

# EE2012 Tutorial 2

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# Instructor

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Location:

- E1A-04-01
- Level 6, #06-02, T-Lab Building

A fair dice is rolled twice and all the possible outcomes are equally likely. Let  $X$  and  $Y$  be the results of the two rolls, respectively.

- (a) Let  $A = \{\max(X, Y) = m\}$  and  $B = \{\min(X, Y) = 2\}$ . Find  $P(A|B)$  for  $m = 1, 2, 5$ .
- (b) Find the probability  $P(X < Y | Y \geq 2)$ .

## Q1 (a) Find $P(A|B)$ for $m = 1, 2, 5$ .

$$A = \{\max(X, Y) = m\} \text{ and } B = \{\min(X, Y) = 2\}$$

By conditional probability, we have  $P(A|B) = P(AB)/P(B)$ . We need to find  $P(AB)$  and  $P(B)$ .

### Step 1: Find $P(B)$

- Denote an outcome by  $(X, Y)$ . There are  $6 \times 6 = 36$  outcomes. They can be illustrated in Figure 1 below.
- For event  $B$ , the minimum can be  $X$  and/or  $Y$ . The outcomes in event  $B = \{(6, 2), (5, 2), (4, 2), (3, 2), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)\}$
- The outcomes in event are marked in the Figure 1.
- Since all outcomes have the same probability  $1/36$ , we have  $P(B) = \frac{9}{36} = \frac{1}{4}$ .

### • Step 2: Find $P(A \cap B)$ for $m = 1, 2, 5$

- When  $m = 1, A \cap B = \emptyset$ .  $P(A|B) = P(AB)/P(B) = 0$ .
- When  $m = 2, A \cap B = \{(2, 2)\}$ .  $P(A|B) = P(AB)/P(B) = \frac{\frac{1}{36}}{\frac{1}{4}} = \frac{1}{9}$ .
- When  $m = 5, A \cap B = \{(5, 2), (2, 5)\}$ .  $P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{2}{36}}{\frac{1}{4}} = \frac{2}{9}$ .

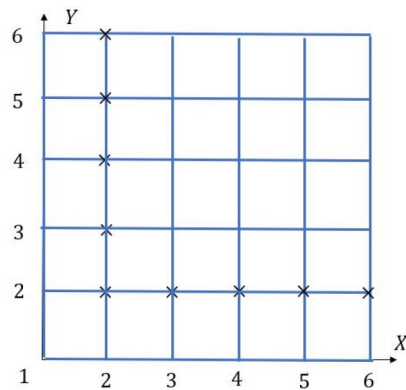


FIG. 1. Outcomes of two dices.

## Q1 (b) Find the probability $P(X < Y | Y \geq 2)$

### Step 1: Find $P(Y \geq 2)$

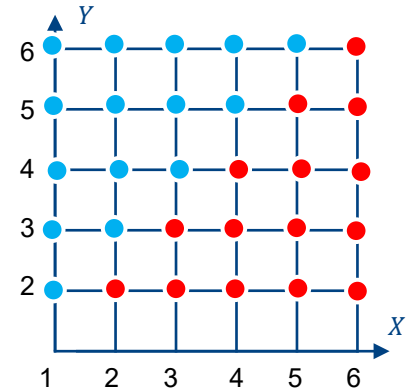
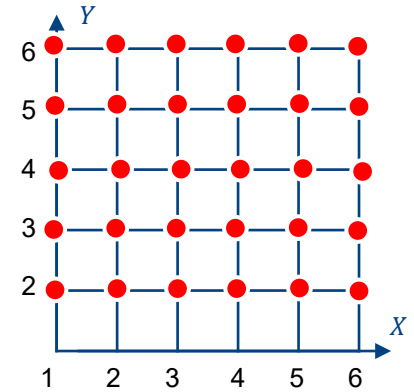
- For  $Y \geq 2$ , the outcomes for  $Y$  are 2, 3, 4, 5, or 6.
- This includes all possible outcomes except those where  $Y = 1$ .
- There are 30 outcomes where  $Y \geq 2$
- Therefore,  $P(Y \geq 2) = \frac{30}{36} = \frac{5}{6}$

### Step 2: Find $P(X < Y \cap Y \geq 2)$

- In the left-top triangle, there are 15 outcomes in the event  $\{X < Y\} \cap \{Y \geq 2\}$ .
  - (1,6), (2,6), (3,6), (4,6), (5,6)
  - (1,5), (2,5), (3,5), (4,5)
  - (1,4), (2,4), (3,4)
  - (1,3), (2,3)
  - (1,2)
- Therefore,  $P(X < Y \cap Y \geq 2) = \frac{15}{36} = \frac{5}{12}$

### Step 3: Compute $P(X < Y | Y \geq 2)$

$$P(X < Y | Y \geq 2) = \frac{P(X < Y \cap Y \geq 2)}{P(Y \geq 2)} = \frac{\frac{5}{12}}{\frac{5}{6}} = \frac{1}{2}$$



## Q2

Suppose you have two coins, one biased and one fair, but you don't know which coin is which.

