Polynomial Interpolation

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Question A

Test：

```
Please enter the number of data:4
x[0]=0
f(x[0])=5
x[1]=1
f(x[1])=3
x[2]=3
f(x[2])=5
x[3]=4
f(x[3])=12
Please enter the x:5
Newton Table:
0       5               0               0               0
1       3               -2              0               0
3       5               1               1               0
4       12              7               2               0.25
The value at 5 is 25
The Polynomial is 5-2x+1x(x-1)+0.25x(x-1)(x-3)
```

能够输出差商表、多项式以及得到多项式在某个点的值。

Question B

当 n=2 时，输出的差商表为：

```
Please enter the n:2
Newton Table:
-5      0.0384615               0               0
0       1               0.192308                0
5       0.0384615               -0.192308               -0.0384615
```

多项式为： $f(x) = 0.0384615 + 0.192308(x + 5) - 0.0384615(x + 5)x$

当 n=4 时，输出的差商表为：

```
Please enter the n:4
Newton Table:
-5      0.0384615               0               0               0               0
-2.5    0.137931                0.0397878               0               0               0
0       1               0.344828                0.061008                0               0
2.5     0.137931                -0.344828               -0.137931               -0.0265252              0
5       0.0384615               -0.0397878              0.061008                0.0265252               0.00530504
```

多项式为:

$$f(x) = 0.0384615 + 0.0397878(x + 5) + 0.061008(x + 5)(x + 2.5) - 0.0265252(x + 5)(x + 2.5)x + 0.00530504(x + 5)(x + 2.5)x(x - 2.5)$$

当 n=6 时，输出的差商表为：

```
Please enter the n:6
Newton Table:
-5      0.0384615               0               0               0               0               0               0
-3.33333        0.0825688               0.0264644               0               0               0               0               0
-1.66667        0.264706                0.109282                0.0248454               0               0               0               0
0       1               0.441176                0.0995683               0.0149446               0               0               0
1.66667 0.264706                -0.441176               -0.264706               -0.0728548              -0.0131699              0               0
3.33333 0.0825688               -0.109282               0.0995683               0.0728548               0.0218564               0.00420316              0
5       0.0384615               -0.0264644              0.0248454               -0.0149446              -0.0131699              -0.00420316             -0.000840633
```

多项式为:

$$f(x) = 0.0384615 + 0.0264644(x + 5) + 0.0248454(x + 5)(x + 3.3333) + 0.0149446(x + 5)(x + 3.3333)(x + 1.6667) - 0.0131699(x + 5)(x + 3.3333)(x + 1.6667)x + 0.00420316(x + 5)(x + 3.3333)(x + 1.6667)x(x - 1.6667) - 0.000840633\ x + 5)(x + 3.3333)(x + 1.6667)x(x - 1.6667)\ (x - 3.3333)$$

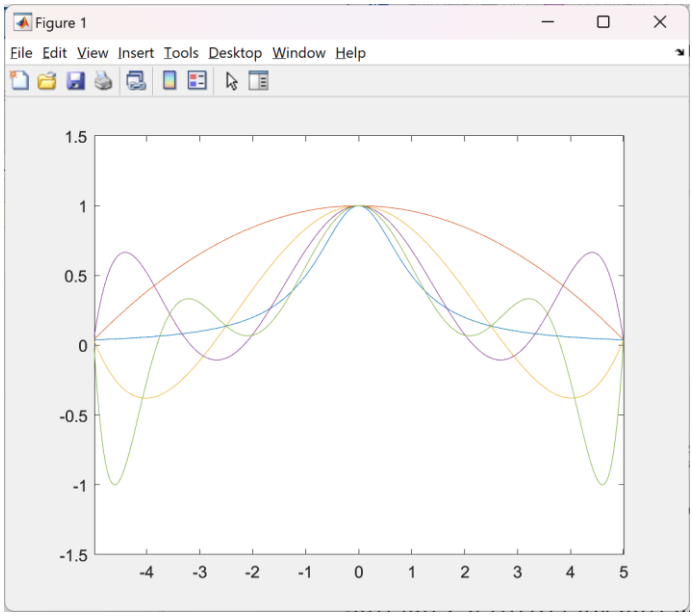
当 n=8 时，输出的差商表为：

```
Please enter the n:8
Newton Table:
-5      0.0384615            0            0            0            0            0            0            0            0
-3.75   0.06639         0.0223428            0            0            0            0            0            0
-2.5    0.137931               0.0572328            0.013956            0            0            0            0            0
-1.25   0.390244               0.20185         0.057847            0.0117043            0            0            0            0
0       1                    0.487805         0.114382            0.015076            0.000674338            0            0            0            0
1.25    0.390244               -0.487805            -0.390244            -0.134567            -0.0299286            -0.00489646            0            0 0
2.5     0.137931               -0.20185         0.114382            0.134567            0.0538267            0.0134008            0.00243964            0
3.75    0.06639         -0.0572328           0.057847            -0.015076            -0.0299286            -0.0134008            -0.00357356            -0.
000687223            0
5       0.0384615            -0.0223428           0.013956            -0.0117043           0.000674338          0.00489646           0.00243964           0
.000687223           0.000137445
```

