Import-Export Linkages as a Channel for Exchange Rate Hedging*

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Abstract

This paper shows that firms hedge exchange rate risk through their import–export linkages. Using Chinese customs micro data, I document that exchange rate shocks from import origins and export markets have opposite effects on export growth, and that these elasticities are markedly attenuated for two-way traders. Moreover, greater overlap between a firm's supplier and destination portfolios further dampens the shock transmission. I develop a tractable trade model with pricing-to-market exports and global sourcing of inputs. Multilateral currency covariance and firm network shares jointly shape exposure of price competitiveness and cost channels. For risk averse firms under mean–variance trade-off, exchange rate uncertainty creates risk-adjusted pricing premia and reallocates firms' import shares toward currencies that offset revenue exposure. I also provide counterfactual experiments to illustrate how natural hedging works in alternative trade structure and exchange rate environments.

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1 Introduction

A core question in international economics concerns the impacts of exchange rates on trade. Firms in global markets operate under persistent exchange rate volatility, which create uncertainty in both their revenue and costs. A few percentage points of exchange rate fluctuations can wipe out an exporter's profit margins. Therefore, trade participants need to consider how to hedge against exchange rate risks to protect themselves from their impact. Yet conventional trade models either assume complete financial hedging with foreign exchange derivatives or treat risk as an ex-post nuisance. In developing economies, regulation, market depth, and contracting frictions limit access to affordable derivatives.[1] At a time when the degree of corporate participation in international markets and the level of international financial uncertainty are both unprecedentedly high, a natural question is: can firms use alternative hedging strategy reducing the impact of exchange rates on firms' export performance? Does this situation renders non-financial hedging an appealing risk management tool?

This paper proposes that import–export linkages could work as an effective channel to hedge exchange rate risk even when financial hedging is incomplete. My interest in import-export linkages stems from two aspects. First, previous literature find that large exporters are often large importers (Amiti, Itskhoki and Konings, 2014), and that export and import will enhance each other (Li et al., 2024). In additions, large exporters export to more markets and import from more origins and large importers import from more origins and export to more markets.[2] Therefore, multilateral import-export relationships constitute one of the defining characteristics of firms engaged in trade today. Second, exchange rates have two opposing force on firms' export performance: the price effect and cost effect. For example, if there is a home currency depreciation, firms will benefit from a lower local currency price or higher domestic (producer) currency price. However, the depreciation makes imported inputs more expensive, thus firms have to bear higher costs. Given that multinational trade and multilateral exchange rate risks share a similar structure, and that exchange rates directly impact firms' import and export pricing, simple intuition suggests one should examine whether their interaction effectively hedges against risk.

To motivate the studies on exchange rate hedging, I document three empirical facts using Chinese customs micro data (2000–2015) regarding the relationship between import-export linkages and exchange rate sensitivity. First, two-way traders that both import and export in the same year exhibit significantly weaker bilateral export value responses to exchange rate fluctuations than those pure exporters. Among those two-way traders, the sensitivity of

[1] Background evidence on the limited availability and high cost of financial hedging for trade-exposed firms in China and other emerging markets is provided in Appendix A.1.

[2] Refers to Appendix Figure A2 for more details about the relationship between firm size and number of export markets and import origins.

export revenue to exchange rates is further weaker if the firm imports and exports from the same country (i.e. country-specific import-export linkage). Second, if a two-way trading firm have similar sets of import origins and export markets, measured by Jaccard index or phi coefficient, its export revenue will be less sensitive to exchange rates. This indicates that a similar network (or greater overlap) of import origins and export markets further weakens the risk exposure and enhance the resilience to exchange rate shocks. Third, firm-level effective exchange rates weighted by import and export value have opposite effects on firms' export. Export-weighted effective shocks raise exports while import-weighted effective shocks lower them. When import and export effective shocks enter jointly, the magnitude of both shocks are weaker for two-way traders compared with the average level. Together, these patterns point to a mechanical exposure offset embedded in trade networks and suggest that pricing and sourcing jointly shape firms' exchange rate exposure.

To explain these stylized facts, I construct a tractable trade model with pricing-to-market exports and global sourcing of intermediate inputs. Without loss of generality, we assume a firm's domestic price are subject to a constant markup without exchange rate risk while the export price is a mix of producer currency pricing and local currency pricing. Firms face nominal price wedges from exchange rate changes in both import and export side. Assuming other currency pairs remain unchanged, a depreciation of the domestic currency against the export market currency confers a local price advantage, thereby stimulating exports. Conversely, a depreciation against the currency of the import source raises input prices, leading to increased production costs. The opposite holds true for appreciation. Therefore, a bilateral exchange rate shock will have opposite impacts on export revenues and imported input costs. Static exchange rate elasticities show the partial effect of each bilateral exchange shock on firms' endogenous price, cost and revenue, which clarifies the two channels that link exchange rates to sales through a pricing-to-market channel and an import cost channel. Where firms have multiple import sources and export markets, multilateral exchange rate shocks produce more complicated effects.

In the simple risk-neutral setting where firms only optimize expected profit, any given import-export network could deliver a mechanical natural hedge: a firm's initial origin-market set generates exposure offset. In the linear linear system, currency shocks are mapped into changes in prices and trade shares with slopes pinned down by demand and substitution elasticities together with the invoicing currency mix. To quantify the risk, I summarize each firm's operating profits affected by exchange rate shocks with a linear profit exposure vector, which records how the firm's revenues and input costs co-move with shocks to each trading partner's currency. This object is constructed from the above bilateral revenue and cost sensitivities, so it encodes both the pricing-to-market channel and the imported-input cost channel. I then use the risk exposure to build firm-level measures of natural hedging. The exposure-offset share (EOS) captures how much of the revenue exposure is neutralized by cost exposure: EOS is high when revenues and costs respond in opposite directions to the same currency shocks, leaving little residual profit risk. The variance-reduction ratio (VRR) measures how much profit volatility is eliminated by that alignment, given the observed covariance structure of exchange rates. These objects play two roles in the paper. Empirically, they provide interpretable firm-level measures of hedging capacity linked to observable import–export structure. In the model, they are sufficient statistics for effective risk faced by firms.

I then extend the model framework to risk-averse firms by embedding a constant absolute risk aversion (CARA) mean–variance objective. I propose an endogenous natural hedging strategy with two-way trade linkages. The key hypothesis is that firms may align revenues and input costs across different currencies through strategically adjust their pricing and sourcing decisions. Exchange rate risk enters the linear system by tilting slopes through two endogenous margins: (i) risk-adjusted pricing premia that compensate currency exposure and (ii) risk-adjusted sourcing that reallocates imports toward currencies that offset revenue exposure. Risk-averse firms do not only passively accept the natural hedging benefits arising from trade structures but also proactively adjust trade structures by aligning import and export shares to reduce net profit exposure, even at the expense of sacrificing expected profits. We should expect that the stronger incentive of natural hedging have a increased exposure offset shares and a reduced profit variance when firms are more risk averse. The risk aversion parameter here should determine the risk premium embedded in prices and guide the reallocation of sourcing toward currencies that offset existing exposure.

The model treats risk neutrality and risk aversion as two points on the same object. When the risk-aversion parameter $\gamma = 0$, the firm maximizes expected profits only. In that case, natural hedging already emerges mechanically: imports and exports load with opposite signs on profits, so the firm's existing sourcing and pricing structure partially offsets currency risk. When $\gamma > 0$, the firm additionally values lower profit variance. This does not create a new mechanism but makes the same margins moving toward reducing exposure. In other words, risk aversion endogenizes the strength of natural hedging: it increases the weight the firm puts on aligning revenue and cost exposures, and it disciplines which currencies it buys from and sells to. Presenting the $\gamma = 0$ benchmark is useful because it isolates the purely mechanical component of exposure offset; letting $\gamma > 0$shows how firms actively amplify that offset when volatility itself enters the objective.

With the structural model, I perform a numerical analysis that uses observed exchange rate data and firm-level trade structure to study how the mechanism operates. I estimate the covariance matrix of bilateral exchange rate movements across major trading currencies, assign structural parameters (demand and sourcing elasticities, invoicing mix, and the risk aversion parameter γ), and simulate small exchange rate shocks. For each firm, I translate these shocks into profit exposure using the linear exposure vector, then compute hedging metrics such as the exposure-offset share (EOS), the variance-reduction ratio (VRR), and revenue-cost correlation.

This framework allows me to evaluate how sourcing diversity, overlap between import origins and export destinations, and pricing currency choices jointly determine residual risk. I also conduct sensitivity tests by varying γ and the producer currency pricing share λ to trace how stronger risk aversion or different invoicing structures tighten the alignment between revenue and cost exposures and reduce normalized profit variance.

I conduct two groups of counterfactual experiments. The first counterfactual perturbs the trade structure by reallocating import and export intensities and shares to measure how network geometry alone changes the hedging effectiveness. The second counterfactual perturbs the exchange rate environment itself by modifying the volatility and correlation structure of currencies. Specifically, I have assumed scenarios where the Chinese RMB is pegged to the US dollar or a basket of currencies, where fluctuations in the RMB exchange rate against global currencies or specific currencies are amplified, or where individual currencies experience low-probability but substantial magnitude jumps in exchange rates (i.e. large devaluation or revaluation). Comparing these scenarios shows how much of natural hedging could be altered by firms' own trade network versus features of the global currency system, and quantifies how policy-relevant environments such as different invoicing regimes or alternative currency blocks affect the stability of exporters' profits.

This paper makes three contributions. First, I treats exchange rate risk through a network–covariance lens and shows that "natural hedging" is a systematic and quantifiable mechanism: firms' import–export networks generate offsetting revenue and cost exposures to currency shocks, and this can be measured in the data using effective exchange rates, network overlap, and firm-level exposure vectors. Second, it provides a unified modeling framework in which that same mechanism is both mechanical (under risk neutrality) and strategic (under risk aversion): the risk-neutral case explains why two-way traders already look insulated, and the risk-averse case shows how firms actively tilt prices and sourcing to amplify that insulation. Third, it delivers a way to evaluate policy and counterfactual environments—pricing regimes, supplier–destination overlap, and the global exchange rate covariance structure—in terms of profit stability, not just average trade flows.

Literature Review. A growing trade literature about import-export linkages shows that exporters' responses to global shocks including exchange rate shocks depend critically on imported input costs. Amiti, Itskhoki and Konings (2014) document that large exporters are simultaneously large importers. Two-way traders' reliance on imported inputs raises marginal costs when the home currency depreciates, dampening export pass-through. Quantitative works by Gopinath and Neiman (2014), Halpern, Koren and Szeidl (2015) and Lu, Mariscal and Mejía (2024) confirm that imported inputs shape revenue responses during shocks. Recently, Blaum (2024) show that after large devaluations import-intensive exporters will expand and the aggregate import share will rise. These studies, however, treat firms as risk-neutral and focus on the expected profit. My paper contributes to this literature by embedding the bilateral exchange rate structure into trade networks. Since exports and imports are both exposed to currency risk, import-export complementarities arise from risk reduction motives in addition to productivity and fixed costs. Incorporating risk aversion provides a new comparative static analysis for examining the value of import-export linkages to firms.

Early research paper (Allayannis and Ofek, 2001) about hedging of exchange rate risk argues that operational hedging is not an effective substitutes but a complement for financial hedging. Later empirical papers such as Raddatz (2011) and Fauceglia, Shingal and Wermelinger (2014) show that firms facing volatile exchange rates shift toward activities where revenues and costs move in opposite directions, implying firms' incentive for risk hedging in operation. Alfaro, Calani and Varela (2021) argued that a firm would use operational hedging strategy in which it matches its cash flows in and out within the same maturity and currency pair to reduce risk. They also found that exporters in emerging markets with high foreign exchange volatility increase their import intensity.[3] Yet most quantitative trade models assume complete financial hedging or treat risk as ex-post consequences while have not explicitly fit risk hedging into the import–export decisions. My paper decomposes multilateral exchange rate risk into bilateral trade relationships, then aggregating them at the firm level using exchange rate covariances. Our model add a mean–variance component to the firm problem and letting import sourcing and export pricing jointly determine the covariance between profits and exchange rates. My model shows how pricing-to-market strategy and trade network works together to reduce profit variance given exchange rate covariance, which is new in the literature.

The heterogeneous responses of exporters to exchange rate shocks have long been one of the central puzzles in international economics. The "dominant currency" framework of Gopinath, Itskhoki and Rigobon (2010) highlights how partial local currency pricing explains low aggregate pass-through. Berman, Martin and Mayer (2012) show that more productive exporters raise markups during depreciation while smaller firms cut foreign prices. Amiti, Itskhoki and Konings (2019) argues that imported input costs and competitor prices pass global shocks into domestic producer prices, with stronger effects for firms exposed more to imports. In Amiti, Itskhoki and Konings (2022), firms internalize these linkages through currency invoicing: higher import intensity and peer use of dominant currencies push exporters to price in those currencies, aligning cost and revenue currencies and reshaping short-run price and quantity responses.[4] My model nests a continuum of pricing regimes by allowing a PCP share λ and derives analytical revenue elasticities that decompose into a direct export pricing channel and an indirect import cost channel. Because risk-averse firms value lower variance, the market-specific prices become functions of trade network and exchange rate covariance, linking

[3] Juvenal and Monteiro (2025) find that under LCP, an increase in the local currency risk premium elevates the relative value of current cash flows, which raise markups and reduce exchange rate pass-through.

[4] More about exchange rate pass-through and invoice currency, refer to Gopinath and Rigobon (2008), Goldberg and Tille (2008), Devereux, Dong and Tomlin (2017), Gopinath et al. (2020).

pricing-to-marketing strategy to natural hedging in a novel way.

The remainder of the paper is organized as follows. Section 2 contains the empirical evidence for motivations while Sections 3 and 4 develop the risk-neutral and risk-averse models to explain the mechanism of natural hedging, respectively. Section 5 and introduce numerical analysis and counterfactual experiments. Section 6 concludes.

2 Empirical Motivations

To motivate our structural analysis, I document several stylized facts concerning export values in response to exchange rate fluctuations. Using Chinese customs micro-level data, I find that which that import and export at the same time and from a similar set of countries are significantly affected less by exchange rate changes. Import-export linkages should have reduced these firms' risk exposure to exchange rate uncertainty. These facts regarding heterogeneous exporters requires a trade model with both nominal price wedge caused by exchange rates and global sourcing.

2.1 Data and Measurements

Our empirical facts combine granular trade data and international macro statistics. The primary data source is the Chinese Customs Trade Statistics (CCTS) from the General Administration of Customs of China. This dataset includes the most comprehensive information on all Chinese trade transactions, including each firm's import or export value, quantity, unit, product name, and code, and source or destination country.

For macro variables, I also use International Financial Statistics (IFS) from IMF, for exchange rates, inflation, and other financial indicators. It includes 194 countries and regions worldwide since 1948. Besides, I also use Penn World Table (PWT) with relative income levels, output, input, productivity and etc. It covers 183 countries during 1950-2019.

The bilateral nominal exchange rate e_{ik} is defined as the nominal exchange rate measured as units of currency in i for one unit of currency in k. The bilateral real exchange rate (RER) with respect to home country i is:

$$RER_{kt} = e_{kt} \times \frac{CPI_{kt}}{CPI_{it}}$$

our sample period is from 2000 to 2015 at the annual frequency. Below Figure 1 plots monthly bilateral nominal exchange rates against the Chinese yuan (CNY) for the four major trading currency of China: USD, EUR, GBP, and JPY over 2000–2015.

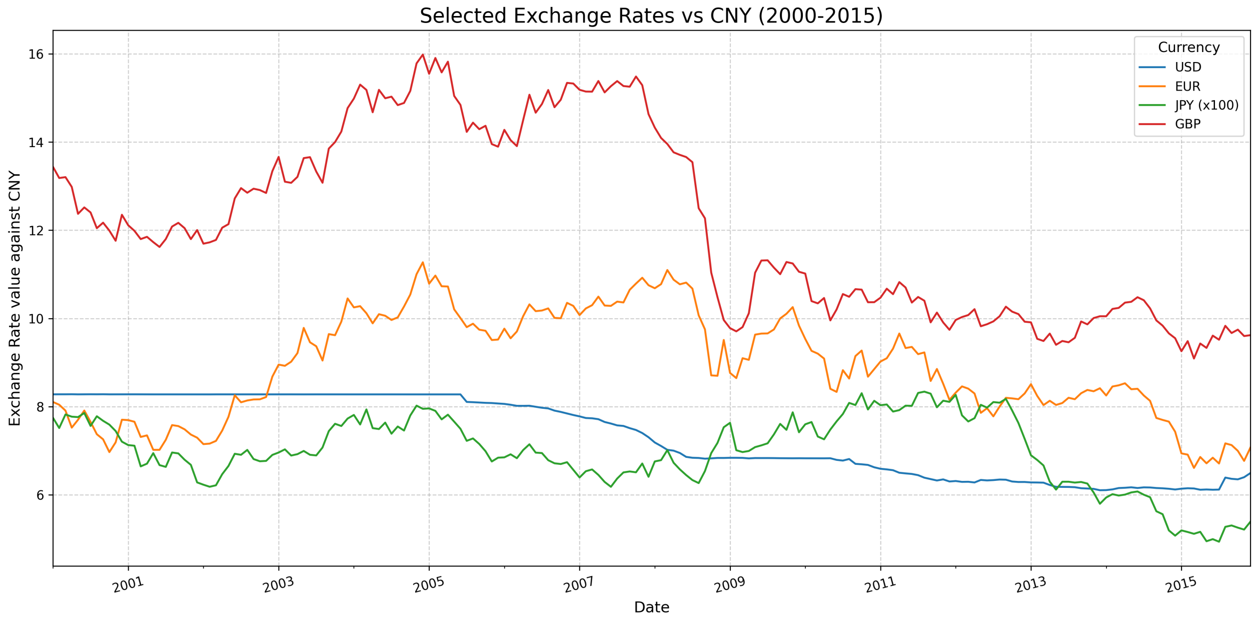


Figure 1: Nominal exchange rate series of USD, Euro, JPY and GBP against CNY

Note: Monthly bilateral exchange rates against the Chinese yuan (CNY), 2000–2015. Higher values indicate a stronger foreign currency (CNY depreciation). The JPY series is scaled by 100 for readability.

The most important dominant currency, USD, is nearly flat around 8.3 before 2005, reflecting the effective nominal peg, and then trends downward as the RMB appreciates to roughly 6.1 by 2014–15. Euro and British pound both rise through the early-to-mid 2000s, fall sharply around the 2008–09 global financial crisis, and subsequently fluctuate with a gradual downward drift. Japaneses yen declines in the early 2000s, appreciates markedly during 2008–11 which is a typical safe-haven pattern, and depreciates again after 2012. The synchronized turning points around 2008 and 2009 highlight the common global shock, while the smoother USD series reflects China's exchange rate regime shift from a dollar peg to a managed floating regime. I use these series to summarize the external price environment and to construct effective exchange rate and risk measures in subsequent analysis.

In practice, I focus on the top 30 export markets and top 30 import origins of China, in total 36 countries with 20 foreign currencies in the sample. The currencies are USD, EUR, JPY, KRW, AUD, SGD, CAD, CHF, GBP, RUB, THB, MYR, BRL, INR, PKR, VND, IDR, PHP, MXN, ZAR. These currencies are either the most liquid in the global market, or those used by Chinese major emerging trading partners. Appendix Figure A1 presents pie charts showing the distribution of China's export and import values by country or region from 2000 to 2015.

After merging the custom data with exchange rate series of 30 countries and combining the import and export records, we obtain an unbalanced panel with 9,701,943 firm-country-year observations with either direction of trade and 3,127,610 with two-way trade. It covers 28,000 firms in 2000 to 186,000 firms in 2015.

2.2 Two-way Linkages and Export Value

The intuition of natural hedging for two-way traders is that exchange rates will change a firm's export competitiveness and its cost advantages in different directions. For example, a depreciating domestic currency should benefit exporters, but higher import costs could lead to lower profit margins, undermining the firm's competitiveness and, consequently, reducing its export sales. My hypothesis is that export responses to bilateral exchange rates depend on both whether the firm is a two-way trader and the destination country is also its import origin.

To show the existence of natural hedging, I regress bilateral export value changes on real exchange rate changes and the interaction term with the firm-level two-way trader indicator which takes 1 if the firm simultaneously imports and exports in the same year. Among two-way traders, I further use import intensity (import value over the sum of imports and exports) to measure the dependence on imported inputs. I should expect two-way traders, especially those with a higher import intensity, can hedge the exchange rate risk more efficiently and therefore have a less volatile export value compared to pure exporters.

$$\Delta V_{fkt}^{X} = \beta_1 \Delta RER_{kt} + \beta_2 \Delta RER_{kt} \times \tilde{s}_{f(k)t-1}^{M} + \gamma Z_{ft-1} + \eta G_{kt} + \delta_f + \epsilon_t \tag{1}$$

where $\tilde{s}_{ft-1}^{M}$ includes different measures firm-level and firm-country-level import status. Specifically, I use $TW_{ft-1} = \mathbf{1}\{V_{ft-1}^{X} > 0, V_{ft-1}^{M} > 0\}$ (both imports and exports), and $TW_{fkt-1} = \mathbf{1}\{V_{fkt-1}^{X} > 0, V_{fkt-1}^{M} > 0\}$ (both imports from and exports to country k), and $s_{f(k)t-1}^{M}$ is the firm-level or firm-country-level import intensity in the previous period. In the customs data, I measure import intensity by the ratio of a firm's import value over the sum of its import value and export value, which is naturally standardized in the range between 0 and 1.[5]

[5]Unless otherwise specified, all export amounts below are denominated in the domestic currency (RMB).

