EC3101 Microeconomics II

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Inter-temporal Choice

Future value, FV $= m(1+i)$
Present value, PV $= \frac{FV}{(1+i)}$

Inter-temporal Utility

$u(C_1, C_2) = C_1 + \delta C_2$, δ is discount rate

Inter-temporal Budget Constraint

$(C_1, C_2) = (m_1, m_2) \Rightarrow$ consumer don't save/borrow
$C_1 + \frac{C_2}{1+i} = m_1 + \frac{m_2}{1+i}$, m_i in units of goods

Changes in Nominal Interest Rate

$MRS_{C_1,C_2} = 1 + i$
changes in r affects slope of curve, higher steeper

	saver	borrower	neither
$\uparrow i$	$\uparrow U$	$\downarrow U$	unchanged
$\downarrow i$	$\downarrow U$	$\uparrow U$	unchanged

Neither result in unchanged utility if and only if income adjust accordingly
With increased i, borrower might become saver

Changes in Price

If $p \neq 1$: $p_1 C_1 + \frac{p_2}{1+r} C_2 = m_1^* + \frac{m_2^*}{1+i}$, $m_i^* = m_i$ money
new MRS $= (1+i)\frac{p_1}{p_2}$

	C_1	C_2
$\uparrow p_1$	$\downarrow$	$\uparrow$
$\downarrow p_1$	$\uparrow$	$\downarrow$
$\uparrow p_2$	$\uparrow$	$\downarrow$
$\downarrow p_2$	$\downarrow$	$\uparrow$

Changes in inflation

$p_1(1+\pi) = p_2$ or $\pi = \frac{p_2 - p_1}{p_1}$
If $\pi \neq 0, p_1 = 1$: $c_1 + \frac{1+\pi}{1+i} c_2 = m_1^* + \frac{m_2^*}{1+i}$

	saver	borrower	neither
$\uparrow \pi$	$\downarrow U$	$\uparrow U$	unchanged
$\downarrow \pi$	$\uparrow U$	$\downarrow U$	unchanged

Neither result in unchanged utility if and only if income adjust accordingly

Real Interest Rate

$1 + \rho = \frac{1+i}{1+\pi}$, ρ is real interset rate
$\rho = \frac{i-\pi}{1+\pi} \approx i - \pi$ for small π

Uncertainty

Expected Utility: $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \nu(c_1) + \pi_2 \nu(c_2)$
Expected Money: $\pi_1 c_1 + \pi_1 c_1$
Given concave utility $\Rightarrow \frac{\partial U}{\partial c} = MU_c(c) < 0, \uparrow c \downarrow U$

Risk Measurement

Risk aversion	: EU < U(EM)
Certainty equivalence	: $U^{-1}(EU)$

Risk Premium

RP = EM - CE
RP is 'willingness-to-pay' to avoid risk, higher more risk averse
CE is the certainty amount needed to achieve the utility under risk

Arrow-Pratt

$-\frac{u''(w)}{u'(w)}$, higher more risk averse

State-Contingent Budget Constraints

States of Nature: outcomes of random events
Contingent consumption plan: specification of amount consumed in different state of nature
Insurance premium: price of insurance, γ

w/o insurance	$C_- = m - L, C_+ = m$
w insurance	$C_- = m - L + (1-\gamma)K$
	$C_+ = m - \gamma K$
budget constraint	$C_+ = -\frac{\gamma}{1-\gamma} C_- + \frac{m - \gamma L}{1-\gamma}$
MRS_{c_-,c_+}	$\frac{\pi_- MU(C_-)}{\pi_+ MU(C_+)} = \frac{\gamma}{1-\gamma}$, price ratio

Market for Insurance

$\pi_{\text{insurance}} = \gamma K - (\pi_- K + \pi_+ 0) = (\gamma - \pi_-)K$
Buy insurance when: $U(C_+ - \gamma K) \geq EU$

Insurance pricing

competitive pricing	$\pi_{\text{insurance}} = 0 \Rightarrow \gamma = \pi_-$
fully insurance	$\frac{MU(C_-)}{MU(C_+)} = 1 = \frac{C_-}{C_+}$
unfair pricing	$\pi_{\text{insurance}} > 0 \Rightarrow \gamma > \pi_-$
partial insurance	$\frac{MU(C_-)}{MU(C_+)} > 1 \Rightarrow \frac{C_-}{C_+} < 1$
Full insurance	$C_- = C_+$, 45^o line
Partial insurance	$C_- < C_+$, nearer to C_+

Monopoly

Single seller for the market with downward sloping market demand
Producer surplus PS below price above supply
Consumer surplus CS above price below demand

Profit Maximisation

$\max_q \pi = p(q)q - c(q)$ till $MR = MC$

Price Elasticity of Demand, PED

PED: $\epsilon = \left[\frac{dq}{q} / \frac{dp}{p}\right] \leq 0, dq = 1$

$p'(q)q + p(q) = c'(q) = p(q)\left[1 + \frac{dp}{p}\frac{q}{dq}\right] = p(q)[1 - \frac{1}{|\epsilon|}]$
In demand curve, ϵ decrease from elastic to inelastic

