# Evidence on Nominal Rigidities

Johannes Wieland
jfwieland@ucsd.edu

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# Outline

# Empirical Evidence on Role of Money

- Big questions in monetary economics:
  - How is an economy with money different from an economy without money?
  - What effects do change in monetary policy have on real activity and inflation?
- Our standard monetary model implied money was neutral. What is the empirical evidence?
  1. Today: VARs
  2. Next class: Other approaches

# Outline

# Introduction to VARs

- Before we get started, introduce a key econometric tool: Vector Autoregression or VAR.
  - Proposed by Sims (1980), who won the Nobel Prize for it.
  - Wanted a way to describe economic time series with minimal theoretical restrictions.
- This is a key tool in macro to summarize relationships between macroeconomic time series.
  - To motivate / test models.
  - Examine response to structural shocks.
  - Frequently used at central banks.

# Stationarity and White Noise

- Cannot find regularities if things do not repeat themselves.
- Leads to concept of stationarity.
  - A time series $\{x_t\}$ is stationary if the mean, variance, and autocorrelation can be well approximated by sufficiently long time averages.
  - $\{x_t\}$ is covariance stationary iff:

$$Ex_t = \mu \; \forall \; t \text{ and } E\{(x_t - \mu)(x_{t-k} - \mu)\} = g_k \; \forall \; t, k$$

  - Sometimes not a great assumption (e.g., economies in transition), but for post-war US GDP, it works.
  - Otherwise, detrend or difference.
- A *white noise process* has mean zero, a constant variance, and is serially uncorrelated.

# AUTOREGRESSIONS

- An *autoregression* is a regression of a time series $\{x_t\}$ on lags of itself.
- Example: AR(1)

$$x_t = \beta_0 + \beta_1 x_{t-1} + \varepsilon_t$$

  - Stationary and stable if $|\beta_1| < 1$ and $\varepsilon_t$ is white noise.
  - Otherwise goes off to infinity and never mean reverts.

- Can estimate $AR(N)$

$$x_t = \beta_0 + \sum_{s=1}^{N} \beta_s x_{t-s} + \varepsilon_t$$

by OLS if $\{x_t\}$ is stationary.

# Vector Autoregression

- A vector autoregression is a generalization of an autoregression in which $x_t$ is a *vector* of time series.
- Simple example we will use:

$$x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

  - Can be of arbitrary size $n$.

- The *reduced-form* of a VAR(1) is then

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

where $A_0$ is an $n \times 1$ vector and $A_1$ is an $n \times n$ matrix.

# Reduced-Form VAR: Estimation

- More generally, for a VAR of size n with P lags,

$$x_t = A_0 + \sum_{s=1}^{P} A_s x_{t-s} + e_t$$

where $x_t, A_0, e_t$ are $n \times 1$ vectors and $A_i$ are $n \times n$ matrices.

- There are thus $n + pn^2$ coefficients and $(n+1)n/2$ in the variance-covariance matrix.

- The right hand side only contains predetermined variables of a stationary process, and the error terms are assumed to be serially uncorrelated with constant variance (can relax).
  - So can estimate each equation by OLS.
  - Application of seemingly unrelated regression.

# Forecasting

- Can forecast using VAR:

$$E_t x_{t+1} = A_0 + A_1 x_t$$
$$E_t x_{t+2} = A_0 + A_1 x_{t+1} = A_0 + A_1[A_0 + A_1 x_t]$$

- Often-used diagnostic tool is the forecast error variance decomposition (FEVD).
  - Tells us proportion of variance of moments in $\{y_t\}$ or $\{z_t\}$ due to $e_{1,t}$ and $e_{2,t}$.
  - Like a partial $R^2$ of forecast error by forecast horizon.

# Impulse Response Functions

$$IR(n) = A_1^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- This is called an *impulse response function* to $e_{1t}$ and is a convenient way to represent how shocks $\{e_t\}$ affect $\{x_t\}$
  - Can plot graphically and create standard error bands.
- Intuitively, this is the difference between two processes $\{x_t\}$ that are made up of identical shocks $\{e_t\}$ except in period $t$, where an additional unit one shock is added to $e_{1t}$.

# Structural VARs

- Unfortunately, reduced form VARs are restrictive.
  1. No simultaneous causality.
  2. Shocks have to be uncorrelated and white noise.
- Generalize to a structural VAR.
- Tackle problem 1 first and allow for simultaneous causality:

$$y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}$$
$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}$$

  where $\varepsilon_{yt}$ and $\varepsilon_{zt}$ are independent white noise processes.
  - Cannot directly estimate because $z_t$ is correlated with $\varepsilon_{yt}$ and $y_t$ is correlated with $\varepsilon_{zt}$, violating exclusion restriction.
  - E.g., the current interest rate is a function of the current output gap, but the current output gap is also a function of the current interest rate.

