EE2211 Tutorial 10

Dr Feng LIN

Q1

We have two classifiers showing the same accuracy with the same cross-validation. The more complex model (such as a 9th-order polynomial model) is preferred over the simpler one (such as a 2nd-order polynomial model).

a) True

b) False

Q1

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a) True

b) False

Q2

We have 3 parameter candidates for a classification model, and we would like to choose the optimal one for deployment. As such, we run 5-fold cross-validation.

Once we have completed the 5-fold cross-validation, in total, we have trained ________ classifiers. Note that, we treat models with different parameters as different classifiers.

A) 10

B) 20

C) 25

D) 15

Q2

We have 3 parameter candidates for a classification model, and we would like to choose the optimal one for deployment. As such, we run 5-fold cross-validation.

Once we have completed the 5-fold cross-validation, in total, we have trained ________ classifiers. Note that, we treat models with different parameters as different classifiers.

A) 10

B) 20

C) 25

D) 15

In each fold, we train 3 classifiers, so 5 folds give 15 classifiers.

Q3

Suppose the binary classification problem, which you are dealing with, has highly imbalanced classes. The majority class has 99 hundred samples and the minority class has 1 hundred samples. Which of the following metric(s) would you choose for assessing the classification performance?

a) Classification Accuracy
b) Cost sensitive accuracy
c) Precision and recall
d) None of these

$$\text{Cost-Sensitive Accuracy} = 1 - \frac{\text{Total Cost}}{\text{max posible cost}}$$

$$\text{Max possible cost} = C_{p,n} * P + C_{n,p} * N$$

Cost Matrix for Binary Classification

	$\hat{\mathbf{P}}$ (predicted)	$\hat{\mathbf{N}}$ (predicted)
P (actual)	$C_{p,p}$ * TP	$C_{p,n}$ * FN
N (actual)	$C_{n,p}$ * FP	$C_{n,n}$ * TN

Total cost:
$C_{p,p}$ * TP +
$C_{p,n}$ * FN +
$C_{n,p}$ * FP +
$C_{n,n}$ * TN

Main Idea: To assign different ***penalties*** for different entries. Higher penalties for more severe results. Smaller costs are preferred.

Usually, $C_{p,p}$ and $C_{n,n}$ are set to 0; $C_{p,n}$ and $C_{n,p}$ may and may not equal

Q3

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	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)	
P (actual)	TP	FN	Recall TP/(TP+FN)
N (actual)	FP	TN	
	Precision TP/(TP+FP)		Accuracy (TP+TN)/(TP+TN+FP+FN)

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The goal is to highlight the problems of the results!

In this case, we shall

1) Use cost matrix, assign different costs for each entry
2) Use Precision and Recall! Precision = 0.5 and Recall = 0.5

Q4

Given below is a scenario for Training error rate Tr, and Validation error rate Va for a machine learning algorithm. You want to choose a hyperparameter (P) based on Tr and Va. Which value of P will you choose based on the above table?

a) 10
b) 9
c) 8
d) 7
e) 6

P	Tr	Va
10	0.10	0.25
9	0.30	0.35
8	0.22	0.15
7	0.15	0.25
6	0.18	0.15

Q4

Given below is a scenario for Training error rate Tr, and Validation error rate Va for a machine learning algorithm. You want to choose a hyperparameter (P) based on Tr and Va. Which value of P will you choose based on the above table?

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e) 6

P	Tr	Va
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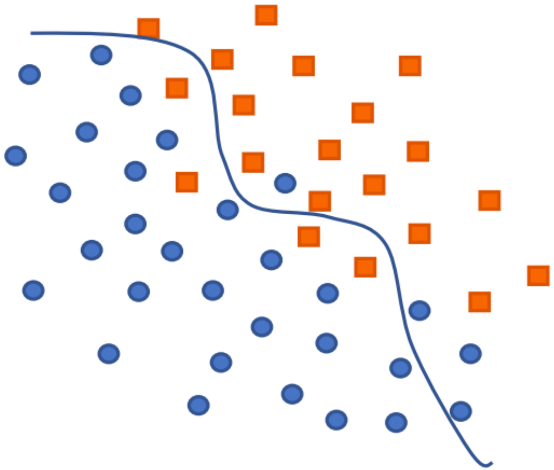
Q5

(Binary and Multicategory Confusion Matrices)

Tabulate the confusion matrices for the following classification problems.

a) Binary problem (the class-1 and class-2 data points are respectively indicated by squares and circles)



Q5

Tabulate the confusion matrices for the following classification problems.

