CG2023 Selected Tutorial Questions

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Tutorial 5

Q2

Q.2 Solve the following linear second order differential equation using Laplace Transform:

$$\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=2u(t)$$

where $y(0^-)=1$ and $y'(0^-)=0$.

Solution to Q.2

Given: $\dfrac{d^2y(t)}{dt^2}+4\dfrac{dy(t)}{dt}+3y(t)=2u(t)$, $\quad y(0^-)=1 \quad$ and $\quad y'(0^-)=0$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = s\tilde{F}(s) - f(0^-)$$

$$\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\} = s^2\tilde{F}(s) - sf(0^-) - f'(0^-)$$

Apply Laplace transform:

$$s^2Y(s) - \underbrace{y(0^-)}_{1}s - \underbrace{y'(0^-)}_{0} + 4sY(s) - 4\underbrace{y(0^-)}_{1} + 3Y(s) = \frac{2}{s}$$

$$\left(s^2+4s+3\right)Y(s) = \frac{2}{s} + s + 4$$

$$Y(s) = \frac{s^2+4s+2}{s\left(s^2+4s+3\right)} = \frac{s^2+4s+2}{s(s+1)(s+3)} = \frac{2}{3s} + \frac{1}{2(s+1)} - \frac{1}{6(s+3)}$$

Apply inverse Laplace transform:

$$y(t) = \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = \left[\frac{2}{3} + \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}\right]u(t)$$

Tutorial 6

Q3

At steady-state, the input to a 1st-order system $\tilde{H}(s)=\dfrac{K}{sT+1}$ is $x(t)=0.4+2\sin(0.25t)$, and $y(t)$ is the corresponding system output.

(a) Find an expression for $y(t)$ in terms K and T.

(b) If $y(t)=2+8\sin\left(0.25(t-2.574)\right)$, find the values of K and T.

(a)

Sinusoidal Response at Steady-State

$$x(t) = Ae^{j(\omega_0 t+\psi)} \rightarrow \boxed{\tilde{H}(j\omega)} \rightarrow y(t) = A\left|\tilde{H}(j\omega_0)\right|e^{j(\omega_0 t+\psi+\angle\tilde{H}(j\omega_0))}$$

Frequency response of the system $\tilde{H}(s) = \dfrac{K}{sT+1}$ is given by

$$\tilde{H}(j\omega) = \tilde{H}(s)\Big|_{s=j\omega} = \frac{K}{j\omega T+1} \rightarrow \begin{cases} \left|\tilde{H}(j\omega)\right| = \dfrac{K}{\left[\omega^2 T^2+1\right]^{1/2}} & \cdots\cdots \text{ Magnitude response} \\ \angle\tilde{H}(j\omega) = -\tan^{-1}\left(\dfrac{\omega T}{1}\right) & \cdots\cdots\cdots\cdots \text{ Phase response} \end{cases}$$

The system input $x(t)=0.4+2\sin(0.25t)$ consists of a DC component which has a frequency of 0 rad/s and a sine wave of frequency 0.25 rad/s. The sinusoidal responses of the system at these frequencies are:

$$@\ 0\ \text{rad/s} \ \rightarrow \begin{cases} \left|\tilde{H}(j0)\right| = K \\ \angle\tilde{H}(j\omega) = 0 \end{cases} \quad \text{and} \quad @\ 0.25\ \text{rad/s} \ \rightarrow \begin{cases} \left|\tilde{H}(j0.25)\right| = \dfrac{K}{\left[0.25^2T^2+1\right]^{1/2}} \\ \angle\tilde{H}(j0.25) = -\tan^{-1}\left(\dfrac{0.25T}{1}\right) \end{cases}$$

Hence,

$$\begin{aligned} y(t) &= 0.4\left|\tilde{H}(j0)\right| + 2\left|\tilde{H}(j0.25)\right|\sin\left(0.25t+\angle\tilde{H}(j0.25)\right) \\ &= 0.4K + \frac{2K}{\left(0.25^2T^2+1\right)^{1/2}}\sin\left(0.25t-\tan^{-1}\left(\frac{0.25T}{1}\right)\right) \qquad \cdots\cdots\cdots \ (\clubsuit) \end{aligned}$$

(b)

$$y(t)=0.4\left|\tilde{H}(j0)\right|+2\left|\tilde{H}(j0.25)\right|\sin\left(0.25t+\angle\tilde{H}(j0.25)\right)$$

$$=0.4K+\frac{2K}{\left(0.25^2T^2+1\right)^{1/2}}\sin\left(0.25t-\tan^{-1}\left(\frac{0.25T}{1}\right)\right) \quad \cdots\cdots\cdots \ (\clubsuit)$$

Comparing $(\clubsuit)$ with the given output $y(t)=2+8\sin\left(0.25(t-2.574)\right)$, we get

$$0.4K=2 \quad \rightarrow \quad K=5$$

$$\left.\begin{aligned} &\text{Using } \frac{2K}{\sqrt{0.25^2T^2+1}}=8 \ \text{ or} \\ &\tan^{-1}(0.25T)=0.25\times 2.574 \end{aligned}\right\} \quad \rightarrow \quad T=3$$

Q4

At steady-state, $y(t) = 5 + 4\cos(3t + 0.3142)$ is the output of a 1st-order system $\tilde{H}(s) = \dfrac{20}{s+4}$. Derive the corresponding system input $x(t)$.

$$\left.\begin{aligned}
x(t) = Ae^{j(\omega_o t+\psi)} \longrightarrow \boxed{\tilde{H}(j\omega)} \longrightarrow y(t) &= A\left|\tilde{H}(j\omega_o)\right| e^{j(\omega_o t+\psi+\angle\tilde{H}(j\omega_o))} \\
x(t) = A\cos(\omega_o t+\psi) \longrightarrow \boxed{\tilde{H}(j\omega)} \longrightarrow y(t) &= A\left|\tilde{H}(j\omega_o)\right| \cos\left(\omega_o t+\psi+\angle\tilde{H}(j\omega_o)\right) \\
x(t) = A\sin(\omega_o t+\psi) \longrightarrow \boxed{\tilde{H}(j\omega)} \longrightarrow y(t) &= A\left|\tilde{H}(j\omega_o)\right| \sin\left(\omega_o t+\psi+\angle\tilde{H}(j\omega_o)\right)
\end{aligned}\right\} \quad (7.20)$$

Steady-state Sinusoidal Response of a LTI System in ω-domain

Frequency response of the system $\tilde{H}(s) = \dfrac{20}{s+4}$ is given by

$$\tilde{H}(j\omega) = \tilde{H}(s)\Big|_{s=j\omega} = \frac{20}{j\omega+4} \rightarrow \begin{cases} \left|\tilde{H}(j\omega)\right| = \dfrac{20}{\left[\omega^2+16\right]^{1/2}} & \cdots\cdots \text{ Magnitude response} \\ \angle\tilde{H}(j\omega) = -\tan^{-1}\left(\dfrac{\omega}{4}\right) & \cdots\cdots\cdots\cdots \text{ Phase response} \end{cases}$$