Coin A is biased, i.e., it comes up heads with a probability  $3/4$ , while Coin B is fair, i.e., it comes up heads with a probability  $1/2$ . Suppose you pick a coin at random and flip it. Let  $C_X$  denote the event that coin  $X$  is picked and let  $H$  and  $T$  be the possible outcomes of the flip.

- (a) Given that the outcome of the flip is a head, what is  $P(C_A|H)$ ? That is, what is the conditional probability of picking coin A given the outcome is a head?
- (b) What is  $P(C_A|T)$ ?

# Q2

## Understand the problem

You have two coins:

- **Coin A (biased):** The probability of getting heads ( $H$ ) is  $P(H \mid C_A) = \frac{3}{4}$ .
- **Coin B (fair):** The probability of getting heads is  $P(H \mid C_B) = \frac{1}{2}$ .

Suppose you pick a coin **at random** and flip it

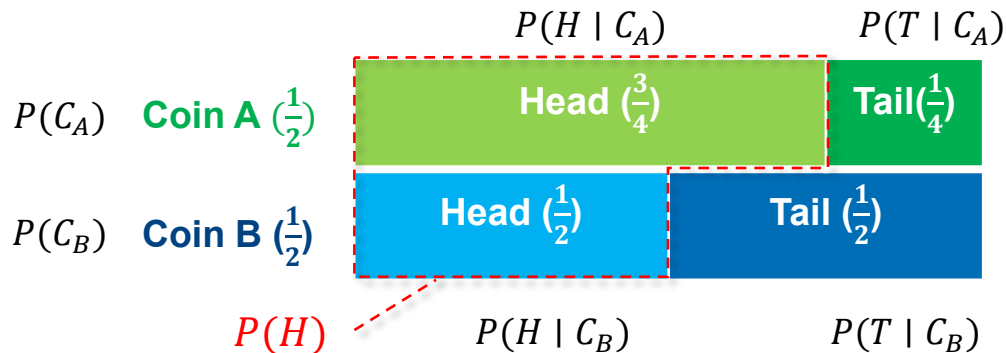
		$P(H \mid C_A)$	$P(T \mid C_A)$
$P(C_A)$	<b>Coin A</b> ( $\frac{1}{2}$ )	Head ( $\frac{3}{4}$ )	Tail ( $\frac{1}{4}$ )
$P(C_B)$	<b>Coin B</b> ( $\frac{1}{2}$ )	Head ( $\frac{1}{2}$ )	Tail ( $\frac{1}{2}$ )
		$P(H \mid C_B)$	$P(T \mid C_B)$

## Q2 (a) Finding $P(C_A | H)$

The probability  $P(C_A | H)$  is the chance that Coin A is picked after we know the result of the flipping is a head. Note that when the picked coin shows a head, it can be Coin A or Coin B.

To calculate  $P(C_A | H)$ , we need the help of Bayes' rule

$$P(C_A | H) = \frac{P(H | C_A) \cdot P(C_A)}{P(H)}$$



### Step 1: Find $P(H)$

The total probability of getting heads ( $P(H)$ ) is a weighted sum of the probabilities of getting heads from each coin. Since you pick either coin with equal probability,  $P(C_A) = P(C_B) = \frac{1}{2}$ .

$$P(H) = P(H | C_A) \cdot P(C_A) + P(H | C_B) \cdot P(C_B) = \frac{3}{4} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{5}{8}$$

### Step 2: Apply Bayes' Theorem to find $P(C_A | H)$

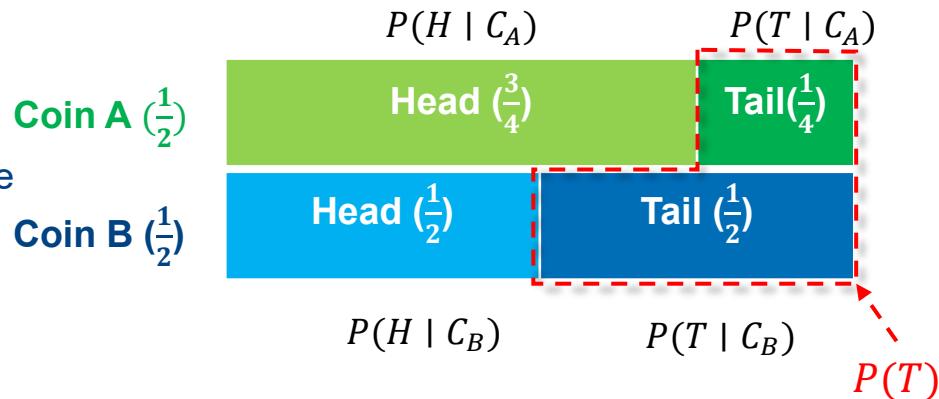
$$P(C_A | H) = \frac{P(H | C_A) \cdot P(C_A)}{P(H)} = \frac{\frac{3}{4} \times \frac{1}{2}}{\frac{5}{8}} = \frac{3}{5}$$



