks into as many small subtasks as we want, we can split the total task time evenly across the five stations. Since the processing times are the same at each station, all stations would have the same cycle time and the total idle time would be zero. The cycle time per station is thus the sum of the processing times divided by the five stations, or (840 seconds)/(5 stations) = 168 seconds/station. This means the conveyor-paced line would operate at 168 seconds per station, and that the output rate of the assembly process would be one car every 168 seconds (or 2.8 minutes). Thus, the hourly capacity of the process would be (60 min/hour)/(2.8 min/car) = 21.4 cars/hour, which is approximately 21 cars/hour.

Part D: Airport Security

You have been hired as a consultant to assess Harvard Airport's airline passenger screening. Passengers seeking to enter the gate area of a terminal must pass through entry lines that consist of a four-step security screening process. Each step can process only one passenger at a time. For each entry line in this airport, there is no space in between steps in which a passenger can wait.

In Terminal 1, each passenger goes through the following four steps:

1. The passenger approaches an airport worker who verifies that their ticket's gate is in Terminal 1.
2. The passenger then approaches a security officer who examines their government ID to verify their identity.
3. The passenger proceeds to the Bag Inspection step, where an officer inspects their carry-on luggage.
4. After the Bag Inspection station, the passenger proceeds to the last step, Security Pat Down, where a security guard passes their hands over the passenger's body to search for prohibited items.

These steps occur in this fixed sequence (for example, a passenger's security pat down occurs only after the passenger completes Bag Inspection), and a passenger cannot exit their current step to enter the next step until the passenger at the next step has left it. Assume that there is always a queue of passengers waiting to enter the Ticket/Gate Verification process.

Terminal 1 has two entry lines, Entry Line A and Entry Line B, and passengers can choose to use either one. Each of these entry lines has one worker dedicated exclusively to each step, and thus each entry line has 4 workers. The steps and processing times are provided in the table below. The time required to complete each task at each entry line varies uniformly around the average time noted. For example, at Entry Line A, for Ticket/Gate Verification, the mean processing time is 25 seconds, and any processing time between 20 seconds and 30 seconds is equally likely.

Step	**Terminal 1** **Entry Line A**	**Terminal 1** **Entry Line B**
Ticket/Gate Verification	25 $\pm$ 5 seconds/passenger	25 $\pm$ 3 seconds/passenger
Government ID Check	25 $\pm$ 5 seconds/passenger	25 $\pm$ 3 seconds/passenger
Bag Inspection	25 $\pm$ 5 seconds/passenger	25 $\pm$ 3 seconds/passenger
Security Pat Down	25 $\pm$ 5 seconds/passenger	25 $\pm$ 3 seconds/passenger

20. On average, can Entry Line A or Entry Line B process more passengers per hour?

a. Entry Line A
b. Entry Line B
c. The two entry lines can process the same number of passengers per hour

d. Not enough information to determine

Answer: (b) Entry Line B

Although the mean processing time for each step is identical across both entry lines, each step in Entry Line B has less variability. Recall from the Process Analytics Simulation exercise that all else equal, less variability in processing time translates to less blocking and starving, which in turn results in a greater output rate. Thus, Entry Line B has a higher average output rate and hence can process more passengers per hour.

21. Suppose the airport was able to create space in Entry Line A for one passenger to stand after completing the Bag Inspection while the Security Pat Down was still processing the passenger ahead of them. How would this influence the average output rate of Entry Line A?

a. It would increase its average output rate
b. It would decrease its average output rate
c. It would have no effect on its average output rate
d. Not enough information to determine

Answer: (a) It would increase its average output rate

The overlapping processing times between successive steps in Entry Line A will lead to blocking and starving. Recall from the Process Analytics simulation exercise that blocking and starving reduces the average output rate of a process. Buffers can reduce instances of blocking and starving. In this context, the place for passengers to stand after they have completed the Bag Inspection step acts as a buffer. It would sometimes free the Bag Inspection to process for the next passenger, eliminating a possible block. Also, by sometimes "storing" a passenger, the buffer can prevent some instances in which the Security Pat Down would have been starved. Because this buffer in Entry Line A would reduce blocking and starving, it would increase the average output of Entry Line A.