The system output $y(t) = 5 + 4\cos(3t + 0.3142)$ consists of a DC component which has a frequency of 0 rad/s and a cosine wave of frequency 3 rad/s. The sinusoidal responses of the system at these frequencies are:

$$@\ 0 \text{ rad/s} \rightarrow \begin{cases} \left|\tilde{H}(j0)\right| = 5 \\ \angle\tilde{H}(j\omega) = 0 \end{cases} \quad \text{and} \quad @\ 3 \text{ rad/s} \rightarrow \begin{cases} \left|\tilde{H}(j3)\right| = \dfrac{20}{[9+16]^{1/2}} = 4 \\ \angle\tilde{H}(j3) = -\tan^{-1}\left(\dfrac{3}{4}\right) = -0.6435 \end{cases}$$

Hence, the input at steady-state is

$$\begin{aligned} x(t) &= \frac{5}{\left|\tilde{H}(j0)\right|} + \frac{4}{\left|\tilde{H}(j3)\right|}\cos\left(3t + 0.3142 - \angle\tilde{H}(j3)\right) \\ &= \frac{5}{5} + \frac{4}{4}\cos\left(3t + 0.3142 - \left[-\tan\frac{3}{4}\right]\right) \\ &= 1 + \cos(3t + 0.9577) \end{aligned}$$

Q5

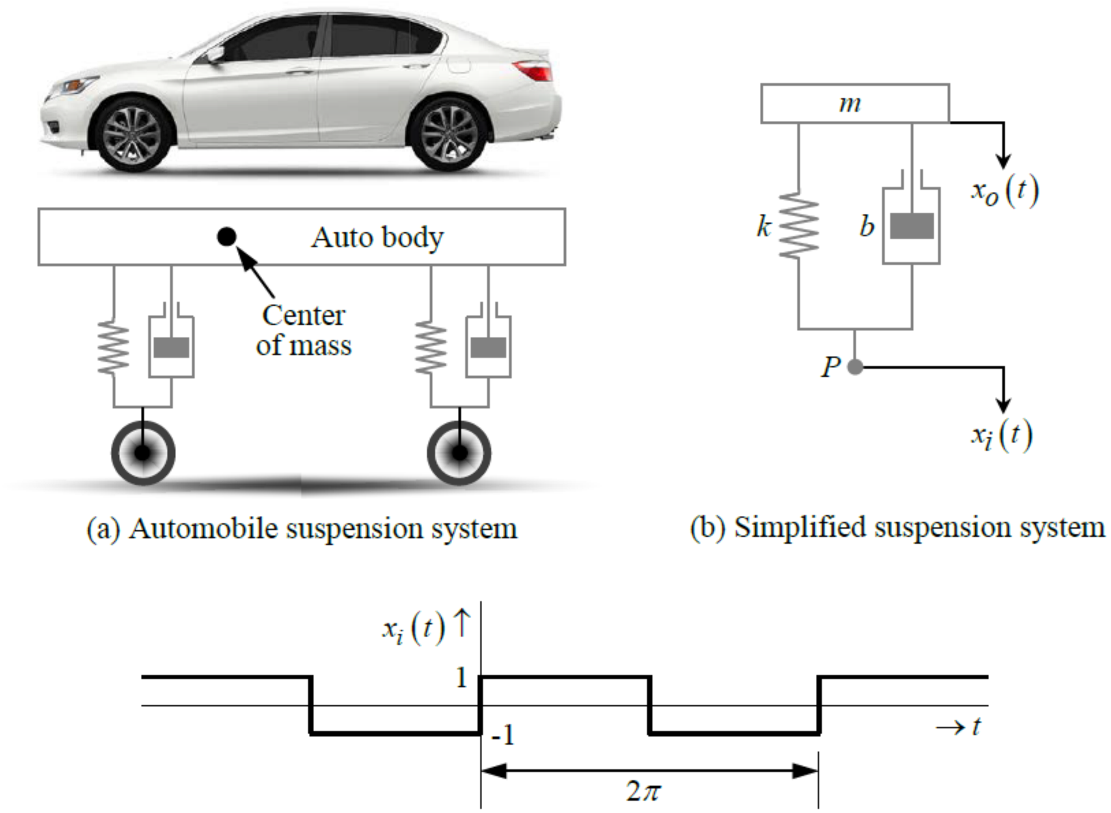
A car suspension system and a very simplifed version of the system are shown in Figure 5(a) and 5(b) respectively.

The transfer function of the simplifed car suspension system is

$$\frac{X_o(s)}{X_i(s)} = \frac{bs+k}{ms^2+bs+k}$$

Suppose a car $\left(m = 4\ kg,\ k = 1\ Nm^{-1},\ b = 1\ Nm^{-1}s\right)$ is travelling on a road that has speed reducing stripes and the input to the simplifed car suspension system, $x_i(t)$, may be modelled by the periodic square wave, of frequency $\omega = 1\ \text{rad/s}$, shown in Figure 5(c).

Q5



(a) Automobile suspension system

(b) Simplified suspension system

(c) Input waveform, $x_i(t)$

Figure 5

Q5

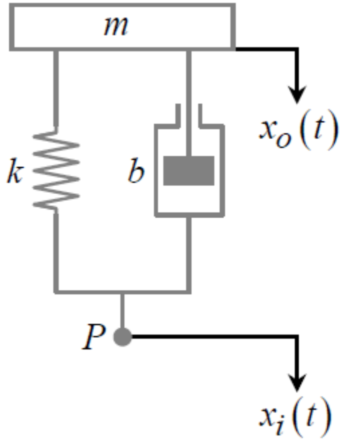
Determine the displacement of the car body, $x_o(t)$.

Hint : The Fourier Series representation of the periodic square wave shown in Figure 5(c) is

$$x_i(t) = \frac{4}{\pi}\left[\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots\cdots\right]$$

Transfer function of suspension system:

$$\tilde{H}(s)=\frac{\tilde{X}_o(s)}{\tilde{X}_i(s)}=\frac{bs+k}{ms^2+bs+k}$$



Substuting $m=4\ kg,\ \ k=1\ Nm^{-1},\ \ b=1\ Nm^{-1}s$, we get

$$\tilde{H}(s)=\frac{s+1}{4s^2+s+1}$$

Frequency response of suspension system:

$$\tilde{H}(j\omega)=\tilde{H}(s)\Big|_{s=j\omega}=\frac{s+1}{4s^2+s+1}\Bigg|_{s=j\omega}=\frac{1+j\omega}{1-4\omega^2+j\omega}$$

$$\rightarrow\begin{cases}\text{Magnitude Response:} & \left|\tilde{H}(j\omega)\right|=\left(\dfrac{1+\omega^2}{\left(1-4\omega^2\right)^2+\omega^2}\right)^{1/2}\\[2ex] \text{Phase Response:} & \angle\tilde{H}(j\omega)=\tan^{-1}\left(\dfrac{\omega}{1}\right)-\tan^{-1}\left(\dfrac{\omega}{1-4\omega^2}\right)\end{cases}$$

$$\left.\begin{array}{l}
x(t)=Ae^{j(\omega_o t+\psi)} \longrightarrow \boxed{\tilde{H}(j\omega)} \longrightarrow y(t)=A\left|\tilde{H}(j\omega_o)\right|e^{j(\omega_o t+\psi+\angle\tilde{H}(j\omega_o))} \\
x(t)=A\cos(\omega_o t+\psi) \longrightarrow \boxed{\tilde{H}(j\omega)} \longrightarrow y(t)=A\left|\tilde{H}(j\omega_o)\right|\cos(\omega_o t+\psi+\angle\tilde{H}(j\omega_o)) \\
x(t)=A\sin(\omega_o t+\psi) \longrightarrow \boxed{\tilde{H}(j\omega)} \longrightarrow y(t)=A\left|\tilde{H}(j\omega_o)\right|\sin(\omega_o t+\psi+\angle\tilde{H}(j\omega_o))
\end{array}\right\} \quad (7.20)$$

