CG2023 Tutorial 7

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Review of LTI Systems

Impulse Response

$h(t)$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$

Step Response

$o(t) = \int_{-\infty}^{t} h(\tau)d\tau$

$\tilde{H}(s) = \mathcal{L}\{h(t)\} = \int_{0}^{\infty} h(t)\, e^{-jst}dt$

$H(f) = \mathfrak{I}\{h(t)\} = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft}dt$

Differential Equations

$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m y(t)}{dt^m}$$

$\xrightarrow{\mathcal{L}}$

Transfer Function

$\tilde{H}(s)$ $\xrightarrow{s = j\omega}$ $\tilde{H}(j\omega)$

$\tilde{X}(s) \rightarrow \boxed{\tilde{H}(s)} \rightarrow \tilde{Y}(s) = \tilde{X}(s) \cdot \tilde{H}(s)$

$$\tilde{H}(s) = \frac{\tilde{Y}(s)}{\tilde{X}(s)} = \frac{b_M s^M + b_{M-1}s^{M-1} + \cdots + b_0}{a_N s^N + a_{N-1}s^{N-1} + \cdots + a_0}$$

Frequency Response

$\tilde{H}(j\omega)$ $\xrightarrow{\omega = 2\pi f}$ $H(f)$

$\tilde{X}(j\omega) \rightarrow \boxed{\tilde{H}(j\omega)} \rightarrow \tilde{Y}(j\omega) = \tilde{X}(j\omega) \cdot \tilde{H}(j\omega)$

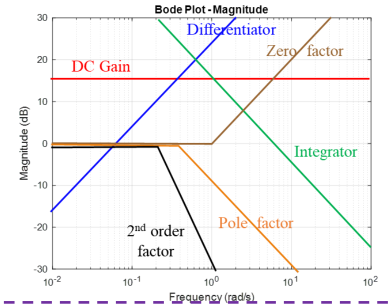
$|\tilde{H}(j\omega)|$

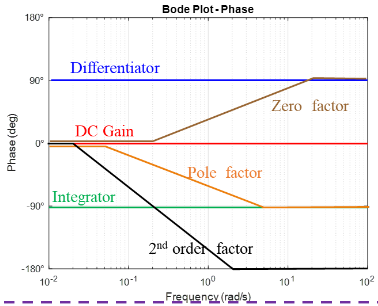
Magnitude response

$e^{j\angle \tilde{H}(j\omega)}$

Phase response

Bode Plots





Transient Response

System Stability

Locations of System Poles

Typical Systems

First order system

Second order system

Review of Bode Diagrams

Transfer Function

$\tilde{H}(s)$

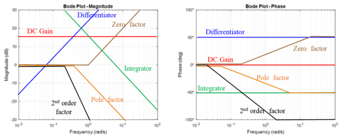
Decompose to a cascade form

- System without differentiator or integrator
- System with a differentiator
- System with an integrator

Construct Straight-line Bode plots using basic systems

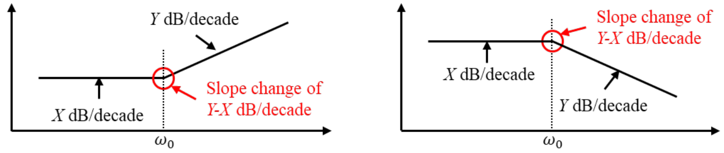
- DC gain
- Differentiator
- Integrator
- Zero factor
- Pole factor
- Second order factor

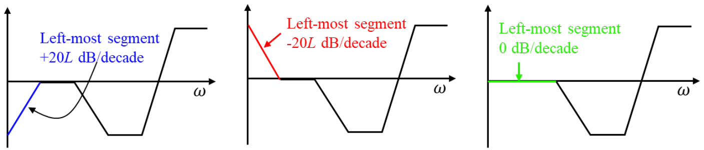
Bode Plots



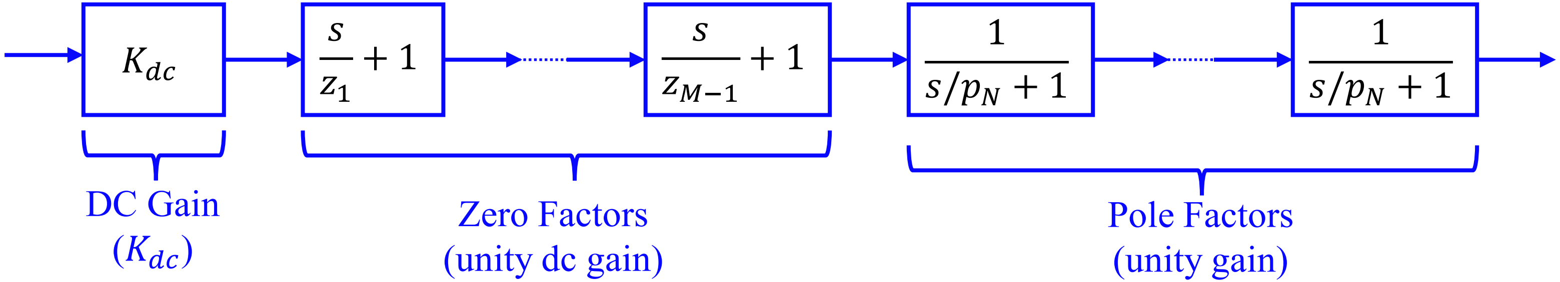
Step1: Examine the **slope of the low-frequency asymptote** of the Bode magnitude plot

Step2: Check for **slope change while proceeding from low-frequency to high-frequency** along the Bode magnitude plot

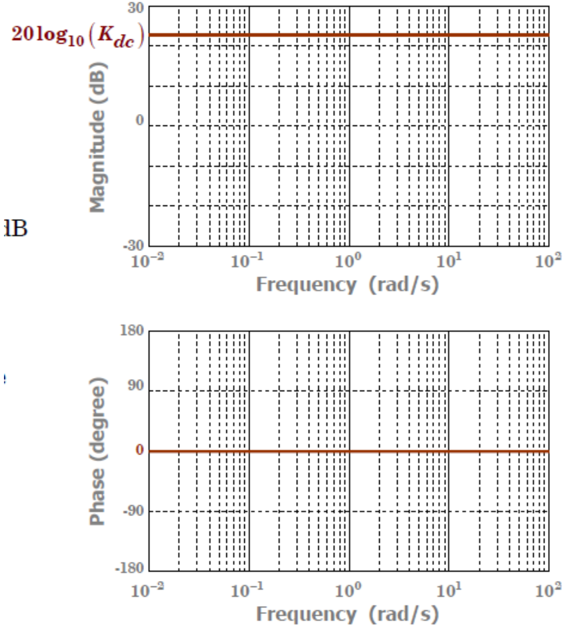




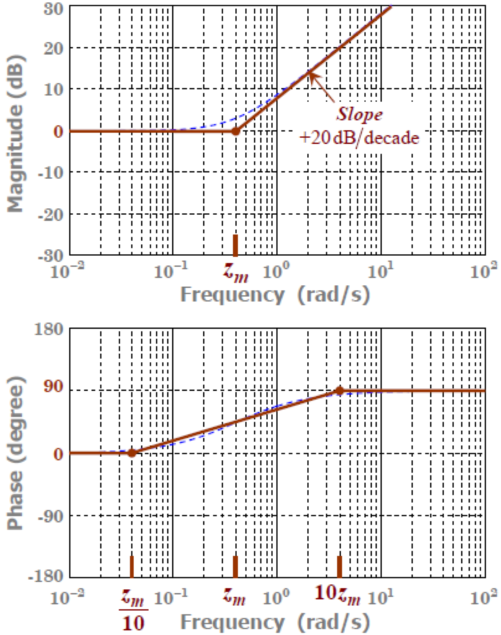
System without Differentiator or Integrator



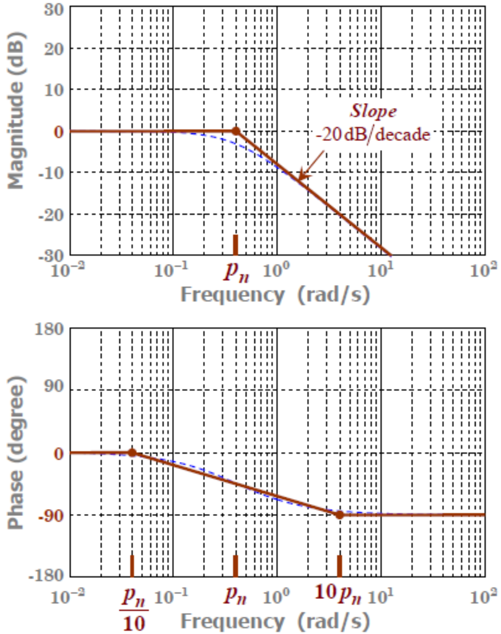
DC Gain $\widetilde{H} = K_{dc}$



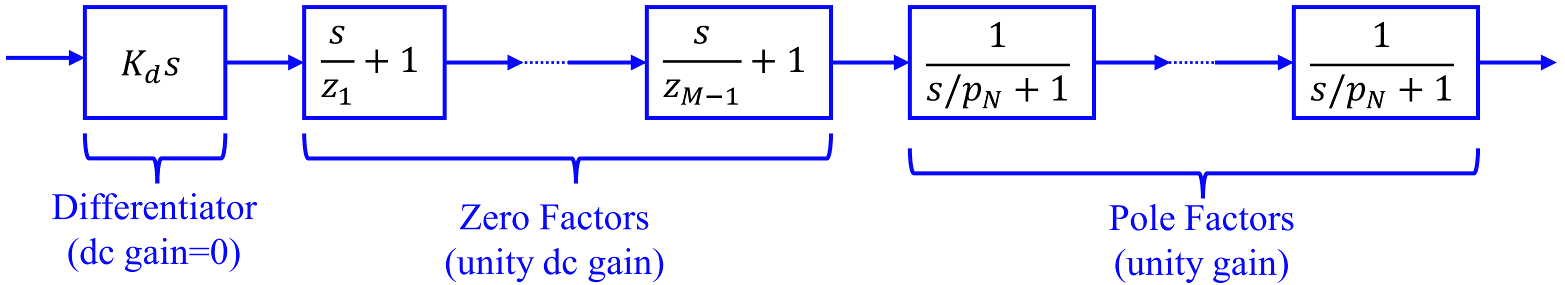
Zero Factor $\widetilde{H} = \frac{s}{z_m} + 1$



