

Decomposition of total variability

$$\underbrace{(y_{ij} - \bar{y}_{..})}_{\text{deviation from grand mean}} = \underbrace{(y_{ij} - \bar{y}_{i.})}_{\text{deviation from group mean (= residual)}} + \underbrace{(\bar{y}_{i.} - \bar{y}_{..})}_{\text{deviation of group mean from grand mean (= treatment effect)}}$$

Pyth. \Rightarrow cross-prod. vanishes

$$\sum_{i=1}^g \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^g \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 + \sum_{i=1}^g \sum_{j=1}^{n_i} (\bar{y}_{i.} - \bar{y}_{..})^2$$
$$=: SS_T \qquad \qquad \qquad =: SS_E \qquad \qquad \qquad =: SS_{Trt}$$

where

SS_T = total sum of squares

SS_{Trt} = treatment sum of squares = $\sum_{i=1}^g n_i \bar{y}_{i.}^2$
("between")

SS_E = error sum of squares ("within")
(residual sum of squares)

$$\Rightarrow SS_T = SS_{Trt} + SS_E$$