Table 1: Firm-level Two-way Linkages and Export Value Elasticity

	(1)	(2)	(3)	(4)	(5)	(6)
Sample		All Exporters			Two-way Traders	
Dependent Var			ΔlnV_{fkt}^{X}			
$\Delta lnRER_{kt}$	0.309***	0.346***	0.502***	0.651***	0.511***	0.623***
	(0.078)	(0.084)	(0.075)	(0.077)	(0.092)	(0.118)
$\Delta lnRER_{kt} \times TW_{ft-1}$			-0.533***	-0.747***		
			(0.117)	(0.121)		
$\Delta lnRER_{kt} \times s_{ft-1}^{M}$					-0.963***	-1.318***
					(0.303)	(0.367)
$\Delta lnRGDP_{kt}$	1.386***	2.102***	1.358***	2.076***	1.440***	2.148***
	(0.212)	(0.271)	(0.210)	(0.266)	(0.246)	(0.312)
Lag Firm Controls	Yes	Yes	Yes	Yes	Yes	Yes
Gravity Controls	Yes	No	Yes	No	Yes	No
Firm-country FE	No	Yes	No	Yes	No	Yes
Firm FE	Yes	No	Yes	No	Yes	No
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5019959	4375637	5019959	4375637	1492001	1305681
R^2	0.117	0.196	0.117	0.196	0.102	0.187

Notes: The dependent variables are firm-country-level export value changes. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

This table estimates the elasticity of firm–country export value growth to bilateral real-exchange rate changes. The interaction terms show that firms that were two-way traders last period and those with higher lagged import intensity exhibit significantly lower export sensitivity to real exchange rate changes. The negative and highly significant coefficients on interaction terms indicate a natural hedging effect from input sourcing, while coefficients on real GDP remains strongly positive across specifications. Fixed effects and control sets are varied across columns as labeled.

Table 2: Market-specific Two-way Linkages and Export Value Elasticity

	(1)	(2)	(3)	(4)
Sample	Two-way Traders			
Dependent Var	ΔlnV^X_{fkt}			
$\Delta lnRER_{kt}$	0.450***	0.502***	0.938***	1.229***
	(0.079)	(0.084)	(0.109)	(0.162)
$\Delta lnRER_{kt} \times TW_{fkt-1}$	-0.472***	-0.512***		
	(0.106)	(0.127)		
$\Delta lnRER_{kt} \times s^M_{fkt-1}$			-4.440***	-5.700***
			(0.574)	(0.802)
$\Delta lnRGDP_{kt}$	1.423***	2.102***	1.322***	1.882***
	(0.243)	(0.310)	(0.248)	(0.319)
Lag Firm Controls	Yes	Yes	Yes	Yes
Gravity Controls	Yes	No	Yes	No
Firm FE	Yes	No	Yes	No
Firm-country FE	No	Yes	No	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	1492001	1305681	1492001	1305681
R^2	0.102	0.187	0.105	0.190

Notes: The dependent variables are firm-country-level export value changes. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

Focusing on two-way traders, this table replaces firm-level measures with partner-specific ones. The interactions terms are negative and significant—larger in magnitude than the firm-level results—showing that importing from country k specifically mutes the pass-through of RER movements to exports to the same country. Together, the evidence supports that tighter import–export linkages attenuate exchange rate exposure of export values.

Motivating fact 1: Firms with two-way import–export linkages at both the firm level and the firm–country level exhibit significantly lower export value elasticity to bilateral exchange rate shocks with stronger attenuation for market-specific linkages.

2.3 Firm-level Network Similarity

A motivating fact from Chinese firms is that large exporters are not only large importers (Amiti, Itskhoki and Konings, 2014), but also more diverse importers. I can see a significantly positive correlation between export size and the number of import origins as well as the number of export markets in Appendix Figure A2. Larger traders are expanding their presence in more countries for both imports and exports, while top exporters are also highly diversified importers (and vice versa). This indicates that large two-way traders will have a greater variety of import-export combinations, which provide them with more hedging options.

To quantify the degree of similarity between a firm's import source countries and export destination countries, I employ two complementary measures: the Jaccard similarity coefficient and the Phi coefficient. The Jaccard similarity coefficient is defined as the ratio of the number of countries with which a firm both imports from and exports to over the total number of countries with which the firm trades (i.e., the union of its import and export partners). For a given firm-year, the Jaccard coefficient is defined as:

$$Jaccard_{ft} = \frac{|\mathcal{J}_{ft} \cap \mathcal{K}_{ft}|}{|\mathcal{J}_{ft} \cup \mathcal{K}_{ft}|}$$

where $\mathcal{J}_{ft}$ and $\mathcal{K}_{ft}$ are the sets of firm f's import origins and export markets, respectively.

This indicator reflects the proportion of trading partners involved in both imports and exports, reflecting the direct overlap in a firm's trade network. A higher Jaccard coefficient indicates that a firm's import sources and export destinations are more similar, suggesting a greater potential for natural hedging of exchange rate risk through bilateral trade relationships.

The Phi coefficient ϕ (also known as the Matthews correlation coefficient) provides a measure of similarity for the firm's import-export network. It is calculated from the joint and marginal distributions of binary indicators for import and export relationships with each country:

$$\phi_{ft} = \frac{N_{11}N_{00} - N_{10}N_{01}}{\sqrt{N_{1.}N_{0.}N_{.1}N_{.0}}}$$

where N_{11} is the number of countries with both import and export, N_{00} is the number with neither, and N_{10} and N_{01} denote unilateral links. The phi coefficient ranges from –1 (perfect negative correlation) to 1 (perfect positive correlation), with 0 indicating no association. In this context, it reflects the overall similarity in the firm's patterns of import and export relationships across all potential partner countries.

Table 3: Firm-level Network Similarity and Export Value Elasticity

	(1)	(2)	(3)	(4)
Sample	Two-way Traders			
Dependent Var	ΔlnV_{fkt}^{X}			
$\Delta lnRER_{kt}$	0.420***	0.484***	0.541***	0.651***
	(0.125)	(0.125)	(0.161)	(0.163)
$\Delta lnRER_{kt} \times Jaccard_{ft-1}$	-0.536*	-0.673**		
	(0.308)	(0.315)		
$\Delta lnRER_{kt} \times \phi_{ft-1}$			-0.594**	-0.777**
			(0.298)	(0.315)
$\Delta lnRGDP_{kt}$	1.435***	2.138***	1.435***	2.138***
	(0.248)	(0.315)	(0.248)	(0.315)
Lag Firm Controls	Yes	Yes	Yes	Yes
Gravity Controls	Yes	No	Yes	No
Firm FE	Yes	No	Yes	No
Firm-country FE	No	Yes	No	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	1492001	1305681	1492001	1305681
R^2	0.102	0.187	0.102	0.187

Notes: The dependent variables are firm-country-level export value changes. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

This table tests whether similarity between a firm's import and export country networks dampens the exchange rate elasticity of its bilateral exports. Among two-way traders, the interactions with the lagged Jaccard coefficient and the lagged Phi/Matthews correlation are negative and statistically significant, implying that firms with more aligned supplier–destination portfolios are less sensitive to bilateral real-exchange rate shocks. The effect is robust across control sets and fixed effects.

Using both indices provides a comprehensive assessment of both the overlap of trading partners and the full pattern of bilateral trade presence and absence. Together, these measures offer a nuanced view of the extent to which a firm's imports and exports are geographically aligned, which is central to understanding natural hedging and the structure of two-way trade.

Motivating fact 2: Conditional on two-way trading, firms whose import and export portfolios span more similar sets of countries exhibit significantly lower export-value elasticity to bilateral real-exchange rate shocks.

2.4 Effective Exchange Rates and Exposure Offset

To study more about the impact of trade structure on risk exposure, I denote that a representative firm exports to k with shares $\vec{\chi}^X_{n\times 1} = [\chi^X_1, \chi^X_2, \cdots, \chi^X_n]'$ and imports from j with shares $\vec{\chi}^M_{n\times 1} = [\chi^M_1, \chi^M_2, \cdots, \chi^M_n]'$. The firm-level export and import shares can be calculated using export (X_{fkt-1}) and import (M_{fjt-1}) values by firm-country pair in the customs data:

$$\chi^X_{fkt} = \frac{V^X_{fkt}}{\sum_{k'\in\mathcal{K}} V^X_{fk't}}, \quad \chi^M_{fjt} = \frac{V^M_{fjt}}{\sum_{j'\in\mathcal{J}} V^M_{fj't}} \tag{2}$$

Then I can construct the firm effective exchange rates as the value-weighted average of bilateral exchange rate shocks of trading partners following Dai and Xu (2017) using the above shares. The effective exchange rates can be understood intuitively as average exchange rate changes of home currency relative to all export markets or import origins, weighted by trade value.[6]

$$\begin{aligned} \Delta FEER^X_{ft} &= \sum_i \chi^X_{fkt} \Delta RER_{kt} \\ \Delta FEER^M_{ft} &= \sum_j \chi^X_{fjt} \Delta RER_{jt} \end{aligned} \tag{3}$$

We specify a firm-level counterpart of Equation 1 with firm-level export value changes and firm-effective exchange rates as follows:

$$\Delta V^X_{ft} = \beta_1 \Delta FEER^X_{ft} + \beta_2 \Delta FEER^M_{ft} + \gamma Z_{ft-1} + \delta_f + \epsilon_t \tag{4}$$

[6]We can also construct FEER using the leave-one-out method (excluding the exchange rate of the country itself) to eliminate self-feedback from individual countries, thereby making the results more robust.

Table 4: Firm Effective Exchange Rates and Export Value Elasticity

	(1)	(2)	(3)	(4)	(5)	(6)
Sample		All Exporters			Two-way Traders	
Dependent Var			ΔlnV^X_{ft}			
$\Delta lnFEER^X_{ft}$	0.165***		0.421***	0.036		0.108*
	(0.029)		(0.029)	(0.057)		(0.063)
$\Delta lnFEER^M_{ft}$		-1.130***	-1.256***		-0.100**	-0.144***
		(0.035)	(0.036)		(0.047)	(0.052)
Lag Firm Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1285518	1285518	1285518	344163	344163	344163

Notes: The dependent variables are firm-level export value changes. Column (1)-(3) include all exporters, column (4)-(6) include two-way traders who both export and import. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively. Standard errors are clustered at the firm level and country-year level.

This table relates firm-level export value to two portfolio-weighted exchange rate shocks: an export-weighted FEER and an import-weighted FEER. For all exporters in columns (1)–(3), export FEER enters positively while import FEER enters negatively; when included jointly, both signs remain, consistent with a demand channel (destinations' currencies) boosting exports and a cost channel (suppliers' currencies) dampening them. Among two-way traders in columns (4)–(6), both elasticities are attenuated in magnitude—the export-weighted coefficient becomes small (and only marginally significant), and the import-weighted coefficient is markedly smaller than in the full sample—indicating reduced net exchange rate exposure once firms' import and export portfolios are accounted for. All specifications include lagged firm controls, firm and year fixed effects and standard errors are clustered at the firm level and country-year level.

Motivating fact 3: Conditional on two-way trading, value-weighted exchange rate shocks have opposite-signed elasticities: export-weighted FEER shocks are positively related to firm export value, while import-weighted FEER shocks are negatively related. When both FEERs entered jointly for two-way traders, the directions persist but magnitudes are attenuated.

3 Model Framework

This section develops a theoretical framework with global sourcing of inputs and pricing-to-market exports based on Gopinath and Neiman (2014), Halpern, Koren and Szeidl (2015), and Blaum (2024). This model contributes to the literature by adding new country-specific price wedges created by exchange rates. Different combinations of import origins and export markets will lead to heterogeneity in firms' exchange rate risk exposure. I will start with the stylized model with risk neutrality to describe two-way traders' decisions. Then, I will describe how firms are exposed to exchange rate risks. This part will illustrate how existing import-export linkages mechanically help mitigate the effects from price wedges created by exchange rate shocks even without proactive adjustments.

3.1 Preference and Technology

Let's consider a world consisting of N countries, in which I use i to denote the home country, j to denote the import origin, and k to denote the export market. For exchange rate and other shocks, I use l to indicate any country that does not explicitly point to exports or imports.

The representative consumer in the home country maximizes its utility by consuming a continuum of manufacturing final goods ω in the CES objective:

$$U_i = \left(\int_{\omega \in \Omega_i} q_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \sigma > 1 \tag{5}$$

Each manufacturing firm in home country i produces a unique variety ω using labor L and intermediate inputs X with the idiosyncratic technology φ:

$$y_i(\omega) = \varphi(\omega) L_i^{1-\alpha} X_i^{\alpha} \tag{6}$$

The intermediate input bundle nest a two-layer CES structure between domestic and foreign imported inputs:

$$X_i = \left[X_{D,i}^{\frac{\epsilon-1}{\epsilon}} + X_{M,i}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1 \tag{7}$$

and among foreign imported inputs from different origins:

$$X_{M,i} = \left(\sum_{j \in \mathcal{J}} X_{ij}^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}}, \kappa > 1 \tag{8}$$

The unit cost of production under constant return to scale is:

$$c_i(\omega) = \varphi(\omega)^{-1} \alpha^{-\alpha} (1-\alpha)^{\alpha-1} w_i^{1-\alpha} P_{X,i}(\omega)^{\alpha} \tag{9}$$

3.2 Firm's Exporting Decision

A firm with productivity φ decides whether to sell its final goods ω in domestic and foreign markets. In domestic market i and foreign market k, the demand for each variety ω is subject to Dixit–Stiglitz monopolistic competition:

$$q_{ii}(\omega) = p_{ii}(\omega)^{-\sigma} P_i^{\sigma-1} S_i, \quad q_{ki}(\omega) = p_{ki}(\omega)^{-\sigma} P_i^{\sigma-1} S_k \tag{10}$$

The firm's domestic price is subject to a constant markup $p_{ii}(\omega) = \frac{\sigma}{\sigma-1} c_i$. The domestic sales income is $R_{ii}(\omega) = p_{ii}(\omega)^{1-\sigma} P_i^{\sigma-1} S_i$.

In export markets, under the pricing-to-market (PTM) assumption, each firm could set a different markup for each market k, $p_{ki}(\omega) = \mu_{ki} c_i$. Following Burstein and Gopinath (2014), I assume the total export value to each foreign market k consists of two components: PCP (producer currency pricing) component λ and LCP (local currency pricing) component $1 - \lambda$.

The total export revenue in market k with a combination of PCP and LCP is

$$R_{ki} = \lambda R_{ki}^{PCP} + (1-\lambda) R_{ki}^{LCP} = \lambda e_{ik}^{\sigma} B_{ki}^{PCP} + (1-\lambda) e_{ik} B_{ki}^{LCP} \tag{11}$$

where $B_{ki}^{PCP} \equiv (\frac{\bar{P}_{ki}^i}{P_k})^{1-\sigma} \frac{S_k}{P_k}$ and $B_{ki}^{LCP} \equiv (\frac{\bar{P}_{ki}^k}{P_k})^{1-\sigma} \frac{S_k}{P_k}$. Details derivation of currency pricing paradigm is in Appendix B.2.

The export intensity is firm's optimal share of product selling to foreign markets:

$$s_i^X \equiv \frac{R_i^X}{Y_i} = \frac{\sum_k \left[\lambda e_{ik}^{\sigma} B_{ki}^{PCP} + (1-\lambda) e_{ik} B_{ki}^{LCP}\right]}{Y_i} \tag{12}$$

For a multi-market exporter, the export value share to country k over its tootal export is:

$$\chi_{ki}^X \equiv \frac{R_{ki}}{R_i^X} = \frac{\lambda e_{ik}^{\sigma} B_{ki}^{PCP} + (1-\lambda) e_{ik} B_{ki}^{LCP}}{\sum_j [\lambda e_{ij}^{\sigma} B_{ij}^{PCP} + (1-\lambda) e_{ij} B_{ij}^{LCP}]} \tag{13}$$

3.3 Firm's Sourcing Decision

The firm's intermediate inputs consist of domestic inputs and foreign inputs: $P_{X,i} X_i = P_{D,i} X_{D,i} + P_{M,i} X_{M,i}$. Given constant return to scale, the intermediate input price index P_X is weighted by domestic input price and imported input prices:

$$P_{X,i} = \left(P_{D,i}{}^{1-\varepsilon} + P_{M,i}{}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}, \quad P_{M,i} = \left(\sum_{j \in \mathcal{J}} p_{ij}^{1-\kappa} dj\right)^{\frac{1}{1-\kappa}} \tag{14}$$

For a multi-origin importer, the import value share from country j over total imported input is:

$$\chi_{ij}^M = \frac{p_{ij} \left(\frac{p_{ij}}{P_{M,i}}\right)^{-\kappa}}{\sum_{j' \in \mathcal{J}} p_{ij'} \left(\frac{p_{ij'}}{P_{M,i}}\right)^{-\kappa}} = \frac{(p_{ij}^* e_{ij})^{1-\kappa}}{\sum_{j' \in \mathcal{J}} (p_{ij}^* e_{ij})^{1-\kappa}} \tag{15}$$

where the perceived price $p_{ij} = p^*_{ij}e_{ij}$, which is determined by foreign producer and bilateral nominal exchange rate. We assume foreign suppliers are subject to perfect competition and sell inputs at its marginal cost. The producer currency intermediate input price imported from origin j is $p^*_{ij} = \tau_{ij}w_j a_j$, where w_j and a_j denotes the wage and productivity level of the origin country. I include country-specific iceberg cost denoted by τ_{ij}.

The firm's overall input price $P_X = P_D\left[1 + A_i^{\epsilon-1}\right]^{\frac{1}{1-\epsilon}}$ where $A_i = P_{D,i}/P_{M,i}$ is the cost advantage of foreign inputs. The optimal share of input expenditure on imported inputs (i.e. import intensity) is:

$$s_i^M = \frac{A_i^{\epsilon-1}}{1 + A_i^{\epsilon-1}}, \quad 0 < s_i^M < 1 \tag{16}$$

The unit cost can be expressed as: $c_i(\omega) = c_i^D(1 - s_i^M(\omega))^{\frac{\alpha}{\epsilon-1}}$ where $c_i^D = \varphi_i^{-1}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\left(\frac{p_i}{\alpha}\right)^{\alpha}$ is the non-importer unit cost.

3.4 Static Partial Equilibrium

Firms in country i both serve domestic and foreign markets $\mathcal{J}(\omega)$ and $\mathcal{K}(\omega)$. The firm ω's profit is given as:

$$\pi_i(\omega) = p_{ii}(\omega)q_{ii}(\omega) + \sum_{\mathcal{K}} \frac{p_{ki}(\omega)q_{ki}(\omega)}{e_{ik}} - w_i L - P_{D,i}X_{D,i} - P_{M,i}X_{M,i} - F(\mathcal{J},\mathcal{K}) \tag{17}$$

s.t. $q_{ii}(\omega) + \tau_{ki}q_{ki}(\omega) = Y_i(\omega) = \varphi(\omega)L_i^{1-\alpha}X_i^{\alpha}$.

The firm's operating profit $\pi_i(\omega)$ equals revenue minus production and sourcing costs. The first two terms capture total sales: domestic sales $p_{ii}(\omega)q_{ii}(\omega)$ and exports $\sum_{\mathcal{K}} \frac{p_{ki}(\omega)q_{ki}(\omega)}{e_{ik}}$, where export revenues are converted back into the firm's home currency using the bilateral exchange rate e_{ik}. From this we subtract labor costs $w_i L_i$, the cost of domestic inputs $P_{D,i}X_{D,i}$, imported inputs $P_{M,i}X_{M,i}$, and the fixed cost $F(\mathcal{J},\mathcal{K})$ of maintaining the set of suppliers $\mathcal{J}$ and export markets $\mathcal{K}$. Output is constrained by technology: total sales must equal output $Y_i(\omega)$, which is produced using labor L_i and a composite input X_i with productivity $\varphi(\omega)$, and exporters must meet iceberg trade costs τ_{ki} when selling abroad.

In a partial equilibrium, given exogenous firm productivity φ, foreign price index P_k, domestic and foreign demand S_i, S_k; and the predetermined set of import origins and export markets $\mathcal{J},\mathcal{K} \subseteq \mathcal{N}$, a firm makes the optimal **two-sided intensive margin decision**. Specifically, firms maximize the expected profit by choosing an intermediate input bundle X_i, labor L, local price in domestic and foreign markets p_{ii} and p_{ki} given import origins and export markets.

By solving the partial equilibrium, I can solve the local adjustment of optimal intensive margin decision θ_i including import and export intensity s_i^M and s_i^X, and import and export shares of each country χ_{ij}^M and χ_{ki}^X, given the initial steady state. Those intensive trade margins in equilibrium sufficiently summarize relevant firm heterogeneity regarding exposure to exchange rate shocks.[7]

3.5 Static Exchange Rate Elasticities

The simple logic of the natural hedging mechanism is that marginal cost will affect domestic and foreign price, then affect quantity and revenue (profit). Using static elasticities, I introduce how exchange rates affect export revenue and profits through import costs.

I first look at the first-order exchange elasticities of imported input price $P_{M,i}$, overall input price $P_{X,i}$ if firms' intensive margin decisions are fixed:

$$\varepsilon(P_{M,i}, e_{ij}) \equiv \frac{\partial \ln P_{M,i}}{\partial \ln e_{ij}} = \chi_{ij}^M > 0, \quad \varepsilon(P_{X,i}, e_{ij}) \equiv \frac{\partial \ln P_{X,i}}{\partial \ln e_{ij}} = s_i^M \chi_{ij}^M > 0 \tag{18}$$

Therefore, the unit cost elasticity with respect to the exchange rate e_{il} between home country and any country l is given by:

$$\varepsilon(c_i, e_{il}) \equiv \frac{d \ln c_i}{d \ln e_{il}} = \alpha \frac{d \ln P_{X,i}}{d \ln e_{il}} = \alpha s_i^M \chi_{il}^M > 0 \tag{19}$$

If the domestic currency appreciates against a country from which it imports ($e_{il} \downarrow$, $\chi_{il}^M > 0 \downarrow$), firms' import costs and unit production cost will both decrease. Since the domestic price p_{ii} is subject to a constant markup in partial equilibrium, the firm will sell at a lower price with better competitiveness.