22. How would adding the standing space to Entry Line A described in the prior question influence the overall labor utilization of Entry Line A?

a. Overall labor utilization would increase
b. Overall labor utilization would decrease
c. Overall labor utilization would not change

Answer: (a) Overall labor utilization would increase.

In Entry Line A, blocking and starving results in idle labor. As the solution to the prior question indicated, adding this standing space would reduce instances of blocking and starving. That would result in less idle time for workers in Bag Inspection and Security Pat Down, and increase overall labor utilization.

Terminal 2 also has two entry lines: Entry Line Q and Entry Line R, and passengers can choose to use either one. These entry lines have the same steps as the entry lines in Terminal 1, but they have different processing times. The time required to complete each task at each entry line in Terminal 2 varies uniformly around the average time noted in the following table.

Step	**Terminal 2** **Entry Line Q**	**Terminal 2** **Entry Line R**
Ticket/Gate Verification	20 $\pm$ 7 seconds/passenger	22 $\pm$ 11 seconds/passenger
Government ID Check	38 $\pm$ 7 seconds/passenger	38 $\pm$ 7 seconds/passenger
Bag Inspection	27 $\pm$ 3 seconds/passenger	28 $\pm$ 4 seconds/passenger
Security Pat Down	15 $\pm$ 4 seconds/passenger	14 $\pm$ 3 seconds/passenger

23. On average, can Entry Line Q or Entry Line R process more passengers per hour?

a. Entry Line Q
b. Entry Line R
c. The two entry lines can process the same number of passengers/hour
d. Not enough information to determine the answer

Answer: (a) Entry Line Q

For Entry Line Q, the processing time of the Government ID Check step, which ranges from 31 to 45 seconds, always exceeds the processing time of all other steps. Thus, processing times of other steps do not overlap with the processing time of Government ID Check, and so Government ID Check is always the bottleneck step. Thus, the average cycle time of Government ID Check (38 seconds) is the average cycle time of the four-step process.

For Entry Line R, Government ID Check also has the highest average processing time of all steps, but the range of its processing time (31 to 45 seconds) overlaps with range of processing times of two other steps: Ticket/Gate Verification (11 to 33 seconds) and Bag Inspection (24 to 32 seconds). The overlap reduces the average output rate of Entry Line R. In some cases when Government ID Check processes especially quickly (e.g., at 31.1 seconds), the Ticket/Gate Verification and/or Bag Inspection steps will process especially slowly (e.g., at 31.5 seconds). Starving and blocking sometimes results from this overlap in the processing times between the bottleneck step and the steps before and after it (Ticket/Gate Verification, which can range from 11 to 33 seconds, and Bag Inspection, which can range from 24 to 32 seconds). Hence, the average cycle time of Entry Line R is *longer* than the average processing time of Government ID Check.

Entry Line Q's faster average cycle time than Entry Line R's means that Entry Line Q has a higher average output rate than Entry line R (that is, Entry Line Q processes more passengers per hour than Entry Line R.

The information in this paragraph applies only to the next two questions: The airport has the option to invest in worker training that would eliminate the variability in processing time for that worker's task and thus their step. For example, training Entry Line R's Bag Inspection worker would change that step's processing time from 28 ± 4 seconds per passenger to exactly 28 seconds per passenger. However, training is costly and justified only if it would increase the average output rate of the trained worker's entry line.

24. For Entry Line Q, how many workers should be trained?

a. None
b. One
c. Two
d. Three
e. Four

Answer: (a) None

For Entry Line Q, there is a lack of overlap between the processing time of the bottleneck step (Government ID Check) and any of the other steps. Because of this, Government ID Check is always the bottleneck for Entry Line Q. Because there is no overlap, Government ID Check is never blocked or starved. Thus, the bottleneck step operates as though it were operating in isolation, producing output on average every 38 seconds. Because Government ID Check is always the bottleneck, eliminating variability in any other step on Entry Line Q would not help increase its average output rate. Thus, the average output rate of Entry Line Q would not be increased by sending any of its workers to this training.