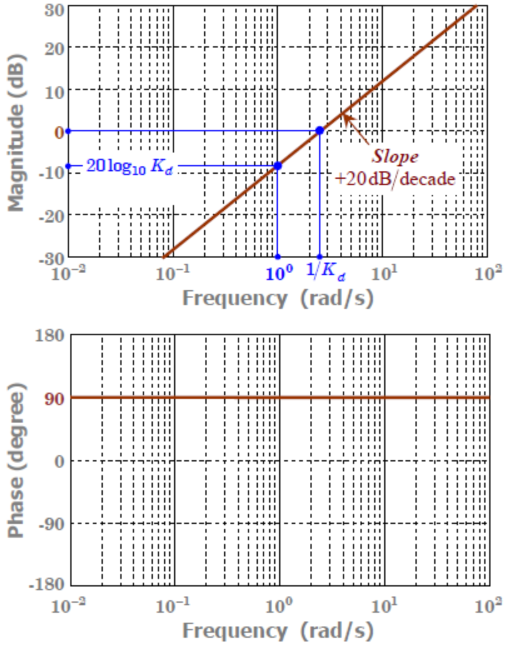
Pole Factor $\widetilde{H} = \frac{1}{s/p_n+1}$



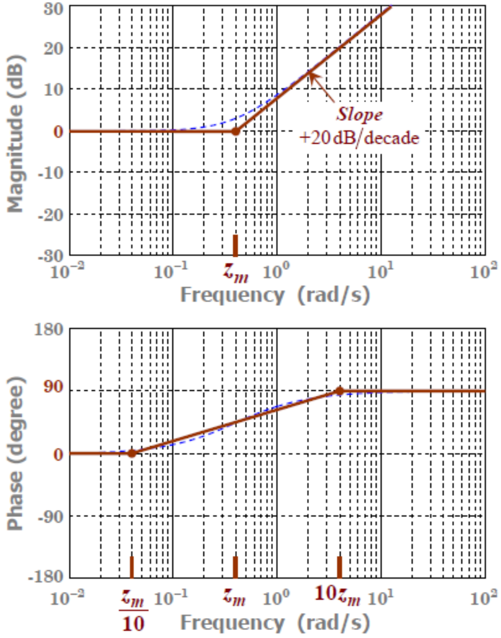
System with a Differentiator



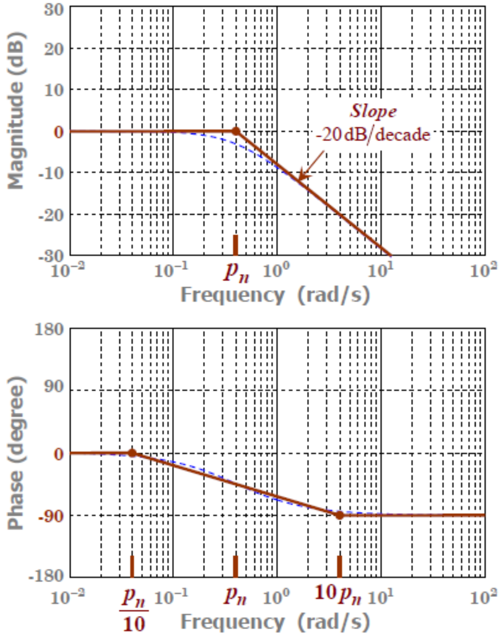
Differentiator $\widetilde{H} = K_d\, s$



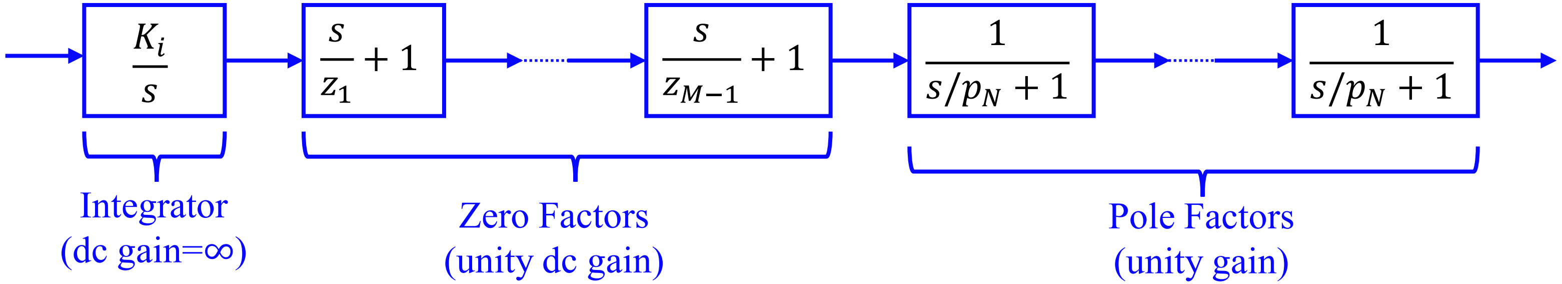
Zero Factor $\widetilde{H} = \frac{s}{z_m} + 1$



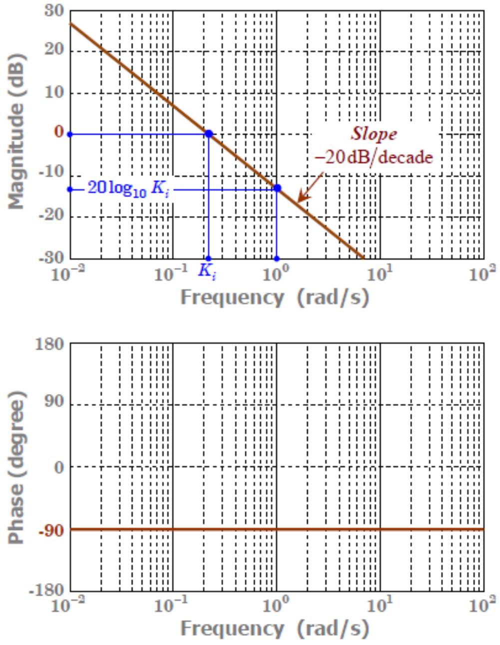
Pole Factor $\widetilde{H} = \frac{1}{s/p_n + 1}$



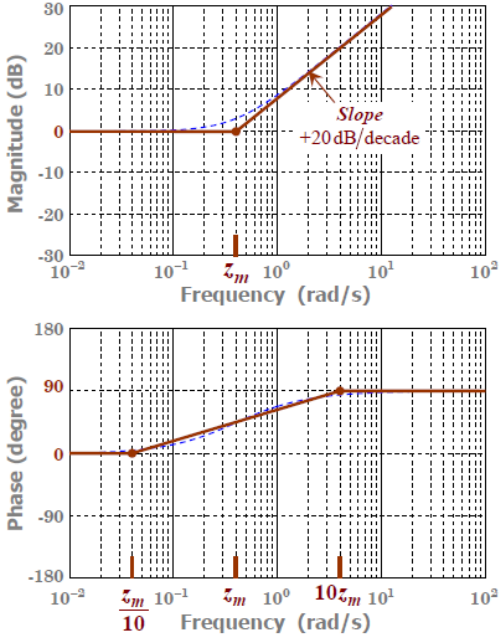
System with an Integrator



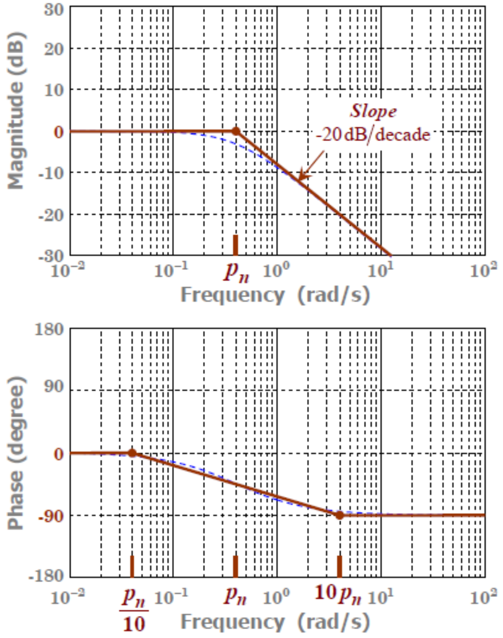
Integrator $\widetilde{H} = K_i \,/\, s$



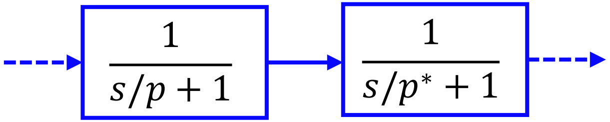
Zero Factor $\widetilde{H} = \frac{s}{z_m} + 1$



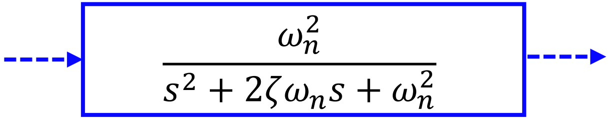
Pole Factor $\widetilde{H} = \frac{1}{s/p_n+1}$