多项式为:

$$f(x) = 0.0384615 + 0.0223428(x+5) + 0.013956(x+5)(x+3.75) + 0.0117043(x+5)(x+3.75)(x+2.5) + 0.000674338(x+5)(x+3.75)(x+2.5)(x+1.25) - 0.00489646(x+5)(x+3.75)(x+2.5)(x+1.25)x + 0.00243964(x+5)(x+3.75)(x+2.5)(x+1.25)x(x-1.25) - 0.000687223(x+5)(x+3.75)(x+2.5)(x+1.25)x(x-1.25)\ (x-2.5) + 0.000137445(x+5)(x+3.75)(x+2.5)(x+1.25)x(x-1.25)\ (x-2.5)(x-3.75)$$

画图结果：



结果显示次数(n)越高，反而函数在两端出现明显动荡、不收敛的现象，相反精度越低。这就是龙格现象。

Question C:

当 n=5 时，输出的差商表为：

```
Newton Table:
0.951057        0.0423501       0       0       0       0
0.587785        0.103764        -0.169057       0       0       0
6.12303e-17     1       -1.52477        1.42548         0       0
-0.587785       0.103764        1.52477         -2.59409        2.61208         0
-0.951057       0.0423501       0.169057        1.42548         -2.61208        2.7465
```

多项式为:

$$f(x) = 0.0423501 - 0.169057(x - 0.951057) + 1.42548(x - 0.951057)(x - 0.587785) + 2.61208(x - 0.951057)(x - 0.587785)x + 2.7465(x - 0.951057)(x - 0.587785)x(x + 0.587785)$$

