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Multivariate Analysis Worksheet

#1) Domain

A) $f(x,y) = \sqrt{16-4x^2-y^2}$

$16-4x^2-y^2 \geq 0$

$\frac{16 \geq 4x^2+y^2}{4}$

$4 \geq x^2 + \left(\frac{y^2}{4}\right)$ constant

$x^2 \leq 4$

$x \leq 2$

$\therefore \boxed{\text{Domain} = -2 \leq x \leq 2}$

B) $f(x,y) = \ln(x^2+y^2)$

$x^2-y^2 > 0$

$x^2 > 0$

$\boxed{x \neq 0 \text{ Domain}}$

#2) Partial Derivative

A) f_x for $f(x,y) = e^{xy}(\cos x \sin y)$ * product rule

$(1D2 - 2D1)$

$e^{xy}(-\sin x \cos \sin y) + (\cos x \sin y)\, e^{xy}$ (y)

$\boxed{-\sin x \sin y\, e^{xy} + e^{xy}(\cos x \sin y)}$

make sure you find the derivative of the exponent

B) f_y for $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ @ $(3, \sqrt{11}, -4)$

$(x^2+y^2+z^2)^{1/2} \rightarrow \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2y)$

$y(x^2+y^2+z^2)^{-1/2} \rightarrow \frac{y}{\sqrt{x^2+y^2+z^2}}$

$\frac{\sqrt{11}}{\sqrt{3^2+\sqrt{11}^2+-4^2}}$ $\frac{\sqrt{11}}{\sqrt{9+11+16}}$ $\boxed{\frac{\sqrt{11}}{\sqrt{36}}} = \frac{\sqrt{11}}{6}$

$\sim \boxed{0.0921}$

(2)

... this ... each ... or 3rd derivative

#2 c) f_{xyz} for $f(x,y,z) = x^3yz - 2x^3y^2z^2 + 3xz - 4y^3z$

$f_x = 3x^2yz - 6x^2y^2z^2 + 3z$

$f_y = x^3z - 4x^3yz^2 - 12y^2z$

$f_z = x^3y - 4x^3y^2z + 3x - 4y^3$

yes, 3rd derivatie

$f_{xy} = 3x^2z - 12x^2yz^2$

$\Rightarrow f_{xyz} = 3x^2 - 24x^2yz$

D) $\frac{\alpha^2 z}{\alpha x \alpha y}$ for $z = x^2 - 3xy + 8xy^2 + 2y^4$

$z' = 2x - 3y + 8y^2$

$\boxed{z'' = -3 + 16y}$

E) $\frac{\alpha^2 z}{\alpha x \alpha x}$ for $z = x^3 - 10xy + \frac{x}{y^2} + 6y^3$

xy^{-2}

$\frac{1}{y^2}$

$z' = 3x^2 - 10y + \frac{1}{y^2}$

$\boxed{z'' = 6x}$

#3) Find Actual Change

A) $f(x,y) = x^2 - xy$ from $(2,4)$ to $(2.16, 3.98)$

(dx) $\Delta x = 0.16$

(dy) $\Delta y = -0.02$

$\Delta z = f(2.16, 3.98) - f(2,4) =$

$(2.16)^2 - (2.16 \cdot 3.98) - [(2^2 - 2 \cdot 4)] =$

$4.6656 - 8.5968$

$-3.9312 + 4 = \boxed{0.0688}\ \Delta z$

B) $f(x,y) = x^3 - 2xy^2$ from $(2,3)$ to $(2.06, 3.10)$

$\Delta x = 0.06$

$\Delta y = 0.10$

$\Delta z = f(2.06, 3.10) - f(2,3) =$

$(2.06)^3 - 2(2.06)(3.10)^2 -$

$2^3 - 2(2)(3)^2$

$8.741816 - 39.5932$

$8 + 28 = 36$

$\boxed{-2.851384}$

$-30.851384 - 36 = \boxed{-66.85}\ \Delta z$

$-(-28)$

(≈ -66.851384)

(x,y) use to find Δx Δy

#4) Approximate change

A) $z = \ln(2x^2+y^2)$ $(0,1) \to (0.02, 1.1)$

$dx = \Delta x = 0.02$
$dy = \Delta y = 0.1$ ok

$f_x = \frac{1}{2x^2+y^2}(4x)$

$f_y = \frac{1}{2x^2+y^2}(2y)$

$dz = \frac{4x}{2x^2+y^2}dx + \frac{2y}{2x^2+y^2}dy$

$\cancel{\frac{0}{1}(0.02)} \frac{4(0)}{2(0)^2+1}(0.02) + \frac{2(1)}{2(0)^2+1^2}$ (0,1) (1.1)