For the global markets, the elasticity of bilateral export price p_{ki} with respect to the exchange rate e_{il} of country l is:

$$\varepsilon(p_{ki}, e_{il}) \equiv \frac{d \ln p_{ki}}{d \ln e_{il}} = \lambda \mathbf{1}\{k = l\} + \alpha s_i^M \chi_{il}^M \tag{20}$$

Therefore, the elasticity of bilateral export revenue R_{ki} with respect to the exchange rate e_{il} of any country l is:

$$\varepsilon(R_{ki}, e_{il}) \equiv \frac{\partial \ln R_{ki}}{\partial \ln e_{il}} = [\lambda\sigma + 1 - \lambda]\mathbf{1}\{k = l\} + (1 - \sigma)\alpha s_i^M \chi_{il}^M \tag{21}$$

[7] The stylized model is static with two simplifying assumptions. First, the local wage is fixed. Second, the price level in domestic country and all foreign countries are exogenous. We can extend this by assuming market clearing condition for domestic input in roundabout production: $P_{D,i} = \int p_{ii}(\omega) d\omega$.

From the above equations, I can decompose the effect of exchange rates on bilateral export revenue into price effect for each export market as PCP ($\lambda\sigma > 0$) + LCP ($1 - \lambda > 0$) and cost effect for each import origin $(1 - \sigma)\alpha s_i^M \chi_{ij}^M \leq 0$.

The cost reductions resulting from domestic currency appreciation will lower export prices p_{ki} and increase export quantities $q_{ki} \uparrow$, thereby boosting export revenues. This cost advantage will offset the direct adverse impact of currency appreciation on price competitiveness.

Moreover, if I aggregate all bilateral export revenue, the elasticity of total export revenue $R_i^X = \sum_k R_{ki}$ with respect to the exchange rate e_{il} of country l is:

$$\varepsilon(R_i^X, e_{il}) \equiv \frac{\partial \ln R_i^X}{\partial \ln e_{il}} = [\lambda\sigma + 1 - \lambda]\chi_{li}^X + (1 - \sigma)\alpha s_i^M \chi_{il}^M \tag{22}$$

Proposition 1. *(Static Effects of Exchange Rate Shocks)*

Holding intensive margins fixed and for $\sigma > 1$: a home-currency depreciation against an import origin raises imported input $X_{M,i}$ and unit costs c_i, which reduces export revenues through the cost channel in proportion to import intensity s^M and the origin share χ_{ij}^M. A depreciation against an export market raises bilateral export revenue via the pricing-competitiveness channel; when a country is both an import source and an export destination, the net revenue effect combines a positive market-competitiveness term and a negative cost term.

Next, I allow firms to adjust their intensive marginal decisions under bilateral exchange rate changes in country l, where s_i^M, s_i^X, χ_{ij}^M, and χ_{ki}^X are all endogenous variables.

Exchange rate elasticity of import intensity:

$$\varepsilon(s_i^M, e_{il}) \equiv \frac{\partial \ln s_i^M}{\partial \ln e_{ij}} = (1 - \epsilon)(1 - s_i^M)\chi_{il}^M \tag{23}$$

Exchange rate elasticity of import share from country j:

$$\varepsilon(\chi_{ij}^M, e_{il}) \equiv \frac{\partial \ln \chi_{ij}^M}{\partial \ln e_{il}} = (1 - \kappa)(\mathbf{1}\{j = l\} - \chi_{il}^M) \tag{24}$$

At the import side, when the exchange rate shock is from the import source $j = l$, I observe the direct adjustment $(1 - \kappa)(1 - \chi_{ij}^M) < 0$, while when the shock source is different from as the import source $j \neq l$, there is only indirect cost effect $(1 - \kappa)(-\chi_{ij}^M) > 0$.

Exchange rate elasticity of export intensity:

$$\varepsilon(s_i^X, e_{il}) = \frac{\partial \ln s_i^X}{\partial \ln e_{il}} = \sigma(1 - s_i^X)\chi_{li}^X \tag{25}$$

Exchange rate elasticity of export share to country k:

$$\varepsilon(\chi_{ki}^X, e_{il}) \equiv \frac{\partial \ln \chi_{ki}^X}{\partial \ln e_{il}} = (\lambda\sigma + 1 - \lambda)(\mathbf{1}\{k = l\} - \chi_{li}^X) \tag{26}$$

I can summarized that at the export side, when the shock is from the export market $k = l$, I observe the direct adjustment $(\lambda\sigma+1-\lambda)(1-\chi^X_{ki}) > 0$, while when the shock source is different from as the export market $k \neq l$, there is only indirect cost effect $(\lambda\sigma + 1 - \lambda)(-\chi^X_{li}) < 0$. The export response shows that a lower input cost will increases the firm's profit and makes it more attractive to export to any destination which is consistent with Blaum (2024).

Proposition 2. *Intensive Margin Responses to Exchange Rate Shocks*

Allowing intensive margins to adjust: a depreciation against an import origin reduces the firm's import intensity s^M and that origin's import share χ^M_{ij} (with reallocation toward other origins), whereas a depreciation against an export market increases the firm's export intensity s^X and that market's export share χ^X_{ki}; shocks to other currencies move shares in the opposite direction, with magnitudes governed by substitution elasticities and the pricing/invoicing mix.

The static exchange rate elasticities described above echo the structural determinants of the relationship between exchange rate pass-through and import intensity in Amiti, Itskhoki and Konings (2014). As firms face exchange rate shocks from multiple countries, the covariance between export and import exchange rates and the degree of exchange rate pass-through to the relative price of imported intermediate goods will ultimately lead to heterogeneity in firms' export pricing and revenues.

3.6 Profit Exposure to Exchange Rate Risk

The firm export revenue R and profit π is subject to N pairs of bilateral exchange rates under the multiple bilateral elasticities. Exchange rates of each country-pair fluctuate around the expectation. I define a small exchange rate shock with a first-order linear approximation around the expectation $\Delta e \in \mathbb{R}^N$ in which $\Delta e_{ij} = e_{ij} - \mathbb{E}[e_{ij}]$.[8] Each bilateral exchange rate series between home country i and foreign country j follows a random-walk process: $\ln(e_{t+1}) = \ln(e_t) + \varepsilon^e_t$.

However, the assumption of independent (or more strictly, i.i.d.) exchange rate shocks may not be true in reality since exchange rates across countries may be correlated. Therefore, I formally assume the noise term $\boldsymbol{\varepsilon}^e_t \in \mathbb{R}^N$ is a co-correlated white noise vector $\boldsymbol{\varepsilon}^e_t \sim \mathcal{N}(\mathbf{0}, \Sigma^e)$ where the exchange rate covariance matrix allows for the co-movement of different currencies $\Sigma^e_{N\times N}$ where each element is a pair-wise covariance: $\sigma_{jk} = \mathrm{Cov}(\Delta e_{ij}, \Delta e_{ik}) = \mathbb{E}[(e_{ij} - \bar{e}_{ij})(e_{ik} - \bar{e}_{ik})]$.

Intuitively, when exchange rate dynamics between different currency pairs are correlated, positively correlated pairs will offset each other across imports and exports, while reinforcing each other when appearing on the same side of either imports or exports. Conversely, negatively

[8] For the static exchange rate covariance matrix using the currency of home country i as the base currency, I usually omit the subscripts i and t to simplify the notation.

correlated pairs will reinforce each other across imports and exports, while offsetting each other when appearing on the same side. Conversely, negatively correlated pairs will reinforce each other on the opposite sides, while offsetting each other when appearing on the same side. To illustrate how profits are exposed to exchange rate risk given the above mechanism, I use the first-order approximate linearization to divide profit into two parts: the "expected value" and the "stochastic term". The stochastic terms are profit deviations caused by exchange rate shocks:

$$\tilde{\pi}_i \equiv \pi_i(\Delta e) - \mathbb{E}[\pi_i] = \tilde{R}_{ii} + \sum_k \tilde{R}_{ki} - \tilde{C}_{M,i} \tag{27}$$

with stochastic terms in domestic sales $\tilde{R}_{ii} = p_{ii}q_{ii} - \mathbb{E}[p_{ii}q_{ii}]$, export sales $\tilde{R}_{ki} = \frac{p_{ki}q_{ki}}{e_{ik}} - \mathbb{E}\left[\frac{p_{ki}q_{ki}}{e_{ik}}\right]$, and import costs $\tilde{C}_{M,i}(\omega) = P_{M,i}X_{M,i} - \mathbb{E}[P_{M,i}X_{M,i}]$.

On the other hand, I can also decompose firm-level exchange rate exposure of profits into exposure to each bilateral exchange shock:

$$\tilde{\pi}_i(\omega) = \sum_l u_{il}\Delta e_{il}$$

in which u_{il} is the linearized exposure of firms' profits to exchange rate shocks Δe_{il}.

The firm-level exposures of revenue and cost can be expressed as scalar products of exposures to each single shock Δe_{il}:

$$\widetilde{R}_{ii} = \sum_{l \in \mathcal{L}} a_{il}\Delta e_{il}, \quad \widetilde{R}_{ki} = \sum_{l \in \mathcal{L}} b_{il}^k \Delta e_{il}, \quad \widetilde{C}_{M,i} = \sum_{l \in \mathcal{L}} c_{il}\Delta e_{il}.$$

In this way, the firm-level profit exposure to bilateral exchange rates u_{il} can be further decomposed into three parts:

$$u_{il} = a_{il} + \sum_k b_{il,k} - c_{il} \tag{28}$$

Following the static exchange rate elasticities derived in Section 3, I can solve the linearized sensitivities for domestic sales, export sales and import costs as:

$$a_{il} \equiv \frac{\partial \tilde{R}_{ii}}{\partial e_{il}} = (1-\sigma)\alpha s_i^M \chi_{il}^M R_{ii}, \quad b_{il}^k \equiv \frac{\partial \tilde{R}_{ki}}{\partial e_{il}} = \varepsilon_{il}^{R_k} R_{ki}, \quad c_{il} \equiv \frac{\partial \tilde{C}_{M,i}}{\partial e_{il}} = \chi_{il}^M C_{M,i} = \alpha s_i^M \chi_{il}^M C_i$$

in which $\varepsilon_{il}^{R_k} \equiv \varepsilon(R_{ki}, e_{il})$ as in Equation 21.[9]

[9] Although all exposure levels a_{il}, b_{il}^k, and c_{il} are correlated with firm size, we could standardize them using the previous period's levels as a baseline for linear approximation.

Therefore, I can summarize the profit exposure in a $N \times 1$ vector $\mathbf{u}_i$:

$$\mathbf{u}_i = (u_{i1}, u_{i2}, ..., u_{iN})^\top = (a_{i1} + \sum_k b_{i1}^k - c_{i1}, \ldots, a_{iN} + \sum_k b_{iN}^k - c_{iN})^\top \tag{29}$$

The exposure vector $\mathbf{u}_i$ describes how a firm's revenue and costs are exposed to multilateral exchange rate shocks, determined by structural parameters (risk aversion γ, elasticity of substitution σ, and etc. The firm-level effective risk exposure term $\Sigma^e \mathbf{u}_i$ summarizes information at two aspects: exchange rate covariance and firm's trade structure. Import sources and export markets with high overlap or highly correlated exchange rates leads to reduce the net exposure.

Furthermore, the profit variance can be written in a linear combination of exchange rate exposures and their covariance matrix:

$$Var(\pi_i) = \mathbf{u}_i^\top \Sigma^e \mathbf{u}_i \tag{30}$$

where the firm-level exposure $\mathbf{u}_i \equiv \mathbf{u}_i^R - \mathbf{u}_i^C$ is defined as Equation 29. In subsequent analysis of natural hedging, we shall focus on how firms strategically adjust exposure vectors based on exchange rate covariance matrices to reduce profit variance.

3.7 Metrics of Hedging Effectiveness

In our model, if an import cost increase is offset by the export revenue decrease in the same currency, then it will reduce the profit variance from exchange rate fluctuations. After log linearization, firm's profit exposure can be expressed as export revenue exposure minus import cost exposure $\mathbf{u}_i = \mathbf{u}_i^R - \mathbf{u}_i^C$. The revenue exposures are summarized in the vector $\mathbf{u}_i^R \equiv \vec{a}_i + \sum_k \vec{b}_i^k$ and the cost exposures in the vector $\mathbf{u}_i^C \equiv \vec{c}_i$. The analytic exposure vectors using average trade shares in 2005 $\mathbf{u}_{i,t=2005}^R$ and $\mathbf{u}_{i,t=2005}^C$ are shown in Figure B1.

In this part, I will construct several indicators to measure the effectiveness of natural hedging. I first measure the effectiveness of natural hedging using **exposure-offset share (EOS)** with revenue and cost exposures $\mathbf{u}_i^r$ and $\mathbf{u}_i^c$:

$$\text{EOS}_i = 1 - \frac{\|\mathbf{u}_i\|}{\|\mathbf{u}_i^R\| + \|\mathbf{u}_i^C\|} = 1 - \frac{\|\vec{a}_i + \sum_k \vec{b}_{ik} - \vec{c}_i\|}{\|\vec{a}_i + \sum_k \vec{b}_{ik}\| + \|\vec{c}_i\|} \in [0, 1] \tag{31}$$

When $\text{EOS}_i = 1$, there is no uncertainty in profits from exchange rate shocks because the revenue and cost exposure vectors are equally inverse ($\|\mathbf{u}_i\| \approx 0$), indicating there is perfect hedging (i.e., exchange rate fluctuations are perfectly neutralized on profits). When $\text{EOS}_i = 0$, the firm's revenue moves independently with its cost ($\|\mathbf{u}_i\| = \|\mathbf{u}_i^r\| + \|\mathbf{u}_i^c\|$), there is no hedging, which means profit fluctuations are fully exposed to exchange rate shocks.

Second, I also report the **variance-reduction ratio (VRR)**, which is 1 minus the normalized profit variance to describe how much the profit risk is reduced by import-export linkages:

$$VRR_i = \frac{2\text{Cov}(R_i, C_i)}{\text{Var}(R_i) + \text{Var}(C_i)} = \frac{2(\mathbf{u}_i^R)^\top \Sigma^e (\mathbf{u}_i^C)}{(\mathbf{u}_i^R)^\top \Sigma^e \mathbf{u}_i^R + (\mathbf{u}_i^C)^\top \Sigma^e \mathbf{u}_i^C} \in [-1, 1] \tag{32}$$

which is unit-free and scale-invariant measure which rises with the intensity of natural hedging. It equals 1 under a perfect natural hedge, 0 when R_i and C_i are uncorrelated, and approaches -1 under perfectly negative co-movement. If revenue and cost respond linearly to a vector of currency shocks Δe_t with exposure vectors, the profit variance can be written as $\text{Var}(\tilde{\pi}_i) = (\mathbf{u}_i^R - \mathbf{u}_i^C)' \Sigma^e (\mathbf{u}_i^R - \mathbf{u}_i^C)$, and the analytic expression of VRR follows the second equal sign.[10]

In addition, I also use the Pearson **revenue-cost correlation** (ρ^{RC}) to measure the co-movement of revenue and costs. If firms' import costs and export sales revenues are hedged under the same set of exchange rate shocks, revenues and costs are positively correlated in the shock space; their direction and magnitude reflect the structure of firms' exposures to shocks in different currencies as determined by structural parameters.

$$\rho_i^{RC} = \frac{\text{Cov}(\tilde{R}_i, \tilde{C}_i)}{\sqrt{\text{Var}(\tilde{R}_i)\text{Var}(\tilde{C}_i)}} = \frac{(\mathbf{u}_i^R)^\top \Sigma^e \mathbf{u}_i^C}{\sqrt{[(\mathbf{u}_i^R)^\top \Sigma^e \mathbf{u}_i^R][(\mathbf{u}_i^C)^\top \Sigma^e \mathbf{u}_i^C]}} \in [-1, 1] \tag{33}$$

I should expect risk-averse firms to prefer to arrange revenues positively correlated with costs to reduce profit volatility. However, the linear correlation does not necessarily reflect a one-on-one (45-degree) relationship. Therefore, I keep it as an auxiliary measure of hedging.

From all analytic indicators, natural hedging corresponds to alignment of the exposure vectors: the closer $\mathbf{u}_i^R$ is to $\mathbf{u}_i^C$ in both direction and magnitude, the smaller the residual profit variance. This geometric view of the structural model illustrates firms can hedge naturally through two channels: (i) price risk premium adjustments that alter $\mathbf{u}_i^R$ and $\mathbf{u}_i^C$ given import and export shares, and (ii) share adjustments that change the composition of import origins and export destinations.

For comparative statics, the import-export structure (χ_{ij}^M and χ_{ki}^X), the PCP weight λ, and the risk aversion parameter γ all affect the effectiveness of hedging. The more they import (higher αs_i^M) and more overlapping between import origins and export markets or more local currency pricing ($\lambda \to 0$) will lead to stronger natural hedging capabilities.

[10]The normalized profit variance is the profit variance over the sum of revenue variance and cost variance: $\text{Var}_i^N(\pi) \equiv \frac{\text{Var}(\tilde{\pi}_i)}{\text{Var}(\tilde{R}_i)+\text{Var}(\tilde{C}_i)} = \frac{(\mathbf{u}_i^R - \mathbf{u}_i^C)^\top \Sigma^e (\mathbf{u}_i^R - \mathbf{u}_i^C)}{(\mathbf{u}_i^R)^\top \Sigma^e \mathbf{u}_i^R + (\mathbf{u}_i^C)^\top \Sigma^e \mathbf{u}_i^C} \in [0, 2]$.

4 Risk-Averse Extension

In the basic risk-neutral model, firms only maximize their expected profit. This assumption should be reasonable in the short run. However, the assumption of risk neutrality prevents firms from actively adjusting their trade networks to mitigate risk. To explain the medium-run responses, I relax this assumption and allow firms to modify export pricing and import share allocation. Introducing risk aversion could offer a structural interpretation of the role played by natural hedging in changing firms' trade behaviors in response to exchange rate uncertainty.

In this section, I will first extend the optimization problem for risk-averse firms with parameter γ. One can understand the risk-neutral model as a special case of the full model in which $\gamma = 0$. I will then solve the risk-adjusted sourcing and pricing equations using first-order conditions (FOCs) under risk aversion and describe how risk-averse optimal decision deviate from risk-neutral profit optimization.

4.1 Optimization under Risk Aversion

I assume that firm's utility function exhibits constant absolute risk aversion (CARA). Therefore, I can have a "certainty equivalence" (CE) in the firm's optimization problem:

$$\max \mathrm{J} = \underbrace{\mathbb{E}[\pi_i]}_{\text{Expected Profit}} \underbrace{- \frac{\gamma}{2} \cdot Var(\pi_i)}_{\text{Risk Cost}} \tag{34}$$

where $\gamma > 0$ is the risk aversion parameter. Since profits can be decomposed into domestic and foreign sales and production costs, the variance of profits due to exchange rate fluctuations could be written in stochastic terms and decomposed as $Var(\tilde{\pi}_i) = Var(\sum_k \tilde{R}_{ki}) + Var(\tilde{C}_{M,i}) + Var(\tilde{R}_{ii}) - 2Cov(\sum_k \tilde{R}_{ki}, \tilde{C}_{M,i}) + 2Cov(\sum_k \tilde{R}_{ki}, \tilde{R}_{ii}) - 2Cov(\tilde{R}_{ii}, \tilde{C}_{M,i})$.

Firms' decisions are no longer determined solely by expected profits, but are also driven by the risk cost. Now the optimization problem under risk aversion becomes:

$$\max_{\theta} J(\theta) = \mathbb{E}[\pi_i(\theta)] - \frac{\gamma}{2} \mathbf{u}_i(\theta)^\top \Sigma^e \mathbf{u}_i(\theta) \tag{35}$$

where firms make intensive margin decision $\theta_i \in \{\vec{p}_i, \vec{\chi}_i^M, \vec{\chi}_i^X, s_i^X, s_i^M\}$ based on given trade network $\mathcal{J}$ and $\mathcal{K}$.

For any decision θ_i, I have the general form of the first-order conditions (FOCs) of intensive margin decisions:

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial \theta} = \gamma(\frac{\partial u_i}{\partial \theta})^{\top} \Sigma^e \mathbf{u}_i \tag{36}$$

The left side is the expected marginal profit margin from adjustment. On the right is the risk penalty $(\partial u_i/\partial\theta)^{\top}\Sigma^e\mathbf{u}_i = \mathrm{Cov}[(\partial\mathbf{u}_i/\partial\theta)^{\top}\Delta e, \mathbf{u}_i^{\top}\Delta e]$, i.e., the covariance between "the marginal exchange rate exposure" and "the firm's current net exchange rate exposure". The effective exchange risk factor $\Sigma^e\mathbf{u}_i$ describes the "overall foreign currency risk exposures under known exchange rate covariance", which embeds the interactive effect of a firm's "marginal trade exposure" and "currency covariance structure".

If I express the optimization problem in a compact matrix form, The optimality of each decision is equal to the inner product of its marginal exposure and the risk penalty, $\nabla_\theta J = \nabla_\theta \mathbb{E}[\pi_i] - \gamma[\nabla_\theta \mathbf{u}_i(\theta)]^{\top}\Sigma^e\mathbf{u}_i(\theta)$. which allows us to numerically solve it with a projection-gradient update method.

In summary, risk-averse firms could use price premium and share allocation to offset risk exposure, thereby reducing variance and decreasing the risk penalty. For price risk premium, when $\partial u/\partial p$ moves in the same direction as u, the premium requires risk compensation (upward adjustment); when moving in opposite directions, it is adjusted downward. For share allocation, shifting allocations toward currencies negatively correlated with u reduces variance, thereby increasing marginal net returns.