Steady-state Sinusoidal Response of a LTI System in ω-domain

Fourier series expansion of input: $x_i(t)=\frac{4}{\pi}\left[\sin(t)+\frac{1}{3}\sin(3t)+\frac{1}{5}\sin(5t)+\cdots\cdots\right]$

- Response of system due to sinusoidal input $\sin(t)$ is given by

$$\left|\tilde{H}(j1)\right|\cdot\sin\left(t+\angle\tilde{H}(j1)\right)=0.4472\sin(t-2.0344)$$

- Response of system due to sinusoidal input $\sin(3t)$ is given by

$$\left|\tilde{H}(j3)\right|\cdot\sin\left(3t+\angle\tilde{H}(j3)\right)=0.09\sin(3t-1.807)$$

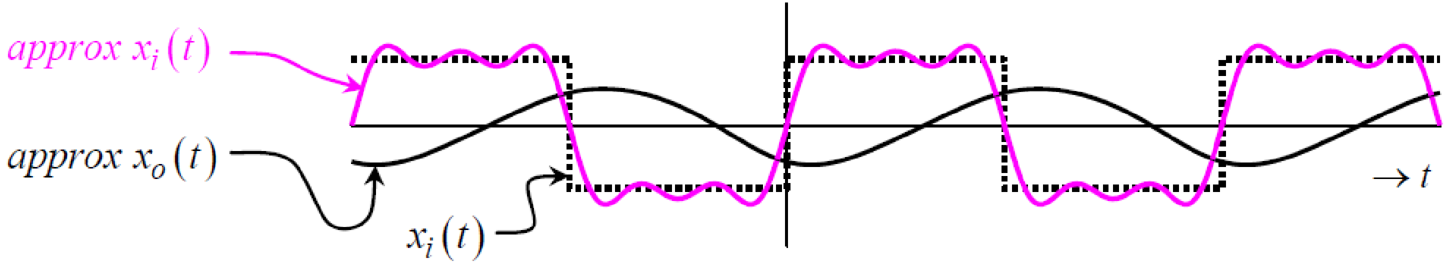
- Response of system due to sinusoidal input $\sin(5t)$ is given by

$$\left|\tilde{H}(j5)\right|\cdot\sin\left(5t+\angle\tilde{H}(j5)\right)=0.0514\sin(5t-1.7177)$$

Since system is linear, the output of the system can be obtained by superposition. Hence,

$$x_o(t) = \frac{4}{\pi}\left[0.4472\sin(t-2.0344) + \frac{1}{3}\times 0.09\sin(3t-1.807) + \frac{1}{5}\times 0.0514\sin(5t-1.7177) + \cdots\right]$$

$$= 0.5694\sin(t-2.0344) + 0.0382\sin(3t-1.807) + 0.0131\sin(5t-1.7177) + \cdots\cdots\cdots$$



Tutorial 7

Q3

The transfer function of a 2nd-order system is given by $\tilde{H}(s)=\dfrac{s+50}{0.2s^2+1.2s+5}$.

(a) Determine the DC gain (in dB), the damping ratio, ζ, and the natural frequency, ω_n, of the system.

(b) Is the system overdamped, critically damped, underdamped or undamped, and why?

(c) Draw the Bode magnitude and phase plots for the system using the semilog-x grid provided below.

(a)

$$\text{DC gain} = \tilde{H}(s)\Big|_{s=0} = \left.\frac{s+50}{0.2s^2+1.2s+5}\right|_{s=0} = 10$$

$$\equiv 20\log_{10}(10) = 20 \text{ dB}$$

To determine ζ and ω_n, first rewrite the denominator of $\tilde{H}(s)$ as $0.2\left(s^2+6s+25\right)$ so that the coefficient of s^2 is equal to 1. Then compare the bracketted term $s^2+6s+25$ with $s^2+2\zeta\omega_n s+\omega_n^2$. This results in

$$\left.\begin{aligned} 2\zeta\omega_n &= 6 \\ \omega_n^2 &= 25 \end{aligned}\right\} \text{ solving} \rightarrow \quad \omega_n = 5 \text{ and } \zeta = 0.6$$

(b)

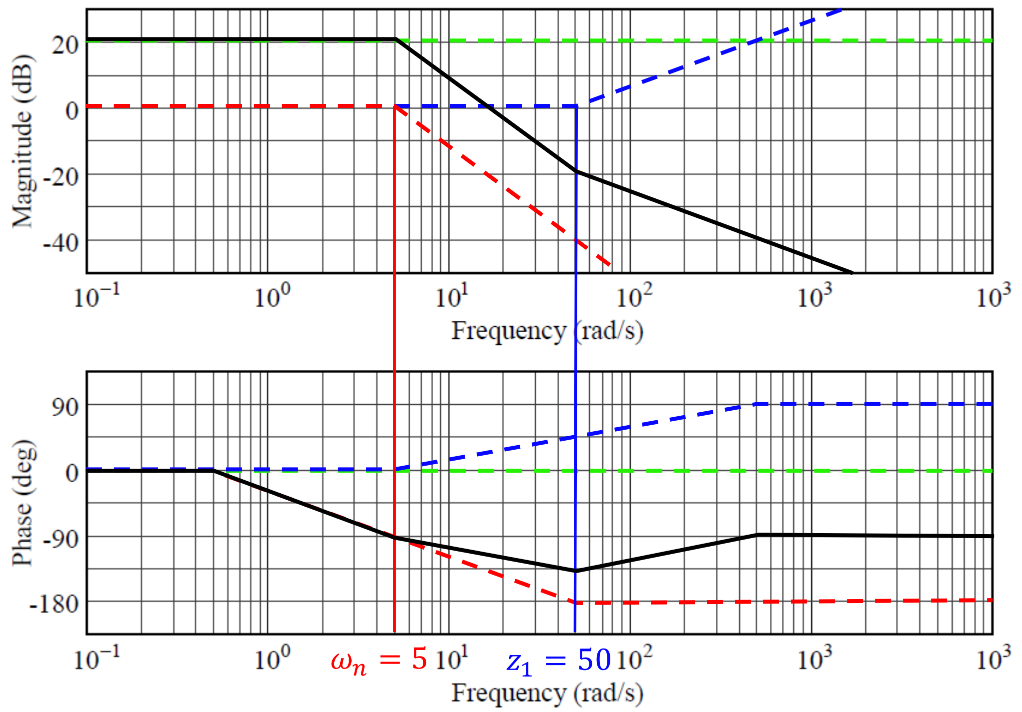
$$\left.\begin{aligned} 2\zeta\omega_n &= 6 \\ \omega_n^2 &= 25 \end{aligned}\right\} \text{ solving} \rightarrow \quad \omega_n = 5 \text{ and } \zeta = 0.6$$

The system UNDERDAMPED because $0 < \zeta < 1$.