25. For Entry Line R, how many workers should be trained?

a. None
b. One
c. Two
d. Three
e. Four

Answer: (b) One

For Entry Line R, Government ID Check has the highest average processing time. Its range of processing times overlap with two other steps, the Ticket/Gate Verification step (which results in Government ID Check sometimes being starved), and the Bag Inspection step (which results in Government ID Check sometimes being blocked). For Entry Line R, eliminating the processing time variability for Government ID Check would remove the overlap in its processing time with these two other steps. By eliminating the blocking and starving of the Government ID Check, we help increase the line's average output rate. Therefore, sending Entry Line R's Government ID Check worker to training would increase the entry line's average output rate.

Once the Government ID Check worker is trained, that step's processing time would be exactly 38 seconds with no variability. In that case, examining Entry Line R's other steps reveals that no other step has a processing time range that exceeds 38 seconds, and so the Government ID Check will always be the bottleneck. Therefore, training any other Entry Line R worker would not further improve the line's average output rate. Thus, for Entry Line R, we would choose to send only the Government ID Check worker to training.

Part E: Apple Picking and Cider Production

It's fall in New England, which means it is peak season for Maciej's family farm to produce apple cider. The farm has a small facility that processes locally-grown apples to produce delicious cider. The process begins with workers unloading apple delivery trucks and depositing the apples into a holding bin that automatically feeds the apples into a pressing machine. The holding bin is large enough so that it is never full. The pressing machine's capacity to convert apples into finished cider is 100 kg of apples per hour. Assume that the pressing machine has no other inputs, and that there is always an ample supply of apples waiting in the delivery trucks.

In the morning from 9:00 am to 12:00 pm (noon), four of Maciej's brothers and sisters ("siblings") deposit apples into the holding bin, and each sibling works at a constant rate of 50 kg per hour. During 12:00 pm to 1:00 pm, all four siblings take a break and no apples are added to the holding bin while the pressing machine continues to run. At 1:00 pm, three of the siblings leave the cider production facility to carry out other work on the farm. Thus, from 1:00 pm to 5:00 pm, only one sibling deposits apples into the holding bin, working at a constant rate of 50 kg per hour. At 5:00 pm, when all of Maciej's siblings go home, the pressing machine continues processing any apples that remain in the holding bin and then automatically shuts off. The holding bin feeding the pressing machine starts empty each morning at 9:00 am, with no WIP from the previous day.

26. How many kilograms of apples are processed per day?

a. 500 kg
b. 600 kg
c. 700 kg
d. 800 kg
e. 900 kg

Answer: (d) 800 kg

Each day, the siblings collectively deposit the following amount of apples into the holding bin feeding the pressing machine:

Morning (9:00 am–12:00 pm): 3 hours * 50 kg/hour per sibling * 4 siblings = 600 kg

Afternoon (1:00–5:00 pm): 4 hours * 50 kg/hour per sibling * 1 sibling = 200 kg.

