EC2104 Tutorial 4 solution

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Section 1

Question 1 and 2

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Some important things to know

1. $\frac{d}{dx}a^{f(x)} = f'(x)a^{f(x)}\ln(a)$
2. Reading product symbol ($\prod$):
   $\prod_{i=1}^{n}(x_i - \gamma_i)^{\beta_i} = (x_1 - \gamma_1)^{\beta_1}(x_2 - \gamma_2)^{\beta_2}\cdots(x_n - \gamma_n)^{\beta_n}$
3. Reading summation symbol ($\sum$):
   $\sum_{i=1}^{n}(x_i - \gamma_i)^{\beta_i} = (x_1 - \gamma_1)^{\beta_1} + (x_2 - \gamma_2)^{\beta_2} + \cdots + (x_n - \gamma_n)^{\beta_n}$
4. If the question ask to prove (or show) two statement are the same
   1. do either of the following, depending on which is easier:
      1. Show right hand side = left hand side
      2. Show left hand side = right hand side
   2. Always check what do you need, and what do you have
5. C^1 := class 1 smoothness, zeroth and first derivatives are continuous. wiki here

Question 2a

Given $z = \sqrt{y} f(y^2 - 2x)$, WTS: $y^2 \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{2} z$

1. Should I show
   1. RHS = LHS $\Rightarrow$ show $y^2 \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{2} z$
   2. LHS = RHS $\Rightarrow$ show $\frac{1}{2} z = y^2 \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$
2. Think
   1. $\frac{1}{2} z = \frac{1}{2} \sqrt{y} f(y^2 - 2x)$, missing $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$
   2. However, finding $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ is a manageable task
3. WTS RHS = LHS, $\frac{\partial z}{\partial x} = -2\sqrt{y} f'$ and $\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{y}} f(y^2 - 2x) + 2y^{\frac{3}{2}} f'$
   1. Check and compare with RHS $\frac{1}{2} z$:
      1. what is missing? $\Rightarrow$ $\frac{1}{\sqrt{y}}$ instead of $\sqrt{y}$
      2. what is extra? $\Rightarrow$ we should not have f'$
   2. Can we manipulate the equations to get what we want?
      - yes, $y^2 \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{2} z$
4. This question is easy in the sense if you follow closely to the LHS equations, you can solve the problem. But in general, some clues is all you need to solve the problem.