Inference for contrasts

$\underline{c} = (c_1, c_2, \dots, c_g) \in \mathbb{R}^g$

$E\left[\sum_{i=1}^{g} c_i \bar{y}_{i.}\right] = \sum_{i=1}^{g} c_i \cdot \mu_i \implies$ unbiased estimator

$Var\left(\sum_{i=1}^{g} c_i \bar{y}_{i.}\right) = \sum_{i=1}^{g} c_i^2 \cdot \frac{\sigma^2}{n_i}$ ("estimation accuracy", standard error)

$$\implies \text{Use } T = \frac{\sum_{i=1}^{g} c_i \bar{y}_{i.} - \underbrace{\sum_{i=1}^{g} c_i \mu_i}_{= 0 \text{ under } H_0}}{\hat{\sigma}\sqrt{\sum_{i=1}^{g} c_i^2 \cdot \frac{1}{n_i}}} \sim t_{N-g}$$

as test statistics, where $\hat{\sigma} = \sqrt{MS_E}$.

CI for $\sum_{i=1}^{g} c_i \cdot \mu_i$:

$$I = \underbrace{\sum_{i=1}^{g} c_i \cdot \bar{y}_{i.}}_{\text{estimate}} \pm \underbrace{t_{N-g,\,1-\frac{\alpha}{2}}}_{\text{quantile}} \underbrace{\sqrt{MS_E}\sqrt{\sum_{i=1}^{g} \frac{c_i^2}{n_i}}}_{\text{standard error}}$$

(like t-test!)