## Q2 (b) Finding $P(C_A | T)$

Similarly, to calculate  $P(C_A|T)$  by using Bayes' rule

$$P(C_A | T) = \frac{P(T | C_A) \cdot P(C_A)}{P(T)}$$



### Step 1: Find $P(T)$

The total probability of getting tails  $P(T)$  is the complement of  $P(H)$ , so:

$$P(T) = 1 - P(H) = 1 - 5/8 = 3/8$$

### Step 2: Apply Bayes' Theorem to find $P(C_A | T)$

Now we substitute the values into Bayes' Theorem:

$$P(C_A | T) = \frac{P(T | C_A) \cdot P(C_A)}{P(T)} = \frac{\frac{1}{4} \times \frac{1}{2}}{\frac{3}{8}} = \frac{1}{3}$$

## Q3

A machine produces photo-detectors in pairs. Tests show that the first photo-detector is acceptable with probability  $3/5$ . When the first photo-detector is acceptable, the second photodetector is acceptable with probability  $4/5$ . If the first photodetector is defective, the second photodetector is acceptable with probability  $2/5$ .

(a) What is the probability that exactly one photodetector of a pair is acceptable?

That is, only one photo-detector is acceptable.

(b) What is the probability that both photo-detectors in a pair are defective?

**Q3 (a) What is the probability that exactly one photodetector of a pair is acceptable? That is, only one photo-detector is acceptable.**

The whole process can be described by a tree diagram of three layers

- From this tree diagram, we can see that the two desired outcomes can be represented by the two paths  $A_1D_2$  and  $D_1A_2$ .
- Since these two outcomes are mutually exclusive, the probability of **only one acceptable** is

$$P((A_1D_2) \cup (D_1A_2)) = P(A_1D_2) + P(D_1A_2)$$

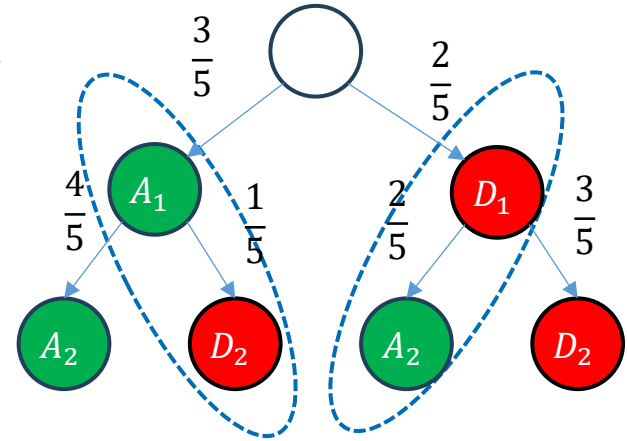
To compute the probability of path  $P((A_1D_2)$  and  $P(D_1A_2)$

$$P(A_1D_2) = P(A_1) \times P(D_2|A_1) = 3/5 \times 1/5 = 3/25$$

$$P(D_1A_2) = P(D_1) \times P(A_2|D_1) = 2/5 \times 2/5 = 4/25$$

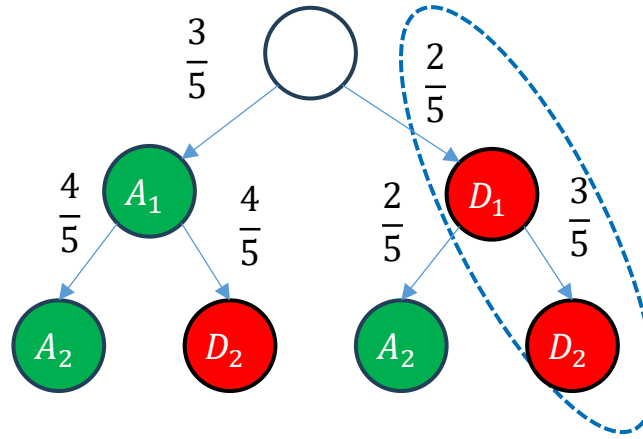
Therefore

$$P((A_1D_2) \cup (D_1A_2)) = 3/25 + 4/25 = 7/25$$



### Q3 (b) What is the probability that both photo-detectors in a pair are defective?

Applying the same method as Q3(a)



- We can observe from the tree diagram that it corresponds to the path  $D_1D_2$ .
- The probability is the product of the probabilities associated with the segments along this path

$$P(D_1D_2) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$$

# Q4

People may be infected by a virus with a chance of  $10^{-4}$ . To detect the infection, a testing kit is used. If one is infected, it is detected as positive by the testing kit with probability 99%. The testing kit responds positive with probability 10% when people, who do not infect, are tested by the kit. Those tested as positive will be treated in the hospital.

- (a) What is the probability of one being hospitalized based on the testing result?
- (b) Do you think this testing kit is useful? Why?

