Stdev — in excel (# hw)
$S' = \{FFFF, FFFV \ldots etc\}$
16 combos

$A, P(A') = 1 - P(A)$

chip - ① dye drive etching is .12
② crack is .29
③ both = 0.07

E = 1
C = 2

[.12 × .29] = not .07, so not independent

1) prob chip doesn't have dye etching $1 - P(E) = 0.88$

2) prob at least 1 defect
$P(E \cup C) = P(C) + P(E) - P(E \cap C)$
$= .34$

3) prob of nothing wrong? not E or C
$1 - P(E \cup C) = 1 - .34$
$= 0.66$

Counting

$\{ {52 \atop 5} \}$ # of poker hands

$\frac{\text{\# of ways E occurs}}{\text{\# outcomes}}$

Product Rule
poss. @ first step × poss. 2nd ... etc —

ex: coin toss
$2 \times 2 \times 2 \times 2 = 16$
$k_1\ k_2\ k_3$

K = step

6 models
8 colors
2 transmission
3 packages

$8 \times 6 \times 2 \times 3$

License Plates
_ _ _ _ _ _
possible? $26^6 = S$
no vowels $= 21^6 = E$
$\frac{P(E)}{P(S)} = \frac{21^6}{26^6} = 0.2776$

no ~~license~~ plate w/ ALL vowels
$26^6 - 5^6$

4 cards, 1, 2, 3, 4
choose one, then 2nd _
$4 \times 3 \times 2 \times 1$

12 members
5 officers
$12 \times 11 \times 10 \times 9 \times 8 = 95{,}040$
$\frac{12!}{7!} = \frac{12!}{(12-5)!}$

$P_{k,n} = \frac{n!}{(n-k)!}$

$0! = 1$

layout 5 cards
$\frac{52!}{(52-5)!}$

Permutations —
order matters — locker combo

order doesn't matter?
$C_{k,n} = \{ {n \atop k} \}$ k things from a

$= \frac{n!}{k!(n-k)!}$

num of 5 card : E
poker hands?
$\binom{52}{5} \left(\frac{52!}{5!(52-5)!} \right)$

Combination

$E = \{\text{all 5 cards are ◇'s}\}$
$|E| = \{ {13 \atop 5} \}$
$P(E) = \frac{\{ {13 \atop 5} \}}{\{ {52 \atop 13} \}}$

$P(E) = \frac{\{ {13 \atop 5} \}}{\{ {52 \atop 13} \}} \times 4$ = flush in any suit

February 3rd

30 items
7 defective
5 randomly

\# of ways to select 5? $= \binom{30}{5} = 142{,}506$ combination $\binom{n}{k}$

$_{30}C_5 = C_{5,30}$ $\left(= \frac{30!}{5!(30!-5!)}\right)$

E = all five are good

$P(E) = \frac{|E|}{|S|} = \frac{\binom{23}{5}}{\binom{30}{5}}$

F = exactly 2 are bad

$P(F) = \frac{|F|}{|S|} = \frac{\binom{7}{2}\binom{23}{3}}{\binom{30}{5}} \approx 0.2609$

Crosscheck

one is bad

$\binom{7}{0}\binom{23}{5} + \binom{7}{1}\cdot\binom{23}{4} + \binom{7}{2}\cdot\binom{23}{3} + \binom{7}{3}\binom{23}{2} + \binom{7}{4}\binom{23}{1} + \binom{7}{5}\cdot\binom{23}{0} = \binom{30}{5}$ should equal denominator

Conditional Probability

roll odd #?

ex: $A = \{1\,3\,5\}$ $\quad P(A) = 1/2$

$B = \{1\,2\,3\}$ $\quad P(B) = 1/2$

B happens, prob changes to $2/3$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{1/2} = \frac{2}{3}$$

0.12 — default 1 = Etching
0.29 — default 2 = Crack
0.07 — both

if C, prob also E?

$$\frac{P(A \cap B)}{P(B)}$$

$$\frac{0.07}{0.29} = 0.2414$$

- What is the probability that a chip has a crack defect but satisfactory etching?

and

- If a chip has a crack defect, what is the probability that it has satisfactory etching?

given

$P(C \cap E')$

$P(C) \cdot P(E'|C)$

$0.29 \cdot .7586$

$\approx = 0.22$

0.21999...

not given

$$P(E'|C) = \frac{P(E' \cap C)}{P(C)} =$$

$$1 - \frac{P(E|C)}{P(C)} = 1 - 0.2414$$

$$= 0.7586$$

If a chip has defective etching, what is the probability that it also has a crack defect?