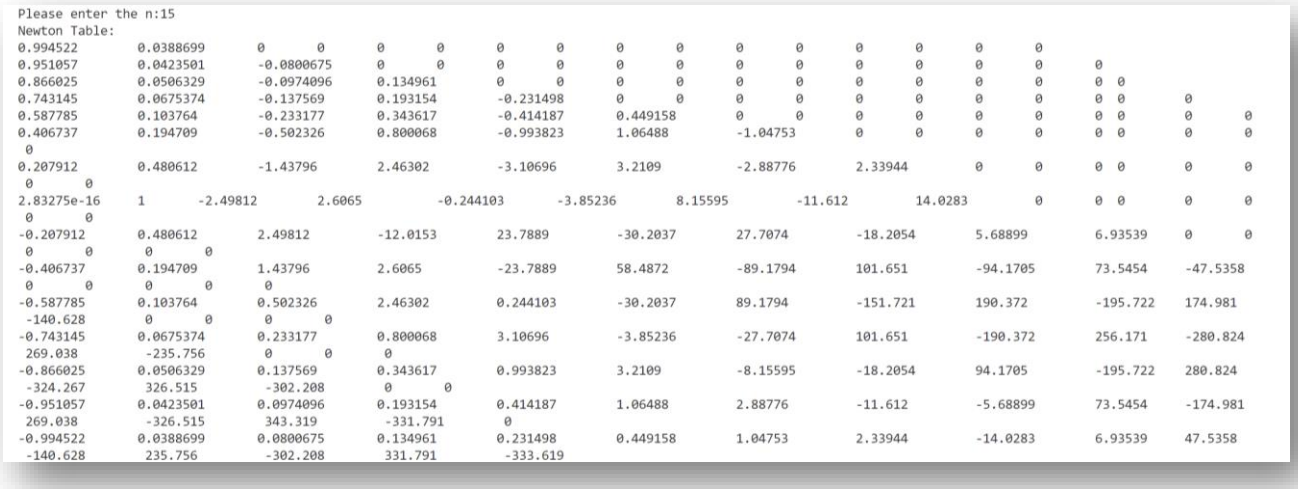
当 n=10 时，输出的差商表为：

```
0.987688        0.0393884       0       0       0       0       0       0       0       0
0.891007        0.0479678       -0.0887389      0       0       0       0       0       0       0
0.707107        0.0740741       -0.141959       0.189679        0       0       0       0       0       0       0
0.45399         0.162531        -0.34947        0.474836        -0.534305       0       0       0       0       0       0
0.156434        0.620427        -1.53886        2.15988         -2.29391        2.11681         0       0       0       0       0
-0.156434       0.620427        0       -2.52096        5.42052         -7.36503        8.28743         0       0       0       0       0
-0.45399        0.162531        1.53886         -2.52096        0       4.66844         -8.94684        11.9543         0       0       0
-0.707107       0.0740741       0.34947         2.15988         -5.42052        4.66844         0       -5.59838        10.3568         0       0
-0.891007       0.0479678       0.141959        0.474836        2.29391         -7.36503        8.94684         -5.59838        0       5.51277         0
-0.987688       0.0393884       0.0887389       0.189679        0.534305        2.11681         -8.28743        11.9543         -10.3568        5.51277         0
```

多项式为：

$$f(x) = 0.0393884 - 0.0887389(x - 0.987688) + 0.189679(x - 0.987688)(x - 0.891007) - 0.534305(x - 0.987688)(x - 0.891007)(x - 0.707107) + 2.11681(x - 0.987688)(x - 0.891007)(x - 0.707107)(x - 0.45399) + 8.28743(x - 0.987688)(x - 0.891007)(x - 0.707107)(x - 0.45399)(x - 0.156434) + 11.9543(x - 0.987688)(x - 0.891007)(x - 0.707107)(x - 0.45399)(x - 0.156434)(x + 0.156434) + 10.3568(x - 0.987688)(x - 0.891007)(x - 0.707107)(x - 0.45399)(x - 0.156434)(x + 0.156434)(x + 0.45399) + 5.51277(x - 0.987688)(x - 0.891007)(x - 0.707107)(x - 0.45399)(x - 0.156434)(x + 0.156434)(x + 0.45399)(x + 0.707107)$$

