

Linear Algebra

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Lecture #20

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Chapter 5

$Eigenvalues\ and\ Eigenvectors$

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Review: Eigenvectors and eigenvectors

Definition

An eigenvector or characteristic vector v for a square matrix A is a nonzero vector that changes at most by a scalar factor when that this matrix is applied to it, i.e., $Av = \lambda v$. The corresponding eigenvalue λ is denoted by as an eigenvalue of A.

P(X) s det (AE.A)

 $(\lambda \overline{L} - A) \times = 0 \qquad \longrightarrow \bigwedge (\lambda \overline{L} - A) \neq \emptyset$ $(\lambda \overline{L} - A) \times = 0$ $\det(A - \lambda I) = 0. \text{ This roots}$ Review: solving $Ax = \lambda x$ Λ

- Find the roots of the polynomial $det(A \lambda I) = 0$. This roots are the eigenvalues of A.
- The sum of the n eigenvalues equals the sum of the n diagonal entries:

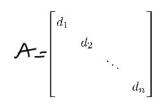
Trace of
$$A := a_{11} + \cdots + a_{nn} = \frac{\lambda_1 + \cdots + \lambda_n}{\lambda_1 + \cdots + \lambda_n}$$
.

• The product of the n eigenvalues equals the determinant of A, that is

$$\lambda_1 \times \cdots \times \lambda_n = \det \Lambda.$$

Example

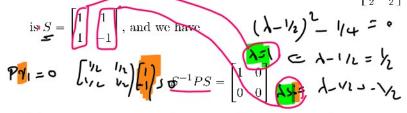
Find the eigenvalues and eigenvectors for diagonal matrices.



Diagonalization of a Matrix

• Example. The eigenvector matrix of the projection $\underline{\underline{P}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$



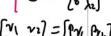


ullet The eigenvector matrix S converts A into its eigenvalue matrix 1/2 [1/2 LAL 1/4] [] P[V(V2) = [PV, PL] P^2 = P PEMn((R) which is diagonal.









A= [T]R

 $T(z) = \lambda x$

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f(x) = 2 (x-1) B

90° rotation

• The eigenvalues themselves are not so clear for a rotation:



has $det(K - \lambda I) = \frac{\lambda^2 + 1}{\lambda^2}$.

- The eigenvalues of K are imaginary number $\lambda = \pm i$ with eigenvectors $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ i \end{bmatrix}$
- The eigenvectors are also not real.
- The eigenvalues are distinct, even if imaginary, and the eigenvectors are independent. They go into the columns of S:



The eigenvalues are disonice, even it imaginary, and one eigenvectors are independent. They go into the columns of S:

$$S = \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix} \qquad \underbrace{S^{-1}KS}_{0} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}.$$

PEMM (18)

7/15/B= {V11 -- 1 Vnv) Vn-r+1, -1 Vn}

. N(P)

$$T(x) \stackrel{3}{\rightarrow} P_{2}$$

$$\begin{bmatrix} 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} C$$

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det (Dr - P) = det (Dr. []

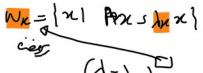
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D2 = 0

F(X) 5 X / (1-1)

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Linear Spaces on \mathbb{C}

A Linear space is a set V along with an addition on V and a scalar multiplication on C

such that the following properties hold:

- 1. **commutativity** u + v = v + u for all $u, v \in V$;
- 2. associativity (u+v)+w=u+(v+w) and (ab)v=a(bv) for all $u, v, w \in V$ and all $a, b \in \mathbb{C}$;
- 3. additive identity there exists an element $0 \in V$ such that v + 0 = v for all $v \in V$;
- 4. additive inverse for every $v \in V$, there exists $w \in V$ such that v + w = 0;
- 5. multiplicative identity 1v = v for all $v \in V$;
- 6. distributive properties a(u+v) = au + av and (a+b)u = au + bu for all $a,b \in \mathbb{C}$ and all $u,v \in V$.