$P(E) = 0.12$

$P(C) = 0.29$

$P(E \cap C) = 0.07$

$$P(C|E) = \frac{P(E \cap C)}{P(E)} = \frac{0.07}{0.12} = 0.5833$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A \cap B) = P(A|B)\,P(B)$$

$$P(A_2 \cap A_1) = P(A_1)\,P(A_1|A_2)$$

Deck w/ 4 spades, 2 ♡s

Draw 2 w/o replacement

H_i = ith card is ♡

S_i = ith card is spade

✓ now use

$P(\text{both spades}) = P(S_1 \cap S_2)$ formula

$P(S_1) \cdot P(S_2|S_1) = \frac{4}{6} \cdot \frac{3}{5} = \frac{12}{30} \approx \frac{2}{5}$

$P(\text{both } \heartsuit s) = P(H_1 \cap H_2)$

$P(H_1) \cdot P(H_2|H_1) = \frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30} = \frac{1}{15}$

ex: standard deck = 13 spades

$P(S_1 \cap S_2) = P(S_1) \cdot P(S_2|S_1)$

$= \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{4} \cdot \frac{12}{51} = \frac{3}{51}$

$P(\text{2 suited})$

each suit

$4\left(\frac{3}{51}\right) = \frac{12}{51} \sim 0.235$

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ex: 5 cards w/o replacement
Prob (all spades?)

$P(S_1 \cap S_2 \cap S_3 \cap S_4 \cap S_5)$

$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48}$

$\frac{33}{66{,}640} = \approx 0.000495$

equation "given"

$= P(S_1) \cdot$
$P(S_2|S_1) \cdot$
$P(S_3|S_1 \cap S_2) \cdot$
$P(S_4|S_1 \cap S_2 \cap S_3) \cdot$
$P(S_5|S_1 \cap S_2 \cap S_3 \cap S_4)$

$\left(\frac{{}_{13}C_5}{{}_{52}C_5}\right) \quad \frac{\binom{13}{5}}{\binom{52}{5}}$

$\left(\frac{n!}{k!(n-k)!}\right)$

not all represented – at least one not there

D = Day shift – 10
S = swing shift – 8
G = graveyard – 6

$|D| + |S| + |G| + |S \cap D| + |S \cap G| + |G \cap D|$

opposite – none there

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

also for $P(A \cap B)$

Law of total Probability

- event can happen multiple ways

ex: 6 card deck (4♠, 2♥)

S_i = i^{th} card spade

H_i = i^{th} card ♥

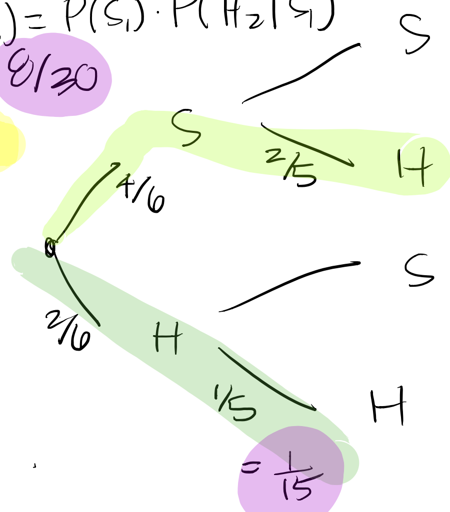
S = all possible outcomes = $S_i \cup H_i$