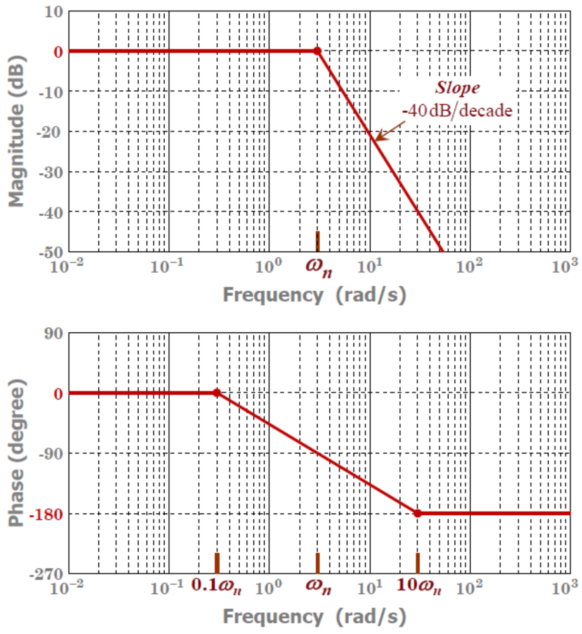
(c)

Rewrite $\tilde{H}(s) = \dfrac{s+50}{0.2s^2+1.2s+5}$ as

$$\tilde{H}(s) = \frac{50}{0.2}\cdot\frac{\frac{s}{50}+1}{s^2+6s+25} = 250\cdot\left(\frac{s}{50}+1\right)\cdot\left(\frac{1}{s^2+6s+25}\right) = \underbrace{10}_{\text{dc}}\cdot\underbrace{\left(\frac{s}{50}+1\right)}_{\text{zero factor}}\cdot\underbrace{\left(\frac{25}{s^2+6s+25}\right)}_{\text{2nd-order factor}}$$



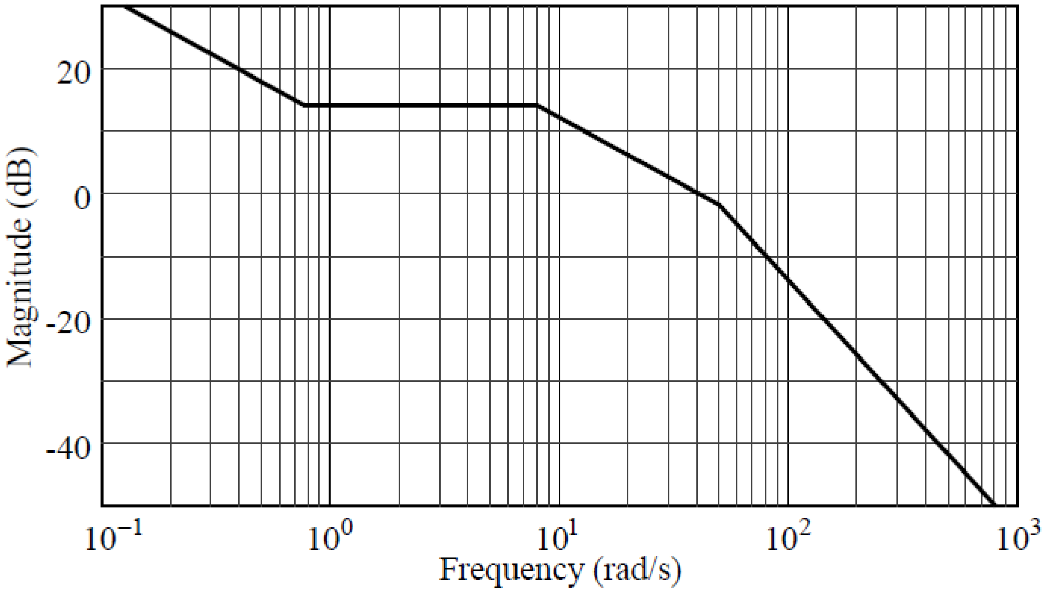
2nd-order Factor



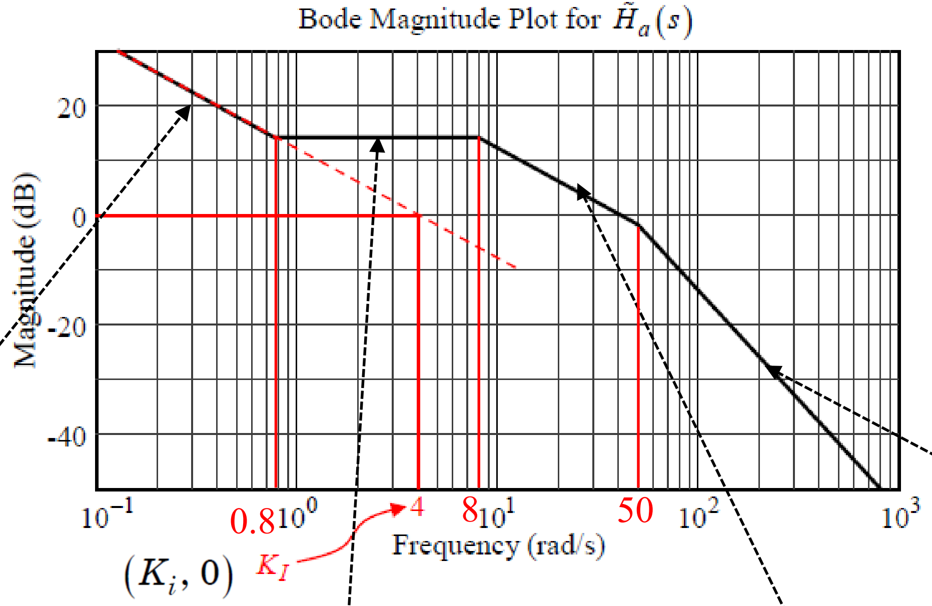
Q5

(a) Identify the system $\tilde{H}_a(s)$ from its Bode magnitude plot shown below. Describe the behavior of the system at very low frequency $(\omega << 0.8 \text{ rad/s})$ and at very high frequency $(\omega >> 50 \text{ rad/s})$.

