Notes on AFFGT(2021) Replication

Linghui Wu

May 17, 2021

1 Open Economy Equilibrium

1.1 Equilibrium Conditions

Calibrated Parameters

The values for the calibrated parameters are taken from Table 1 on page 20 when solving the equilibrium. To be specific, they are substitution elasticities $\theta = 4$ and $\sigma = 4$, fixed costs $f^u = 1$ and $f^d = 1$, labor intensities $\alpha^u = 1$ and $\alpha^d = 0.5483$, scaled population $L^{us} = 0.4531$ and $L^{row} = 9.5469$, productivities $A^u_{us} = A^d_{us} = 1$, $A^u_{row} = 0.1121$ and $A^d_{row} = 0.2752$, and iceberg trade costs $\tau_d = 3.0066$ and $\tau_u = 2.5731$. Tax t^s_{ij} and subsidises v^s_{ij} where $s \in \{u, d\}$ and $i, j \in \{H, F\}$ are exogenous and set to be zeros. I consider solving the model with both U.S. wages being normalized as $w_{us} = 1$ and not being normalized.

Characterizing Equations

I follow Appendix A to pin down the equations that characterize the open economy equilibrium. Here, I further simplify and reorder the non-linear system of equations so that the variables can be expressed in terms of the known parameters and the pre-defined variables.

- Price in the upstream sectors: $(p^u_{ii}, p^u_{ij}, p^u_{jj}, p^u_{ji})$.

$$\begin{aligned} mc^u_i &= \frac{\bar{\alpha}^u}{A^u_i} w^{\alpha^u} (P^u_i)^{1-\alpha^u} = \frac{\bar{\alpha}^u}{A^u_i} w^{\alpha^u} \\ p^u_{ij} &= \frac{\mu^u \tau^u_{ij} mc^u_i}{1 + v^u_{ij}} = \frac{\mu^u \tau^u}{1 + v^u_{ij}} \frac{\bar{\alpha}^u}{A^u_i} w^{\alpha^u} \end{aligned} \tag{1}$$

since $\alpha^u = 1$.

- Price indexes in the upstream sectors: $(P^u_{ii}, P^u_{ij}, P^u_{jj}, P^u_{ji})$ and (P^u_i, P^u_j).

$$\begin{aligned} P^u_{ji} &= [\int_0^{M^u_j} [(1 + t^u_{ji}) p^u_{ji}(\omega)]^{1-\theta} d\omega]^{\frac{1}{1-\theta}} = (M^u_j)^{\frac{1}{1-\theta}} (1 + t^u_{ji}) p^u_{ji} \\ P^u_i &= [\Sigma_{j\in\{H,F\}} (P^u_{ji})^{1-\theta}]^{\frac{1}{1-\theta}} \end{aligned} \tag{2}$$

- Price in the downstream sectors: $(p^d_{ii}, p^d_{ij}, p^d_{jj}, p^d_{ji})$.

$$
\begin{aligned}
mc^d_i &= \frac{\bar{\alpha}^d}{A^d_i} w^{\alpha^d} (P^u_i)^{1-\alpha^d} \\
p^d_{ij} &= \frac{\mu^d \tau^d_{ij} mc^d_i}{1 + v^d_{ij}} = \frac{\mu^d \tau^d}{1 + v^d_{ij}} \frac{\bar{\alpha}^d}{A^d_i} w^{\alpha^d} (P^u_i)^{1-\alpha^d}
\end{aligned}
\qquad [3]
$$

by plugging P^u_i and P^u_j derived from Equation 2.

