

Linear Algebra

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Triangularizable matrices

F=+ LR

Let T be a linear function over V with dimension $n < \infty$ over $\mathbb{F} = \mathbb{R}$ or $\underline{\underline{\mathbb{C}}}$. The linear function $\overline{\underline{\mathbb{C}}}$ is triangularizable if and only if the minimal polynomial of T splits in $\mathbb{F}(x)$ into linear factors.

$$\begin{array}{ll}
-2 \cos y \cos y \cos y \cos y \cos z \sin z - \sin z \cos z \cos y \cos y \cos y \cos z
\end{array}$$

$$\begin{bmatrix}
T
\end{bmatrix}_{\mathcal{B}} = \begin{bmatrix}
\alpha_{11} & * \\
0 & \alpha_{nm}
\end{bmatrix}$$

$$= (m_{-\alpha_{11}}) - -(m_{-\alpha_{1n}})$$

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Proof.

$$PAP = \begin{bmatrix} \lambda & * \\ 0 & \lambda_2 \end{bmatrix}$$

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P(a) ∈ R[2]





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Lecture #24

3 / 17

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Proof.

$$\begin{bmatrix}
T \end{bmatrix}_{R}^{2} = \begin{bmatrix}
\lambda_{1} \\
\lambda_{1} \\
\lambda_{2} \\
\lambda_{3}
\end{bmatrix}$$

$$\begin{bmatrix}
\lambda_{1} \\
\lambda_{4} \\
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Review: Diagonalizable linear transformations

Theorem

Let $T: V \to V$ be a linear transformation where V is finite dimensional, and T has different eigenvalues $\lambda_1, \ldots, \underline{\lambda_k}$. Suppose that W_i is the null space of $\lambda_i I - T$ for each $1 \le i \le k$. Then the following statements are equivalent:

- i. T is diagonalizable. \Longrightarrow $\bigvee_{\mathbf{k}} = \mathbf{W}_{\mathbf{k}} \oplus ... \oplus \mathbf{W}_{\mathbf{k}}$
- ii. The characteristic polynomial of T is

WisN(X:I-T)

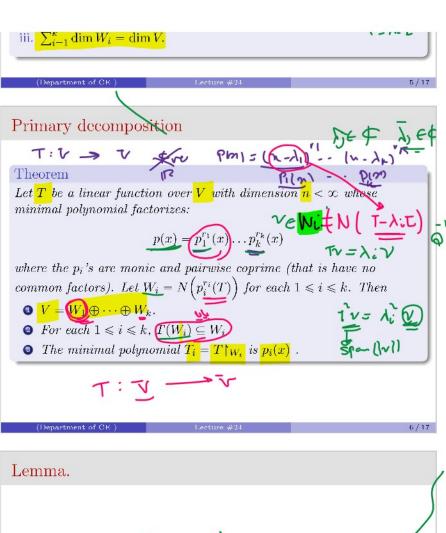
 $f(\lambda) = (\lambda - \lambda_1)^{n_1} \cdots (\lambda - \lambda_k)^{n_k},$

and dim $W_i = n_i$.

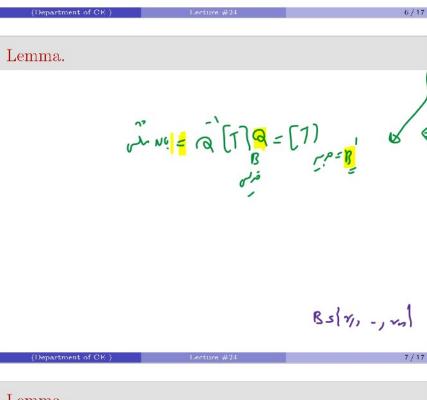
iii. $\sum_{i=1}^k \dim W_i = \dim V.$

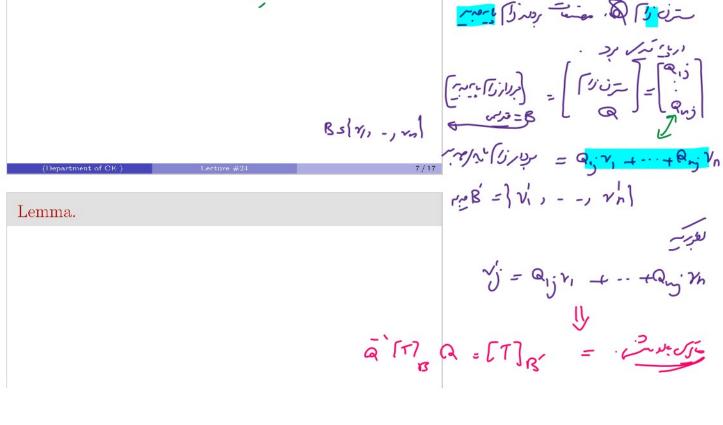
X-1:

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Q = [] P





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Lemma.		
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(Department of CE)	Lecture #24	10 / 17
Lemma.		

(Department of CE) Lecture #24	2 / 17
Lemma.	

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13 / 17

Jordan Form

Suppose that T is a linear function on V with the characteristic polynomial is

$$f(x) = (x - \lambda_1)^{d_1} \dots (x - \lambda_k)^{d_k}$$

where $\lambda_1, \ldots, \lambda_k$ are distinct elements and $d_i \ge 1$. Then the minimal polynomial for T will be

$$p(x) = (x - \lambda_1)^{r_1} \dots (x - \lambda_k)^{r_k}$$

where $1 \leq r_i \leq d_i$ based on the Cayley–Hamilton theorem. If W_i is the null space of $(T - \lambda_i I)^{r_i}$, then the primary decomposition theorem tells us that

$$V = W_1 \oplus \cdots \oplus W_k$$

such that the linear function $T_i = T \upharpoonright_{W_i} : W_i \to W_i$ has minimal polynomial $(x - \lambda_i)^{r_i}$.

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14 / 17

Jordan Form

$$V = W_1 \oplus \cdots \oplus W_k$$

Suppose that B_i is a basis for W_i . It has been proved that $B = \bigcup_{i=1}^k B_i$ is a basis for V. Based on primary decomposition theorem,

$$T(W_i) \subseteq W_i$$
.

Thus

$$[T]_B = \begin{bmatrix} [T_1]_{B_1} & & \\ & \ddots & \\ & & [T_k]_{B_k} \end{bmatrix}.$$

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15 / 17

Jordan Form

$$V = W_1 \oplus \cdots \oplus W_k$$

and

$$[T]_B = \begin{bmatrix} [T_1]_{B_1} & & \\ & \ddots & \\ & & [T_k]_{B_k} \end{bmatrix}.$$

- Let N_i be the linear function on W_i defined by $N_i = T \lambda_i I$.
- Then N_i is nilpotent and has minimal polynomial x^{r_i} .
- Thus, T on W_i acts as N_i plus the scalar λ_i times the identity function I.
- Suppose we choose a basis for the subspace W_i and then find the representation matrix of N_i on W_i .

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Lecture #24

16/17

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Thank You!

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17 / 1