当 n=15 时，输出的差商表为：

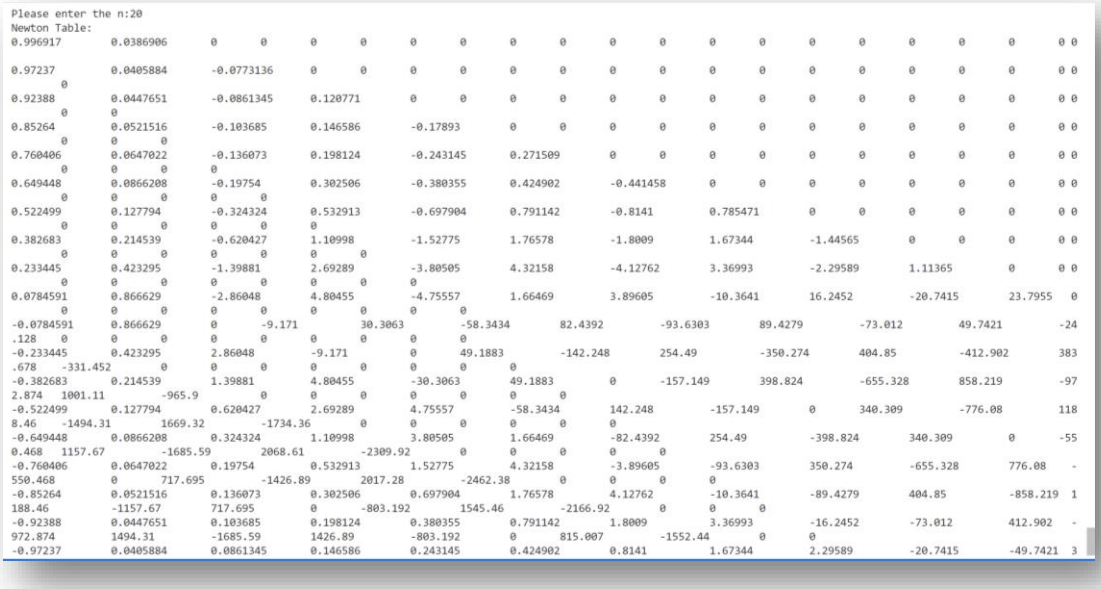


```
Please enter the n:15
Newton Table:
0.994522        0.0388699       0       0       0       0       0       0       0       0       0       0       0       0       0
0.951057        0.0423501       -0.0800675      0       0       0       0       0       0       0       0       0       0       0       0
0.866025        0.0506329       -0.0974096      0.134961        0       0       0       0       0       0       0       0       0       0       0
0.743145        0.0675374       -0.137569       0.193154        -0.231498       0       0       0       0       0       0       0       0       0       0
0.587785        0.103764        -0.233177       0.343617        -0.414187       0.449158        0       0       0       0       0       0       0       0       0
0.406737        0.194709        -0.502326       0.800068        -0.993823       1.06488         -1.04753        0       0       0       0       0       0       0       0
 0
0.207912        0.480612        -1.43796        2.46302         -3.10696        3.2109          -2.88776        2.33944         0       0       0       0       0       0
 0       0
2.83275e-16     1       -2.49812        2.6065          -0.244103       -3.85236        8.15595         -11.612         14.0283         0       0       0       0       0
 0       0
-0.207912       0.480612        2.49812         -12.0153        23.7889         -30.2037        27.7074         -18.2054        5.68899         6.93539         0       0
 0       0       0       0
-0.406737       0.194709        1.43796         2.6065          -23.7889        58.4872         -89.1794        101.651         -94.1705        73.5454         -47.5358
 0       0       0       0       0
-0.587785       0.103764        0.502326        2.46302         0.244103        -30.2037        89.1794         -151.721        190.372         -195.722        174.981
 -140.628        0       0       0       0
-0.743145       0.0675374       0.233177        0.800068        3.10696         -3.85236        -27.7074        101.651         -190.372        256.171         -280.824
 269.038         -235.756        0       0       0
-0.866025       0.0506329       0.137569        0.343617        0.993823        3.2109          -8.15595        -18.2054        94.1705         -195.722        280.824
 -324.267        326.515         -302.208        0       0
-0.951057       0.0423501       0.0974096       0.193154        0.414187        1.06488         2.88776         -11.612         -5.68899        73.5454         -174.981
 269.038         -326.515        343.319         -331.791        0
-0.994522       0.0388699       0.0800675       0.134961        0.231498        0.449158        1.04753         2.33944         -14.0283        6.93539         47.5358
 -140.628        235.756         -302.208        331.791         -333.619
```