- Price indexes in the downstream sectors: $(P^d_{ii}, P^d_{ij}, P^d_{jj}, P^d_{ji})$ and (P^d_i, P^d_j).

$$
\begin{aligned}
P^d_{ji} &= [\int_0^{M^d_j} [(1 + t^d_{ji}) p^d_{ji}(\omega)]^{1-\sigma} d\omega]^{\frac{1}{1-\sigma}} = (M^d_j)^{\frac{1}{1-\sigma}} (1 + t^d_{ji}) p^d_{ji} \\
P^d_i &= [\Sigma_{j \in \{H,F\}} (P^d_{ji})^{1-\sigma}]^{\frac{1}{1-\sigma}}
\end{aligned}
\qquad [4]
$$

- Production levels by free entry condition: $(y^u_i, y^u_j, y^d_i, y^d_j)$.

$$
\begin{aligned}
y^d_i &= (\sigma - 1) f^d_i = (\sigma - 1) f^d \\
y^u_i &= (\theta - 1) f^u_i = (\theta - 1) f^u
\end{aligned}
\qquad [5]
$$

- Labor demand in the upstream and downstream sectors: $(\ell^u_i, \ell^u_j, \ell^d_i, \ell^d_j)$.

$$
\begin{aligned}
\ell^u_i &= \frac{f^u_i + y^u_i}{A^u_i} \\
\ell^d_i &= \alpha^d \frac{mc^d_i (f^d_i + y^d_i)}{w_i}
\end{aligned}
\qquad [6]
$$

where y^u_i, y^d_i and mc^d_i are from Equations 5 and 3.

- Quantities and outputs in the upstream sectors: $(Q^u_{ii}, Q^u_{ij}, Q^d_{ji}, Q^d_{jj})$ and $(x_{ii}, x_{ij}, x_{ji}, x_{jj})$.

$$
\begin{aligned}
Q^u_{ij} &= (1 - \alpha^d) \frac{mc^d_i (f^d_i + y^d_i)}{P^u_i} (\frac{P^u_{ji}}{P^u_i})^{-\theta} = (1 - \alpha^d) \frac{mc^d_i (f^d + y^d_i)}{P^u_i} (\frac{P^u_{ji}}{P^u_i})^{-\theta} \\
x_{ji} &= Q^u_{ji} [\frac{(1 + t^u_{ji}) p^u_{ji}}{P^u_{ji}}]^{-\theta}
\end{aligned}
\qquad [7]
$$

- Outputs in the downstream sectors: $(c_{ii}, c_{ij}, c_{ji}, c_{jj})$.

$$
c_{ji} = \frac{w_i L_i + T_i}{(P^d_i)^{1-\sigma}} [(1 + t^d_{ji}) p^d_{ji}]^{-\sigma} \qquad [8]
$$

1.2 Solving Non-linear Systems

To solve the equilibrium allocations, I need $\boldsymbol{x} = (w_j, M^u_i, M^u_j, M^d_i, M^d_j, T_i, T_j)$ be a vector of seven endogenous variables if the wage is normalized and $\boldsymbol{x} = (w_i, w_j, M^u_i, M^u_j, M^d_i, M^d_j, T_i, T_j)$ be a vector of eight endogenous variables otherwise[1]. The equilibrium constraints that I employ for optimizations are as follows[2].

- Labor market clearing: $(LMC_i, LMC_j) = \mathbf{0}$.

$$LMC_i = L_i - M_i^d \cdot \ell_i^d - M_i^u \cdot \ell_i^u \tag{9}$$

- Good market clearing: $(GMC_i^u, GMC_j^u, GMC_i^d, GMC_j^d) = \mathbf{0}$.

$$\begin{aligned} GMC_i^u &= y_i^u - M_i^d x_{ii} - M_j^d \tau_{ij}^u x_{ij} = y_i^u - M_i^d x_{ii} - M_j^d \tau^u x_{ij} \\ GMC_i^d &= y_i^d - c_{ii} - \tau_{ij}^d c_{ij} = y_i^d - c_{ii} - \tau^d c_{ij} \end{aligned} \tag{10}$$

- Budget balance: $(BB_i, BB_j) = \mathbf{0}$.

$$BB_i = T_i - \Sigma_{j\in\{H,F\}}[t_{ji}^d M_j^d c_{ji} p_{ji}^d + t_{ji}^u M_i^d M_j^u x_{ji} p_{ji}^u - v_{ij}^d M_i^d c_{ij} p_{ij}^d - v_{ij}^u M_i^u M_j^d x_{ij} p_{ij}^u]. \tag{11}$$

1.3 Results and Allocations

I implement `fsolve` in MATLAB as the numerical solver [3] with the $\boldsymbol{x} = (\sqrt{\boldsymbol{x}})^2$ technique to solve the equilibrium allocations. Here, I only report the results when the wage is normalized.