## Q4 (a) What is the probability of one being hospitalized based on the testing result?

### Step 1: Define the events

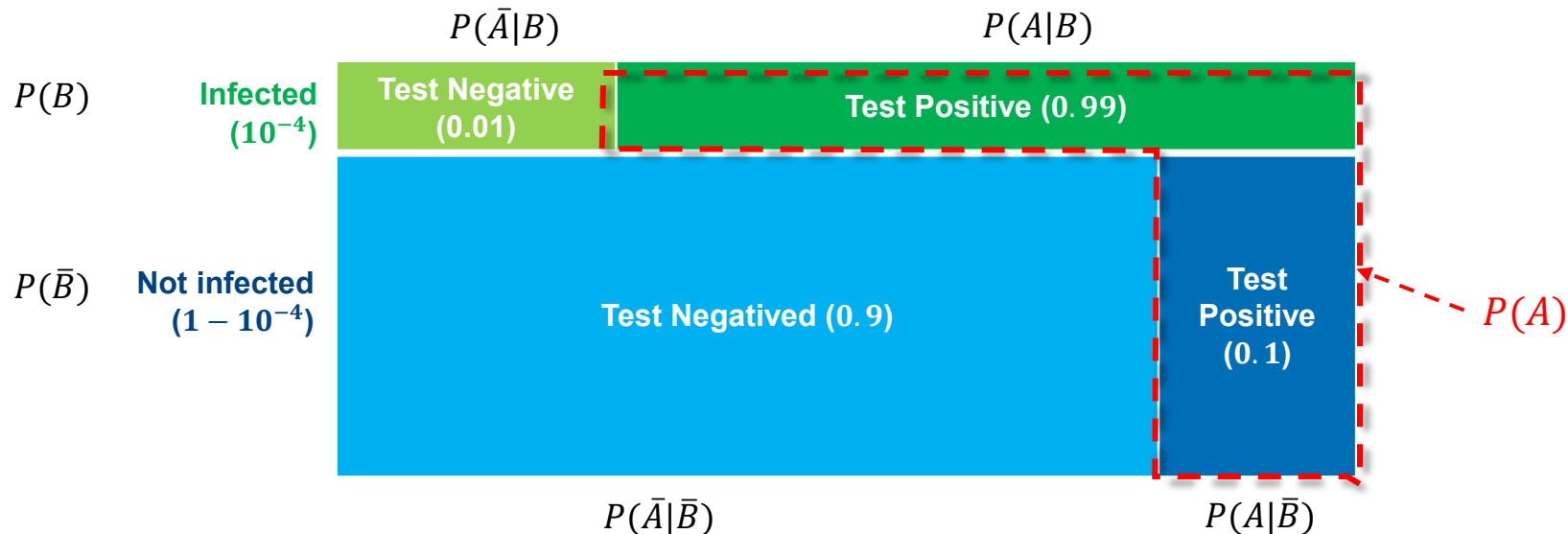
- Let  $A$  be “one is hospitalized”, which is the same as “one’s testing is positive”.
- Let  $B$  be “one is infected”.

### Step 2: Define the probabilities

- $P(B) = 10^{-4}$  : prior probability that a person is infected.
- $P(A | B) = 0.99$ : probability that the test is positive given the person is infected (sensitivity).
- $P(A | \bar{B}) = 0.10$  : probability that the test is positive given the person is not infected (false positive rate).
- $P(\bar{B}) = 1 - P(B) = 1 - 10^{-4} = 0.9999$  : probability that a person is not infected.

		$P(\bar{A} B)$	$P(A B)$
$P(B)$	Infected ( $10^{-4}$ )	Test Negative (0.01)	Test Positive (0.99)
$P(\bar{B})$	Not infected ( $1 - 10^{-4}$ )	Test Negative (0.9)	Test Positive (0.1)
		$P(\bar{A} \bar{B})$	$P(A \bar{B})$

## Q4 (a) What is the probability of one being hospitalized based on the testing result?



**Step 3: Compute  $P(A)$ :** The total probability of testing positive

$P(A)$  is the sum of two mutually exclusive cases:

1. The person is infected and tests positive.

2. The person is not infected and still tests positive (false positive)

$$\begin{aligned}
 P(A) &= P(AB) + P(A\bar{B}) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B}) \\
 &= 0.99 \times 10^{-4} + 0.1 \times (1 - 10^{-4}) = 0.000099 + 0.099999 \approx 0.100098
 \end{aligned}$$

## Q4(a)

### Step 4: Apply Bayes' Theorem to find $P(B | A)$

Now we substitute the values into Bayes' Theorem:

Probability that a person is infected given the person's test is positive

$$P(B | A) = \frac{P(A | B) \cdot P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})} = \frac{0.99 \times 10^{-4}}{0.99 \times 10^{-4} + 0.1 \times (1 - 10^{-4})} \approx 0.00098912$$

If a person is tested positive the 2<sup>nd</sup> time

$P(B) = 0.00098912$  (The current Prior is previous Posterior probability: tested positive the 1<sup>st</sup> time )

$$P(\bar{B}) = 1 - 0.00098912$$

$P(A|B)$ : Sensitivity = 0.99

$P(A|\bar{B})$ : False positive rate = 0.1

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})} = \frac{0.99 \times 0.00098912}{0.99 \times 0.00098912 + 0.1 \times (1 - 0.00098912)} = 0.0097$$