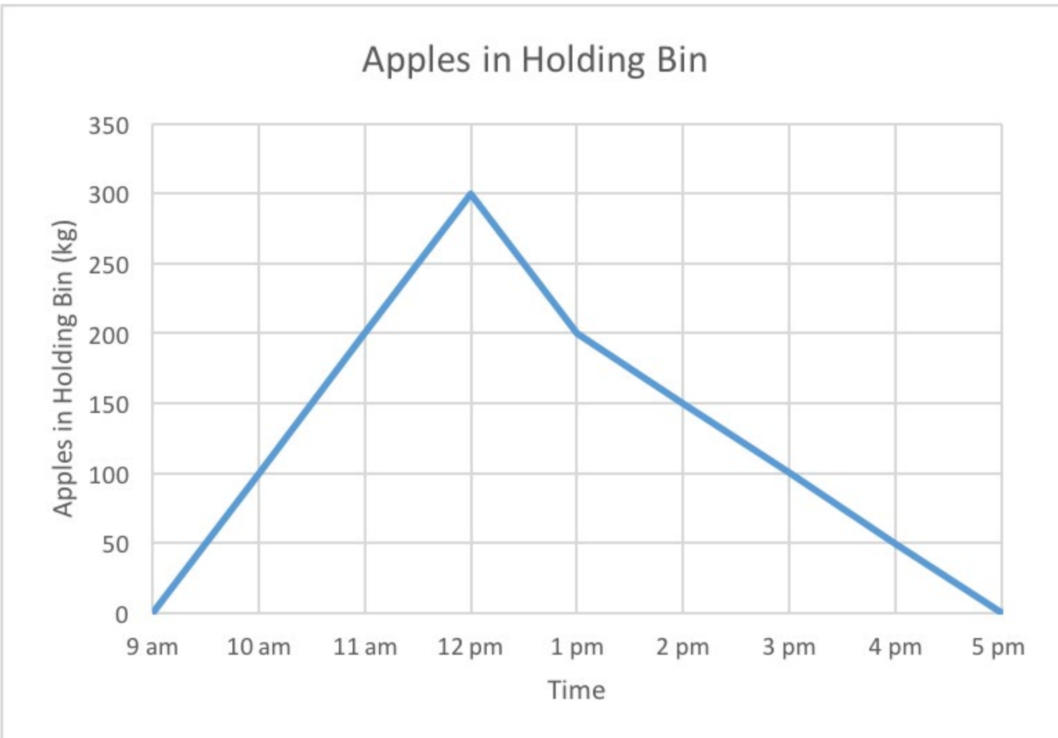
Since all of these apples are put into the pressing machine and converted to cider, the pressing machine processes 600 + 200 = 800 kg of apples per day.

27. What is the maximum amount (in kg) of apples that the holding bin holds at any particular moment on any given day?

a. 150 kg
b. 200 kg
c. 250 kg
d. 300 kg
e. 350 kg

Answer: (d) 300 kg

Let us begin by plotting the accumulation of apples over the course of the day:



In the morning (9:00 am to 12:00 pm), apples are being deposited in the holding bin at a rate of 50 kg/hour per sibling * 4 siblings = 200 kg/hour, and are being processed by the pressing machine at a rate of 100 kg/hour. Because more is being deposited in the holding bin than is being processed, the holding bin is accumulating at a rate of 100 kg/hour (that is, 200 kg/hour – 100 kg/hour). By 12:00 pm, three hours after this period started, 100 kg/hour * 3 hours = 300 kg of apples have accumulated in the holding bin.

From 12:00 pm to 1:00 pm, no apples are deposited into the holding bin, but the pressing machine continues to process apples at a rate of 100 kg/hour. Thus, the net accumulation rate in the afternoon is 0 kg/hour (deposit) – 100 kg/hour (pressing machine processing) = -100 kg/hour. Over the one hour, this reduces the accumulation in the bin by 100 kg (calculated as 100 kg/hour * 1 hour), from a total of 300 kg to 200 kg.

From 1:00 pm to 5:00 pm, one sibling returns to deposit apples in the holding bin, and thus the input rate is 50 kg/hour. Meanwhile, the pressing machine continues to process at 100 kg/hour. Thus, the net accumulation rate from 1:00 pm to 5:00 pm is 50 kg/hour (deposited) – 100 kg/hour (pressing machine processing) = -50 kg/hour.

Operating at a net accumulation rate of -50 kg/hour over the four hours between 1:00 pm and 5:00 pm, the pressing machine will consume 200 kg. This means that by 5:00 pm, the 200 kg of apples that were in the bin at 1:00 pm will have been processed, with no accumulated inventory remaining at 5:00 pm.

The maximum accumulation is 300 kg, which occurs at 12:00 pm.

28. What is the average inventory (in kg) of apples in the holding bin over the course of a day when the pressing machine is operating? Please round to the nearest 10 kg.

a. About 100 kg
b. About 110 kg
c. About 120 kg
d. About 140 kg
e. About 160 kg

Answer: (d) About 140 kg

One way to calculate the average inventory level throughout the day is to calculate how many kg-hours of inventory exists in the bin throughout the day, and then divide this by the number of hours when the pressing machine is operating. Note that the pressing machine operates whenever any apples are being processed, which includes whenever any inventory exists; a previous solution showed how this occurs for eight hours per day.