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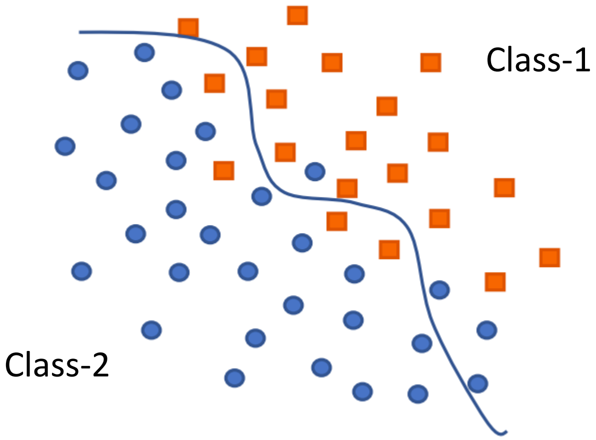
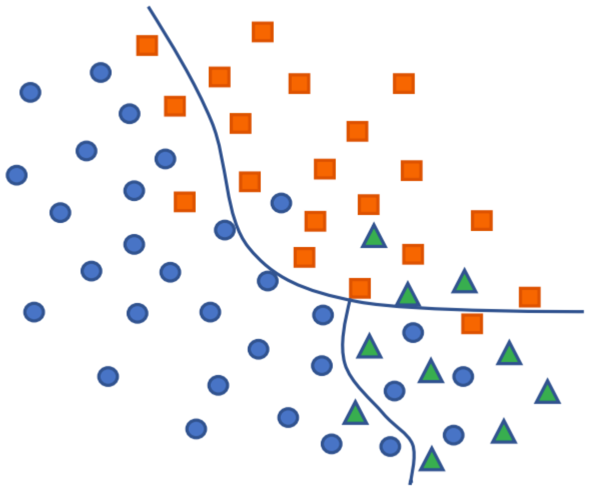


	$P_{\hat{1}}$	$P_{\hat{2}}$
P_1	16	4
P_2	4	26

Q5

Tabulate the confusion matrices for the following classification problems.

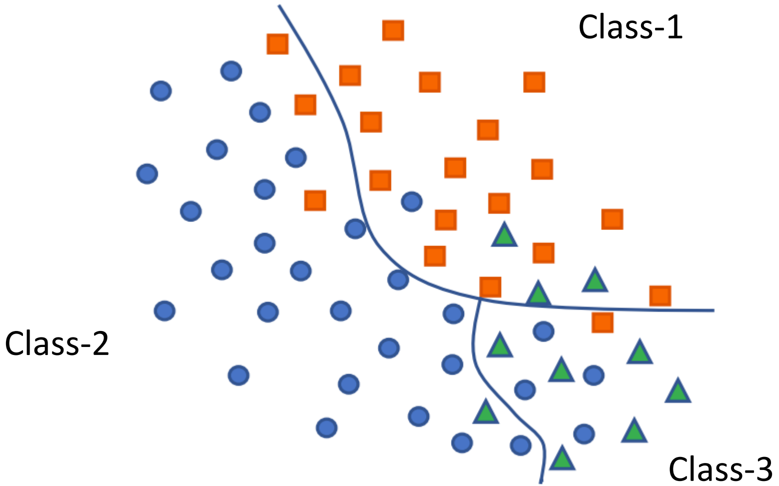
b) Three-category problem (the class-1, class-2 and class-3 data points are respectively indicated by squares, circles and triangles)