# Reduced-Form Representation

- Write structural VAR as a matrix:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

- Premultiply by $B^{-}1$ to get reduced-form representation:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

  where $A_0 = B^{-1}\Gamma_0$, $A_1 = B^{-1}\Gamma_1$, and $e_t = B^{-1}\varepsilon_t$
- Note reduced form errors $e_t$ are of form:

$$e_{1t} = \frac{\varepsilon_{yt} - b_{12}\varepsilon_{zt}}{1 - b_{12}b_{21}}$$

  - Stationary white noise, but correlated with one another.
  - For IRs and FEVDs, want responses to $\varepsilon_t$ not $e_t$.

# The Identification Problem

- *Cannot invert from reduced form to structural VAR unless add restrictions.*
- Reduced form has 9 unknowns, six as plus 3 terms of var-covar matrix:

$$y_t = a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + e_{1t}$$
$$z_t = a_{20} + a_{21} y_{t-1} + a_{22} z_{t-1} + e_{2t}$$

- Structural form has 10 unknowns, 8 $b$'s and $\gamma$'s plus 2 terms of var-covar matrix (uncorrelated shocks):

$$y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}$$
$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}$$

- Fundamentally under-identified.
  - Intuitively, $e$'s depend on both $\varepsilon_{yt}$ and $\varepsilon_{zt}$ so cannot invert from $e$'s to $\varepsilon$'s. Extra parameters determine this relationship.

# Recursive VARs and Identification

- Solution is recursive system:
  - Assume $y_t$ has contemporaneous effect on $z_t$, but $z_t$ has no contemporaneous effect on $y_t$.
  - Jargon: "order" $y_t$ first, in the sense that it is "causally prior."
  - E.g., monetary policy affects output with a lag.
- System is:

$$
\begin{aligned}
y_t &= b_{10} + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt} \\
z_t &= b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}
\end{aligned}
$$

so

$$
e_{1t} = \varepsilon_{yt} \text{ and } e_{2t} = \varepsilon_{zt} - b_{21}\varepsilon_{yt}
$$

- Exactly identified because one parameter ($b_{12}$) is now a zero. 9 parameters in both structural VAR and reduced form.
- Intuition: Can now distinguish $\varepsilon_{yt}$ and $\varepsilon_{zt}$ shocks.
  - Only $\varepsilon_{yt}$ shocks affect contemporaneous values of $y_t$.
  - $e_{1t}$ attributed completely to $\varepsilon_{yt}$; can invert $e$'s to get $\varepsilon$'s.

# Cholesky Decomposition

- Lower triangular assumption on the structural residuals is called a Cholesky decomposition.
  - Most common identification scheme for VAR.
- Generalize this to $n$ variable and $p$ lag VAR.
  - $B$ is then an $n \times n$ matrix.
  - Exact identification requires $(n^2 - n)/2$ restrictions between the regression residuals and structural innovations.
  - Cholesky does this by setting exactly $(n^2 - n)/2$ values of the $B$ matrix to zero.

# Error Correlation Version

- Assume instead there is no simultaneous causality but $e_t = C\varepsilon_t$ where $\varepsilon_t$ are the independent structural shocks and C is an $n \times n$ matrix:

$$x_t = A_0 + A_1 x_{t-1} + C\varepsilon_t$$

- Equivalent to structural VAR

$$C^{-1}x_t = C^{-1}A_0 + C^{-1}A_1 x_{t-1} + \varepsilon_t$$

  - Same as before. Recursive VAR if $C^{-1}$ is lower triangular.
  - Same problem — cannot tell apart shocks.
  - Now direct relationship between $e$'s and $\varepsilon$'s instead of relationship arising through simultaneous causality.
- Alternate interpretation of Cholesky: Assume $\varepsilon_{zt}$ affects both $y_t$ and $z_t$ but $\varepsilon_{yt}$ has no contemporaneous effect on $z_t$.

# Cholesky Decomposition: Key Assumptions

- Cholesky is a STRONG assumption.
  - No reverse-causality.
  - No omitted variables correlated with "lower ordered" variables and "higher ordered" variables.
  - Strong exclusion restriction.
- "Ordering" sounds innocuous. It's not.
  - $n!$ possible ordering.

