Linear Algebra

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Review: Classification of n-alternating multilinear maps

- For an n-alternating multilinear map

$$\phi : \underbrace{V \times \cdots \times V}_{n} \to \mathbb{R}$$

we have

$$\begin{aligned}
\phi\left(a_1, \ldots, a_n\right) &= \sum_{j_1=1}^{n} \cdots \sum_{j_n=1}^{n} a_{1j_1} \cdots a_{nj_n} \phi\left(e_{j_1}, \ldots, e_{j_n}\right) \\
&= \sum_{\sigma \in S_n} \left(\prod_{i=1}^{n} a_{i\sigma(i)} \phi\left(e_{\sigma(1)}, \ldots, e_{\sigma(n)}\right) \right) \\
&= \left(\sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i\sigma(i)} \right) \operatorname{sgn}(\sigma) \phi\left(e_1, \ldots, e_n\right) \\
&= \left(\sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{i\sigma(i)} \right) \phi\left(e_1, \ldots, e_n\right)
\end{aligned}$$

Determinant

- Let a_i be the i-th row of $A = [a_{ij}]$.
- The determinant of A is defined by

$$\det A = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{i\sigma(i)}.$$

- For 2 by 2 matrix

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Properties of the Determinant

1. The determinant changes sign when two rows are exchanged.
2. The determinant of the identity matrix is 1.
3. The determinant depends linearly on the each row.

Properties of the Determinant

4. If two rows of A are equal, then $\det A = 0$ (why?)

5. Subtracting a multiple of one row from another row leaves the same determinant.(why?)

6. If A has a row of zeros, then $\det A = 0$ since the map det is n-multilinear.

Properties of the Determinant

7. If A is triangular then $\det A = a_{11}a_{22}\cdots a_{nn}$ (why?)

8. If A is singular, then $\det A = 0$. If A is invertible, then $\det A \neq 0$ (why?)

Properties of the Determinant

9. The transpose of A has the same determinant as A itself: $\det A = \det A^T$ (why?)

10. The determinant of AB is the product of $\det A$ times $\det B$ (why?)

11. Let A be an invertible matrix. Then $\det A \neq 0$.

Thank You!