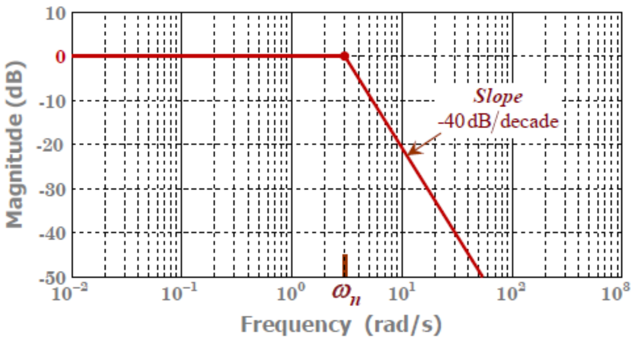
2nd -order Factor $\widetilde{H} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega^2}$

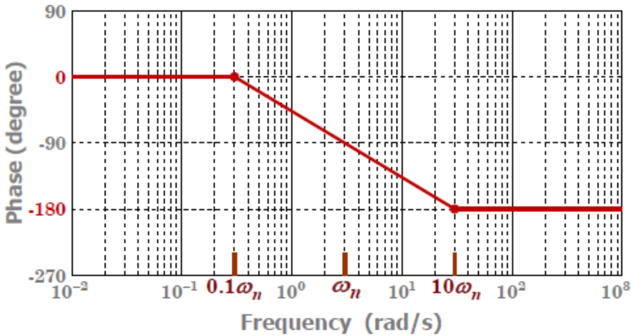


Complex Conjugate Pole Factors
(unity dc gain)



Under-damped 2nd –order Factor
(unity dc Gain)
$\omega_n = |p|$ and $\zeta = \mathrm{Re}[p]/|p|$



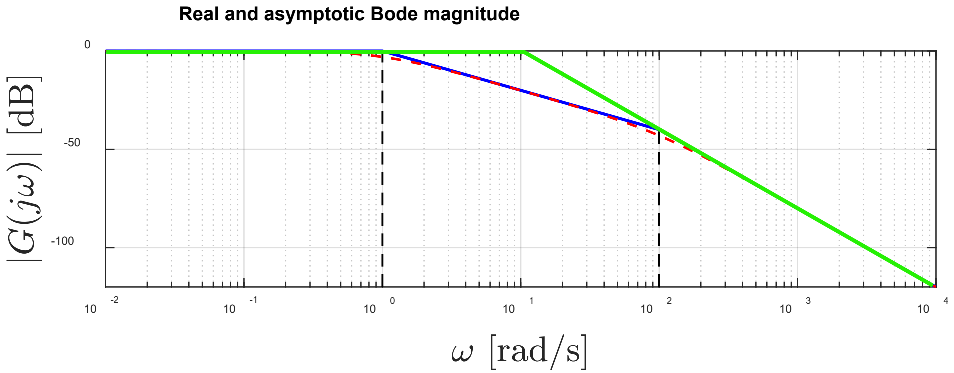


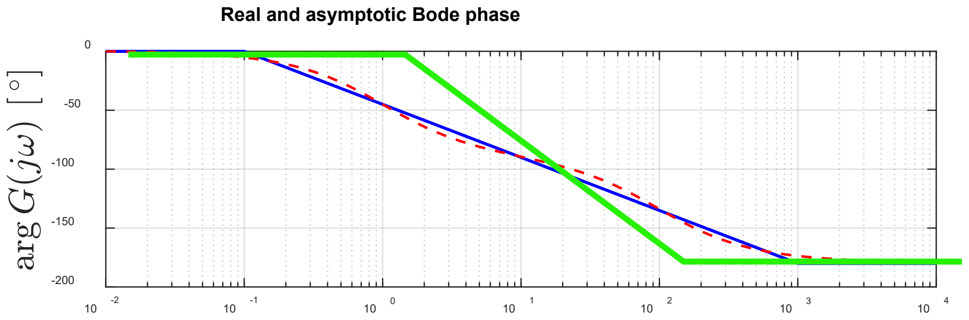
a) When $\zeta > 1$ (***over-damped 2nd -order systems***), treat the system as two cascaded pole factors.

b) When $\zeta = 1$ (***critically-damped 2nd -order systems***), treat as two repeated pole factors.

c) When $0 \leq \zeta < 1$ (***underdamped and undamped 2nd -order systems***), approximate the system as $\zeta = 1$.

Over-damped 2nd –order System





$$\tilde{H}(s) = \frac{100}{s^2 + 101s + 100}$$

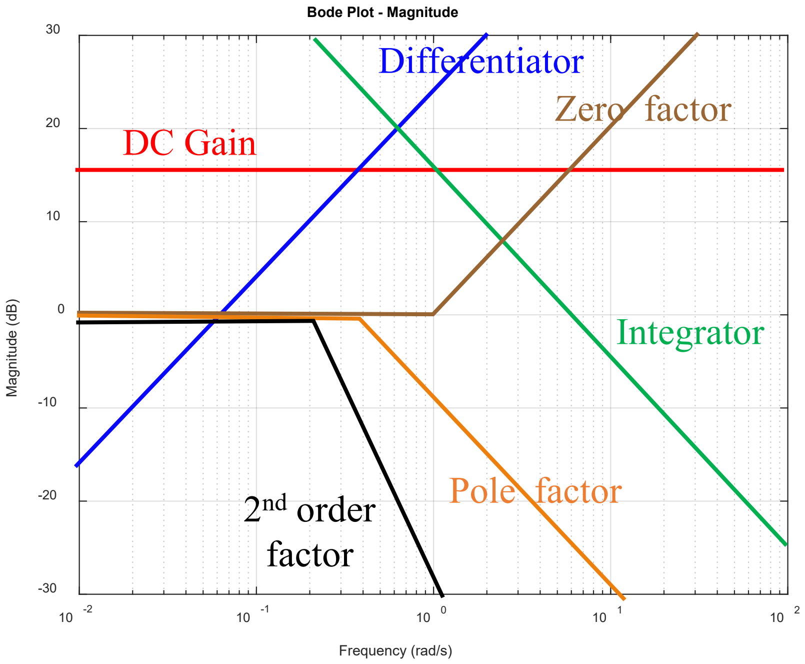
$$= \frac{1}{s+1} \times \frac{1}{\frac{s}{100} + 1}$$

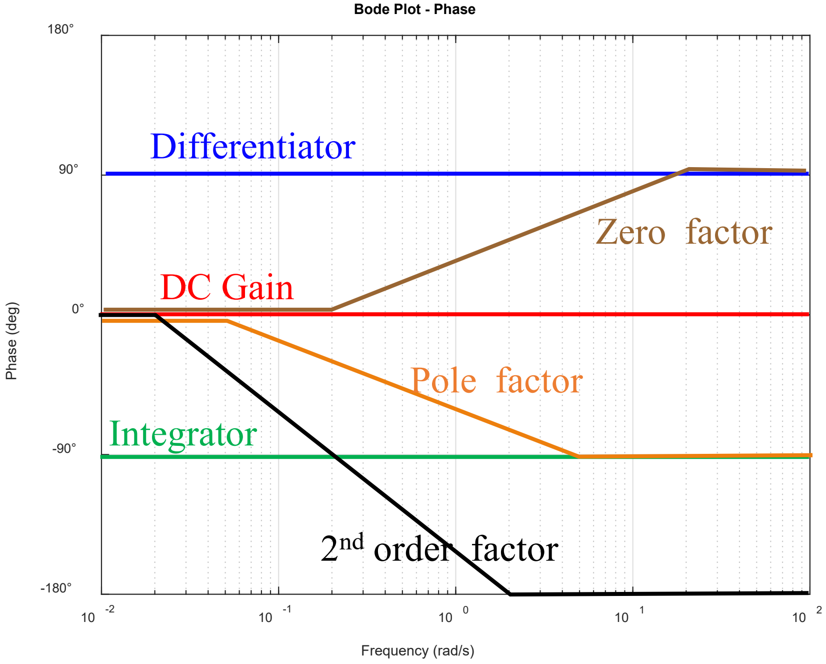
The real bode plot

Asymptotic bode plot using two pole factors

Asymptotic bode plot using critically-damped 2nd-order system

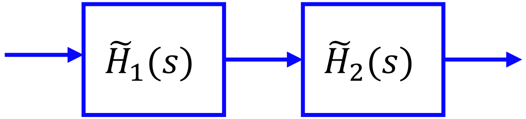
Bode Plots of Basic Systems





Frequency Response of Cascaded Systems

Cascaded Systems



Let $\tilde{H}(s) = \tilde{H}_1(s)\,\tilde{H}_2(s)$ be a cascade of two systems. The frequency response is given by

$$\tilde{H}(j\omega) = \tilde{H}_1(j\omega)\,\tilde{H}_2(j\omega)$$

$$|\tilde{H}(j\omega)|e^{j\angle\tilde{H}(s)} = |\tilde{H}_1(j\omega)|e^{j\angle\tilde{H}_1(s)}\,|\tilde{H}_2(j\omega)|e^{j\angle\tilde{H}_2(s)}$$

$$= \left|\tilde{H}_1(j\omega)\right|\left|\tilde{H}_2(j\omega)\right|e^{j[\angle\tilde{H}_1(j\omega)+\angle\tilde{H}_2(j\omega)]}$$

Phase:

$$\angle\tilde{H}(s) = \angle\tilde{H}_1(s) + \angle\tilde{H}_2(s)$$

Magnitude:

$$\underbrace{20\log_{10}\left|\tilde{H}(j\omega)\right|}_{\left|\tilde{H}(j\omega)\right|_{\mathrm{dB}}} = \underbrace{20\log_{10}\left|\tilde{H}_1(j\omega)\right|}_{\left|\tilde{H}_1(j\omega)\right|_{dB}} + \underbrace{20\log_{10}\left|\tilde{H}_2(j\omega)\right|}_{\left|\tilde{H}_2(j\omega)\right|_{dB}}$$

