

## ch2

ideally find cause-effect relationships.

but usually we get only OBSERVATION data, we can only make inference about its association.

Experimental unit: X "Apply treatment by randomization"  
Response unit: Y surrogate response.

Experimental error:

prospective vs case-control (retro-spective)

Cohort study

$$\hat{\alpha}_i = \frac{1}{N-g} \sum_{j=1}^g \sqrt{\frac{1}{n_i} - \frac{1}{N}} F_{in} = t_n$$

## ch3 Completely Randomized Designs.

Balanced if the # experimental units are the same

cell-mean Model

each treatment group to have its own expectation.

Regression with a categorical response.

sum to zero

contr. treatment. compare to the reference group

Test:  $SST = S_{treat} + SSE$   
 $g-1 \quad N-g$

$$F_{ratio} = \frac{MS_{treat}}{MSE}$$

F-test compare "Is a treatment effect or not"

## Checking model assumption

const. Error

Error normally

Error mean

T

Index-plot to check for serial structure.

## Interpretation After transformation

## ch4 contrasts and multiple testing

Each contrast can be made via the sum of squares

$$\frac{MS_C}{MS_E} \sim F_{1, N-g}$$

Two contrast are orthogonal if

$$\sum_{i=1}^g \frac{c_i c_j}{n_i} = 0$$

If we have g treatment  $\Rightarrow$  only (g-1) different orthogonal

## Multiple testing

$$\text{Prob}[\geq 1 \text{ False rejection}] = 1 - (1-\alpha)^m$$

Family-wise-error rate.

rejecting at least one of the true  $H_0$ :

$$FWER = \text{Prob}[\geq 1]$$

Bonferroni

$$-\frac{\alpha}{m}$$

Bonferroni-Holm

1. Rank them

2.  $P_{(j)} \leq \frac{\alpha}{m-j+1}$  Reject.

3. Stop until first insignificant

Scheffe

using (g-1)  $F_{g-1, N-g}$

Tukey comparison of all possible combinations

Multiple compare to control

## ch5 Factorial Treatment Structure

crossed: Observed all of the combinations of the factors  
Visualize using interaction plot

Two-way ANOVA ratio  $\tau$  vs  $\beta$ ;  $\alpha$  vs  $\beta$  interaction

interaction: How much in response when we switch on another variables.

Additive without any interaction

Test for Two way anova:

$$SST = SSB + SSB + SSB + SSE$$

$$ab(n-1)$$

Interaction

$$\frac{MS_{AB}}{MSE}$$

$$F_{(a-1)(b-1), ab(n-1)}$$

main effect A

$$F_{a-1, ab(n-1)}$$

B similar

Single Replicates

Cannot do inference due to we cannot estimate experimental error.

Can still fit a main-effect model

Change of scale by logarithm may drop the interaction

Unbalanced data.

Regression: still OK. coefficient a bit ugly.

Error: cannot attribute which part of error to which.

How: Reduction in residual sum of squares

$$SSCB(1, A)$$

reduction in residual sum of squares

$$(1, A, B) \text{ and } (1, A)$$

Type I sequential

Type II hierarchical

$$SSCA(1)$$

$$SSCA(1, B)$$

$$SSCB(1, A)$$

$$SSCB(1, A)$$

$$SSCAB(1, A, B)$$

$$SSCAB(1, A, B)$$

Type II fully adjusted

$$SSCAB(1, A, B)$$

$$SSCA(1, A, B)$$

$$SSCB(1, A, B)$$

classical rule

$$\text{dof of SSE} \geq 10$$

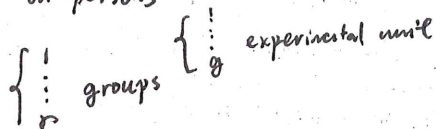
Incomplete Block  $\rightarrow$  Balanced Block design Variance

## ch 6 Block Design

Idea: Apply different treatment on same "person" to remove the person to person variation.

"Block" on persons

### RCBD



在每个 r 里面, treatments are randomized.  
complete:  $\{1 \dots g\}$  cover all set of treatments.

- It can be the case where only 2 replicates (No interaction term can be fit)

$$Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$$

If MS of the block factor  $\gg$  MSE  
we can conclude blocking was efficient.

More than one factor

$$Y \sim \text{Block} + A + B$$

### Multiple Block Factors

Latin square Design: 2 blocks factors one treatment.

## ch 7 Random and mixed Effects Models

Not fixed  $d_i \sim N(0, \dots)$   
For each i / treatment,  $\epsilon_i$  is a RV.

$$\text{Var}(Y_i) = \text{Var}(\alpha) + \sigma^2$$

$$\text{lmer}(Y \sim (1|x))$$

Two way ANOVA with random effects

$$\text{lmer}(Y \sim (1|x) + (1|y) + (1|x:y))$$

$$F = \frac{MS_A}{MS_{ABO}} \sim F_{a-1, a(b-1)}$$

### Nesting

combining factors in different way

cash 1 batch 1 ||  
cash 1 batch 2

$$\text{lmer}(1| \text{batch/cash})$$

random effect per batch and per cash within cash

$$\text{lmer}(1| \text{batch}) + (1| \text{cash:batch})$$

random effect per combination

CANNOT use

$$\text{lmer}(1| \text{batch}) + (1| \text{cash})$$

### Mixed Effects model.

## ch 8 Split Plot Design

fertilizer  $\rightarrow$  plot  
variety  $\rightarrow$  subplot

whole plot fact  
split  $\rightarrow$

$$Y_{ijk} = \mu + \alpha_i + n_{ki} + \beta_j + (\alpha\beta)_{ij} + \epsilon$$

↑  
whole plot error.

ensure we can get a whole plot error per plot

$$\text{lmer}(Y \sim \text{fertil} + \text{variety} + (1|\text{plot}))$$

Basically RCBD on the plot level.  
whole

Special case of Factorial treatment structure.

split-split

## ch 9. Incomplete Block Design

disconnected design: Grouping / Blocking 会导致没有所有组合

BIBD

$$N = b \cdot k = g \cdot r$$

↑  
units per block  
↑  
replicate treatment

Blocked: 如字所有组合用样经常发生 (1)

$$r(k-1)/(g-1) \text{ 一定要是整数}$$

## ch 10. Power

$$\text{Power} = 1 - \beta = 1 - \text{Type II}$$

是否 reject  $H_0$  的概率

MS

F-test for fully nested model.

$$F = \frac{MS_A}{MS_{B(A)}}$$