

$$V(\bar{X}) = \sigma^2 \frac{\bar{X}}{n} = \frac{\sigma^2}{n} = \frac{V(X)}{n}$$

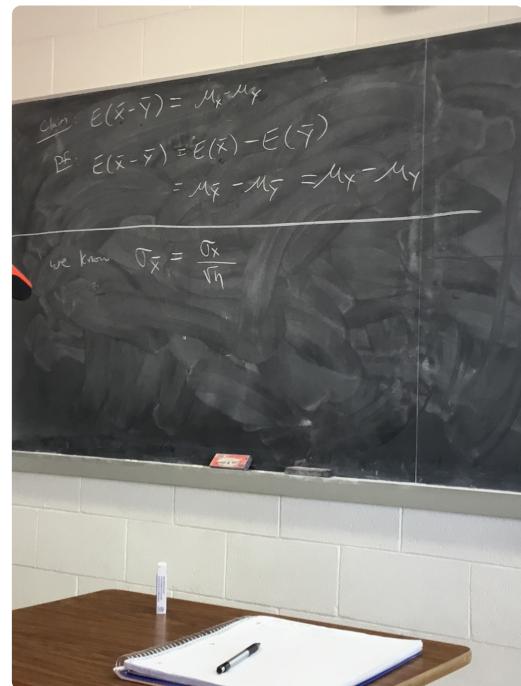
$$\sigma_{\bar{x}-\bar{y}}^2 = V(\bar{X} - \bar{Y})$$

$$V(\bar{X}) + V(\bar{Y})$$

$$\frac{V(X)}{m} + \frac{V(Y)}{n} = \frac{\sigma^2}{m} + \frac{\sigma^2}{n}$$

$\bar{X} - \bar{Y}$ is unbiased estimator
of $\mu_1 - \mu_2$

* both pips are normal, then so
are samples



2 Sample t-test

$$\text{If } H_0, \Delta_0 = 0$$

$$\therefore H_0 \text{ is } \mu_1 = \mu_2$$

$$H_0: \mu_1 - \mu_2 = \Delta_0$$

$$H_a: \mu_1 - \mu_2 \neq \Delta_0$$

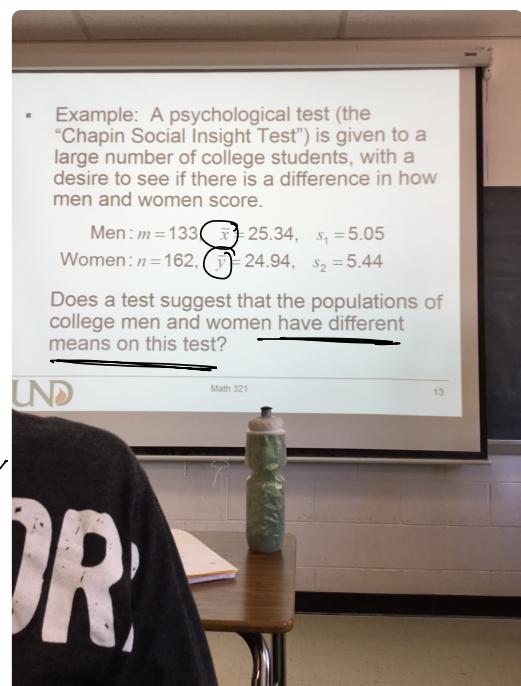
evident
vs
the difference
is diff. than
you thought

Ex $H_0: \mu_x - \mu_y = 0$

$$H_a: \mu_x - \mu_y \neq 0$$

$$z = \frac{\text{point est.} - \text{null value}}{\text{standard error}}$$

$$pe = \bar{X} - \bar{Y} \quad \text{null} = \Delta_0$$



$$\text{st error } \sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}$$

$$z = \frac{(25.34 - 24.94) - 0}{\sqrt{\frac{5.05^2}{133} + \frac{5.44^2}{162}}}$$

$$= \boxed{0.6537}$$

not
extreme
enough
(1.5 ...)