The inventory graph provided in the solutions to the prior problem reveals three time periods, and we can calculate the kg-hours for each period based on the shapes under the curve for each period: from 9:00 am to noon, a triangle; from noon to 1:00 pm, a triangle atop a rectangle; and from 1:00 to 5:00 pm, another triangle. Recall the area of the triangle is ½ * length * height, and the area of a rectangle is length * height.

> 9:00 am–12:00 pm: The length here is 3 hours (that is, 9:00 am to noon), and the height is 300 kg. Thus, the area of the triangle during this period = ½ * length * height = ½ * 3 hours * 300 kg = 450 kg-hours.
>
> 12:00–1:00 pm: For the rectangle portion, the length is 1 hour and the height is 200 kg, and so the area of the rectangle portion is length * height = 1 hour * 200 kg = 200 kg-hours. The triangle above it, the length is still 1 hour, and the height is 100 kg (calculated as 300 kg – 200 kg). Thus the area of the triangle = ½ * length * height = ½ * 1 hour * 100 kg = 50 kg-hours. The total during this period is the sum of these two areas, 250 kg-hours.
>
> 1:00–5:00 pm: The length here is 4 hours (that is, 1:00 pm to 5:00 pm), and the height is 200 kg. Thus, the area of the triangle during this period = ½ *length * height = ½ * 4 hours * 200 kg = 400 kg-hours.

Thus, the total area under the curve is the sum of these three time periods = 450 + 250 + 400 kg-hours = 1,100 kg-hours.

As the prior graph indicates, the inventory accumulates from zero and diminishes to zero over eight hours, and thus inventory exists for 8 hours each day, and pressing machine runs for eight hours. The average inventory throughout the day is therefore 1,100 kg-hours/8 hours = 137.5 kg, or approximately 140 kg.

A second way to calculate the overall average inventory throughout the day is to first calculate the average inventory during each of three periods of the day, and then weight these averages based on their duration. Returning to the above graph, we find:

9:00 am–12:00 pm: The bin held 0 kg at 9:00 am and 300 kg at 12:00 pm, with a constant rate of increase. When an inventory increases or decreases at a constant rate, the average inventory is just the average of the starting and ending inventories, in this case, 150 kg.

12:00–1:00 pm: The bin held 300 kg at 12:00 pm and 200 kg at 1:00 pm, decreasing at a constant rate, so the average inventory during this hour was 250 kg.

1:00–5:00 pm: The bin held 250 kg at 1:00 pm and 0 kg at 5:00 pm, decreasing at a constant rate, so the average inventory during this period was 125 kg.

The average over the day is calculated as the weighted average of these three periods over the eight hours during which there is inventory. The weights are the proportion of this time during which each period occurs. Thus, the first period represents 3/8 of the time, the second is 1/8 of the time, and the third 4/8 of the time, so the average inventory over the course of a day is:

(150 kg * 3/8) + (250 kg * 1/8) + (100 kg * 4/8) = 137.5 kg, or approximately 140 kg.

29. Maciej worries about the freshness of his cider and wonders whether the apples spend too much time in the holding bin. How long, on average, do the apples spend in the holding bin?

a. about 1 hour
b. about 1.4 hours
c. about 2 hours
d. about 2.2 hours
e. about 2.5 hours

Answer: (b) About 1.4 hours

The solution to the prior question showed that over the course of the day, the total accumulated apples in the holding bin in kg-hours is 1,100 kg-hours each day. We have earlier calculated that 800 kg are processed each day, and so apples spend an average of 1,100 kg-hours/800 kg = 1.375 hours (that is, about 1.4 hours) in the holding bin before being pressed each day.