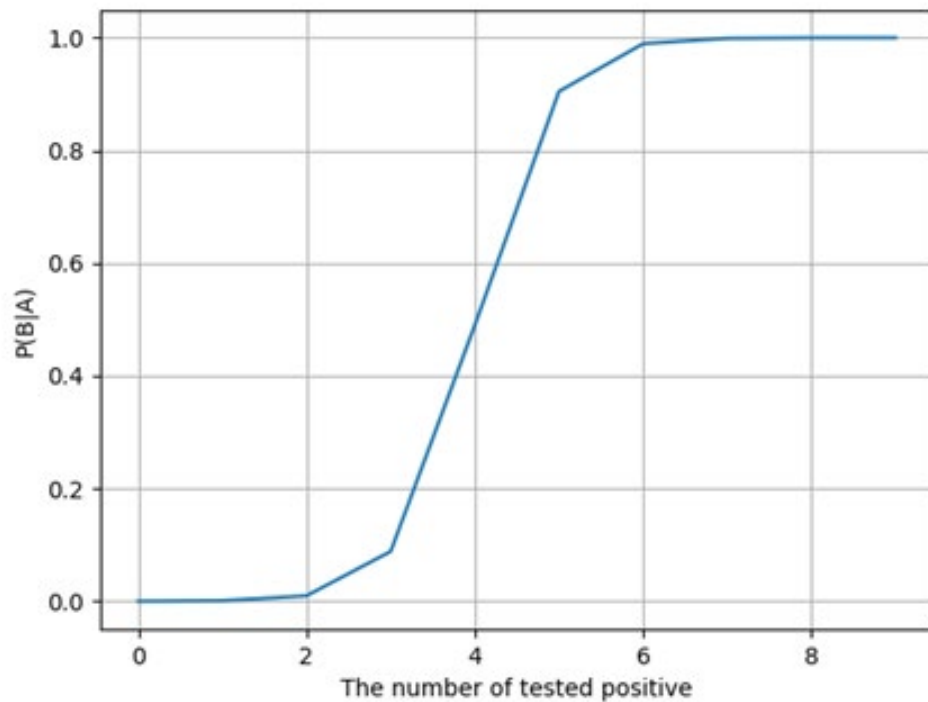
If a person is tested positive the 5<sup>th</sup> time

$$P(B|A) = 0.9048$$



## Q4 (a)

The plot illustrates the change of  $P(B|A)$  with respect to the number of tested positive.



## Q4 (b) Do you think this testing kit is useful? Why?

- Let  $\bar{A}$  be the complement event of  $A$ . Since  $P(A|B) = 0.99$  and  $P(\bar{A}|B) = 0.01$ , this testing kit is good at identifying the infected cases.
- But if we check the intermediate steps above when calculating  $P(A)$ , we can see that the second term  $P(A|\bar{B})P(\bar{B})$  contributes much more than the first term  $P(A|B)P(B)$ . More precisely, the second term actually contributes more than 99.9% to  $P(A)$ . However, the second term is the probability for “false positive” cases that people are not infected but tested positive.
- That is more than 99.9% of the people hospitalized actually do not need to be hospitalized. In this sense, this testing kit is not very successful.

## Q5

A fair dice is rolled **twice** and the sum of the numbers is observed.

- (a) What is the probability that the **sum is at least 9** if it is known that **a 5** was rolled?
- (b) What is the probability that **a 5** was rolled given that **the sum is at least 9**?

## Q5 (a) What is the probability that the sum is at least 9 if it is known that a 5 was rolled?

It is rolled twice and the outcomes can be denoted by a 2-tuple  $(X_1, X_2)$ . There are 36 outcomes and each occurs with the same probability  $1/36$  because of fair dice.

### Step 1: Define the events

- Let  $A$  = “the sum is at least 9”
- Let  $B$  = “a 5 is rolled”.
- We want to compute  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{|A \cap B|}{|B|}$

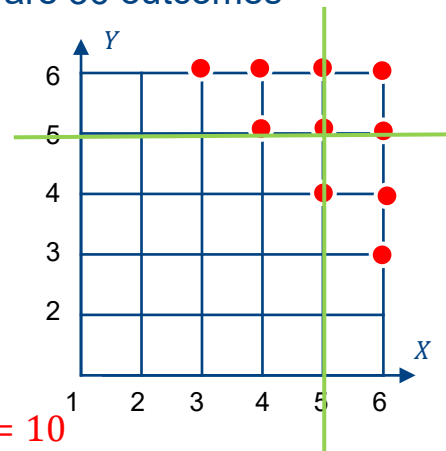
### Step 2: Counting

- $A = \{(3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)\}$ ,  $|A| = 10$
- $B = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)\}$ ,  $|B| = 11$ .
- $A \cap B = \{(4, 5), (5, 5), (6, 5), (5, 4), (5, 6)\}$ ,  $|A \cap B| = 5$ .

### Step 3: Compute Conditional Probability $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{|A \cap B|}{|B|} = \frac{5}{11}$$

because all the elementary outcomes have the same probability.



## Q5 (b) What is the probability that a 5 was rolled given that the sum is **at least 9**?

This question asks for  $P(B|A)$ . It seems like a Bayes probability problem. But it turns out we don't need to find out the likelihood  $P(A|B)$  and a prior probability  $P(B)$ .