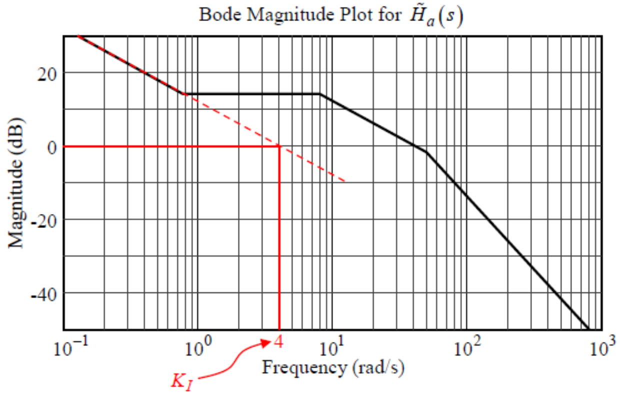
Bode Magnitude Plot for $\tilde{H}_a(s)$



Q5(a)



The low frequency asymptote is a line with slope -20 dB/decade, which indicates the presence of an integrator $\frac{K_I}{s} = \frac{4}{s}$	A slope change of +20 dB/decade occurs at 0.8 rad/s, which indicates the presence of a zero factor $\frac{s}{0.8} + 1$	A slope change of -20 dB/decade occurs at 8 rad/s, which indicates the presence of a pole factor $\frac{1}{\frac{s}{8} + 1}$	A slope change of -20 dB/decade occurs at 50 rad/s, which indicates the presence of a pole factor $\frac{1}{\frac{s}{50} + 1}$



Therefore,

$$\tilde{H}_a(s)=\frac{4}{s}\cdot\left(\frac{s}{0.8}+1\right)\cdot\left(\frac{1}{\frac{s}{8}+1}\right)\cdot\left(\frac{1}{\frac{s}{50}+1}\right)=\frac{4}{s}\cdot\frac{1}{0.8}(s+0.8)\cdot 8\left(\frac{1}{s+8}\right)\cdot 50\left(\frac{1}{s+50}\right)$$

$$=\frac{2000}{s}\cdot\frac{s+0.8}{(s+8)(s+50)}$$

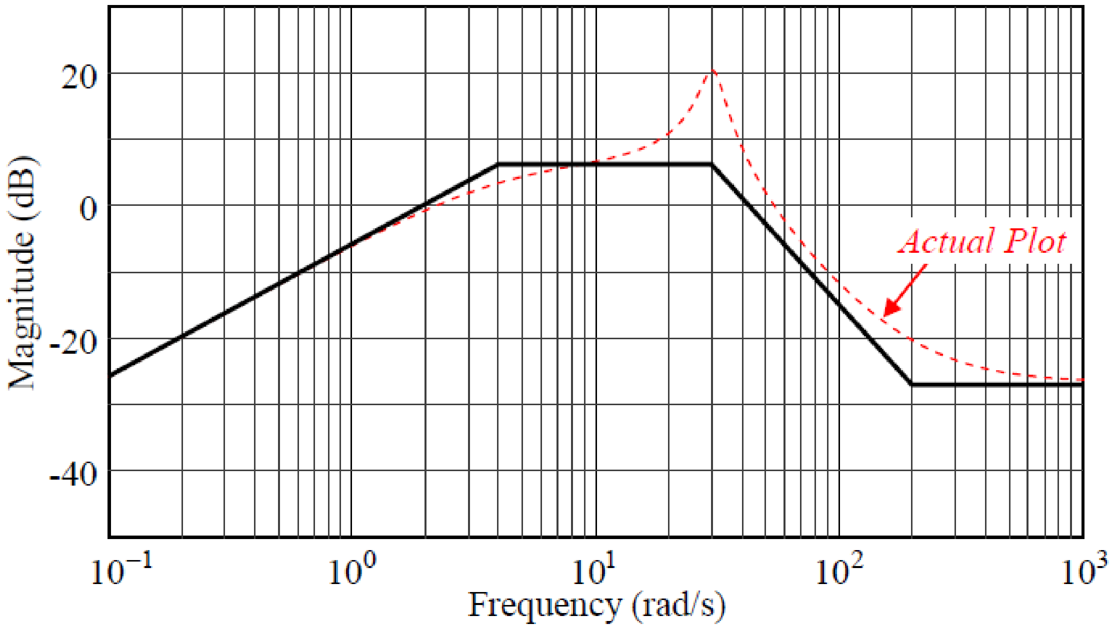
At $\omega << 0.8$ rad/s: $\tilde{H}_a(s)\cong\frac{2000}{s}\cdot\frac{\cancel{s}+0.8}{(\cancel{s}+8)(\cancel{s}+50)}=\frac{4}{s}$ $\leftarrow$ (Integrator of gain 4).

At $\omega >> 50$ rad/s: $\tilde{H}_a(s)\cong\frac{2000}{s}\cdot\frac{s+\cancel{0.8}}{(s+\cancel{8})(s+\cancel{50})}=\frac{2000}{s^2}$ $\leftarrow$ (double integrator of combined gain 2000).

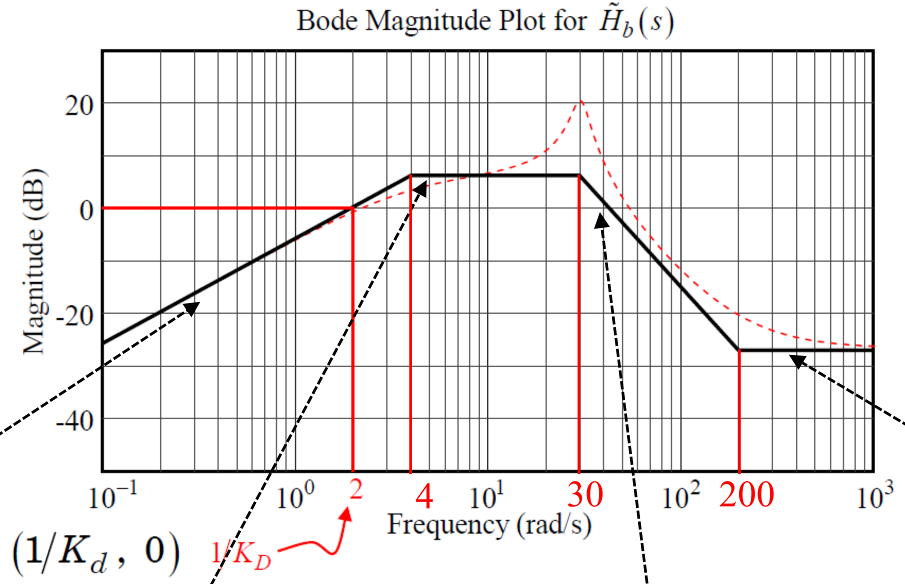
(b) The Bode magnitude plot for the system $\tilde{H}_b(s) = K\dfrac{(s+a)(s+b)(s+c)}{(s+d)\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$ is shown below.

Find the values of K, a, b, c, d and ω_n.

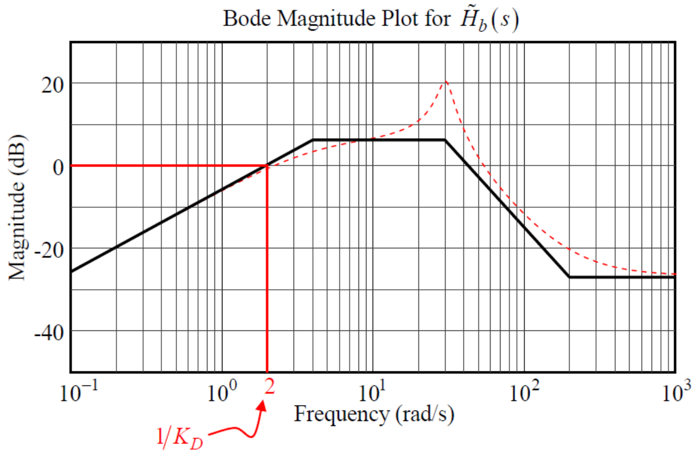
Bode Magnitude Plot for $\tilde{H}_b(s)$



Q5(b)