Asymptotic Frequency Response Tables

- **Asymptotic PHASE of Phase Plot** $\left[\angle\tilde{H}(j\omega)\right]$

High frequency:

$$\lim_{\omega\to\infty}\angle\tilde{H}(j\omega)=\overbrace{[\text{Nº of Poles}-\text{Nº of Zeros}]}^{\text{POLE-ZERO EXCESS}}\times(-90^\circ)$$

Low frequency:

$$\lim_{\omega\to 0}\angle\tilde{H}(j\omega)=\left[\text{Nº of }\int dt-\text{Nº of }\frac{d}{dt}\right]\times(-90^\circ)$$

- **Asymptotic SLOPE of Magnitude Plot** $\left[\left|\tilde{H}(j\omega)\right|_{dB}\right]$

High frequency:

$$\lim_{\omega\to\infty}\left[slope\ of\left|\tilde{H}(j\omega)\right|_{dB}\right]=\overbrace{[\text{Nº of Poles}-\text{Nº of Zeros}]}^{\text{POLE-ZERO EXCESS}}\times(-20\ \ dB/decade)$$

Low frequency:

$$\lim_{\omega\to 0}\left[slope\ of\left|\tilde{H}(j\omega)\right|_{dB}\right]=\left[\text{Nº of }\int dt-\text{Nº of }\frac{d}{dt}\right]\times(-20\ \ dB/decade)$$

ASYMPTOTIC VALUE OF PHASE RESPONSE

Component	Low Frequency	High Frequency	No. of Zero	No. of Pole
Integrator	–90°	–90°	0	1
Differentiator	+90°	+90°	1	0
Pole factor	0°	–90°	0	1
Zero factor	0°	+90°	1	0
2nd-order factor	0°	–180°	0	2

ASYMPTOTIC SLOPE OF MAGNITUDE RESPONSE

Component	Low Frequency (dB/decade)	High Frequency (dB/decade)	No. of Zero	No. of Pole
Integrator	–20	–20	0	1
Differentiator	+20	+20	1	0
Pole factor	0	–20	0	1
Zero factor	0	+20	1	0
2nd-order factor	0	–40	0	2

Deriving System Transfer Function from Bode Magnitude Plot

Step 1: Examine the **slope of the low-frequency asymptote** of the Bode magnitude plot:

$+20L$ dB/decade $\rightarrow$ Existence of a cascade of L identical differentiators, $(K_d s)^L$. Equation of low frequency asymptote:

$$\left|\tilde{H}(j\omega)\right|_{dB} = 20L\log_{10}(K_d) + 20L\log_{10}(\omega)$$

which cuts the coordinates $(1/K_d,\ 0)$ and $(1,\ 20L\log_{10}(K_d))$.

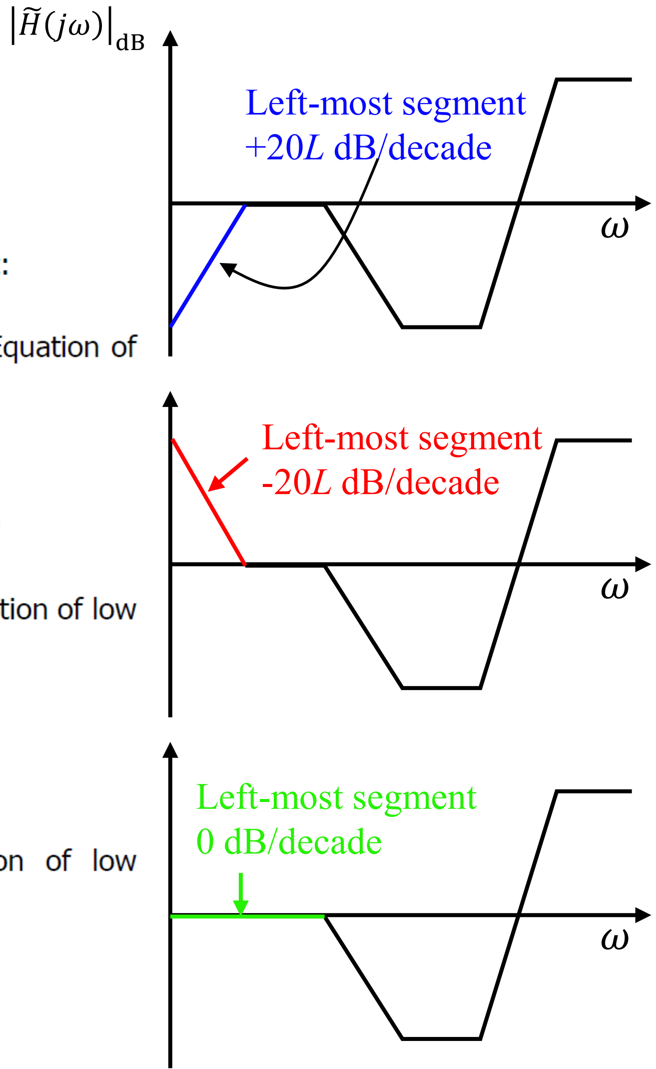
$-20L$ dB/decade $\rightarrow$ Existence of a cascade of L identical integrators, $(K_i/s)^L$. Equation of low frequency asymptote:

$$\left|\tilde{H}(j\omega)\right|_{dB} = 20L\log_{10}(K_i) - 20L\log_{10}(\omega)$$

which cuts the coordinates $(K_i,\ 0)$ and $(1,\ 20L\log_{10}(K_i))$.

0 dB/decade $\rightarrow$ DC gain, K_{dc} (without differentiator or integrator). Equation of low frequency asymptote:

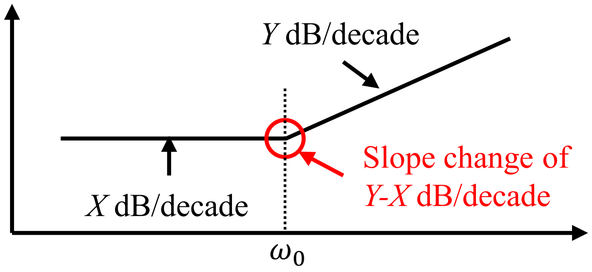
$$\left|\tilde{H}(j\omega)\right|_{dB} = 20\log_{10}(K_{dc}).$$



Deriving System Transfer Function from Bode Magnitude Plot

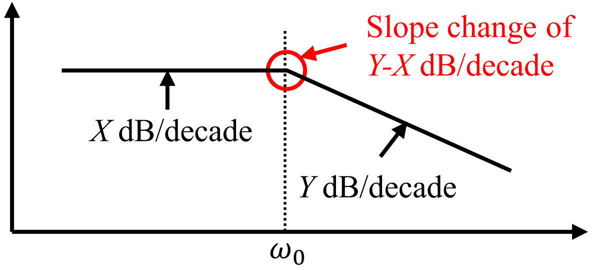
Step 2: Check for **slope change while proceeding from low-frequency to high-frequency** along the Bode magnitude plot:

$+20L$ dB/decade **slope change** at $\omega = \omega_o$ $\rightarrow$ Existence of a cascade of L identical zero factors, $(s/\omega_o + 1)^L$, assuming that all the zeros of $\tilde{H}(s)$ are real.



$-20L$ dB/decade **slope change** at $\omega = \omega_o$ $\rightarrow$

- $L = 1$: Existence of a pole factor $\dfrac{1}{s/\omega_o + 1}$.
- $L = 2$: Existence of a repeated pole factor $\left(\dfrac{1}{s/\omega_o + 1}\right)^2$, or a 2nd-order factor $\dfrac{\omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2}$
- $L > 2$: May be a hybrid of pole factors and 2nd-order factors, $\left(\dfrac{1}{s/\omega_o + 1}\right)^p \left(\dfrac{\omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2}\right)^q$, where $p + 2q = L$.



Definitions

$$\textbf{DC Gain} = \tilde{H}(j\omega)\Big|_{\omega=0} = \tilde{H}(0)$$

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ii. Bode plots for Differentiator: $\tilde{H}(s) = K_d s$

Frequency Response : $\tilde{H}(j\omega) = K_d j\omega = K_d \omega \underbrace{e^{j90^\circ}}_{j}$

Magnitude Response : $\begin{cases} \left|\tilde{H}(j\omega)\right| = K_d \omega \\ \left|\tilde{H}(j\omega)\right|_{dB} = 20\log_{10}(K_d) \\ \qquad\qquad \underbrace{+20\log_{10}(\omega)\ \text{dB}}_{\text{20 dB/decade slope}} \end{cases}$

20 dB/decade slope

Increase ω by a factor of 10 (i.e. 1 decade) will cause to $\left|\tilde{H}(j\omega)\right|_{dB}$ change by:

$$\left|\tilde{H}(j10\omega)\right|_{dB} - \left|\tilde{H}(j\omega)\right|_{dB}$$
$$= 20\log_{10}(K_d) + 20\log_{10}(10\omega) - 20\log_{10}(K_d) - 20\log_{10}(\omega)$$
$$= 20\log_{10}(10) + 20\log_{10}(\omega) - 20\log_{10}(\omega)$$
$$= 20\log_{10}(10) = 20dB$$

Q1

Consider the LTI system: $\tilde{H}(s) = \dfrac{3s(2s+1)(0.6s+3)^2}{(4s^2+10s+4)(2s^2+8s+12.5)(2s+3)^2}$

(a) Express $\tilde{H}(s)$ as a cascade of basic systems such as dc-gain, integrator, differentiator, zero-factor, pole-factor and/or 2nd-order factor.

(b) How many zeros and poles are present in $\tilde{H}(s)$?

(c) Determine the low-frequency and high-frequency asymptotic slopes (in dB/decade) of its magnitude response.

(d) Determine the low-frequency and high-frequency asymptotic phases (in degrees) of its phase response.