z must be > 1
for significance

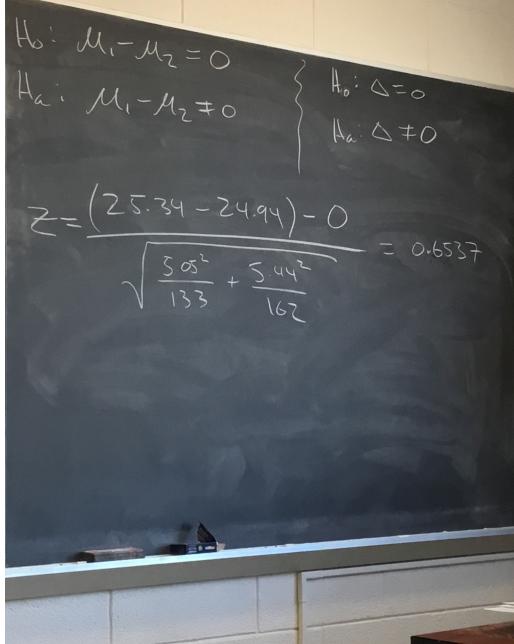
variances

re $z = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$.

5.34, $s_1 = 5.05$
24.94, $s_2 = 5.44$

$$H_0: \Delta = 0$$

$$H_a: \Delta \neq 0$$



Our z statistic is therefore $z = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$.

Use σ_1 and σ_2 if known.

Example: Students:

Men: $m = 133, \bar{x} = 25.34, s_1 = 5.05$
 Women: $n = 162, \bar{y} = 24.94, s_2 = 5.44$

$z = ?$

Math 321

* then compute p-value

reject if p is small

$\alpha = 0.05$, do not reject b/c $p(z^* < -0.6537) = 2(0.2578) = 0.5156$

$P > \alpha$
 if $P < \alpha$, stronger ev. H_0 is true
 Do we know which is true?
NO

NO evidence to suggest any diff between men & women scores

if true, $\mu_x - \mu_y \leq 8$
 (one sided) vs
 $H_a: \mu_x - \mu_y > 8$

$$z = \frac{(562 - 545) - 8}{\sqrt{\frac{18^2}{50} + \frac{24^2}{50}}} = 2.12$$

is diff more than 8 or not?

$$P(z^* > 2.12) \text{ or } P(z^* < -2.12)$$

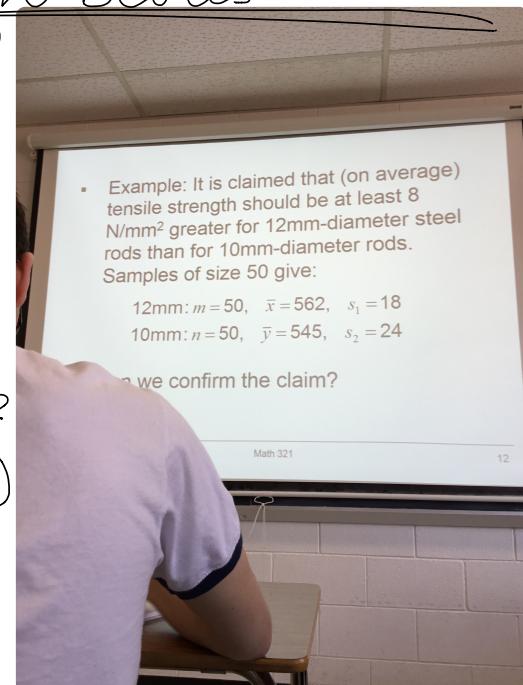
$$= 0.017$$

at $\alpha = 0.05$, we reject H_0 b/c any $P < \alpha = 0.05$ ($0.017 < 0.05$)

suff. evidence to conclude 12 mm rods are $>$ than 8 stronger than 10 rod

Test @ $\alpha = 0$?