0

$0 + 2(1.1) = \boxed{2.2}$ approximately

B) $z = \cos(4x-y)$ $(0.25, 0.5) \to (0.3, 0.75)$

$\Delta x = 0.05$
$\Delta y = 0.25$

$dx = -\sin(4x-y)4$

$dy = -\sin(4x-y)-1$

$dz = -4\sin(4x-y)dx + \sin(4x-y)$

$-4\sin(4(0.25)-0.5)\boxed{0.2} + \sin(4(0.25)-0.5)\boxed{0.75}$

$-0.01047842 + 0.00654901$

$= \boxed{-0.00392941}$ approx.

$\Delta x = dx \Rightarrow$ use 0.05 in dz evaluation

$\Delta y = dy \Rightarrow$ use 0.25 in dz evaluation

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#5) volume - % error

$V = \frac{1}{3}\pi r^2 h$

$R = 10$ in

$H = 16$ in

error .02 inches

errors of R + H

$\frac{dv}{v} = \frac{\text{prop error}}{\text{volume}}$

$\frac{dr}{r} = \frac{0.02}{10} = \boxed{.2}$ $\frac{dh}{h}$ $\frac{0.02}{16} = \boxed{.125}$

$\frac{Dv}{v} = \frac{\frac{2}{3}\pi r h\, dr + \frac{1}{3}\pi r^2}{\frac{1}{3}\pi r^2 h}$

$dr = \frac{2}{3}\pi r h$

$dh = \frac{1}{3}\pi r^2$

$2 \frac{\cancel{2}dr}{r} + 1\frac{\cancel{3}dh}{h} = \frac{2}{\cancel{3}}(.2) + \frac{1}{\cancel{3}}(.125) = \boxed{\cancel{0.175}}$

0.525%

$\therefore \frac{dv}{v} = \boxed{17.5\%}$

#6) Partial Derivatives of w + ev. @ t $\frac{dw}{dt}$

a) $w = xyz^2 + y^2 z$. $x = t$, $y = t^2$, $z = t^3$

$t = 1$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

$$\begin{array}{ccc} & w & \\ \downarrow & \downarrow & \downarrow \\ x & y & z \\ \downarrow & \downarrow & \downarrow \\ t & t & t \end{array}$$

$\frac{dw}{dt} = yz^2(1) + (xz^2 + 2yz)(2t) + (2xyz + y^2)(3t)$

when $t = 1$ $x = 1$ $y = 1$ $z = 1$

$1 \cdot 1(1) + [1 \cdot 1 + 2(1 \cdot 1)](2(1)) + [2(1)(1)(1) + (1)^2](3)$

$1 + (1 + 4) + (2 + 3) = \boxed{11}$

$1 + 6 + 9 = \boxed{16}$

B) $w = \sin(2x+y)$ $x = s^2$ $y = t^2$

dw/dt when $s=2$, $t=1$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{ds} + \frac{\partial w}{\partial y}\frac{dy}{dt}$

should this be dt, or 0?

$dw/dt = \cos(2x+y)2(\cancel{2s}) + \cos(2x+y)(1)(2t)$ 0

when $s=2$ & $t=1$ $x=4$ $y=1$

$\cos(2(4)+1)2(2\cdot 2) + \cos(2(4)+1)(1)(2(1))$

$8\cos(9) + \underline{2\cos(9)}$ → make sure calculator is in radians

$\cancel{7.9015067}25 + \cancel{1.97537}6681 = 9.876883406$

$\sim \boxed{9.88}$

$2\cos 9 = -1.82226$

C) $w = \sqrt{x+3y}$, $x = 2t$ $y = t$

Find dw/dt when $t=2$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$

$(x+3y)^{1/2}$

$\frac{1}{2}(x+3y)^{-1/2}(1)(2) + \frac{1}{2}(x+3y)^{-1/2}(3)(1)$

$(x+3y)^{-1/2} + \frac{3}{2}(x+3y)^{-1/2}$

when $t=2$ $x=4$ $y=2$

$(4+3(2))^{-1/2} + \frac{3}{2}(4+6)^{-1/2}$

ok $\frac{1}{\sqrt{10}} + \frac{3}{2\sqrt{10}} = \cancel{5.0596}44256$

0.790569415

$\sim \cancel{\boxed{5.06}}$

D) $w = \ln(x-y)$ $x = st$ $y = 3t$

Find dw/ds when $s=4$ $t=1$

① $dw/dt = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$

② $dw/ds = \frac{\partial w}{\partial x}\frac{dx}{ds}$

① $dw/dt = \left(\frac{1}{x-y}\right)(1)(s) + \left(\frac{1}{x-y}\right)(-1)(3)$

when $s=4$ + $t=1$ $x=4(1)$ $y=3(1)$

$\left(\frac{1}{4-3}\right)(1)(4) + \left(\frac{1}{4-3}\right)(-1)(3)$

$4 + -3 = \boxed{1}$

② dw/ds $\left(\frac{1}{x-y}\right)(1)(t)$ when $t=1$

$\left(\frac{1}{4-3}\right)(1)(1) = \boxed{1}$

#7) Find ∇f, gradient of f:

A) $f(x,y,z) = 2x^2 + 2y^2 + z^3e^z$ $(0,4,-1)$

$\nabla f(x,y,z) = (4x)i + (4y)j + (z^3e^z + e^z 3z^2)k$ ok

$0i + 16j + (-1 \cdot 156.5344649 + 156... \cdot 3(1))k$

$16j + (0.3678794411 + 3.672200628)k$

$\boxed{16j + \cancel{3.72}k}$ $+ 0.7357588823\hat{k}$

B) $f(x,y) = 2x\ln(x^2y)$ @ $(-2,6)$

$\nabla f(x,y) = \left[2 \cdot \frac{1}{x^2y}(2x)\right]i + \left[\left(\frac{1}{x^2y}\right)(1)\right]j$

f_x

$\left(\cancel{\frac{4x}{x^2y}}\right)i + \left(\frac{1}{x^2y}\right)j$

$\frac{4(-2)}{(-2)^2(6)}i + \frac{1}{(-2)^2(6)} \Rightarrow \frac{-8}{24}i + \frac{1}{24}j$

or $\boxed{-\frac{3}{8}i + \frac{1}{24}j}$

f_x is product rule $\left[\frac{2x}{x^2y}(2xy) + \ln(x^2y)(2)\right]\hat{i}$

$+ \frac{2x}{x^2y}(x^2)\hat{j}$

8) calculate directional derivative

A) $F(x,y) = 2 - x^2 - y^2/2$ @ $(1,2)$ in dir $u = \cos \pi/4\, i + \sin \pi/4\, j$

$D_u f(x,y) = -2x\cos\theta - y\sin\theta$

$\theta = \pi/4 \quad x=1 \quad y=2$

$-2(1)\cos\pi/4 - 2\sin\pi/4$

$-2\,\sqrt{2}/2 - 2\,\sqrt{2}/2$

$-\frac{2\sqrt{2}}{2} - \frac{2\sqrt{2}}{2}$ $\quad -\sqrt{2} - \sqrt{2} = \cancel{\boxed{0}}$ $\boxed{-2\sqrt{2}}$

B) $f(x,y,z) = z^3 e^{xy}$ @ $(-1,0,3)$ in $v = 3i - j - 5k$

$(z^3 e^{xy})i + (z^3 e^{xy}(x))j + (3z^2 e^{xy})k$

$(3^3 e^{-1\cdot 0})i + (3^3 e^{-1\cdot 0})j + (3(3)^2 \cdot e^{-1\cdot 0})k$

$(27\cdot 1)i + (27\cdot 1)j + (27\cdot 1)k$

$\boxed{27i + 27j + 27k}$ → gradient $= \langle 0, -27, 27\rangle$

$\langle 27, 27, 27\rangle$

$\vec{u} = \frac{\langle -1, 0, 3\rangle}{\sqrt{-1^2 + 0^2 + 3^2}} = \sqrt{1+9} = \sqrt{10}$ $\quad \frac{\langle -1, 0, 3\rangle}{\sqrt{10}} \langle \frac{-1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}} \rangle$

$D_u f(x,y) = \langle 27, 27, 27\rangle \cdot \langle \frac{-1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}} \rangle$

$\langle -27/\sqrt{10} + 0 + 81/\sqrt{10} \rangle$ $\quad 81 - 27 = 54$

$\boxed{54/\sqrt{10}}$

to find unit vector of $v = 3\hat{i} - \hat{j} - 5\hat{k}$

$\|v\| = \sqrt{9+1+25} = \sqrt{35}$ $\quad u_v = \frac{3}{\sqrt{35}}\hat{i} - \frac{1}{\sqrt{35}}\hat{j} - \frac{5}{\sqrt{35}}\hat{k}$

so $\nabla f \cdot u = 0 + 27/\sqrt{35} - 135/\sqrt{35} = \boxed{\frac{-108}{\sqrt{35}} = -18.2553319}$