4.2 Risk-adjusted Pricing Equations

Firms will face a trade-off between expected revenue against price volatility in pricing decisions. The higher the risk, the more conservative the firm's pricing — firms will reduce volatility at the cost of expected profits i.e. a higher expected marginal profit is required to offset this risk penalty. Therefore, the adjusted pricing equations could be a starting point for how risk aversion differs from the conventional risk-neutral framework.

From the risk-averse optimization, I can derive the new first-order conditions (FOCs) of domestic price p_{ii} and export price p_{ki} as:

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial p_{ii}} - \frac{\gamma}{2}\frac{\partial Var(\pi_i)}{\partial p_{ii}} = 0, \quad \frac{\partial \mathbb{E}[\pi_i]}{\partial p_{ki}} - \frac{\gamma}{2}\frac{\partial Var(\pi_i)}{\partial p_{ki}} = 0 \tag{37}$$

I define risk premia ψ_{ii} and ψ_{ki} as the additional price markup charged by risk-averse firms to reduce the profit variance (uncertainty) from exchange rate fluctuations at the cost of expected profits:

$$\psi_{ii} \equiv \frac{\frac{\gamma}{2}\frac{\partial Var(\pi_i)}{\partial p_{ii}}}{\frac{\partial E[\pi_i]}{\partial p_{ii}}}, \quad \psi_{ki} \equiv \frac{\frac{\gamma}{2}\frac{\partial Var(\pi_i)}{\partial p_{ki}}}{\frac{\partial E[\pi_i]}{\partial p_{ki}}}$$

I can solve the domestic risk premium and the export risk premium ψ_{ki}

$$\psi_{ii} = \gamma(1-\sigma)\alpha s_i^M \sum_{j,l} \chi_{ij}^M u_{il}\sigma_{jl}^e, \quad \psi_{ki} = \gamma \sum_{j,l} \varepsilon_{ij}^{R_k} u_{il}\sigma_{jl}^e \tag{38}$$

The double weighted summation indicates that overall firm-level exchange risk equals to risk of each currency exposure times the sum of weighted effects of covariance between that currency and other currencies.

Therefore, I can write the domestic and export risk premia in matrix form with the profit exposure factor $\mathbf{u}_i$, :

$$\psi_{ii} = \gamma\phi_i^M (\vec{\chi}_i^M)^\top (\Sigma^e \mathbf{u}_i), \quad \psi_{ki} = \psi_{ii} + \gamma\phi_i^X (\Sigma^e \mathbf{u}_i)_k \tag{39}$$

where $\phi_i^M = (1-\sigma)\alpha s_i^M$ and $\phi_i^X = \lambda\sigma + 1 - \lambda$ are sensitivity factors that map trade shares and structural parameters to price risk premium. The risk premium for the overall export price is a weighted sum of risk premium to each market $\psi_i^X = \frac{1}{K}\sum_K \chi_{ki}^X \psi_{ki} = \psi_{ii} + \gamma\phi_i^X (\vec{\chi}_i^X)^\top (\Sigma^e \mathbf{u}_i)$.

The destination component of the premium reflects pricing-to-currency behavior (a PCP/LCP mix), and the common component reflects the firm's extra cost from imported inputs; together they generate country-specific price premia. Under the same exposure structure, higher γ leads to higher risk compensation requirement, and thus the price level wedge is larger.

4.3 Risk-Adjusted Import Allocation

Our model allows firms to freely shift import and export shares from the existing trade network. The export shares are endogenously determined by the export pricing (with risk premium). In this part, I focus on the endogenous import share allocation with risk-adjusted "effective unit sourcing cost", which deviates from the risk-neutral import price.

Firms selects import shares χ_{ij}^M from each source $j \in \mathcal{J}$ to minimize the certainty equivalent sourcing cost under exchange rate risk. The first-order conditions (FOCs) of import shares under risk aversion:

$$\frac{\partial \mathbb{E}[C_{M,i}]}{\partial \chi_{ij}^M} + \gamma (d_{ij}^M)^\top \Sigma^e \mathbf{u}_i = \lambda_M \tag{40}$$

where λ_M is the shadow price from Kuhn-Tucker condition under $\sum_j \chi_j^M = 1$ and d_{ij}^M is the marginal impact of import shares on cost exposure:

$$d_{ij}^M \equiv \frac{\partial u_i^C}{\partial \chi_{ij}^M} \in \mathbb{R}^J, \quad \mathbf{d}_i^M = [d_{i1}^M, \ldots, d_{iJ}^M] \tag{41}$$

Therefore, I define the "effective unit sourcing cost" under which a firm decides its import shares after risk adjustment as

$$\tilde{p}_{ij} \equiv p_{ij} + \Gamma_M \cdot \gamma (d_{ij}^M)^\top \Sigma^e \mathbf{u}_i \tag{42}$$

where $\Gamma^M > 0$ is the unit conversion constant derived from the CES demand system.[11] This is an internal "shadow purchase price" used within the firm for sourcing allocation. It is not the actual price on the invoice and serves solely to drive the selection of χ^M (share channel).

Therefore, the import shares under risk aversion become

$$\chi_{ij}^M = \frac{(\tilde{p}_{ij})^{1-\kappa}}{\sum_{j' \in \mathcal{J}} (\tilde{p}_{ij'})^{1-\kappa}} = \frac{(p_{ij} + \Gamma_M \gamma (d_{ij}^M)^\top \Sigma^e \mathbf{u}_i)^{1-\kappa}}{\sum_{j'} (p_{ij'} + \Gamma_M \gamma (d_{ij'}^M)^\top \Sigma^e \mathbf{u}_i)^{1-\kappa}} \tag{43}$$

When the cost exposure d_{ij}^M from source j moves in the same direction as the firm's existing net exposure u_i (weighted by Σ^e), $\tilde{p}_{ij} > p_{ij}$ implies that the source becomes "more expensive" in the firm's perspective. Conversely, if the exposure moves in the opposite direction, the source becomes "cheaper"– prompting an increase in its share to achieve hedging of net income exposure through imports.

When $(d_{ij}^M)^\top \Sigma^e u_i > 0$, increasing χ_j^M could elevates cost exposure and thereby reduces net exposure u_i because $\frac{\partial u_i}{\partial \chi_{ij}^M} = -\frac{\partial u_i^C}{\partial \chi_{ij}^M} = -d_{ij}^M$. In this case, increased imports from j can hedge existing net income exposure, particularly when export is exposed to currencies highly correlated with j, making such imports more desirable.

In summary, import adjustments will consider whether this new import cost exposure aligns with the direction of existing net exposure. Firms will favor import sources that are negatively correlated with their existing net exposure to achieve an optimal trade-off between expected profit and variance. This enables the "natural hedge" to directly enter the marginal conditions of share allocation without additional constraints or penalty.

4.4 Linear Equation System under Risk Aversion

In this part, I will provide a quantifiable representation of risk-averse model to how exchange rate risk enters the linear system using the "certainty equivalence" under the small-risk linear approximation. First, using the local linear approximation of the first-order conditions, the risk-neutral structural model can be written in a linear system:

$$A(\theta)\Delta z = B(\theta)\Delta s + C(\theta)\Psi$$

[11] When using the most common CES aggregation (equal weights, scale normalization), Γ_M is a constant; in structural estimation, either identify Γ_M and γ as a product or normalize Γ_M to 1.

where Δs is the exogenous exchange rate shock vector with country dimension N. Δz includes changes in endogenous variables such as prices, quantities and revenue. Ψ captures level factors and other unmodeled shocks which are irrelevant to the exchange rate pass-through.

The risk-adjusted pricing equation generates an additional risk term of markups, which is absent in risk neutrality. After inserting risk-loading block into the first-order linearized risk-neutral pricing equation, I have the risk-averse pricing equation as:

$$A_p \Delta z + \gamma R_p \Sigma^e \Delta u = B_p \Delta s + C_p \Psi \tag{44}$$

in which the risk-loading block R_p describe price sensitivity to changes in exposure and will be derived from demand elasticity and risk-adjusted pricing equations. The intuition of price channel is that γ amplifies the covariance difference between marginal revenue exposure and net profit exposure, meaning greater risk yields higher compensation. The price risk premium alters the slope of the linear equation for the endogenous variable Δz, because the risk term itself depends on the endogenous change in net exposure $\Delta u(\Delta z, \Delta s, \psi)$, which is a linear function of Δz.

The import and export shares are determined endogenously by import sourcing shadow prices and export pricing premia:

$$\Delta \chi^X = S_z^X \Delta z + S_s^X \Delta s + S_\Psi^X \Psi, \quad \Delta \chi^M = S_z^M \Delta z + S_s^M \Delta s + S_\Psi^M \Psi + S_u^M \gamma \Sigma^e \Delta u \tag{45}$$

where $S_\cdot^X$ and $S_\cdot^M$ are derived from the first order conditions of optimal trade shares. Specifically, S_M^u is the matrix assembled from the derivatives of the effective import share with respect to $\tilde{p}_j = p_j + \Gamma_M \gamma (d_j^M)^\top \Sigma^e u$.[12]

The net profit exposure change is a linear combination of endogenous variables Δz and import and export shares (χ^X, χ^M):

$$\Delta u = U_z \Delta z + D^X \Delta \chi^X - D^M \Delta \chi^M \tag{46}$$

in which U_z denotes exchange rate elasticities of price-related endogenous variables, while $D^X = [D_k^X] \in \mathbb{R}^{N \times K}$ and $D^M = [D_j^M] \in \mathbb{R}^{N \times J}$ collect "marginal profit exposure" of each export or import share, respectively. Since all risk aversion terms enter the linear equations solely through $\Sigma^e \Delta u$, this equation serves as the connecting hinge for all risk blocks.

Now we can understand Δu as a sufficient statistic for the two-channel risk block. This approach maps the price risk premium (Δz) and the share adjustment channels ($\Delta \chi^X, \Delta \chi^M$) to the net exchange rate exposure change Δu to obtain a unified representation:

[12]This eliminates the need for a separate "export share KKT" and avoids solving import shares as distinct multipliers. Instead, import shares enter Δu via the share mapping of "risk-adjusted sourcing decision".

$$\Delta u = T_u(\gamma)(N_z \Delta z + N_s \Delta s + N_\Psi \Psi) \tag{47}$$

in which I denote structural coefficients $N_z = U_z + D^X S^X_z - D^M S^M_z$, $N_s = D^X S^X_s - D^M S^M_s$, $N_\Psi = D^X S^X_\Psi - D^M S^M_\Psi$. The Schur complement $T_u(\gamma) \equiv (I + \gamma D^M S^M_u \Sigma^e)^{-1}$ is the linear fixed point of the imported share's feedback to risk. As γ increases, Δu tends to be pushed toward a direction negatively correlated with existing exposure for stronger natural hedging.

Finally, I can absorb two channels simultaneously in the price equation and derive a final master equation:

$$\underbrace{(A_z + \gamma R_p \Sigma^e T_u N_z)}_{A^*(\gamma)} \Delta z = \underbrace{(B_p - \gamma R_p \Sigma^e T_u N_s)}_{B^*(\gamma)} \Delta s + \underbrace{(C_p - \gamma R_p \Sigma^e T_u N_\psi)}_{C^*(\gamma)} \psi \tag{48}$$

This is the merged equation in which I can solve Δz. All risk effects from both channels have been incorporated in this equation through export risk loading $S^X_\cdot$ (price risk premium) and import risk loading S^M_u (risk-adjusted sourcing shares).

5 Numerical Analysis

Our structural model aims to reveal how exchange rate risk reshapes trade patterns in ways distinct from actual trade models without nominal prices or risk-neutral models without risk aversion. In this section, I will use the exact-hat algebra (Dekle, Eaton and Kortum, 2008) approach to quantify how firms use import and export linkages to hedge exchange rate risk.

First, I will estimate the exchange rate covariance matrix Σ^e using historical exchange rate data. Second, a set of conventional parameters is obtained from the literature or calibration. Finally, I will conduct Monte-Carlo simulations and sensitivity tests to show firms' responses under different risk-aversion parameters.

5.1 Estimation of Exchange Covariance Matrix

In the structural model, I need exchange rate covariance matrix $\Sigma^e \in \mathbb{R}^{N\times N}$ to describe the co-movement of multilateral exchange rates, in which each element σ_{jk} is the bilateral covariance between import and export exchange rates.

$$\Sigma^e_{N\times N} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix}$$

In practice, I will estimate the exchange rate covariance matrix Σ using expected (average) monthly log foreign exchange returns from IFS monthly exchange rate time series.[13] Using the historical exchange rate time series, we can sample covariance matrix: $\hat{\Sigma}_S = \frac{1}{T-1}\sum_{t=1}^{T} X_t X_t'$ where $r_t = (r_{1t}, \ldots, r_{Nt})'$ include logarithmic returns of currencies in period t and $X_t = r_t - \frac{1}{T}\sum_{t=1}^{T} r_t$: observed deviation from sample mean of log returns. I show the estimated sample exchange rate correlation matrix (covariance matrix divided by the diagonal mean variance of each currency) in Appendix Figure C1 and Figure C2.

The sample variance matrix is unbiased, yet it may contain noise due to sampling issues. To maintain the strict positive definiteness of the matrix and significantly reduce the mean square error at the expense of a slight loss of unbiasedness, I use Ledoit-Wolf shrinkage covariance $\hat{\Sigma}_{\mathrm{LW}} = \delta F_{cc} + (1-\delta)\hat{\Sigma}_S$. Details of weight δ and constant-correlation matrix F_{cc} is introduced in Appendix C.2.

5.2 Parameterization

For key parameters, I adopt values calibrated from standard references in the literature. I set the demand elasticity to $\sigma = 3.8$ and the output elasticity (input share) of material $\alpha = 0.6$ from Oberfield and Raval (2021).[14] Besides, The elasticity of substitution between domestic and foreign inputs is set at $\epsilon = 4$ following Halpern, Koren and Szeidl (2015) and Gopinath and Neiman (2014).

The PCP share λ is identified by the average empirical exchange rate pass-through among Chinese exporters (Li, Ma and Xu, 2015) and (Li, Lu and Zhao, 2025). The higher the exchange rate elasticity of the destination currency price, the closer the pricing regime is to PCP, λ will be closer to 1. The benchmark import elasticity of substitution between different origins κ is 3.0 from Feenstra et al. (2018). I can also estimate it from the sensitivity of import shares to relative import prices and exchange rate shocks for cross-validation.

The CARA risk aversion parameter γ starts with the baseline value 2.0 from Chetty (2006).From the utility-equivalent trade-off implied by the CARA mean–variance approximation, the original γ represents absolute risk aversion with the unit of one over the currency unit. [15] I will a series of sensitivity tests using different values to show the heterogeneous hedging behaviors

[13]The time length $T = 16 \times 12 = 192 \gg 1$ and the dimension is $N = 20$ countries.

[14]I can also estimate σ, the demand elasticity for final products, using firm-country-level price and quantity regressions, instrumenting for endogenous prices with exchange rate or cost shocks following the approach of Berman, Martin and Mayer (2012).

[15]In CARA definition, the value of γ changes proportionally when the monetary unit or dimension of the profit change. In practice, I can also generate relative risk aversion (CRRA) by rendering the profit dimensionless: let $\tilde{\pi} = \pi/\bar{\pi}$ (scaled by the benchmark profit). Then, for example, when $\mathrm{sd}(\tilde{\pi}) \approx 0.2$, $\tilde{\gamma} = 1$ implies that "a profit fluctuation of one standard deviation incurs an equivalent penalty of approximately $(1/2) \times 0.2^2 = 2\%$."

of firms with different degree of risk aversion. Calibration strategy using cross-sectional sensitivity of hedging effectiveness indicators to exchange rate volatility is explained in Appendix C.4. The calibration results approached 2.0, confirming the above parametric selection.

Table 5: Parameterization

Parameter	Symbol	Value	Method & Source
Demand elasticity	σ	3.8	Oberfield and Raval (2021)
Output elasticity of materials	α	0.6	Oberfield and Raval (2021)
PCP share	λ	0.95	Li, Ma and Xu (2015), Li, Lu and Zhao (2025)
Elasticity of substitution between domestic and foreign inputs	ϵ	4.0	Halpern, Koren and Szeidl (2015), Gopinath and Neiman (2014)
Elasticity of substitution between inputs from different origins	κ	3.0	Feenstra et al. (2018)
Risk aversion	γ	2	Chetty (2006) Elminejad, Havranek and Irsova (2022)

Note: This table lists the parameter values assigned from literature and calibration.

5.3 Simulation and Sensitivity Tests

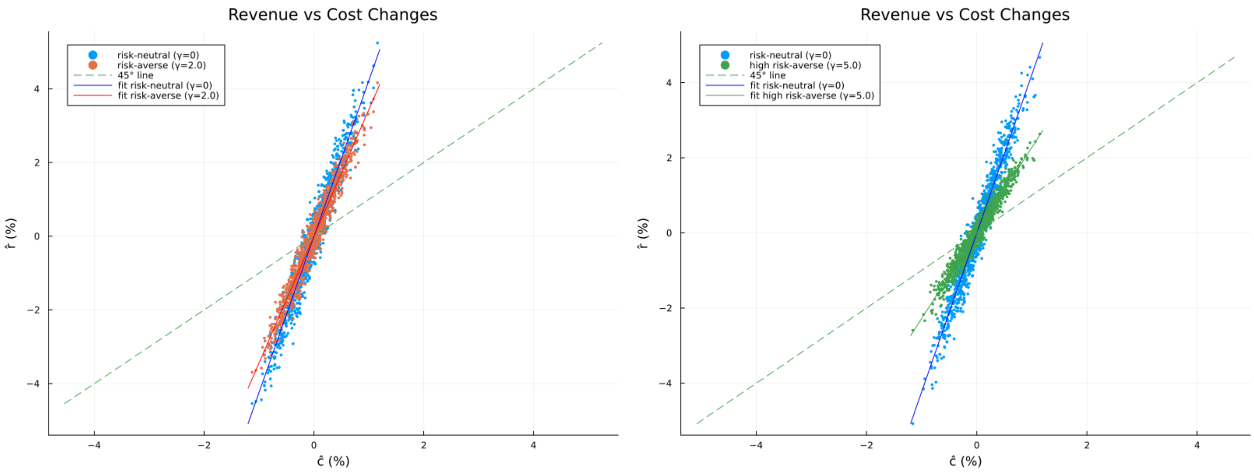
Given the estimated exchange rate covariance matrix and key parameters, I can conduct Monte-Carlo simulations with given initial import and export shares by hypothetical exchange rate shocks and observe responses in revenue and cost and corresponding indicators of effectiveness of natural hedging.

I simulate exchange rate risk by drawing 10000 shocks $\Delta e \sim \mathcal{N}(0, \hat{\Sigma})$ over 20 currencies, where $\hat{\Sigma}$ is a Ledoit–Wolf shrinkage covariance estimated from the data and factorized via a robust Cholesky. For each draw, revenue and cost changes are computed from the exposure vectors from the exact-hat algebra under risk-adjusted pricing and sourcing. In addition, I conduct sensitivity tests to show how risk aversion parameter γ and pricing scheme λ would affect firms' hedging behavior through prices and shares.

Across draws I summarize the mean of natural hedging indexes: exposure-offset share (EOS), the revenue–cost correlation $\rho(r, c)$, and the normalized profit variance used for comparability.

Risk-neutrality vs risk-aversion

I use risk aversion parameter $\gamma = 2$ as our benchmark calibrated value to compare with those with $\gamma = 0$ as risk-neutral case and those with d $\gamma = 5$ as the highly risk-averse case. The two scatterplots (with 1000 points) report firm-level percent changes in revenue $\hat{r}$ and corresponding percent changes in cost $\hat{c}$ under alternative risk preferences. I vary only the risk-aversion parameter γ and keep other parameters fixed. Dots denotes firms in the simulation and the dashed line is the 45-degree benchmark (one-for-one co-movement), and solid lines are OLS fits for each group.

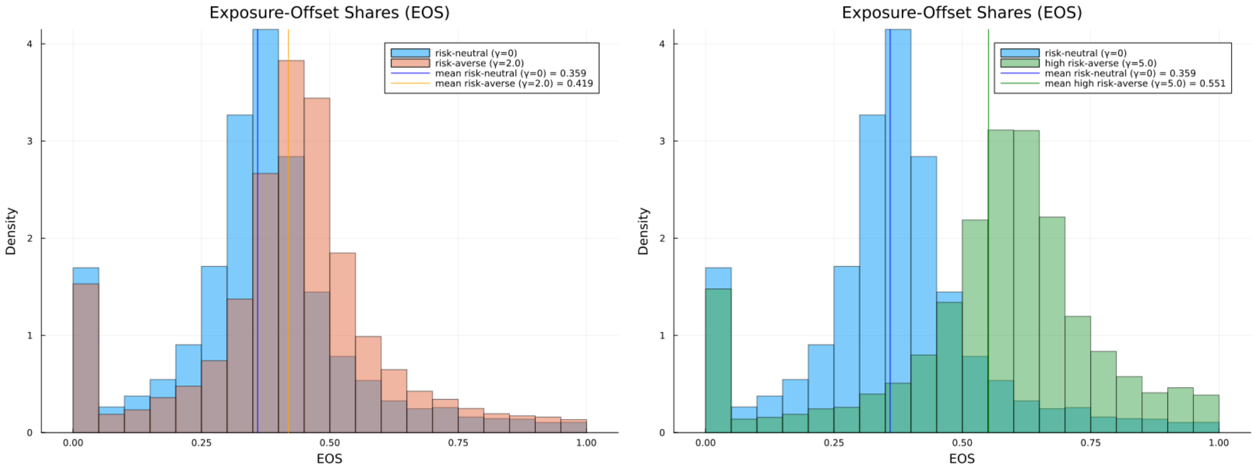


(a) Risk-neutral $\gamma = 0$ vs risk-averse $\gamma = 1$ (b) Risk-neutral $\gamma = 0$ vs highly risk-averse $\gamma = 5$

Figure 2: Scatterplot of simulated revenue and cost changes

In both panels, the simulated point cloud is upward-sloping, with fitted lines passing close to the origin—evidence of strong revenue–cost co-movement. As γ rises the fitted line rotates slightly toward the 45-degree benchmark and the scatter around it narrows, especially for $\gamma = 5$. This rotation and tightening indicate stronger natural hedging: risk-averse firms choose trade portfolios that bring revenue movements closer to cost movements, alleviating profit fluctuations for a given shock. Substantial overlap remains, reflecting persistent cross-firm heterogeneity.