An alternative approach is to use Little's Law, which in this context can be stated as:

Average throughput time (TPT) = Average WIP/Output Rate

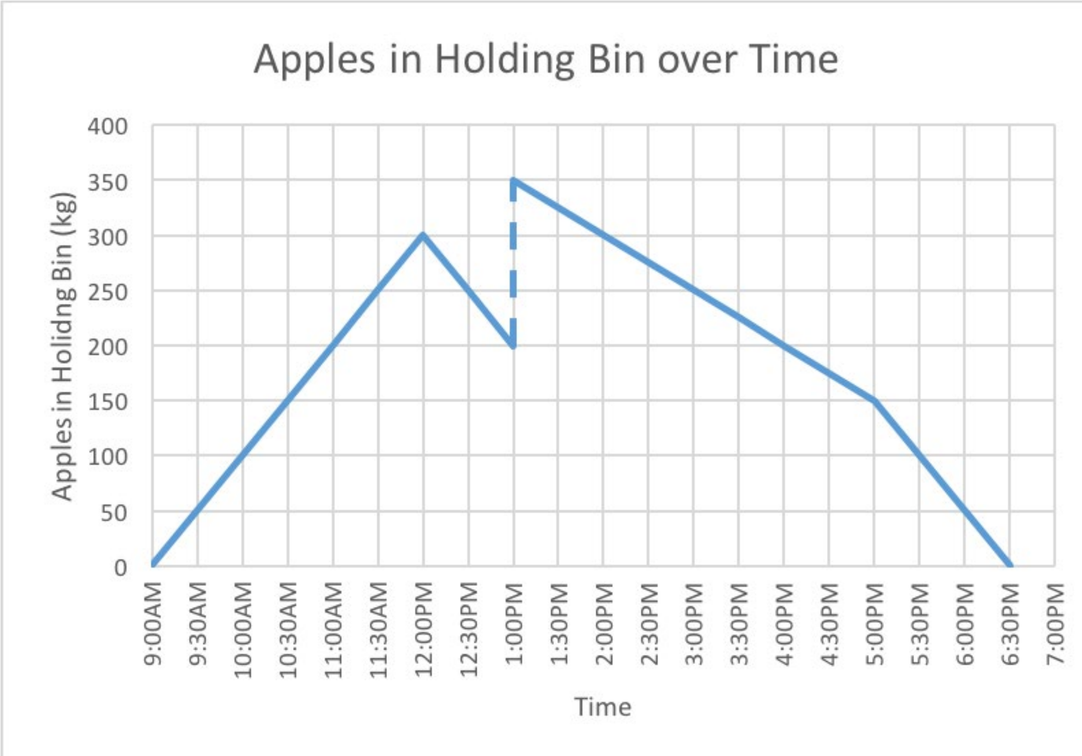
In the previous question, average work in progress inventory during the day (average WIP) was calculated to be 137.5 kg or approximately 140 kg. The output rate (determined by the presses) is 100 kg per hour. We can divide average WIP by the average output rate to obtain average TPT:

Average TPT = 140 kg/100 kg per hour = 1.4 hours.

30. In addition to the operations described above, one morning, Maciej allowed visitors to go apple picking at his orchards until noon, as long as they agreed to give half the apples they picked to him. At 1:00 pm, Maciej brought 150 kg of apples from the visitors and dumped them in the holding bin all at once. The siblings continue to staff the operation as described above, with four working in the morning and one working after lunch. The extra activity with visitors had no effect on the siblings' performance of their apple-related tasks as described above. At what time will the pressing machine automatically shut off on this day?

a. 5:00 pm
b. 5:30 pm
c. 6:00 pm
d. 6:30 pm
e. 7:00 pm

Answer: (d) 6:30 pm



In the morning (9:00 am to 12:00 pm), apples are being deposited in the holding bin at a rate of 50 kg/hour per sibling * 4 siblings = 200 kg/hour, and are being processed by the pressing machine at a rate of 100 kg/hour. Because more is being deposited in the holding bin than is being processed, the holding bin is accumulating the difference at a rate of 100 kg/hour (that is, 200 kg/hour – 100 kg/hour). By 12:00 pm, three hours after this day started at 9:00 am, 100 kg/hour * 3 hours = 300 kg of apples have accumulated in the holding bin.

From 12:00 pm to 1:00 pm, no apples are deposited into the holding bin, but the pressing machine continues to process apples at a rate of 100 kg/hour. Thus, the net accumulation rate from 12:00 pm to 1:00 pm is 0 kg/hour (deposit) – 100 kg/hour (pressing machine processing) = -100 kg/hour. Thus, over this one hour, the accumulation in the bin is reduced by 100 kg (calculated as 100 kg/hour * 1 hour), from a total of 300 kg to 200 kg.