### Step 1: Define Events

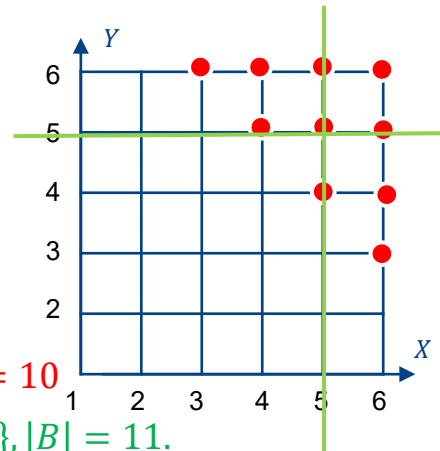
- Let  $A$  = “the sum is at least 9”
- Let  $B$  = “a 5 is rolled”.
- We want to compute  $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{|A \cap B|}{|A|}$

### Step 2: Counting

- $A = \{(3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)\}$ ,  $|A| = 10$
- $B = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)\}$ ,  $|B| = 11$ .
- $A \cap B = \{(4, 5), (5, 5), (6, 5), (5, 4), (5, 6)\}$ ,  $|A \cap B| = 5$ .

### Step 3: Compute the Conditional Probability $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{|A \cap B|}{|A|} = \frac{5}{10} = 0.5$$



## Q6

Symbols 0 and 1 are transmitted in a noisy channel with probability  $p$  and  $1 - p$ , respectively. Due to the channel noise, they are received correctly with probability  $u$  and  $v$ , respectively. Assume the transmissions for any two symbols are independent.

- (a) What is the probability that the  $k^{th}$  symbol is received correctly?
- (b) What is the probability that the sequence 1101 is received correctly?
- (c) To reduce the error probability, each symbol is transmitted three times and the receiver applies a majority vote to decode it. What is the probability that 0 is correctly decoded? If 101 is received, what is the probability this symbol is 0?

## Q6 (a) What is the probability that the $k^{th}$ symbol is received correctly?

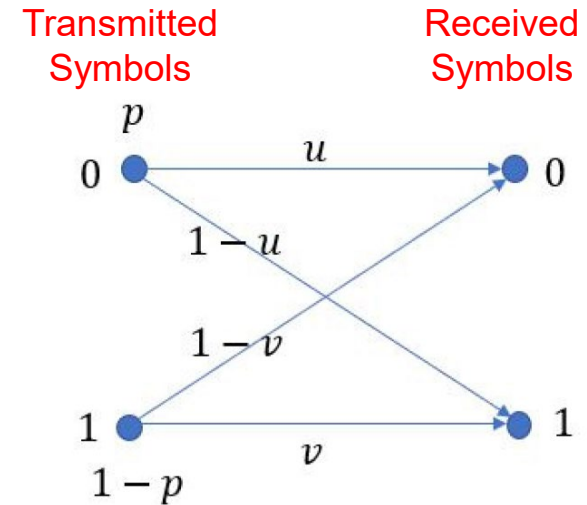
Understand the information:

- **Symbol 0**

- transmitted with probability  $p$ ,
- received correctly with probability  $u$
- received incorrectly with probability  $1 - u$ .

- **Symbol 1**

- transmitted with probability  $1 - p$ ,
- received correctly with probability  $v$  and
- received incorrectly with probability  $1 - v$ .



Noisy channel model.

## Q6 (a) What is the probability that the $k^{th}$ symbol is received correctly?

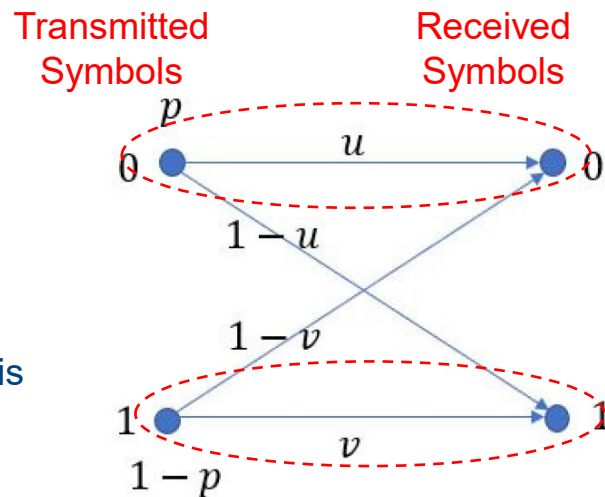
We define

$S^k$  be the  $k^{th}$  transmitted symbol.