(a)

First check the bracketted 2[nd]-order terms in the denominator to see if they have real or complex roots. Factorize those with real roots.

$$\tilde{H}(s) = \frac{3s(2s+1)(0.6s+3)^2}{\underbrace{\left(4s^2+10s+4\right)}_{\substack{\text{factorize this}\\ \text{because this}\\ \text{has real roots}\\ \overbrace{4(s+2)(s+0.5)}}} \underbrace{\left(2s^2+8s+12.5\right)}_{\substack{\text{do not factorize}\\ \text{this because this}\\ \text{has complex roots}}}(2s+3)^2} = \frac{3s(2s+1)(0.6s+3)^2}{4(s+2)(s+0.5)\left(2s^2+8s+12.5\right)(2s+3)^2}$$

$\cdots\cdots$ Rewrite to make the constant term in the bracketted 1st-order terms equal to 1 and the coefficient of s^2 in the bracketted 2nd-order term equal to 1.

$$= \frac{3s \cdot (2s+1) \cdot 9\left(\frac{0.6}{3}s + \frac{3}{3}\right)^2}{4 \cdot 2\left(\frac{s}{2} + \frac{2}{2}\right) \cdot 0.5\left(\frac{s}{0.5} + \frac{0.5}{0.5}\right) \cdot 2\left(\frac{2}{2}s^2 + \frac{8}{2}s + \frac{12.5}{2}\right) \cdot 9\left(\frac{2}{3}s + \frac{3}{3}\right)^2}$$

$$= 0.375s \cdot \cancel{\left(\frac{s}{0.5}+1\right)} \cdot \left(\frac{s}{5}+1\right)^2 \cdot \left(\frac{1}{\frac{s}{2}+1}\right) \cdot \cancel{\left(\frac{1}{\frac{s}{0.5}+1}\right)} \cdot \left(\frac{1}{s^2+4s+6.25}\right) \cdot \left(\frac{1}{\frac{s}{1.5}+1}\right)^2$$

$\cdots\cdots$ Next, multiply the numerator and denominator of $\tilde{H}(s)$ with the constant term of the bracketted 2nd-order term.

$$= \underbrace{0.06s}_{\text{diff}} \cdot \cdot \underbrace{\left(\frac{s}{5}+1\right)^2}_{\substack{\text{double}\\ \text{zero}\\ \text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{2}+1}\right)}_{\substack{\text{pole}\\ \text{factor}}} \cdot \cdot \underbrace{\left(\frac{6.25}{s^2+4s+6.25}\right)}_{\substack{\text{2nd-order}\\ \text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{1.5}+1}\right)^2}_{\substack{\text{double}\\ \text{pole}\\ \text{factor}}}$$

(b)

$$\underbrace{0.06s}_{\text{diff}} \cdot \cdot \underbrace{\left(\frac{s}{5}+1\right)^2}_{\substack{\text{double}\\ \text{zero}\\ \text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{2}+1}\right)}_{\substack{\text{pole}\\ \text{factor}}} \cdot \cdot \underbrace{\left(\frac{6.25}{s^2+4s+6.25}\right)}_{\substack{\text{2nd-order}\\ \text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{1.5}+1}\right)^2}_{\substack{\text{double}\\ \text{pole}\\ \text{factor}}}$$

There are 3 zeros and 5 poles present in $\tilde{H}(s)$.

(c)

$$\underbrace{0.06s}_{\text{diff}} \cdot \cdot \underbrace{\left(\frac{s}{5}+1\right)^2}_{\substack{\text{double}\\\text{zero}\\\text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{2}+1}\right)}_{\substack{\text{pole}\\\text{factor}}} \cdot \cdot \underbrace{\left(\frac{6.25}{s^2+4s+6.25}\right)}_{\substack{\text{2nd-order}\\\text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{1.5}+1}\right)^2}_{\substack{\text{double}\\\text{pole}\\\text{factor}}}$$

Asymptotic slope of magnitude response:

$$@\ \text{Low-frequency} = \left(\text{№ of} \int \text{'s} - \text{№ of} \frac{d}{dt}\text{'s} \right) \times (-20) = (0-1) \times (-20) = \quad 20 \ \text{db/decade}$$

$$@\ \text{High-frequency} = (\text{№ of poles} - \text{№ of zeros}) \times (-20) = (5\ \text{-}3) \times (-20) = -40 \ \text{db/decade}$$

(d)

$$\underbrace{0.06s}_{\text{diff}} \cdot \underbrace{\left(\frac{s}{5}+1\right)^2}_{\substack{\text{double}\\\text{zero}\\\text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{2}+1}\right)}_{\substack{\text{pole}\\\text{factor}}} \cdot \underbrace{\left(\frac{6.25}{s^2+4s+6.25}\right)}_{\substack{\text{2nd-order}\\\text{factor}}} \cdot \underbrace{\left(\frac{1}{\frac{s}{1.5}+1}\right)^2}_{\substack{\text{double}\\\text{pole}\\\text{factor}}}$$

Asymptotic phase of phase response:

$$@\ \text{Low-frequency} = \left(\text{№ of} \int\text{'s} - \text{№ of} \frac{d}{dt}\text{'s}\right) \times (-90) = (0-1)\times(-90) = 90^\circ$$

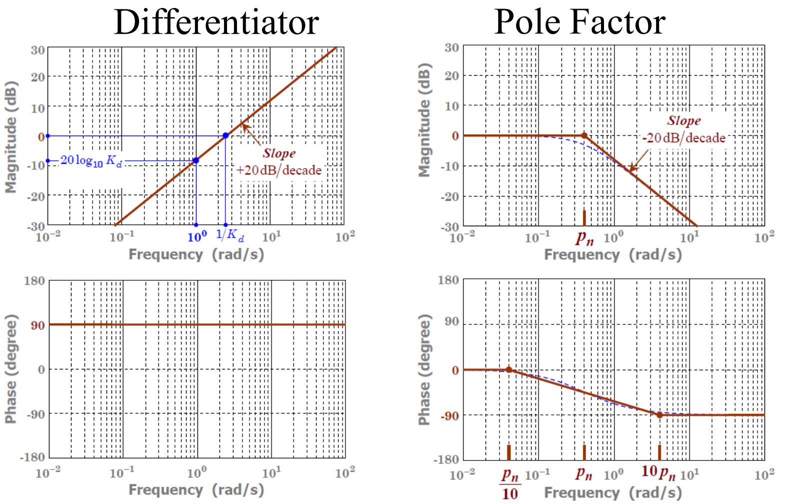
$$@\ \text{High-frequency} = (\text{№ of poles} - \text{№ of zeros}) \times (-90) = (5 - 3) \times (-90) = -180^\circ$$

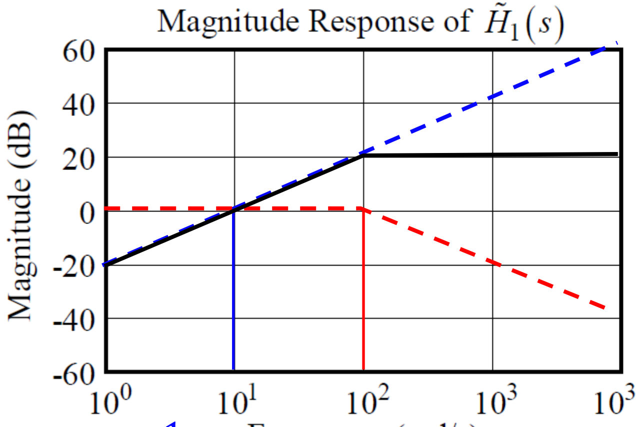
Q2

Draw the Bode magnitude and phase plots for the system $\widetilde{H}_1(s) = \frac{10s}{s+100}$ and $\widetilde{H}_2(s) = \frac{s+100}{10s}$ using the semilog-x grid provided below

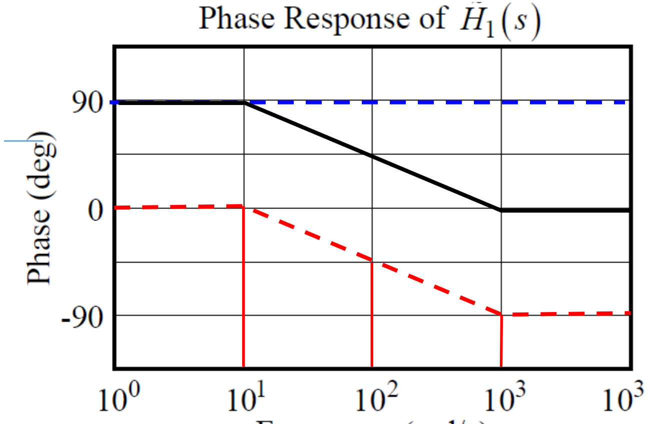
$$\tilde{H}_1(s) = \frac{10s}{s+100} = 0.1s\left(\frac{1}{\frac{s}{100}+1}\right) = K_d s\left(\frac{1}{\frac{s}{p_1}+1}\right)$$