PED cases ($\epsilon \leq 1$ and Markup Pricing)

$\epsilon = \infty$: $p(q) = c'(q)$	perfect competition
$\epsilon \leq 1$: $q \downarrow 0 = 0$	near 0 production
$\epsilon > 1$: $p(q) = \frac{c'(q)}{[1 - \frac{1}{\lvert\epsilon\rvert}]}$	markup prices

Inefficiency of Monopoly

Since $Q^* < Q_m$, monopoly produce less than market equilibrium, $CS^* < CS_m$ and DWL exist

First-degree Price Discrimination

P = demand, PS = market surplus

Third-degree Price Discrimination

Assumptions

- Perfect identification of different markets
- No arbitrage across groups

solve $\max_{q_1,q_2} p_1(q_1)q_1 + p_2(q_2) - c_1(q_1) - c_2(q_2)$

Uniform Pricing

Firm charging the same price for both markets
$p_1(q) > p_2(q)$

$$Q = p_1^{-1} 1_{\{P > p_2(0)\}} + [p_1^{-1} + p_2^{-1}] 1_{\{P \leq p_2(0)\}}$$
$$P = p_1 1_{\{Q > Q(p_2(0))\}} + p(p_1^{-1} + p_2^{-1}) 1_{\{Q \leq Q(p_2(0))\}}$$

Solve for MR = MC and decide on Q, P, π

Barriers to Entry

Types: legal, strategic, structural

Natural Monopoly

Firm with Economies of scale and low demand (relative to cost) $\Rightarrow$ downward sloping $ATC > MC$

Regulating Natural Monopoly

MC pricing	$MC = P < ATC \Rightarrow \pi < 0$
ATC pricing	$MC < P = ATC \Rightarrow \pi = 0$
Qty tax,τ	$\uparrow P, \downarrow Q$, distort market

However, ATC pricing incur high regulatory cost and no firms have no incentive to be cost efficient

Quantity Tax Levied on Monopolist

w/o τ : $p(q)\left[1+\frac{1}{\epsilon}\right] = k \Rightarrow p(q) = \frac{\epsilon k}{1+\epsilon}$

w τ : $\frac{\epsilon(k+\tau)}{1+\epsilon} \Rightarrow \frac{\epsilon k}{1+\epsilon} + \frac{\epsilon \tau}{1+\epsilon}$

Since monopolist only operates when $\epsilon < 1$, $\frac{\epsilon\tau}{1+\epsilon} > 1$, more elastic demand means firms pass even more than full time to consumer.
However, in reality firms do not pass more than tax to consumer. This result also assume whole demand curve to be constant elasticity (not true)

Oligopoly

Cournot

Assumptions:

- identical product
- independent and simultaneously strategy
- firms choose output level

Payoffs and best response function:

$$\max_{q_i} P(q_1 + q_2)q_i - C_i(q_i) \;\forall\; i \in N$$

Stackelberg

Assumptions:

- Stackelberg leader set quantity independently
- follower observe leader and set quantity

1. Solve followers' best response per *Cournot*
2. Solve leaders' best response function

$$\max_{q_i} P(q_i + q_j(q_i))q_i - C_i(q_i) \;\forall\; i, j \in N$$

Collusion

Firms collude to form cartel and choose Q_{mkt}

$$\max_{Q} P(Q)Q - C(Q)$$

Decision to produce

$C_i(Q) < C_j(Q)$	only firm j will produce
$C_i(Q) = C_j(Q)$	firms share Q till $MC_i = MC_j$

Collusion is not sustainable in static game
In dynamic game, firms punish by 'grim trigger strategy', not cooperate again
α = payoff in collusion
β = payoff in Cournot
γ = payoff in Cournot with opp in collusion
δ = discount factor

$$\pi_{coop} = \pi_{cheat}$$
$$\alpha + \delta\alpha + \delta^2\alpha + \cdots = \gamma + \delta\beta + \delta^2\beta + \cdots$$
$$\sum_{i=[0,\infty]} \delta^i \alpha = \gamma + \sum_{i=[1,\infty]} \delta^i \beta$$
$$\frac{\alpha}{1+\delta} = \frac{\gamma}{1+\delta}$$

Bertrand

Assumptions:

- identical product
- constant $MC = c$, no fixed cost
- firms choose price level

Cases:

$P_i > P_j$	$Q_i = 0, P_i = P_j - \delta \downarrow_0$
$P_i = P_j$	$Q_i = Q_j$
$P_j > P_{monopoly}$	$P_i = P_{monopoly}$
$P_j < MC$	$P_i = MC$

Bertrand, Cournot, Stackelberg, Collusion

	Bertrand	Cournot	Stackelberg	Collusion
P^*	1	2	3	4
Q^*	4	3	2	1
π	0	1	2	3

best for consumers $\longrightarrow$ best for firms

Capacity Constraint

In this mod we assume single firm can satisfy the entire market