Q5

Tabulate the confusion matrices for the following classification problems.

b) Three-category problem (the class-1, class-2 and class-3 data points are respectively indicated by squares, circles and triangles)



	$\hat{P_1}$	$\hat{P_2}$	$\hat{P_3}$
P_1	16	3	1
P_2	1	25	4
P_3	3	1	6

Q6 (python)

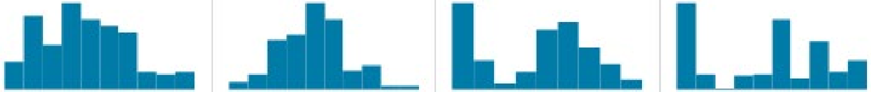
(5-fold Cross-Validation)

Get the data set “from sklearn.datasets import load_iris”. Perform a 5-fold Cross-validation to observe the best polynomial order (among orders 1 to 10 and without regularization) for validation prediction. Note that, you will have to partition the whole dataset for training/validation/test parts, where the size of validation set is the same as that of test. Provide a plot of the average 5-fold training and validation error rates over the polynomial orders. The randomly partitioned data sets of the 5-fold shall be maintained for reuse in evaluation of future algorithms

Q6

About this file

The dataset is a CSV file which contains a set of 150 records under 5 attributes - Petal Length, Petal Width, Sepal Length, Sepal width and Class(Species)