The low frequency asymptote is a line with slope +20 dB/decade, which indicates the presence of a differentiator $K_D\,s = 0.5s$	A slope change of -20 dB/decade occurs at 4 rad/s, which indicates the presence of a pole factor $\frac{1}{\frac{s}{4}+1}$	A slope change of -40 dB/decade occurs at 30 rad/s, which indicates the presence of a 2nd-order factor due to the existence of a resonant peak $\frac{30^2}{s^2+60\zeta s+30^2}$	A slope change of +40 dB/decade occurs at 200 rad/s, which indicates the presence of a double zero factor $\left(\frac{s}{200}+1\right)^2$



Therefore,

$$\tilde{H}_b(s) = 0.5s \cdot \left(\frac{1}{\frac{s}{4}+1}\right) \cdot \left(\frac{900}{s^2 + 60\zeta s + 900}\right) \cdot \left(\frac{s}{200}+1\right)^2$$

$$= 0.5s \cdot 4\left(\frac{1}{s+4}\right) \cdot 900\left(\frac{1}{s^2 + 60\zeta s + 900}\right)\left(\frac{1}{s+30}\right)^2 \cdot \frac{1}{200^2}(s+200)^2$$

$$= 0.045 \frac{s(s+200)^2}{(s+4)\left(s^2 + 60\zeta s + 900\right)} = K\frac{(s+a)(s+b)(s+c)}{(s+d)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

which results in $K = 0.045$, $a = 0$, $b = c = 200$, $d = 4$ and $\omega_n = 30$.

Tutorial 8

Q.3 The spectrums of two real bandpass signals, $x(t)$ and $y(t)$, are shown in Figure Q3.

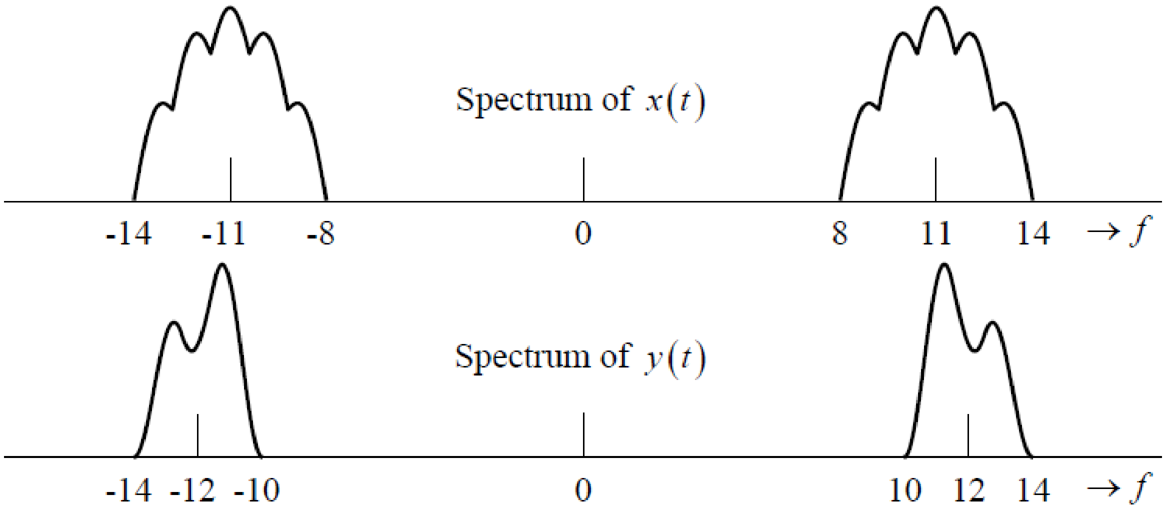


Figure Q3

For each of $x(t)$ and $y(t)$:

(a) What is the Nyquist sampling frequency?

(b) Find the smallest sampling frequency that allows for exact reconstruction of the signal from its sampled version and determine the frequency response, $H(f)$, of the corresponding ideal reconstruction filter.

Q.3 The spectrums of two real bandpass signals, $x(t)$ and $y(t)$, are shown in Figure Q3.

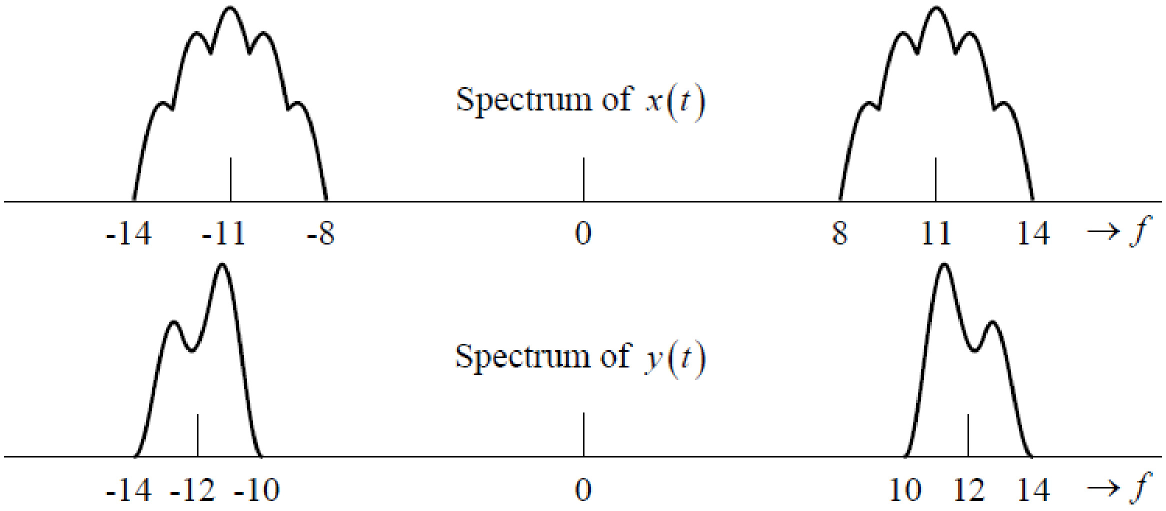


Figure Q3

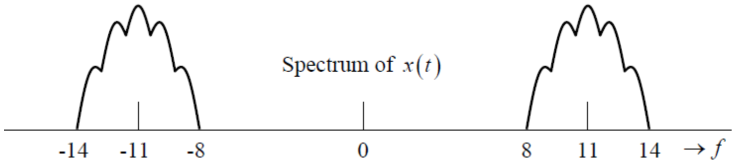
For each of $x(t)$ and $y(t)$:

(a) What is the Nyquist sampling frequency?

Solution to Q.3

(a) Nyquist sampling frequency for both $x(t)$ and $y(t)$ is $2 \times 14 = 28$ Hz.

(b) Find the smallest sampling frequency that allows for exact reconstruction of the signal from its sampled version and determine the frequency response, $H(f)$, of the corresponding ideal reconstruction filter.



SIGNAL $x(t)$:

Due to symmetry of the positive-frequency and negative-frequency spectral images about the center frequencies 11 Hz and −11 Hz, respectively, both the Overlapping Spectral Images formula and the Un-aliased Spectral Images formula are applicable. Of these, the Overlapping Spectral Images formula always yield the lowest sampling frequency.