The values for the eight params are

$$\begin{aligned} \left[w_j\right] &= \left[0.1285\right] \\ \left[M_i^u, M_j^u, M_i^d, M_j^d\right] &= \left[0.0516, 0.1205, 0.0322, 0.0790\right] \\ \left[T_i, T_j\right] &= \left[4.63e^{-10}, 1.75e^{-10}\right] \end{aligned}$$

The optimized parameters are independent of the initial guesses that passed in. "fsolve" finds the right solution and exits because the first-order optimality is small. The diagnostics of the optimization results shows that the sum of the squared function errors "fval" is 7.93e-18 and the sum of absolute "fval" is 4.79e-9.

Next, I compare the statistics around the zero-tariff equilibrium with those in Table 3 of the draft. Except for discrepency from the numerical precision, the two sets of statistics are nearly the same.

[1] The corresponding MATLAB scripts are `est.m` and `solve_eq.m` for the open economy where the wage is not normalized. `est_nw.m` and `solve_eq_nw.m` are for the open economy where the wage is normalized.

[2] When the wage is not normalized, the non-linear system is squared with eight unknowns and eight equations and it yields a unique solution. On the other hand, when the wage is not normalized, I utilize the same set of eight equations. Though the number of unknown parameters is fewer than that of the equations, the system still yields a unique solution

[3] Due to the restricted access to `Knitro`.

Table 1: Comparison of Statistics around the Zero Tariff Equilibrium

Statistics	$\Omega_{H,H}$	$\Omega_{F,H}$	$\Omega_{F,F}$	$\Omega_{H,F}$	b_H^H	b_F^H	λ_H^d
Table 3	0.4149	0.0368	0.4356	0.016	0.9363	0.0617	0.993
My Solutions	0.4150	0.0367	0.4357	0.016	0.9383	0.0617	0.993

Note: This table contains summary statistics for the endogeneous aggregate variables relevant for the first order approximation around the zero tariff equilibrium.

$\Omega_{F,H} \equiv \frac{M_F^u M_H^d p_{F,H}^u x_{F,H}}{M_H^d (p_{H,F}^d c_{H,F} + p_{H,H}^d c_{H,H})}$ is the share of Home final-good revenue spend on intermediate input varieties from F.

$b_F^H \equiv \frac{M_F^d p_{F,H}^d c_{F,H}^d}{w_H L_H}$ is the share of Home income spend on foreign varieties.

$\lambda_H^d \equiv \frac{M_H^d (p_{H,F}^d c_{H,F} + p_{H,H}^d c_{H,H})}{w_H L_H}$ is the ratio of domestic final-good revenue to national income in country H.

2 Minor Comments

A few grammatical errors and typos in the April 24 draft are as follows.

1. On page 6, it says "thus assuming that all firm's output can used as either for final consumption or as an intermediate input". It should be "[...] can be used as [...]".

2. On page 36, the last sentence of the first paragraph begins with "Nota that". It should be "Note that".

3. In equation (29) on page 36, the integration domain of P_{ji}^u should be from 0 to M_j^u not to M_j^d.

4. In equation (32) on page 37, the subscript of c_{ji} is flipped. It should be $y_i^d = c_{ii} + tau_{ij}^d c_{ij}$.

5. In equation (34) on page 37, the last term of T_i should be $-v_{ij}^u M_j^d M_i^u x_{ij} p_{ij}^u$.

To improve readability, I would suggest specifying on page 36 that

1. $\mu_u = \frac{\theta}{\theta-1}$ and $\mu_d = \frac{\sigma}{\sigma-1}$ in equation (26).
2. $\bar{\alpha}^s = \frac{1}{(\alpha^s)^{\alpha^s}(1-\alpha^s)^{1-\alpha_s}}$ where $s \in \{u, d\}$ in equation (26).
3. α in equations (28) for Q_{ij}^u and $\ell_i^d$is α^d.