At 1:00 pm, Maciej dumps 150 kg of apples into the holding bin, represented in the graph below as a jump of +150 kg from 200 kg to 350 kg.

From 1:00 pm to 5:00 pm, one sibling returns to deposit apples in the holding bin, and thus the input rate is 50 kg/hour. Meanwhile, the pressing machine continues to process at 100 kg/hour. Thus, the net accumulation rate from 1:00 pm to 5:00 pm is 50 kg/hour (deposited) – 100 kg/hour (pressing machine processing) = -50 kg/hour. Over these four hours, the accumulation in the bin is reduced by -50 kg/hour * 4 hours = 200 kg, from a total of 350 kg at 1:00 pm to 150 kg at 5:00 pm.

After 5:00 pm, the one sibling goes home while the pressing machine continues to process at 100 kg/hour. Thus, the net accumulation rate is 0 kg/hour (deposited) – 100 kg/hour (pressing machine processing) = -100 kg/hour.

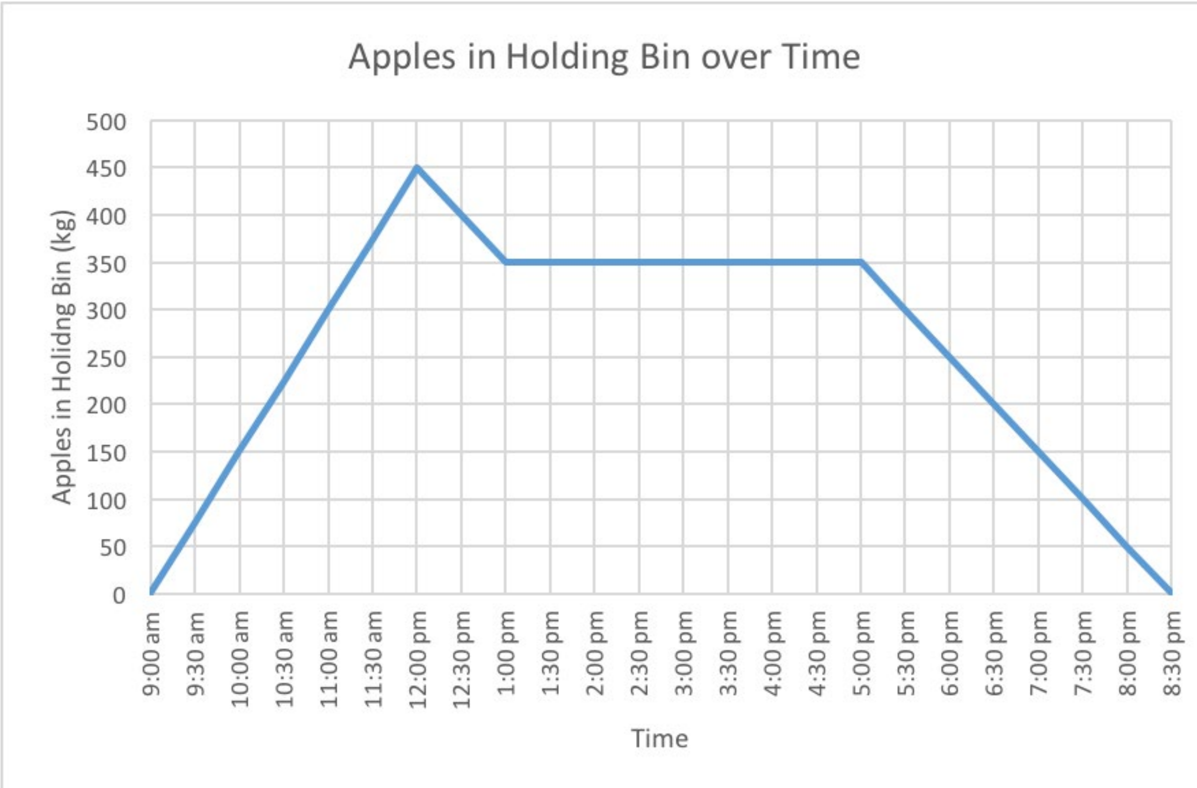
We can now find how long it takes to process the remaining apples by dividing the accumulation of apples in the bin at 5:00 pm by the rate at which it takes the pressing machine to process them. That is, 150 kg/100 kg per hour = 1.5 hours. Thus, it takes an additional 1.5 hours after 5:00 pm to process the remaining apples, meaning that the pressing machine finishes its work and automatically shuts off at 6:30 pm.