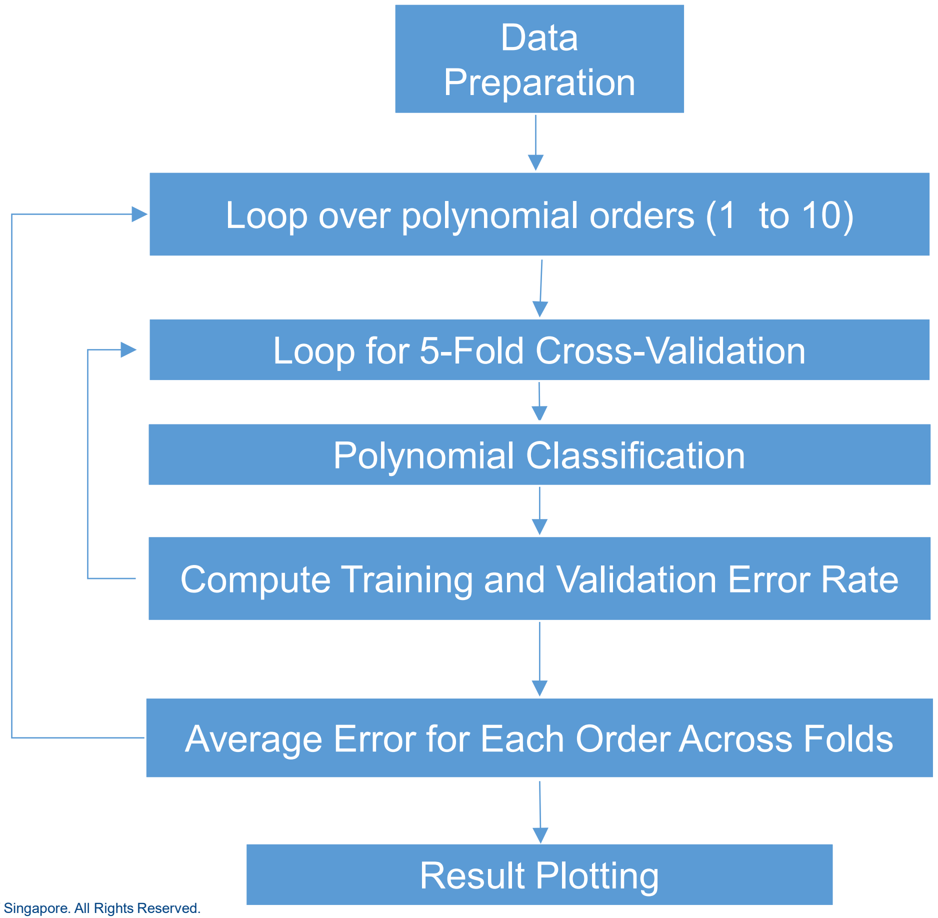


# sepal_length	# sepal_width	# petal_length	# petal_width	species
4.3 – 7.9	2 – 4.4	1 – 6.9	0.1 – 2.5	3 unique values
5.1	3.5	1.4	0.2	Iris-setosa
4.9	3	1.4	0.2	Iris-setosa
4.7	3.2	1.3	0.2	Iris-setosa
4.6	3.1	1.5	0.2	Iris-setosa
5	3.6	1.4	0.2	Iris-setosa
5.4	3.9	1.7	0.4	Iris-setosa
4.6	3.4	1.4	0.3	Iris-setosa
5	3.4	1.5	0.2	Iris-setosa
4.4	2.9	1.4	0.2	Iris-setosa
4.9	3.1	1.5	0.1	Iris-setosa
5.4	3.7	1.5	0.2	Iris-setosa
4.8	3.4	1.6	0.2	Iris-setosa
4.8	3	1.4	0.1	Iris-setosa