多项式为：

$$
\begin{aligned}
f(x) = {} & 0.0388699 - 0.0800675(x - 0.994522) + 0.134961(x - 0.994522)(x - 0.951057) \\
& - 0.231498(x - 0.994522)(x - 0.951057)(x - 0.866025) + 0.449158(x \\
& - 0.994522)(x - 0.951057)(x - 0.866025)(x - 0.743145) - 1.04753(x \\
& - 0.994522)(x - 0.951057)(x - 0.866025)(x - 0.743145)(x - 0.587785) \\
& + 2.33944(x - 0.994522)(x - 0.951057)(x - 0.866025)(x - 0.743145)(x \\
& - 0.587785)(x - 0.406737) + 14.0283(x - 0.994522)(x - 0.951057)(x \\
& - 0.866025)(x - 0.743145)(x - 0.587785)(x - 0.406737)(x - 0.207912) \\
& + 6.93539(x - 0.994522)(x - 0.951057)(x - 0.866025)(x - 0.743145)(x \\
& - 0.587785)(x - 0.406737)(x - 0.207912)x - 47.5358(x - 0.994522)(x \\
& - 0.951057)(x - 0.866025)(x - 0.743145)(x - 0.587785)(x - 0.406737)(x \\
& - 0.207912)x(x + 0.207912) - 140.628(x - 0.994522)(x - 0.951057)(x \\
& - 0.866025)(x - 0.743145)(x - 0.587785)(x - 0.406737)(x - 0.207912)x(x \\
& + 0.207912)(x + 0.406737) - 235.756(x - 0.994522)(x - 0.951057)(x \\
& - 0.866025)(x - 0.743145)(x - 0.587785)(x - 0.406737)(x - 0.207912)x(x \\
& + 0.207912)(x + 0.406737)(x + 0.587785) - 302.208(x - 0.994522)(x \\
& - 0.951057)(x - 0.866025)(x - 0.743145)(x - 0.587785)(x - 0.406737)(x \\
& - 0.207912)x(x + 0.207912)(x + 0.406737)(x + 0.587785)(x + 0.743145) \\
& - 331.791(x - 0.994522)(x - 0.951057)(x - 0.866025)(x - 0.743145)(x \\
& - 0.587785)(x - 0.406737)(x - 0.207912)x(x + 0.207912)(x + 0.406737)(x \\
& + 0.587785)(x + 0.743145)(x + 0.866025) - 333.619(x - 0.994522)(x \\
& - 0.951057)(x - 0.866025)(x - 0.743145)(x - 0.587785)(x - 0.406737)(x \\
& - 0.207912)x(x + 0.207912)(x + 0.406737)(x + 0.587785)(x + 0.743145)(x \\
& + 0.866025)(x + 0.951057)
\end{aligned}
$$

当 n=20 时，输出的差商表为：



```
Please enter the n:20
Newton Table:
0.996917        0.0386906       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0 0
0.97237         0.0405884       -0.0773136      0       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0 0
        0
0.92388         0.0447651       -0.0861345      0.120771        0       0       0       0       0       0       0       0       0       0       0       0       0       0 0
        0       0
0.85264         0.0521516       -0.103685       0.146586        -0.17893        0       0       0       0       0       0       0       0       0       0       0       0 0
        0       0       0
0.760406        0.0647022       -0.136073       0.198124        -0.243145       0.271509        0       0       0       0       0       0       0       0       0       0 0
        0       0       0       0
0.649448        0.0866208       -0.19754        0.302506        -0.380355       0.424902        -0.441458       0       0       0       0       0       0       0       0 0
        0       0       0       0       0
0.522499        0.127794        -0.324324       0.532913        -0.697904       0.791142        -0.8141         0.785471        0       0       0       0       0       0 0
        0       0       0       0       0       0
0.382683        0.214539        -0.620427       1.10998         -1.52775        1.76578         -1.8009         1.67344         -1.44565        0       0       0       0 0
        0       0       0       0       0       0       0
0.233445        0.423295        -1.39881        2.69289         -3.80505        4.32158         -4.12762        3.36993         -2.29589        1.11365         0       0 0
        0       0       0       0       0       0       0       0
0.0784591       0.866629        -2.86048        4.80455         -4.75557        1.66469         3.89605         -10.3641        16.2452         -20.7415        23.7955  0
        0       0       0       0       0       0       0       0       0
-0.0784591      0.866629        0       -9.171          30.3063         -58.3434        82.4392         -93.6303        89.4279         -73.012         49.7421         -24
.128    0       0       0       0       0       0       0       0       0
-0.233445       0.423295        2.86048         -9.171          0       49.1883         -142.248        254.49          -350.274        404.85          -412.902        383
.678    -331.452        0       0       0       0       0       0       0       0       0
-0.382683       0.214539        1.39881         4.80455         -30.3063        49.1883         0       -157.149        398.824         -655.328        858.219         -97
2.874   1001.11         -965.9          0       0       0       0       0       0       0       0
-0.522499       0.127794        0.620427        2.69289         4.75557         -58.3434        142.248         -157.149        0       340.309         -776.08         118
8.46    -1494.31        1669.32         -1734.36        0       0       0       0       0       0       0
-0.649448       0.0866208       0.324324        1.10998         3.80505         1.66469         -82.4392        254.49          -398.824        340.309         0       -55
0.468   1157.67         -1685.59        2068.61         -2309.92        0       0       0       0       0       0
-0.760406       0.0647022       0.19754         0.532913        1.52775         4.32158         -3.89605        -93.6303        350.274         -655.328        776.08   -
550.468         0       717.695         -1426.89        2017.28         -2462.38        0       0       0       0       0
-0.85264        0.0521516       0.136073        0.302506        0.697904        1.76578         4.12762         -10.3641        -89.4279        404.85          -858.219  1
188.46          -1157.67        717.695         0       -803.192        1545.46         -2166.92        0       0       0       0
-0.92388        0.0447651       0.103685        0.198124        0.380355        0.791142        1.8009          3.36993         -16.2452        -73.012         412.902  -
972.874         1494.31         -1685.59        1426.89         -803.192        0       815.007         -1552.44        0       0       0
-0.97237        0.0405884       0.0861345       0.146586        0.243145        0.424902        0.8141          1.67344         2.29589         -20.7415        -49.7421  3
```