Center frequency : $f_c = 11$ Hz

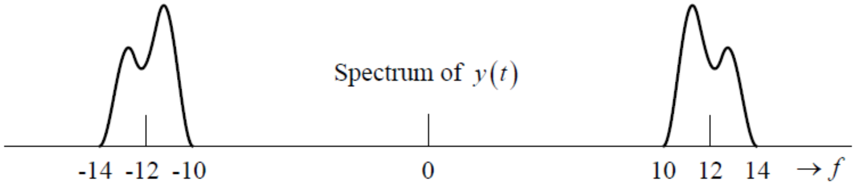
Bandwidth : $B = 6$ Hz

Overlapping spectral images formula ($Possible\ if\ f_c > 0.5B$) :

$$\begin{cases} f_s = \dfrac{2f_c}{k};\quad k = 1,\ 2,\ \cdots,\ \left\lfloor \dfrac{2f_c}{B} \right\rfloor \\ \quad = \dfrac{22}{k};\quad k = 1,\ 2,\ 3 \\ \quad = 22,\ \ 11,\ \ \dfrac{22}{3}\ \text{Hz} \end{cases}$$

- Therefore, the smallest possible sampling frequency is $f_{s,\min} = \dfrac{22}{3}$ Hz
- For exact reconstruction of $x(t)$, the frequency response of the ideal reconstruction filter should be:

$$H(f) = \frac{1}{2f_{s,\min}}\left[\text{rect}\left(\frac{f+f_c}{B}\right) + \text{rect}\left(\frac{f-f_c}{B}\right)\right] = \frac{3}{44}\left[\text{rect}\left(\frac{f+11}{6}\right) + \text{rect}\left(\frac{f-11}{6}\right)\right]$$



SIGNAL $y(t)$:

Due to asymmetry of the positive-frequency and negative-frequency spectral images about the center frequencies 12 Hz and −12 Hz, respectively, only the Un-aliased Spectral Images formula is applicable.

Center frequency : $f_c = 12$ Hz

Bandwidth : $B = 4$ Hz

Un-aliased spectral images formula (*Possible if* $f_c > 1.5B$) :

$$\begin{cases} \dfrac{2f_c + B}{k+1} \le f_s \le \dfrac{2f_c - B}{k}; \quad k = 1,\ 2,\ \cdots,\ \left\lfloor \dfrac{2f_c - B}{2B} \right\rfloor \\ \to \quad \dfrac{28}{k+1} \le f_s \le \dfrac{20}{k}; \quad k = 1,\ 2 \\ \to \quad \underbrace{14 \le f_s \le 20}_{k=1} \text{ and } \underbrace{\dfrac{28}{3} \le f_s \le 10}_{k=2} \end{cases}$$

- Therefore, the smallest possible sampling frequency is $f_{s,\min} = \dfrac{28}{3}$ Hz
- For exact reconstruction of $y(t)$, the frequency response of the ideal reconstruction filter should be:

$$H(f) = \frac{1}{f_{s,\min}}\left[\mathrm{rect}\left(\frac{f+f_c}{B}\right) + \mathrm{rect}\left(\frac{f-f_c}{B}\right)\right] = \frac{3}{28}\left[\mathrm{rect}\left(\frac{f+12}{4}\right) + \mathrm{rect}\left(\frac{f-12}{4}\right)\right]$$

Q.5 Speech studies have shown that perceptual cues pertaining to speech intelligibility and speaker identity are mainly found within the first 4 kHz of the speech spectrum. We shall refer to this frequency band as the *TQ-Band*, where *TQ* stands for '*Telephone Quality*'.

The spectrum, $X(f)$, of a speech signal $x(t)$ is shown in Figure Q5 where the *TQ-Band* is indicated.

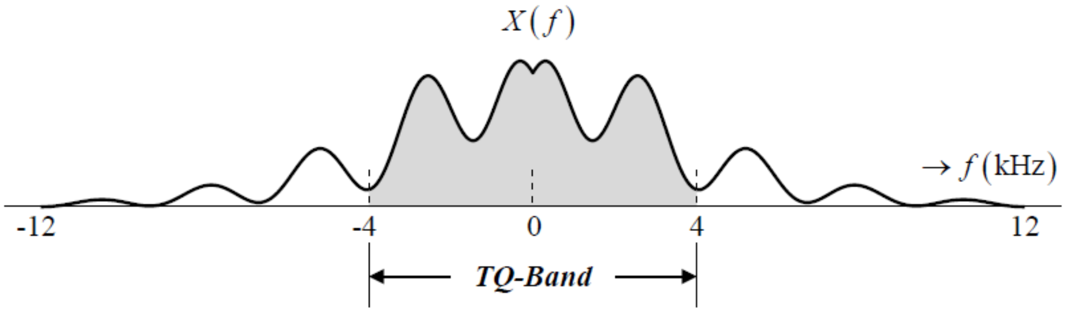


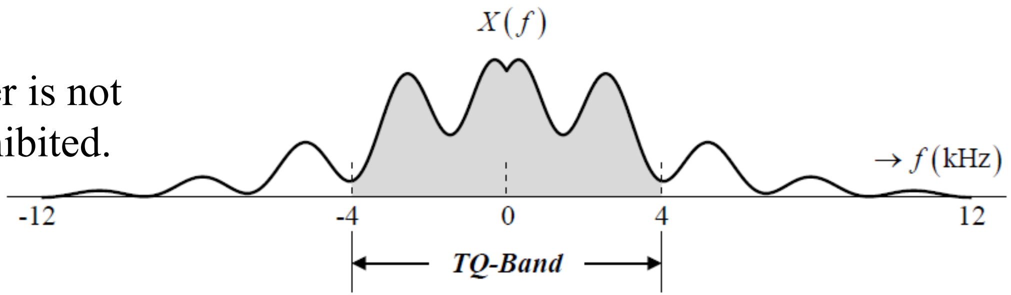
Figure Q5

In order to send $x(t)$ over a digital telephone network, the first step is to sample $x(t)$ without corrupting its *TQ-Band*. Suggest a method for sampling $x(t)$ in each of the following situations.

Situation A:	Anti-aliasing lowpass filter is not available and frequency aliasing is prohibited.
Situation B:	Lowest sampling frequency must be used at all cost.
Situation C:	Anti-aliasing lowpass filter is not available and frequency aliasing is permitted.

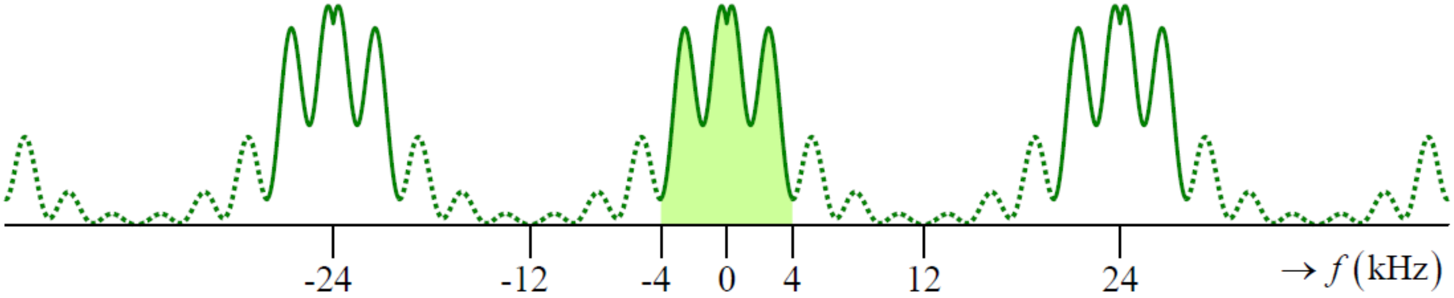
Comment on the advantage and disadvantage of each method.