To motivate our analysis of natural hedging, the next figures plot the cross-sectional distribution of firms' exposure-offset shares (EOS) which summarizes how much revenue and cost exposures offset each other, with higher values indicating stronger hedging. I hold all primitives fixed and vary only the risk-aversion parameter γ in the decision problem, recomputing optimal pricing and sourcing and the implied EOS at the firm level. Histograms display kernel densities of EOS and vertical lines mark group means, which provide a clear mapping from risk preferences to the strength of model-implied natural hedging.



(a) Risk-neutral $\gamma = 0$ vs risk-averse $\gamma = 1$ (b) Risk-neutral $\gamma = 0$ vs highly risk-averse $\gamma = 5$

Figure 3: Distribution of Exposure-Offset Shares (EOS)

Figure (a) and (b) compare the distribution of exposure–offset shares (EOS) across risk preferences. Relative to the risk-neutral benchmark ($\gamma = 0$, mean $= 0.359$), the EOS distribution shifts to the right for $\gamma = 2.0$ (mean $= 0.419$) and more markedly for $\gamma = 5.0$ (mean $=$ 0.551): mass in the low-EOS region (less than 0.2) shrinks, while the right tail (more than 0.6) thickens. This pattern indicates that greater risk aversion is associated with stronger natural hedging — firms choose portfolios with higher exposure offsets—yet the sizeable overlap across curves underscores persistent cross — firm heterogeneity driven by initial exposures and revenue–cost structures.

Finally, we form a grouped indicator dashboard which reports firm-level means of four natural-hedging metrics under three risk preferences ($\gamma \in \{0, 2, 5\}$): average EOS and normalized profit variance (variance of profits divided by the sum of variance of revenue and costs) from Monte-Carlo simulations. In addition, I add the profit level wedge $h^R - h^C$, which is the loss of expected profit to reduce variance and MV ratio $-(h^R - h^C)/\sigma(\pi)$, which normalizes the level adjustment by profit volatility.

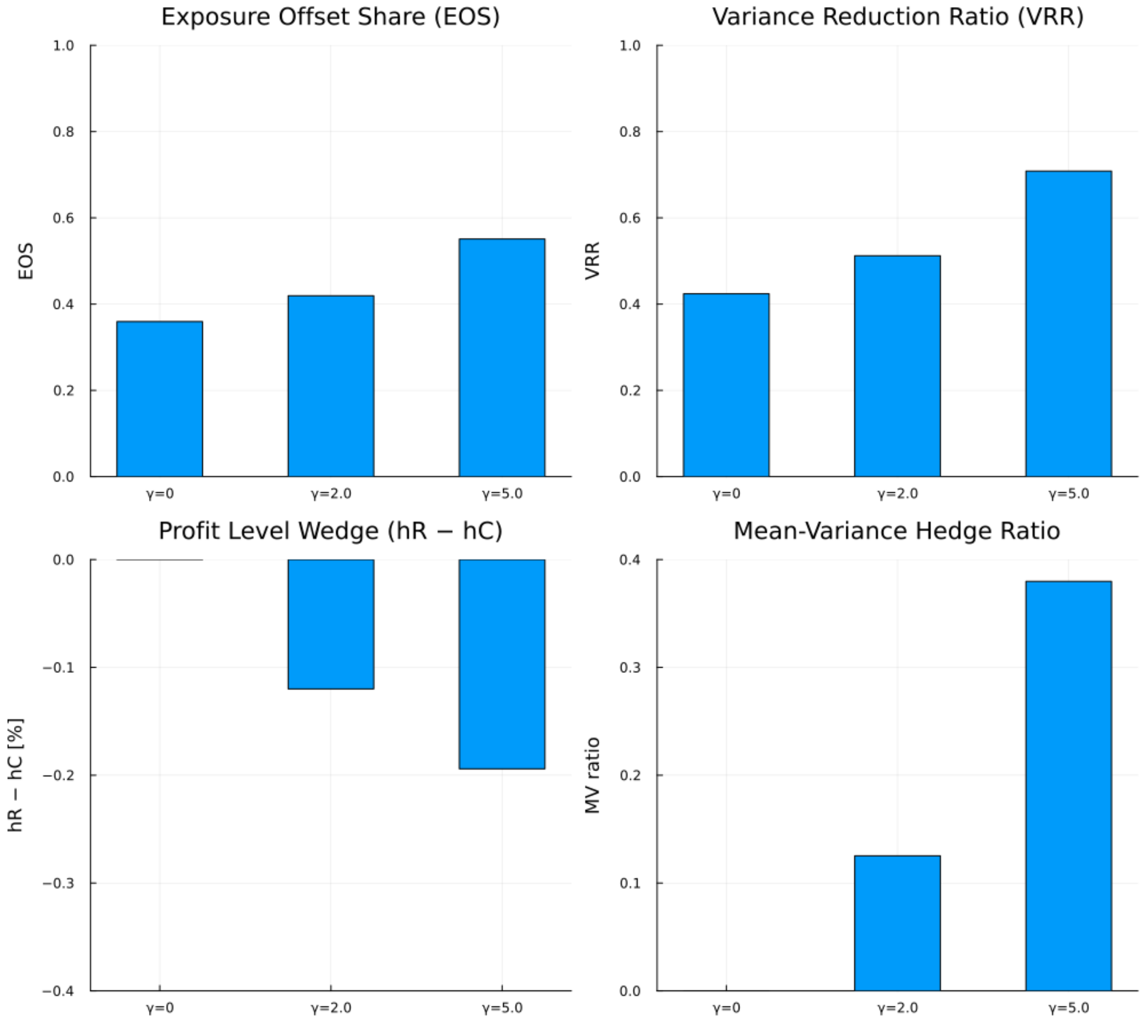


Figure 4: Overview of Natural Hedge Sensitivity to γ

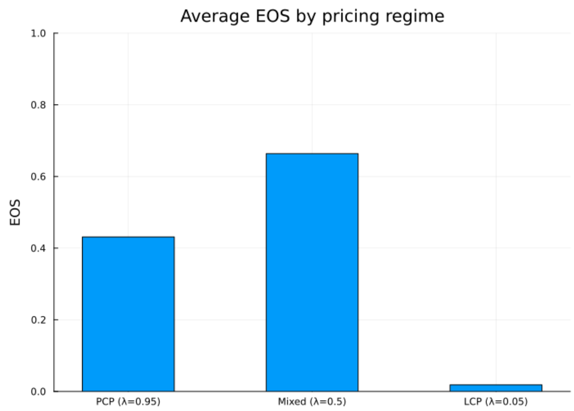
All four panels move in the directions implied by stronger hedging as γ rises. EOS increases, while profit variance falls. The profit level wedge $h^R - h^C$ becomes more negative, indicating a deliberate reduction in mean profit to curb risk. Consistently, the MV ratio $-(h^R - h^C)/\sigma(\pi)$ increases almost linearly, showing a higher willingness to trade off level for variance reduction. Together, these averages summarize a coherent mean–variance shift toward stronger natural hedging at higher risk aversion.

Export Pricing Scheme: PCP to LCP

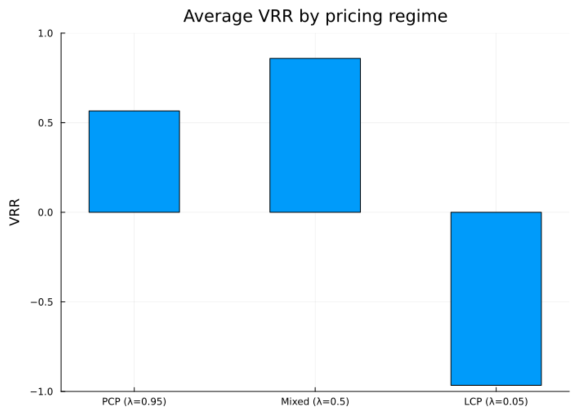
In the model, the export pricing currency scheme play an important role in exchange rate pass-through and could load different degree of risk to exporters. This sensitivity test evaluates how the pricing regime parameter λ (from PCP to LCP) affects exposure alignment. I hold firm structure and the ER covariance Σ^e fixed.

We compare natural hedging performance across three pricing regimes while holding the firm structure and the FX covariance matrix fixed The below figures show exposure-overlap share

(EOS) and the variance-reduction rate VRR $= \frac{2Cov(R,C)}{Var(R)+Var(C)}$ under the balanced trade intensity and baseline matching setup. Higher bars indicate more aligned export and import exposure and more profit variance canceled by revenue–cost co-movement.



(a) Exposure offset by λ



(b) Variance reduction by λ

Figure 5: Export Pricing Scheme: PCP vs LCP

Both indicators show the clear ranking: the highest degree of hedging effectiveness is at the mixed pricing scheme $\lambda = 0.5$, lower under near-PCP ($\lambda = 0.95$), and near zero under LCP ($\lambda = 0.05$). EOS is highest under mixed and moderate under PCP, but collapses under LCP; VRR is strongly positive under mixed, positive under PCP, and negative under LCP.

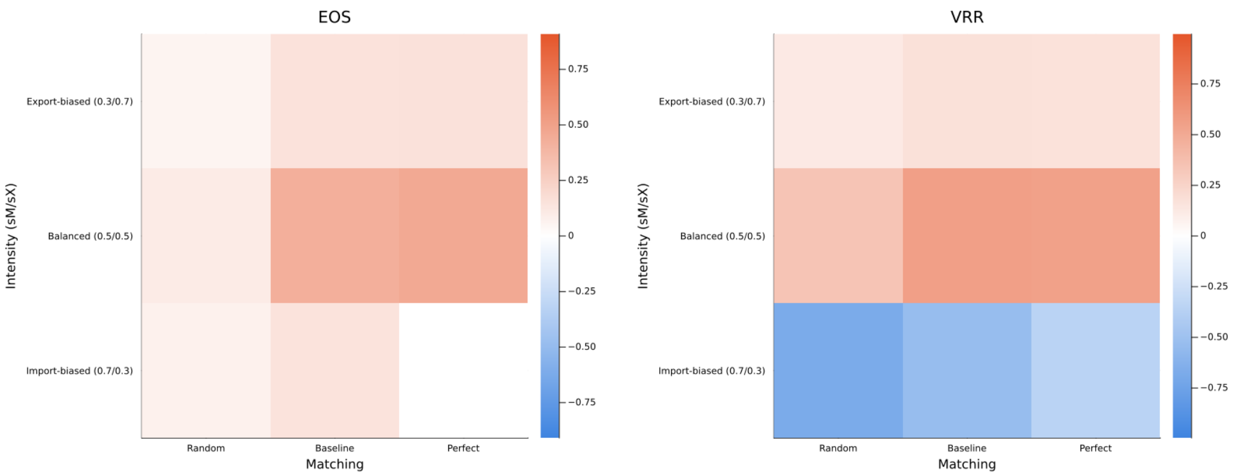
Intuitively, partial exchange pass-through under mixed regime makes both revenue and cost exposures similar in size and currency composition, maximizing maximizes the relative covariance $Cov(R, C)$. With LCP, export prices are insulated from exchange rates while import costs still move. Therefore, the alignment is broken because export side exposure is dampened while import-side exposure remains, which reduces $\text{Cov}(R, C)$, and drives VRR negative. Under PCP, both sides are exposed but not perfectly balanced across currencies, yielding only moderate overlap, so the variance reduction is only intermediate.

5.4 Counterfactual Trade Structure

The first group of counterfactual tests examines the consequences of changing the firms' import-export linkages in trade structure. I change the firms' trade structure in two dimensions. First, I will assign hypothetical import and export intensities. I use the "balanced" firms with both import intensity (import value over total import cost) and export intensity (export value over total sales) equals to 0.5 as the our benchmark. Alternatively, I use import-based firms with higher import intensity ($s^M = 0.8$) and lower export intensity ($s^X = 0.2$) and export-biased firms with with lower import intensity ($s^M = 0.2$) and higher export intensity ($s^X = 0.8$).

Second, I will compare the effectiveness of natural hedging with real-data trade shares with using hypothetical trade shares. Specifically, I keep the total import value unchanged, but force imports from each country to be in proportion to exports to the same country to generate a perfect matching scenario ($\vec{\chi}^X = \vec{\chi}^M$). Then, I randomize the import and export shares of all firms to simulate an "mismatch exacerbation" scenario, in which there is no correlation between firms' sourcing and exporting decisions (no correlation $\vec{\chi}^X$ and $\vec{\chi}^M$). All above counterfactual cases are compared with the baseline case in which $\vec{\chi}_0^X$ and $\vec{\chi}_0^M$ are from real trade data in 2005.

I show two heatmaps for alternative intensity balance and share matching for two-way traders. The first figure plots the exposure-overlap share (EOS) on the grid of trade-intensity — import-biased ($s^M > s^X$), balanced ($s^M \approx s^X$), export-biased ($s^M < s^X$) — against the matching between import and export currency baskets (Random, Baseline, Perfect). The second heatmap shows the variance reduction rate VRR in all above cases. In both figures, I keep the exchange rate covariance Σ^e fixed and solve exposures under $\lambda = 0.95$ (near-PCP) with $\gamma = 0$ (short-run, risk-neutral). Higher value or warmer color always denotes strong natural hedging, either stronger alignment of revenue and cost exposures (higher EOS) or more variance canceled by revenue–cost co-movement (higher VRR).



(a) Exposure offset by s^M, s^X and χ^M, χ^X (b) Variance reduction by s^M, s^X and χ^M, χ^X

Figure 6: Heatmaps for alternative intensity balance and share matching

In the first figure, I observe two clear phenomena. First, improving share matching raises EOS (Random < Baseline < Perfect for any given intensity). Second, balanced intensity delivers the highest EOS, while import- or export-biased cases are weaker because one side dominates the exposure magnitudes. The peak occurs at balanced import-export intensities with baseline or perfect matching, confirming that natural hedging is strongest when the import and export baskets are well-aligned and of comparable size.

In the second figure, the variance reduction VRR shows similar patterns with EOS. Balanced intensities with better matching yields the strongest variance reduction (large positive VRR). Import-biased cases show negative hedging under random matching, but improve toward zero as matching moves to the baseline and perfect matching. Export-biased cases are mildly positive. Overall, natural hedging is the strongest in magnitude when exposures are both aligned (in matching) and balanced in size (in intensity).

5.5 Counterfactual Exchange Rate Environment

In the second group of counterfactual exercise, I focus on different macro scenarios of exchange rate dynamics, either for one currency or for the overall global foreign exchange market. Specifically, I will alter the external exchange rate uncertainty and covariance structure to study changes of firms' risk exposure and corresponding hedging performance.

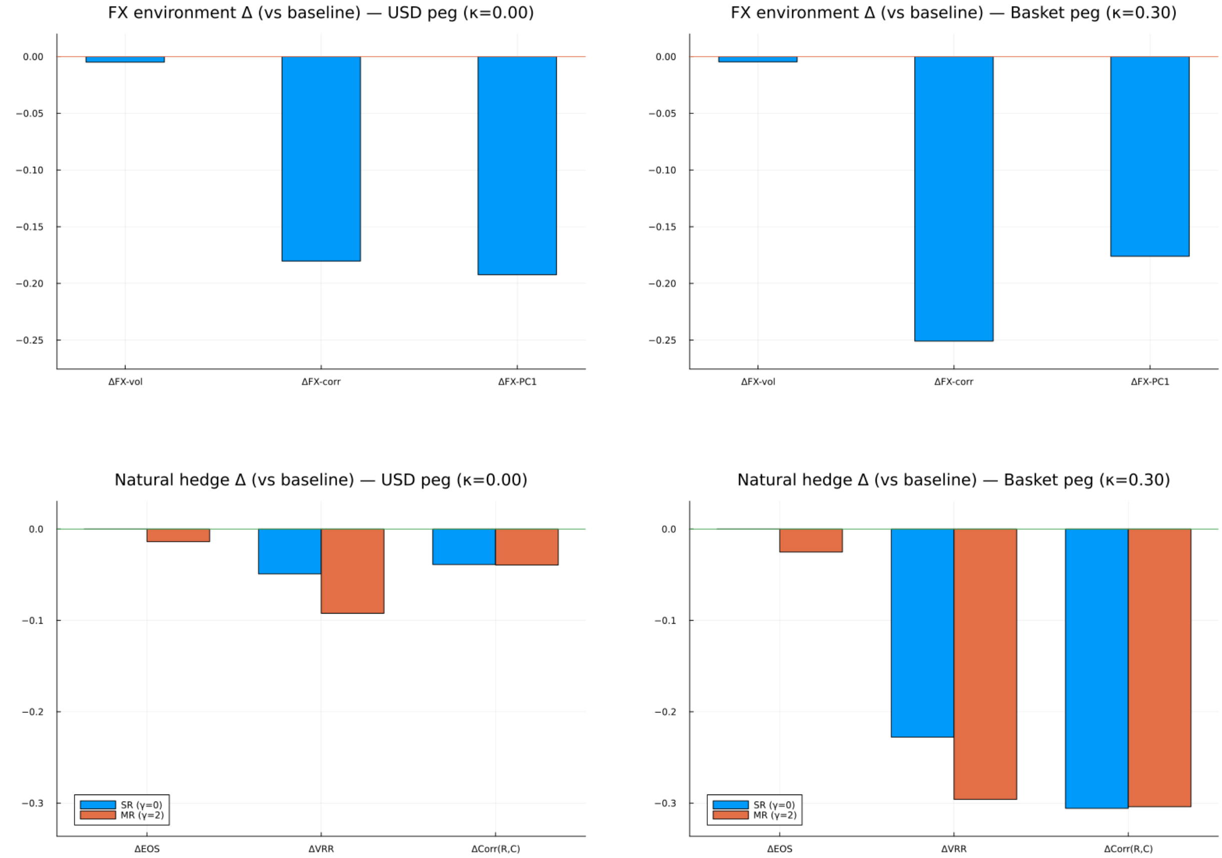
I summarize the ER environment shifts with three indicators: ΔFX-vol, the change in average currency volatility (scale-sensitive); ΔFX-corr, the change in average cross-currency correlation (scale-invariant); and ΔFX-PC1, the change in the variance share of the first principal component (scale-invariant). ΔFX-vol mainly scales risk: global hikes increase absolute risk, whereas idiosyncratic hikes raise volatility while weakening co-movement. By contrast, higher ΔFX-corr or ΔFX-PC1 indicates a stronger common ER factor and raises the co-movement of revenues and costs.

In all figures, the top panels report changes in the ER environment (ΔFX-vol, ΔFX-corr, ΔFX-PC1). The bottom panels report changes in natural hedging effectiveness (ΔEOS, ΔVRR, $\Delta Corr(R, C)$). In all scenarios, I compare a risk-neutral case ($\gamma = 0$) where the trade structure is fixed in the short run (SR) , and a risk-averse case ($\gamma = 2$) in which firms can partially adjust their trade shares and exposure in the medium run (MR). The comparison of SR and MR will highlight the central role of risk aversion and endogenous natural hedging.

Currency pegs

First, I impose hypothetical currency peg by changing certain entries of the exchange rate covariance matrix Σ^e. Two peg regimes are considered: a USD peg and a basket peg. The selected currency basket in this section is identical to 20 currencies in the covariance matrix with weights equal to the average of the base-year import share χ_0^M and the export share χ_0^X (normalized to sum to 1).[16] I compare both pegging schemes against the real-world exchange rate series. The results are shown in Figure 7.

[16] To make the magnitude comparable and allow minor adjustments of currency basket due to trade adjustments, I use partial tracking with strength κ^e: the smaller the value, the tighter the tracking. For example, $\kappa^e = 0.3$ indicates that the standard deviation in that direction is only 30% of the original value.