多项式为：

$$f(x) = 0.0386906 - 0.0773136(x - 0.996917) + 0.120771(x - 0.996917)(x - 0.97237) - 0.17893(x - 0.996917)(x - 0.97237)(x - 0.92388) + 0.271509(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264) - 0.441458(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406) + 0.785471(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448) - 1.44565(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499) + 1.11365(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683) + 23.7955(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445) - 24.128(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591) - 331.452(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591) - 965.9(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445) - 1734.36(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445)(x + 0.382683) - 2309.92(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445)(x + 0.382683)(x + 0.522499) - 2462.38(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445)(x + 0.382683)(x + 0.522499)(x + 0.649448) - 2166.92(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445)(x + 0.382683)(x + 0.522499)(x + 0.649448)(x + 0.760406) - 1552.44(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445)(x + 0.382683)(x + 0.522499)(x + 0.649448)(x + 0.760406)(x + 0.85264) - 788.326(x - 0.996917)(x - 0.97237)(x - 0.92388)(x - 0.85264)(x - 0.760406)(x - 0.649448)(x - 0.522499)(x - 0.382683)(x - 0.233445)(x - 0.0784591)(x + 0.0784591)(x + 0.233445)(x + 0.382683)(x + 0.522499)(x + 0.649448)(x + 0.760406)(x + 0.85264)(x + 0.92388)$$

画图结果：

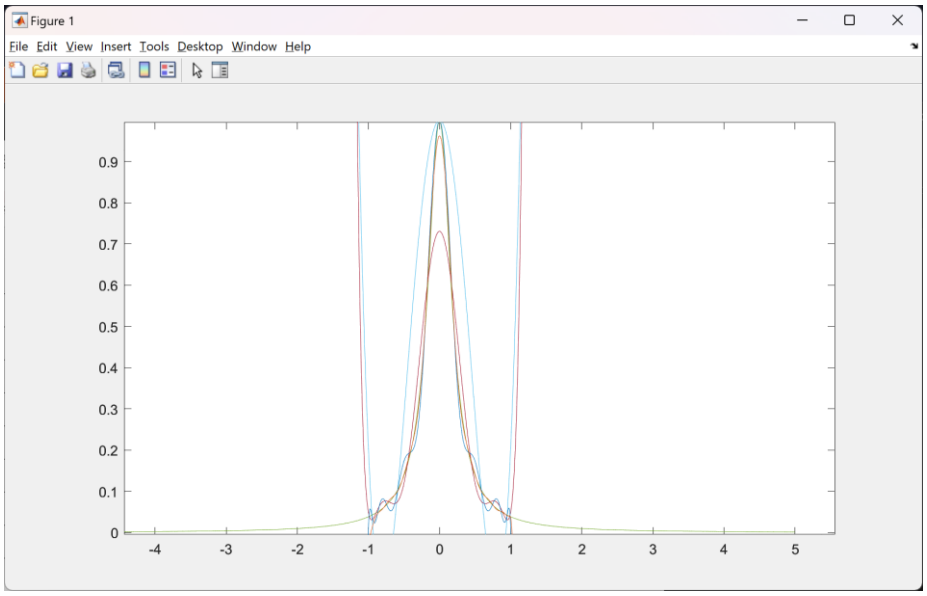


Figure 1
File Edit View Insert Tools Desktop Window Help
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
5