Situation A: Anti-aliasing lowpass filter is not available and frequency aliasing is prohibited.



Solution to Q.5

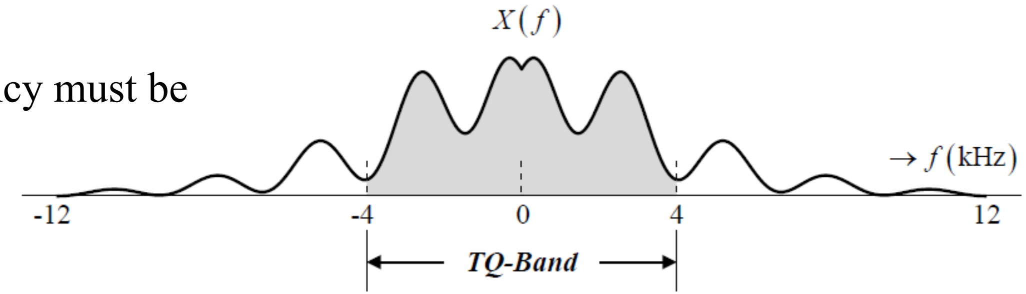
Situation A: $\left(\begin{array}{l}\textbf{No anti-aliasing lowpass filter}\\ \textbf{Frequency aliasing prohibited}\end{array}\right)$: **Sampling frequency** $= 2 \times 12 = 24$ **kHz**



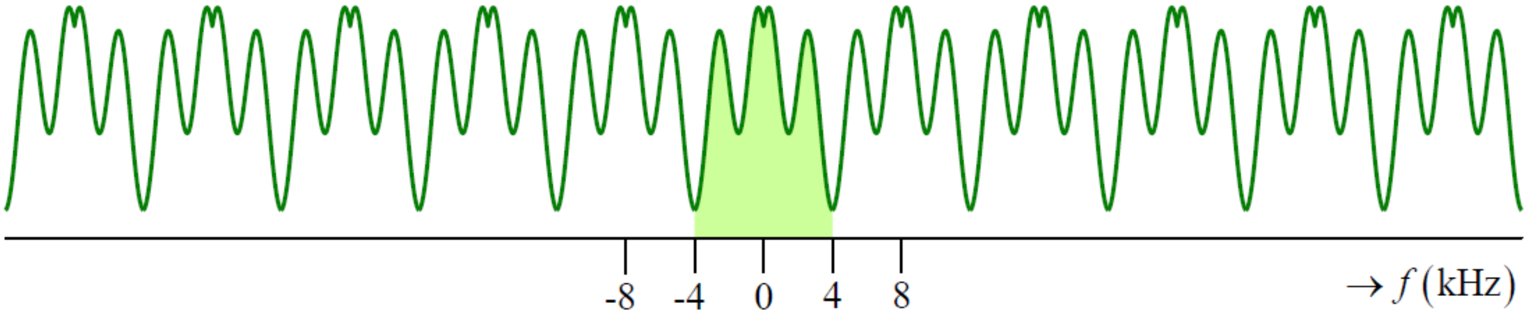
Advantage: No anti-aliasing LPF needed.

Disadvantage: Sampling frequency is higher than necessary to preserve the *TQ-Band*.

Situation B: Lowest sampling frequency must be used at all cost.



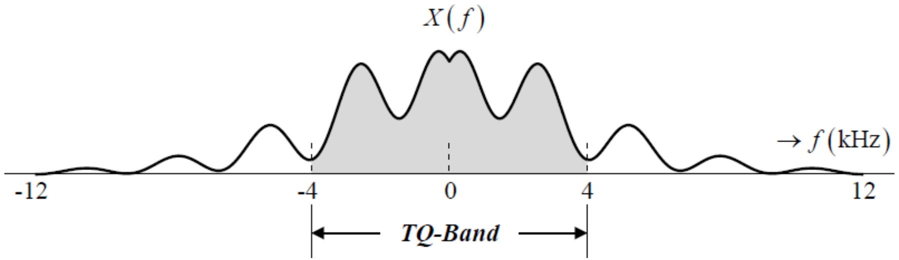
Situation B: Lowest sampling frequency: $\begin{cases} \textbf{Ideal anti-aliasing filter of bandwidth 4 kHz} \\ \textbf{Sampling frequency} = 2 \times 4 = 8 \textbf{ kHz} \end{cases}$



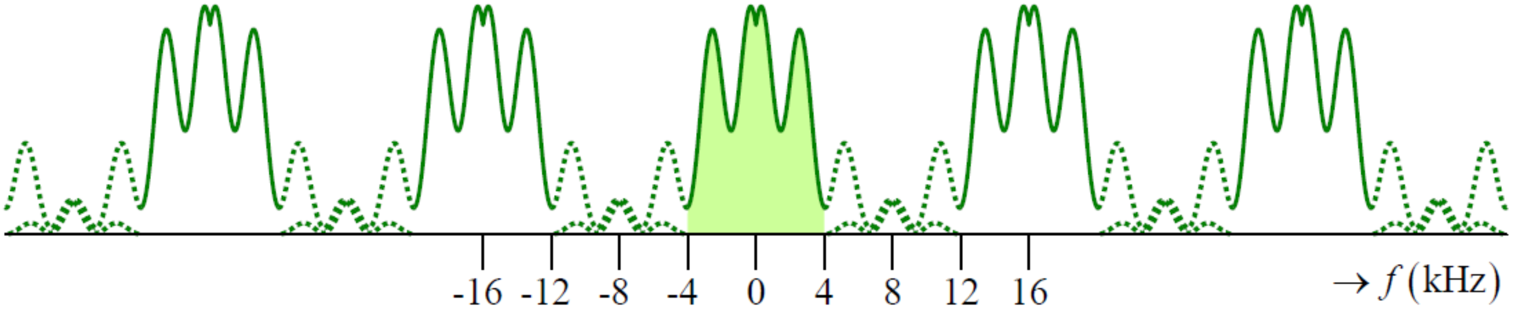
Advantage: Lowest possible sampling frequency to preserve the *TQ-Band*.

Disadvantage: Require anti-aliasing LPF with sharp cutoff.

Situation C: Anti-aliasing lowpass filter is not available and frequency aliasing is permitted.



Situation C: $\begin{pmatrix}\textbf{No anti-aliasing lowpass filter}\\ \textbf{Frequency aliasing permitted}\end{pmatrix}$: **Sampling frequency = 24 − 8 = 16 kHz**



Advantage:	Lowest possible sampling frequency to preserve the *TQ-Band* without requiring anti-aliasing LPF with sharp cutoff.
Disadvantage:	Sampling frequency is still higher than the minimum needed to preserve the *TQ-Band*.

This problem illustrates the trade-off between oversampling and the requirement of expensive anti-aliasing LPF with sharp cutoff frequency.

Thanks!