(a) Peg to USD

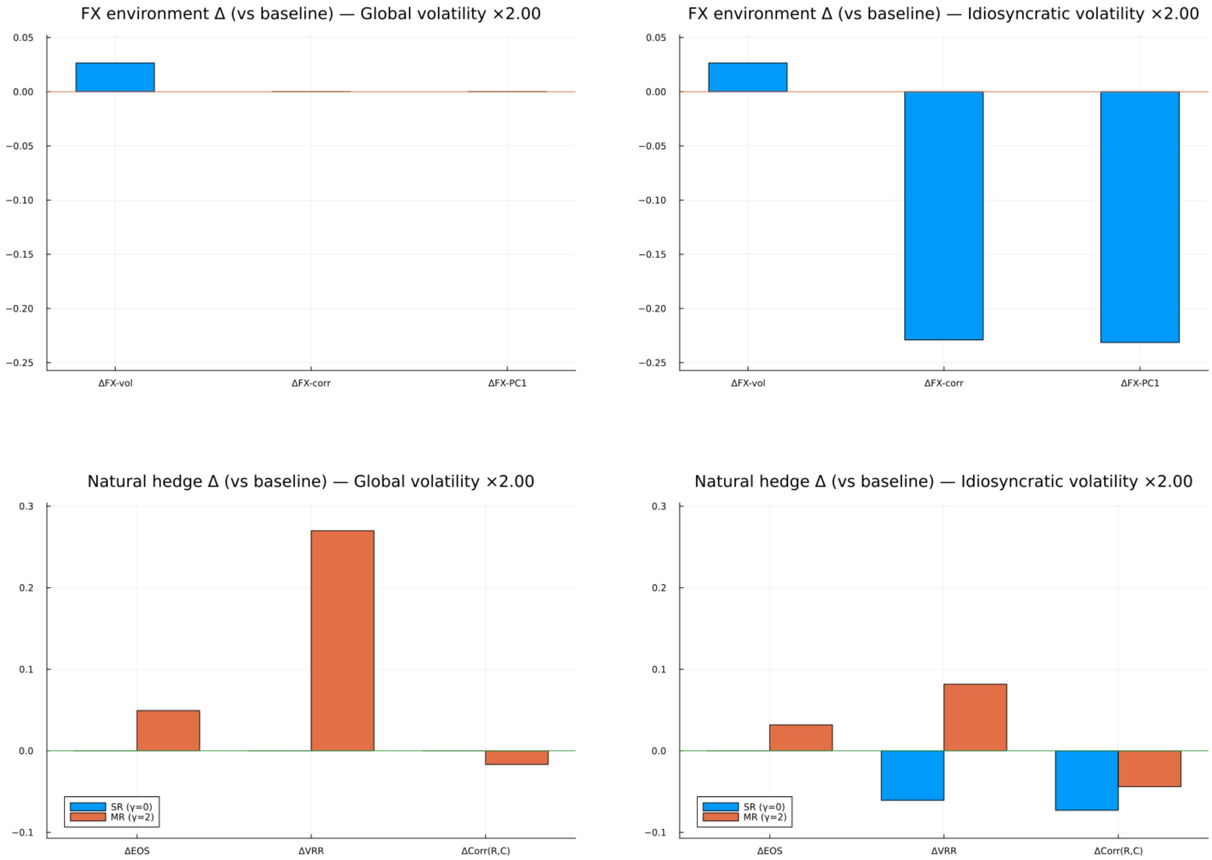
(b) Peg to currency basket

Figure 7: Currency pegs

Both pegs leave volatility roughly unchanged but sharply reduce cross-currency co-movement. The reduction is small under a USD peg and large under a basket peg (ΔFX-corr, ΔFX-PC1 $<$ 0). EOS almost does not change in the short run because the trade structure is fixed, however, in the medium run, risk averse firms cannot recreate the missing common factor; instead they tilt toward the pegged currency and trim the opposite side, so ΔEOS is slightly negative and ΔVRR falls more in MR than in SR (and most under the basket peg). When $Cov(R, C)$ is mechanically compressed by the peg, risk-averse adjustment could not raise co-movement, so the profit variance rises and the hedge weakens.

Volatility amplification

Another hypothetical scenario is the rise of exchange rate volatility. I amplify exchange rate volatility by (i) doubling the global common component and (ii) doubling idiosyncratic currency-specific components. Specifically, the global hikes increase all diagonal and non-diagonal elements in Σ^e while keeping correlations while the idiosyncratic hikes increase only diagonal elements in Σ^e but keep all off-diagonal elements unchanged, and decrease the average correlation between different currency pairs. This counterfactual test aims at showing how a significant rise in exchange rate volatility of the RMB affects firms' optimal import sourcing and export responses.



(a) Global volatility hikes

(b) Idiosyncratic volatility hikes

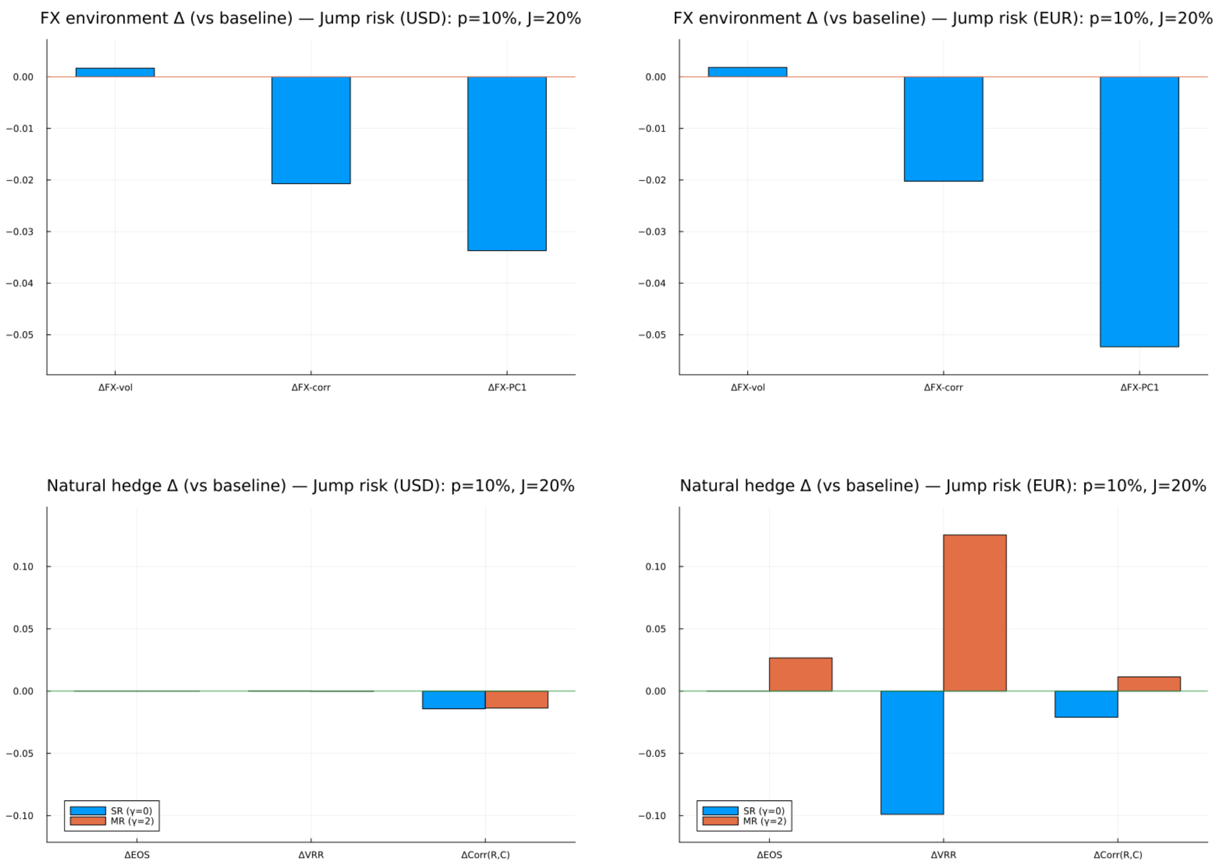
Figure 8: Volatility amplification

Amplifying the global component raises overall ER volatility slightly but leaves the correlation structure and first principal component unchanged (ΔFX-corr ≈ 0, ΔFX-PC1 ≈ 0). ΔVRR and $\Delta\rho(R, C)$ are closed to 0 for SR. However, MR delivers sizable gains ($\Delta EOS > 0$, $\Delta VRR > 0$): firms rebalance their trade shares so revenue and cost magnitudes move more symmetrically, cutting profit variance. By contrast, amplifying idiosyncratic volatility reduces cross-currency co-movement (ΔFX-corr < 0, ΔFX-PC1 < 0) and modestly weakens natural hedging in the SR (negative ΔVRR and $\Delta\rho(R, C)$). However, MR increases both VRR and EOS: risk-averse firms can cut the more volatile side and re-align import-export shares towards better matching, improving the relative covariance even when the correlation declines.

The results illustrate firms' strong incentive to restructure import-export linkages to mitigate profit volatility. Conversely, if the overall exchange rate volatility declines (e.g., more stringent exchange rate intervention), risk-averse firms can save on the costs of hedging risk and pursue higher expected profits.

Jump-mixture Risk

Low-probability but large-magnitude exchange rate jumps (devaluation or revaluation) increase tail risks in trade (Alessandria, Kaboski and Midrigan, 2010; Auer, Burstein and Lein, 2021; Blaum, 2024). I introduce rare but sizable jumps on certain currency to model the extreme cases. First, I assume that with probability $p = 10\%$, the USD could depreciate or appreciate by $|J| = 20\%$. This "jump-mixture" scenario adds a mean shift by pJ and variance shift by $p(1-p)J^2$ only to the bilateral exchange rate with USD. I repeat the experiment but replace jumps on the EUR (same $p = 10\%$, $|J| = 20\%$).



(a) Jump-mix risk of USD

(b) Jump-mix risk of EUR

Figure 9: Jump-mixture risk

In the first panel, USD jumps slightly increase average ER volatility but reduce cross-currency co-movement (ΔFX-corr < 0; ΔFX-PC1 < 0). In the short run, EOS and VRR almost does not change, yet $\Delta\rho(R, C)$ is mildly negative. Firms spread USD exposure across revenue and cost without improving correlations, reflecting the central role of USD in trade invoicing. In the second panel, EUR jumps also lower ER co-movement (ΔFX-corr < 0; ΔFX-PC1 < 0) with only a tiny rise in overall volatility. Unlike the USD case, EUR jumps produce a different response: MR raises EOS and VRR markedly and nudges correlation up, while SR shows a fall in VRR and correlation. The interpretation is that firms' EUR exposures are more unbalanced across imports versus exports. Jump risk on a currency with initially unbalanced import-export loads invites active realignment toward that currency on both sides, so risk-averse behavior strengthens the natural hedging in the medium run.

6 Conclusion

In sum, this paper shows that firms' import–export linkages systematically blunt exchange rate exposure: two-way traders display markedly lower export elasticity to bilateral RER shocks—especially when the partner is both a supplier and a destination—and this attenuation is stronger for firms whose import and export networks closely overlap, consistent with natural hedging at the firm–country level. Building up from these facts, export- and import-weighted effective exchange rates have opposite-signed effects on export growth, and when entered jointly the magnitudes shrink for two-way traders, revealing portfolio offsetting of demand and cost channels.

The structural contribution is to embed a constant absolute risk aversion mean–variance objective into a global sourcing trade model, which delivers closed-form exposure accounting and two endogenous hedging margins — risk-adjusted pricing premia and risk-adjusted sourcing. The risk exposure are assembled in an exact-hat linear system suited for counterfactual analyses. I find higher risk aversion shifts firms toward stronger import-export alignment, lowers normalized profit variance, and raises the level–variance trade-off, while mixed PCP/LCP pricing maximizes hedging by balancing revenue and cost exposures. This framework also clarifies how changes in the exchange rate environment, such as shifts in average cross-currency correlation or common-factor strength, map into the revenue–cost co-movement and thus variance reduction, offering policy and managerial guidance on operational hedging and currency-pricing choices. Together, my approach provides a unified framework of natural hedging of exchange rate risk and a practical toolkit for evaluating sourcing patterns, pricing regimes, and exchange rate regimes from a perspective of network and covariance.

Looking forward, the same machinery scales to (i) a general-equilibrium setting with input–output feedback, (ii) dynamic decisions with adjustment costs and rational expectations, and (iii) a unified treatment that endogenizes invoice-currency choice alongside operational and financial hedging. It is also natural to extend the risk process beyond variance, allowing for jumps, skewness, and time-varying volatility, and to tighten causal identification using external variation in currency-correlation structure, invoice-currency diffusion, or policy shocks. Finally, cross-country and cross-industry applications can map heterogeneity in import intensity, demand elasticities, and pricing regimes into systematic differences in natural hedging.

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A Empirical Appendix

A.1 Constraints on Financial Hedging in China and Emerging Markets

Across major emerging markets and developing economies (EMDEs), the access to foreign-exchange (FX) derivatives by non-financial firms remains limited. Instruments are often short-tenor and unevenly available across currencies, and access is circumscribed by "actual-needs" rules, documentation hurdles, and collateral margin requirements. In China, official statistics show corporate hedge ratios well below full coverage despite recent gains. BIS surveys for Asia-Pacific report that formal hedging policies are far from universal and that hedging tenors concentrate below one year. Empirically, dollar funding frictions and persistent deviations from covered interest parity (CIP) raise the overall cost of forwards and swaps in EMDE currencies, particularly in financial distress. These facts collectively imply that "natural hedging" by matching currency income and payments in operations and asset and liability balance sheets is often the only scalable and affordable risk management channel for exporters and importers in EMDEs.

1. Limited corporate financial hedging in China

Chinese authorities report a rising but still partial reliance on financial hedges. SAFE indicates that in 2021 the corporate FX hedging ratio was 21.7%, up 4.6 p.p. from 2020; by 2024, officials stated the ratio was about 27% (and around 30% in later briefings). Even at these levels, the average firm is far from fully hedged. Two features help explain this: (i) China's "actual-needs" principle limits derivatives usage to documented underlying exposures; and (ii) fragmentation between onshore CNY and offshore CNH markets creates pricing gaps and liquidity asymmetries that firms must navigate.

2. Policy and market-access constraints across major EMDEs

Regulatory requirements in Asian countries typically link derivatives to underlying trade or financial risks, restrict speculative use, and impose documentation or time-limit restrictions.

In China, there is "actual-needs" rules for using financial derivatives which prohibits firms from currency speculation. NDFs are used offshore. Inshore or offshore segmentation persists. In India (RBI), users generally must evidence underlying contracted exposure (with limited de-minimis exceptions), are subject to position limits, and dealers may call for documents. Exchange-traded INR contracts are capped unless exposures are documented. In Indonesia, prudential regulation requires minimum FX hedging ratios for non-bank firms with foreign-currency debt (e.g., 25% of short-term FX liabilities) and imposes liquidity and rating tests. In Malaysia, the central bank does not recognize offshore trading in MYR, including non-deliverable forward (NDF). For Philippines and Thailand, hedging contracts must be coterminous with underlying exposures; non-resident local-currency positions are restricted via dedicated accounts.

These rules, intended to maintain stability, mechanically restrict the proportion of risk exposure that can be hedged. This poses particular challenges for small and medium-sized enterprises (SMEs) that cannot fully prepare documentation, meet credit testing requirements, or dynamically adjust hedging operations in response to order changes.

3. Depth, tenor and transaction costs

BIS's comparative evidence shows material bid-ask spreads and short available tenors in many EMDE currencies. For example, one-month forward/NDF quotes for several Asia-Pacific currencies carry spreads in the 10–50 pips range on institutional clips, far wider than for liquid G10 pairs; hedging tenors are mainly less than one year even for financial institutions and firms. For China specifically, the CNY–CNH segmentation is documented to generate time-varying pricing gaps and liquidity-driven basis swings between onshore and offshore markets. These frictions translate directly into higher and more volatile hedge costs for traders.

4. Funding frictions and the CIP basis

In frictionless markets, forward pricing locks in cross-currency funding at the covered interest parity (CIP) rate. Post-GFC research shows systematic and persistent CIP deviations even in G10 countries; in EMDEs the basis is larger and more volatile, widening in risk-off episodes and when dealer intermediation capacity is impaired. Recent IMF work constructs "purified" EM CIP measures and still finds economically meaningful deviations; separate IMF analysis shows offshore EM CIP deviations co-move with global risk and dealers' risk-bearing capacity, while onshore deviations remain segmented. For EMDE firms, these deviations show up as embedded premia in forwards/swaps, especially beyond very short tenors.

5. Collateral, documentation and operational barriers

Most Over-the-Counter (OTC) FX hedges for firms now sit under global margining frameworks for non-centrally cleared derivatives. Covered entities must exchange daily variation margin and initial margin (bankruptcy-remote) above thresholds—requiring credit support annexes (CSAs), eligible collateral schedules, custodial accounts, and daily operations. Industry surveys show that margin collected for uncleared derivatives is enormous (about 1.5 trillion USD IM+VM in 2024 among leading firms), underscoring the scale of collateral tied up in the system. Negotiating and maintaining the ISDA/CSA stack is time- and resource-intensive: ISDA's 2024 document-negotiation survey reports that timelines have not shortened since 2006, and independent benchmarks indicate a large share of ISDA/IM-CSA negotiations take months, with a non-trivial fraction exceeding six months—a heavy lift for SMEs with episodic FX needs.

6. Anecdotal evidence about financial hedging beyond Asia

The difficulty of obtaining affordable, long-tenor hedges is sufficiently acute that governments and MDBs are experimenting with publicly-supported hedging channels. In Brazil, authorities announced a program—implemented with the Inter-American Development Bank and the central bank—to extend cheaper, longer-dated FX derivatives to crowd in green investment, precisely because market hedges are scarce/expensive at long maturities. Similar assessments by the World Bank, OECD and specialized providers (e.g., TCX) note that long-dated EMDE FX hedges are thinly available and costly, particularly for infrastructure-style exposures.

7. Implications for the role of natural hedging

Taken together, the regulatory tying of hedges to documented exposures, short market tenors, wider spreads and episodic CIP premia, and the fixed costs of ISDA/CSA collateralization and negotiation make financial hedging an incomplete and sometimes prohibitively costly solution for a broad swath of EMDE exporters and importers. This underlines the economic rationale for natural hedges—pricing in trade currency, matching receivables/payables by currency, building FX-balanced supply chains, and structuring balance sheets to align FX cash inflows and outflows—as first-best instruments whenever feasible. In China and other EMDEs, these operational choices are not simply preferences; they are responses to binding market and policy frictions that limit the reach and affordability of FX derivatives.

A.2 Descriptive Figures

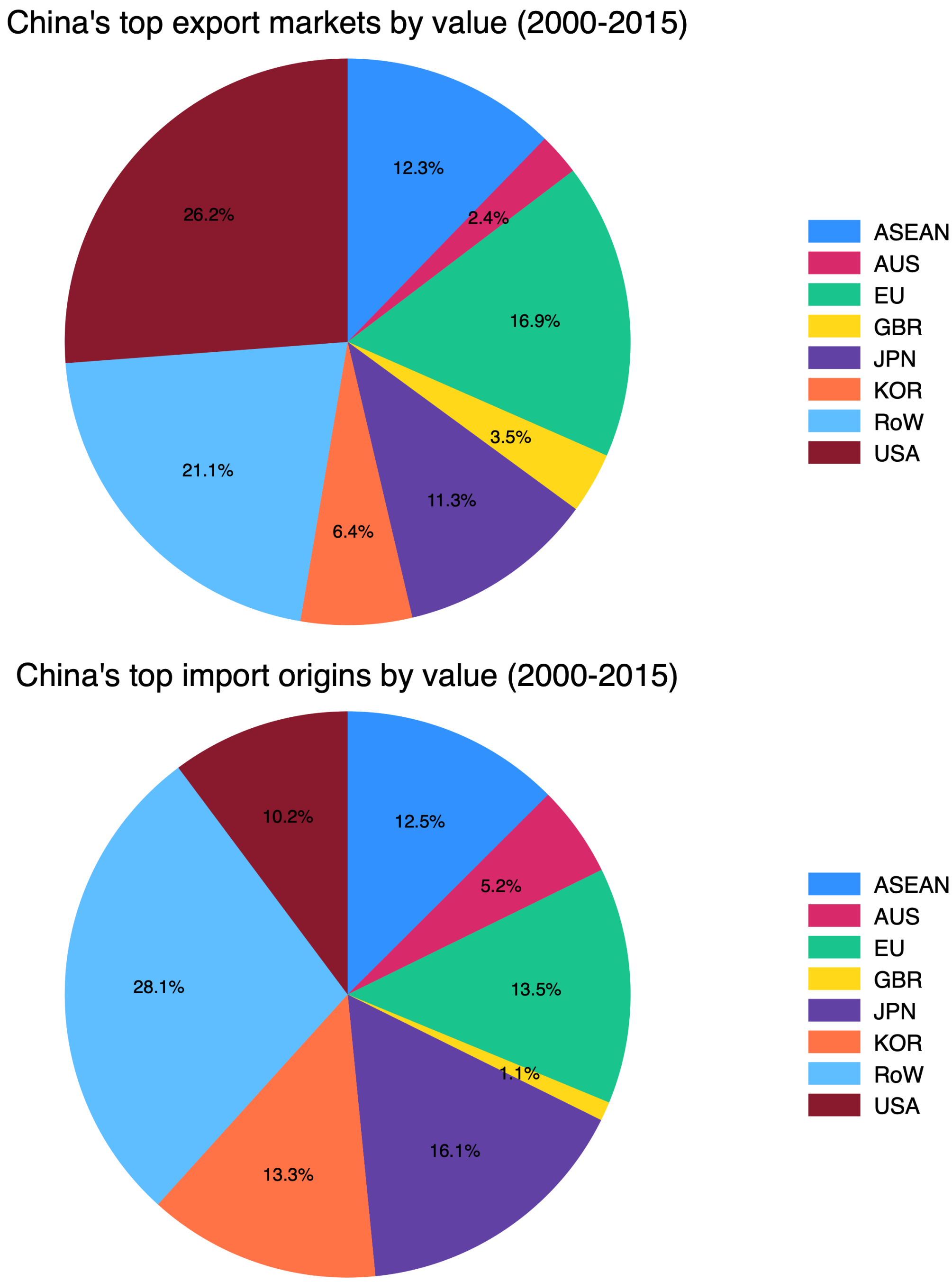

Figure A1: Firm trade size and diversity

Note: Shares of China's total imports and exports are aggregated over 2000–2015 and grouped into major partner blocs (ASEAN, Australia, EU, United Kingdom, Japan, Korea, United States) plus the rest of world (RoW). Values are measured in nominal trade value; percentages indicate each group's share of China's total imports (left) or total exports (right) over the period.

