

Softdrinks:  $X \sim N(\mu_x, (0.05)^2)$

$$\bar{X} \sim N\left(\mu_x, \frac{(0.05)^2}{n}\right)$$

$$\sigma_{\bar{X}} = \frac{0.05}{\sqrt{n}}$$

$$= 0.025$$

mean of  
sample is  
same as  
normal

$$\mu_{\bar{X}} = \mu_x$$

\* always

$$P(\mu_x - 0.04 \leq \bar{X} \leq \mu_x + 0.04)$$

$$= P(\mu_x - 0.04 \leq \bar{X} \leq \mu_x + 0.04)$$

cumulative

$$P(\bar{X} \leq \mu_x + 0.04) - P(\bar{X} \leq \mu_x - 0.04)$$

convert to  $Z$  (standard)

$$P\left(\frac{\bar{Z} \leq \mu_x + 0.04 - \mu_x}{0.025}\right) - P\left(\frac{\bar{Z} \leq \mu_x - 0.04 - \mu_x}{0.025}\right)$$

$$P\left(Z \leq \frac{0.04}{0.025}\right) - P\left(Z \leq \frac{-0.04}{0.025}\right)$$

$$\Phi(1.6) - \Phi(-1.6) = 0.9452 - 0.0548 = \boxed{0.8904}$$

lookup in table

\* we know sampling dist. will be normal

### Central Limit Theorem

As long as sample size is big enough, can assume normal distribution

If our original population has a normal distribution, the sampling distribution of  $\bar{X}$  is also normal, regardless of sample size.

Example: An automated filling machine fills soft drink cans with a volume that has a normal distribution with  $\sigma = 0.05$  ounces

If we sample 4 cans and take the sample mean, what is the probability that  $\bar{X}$  will be within 0.04 ounces of the population mean?

Math 321

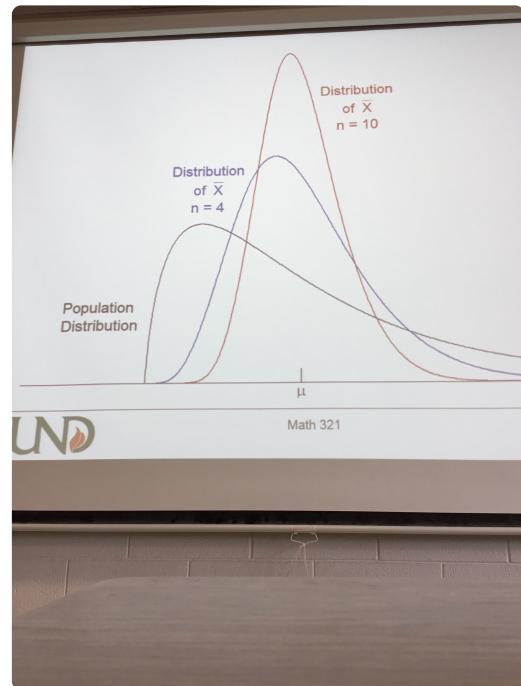
$$\Phi_2 = P(Z \leq z)$$

Theorem (Central Limit Theorem): If  $X_1, X_2, \dots, X_n$  are independent random variables, from a population or process with mean  $\mu$  and standard deviation  $\sigma$ , then as long as  $n$  is sufficiently large,

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{and} \quad T = \sum_{i=1}^n X_i \sim N(n\mu, n\sigma^2)$$

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## Review

#13, LATEST HW

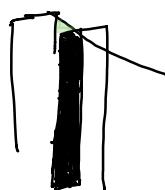
looking for binomial - exactly, at most, at least

a)  $P(X \geq 30)$  AT MOST

approx. discrete by continuous  
 $\Phi\left(\frac{30.5 - \mu}{\sigma}\right)$

$$= \Phi\left(\frac{30.5 - 24}{4.596}\right)$$

add .5 because



$$x \boxed{1.414273281} \\ \text{normalized} \\ P(X \leq 1.41...)$$

$$np \geq 10$$

$$n(1-p) \geq 10$$

$$\mu = np = 24$$

$$\sigma = \sqrt{np(1-p)}$$

$$= \sqrt{24(.08)}$$

$$= 4.59$$

c)  $P(X \leq 25) - P(X \leq 14)$

$\hookrightarrow P(15 \leq X \leq 25)$

$$\approx \Phi\left(\frac{25.5 - 24}{\sigma}\right) - \Phi\left(\frac{14.5 - 24}{\sigma}\right)$$

#35, ch. 3

.6086

Demand for magazines

x	1	2	3	4	5	6
p(x)	1/15	2/15	3/15	4/15	3/15	2/15
${}^3R(x)$	\$4	\$8	\$12	\$12	\$12	\$12
						$\leftarrow 3 \text{ revenue}$

- He pays \$2 per magazine

- sells for \$4

How many should he buy? 3 or 4?

① buys 3 = \$6

$$E(R(x)) = 4 \cdot \frac{1}{15} + 8 \cdot \frac{2}{15} + 3 \cdot \frac{3}{15} \\ 12 \cdot \frac{4}{15} + 12 \cdot \frac{3}{15} + 12 \cdot \frac{2}{15}$$

$$\approx \$10.93$$

$$\therefore \text{profit} = 10.93 - 6 = \$4.93$$

... same for 4

$$\text{profit} = \$5.33$$