$C^k$  be the event the  $k^{th}$  symbol is received correctly

Thus, the total probability that the  $k^{th}$  symbol is received correctly is

$$\begin{aligned} P(C_k) &= \sum_{S_k \in \{0,1\}} P(C_k, S_k) = \sum_{S_k \in \{0,1\}} P(C_k|S_k)P(S_k) \\ &= P(C_k|S_k = 0)P(S_k = 0) + P(C_k|S_k = 1)P(S_k = 1) = up + v(1 - p). \end{aligned}$$

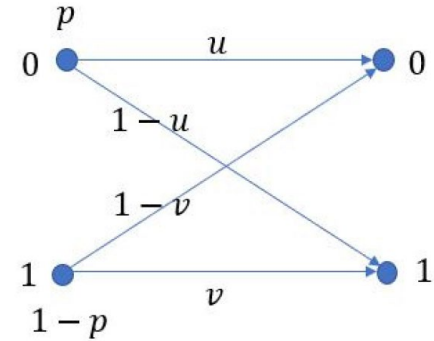
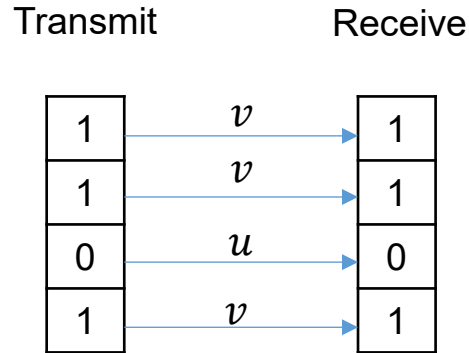


Noisy channel model.



## Q6 (b) What is the probability that the sequence 1101 is received correctly?

There are one zero and three 1 in the sequence 1101.



Noisy channel model.

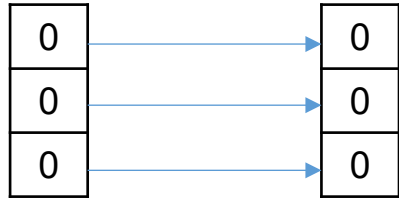
The probability they are received correctly is

$$v \times v \times u \times v = uv^3$$

**Q6 (c) To reduce the error probability, each symbol is transmitted **three times** and the receiver applies a majority vote to decode it. **What is the probability that 0 is correctly decoded?****

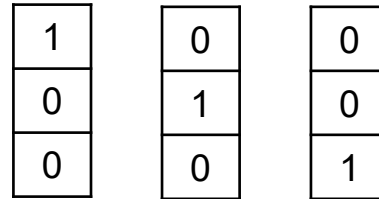
Symbol 0 is transmitted as 000 by the coding method. To decode the received sequence to symbol 0 with a **majority vote**, the received sequence should have at least two 0's out of 3 received symbols.

Transmit

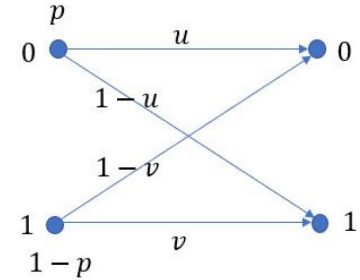


Case I

Receive



Case II



Noisy channel model.

Case I:  $P(\text{three 0's received} | 000 \text{ transmitted}) = u^3$

Case II:  $P(\text{two 0's received} | 000 \text{ transmitted}) = \binom{3}{2} u^2 (1 - u) = 3u^2 (1 - u)$

So the probability 0 is correctly decoded:

$$P(\text{at least two 0's received} | 000 \text{ transmitted}) = P(\text{two 0's received} | 000 \text{ transmitted}) + P(\text{three 0's received} | 000 \text{ transmitted}) = 3u^2 (1 - u) + u^3$$

**Q6 (c) To reduce the error probability, each symbol is transmitted three times and the receiver applies a majority vote to decode it. If 101 is received, what is the probability this symbol is 0?**

- Even  $A$  = "symbol 0 is transmitted"
- Even  $B$  = "101 received".

The probability that symbol 0 is transmitted when 101 is received

**Bayes' theorem.** 
$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

$P(A)$  **Symbol 0**

$P(\bar{A})$  **Symbol 1**

$P(B A)$	$P(\bar{B} A)$
101	$\overline{101}$
$P(B \bar{A})$	$P(\bar{B} \bar{A})$
101	$\overline{101}$

### Step 1: Define probability

- $P(A) = p$ , the prior probability that 0 was transmitted.
- $P(\bar{A}) = 1 - p$ , the prior probability that 1 was transmitted.
- $P(B|A) = u(1 - u)^2$ , the probability that the sequence 101 was received given 0 was transmitted
- $P(B|\bar{A}) = v^2(1 - v)$ , the probability that the sequence 101 was received given 1 was transmitted.

### Step 2: Compute $P(B)$

$$\begin{aligned} P(B) &= P(B, A) + P(B, \bar{A}) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A}) \\ &= u(1 - u)^2p + v^2(1 - v)(1 - p) \end{aligned}$$

### Step 3: Compute $P(A|B)$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{u(1 - u)^2p}{u(1 - u)^2p + v^2(1 - v)(1 - p)}$$



## Q7

An urn contains 7 red balls and 7 blue balls. The experiment asks that you draw from the urn two times and that you need to get a red ball and a blue ball. You can choose to either draw with replacement or without replacement. Which strategy would give you the higher chance of success?

## Q7 Method I

We need to compare the probabilities of success for these two strategies. The desired event of getting a red ball and a blue ball includes the outcomes  $\{r_1b_2, b_1r_2\}$ , where  $r_i$  and  $b_j$  represent the red and blue ball, and subscript  $i$  and  $j$  indicates the pick order.