where $K_d = 0.1$ and $p_1 = 10^2$





$\frac{1}{K_d} = 10$ $p_1 = 10^2$

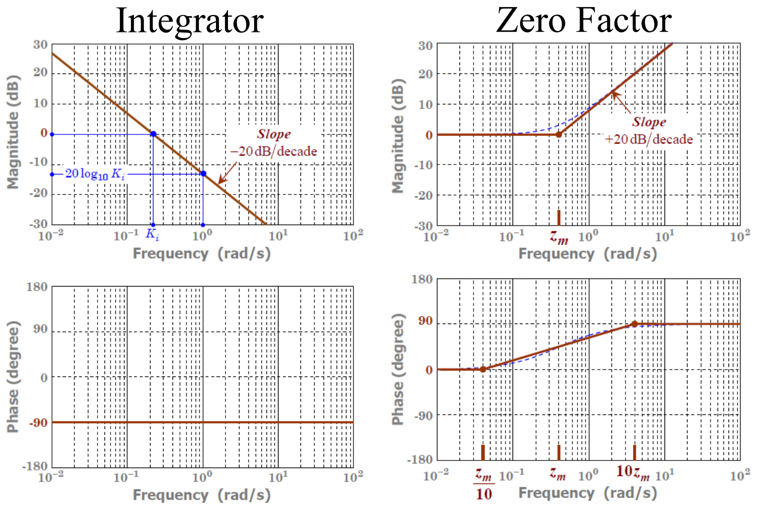


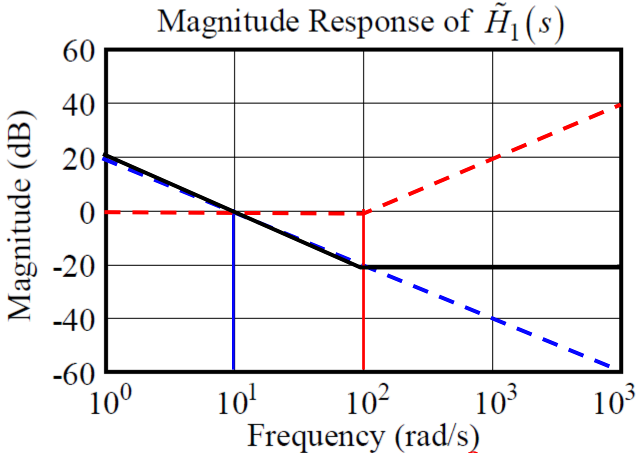
$\frac{p_1}{10}$ p_1 $10\,p_1$

$(1/K_d\,,\ 0)$ and $(1,\ 20\log_{10} K_d)$

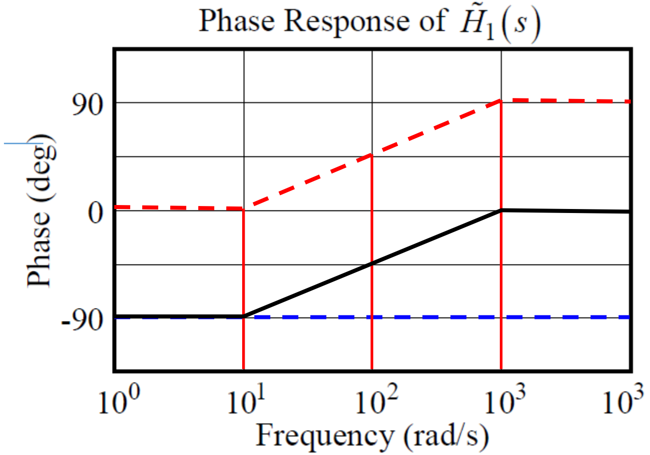
$$\widetilde{H}_2(s) = \frac{s+100}{10s} = \frac{10}{s}\left(\frac{s}{100}+1\right) = \frac{K_i}{s}\left(\frac{s}{z_1}+1\right)$$

where $K_i = 10$ and $z_1 = 10^2$





$K_i = 10$ $p_z = 10^2$



$\frac{z_1}{10}$ z_1 $10\, z_1$

$(K_i, 0)$ and $(1,\ 20\log_{10} K_i)$

Q3

The transfer function of a 2nd-order system is given by $\tilde{H}(s) = \dfrac{s+50}{0.2s^2 + 1.2s + 5}$.

(a) Determine the DC gain (in dB), the damping ratio, ζ, and the natural frequency, ω_n, of the system.

(b) Is the system overdamped, critically damped, underdamped or undamped, and why?

(c) Draw the Bode magnitude and phase plots for the system using the semilog-x grid provided below.

(a)

$$\text{DC gain} = \tilde{H}(s)\Big|_{s=0} = \left.\frac{s+50}{0.2s^2+1.2s+5}\right|_{s=0} = 10$$

$$\equiv 20\log_{10}(10) = 20 \text{ dB}$$

To determine ζ and ω_n, first rewrite the denominator of $\tilde{H}(s)$ as $0.2\left(s^2+6s+25\right)$ so that the coefficient of s^2 is equal to 1. Then compare the bracketted term $s^2+6s+25$ with $s^2+2\zeta\omega_n s+\omega_n^2$. This results in

$$\left.\begin{aligned} 2\zeta\omega_n &= 6 \\ \omega_n^2 &= 25 \end{aligned}\right\} \text{ solving} \rightarrow \quad \omega_n = 5 \text{ and } \zeta = 0.6$$

(b)

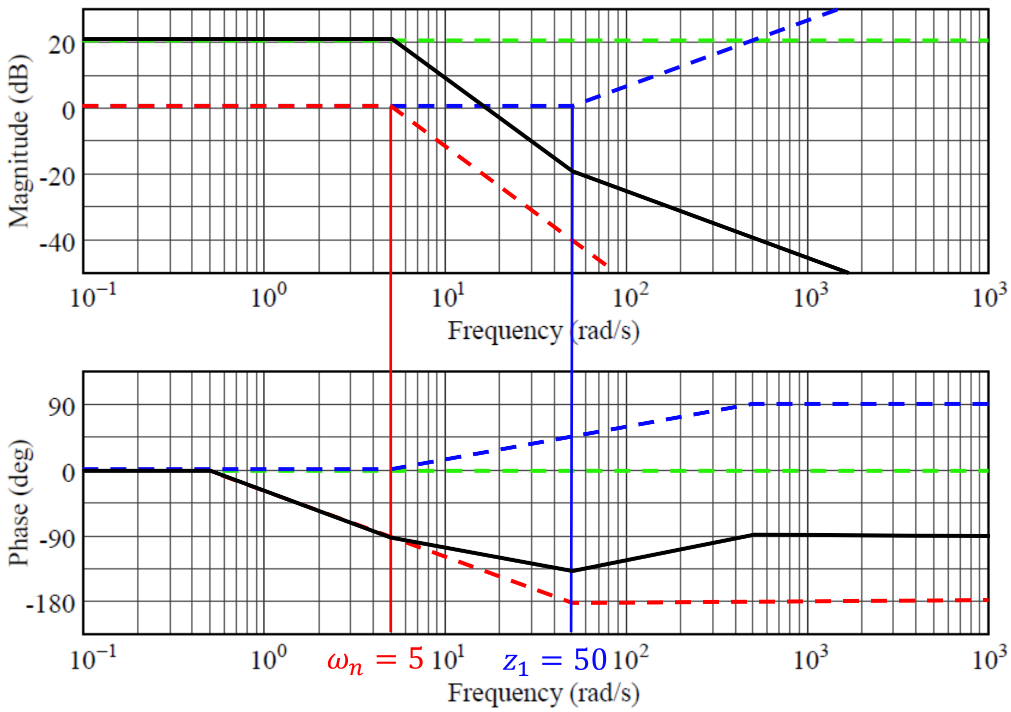
$$\left.\begin{aligned} 2\zeta\omega_n &= 6 \\ \omega_n^2 &= 25 \end{aligned}\right\} \text{ solving} \rightarrow \quad \omega_n = 5 \text{ and } \zeta = 0.6$$

The system UNDERDAMPED because $0 < \zeta < 1$.

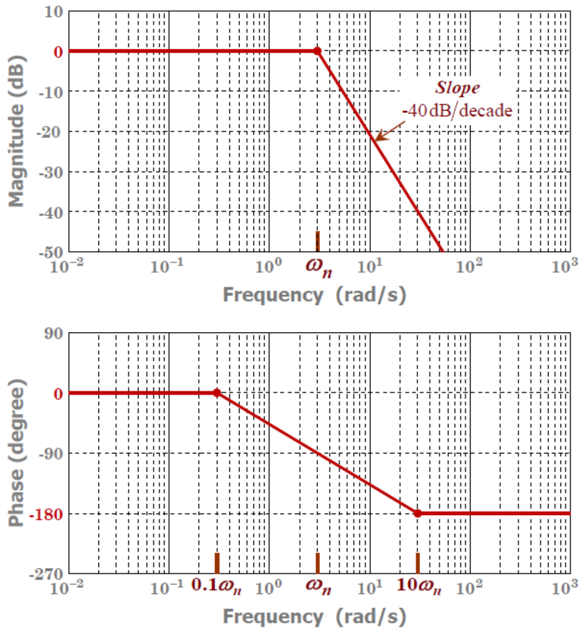
(c)

Rewrite $\tilde{H}(s) = \dfrac{s+50}{0.2s^2+1.2s+5}$ as

$$\tilde{H}(s) = \frac{50}{0.2}\cdot\frac{\frac{s}{50}+1}{s^2+6s+25} = 250\cdot\left(\frac{s}{50}+1\right)\cdot\left(\frac{1}{s^2+6s+25}\right) = \underbrace{10}_{\text{dc}}\cdot\underbrace{\left(\frac{s}{50}+1\right)}_{\text{zero factor}}\cdot\underbrace{\left(\frac{25}{s^2+6s+25}\right)}_{\text{2nd-order factor}}$$