Question D：

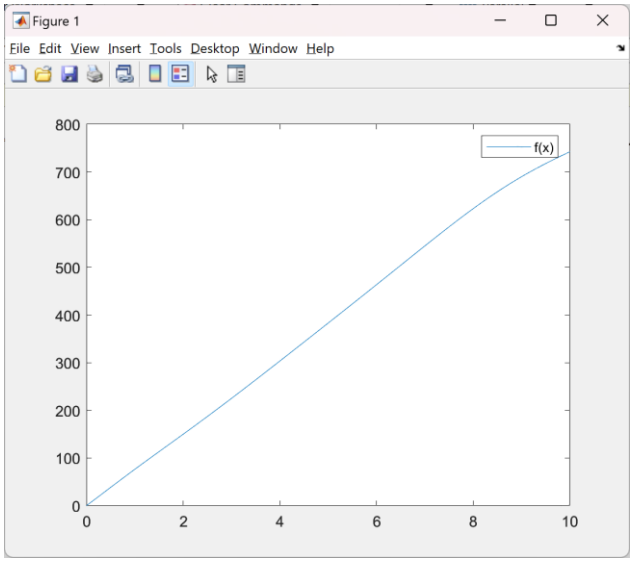
(a).

差商表为：

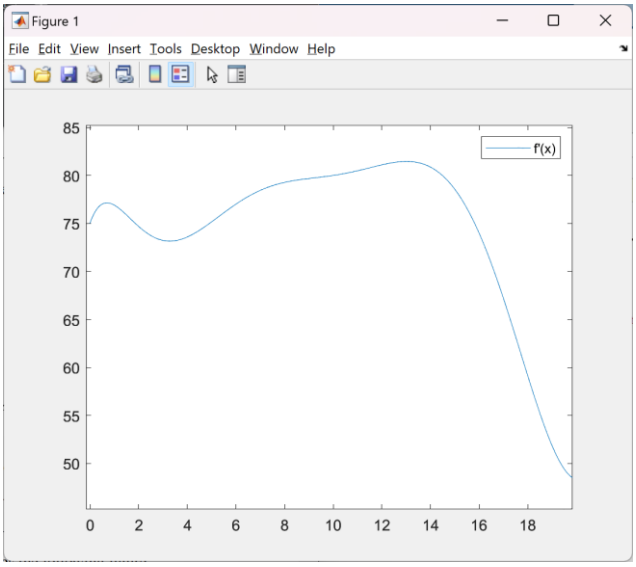
```
Please enter the x:10
Hermite Table:
0       0       0       0       0       0       0       0       0       0
0       0       75      0       0       0       0       0       0       0
3       225     75      0       0       0       0       0       0       0
3       225     77      0.666667        0.222222        0       0       0       0       0       0
5       383     79      1       0.0666667       -0.0311111      0       0       0       0       0
5       383     80      0.5     -0.25   -0.0633333      -0.00644444     0       0       0       0
8       623     80      0       -0.1    0.03    0.0116667       0.00226389      0       0       0
8       623     74      -2      -0.666667       -0.113333       -0.0286667      -0.00504167     -0.000913194    0       0
13      993     74      0       0.25    0.114583        0.0227917       0.00514583      0.000783654     0.000130527     0
13      993     72      -0.4    -0.08   -0.04125        -0.0194792      -0.00422708     -0.000937292    -0.00013238     -2.02236e-05
The value at 10 is 742.503
```

多项式为：

$$f(x) = -2.02236 \times 10^{-5}x^9 + 0.001040589x^8 - 0.021875646x^7 + 0.243041064x^6 - 1.538294618x^5 + 5.50811x^4 - 10.09530x^3 + 7.16191x^2 + 75x$$

