Brief Summary of Chapters 1-5

Time Domain

1. Signals – Classification and Basic Signals

❖ Classification of Signals

- Periodic Signals: $x(t) = x(t+T);\ \forall t$
- Complex Signals: $z = re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$
- Energy Signals

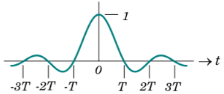
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt,\ 0 < E < \infty$$

- Power Signals

$$P = \lim_{\tau\to\infty} \frac{1}{2\tau}\int_{-\tau}^{\tau} |x(t)|^2 dt, 0 < P < \infty$$

❖ Basic Signals

- $sinc\left(\frac{t}{T}\right) = \begin{cases} \frac{\sin(\pi t/T)}{\pi t/T}; & t \neq 0 \\ 1; & t = 0 \end{cases}$



- $x(t) = \mu e^{j(\omega_0 t+\phi)} = \mu[\cos(\omega_0 t+\phi) + j\sin(\omega_0 t+\phi)]$

2. Time-domain Operations & the Dirac Impulse

❖ Time-domain Operations

- Time-scaling; Sum; Product; Convolution

❖ The Dirac-δ function (Unit Impulse function) $\delta(t)$

$$\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \neq 0 \end{cases} \text{ and } \int_{-\varepsilon}^{\varepsilon} \delta(t)dt = 1, \forall\varepsilon > 0$$

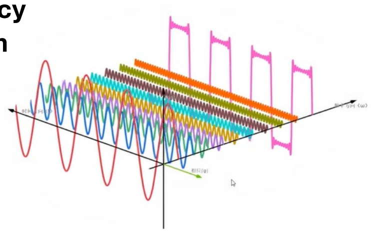
- Properties of $\delta(t)$
  - Symmetry : $\delta(t) = \delta(-t)$
  - Sampling: $x(t)\delta(t-\lambda) = x(\lambda)\delta(t-\lambda)$
  - Sifting: $\int_{-\infty}^{\infty} x(t)\delta(t-\lambda)dt = x(\lambda)$
  - Replication: $x(t) * \delta(t-\lambda) = x(t-\lambda)$
- The Dirac-δ Comb function:

$$comb_\lambda(t) = \sum_n \delta(t - n\lambda)$$

Model a periodic signal: $x_p(t) = x(t) * \sum_n \delta(t-n\lambda)$

Model a sampled signal: $x_s(t) = x(t) \times \sum_n \delta(t-n\lambda)$

Frequency Domain



$$\tilde{x}(t) = \mu e^{j(2\pi f_0 t+\phi)}$$
$$= \mu e^{j\phi} \times e^{j2\pi f_0 t}$$

Spectrum (↑ $\mu e^{j\phi}$)

Normalized complex sinusoid (↑ $e^{j2\pi f_0 t}$)

5. Spectral Density and Bandwidth

❖ Rayleigh Energy Theorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

❖ Energy Spectral Density

$$E_x(f) = |X(f)|^2$$

❖ Parseval Power Theorem

$$P = \lim_{W\to\infty} \frac{1}{2W}\int_{-W}^{W} |x(t)|^2 dt$$
$$= \int_{-\infty}^{\infty} \lim_{W\to\infty} \frac{1}{2W} |X(f)|^2 df = \int_{-\infty}^{\infty} P_x(f) df$$

For periodic signals:

$$P_x(f) = \sum_{k=-\infty}^{\infty} |c_k|^2 \delta(f - kf_p)$$

$$P = \int_{-\infty}^{\infty} P_x(f)df = \sum_{k=-\infty}^{\infty} |c_k|^2$$

3. Discrete-Frequency Spectrum (Fourier Series)

An expansion of a **periodic** function into a sum of sinusoids

❖ Complex Exponential Fourier Series

- Fourier series coefficients

$$c_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t) e^{-j2\pi kt/T_p} dt, k = 0, \pm 1, \pm 2, \cdots$$

- Fourier series expansion

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T_p}$$

4. Continuous-Frequency Spectrum (Fourier Transform)

The spectrum of an **aperiodic** signal in the continuous-frequency domain, f.

❖ Fourier Transform:

- Forward Fourier Transform:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

- Inverse Fourier Transform:

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

❖ Properties of Fourier Transform:

- Linearity; Time Scaling; Duality; Time Shifting; Frequency Shifting; Differentiation; Integration; Convolution.

❖ Spectral Properties of a REAL Signal

❖ Spectrum of Signals that are not Absolutely Integrable

- Impulse: $\Im\{K\delta(t)\} = K$
- DC: $\Im\{K\} = K\delta(f)$
- Complex Exponential: $\Im\{Ke^{j2\pi f_0 t}\} = K\delta(f - f_0)$
- Arbitrary periodic signals:

$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k \delta\left(f - \frac{k}{T_p}\right)$$

- Dirac-δ Comb Function:

$$\Im\{comb_\lambda(t)\} = \frac{1}{\lambda}\sum_k \delta(f - k/\lambda)$$