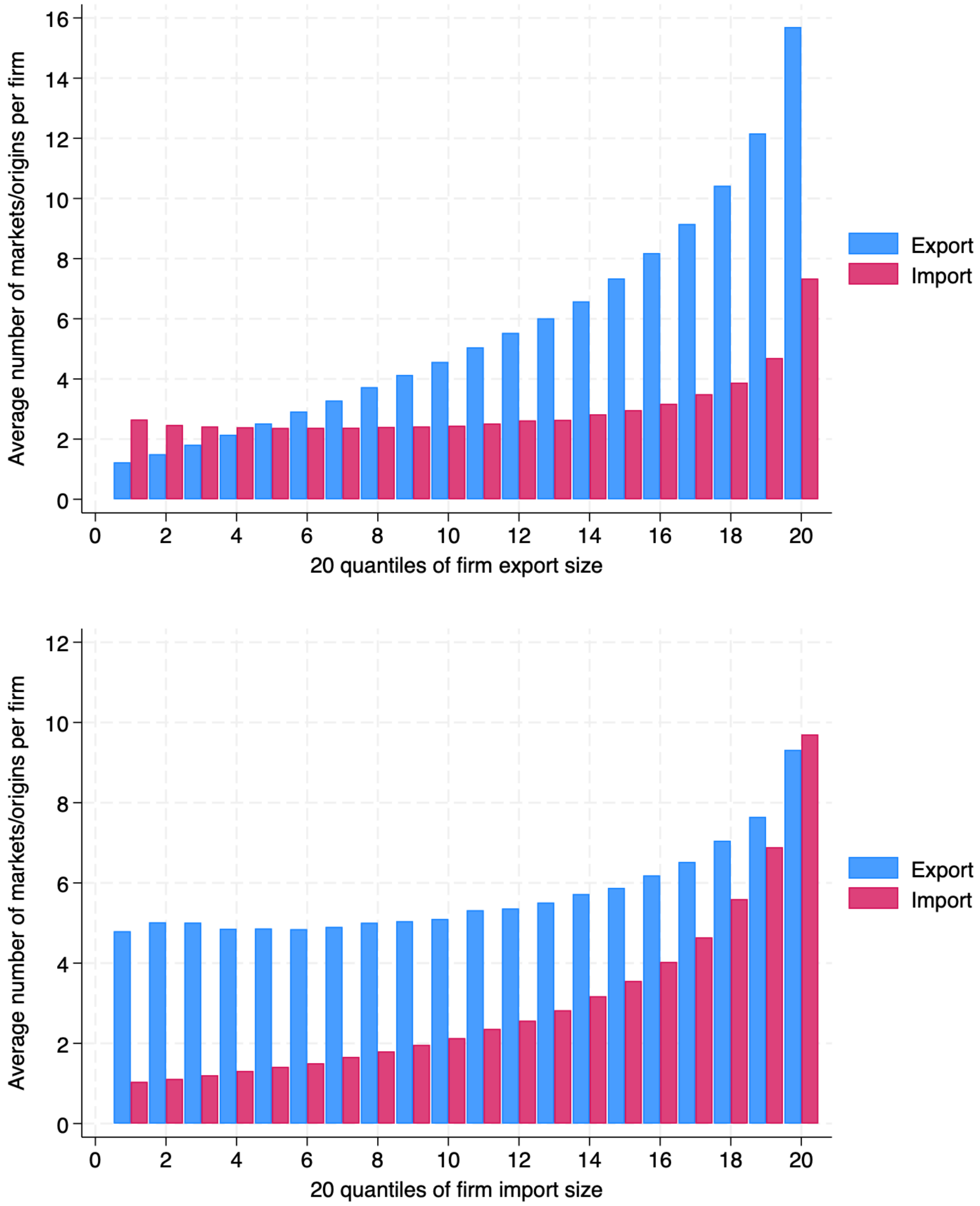

Figure A2: Firm trade size and diversity

Note: The bar charts show that for each 5% bin of firm size (left: by export value; right: by import value), the average number of distinct export destinations (blue) and import origins (red) per firm.

A.3 Notes on Import and Export Intensities

For all empirical moments including trade intensities and shares, I remove outliers (top and bottom 1% tails in export revenue) and only keep continuing firms who exist in the data for three consecutive years (in both the previous and the next year). Ideally, I should use precise firm-level total revenues and total input costs to calculate trade intensity and profits. However, general customs data typically do not provide such variables. Therefore, following standard practices in empirical trade research (e.g., Alfaro, Calani and Varela (2021)), I use the ratio of import (export) values to total imports and exports as a proxy for relative import (export) intensity. Additionally, I employ net trade income (exports minus imports) as a proxy for gross profit. This variable directly reflects the extent to which firms are exposed to exchange rate risk through trade activities. While it excludes domestic cost structures and non-trade income, it provides a unified measure of risk exposure for this study.

B Model Appendix

B.1 List of Variables and Parameters

Endogenous variables:

- p_{ii}: domestic sales price in home market i;
- p_{ki}: export sales price in market k;
- c_i: marginal cost of production;
- $P_{M,i}$: imported intermediate input price index;
- $P_{X,i}$: total intermediate input price index;
- $X_{M,i}$: quantity of imported intermediate inputs;
- X_i: quantity of total intermediate inputs;
- L_i: labor input
- y_i: production output;
- ψ_{ii}: risk premium in domestic sales price;
- ψ_{ki}: risk premium in export price;
- u_{il}: profit exposure of the firm to each currency.

Exogenous variables:

- e_{ij}: exchange rate of local currency i against currency of country j ($\uparrow$ indicates depreciation of local currency);

- $P_{D,i}$: domestic intermediate goods price index;
- P_j: foreign price index;
- τ_{ij}: iceberg trade cost;
- w_i: domestic wage level;
- S_i, S_j: domestic and foreign aggregate demand size.

Parameters (for estimation):

- σ: CES elasticity of demand across markets;
- λ: the weight of PCP pricing in export revenues ($1-\lambda$ is LCP);
- κ: internal CES elasticity of imports across countries.

Parameters (for calibration):

- α: expenditure share of intermediate goods in production function;
- ϵ: CES elasticity parameter for domestic vs imports;
- γ: risk aversion parameters;
- Σ^e: exchange rate covariance matrix.

Trade margins (directly observable):

- s_i^M: imported intermediate inputs over total intermediate inputs (import intensity);
- s_i^X: export revenue over total sales revenue (export intensity);
- χ_{ij}^M: import share of firm from origin country j;
- χ_{ki}^X: export share of firm i to market country k;

B.2 Currency Pricing Paradigm: PCP vs LCP

Exchange rates will affect both export price p_{ki} and import price $P_{M,i}$. Exchange rate pass-through depends on currency pricing paradigm.

Step-by-step modeling paths considering pricing paradigms:

- Step 1: assume PCP at the import side;

- Step 2: allow PCP/LCP mix at the export side;
- Step 3: allow PCP/LCP mix at the import side (extension).

For simplicity, I start with step 1 and 2 in the benchmark model.

The PCP part of export revenue:

$$R_{ki}^{PCP} = e_{ik} p_{ki}^{PCP} q_{ki} = e_{ik} (\frac{\bar{p}_{ki}^{i}}{e_{ik}})^{1-\sigma} (\frac{S_k}{P_k^{\sigma}}) \tag{B1}$$

The LCP part of export revenue:

$$R_{ki}^{LCP} = e_{ik} p_{ki}^{LCP} q_{ki} = e_{ik} (\bar{p}_{ki}^{k})^{1-\sigma} (\frac{S_k}{P_k^{\sigma}}) \tag{B2}$$

The total export revenue in market k:

$$\begin{aligned} R_{ki} &= \lambda R_{ki}^{PCP} + (1-\lambda) R_{ki}^{LCP} \\ &= \lambda p_{ki}^{PCP} q_{ki}^{PCP} + (1-\lambda) p_{ki}^{LCP} q_{ki}^{LCP} e_{ik} \\ &= \lambda (p_{ki}^{PCP})^{1-\sigma} P_k^{\sigma-1} S_k e_{ik}^{\sigma} + (1-\lambda)(p_{ki}^{LCP})^{1-\sigma} P_k^{\sigma-1} S_k e_{ik} \\ &= c_i^{1-\sigma} [\lambda d_{ki} B_k e_{ik}^{\sigma} + (1-\lambda) d'_{ki} B_k e_{ik}] \end{aligned} \tag{B3}$$

where $B_k = P_k^{\sigma-1} S_k$, d_{ki} and d'_{ki} are constant terms independent of exchange rate shocks.

Under the assumption of standard CES + risk neutrality, I have constant markup $\mu_{ki}^{PCP} = \frac{\sigma}{\sigma-1}$ which does not change with exchange rates, and $\mu_{ki}^{LCP} = \frac{\sigma}{\sigma-1} \frac{e_{ik}}{\bar{e}_{ik}}$, which varies with exchange rates. Therefore, $d_{ki} = (\frac{\sigma}{\sigma-1} \tau_{ki})^{1-\sigma}$ and $d'_{ki} = (\frac{\sigma}{\sigma-1} \tau_{ki} \bar{e}_{ik}^{-1})^{1-\sigma}$.

Producer Currency Pricing (PCP):

Local price in export market k from i under PCP:

$$\begin{aligned} p_{ki}^{PCP}(\omega) &= \mu_{ki}^{PCP} \tau_{ki} c_i e_{ki} \\ &= \frac{\sigma}{\sigma-1} \tau_{ki} c_i e_{ik}^{-1} \end{aligned} \tag{B4}$$

The PCP demand function in export market k:

$$q_{ki}^{PCP}(\omega) = p_{ki}^{PCP}(\omega)^{1-\sigma} {P_k}^{\sigma-1} S_k \tag{B5}$$

Export revenue in home currency i under PCP:

$$R_{ki}^{PCP} = (\frac{\sigma}{\sigma-1} \tau_{ki} c_i)^{1-\sigma} P_k^{\sigma-1} S_k e_{ik}^{\sigma} \tag{B6}$$

Local Currency Pricing (LCP):

Local price in export market k from i under LCP:

$$\begin{aligned} p_{ki}^{LCP}(\omega) &= \mu_{ki}^{PCP}\tau_{ki}c_i e_{ki} \\ &= \frac{\sigma}{\sigma-1}\frac{e_{ik}}{\bar{e}_{ik}}\tau_{ki}c_i e_{ik}^{-1} \\ &= \frac{\sigma}{\sigma-1}\tau_{ki}c_i\bar{e}_{ik}^{-1} \end{aligned} \tag{B7}$$

The LCP demand function in export market k:

$$q_{ki}^{LCP}(\omega) = p_{ki}^{LCP}(\omega)^{1-\sigma}P_k{}^{\sigma-1}S_k \tag{B8}$$

Export revenue in home currency i under LCP:

$$R_{ki}^{LCP} = (\frac{\sigma}{\sigma-1}\tau_{ki}c_i\bar{e}_{ik}^{-1})^{1-\sigma}P_k^{\sigma-1}S_k e_{ik} \tag{B9}$$

B.3 Explanations for PTM

Within the framework of risk aversion, I prefer to assume pricing-to-market (PTM, allowing firms to set a different p_{ki} for each market k) because:

1. Allowing for targeted hedging of different exchange rate exposures.
   - Natural hedging reduces overall profit volatility by offsetting "exposures" in different markets.
   - If firms only use a unique cross-market pricing (not PTM), exchange rate exposures to all destinations would be bundled into a single price p_i. There would be no risk management to account for correlations and volatility of currencies across countries.
   - Firms under PTM can proactively raise or lower the $1-\sigma$ multiplier of price elasticity in some markets to optimize their risk exposure in that market.
2. The essence of optimal pricing under risk aversion is "market differentiation".
   - In the FOC with cost of risk, I have $(1-\sigma)\frac{R_{ki}}{p_{ki}} = \frac{\gamma}{2}\frac{\partial Var(\pi_i)}{\partial p_{ki}}$.
   - The right-hand side $\frac{\partial Var}{\partial p_{ki}}$ will vary depending on the covariance of each market with other markets, including the import network.
   - Only by allowing firms to adjust each p_{ki} individually can they truly "risk-optimize" their allocations by keeping exposure low in high covariance markets and increasing exposure in low or negative covariance markets.
3. Reconciling price discrimination with the need for value preservation.
   - Empirically, most multinationals price differently across countries.
   - Risk aversion + natural hedging motives $\rightarrow$ higher risk premiums (markups) in risky markets, lower premiums or higher sales in more hedged markets.

B.4 Derivation of Price Risk Premium

The partial derivative of expected profit to domestic sales price and export prices are:

$$\frac{\partial E[\pi_i]}{\partial p_{ii}} = (1-\sigma)q_{ii}, \quad \frac{\partial E[\pi_i]}{\partial p_{ki}} = (1-\sigma)q_{ki}$$

The partial derivative of profit variance to domestic sales price:

$$\frac{\partial Var(\pi_i)}{\partial p_{ii}} = 2\sum_{j,l} \frac{\partial u_{ij}}{\partial p_{ii}} \Sigma^e_{jl} u_{il} = 2(1-\sigma)^2 \alpha s_i^M q_{ii} \sum_{j,l} \chi^M_{ij} \Sigma^e_{jl} u_{il} \tag{B10}$$

The partial derivative of profit variance to export price:

$$\frac{\partial Var(\pi_i)}{\partial p_{ki}} = 2\sum_{j,l} \frac{\partial u_{ij}}{\partial p_{ki}} \Sigma^e_{jl} u_{il} = 2(1-\sigma)q_{ki} \sum_{j,l} \varepsilon^R_{ij} \Sigma^e_{jl} u_{il} \tag{B11}$$

because derivatives of exchange rate risk exposure on domestic and export prices:

$$\frac{\partial u_{il}}{\partial p_{ii}} = \frac{\partial a_{il}}{\partial p_{ii}} = (1-\sigma)\alpha s_i^M \chi^M_{il} \frac{\partial R_{ii}}{\partial p_{ii}} = (1-\sigma)^2 \alpha s_i^M \chi^M_{il} q_{ii} \tag{B12}$$

$$\frac{\partial u_{il}}{\partial p_{ki}} = \frac{\partial b_{ik,l}}{\partial p_{ki}} = \varepsilon^R_{il} \frac{\partial R_{ki}}{\partial p_{ki}} = (1-\sigma)\varepsilon^R_{il} q_{ki} \tag{B13}$$

Finally, I can solve ψ_{ii} and ψ_{ki} as: $\psi_{ii} = \gamma(1-\sigma)\alpha s_i^M \sum_{j,l} \chi^M_{ij} u_{il} \sigma^e_{jl}$ and $\psi_{ki} = \gamma \sum_{j,l} \varepsilon^{R_k}_{ij} u_{il} \sigma^e_{jl}$.

B.5 Time-varying Risk Premium

In the extension, I could allow risk premium terms to change in the exact-hat linear system. Let's define $\mathcal{D}_{ii} = \sum_{j,l} \chi^M_{ij} \Sigma^e_{jl} u_{il}$ and $\mathcal{D}_{ki} = \sum_{j,l} \varepsilon^{R_k}_{ij} \Sigma^e_{jl} u_{il}$. The risk premium terms can be written as $\psi_{ii} = \gamma(1-\sigma)\alpha s_i^M \mathcal{D}_{ii}$, and $\psi_{ki} = \gamma \mathcal{D}_{ki}$.

Now, the hat variables of risk premiums can be written as:

$$\hat{\psi}_{ii} = \frac{\sum_{j,l} \chi^M_{ij} \Sigma^e_{jl} u_{il} \hat{u}_{il}}{\mathcal{D}_{ii}} \tag{B14}$$

$$\hat{\psi}_{ki} = \frac{\sum_{j,l} \varepsilon^{R_k}_{ij} \Sigma^e_{jl} u_{il} \hat{u}_{il}}{\mathcal{D}_{ki}} \tag{B15}$$

With the exposure decomposition, I have profit risk exposure to country l:

$$\hat{u}_{il} = \frac{a_{il}}{u_{il}}\hat{R}_{ii} + \sum_k \frac{b_{il,k}}{u_{il}}\hat{R}_{ki} - \frac{c_{il}}{u_{il}}\hat{C}_{M,i} \tag{B16}$$

I decompose the risk premium hats into three component shocks (local sales, exports, costs):

$$\hat{\psi}_{ii} = \theta^a_{ii}\hat{R}_{ii} + \sum_k \theta^b_{ii,k}\hat{R}_{ki} + \theta^c_{ii}\hat{C}_{M,i} = \vec{\theta}_{ii}[\hat{R}_{ii}, \{\hat{R}_{ki}\}, \hat{C}_{M,i}] \tag{ψ-dom}$$

$$\hat{\psi}_{ki} = \theta^a_{ki}\hat{R}_{ii} + \sum_k \theta^b_{ki,k}\hat{R}_{ki} + \theta^c_{ki}\hat{C}_{M,i} = \vec{\theta}_{ki}[\hat{R}_{ii}, \{\hat{R}_{ki}\}, \hat{C}_{M,i}] \tag{ψ-exp}$$

The risk premium weight vectors $\vec{\theta}_{ii} = [\theta^a_{ii}, \{\theta^b_{ii,k}\}_k, \theta^c_{ii}]^\top$ and $\vec{\theta}_{ki} = [\theta^a_{ki}, \{\theta^b_{ki,k}\}_k, \theta^c_{ki}]^\top$ are measures of the sensitivity of the "marginal risk exposure". They tell us how much a 1% change in each revenue or cost component feeds into the risk-premium hat.

The risk premium weight vectors $\vec{\theta}_{ii}$ and $\vec{\theta}_{ki}$ include all base-year information from risk aversion γ, exposure vector u_{il}, the exchange rate covariance matrix Σ^e, and the relevant weights (import shares χ^M_{ij} or export elasticities ε^R_{ij}).

For the domestic risk premium weight vector $\vec{\theta}_{ii}$:

$$\theta^a_{ii} = \frac{(1-\sigma)\alpha s^M_i \sum_{j,l} \chi^M_{ij}\Sigma^e_{jl}\chi^M_{il}}{\mathcal{D}_{ii}} \tag{B17}$$

$$\theta^{bk}_{ii} = \frac{\sum_{j,l} \chi^M_{ij}\Sigma^e_{jl}\varepsilon^R_{il}\frac{R_{ki}}{R_{ii}}}{\mathcal{D}_{ii}} \tag{B18}$$

$$\theta^c_{ii} = \frac{\sum_{j,l} \chi^M_{ij}\Sigma^e_{jl}\chi^M_{il}\frac{C_{M,i}}{R_{ii}}}{\mathcal{D}_{ii}} \tag{B19}$$

For the export risk premium weight vector $\vec{\theta}_{ki}$:

$$\theta^{bk}_{ki} = \frac{\sum_{j,l} \varepsilon^R_{ij}\Sigma^e_{jl}\varepsilon^R_{il}}{\mathcal{D}_{ki}}. \tag{B20}$$

All constant factors γ, $(1-\sigma)$, and αs^M_i cancel in the hat.

B.6 Cross-sectional Exposure Vectors

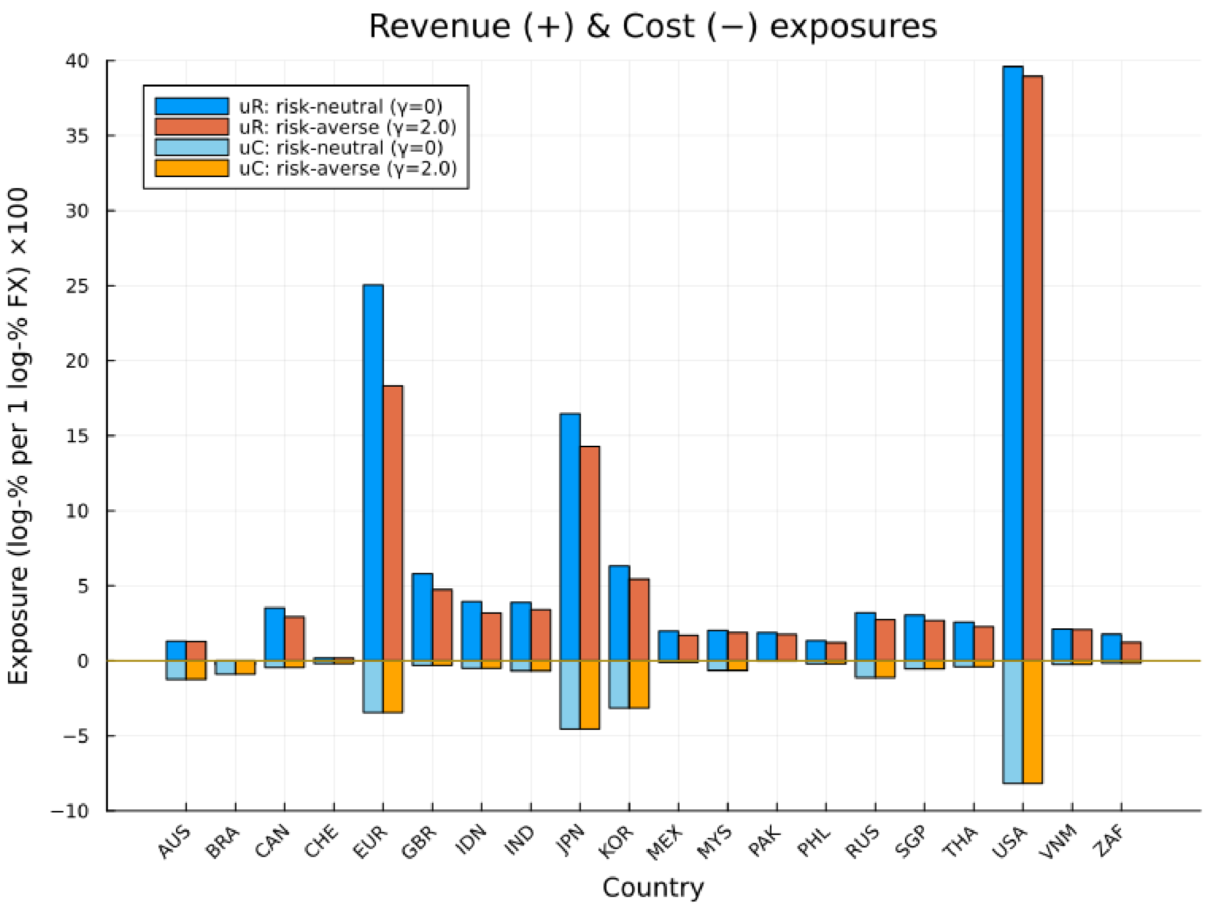

Figure B1: Revenue and cost exposures in 2005

C Numerical Appendix

C.1 Risk-Neutral Exact-hat Algebra Linear Equations

For the baseline model, I will build a static system of exact-hat algebra linear equations.