# Other SVAR Identification Schemes

- VARs criticized for being too reduced form.
- Structural VAR (SVAR) approach uses economic theory rather than Cholesky decomposition invert reduced form VAR to structural VAR (that is, to recover structural innovations from reduced form residuals).
  - Must impose $(n^2 - n)/2$ restrictions.
- Examples:
  - Gali (1999) splits Solow residual into tech and non-tech shocks by assuming that only tech shocks affect long-run productivity.
  - Sign restrictions (Uhlig).
  - Assuming cross-sectional regression holds (e.g., Beraja, Hurst, and Ospina, 2016).

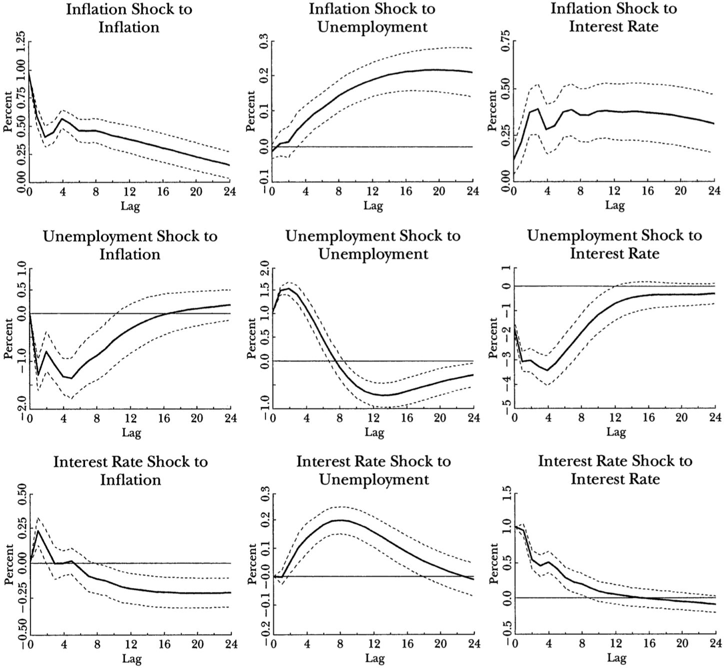
# Outline

# VAR Evidence on Non-Neutrality

- Apply VARs to study the effects of monetary shocks.
- Key challenge is endogeneity.
  - Changes in monetary policy occur for good reasons.
  - Error term $\varepsilon_t$ correlated with outcome:

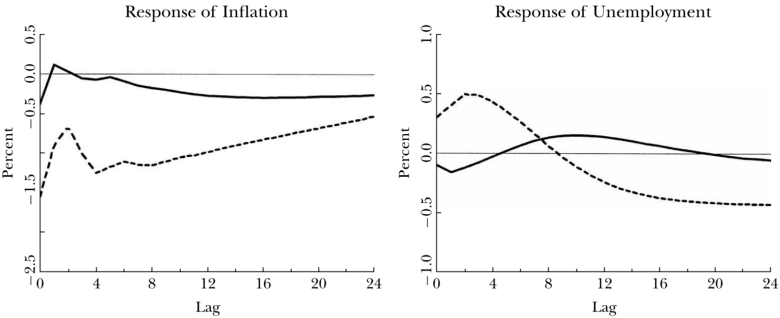
$$\Delta y_t = \alpha + \beta \Delta i_t + \varepsilon_t$$

- Start with simple VAR from Stock and Watson (2001).
  - 3 variables: inflation, unemployment, and Federal Funds interest rate.
  - Order $\pi_t, u_t, R_t$ in recursive VAR.
    - ⋆ $\pi_t$ affects $u_t$ and $R_t$ contemporaneously but not vice-versa.
    - ⋆ $u_t$ affects $R_t$ contemporaneously but not vice-versa.
  - See paper for data description (FEVD, Granger Causality tests).


Inflation Shock to Inflation
Inflation Shock to Unemployment
Inflation Shock to Interest Rate
Unemployment Shock to Inflation
Unemployment Shock to Unemployment
Unemployment Shock to Interest Rate
Interest Rate Shock to Inflation
Interest Rate Shock to Unemployment
Interest Rate Shock to Interest Rate
Percent
Lag