$S_1 \cap H_1 = \emptyset$

= separate events, add probs together

$P(S_1 \cap H_2) = P(S_1) \cdot P(H_2 | S_1)$

$= 8/30$



Prob 2nd is ♥? $= \frac{1}{15} + \frac{4}{15} = \frac{5}{15} = \frac{1}{3}$

$= P(H_2) = P(H_1 \cap H_2) + P(S_1 \cap H_2)$

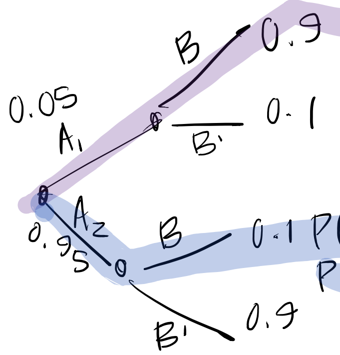
$= P(H_1) \cdot P(H_2 | H_1) + P(S_1) \cdot P(H_2 | S_1)$

ex 2: med condition   5% pop has

A_1 = person has condition

A_2 = person does not have condition

B = test positive



$P(A_1 \cap B) = P(A_1) \cdot P(B | A_1) = 0.045$

$(0.05 \times .9)$

$P(A_2 \cap B) = P(A_2) \cdot P(B | A_2)$

$(0.95 \times 0.1) = 0.095$

$S = A_1 \cup A_2$

$A_1 \cap A_2 = \emptyset$

$P(A_1) \cdot P(B | A_1) + P(A_2) \cdot P(B | A_2)$

$= 0.05 \cdot 0.9 + 0.95 \cdot 0.1 = 0.045 \cdot 0.095$

$= 0.14$

Bayes Rule

*IND tests positive –

$\boxed{* \; P(A_+ | B) = \frac{P(B \cap A_+)}{P(B)}}$

* turn cond'l prob around

actually has condition?

$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)}$

$= \frac{0.05 \cdot 0.9}{(0.05 \cdot 0.9) + (0.95 \cdot 0.1)}$

$\boxed{= 0.321}$

$P(A_1) = 0.01$

$P(A_2) = 0.99$

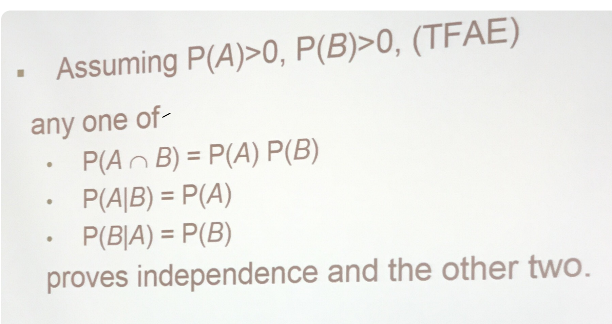
$= \frac{0.01 \cdot 0.9}{(0.01 \cdot 0.9) + (0.99 \cdot 0.1)} = 0.083$

(less than 10% test is accurate)

Independence

If $P(A \cap B) = P(A)P(B)$, = INDEPENDENT

$P(A|B) = \frac{P(A \cap B)}{P(B)}$



DECK:

$P(A) =$ club

$\frac{13}{52} = \frac{1}{4}$

$P(B) =$ an ace

$\frac{4}{52} = \frac{1}{13}$

$P(A \cap B) = \frac{1}{52} =$

$P(A) \cdot P(B) = P(A \cap B)$

= Independent

$C =$ red card

A & C not independent

$P(A \cap C) = 0$ ∴ not independent

$P(C) = 1/4 \cdot 1/2 = 1/8$

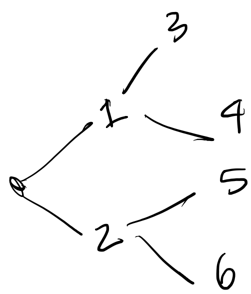
$P(A) = 1/2$

club + red card are mutually exclusive, but not independ.

Suppose A & B = m.e.

$$(A \cap B) = \emptyset$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 = 0 \neq P(A)$$



independent = multiplication