Don't reject H_0 more



(CI)

$$\bar{x} - \bar{y} \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

$$100(1 - \alpha)\% CI \supset$$

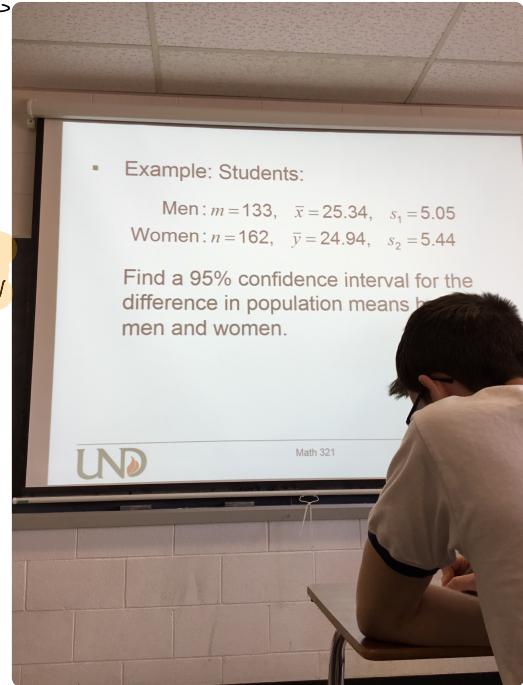
$$(25.34 - 24.94) \pm 1.96 \sqrt{\frac{5.05^2}{133} + \frac{5.44^2}{162}}$$

$$= 0.4 \pm \sim 1.2$$

$$95\% \text{ CI} = (0.8, 1.6)$$

If null value is in interval,
don't reject
otherwise reject

$0 \in 95\% \text{ CI for } \Delta$

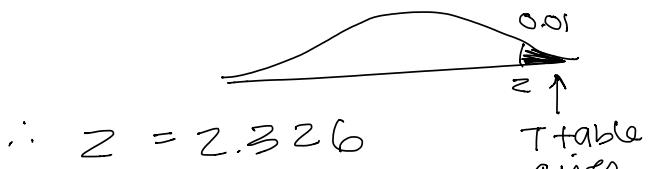


One sided

$$99\% \text{ LB} = \bar{x} - \bar{y} - z_{\alpha} \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

$$562 - 545 - z_{\alpha}$$

\downarrow ↑
sample size



$$\therefore z = 2.326$$

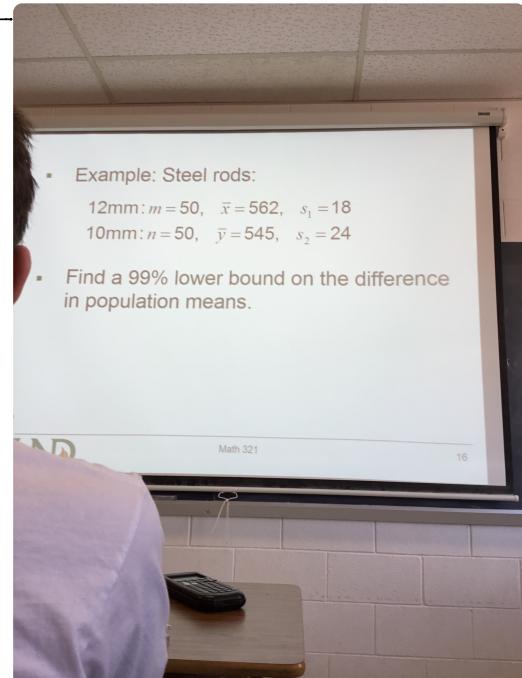
$$562 - 545 - 2.326^*$$

$$\left(\frac{18^2}{50} + \frac{24^2}{50} \right)$$

T table gives critical value
creates right tail

$$= 17 - 9.87 = 7.13$$

$$99\% \text{ LB} = [7.13 < \Delta]$$



↪ at 95% CI, would get a bound bigger than 8