# IRFs: Two Structural Approaches

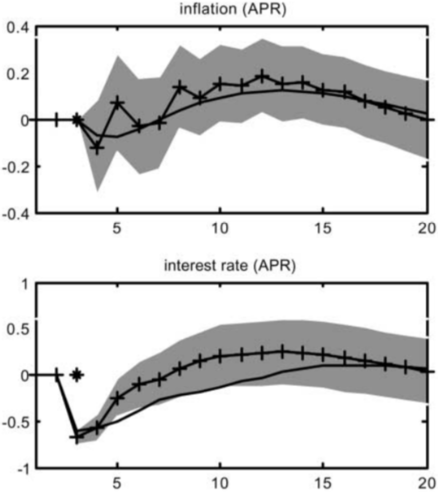
- Stock and Watson then use a structural VAR in which impose a Taylor rule for identification rather than recursive VAR.
  - In solid: backward-looking Taylor rule.
  - In dashed: forward-looking Taylor rule.
  - Structural assumptions (and hence ordering) not innocuous.



## Christiano, Eichenbaum, and Evans (2005)

- Paper does a lot. For now just focus on VAR evidence.
- Run an 9-variable VAR. Ordering:
    1. Real GDP
    2. Real Consumption
    3. GDP Deflator
    4. Real Investment
    5. Real Wage
    6. Labor Productivity
    7. Federal Funds Rate
    8. Real Profits
    9. M2 Growth
- Economic conditions can affect monetary policy, but monetary policy only affects economic conditions with a lag.
    - Trying to get around endogeneity of monetary policy by statistically modeling it. But still Stock-Watson concerns.

# CEE: IRF To Money Shock



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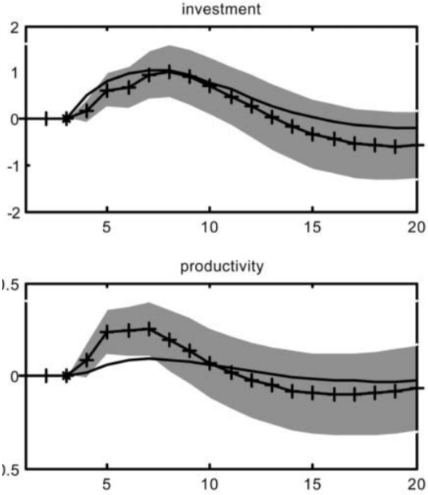

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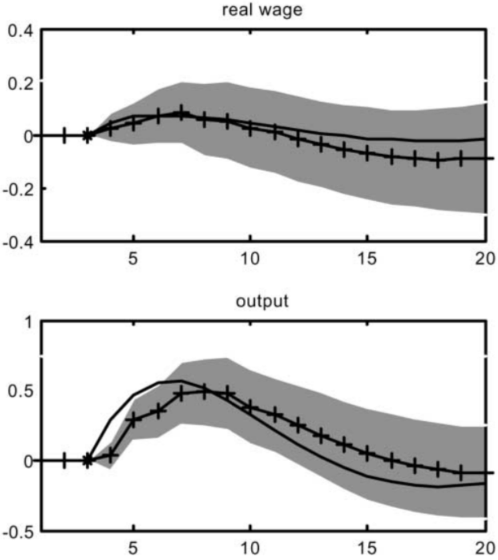

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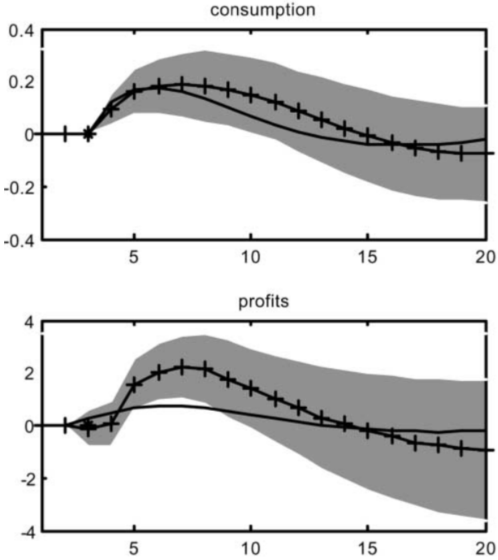

# Christiano, Eichenbaum and Evans (2005) Summary

1. Hump-shaped response of output, consumption and investment, peaking at 1.5 years and returning to trend after 3.
2. Hump-shaped response of inflation, peaking after two years.
3. Interest rate falls for one year
4. Real profits, wages, and labor prod rise.
5. Growth rate of money rises immediately.
    - Phillips curve and Taylor rule as in Stock and Watson still hold.
    - Consistent with significant *monetary non-neutrality* $\Rightarrow$ money affects real outcomes.

# Outline

# NEXT STEPS

- VAR criticisms.
- Other approaches to identify real effects of money.
- Summary of the evidence.