https://www.kaggle.com/datasets/arshid/iris-flower-dataset

Q6

Block diagram

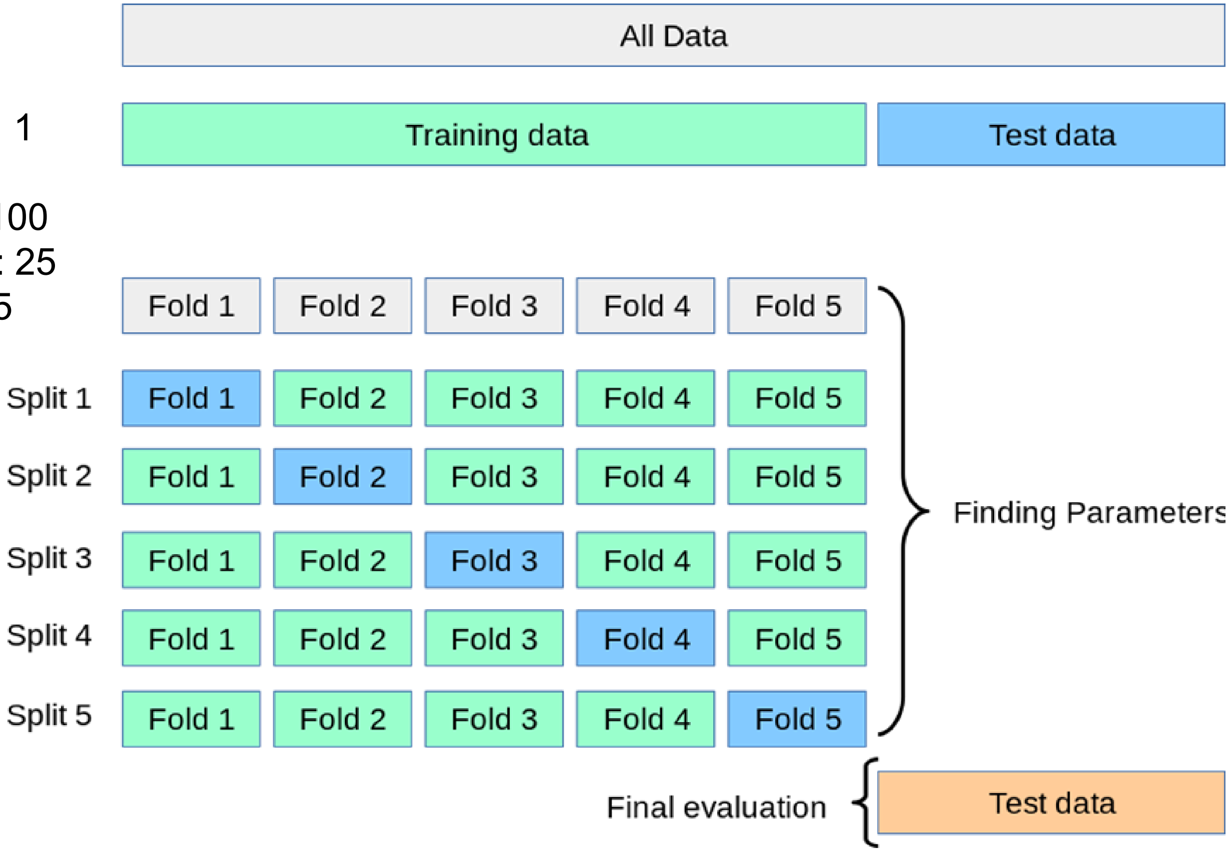


- One-Hot Encoding
- Data Splitting

- 5-Fold Splitting
- Feature Expansion
- Least-Squares Solution

Q6

- Train : Validation : Test = 4 : 1 : 1
- In total, we have 150 samples.
- Number of Training Samples: 100
- Number of Validation Samples: 25
- Number of Testing Samples: 25



https://scikit-learn.org/stable/modules/cross_validation.html

Q6

```python
##--- load data from scikit ---##
import numpy as np
import pandas as pd
print("pandas version: {}".format(pd.__version__))
import sklearn
print("scikit-learn version: {}".format(sklearn.__version__))
from sklearn.datasets import load_iris
iris_dataset = load_iris()
X = np.array(iris_dataset['data'])
y = np.array(iris_dataset['target'])
## one-hot encoding
Y = list()
for i in y:
    letter = [0, 0, 0]
    letter[i] = 1
    Y.append(letter)
Y = np.array(Y)
test_Idx = np.random.RandomState(seed=2).permutation(Y.shape[0])
X_test = X[test_Idx[:25]]
Y_test = Y[test_Idx[:25]]
X = X[test_Idx[25:]]
Y = Y[test_Idx[25:]]
```

- Loads the Iris dataset, a classic dataset with three classes of flowers (Setosa, Versicolor, and Virginica).
- X: A NumPy array of shape (150, 4), containing four features for each sample.
- y: A NumPy array of shape (150,), representing class labels (0, 1, or 2).

- One-Hot Encoding: The target variable, y, is converted to a one-hot encoding format (Y) for each class (e.g., [1,0,0] for class 0, [0,1,0] for class 1, etc.).

- Y is now a NumPy array of shape (150, 3)

- Uses a fixed seed (2) to create a reproducible random permutation of indices for splitting data.
- Selects the first 25 samples for X_test and Y_test, leaving the remaining 125 samples in X and Y for training and validation.

Q6

```
from sklearn.preprocessing import PolynomialFeatures
error_rate_train_array = []
error_rate_val_array = []
##--- Loop for Polynomial orders 1 to 10 ---##
for order in range(1,11):
    error_rate_train_array_fold = []
    error_rate_val_array_fold = []
    # Random permutation of data
    Idx = np.random.RandomState(seed=8).permutation(Y.shape[0])
    # Loop 5 times for 5-fold
    for k in range(0,5):
        ##--- Prepare training, validation, and test data for the 5-fold ---#
        # Prepare indexing for each fold
        X_val = X[Idx[k*25:(k+1)*25]]
        Y_val = Y[Idx[k*25:(k+1)*25]]
        Idxtrn = np.setdiff1d(Idx, Idx[k*25:(k+1)*25])
        X_train = X[Idxtrn]
        Y_train = Y[Idxtrn]
```

The code performs polynomial classification by expanding features to polynomial forms of varying degrees (1 to 10). For each polynomial order, it uses 5-fold cross-validation:

Creates a new random permutation of indices to assign data for 5-fold cross-validation.

Divides data into training and validation sets for each fold:

- X_val and Y_val: Select the next 25 samples for validation.
- X_train and Y_train: Exclude the validation indices, using the remaining samples for training.