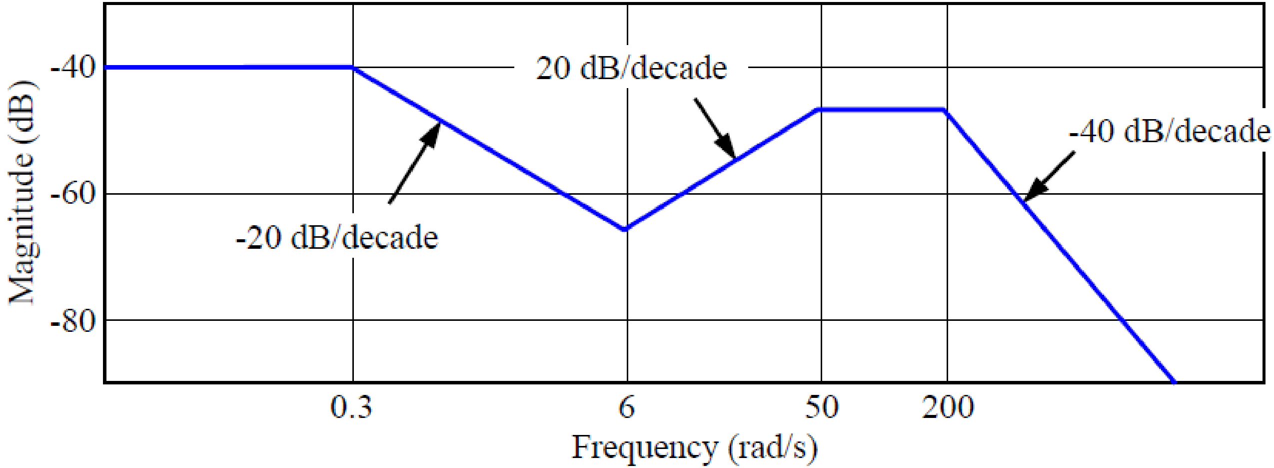
2nd-order Factor



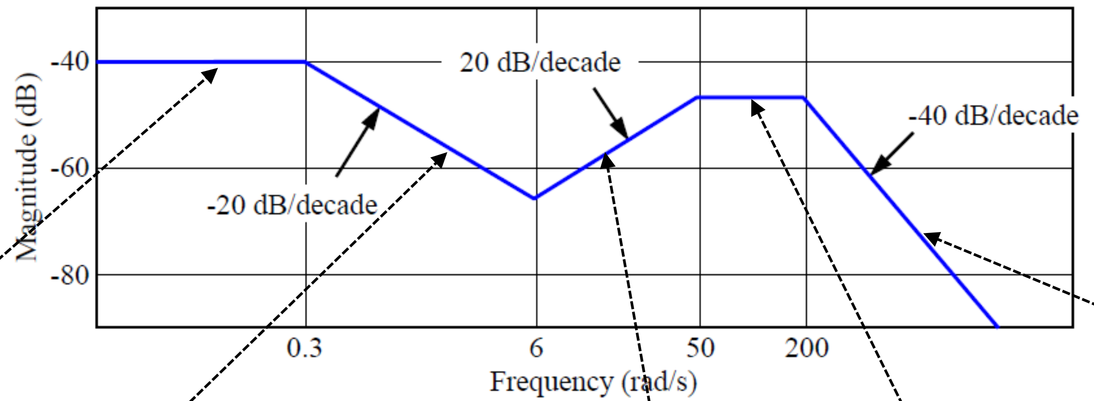
Q4

The straight-line Bode magnitude plot for a LTI system, $\tilde{H}(s)$, which consists of real poles and zeros, is shown below. Identify $\tilde{H}(s)$.

Bode magnitude plot for $\tilde{H}(s)$

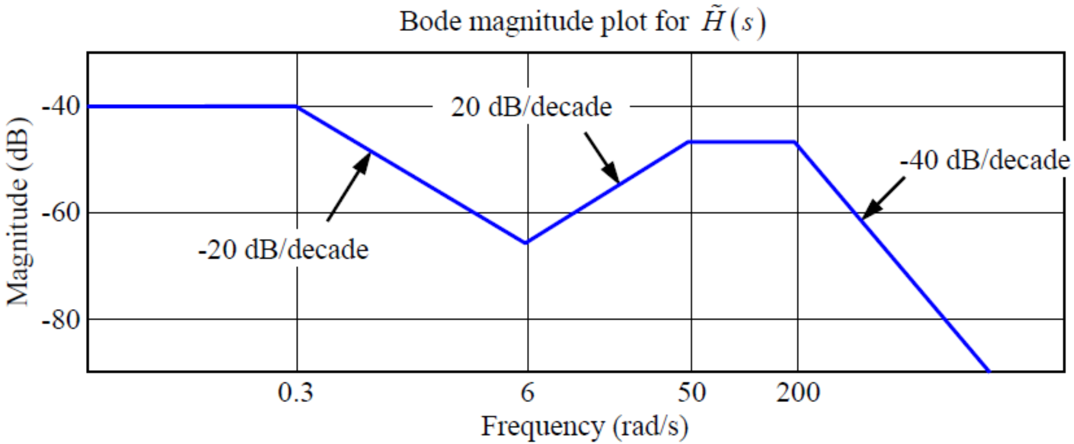


Bode magnitude plot for $\tilde{H}(s)$



The low frequency asymptote is a horizontal line at 40 dB, which indicates the presence of a DC gain factor of $K_{dc} = 10^{-40/20} = 0.01.$	A slope change of -20 dB/ decade occurs at 0.3 rad/s, which indicates the presence of a pole factor of: $\dfrac{1}{\frac{s}{0.3}+1}$	A slope change of +40 dB/decade occurs at 6 rad/s, which indicates the presence of a double zero factor of: $\left(\frac{s}{6}+1\right)^2$	A slope change of -20 dB/decade occurs at 50 rad/s, which indicates the presence of a pole factor of: $\dfrac{1}{\frac{s}{50}+1}$	A slope change of -40 dB/decade occurs at 200 rad/s, which indicates the presence of a double pole factor of: $\left(\dfrac{1}{\frac{s}{200}+1}\right)^2$

$$\begin{cases} \left|\tilde{H}(j\omega)\right| = K_{dc} \\ \left|\tilde{H}(j\omega)\right|_{dB} = 20\log_{10}\left(K_{dc}\right) \text{ dB} \end{cases}$$



Therefore,

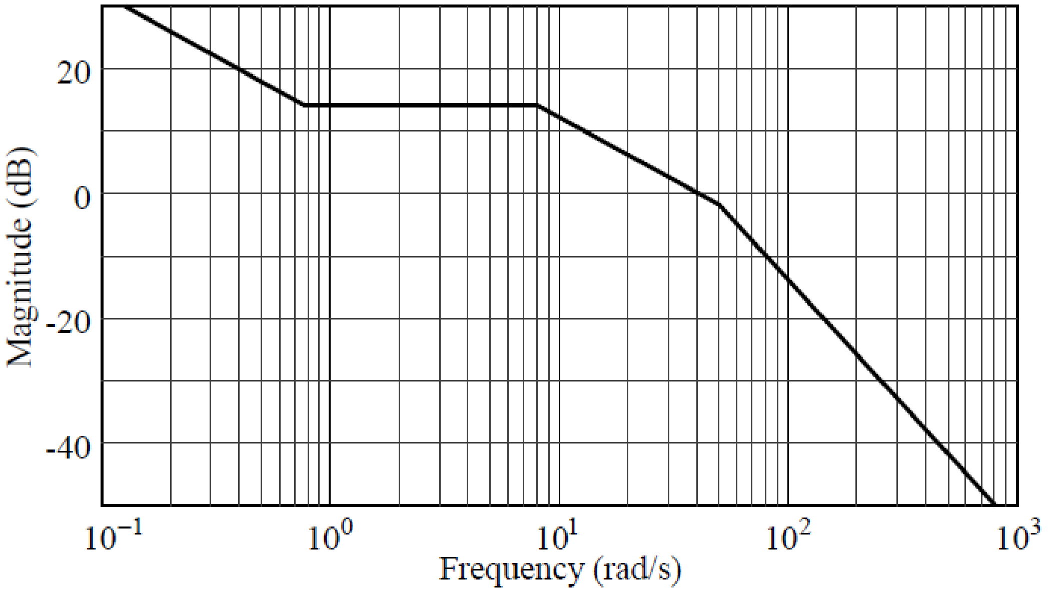
$$\tilde{H}(s) = 0.01 \cdot \frac{1}{\frac{s}{0.3}+1} \cdot \left(\frac{s}{6}+1\right)^2 \cdot \frac{1}{\frac{s}{50}+1} \cdot \left(\frac{1}{\frac{s}{200}+1}\right)^2$$

$$= \frac{0.01}{36}\left(\frac{0.3}{s+0.3}\right)(s+6)^2\left(\frac{50}{s+50}\right)\left(\frac{200}{s+200}\right)^2 = \frac{500}{3}\frac{(s+6)^2}{3(s+0.3)(s+50)(s+200)^2}$$

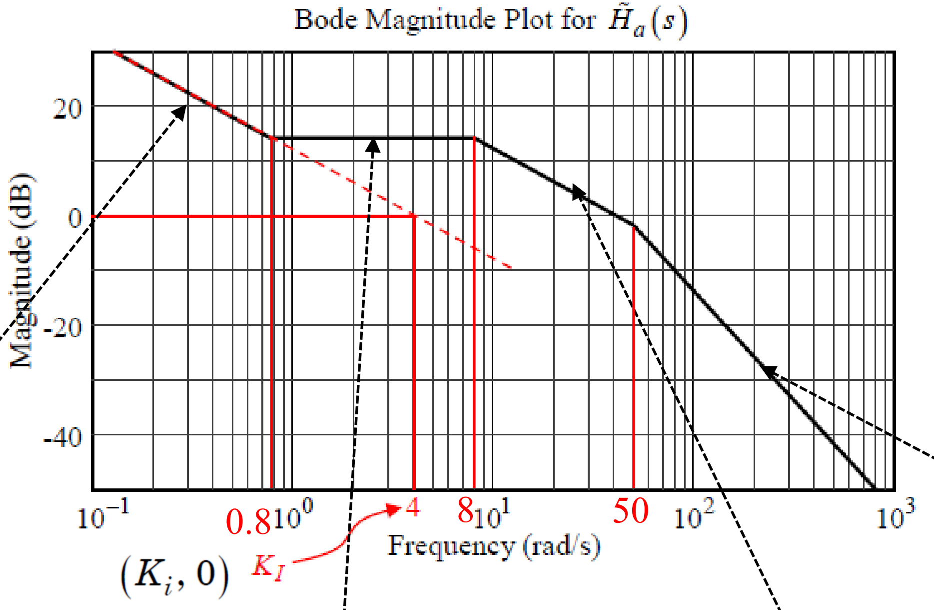
Q5

(a) Identify the system $\tilde{H}_a(s)$ from its Bode magnitude plot shown below. Describe the behavior of the system at very low frequency $(\omega << 0.8 \text{ rad/s})$ and at very high frequency $(\omega >> 50 \text{ rad/s})$.