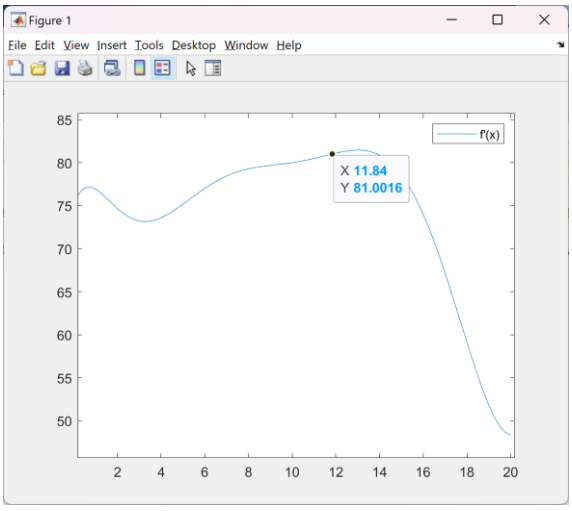


$$f'(x) = -0.0001820124x^8 + 0.008324712x^7 - 0.153129522x^6 + 1.458246384x^5 - 7.69147309x^4 + 22.03244x^3 - 30.2859x^2 + 14.32382x + 75$$



因此在 t=10s 时，汽车的位置为 f(10)=742.503(feet),速度为 f'(10)=48.3537(feet per second)

(b)



由图可以得出当时间接近 11.84s 时，汽车的速度超过 81(feet per second).

Question E:

(a)Sp1:

```
0       6.67            0               0               0               0               0               0
6       17.3            1.77167         0               0               0               0               0
10      42.7            6.35            0.457833                0               0               0               0
13      37.3            -1.8            -1.16429                -0.124778               0               0               0
17      30.1            -1.8            0               0.105844                0.013566                0               0
20      29.3            -0.266667               0.219048                0.0219048               -0.00599567             -0.000978085            0
28      28.7            -0.075          0.0174242               -0.0134416              -0.00196368             0.000183272             4.1477e-05
The value at 43 is 14640.3
```

多项式为：

$$f(x) = 6.67 + 1.77167x + -0.124778x(x-6)(x-10)) + 0.013566x(x-6)(x-10)(x-13) - 0.000978085x(x-6)(x-10)(x-13)(x-17) + 4.1477 \times 10^{-5}x(x-6)(x-10)(x-13)(x-17)(x-20)$$

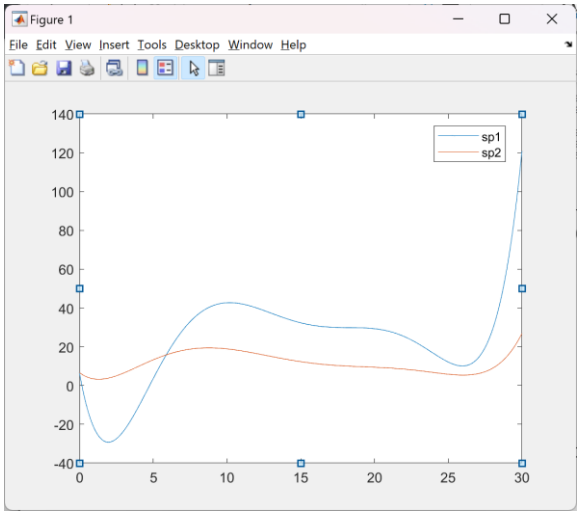
Sp2:

```
Newton Table:
0       6.67            0               0               0               0               0               0
6       16.1            1.57167         0               0               0               0               0
10      18.9            0.7             -0.0871667              0               0               0               0
13      15              -1.3            -0.285714               -0.0152729              0               0               0
17      10.6            -1.1            0.0285714               0.0285714               0.00257908              0               0
20      9.44            -0.386667               0.101905                0.00733333              -0.00151701             -0.000204804            0
28      8.89            -0.06875                0.0289015               -0.00486688             -0.00067779             3.81462e-05             8.67
68e-06
The value at 43 is 2981.48
```

多项式为：

$$f(x) = 6.67 + 1.57167x - 0.0871667x(x-6-0.0152729x(x-6)(x-10) + 0.00257908x(x-6)(x-10)(x-13) - 0.000204804x(x-6)(x-10)(x-13)(x-17) + 8.6768 \times 10^{-6}x(x-6)(x-10)(x-13)(x-17)(x-20)$$

用 matlab 画图出来的结果：



(b).

Sp1 : f(43)=14640.3

Sp2 : f(43)=2981.48