31. Maciej decided to not allow visitors to go apple picking on his farm anymore. Instead, he decided to expand his operation by hiring two neighbors to be additional workers. These neighbors work along with the siblings to deposit apples into the holding bin from 9:00 am to 12:00 pm and from 1:00 pm to 5:00 pm. The siblings continue to staff the operation as described above, with four working in the morning and one working after lunch. Each of these neighbors can deposit apples at a rate of 25 kg/hour. With these two neighbors working along with Maciej's siblings, at what time will the pressing machine automatically shut off?

a. 7:00 pm
b. 8:30 pm
c. 9:30 pm
d. 10:15 pm
e. 11:00 pm

Answer: (b) 8:30 pm

The neighbors add (25 kg/hour/neighbor) * (2 neighbors) * (7 hours) = 350 kg of apples to the holding bin, so the pressing machine must now process a total of 800 kg + 350 kg = 1150 kg each day. The pressing machine is never "starved," so it processes 100 kg each hour. Thus it will take (1150 kg)/(100 kg/hour) = 11.5 hours to process the apples. Thus, the process starts at 9:00 am, and ends at 8:30 pm.



In the morning (9:00 am to 12:00 pm), apples are being deposited in the holding bin at a rate of (50 kg/hour per sibling * 4 siblings) + (25 kg/hour per neighbor * 2 neighbors) = 250 kg/hour, and are being processed by the pressing machine at a rate of 100 kg/hour. Because more is being deposited in the holding bin than is being processed, the holding bin is accumulating the difference at a rate of 150 kg/hour (that is, 250 kg/hour – 100 kg/hour). By 12:00 pm, three hours after this day started at 9:00 am, 150 kg/hour * 3 hours = 450 kg of apples have accumulated in the holding bin.

From 12:00 pm to 1:00 pm, no apples are deposited into the holding bin, but the pressing machine continues to process apples at a rate of 100 kg/hour. Thus, the net accumulation rate during this hour is 0 kg/hour (deposit) – 100 kg/hour (pressing machine processing) = -100 kg/hour. During this hour, the accumulation in the bin is reduced by 100 kg (calculated as 100 kg/hour * 1 hour), reducing the accumulation from 450 kg (at 12:00 pm) to 350 kg (at 1:00 pm).

From 1:00 pm to 5:00 pm, one sibling returns to deposit apples in the holding bin. Joining the one sibling are the two neighbors. Hence, the total input rate is (50 kg/hour per sibling * 4 siblings) + (25 kg/hour per neighbor * 2 neighbors) = 100 kg/hour. Meanwhile, the pressing machine continues to process at 100 kg/hour. Thus, the net accumulation rate from 1:00 pm to 5:00 pm is 100 kg/hour (deposited) – 100 kg/hour (pressing machine processing) = 0 kg/hour. During these four hours, the accumulation in the bin remains at 350 kg of apples.

At 5:00 pm, the one sibling and two neighbors go home while the pressing machine continues to process at 100 kg/hour. Thus, the net accumulation rate starting at 5:00 pm is 0 kg/hour (deposited) – 100 kg/hour (pressing machine processing) = -100 kg/hour.

We can now calculate how long it takes to process the remaining apples by dividing the accumulation of apples in the bin at 5:00 pm by the pressing machine processing rate. That is, 350 kg/100 kg per hour = 3.5 hours. Thus, it takes an additional 3.5 hours after 5:00 pm to process the remaining apples, meaning that the pressing machine finishes its work and automatically shuts off at 8:30 pm.