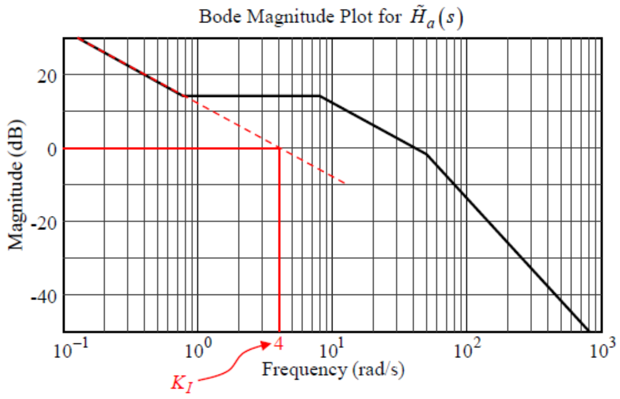
Bode Magnitude Plot for $\tilde{H}_a(s)$



Q5(a)



The low frequency asymptote is a line with slope -20 dB/decade, which indicates the presence of an integrator $\frac{K_I}{s} = \frac{4}{s}$	A slope change of +20 dB/decade occurs at 0.8 rad/s, which indicates the presence of a zero factor $\frac{s}{0.8} + 1$	A slope change of -20 dB/decade occurs at 8 rad/s, which indicates the presence of a pole factor $\frac{1}{\frac{s}{8} + 1}$	A slope change of -20 dB/decade occurs at 50 rad/s, which indicates the presence of a pole factor $\frac{1}{\frac{s}{50} + 1}$



Therefore,

$$\tilde{H}_a(s) = \frac{4}{s}\cdot\left(\frac{s}{0.8}+1\right)\cdot\left(\frac{1}{\frac{s}{8}+1}\right)\cdot\left(\frac{1}{\frac{s}{50}+1}\right) = \frac{4}{s}\cdot\frac{1}{0.8}(s+0.8)\cdot 8\left(\frac{1}{s+8}\right)\cdot 50\left(\frac{1}{s+50}\right)$$

$$= \frac{2000}{s}\cdot\frac{s+0.8}{(s+8)(s+50)}$$

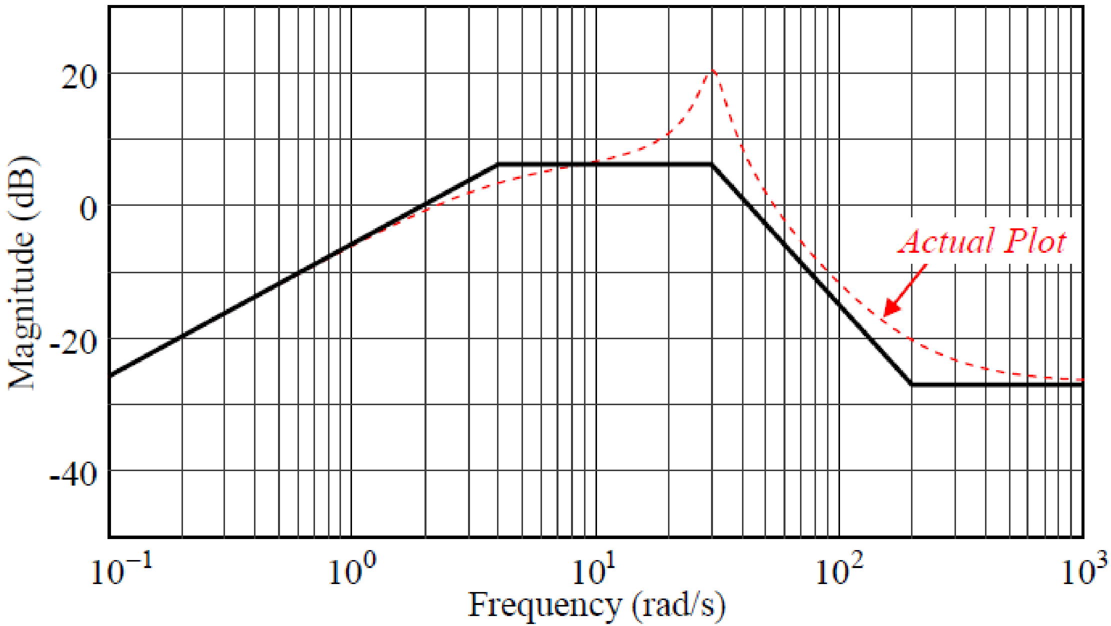
At $\omega << 0.8$ rad/s: $\tilde{H}_a(s) \cong \dfrac{2000}{s}\cdot\dfrac{\cancel{s}+0.8}{(\cancel{s}+8)(\cancel{s}+50)} = \dfrac{4}{s} \quad \leftarrow$ (Integrator of gain 4).

At $\omega >> 50$ rad/s: $\tilde{H}_a(s) \cong \dfrac{2000}{s}\cdot\dfrac{s+\cancel{0.8}}{(s+\cancel{8})(s+\cancel{50})} = \dfrac{2000}{s^2} \quad \leftarrow$ (double integrator of combined gain 2000).

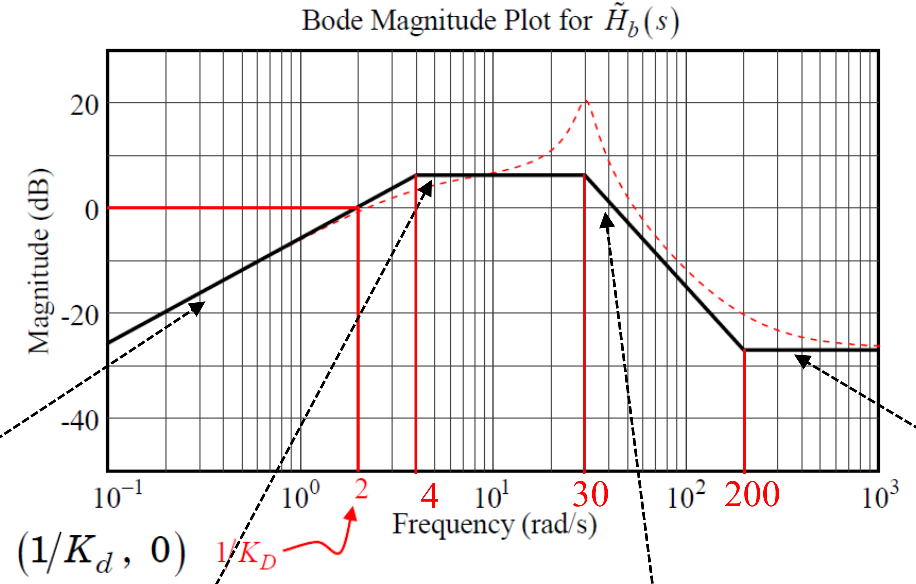
(b) The Bode magnitude plot for the system $\tilde{H}_b(s) = K\dfrac{(s+a)(s+b)(s+c)}{(s+d)\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$ is shown below.

Find the values of K, a, b, c, d and ω_n.

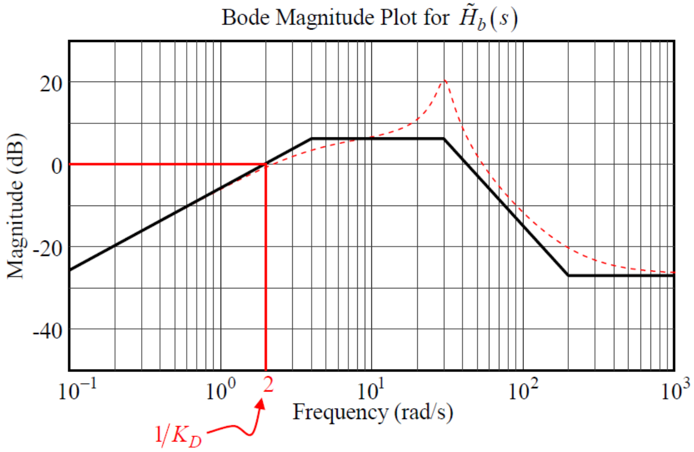
Bode Magnitude Plot for $\tilde{H}_b(s)$



Q5(b)



The low frequency asymptote is a line with slope +20 dB/decade, which indicates the presence of a differentiator $K_D\, s = 0.5s$	A slope change of -20 dB/decade occurs at 4 rad/s, which indicates the presence of a pole factor $\frac{1}{\frac{s}{4}+1}$	A slope change of -40 dB/decade occurs at 30 rad/s, which indicates the presence of a 2nd-order factor due to the existence of a resonant peak $\frac{30^2}{s^2+60\zeta s+30^2}$	A slope change of +40 dB/decade occurs at 200 rad/s, which indicates the presence of a double zero factor $\left(\frac{s}{200}+1\right)^2$



Therefore,

$$\tilde{H}_b(s) = 0.5s \cdot \left(\frac{1}{\frac{s}{4}+1}\right) \cdot \left(\frac{900}{s^2 + 60\zeta s + 900}\right) \cdot \left(\frac{s}{200}+1\right)^2$$

$$= 0.5s \cdot 4\left(\frac{1}{s+4}\right) \cdot 900\left(\frac{1}{s^2 + 60\zeta s + 900}\right)\left(\frac{1}{s+30}\right)^2 \cdot \frac{1}{200^2}(s+200)^2$$

$$= 0.045\frac{s(s+200)^2}{(s+4)\left(s^2 + 60\zeta s + 900\right)} = K\frac{(s+a)(s+b)(s+c)}{(s+d)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

which results in $K = 0.045$, $a = 0$, $b = c = 200$, $d = 4$ and $\omega_n = 30$.

Thanks!