**Case I: If you draw with replacement,**

The probability that you get a red ball and a blue ball is

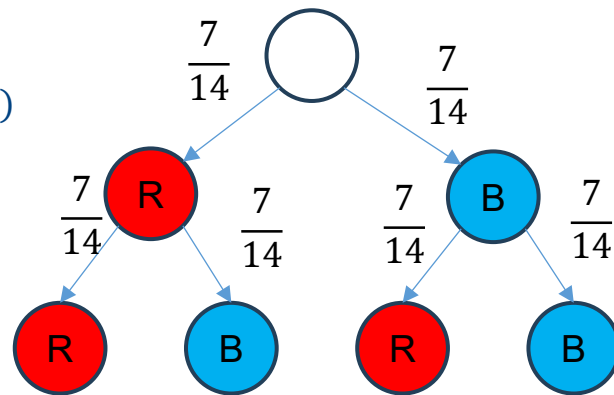
$$\begin{aligned} P(\text{"one red and one blue"}) &= P(\{r_1b_2, b_1r_2\}) = P(\{r_1b_2\}) + P(\{b_1r_2\}) \\ &= P(\{r_1\}) \times P(\{b_2\}) + P(\{b_1\}) \times P(\{r_2\}). \end{aligned}$$

where

$$P(\{r_1\}) = P(\{r_2\}) = P(\{b_1\}) = P(\{b_2\}) = 7/14 = 1/2$$

Then

$$P(\text{"one red and one blue"}) = 1/2 \times 1/2 + 1/2 \times 1/2 = 1/2$$



## Q7 Method I

**Case II: If you draw without replacement**, the desired event is also  $\{r_1 b_2, b_1 r_2\}$ . Its probability is

$$P(\text{"one red and one blue"}) = P(\{r_1 b_2, b_1 r_2\}) = P(\{r_1 b_2\}) + P(\{b_1 r_2\}).$$

But due to no replacement, the picking results are not independent anymore,  
and we need to calculate it based on the conditional probability.

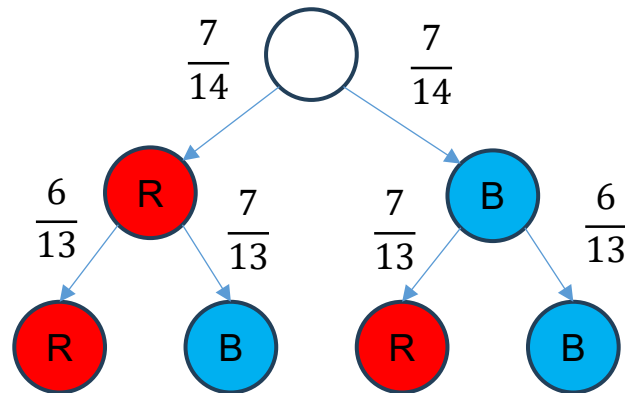
$$P(\{r_1 b_2\}) = P(\{b_2\}|\{r_1\}) \times P(\{r_1\}) = \frac{7}{13} \times \frac{7}{14}$$

and

$$P(\{b_1 r_2\}) = P(\{r_2\}|\{b_1\}) \times P(\{b_1\}) = \frac{7}{13} \times \frac{7}{14}.$$

So

$$P(\text{"one red and one blue"}) = \frac{7}{13} \times \frac{7}{14} + \frac{7}{13} \times \frac{7}{14} = \frac{49}{91} \approx 0.5385$$



**Based on the probabilities of success calculation, you should draw without replacement.**

## Q7 Method II

There is another way to find the probability of success without replacement. Any two out of these 14 balls can be picked with the same probability, which form a uniform sample space. The probability of the desired event can be calculated by the ratio of the desired event size and the sample space size.

### Case I: If you draw with replacement

#### Step 1: Total outcomes:

- Since there are 14 balls (7 red, 7 blue) and you are drawing twice with replacement, each draw has 14 possible outcomes.
- So, the total number of possible outcomes is  $14 \times 14 = 196$ .

#### Step 2: Favorable outcomes (red and blue):

- The number of ways to get red first and blue second:  $7 \times 7 = 49$  (7 choices for red, 7 choices for blue).
- The number of ways to get blue first and red second:  $7 \times 7 = 49$ .

#### Step 3: Probability of success:

- Probability of drawing one red and one blue ball

$$P = \frac{49 + 49}{196} = \frac{1}{2}$$

## Q7 Method II

### Case II: If you draw without replacement

#### Step 1: Total outcomes

- The total number of possible outcomes is  $14 \times 13 = 182$  (because the second draw has only 13 balls to choose from).

#### Step 2: Favorable outcomes (red and blue):

- For the red first, then blue: There are 7 choices for the red ball (out of 14), and once a red is chosen, 7 choices remain for the blue (out of 13 remaining balls).  
favorable outcomes =  $7 \times 7 = 49$ .
- For the blue first, then red: There are 7 choices for the blue ball (out of 14), and once a blue is chosen, 7 choices remain for the red (out of 13 remaining balls). So, favorable outcomes  
favorable outcomes =  $7 \times 7 = 49$ .

#### Step 3: Probability of success:

$$P = \frac{49 + 49}{182} = \frac{98}{182} \approx 0.5358$$





**THANK YOU**