Q6

```
##--- Polynomial Classification ---##
poly = PolynomialFeatures(order)
P = poly.fit_transform(X_train)
Pval = poly.fit_transform(X_val)
if P.shape[0] > P.shape[1]: # over-/under-determined cases
    reg_L = 0.00*np.identity(P.shape[1])
    inv_PTP = np.linalg.inv(P.transpose().dot(P)+reg_L)
    pinv_L = inv_PTP.dot(P.transpose())
    wp = pinv_L.dot(Y_train)
else:
    reg_R = 0.00*np.identity(P.shape[0])
    inv_PPT = np.linalg.inv(P.dot(P.transpose())+reg_R)
    pinv_R = P.transpose().dot(inv_PPT)
    wp = pinv_R.dot(Y_train)
```

$$\hat{\mathbf{w}} = (\mathbf{P}^T\mathbf{P} + \lambda\mathbf{I})^{-1}\mathbf{P}^T\mathbf{y}$$

$$\hat{\mathbf{w}} = \mathbf{P}^T(\mathbf{P}\mathbf{P}^T + \lambda\mathbf{I})^{-1}\mathbf{y}$$

- Creates polynomial features of the specified order for X_train and X_val.
- P and Pval are the transformed features for training and validation, respectively.

- Checks if the system is over- or under-determined (more rows than columns).
- Adds a small regularization term (reg_L or reg_R) for numerical stability in the pseudoinverse calculation.
- Least-Squares Solution: It calculates weights wp to fit the model by solving a system of equations based on whether the system is overdetermined or underdetermined.

Q6

- difference.any(axis=1) check if this row contain any non-zero elements
- np.where(difference.any(axis=1)) gives a tuple with one array — the row indices where the row contains any non-zero element.
- np.where(...)[0] extracts just the array of indices (from the tuple)

```
difference = np.array([[0, 0, 0],   # correct prediction
                       [0, 1, 0],   # incorrect prediction
                       [0, 0, 0],   # correct prediction
                       [1, 0, 0]])  # incorrect prediction
```

$$\hat{f}_{\mathbf{w}}(\mathbf{P}(\mathbf{X}_{new})) = \mathbf{P}_{new}\hat{\mathbf{w}}$$

```
##--- trained output ---##
y_est_p = P.dot(wp);
y_cls_p = [[1 if y == max(x) else 0 for y in x] for x in y_est_p ]
m1tr = np.matrix(Y_train)
m2tr = np.matrix(y_cls_p)
# training classification error count and rate computation
difference = np.abs(m1tr - m2tr)
error_train = np.where(difference.any(axis=1))[0]
error_rate_train = len(error_train)/len(difference)
error_rate_train_array_fold += [error_rate_train]
##--- validation output ---##
yval_est_p = Pval.dot(wp);
yval_cls_p = [[1 if y == max(x) else 0 for y in x] for x in yval_est_p ]
m1 = np.matrix(Y_val)
m2 = np.matrix(yval_cls_p)
# validation classification error count and rate computation
difference = np.abs(m1 - m2)
error_val = np.where(difference.any(axis=1))[0]
error_rate_val = len(error_val)/len(difference)
error_rate_val_array_fold += [error_rate_val]
# store results for each polynomial order
error_rate_train_array += [np.mean(error_rate_train_array_fold)]
error_rate_val_array += [np.mean(error_rate_val_array_fold)]
```

- y_est_p: Predicts continuous outputs by applying wp to the training data.
- y_cls_p: Converts y_est_p to a binary one-hot format for classification (1 for the maximum value, 0 elsewhere).
- m1tr and m2tr represent the true and predicted one-hot encoded labels as matrices for easy comparison.
- Computes the training error rate by comparing y_cls_p to Y_train, identifying misclassified samples in each fold.

- Applies the same classification process to the validation set.
- Appends the validation error rate for this fold to error_rate_val_array_fo

- Average training and validation error rates across the 5 folds for each polynomial order and store results.

Q6


Training and Validation Error Rates
Training
Validation
Error Rates
0.0
0.1
0.2
0.3
0.4
0.5
0.6
2
4
6
8
10
Order

THANK YOU