Diagonalizable matrices

Definition

Similarity. Let $A, B \in M_n(\mathbb{F})$, $\mathbb{F} = \mathbb{R}$ or \mathbb{C} . We say that A and B are similar over \mathbb{F} , if there exists an invertible matrix $S \in \mathbb{F}$ such that $S^{-1}AS = B.$

Definition

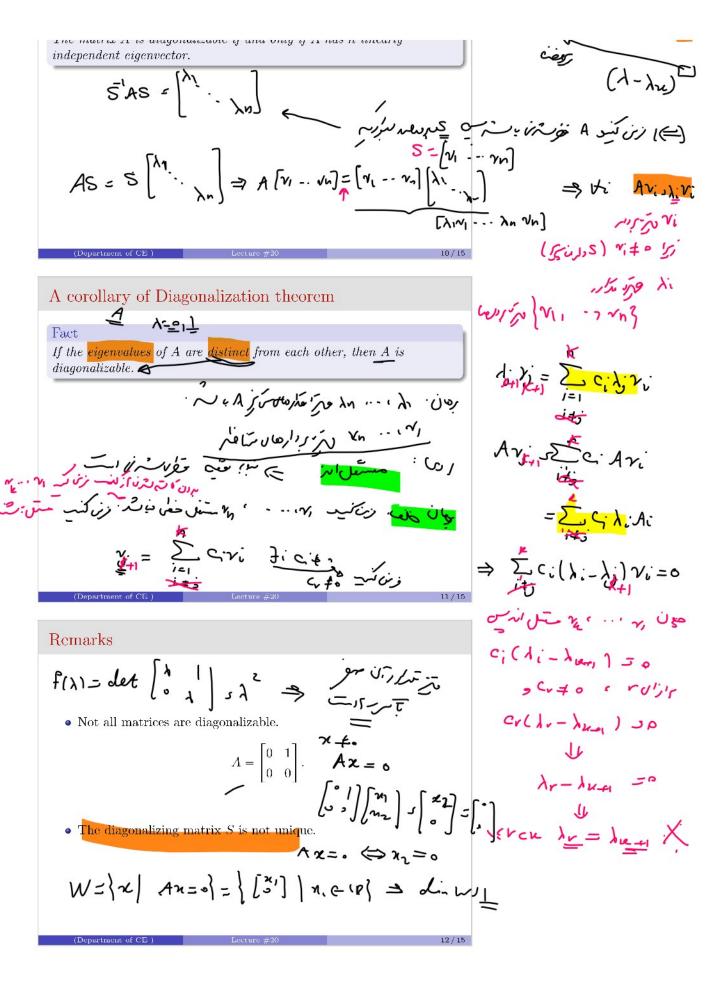
Assume $A \in M_n(\mathbb{R})$. A is called diagonalizable if it is similar to a diagonal matrix, i.e., if there exists an invertible matrix S and a diagonal matrix D such that

$$S^{-1}AS = D.$$

Diagonalization of a Matrix

Theorem

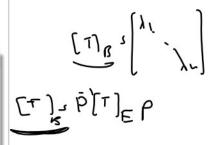
The Diagonalization Theorem. Let $A \in M_n(\mathbb{F})$ where $\mathbb{F} = \mathbb{R}$ or \mathbb{C} . The matrix A is diagonalizable if and only if A has n linearly independent eigenvector.



Diagonalizable linear transformation

Definition

A linear transformation $T:V\to V$ where the dimension of V is finite, is said to be diagonalizable if there exists a basis B such that $[T]_B$ is diagonal matrix.



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Diagonalizable linear transformation

Theorem

Let $T: V \to V$ be a linear transformation where the dimension of V is finite with different eigenvalues $\lambda_1, \ldots, \lambda_k$. Suppose that W_i is null space of $T - \lambda_i I$ for each $1 \le i \le k$. The the following statements are equivalent:

- T is diagonalizable.
- Its eigenvalue vector is $f(x) = (x d_1)^{n_1} \dots (x d_n)^{n_n} \dots (x d_n)^{n_n}$ and $\dim W_i = n_i$.

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