$$\mathbb{P}\left[\text{there is at least one false rejection among all } m \text{ tests}\right]$$

$$= \mathbb{P}\left[\bigcup_{j=1}^{m} \underbrace{\{\text{test } j \text{ falsely rejects } H_{0,j}\}}_{=: A_j}\right] \qquad (*)$$

$$\overset{\substack{\text{union}\\\text{bound}}}{\leq} \sum_{j=1}^{m} \underbrace{\mathbb{P}[A_j]}_{\alpha} = m \cdot \alpha \quad \text{for } \underline{\text{any}} \text{ possible dependency}$$
$$\text{structure.}$$

$$(*) = 1 - \mathbb{P}\left[\bigcap_{j=1}^{m} A_j^c\right] \overset{\text{indep.}}{=} 1 - \prod_{j=1}^{m} \underbrace{\mathbb{P}[A_j^c]}_{1-\alpha}$$

$$= \underline{1 - (1-\alpha)^m}$$