Let's define $\hat{x} = \frac{\tilde{x}}{x} = \ln x' - \ln x$. All nonlinear equations can be first-order linearly approximated in logarithmic terms. By fixing the import intensity and shares s_i^M, χ_{ij}^M as "base-period weights", all the unknowns can be written in the hat variable as a linear system.

- Fix the exogenous elements:$\{\hat{e}_{ij}, \hat{w}_j, \hat{\tau}_{ij}, \hat{P}_i, \hat{P}_k, \hat{S}_i, \hat{S}_k\}$..
- Determine the unknowns: $\{\hat{p}_{ii}, \hat{p}_{ki}, \hat{X}_{M,i}, \hat{L}_i, \hat{c}_i, \ldots\}$

The system of linear equations $A\hat{\mathbf{z}} = B\hat{\mathbf{s}}$ can be solved once for all logarithmic changes.

(F1) FOC for domestic price:

$$\hat{p}_{ii} = \hat{c}_i + \hat{\psi}_{ii} \tag{F1*}$$

(F2) FOC for export price:

$$\hat{p}_{ki} = \hat{c}_i + \hat{\tau}_{ki} + \lambda \hat{e}_{ik} + \hat{\psi}_{ki} \tag{F2*}$$

(P0) Country-specific import price:

$$\hat{p}_{ij} = \hat{w}_j + \hat{\tau}_{ij} + \hat{e}_{ij} \tag{C1}$$

(P1) Import price index:

$$\hat{P}_{M,i} = \sum_j \chi_{ij}^M \hat{p}_{ij} \tag{P1}$$

(P2) Total input price index:

$$\hat{P}_{X,i} = s_i^M \hat{P}_{M,i} + (1 - s_i^M)\hat{P}_{D,i} \tag{P2}$$

(MC) Marginal cost of production:

$$\hat{c}_i = (1 - \alpha)\hat{w}_i + \alpha \hat{P}_{X,i} \tag{MC}$$

(C1) Input under cost minimization:

$$\hat{X}_{M,i} - \hat{X}_i = -\varepsilon(\hat{P}_{M,i} - \hat{P}_{X,i}) \tag{C1}$$

(M) Country-specific import demand:

$$\hat{x}_{ij} - \hat{X}_{M,i} = -\kappa(\hat{p}_{ij} - \hat{P}_{M,i}) \tag{C2}$$

(Y1) Cobb-Douglas production:

$$\hat{y}_i = (1 - \alpha)\hat{L}_i + \alpha \hat{X}_i \tag{Y1}$$

(Y2) Output-demand identity:

$$\hat{y}_i = (1 - s_i^X)\hat{q}_{ii} + s_i^X \sum_k \chi_{ki}^X \hat{q}_{ki} \tag{Y2}$$

(D1) Sales in domestic market:

$$\hat{q}_{ii} = -\sigma \hat{p}_{ii} + (\sigma - 1)\hat{P}_i + \hat{S}_i \tag{D1}$$

(D2) Sales in foreign market k:

$$\hat{q}_{ki} = -\sigma \hat{p}_{ki} + (\sigma - 1)\hat{P}_k + \hat{S}_k + \lambda\sigma\hat{e}_{ik} \tag{D2}$$

For illustration, I can combine (Y2) with (D1) and (D2) and express quantity as the prices under exchange rate pass-through in the adjusted revenue equation (R+):

$$\begin{aligned}\hat{R}_i = &\underbrace{(1-\sigma)[(1-s_i^X)\hat{p}_{ii} + s_i^X \sum_k \chi_k^X \hat{p}_{ki}]}_{\text{price effect}} + \underbrace{(1-s_i^X)\hat{S}_i + s_i^X \sum_k \chi_{ki}^X \hat{S}_k}_{\text{exogenous demand shock}} \\ &\underbrace{+(\sigma-1)[(1-s_i^X)\hat{P}_i + s_i^X \sum_k \chi_{ki}^X \hat{P}_k]}_{\text{price index effect}} + \underbrace{\lambda\sigma \sum_k \chi_{ki}^X e_{ik}}_{\text{PCP direct effect}}\end{aligned} \tag{Y+}$$

C.2 Ledoit-Wolf Shrinkage Covariance

Ledoit-Wolf shrinkage covariance is one of the default practices in modern high-dimensional empirical and financial risk measures.

In the Ledoit-Wolf shrinkage covariance, F_{cc} is the constant-correlation matrix:

$$F_{\text{cc}} = \bar{\sigma}^2 \left[\bar{\rho}\mathbf{1}\mathbf{1}' + (1-\bar{\rho})I_N\right], \quad \bar{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}\sigma_i^2, \quad \bar{\rho} = \frac{1}{N(N-1)}\sum_{i\neq j}\frac{S_{ij}}{\sigma_i\sigma_j} \tag{C3}$$

$\delta \in [0,1]$ is shrinkage intensity.

Ledoit and Wolf (2004) gives the closed-form optimal weighting:

$$\delta^* = \frac{\beta^2}{\alpha^2+\beta^2}, \qquad \begin{cases}\alpha^2 = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\operatorname{Var}(S_{ij}) \\ \beta^2 = \|S-F\|_F^2\end{cases} \tag{C4}$$

in which where $\|\cdot\|_F$ is the Frobenius norm; α^2, β^2 can be used for unbiased estimation of sample moments, with computational complexity is $O(N^2T)$. δ^* decreases to 0 as T increases (large sample shrinkage disappears in longer time period); increases as the ratio of N over T increases (stronger shrinkage in high dimensions in wider panel).

C.3 Estimated Exchange Rate Correlation Matrix

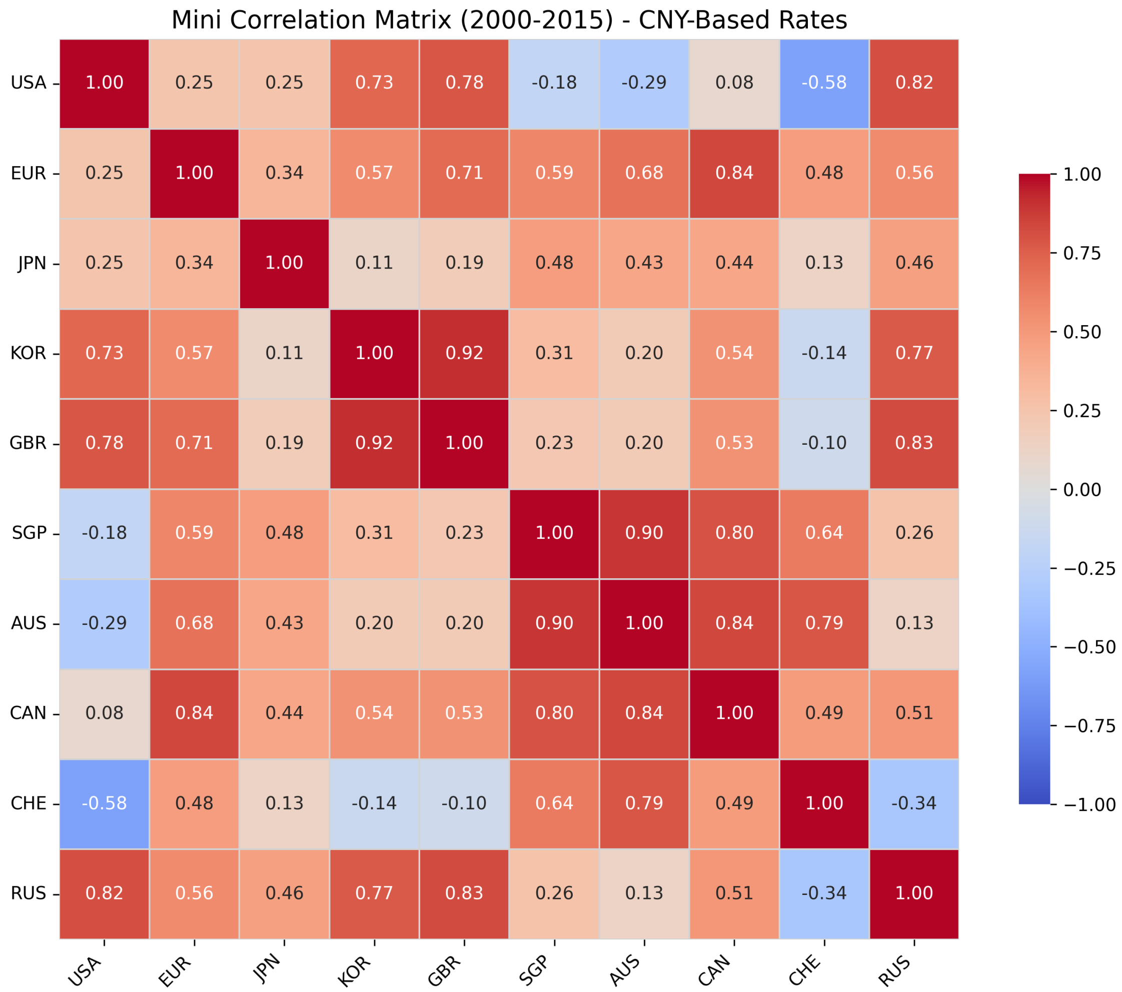

Figure C1: Estimated exchange rate correlation matrix of 10 currencies

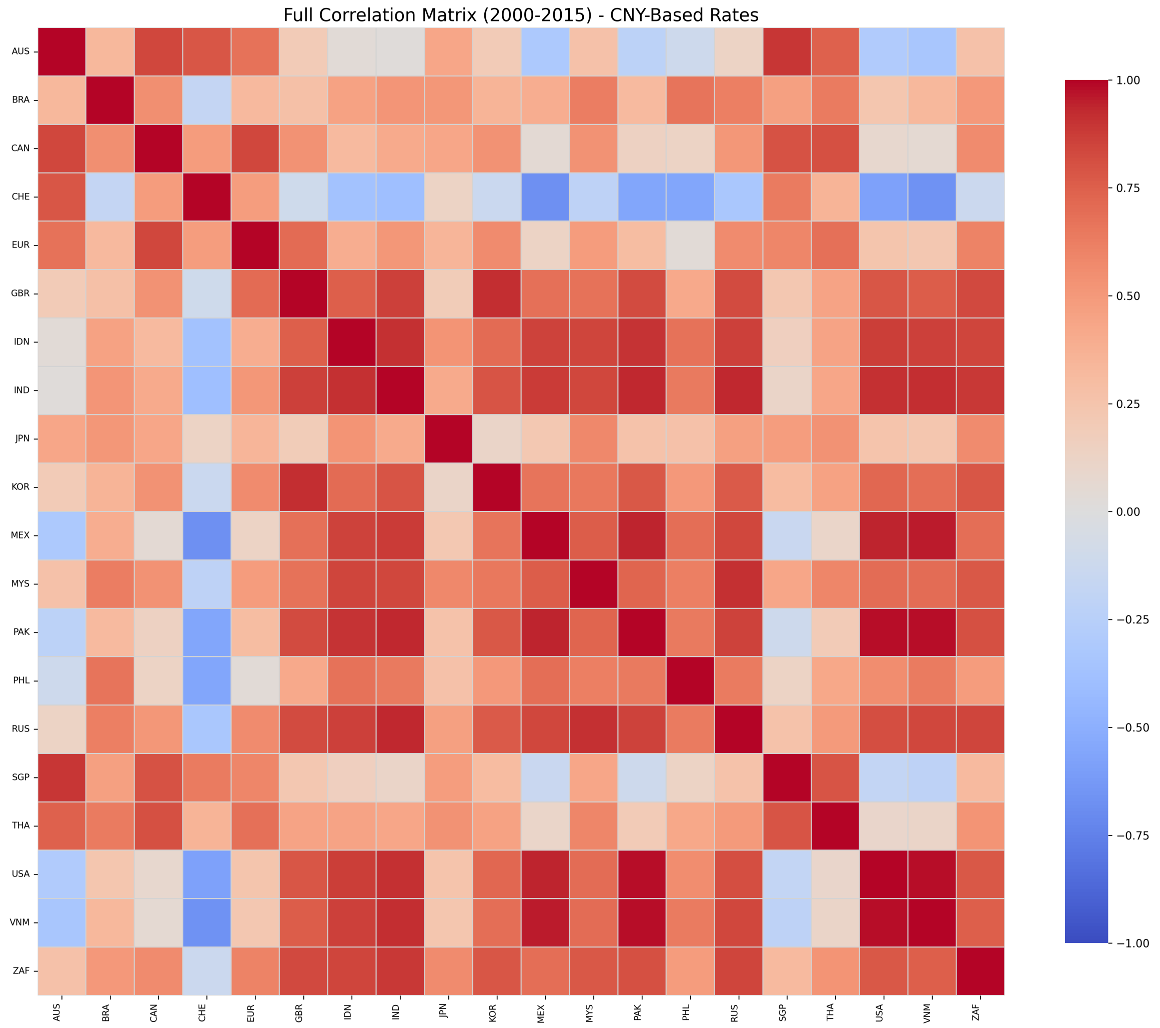

Figure C2: Estimated exchange rate correlation matrix of 30 currencies

C.4 Identifying Risk Aversion from Volatility Sensitivity Moments

This appendix describes how the model disciplines the firm's risk aversion parameter, γ, using cross-sectional sensitivity of natural hedging effectiveness measures to exchange rate volatility. The key idea is that γ does not just affect the overall level of hedging, but the extent to which high-volatility firms hedge more aggressively than low-volatility firms. We therefore target volatility-sensitivity moments rather than levels.

Firms choose import sourcing, export pricing, and market allocation to maximize a mean–variance objective, $\max \mathbb{E}[\pi_i] - \frac{\gamma}{2}\text{Var}(\pi_i)$, where π_i is operating profit in domestic currency. When $\gamma = 0$ (risk neutral), the firm does not distort sourcing or pricing purely to reduce variance: it responds only to expected profitability. When $\gamma > 0$, the firm is willing to reallocate purchases and adjust markups in order to reduce $\mathrm{Var}(\pi_i)$, especially in environments where exchange rate movements are large and volatile. In other words, the effect of γ is strongest precisely when the firm faces high currency risk.

We capture these adjustments using two natural hedging measures: (i) EOS, which measures how closely revenue and cost exposures to bilateral exchange rates offset each other, and (ii) VRR, which measures how much gross revenue–cost co-movement eliminates residual profit variance. For each firm i, let σ_i^e be the volatility of its effective currency exposure. We then estimate cross-sectional relationships of the form

$$\mathrm{EOS}_i = \alpha_E + \beta_E \sigma_i^e + \epsilon_{E,i}, \quad \mathrm{VRR}_i = \alpha_V + \beta_V \sigma_i^e + \epsilon_{V,i}$$

or equivalently, the difference in average EOS (or VRR) between high-σ^e and low-σ^e firms. The slope coefficients β_E and β_V summarize how strongly firms in riskier currency environments raise their natural hedging intensity. In the model, these slopes are increasing in γ: a more risk-averse firm tilts sourcing and pricing more aggressively in response to higher volatility. When $\gamma = 0$, β_E and β_V are close to zero because the firm has no incentive to trade off expected profits for variance reduction.

These slope (or high–low gap) moments are the core moments used to identify γ in simulated method of moments. We choose γ so that the model-generated $\beta_E^{\mathrm{sim}}(\gamma)$and $\beta_V^{\mathrm{sim}}(\gamma)$ match their empirical counterparts β_E^{emp} and β_V^{emp}. This approach has two advantages over matching levels of EOS or VRR directly. First, EOS and VRR levels are influenced by many forces unrelated to risk aversion—pricing conventions (PCP vs LCP), invoicing currency, input specificity, import diversification, and technological constraints. These forces push up or down hedging for all firms, regardless of volatility. Second, differencing across volatility strips out those common drivers and isolates firms' marginal response to risk, which is exactly what γ governs in the mean–variance problem.

In summary, the identification of γ is based not on how hedged the average firm looks, but on how much extra hedging high-volatility firms do relative to otherwise similar low-volatility firms. The volatility-sensitivity of EOS and VRR provides a set of moments that (i) has a clear behavioral interpretation, (ii) maps monotonically into γ in the model, and (iii) is robust to institutional features that shift hedging levels mechanically.

C.5 Empirical Measures of Hedging Effectiveness

Specifically, I measure firm-level exposure-offset share (EOS) using import-export networks, and firm-level profit volatility (standard deviation of profits).

Moment 1: Exposure-offset share (EOS)

Combining the data to the model, the empirical measure of exchange exposure-offset share

(EOS) at the firm level is defined as:

$$EOS_{ft}^{data} = 1 - \frac{\|(\vec{\chi}_{ft-1}^{X} + \vec{\chi}_{ft-1}^{M})^{\top}\vec{e}_{it}\|}{\|(\vec{\chi}_{ft-1}^{X})^{\top}\vec{e}_{it} + (\vec{\chi}_{ft-1}^{X})^{\top}\vec{e}_{it}\|} \tag{C5}$$

where the exposure vectors for revenue and costs are proxies by the firm-level exchange rate shocks weighted by export and import shares.

Moment 2: Variance Reduction Ratio

The profit volatility at the firm level is defined as the standard deviation of profits.

$$\sigma(\pi_f) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(\pi_{ft} - \bar{\pi}_f)^2}, \quad \bar{\pi}_f = \frac{1}{T}\sum_{t}\pi_{ft} \tag{C6}$$

I calculate the average profit volatility across firms as a target moment condition for SMM estimation:

$$\sigma(\pi)^{data} = \frac{1}{F}\sum_{f=1}^{F}\sigma(\pi_f) \tag{C7}$$

Moment 3: Revenue-cost correlation

The risk aversion coefficient γ enters the expression for a_i, b_i via the optimal export adjustment, especially in the risk-adjusted first-order condition. Thus I can use revenue-cost correlation as a target moment for estimation of risk aversion parameter.

I calculate the empirical correlation coefficient using firm-level (log) changes in export revenues and import costs from the data.

$$\rho_{ft}^{\text{data}}(R, C) = \text{Corr}(\Delta \ln R_{ft}, \Delta \ln C_{ft}) \tag{C8}$$

In practice, I use the Fisher-z transformation to improve normality and linear additivity $z(\rho) = \frac{1}{2}\ln(\frac{1+\rho}{1-\rho})$. The Fisher-z transformation maps the bounded ρ to the real number line $(-\infty, \infty)$ and allows its sampling distribution to be closer to normal, with variance almost independent of the true correlation coefficient. Besides, it can avoid boundary problems and improve optimization stability, facilitates weighting and linearization operations. If I need to go back to the correlation coefficient scale after estimation, I can easily convert $\text{Var}(\hat{z})$ to $\text{Var}(\hat{\rho})$ using the delta method $\hat{\rho} = \frac{e^{2\hat{z}}-1}{e^{2\hat{z}}+1}$.

C.6 ER Environment Metrics in Counterfactual Analysis

A1. Change in Average ER Volatility (ΔFX-vol)

Let Σ^e be the covariance matrix of bilateral log exchange rate changes Δe (currencies $1, \ldots, N$). Define currency-level standard deviations $\sigma_i = \sqrt{\Sigma_{ii}}$ and the average volatility $\overline{\sigma} = \frac{1}{N}\sum_{i=1}^{N}\sigma_i$. The statistic is $\Delta\text{FX-vol} = \overline{\sigma}^{(\text{scenario})} - \overline{\sigma}^{(\text{baseline})}$.

ΔFX-vol captures how "turbulent" the ER environment becomes on average. It is sensitive to scale: a global volatility hike (multiplying Σ by s^2) raises ΔFX-vol mechanically. Higher average volatility lifts $\text{Var}(R)$ and $\text{Var}(C)$. If the correlation structure is unchanged (global hike), $VRR = \frac{2\text{Cov}(R,C)}{\text{Var}(R)+\text{Var}(C)}$ may move little in short run. If idiosyncratic variance rises (diagonal-only hike), the denominator grows faster than the numerator, so VRR typically falls.

A2. Change in Average Cross-Currency Correlation (ΔFX-corr)

First, I convert Σ^e to the correlation matrix $\Lambda^e = \text{diag}(\Sigma)^{-1/2}\Sigma^e\text{diag}(\Sigma)^{-1/2}$. Take the off-diagonal average $\bar{\rho} = \frac{1}{K(K-1)}\sum_{i\neq j} C_{ij}$, I can have $\Delta\text{FX-corr} = \bar{\rho}^{(\text{scenario})} - \bar{\rho}^{(\text{baseline})}$.

ΔFX-corr measures how strongly currencies co-move. It is scale-invariant (rescaling Σ leaves ΔFX-corr unchanged). Lower average correlation reduces $\text{Cov}(R, C)$ relative to $\text{Var}(R) + \text{Var}(C)$, pushing VRR down. Pegs that remove a common factor tend to lower ΔFX-corr.

A3. Change in Share of Variance in the First Principal Component (ΔFX-PC1)

Let $\lambda_1 \geq \cdots \geq \lambda_N$ be eigenvalues of Σ. The first principal component PC1 share is defined as $PC1 = \frac{\lambda_1}{\sum_{i=1}^{N}\lambda_i}$. I have $\Delta\text{FX-PC1} = PC1^{(\text{scenario})} - PC1^{(\text{baseline})}$.

ΔFX-PC1 captures the strength of the global ER factor. It is also scale-invariant. A higher PC1 share means risk is more concentrated in one common driver; a lower share means risk is more dispersed. A stronger common factor raises co-movement between revenues and costs, so VRR often rises. Pegs or idiosyncratic hikes that weaken the common factor typically lower ΔFX-PC1 and reduce VRR, even as absolute profit risk may fall under pegs.