Function; Variable	e; Reserved Keywords		done by: Ling 1 Dec 19
	SAS	R	SPSS
Filepath	Filepath = F:\folder\file\	F:/folder/file	
Missing entry		NA	
Comment lines	/* comments */	# comments	-
Importing data	data varname; infile 'filepath' delimiter="," firstobs=2 obs=n; [obs is the final line]	<pre>var1 = read.table('filepath', header=TRUE, sep=",", col.names, row.names) [attach column names] attach(var2)</pre>	"File" -> "Open"
Header names	input col1 col2\$ @@; run;	<pre>[col names] names (df1) = c('name1', 'name2') [row names] row.names(df1) = c('name1', 'name2')</pre>	*edit directly
Manual entry	datalnes; datal data2 data3 ;	<pre>[create vectors] var1 = c(data1, data2, data3); var4 = seq(from=n, to=m, by=x) [create numeric] var2 = numeric(n) [replicates elements] var3 = rep(list, times) [convert vectors to matrix] dim(vector) = c(row, col) [create matrix] var5 = matrix(vector, nrow=n, ncol=m) [create data frames] var6 = data.frame(vector/matrix) [manual entry] var7 = scan()</pre>	"Variable View" -> Define variables "Data View" -> Enter data
Fixed format entry	input col1 1-2 col2\$ 4;	<pre>var1 =   read.fwf('filepath', width=c(1,2,3))</pre>	"File" -> "Open" -> "Fixed width"
Creating subset	set varname; (filepath if SAS dataset) if condition1 then output;	*reassign variable	"Data" -> "Merge Files"
Filter	if condition1 then output;  *note: var must be created first	<pre>var[condition,]</pre>	[Data View] "Data" -> "Select Cases" -> "If condition is satisfied" -> "If" -> enter conditions
Create new col	col_name = (a+b)/2;	[expand dataframe] var \$col = var [bind a row to matrix] var3 = rbind (var1, var2) [bind a col to matrix] var3 = cbind (var1, var2)	"Edit" -> "Insert Variable"
Keeping subset	keep col1 col2 col3;	var1[rows,cols] *could be logics(conditions) too	-
Dropping subset	drop col1 col2 col3;	{note: when filter, apply on rows}	"Edit" -> "Clear"

Function; Variable;	unction; Variable; Reserved Keywords done by: Ling 1 Dec		
	SAS	R	SPSS
Concatenating data set	set var1 var2;	[combining data frames by rows] var3 = rbind(var1, var2)	[Data View] "Data" -> "Merge Files" -> "Add Cases"
If-else	if condition then outcome; else if conditon then output;	<pre>if(condition) {statement} else if (condition) {statement} else {statement}</pre>	*applicable in filter
Loop	<pre>do var = start to end;   statements; end;</pre>	<pre>[while loop] while (condition) { statements} [for loop] for (var in 1:n) { statement}</pre>	-
	e.g. do batch = 1 to n; do treat = 1 to n; input value @@; output; end; end;		
Merging data	[Must be sorted by criteria first] merge var1 var2; by criteria;	<pre>[merge data by extending cols] var6 = merge(var4, var5, by='criteria', all=TRUE)</pre>	[sorted data] "Data View" -> "Data" -> "Merge Files" -> "Add Variables"
Output data	*change the dataname to a permanent address	<pre>[save output on screen] sink('filepath') operations sink() [print output] cat(value1, value2, value3, "\n") [save dataframes] write.table(var1, 'filepath', quote=FALSE)</pre>	"File" -> "Save"
Sorting	proc sort data=name; by descending col1;	<pre>var1[order(criteria),] var2[rev(order(criteria)),] rank(var)</pre>	"Data" -> "Sort Cases"
Writing functions	-	<pre>functionname = function(input1,input2) {   statement   return(output)}</pre>	-
Transformation	*refer to create new col	*create new var *e.g. BMI = height/weigt**2	[Calculation] "Transform" -> "Compute Variable" [Recode] "Transform" -> "Recode into Different Variable" {note: Click "Output variable are strings" if output values are characters}

Function; Variable;	; Reserved Keywords		done by: Ling 1 Dec 19
	SAS	R	SPSS
Format values	proc format; *gives lables to values value \$value_name '1' = 'Name1'; value value_name 1-20 = "range"	<pre>#value labels label = c("label1","label2") var.group = label[var] #making use of indexing  #recode var value = ifelse(test=, yes=, no=) #convert value to index</pre>	
Logical Operators	equal = greater than and equal >= less than	equal == not equal != less than	-
Modify value		<pre>var[row,col] = value</pre>	
Applying functions		list apply, apply function to a list, recoding the list lapply (list, function) table apply tapply (vector, index, function) *the index works like grouping variable apply (array, margin, function) * margin: 1 indicates rows, 2 indicates columns, c(1, 2) indicates rows and columns. Where X has named dimnames, it can be a character vector selecting dimension names.	

ed Keywords	SAS	SPSS
	Describing numerical data	
tion bility r measures	proc means data=var n mean std min max stderr maxdec=n; title "this is a title"; var col_name; (class col_name; *different groups) OR proc univariate data=var; title "this is a title"; var col_name;	"Analyze" -> "Descriptive Statistics" -> "Frequencies"
gram	proc univariate data=var; var col_name; histogram col_name / midpoints=n to m by 10 normal; class col_name; *seperate by group qqplot col_name;	"Analyze" -> "Descriptive Statistics" -> "Explore" OR ~->"QQ Plots" "Graphs" -> "Legacy
olot, Stem and leaf plot, antile-quantile) plot	proc univariate data=var plot; var col_name; OR proc boxplot data=var; plot col_name*col_name;	Dialogs" -> "Histogram"/"Box plot" *add <factor list=""> for 2 variable analysis</factor>
er plot for bivariate data	proc sgplot data=var; title "Title"; by col_name; #different graph scatter x=col_name y=col_name / group=criteria; #same graph	"Graphs" -> "Chart Builder" *need to drag x and y factors, set colour OR in "Legacy Dialogs"
med mean	proc univariate data=var trimmed=0.2 winsorized=0.2;	-
orized mean	var col_name;	-
r's M-estimators	-	"Analyze:" ->
y's bisquare estimator	-	"Descriptive Statistics"
pel's M-estimator	-	-> "Explore"
quartile Range (IQR) an Absolute Deviation (MAD) s mean difference	proc univariate data=var robustscale; var col_name;	
an A	bsolute Deviation (MAD)	bsolute Deviation (MAD) var col_name;

Function; Variable	; Reserved Keywords	SAS	SPSS
Categorical data	- Frequency tables	proc freq data=var; title "Title"; tables col_name col_name;	"Analyze" -> "Descriptive Statistics" -> "Frequencies"
	- Contingency tables (two-way frequency table)	tables col_name*col_name;	"Analyze" -> "Descriptive Statistics" -> "Crosstabs" -> "Stat
	- Chi-square tests	tables col_name*col_name / chisq; weight col_name; #for manual entry summarised table	"Data" -> "Weight Cases" -> contingency
	- Charts		
Categorical data (paired data)	- McNemar's test	proc freq data=var; tables before*after/agree; weight count; #for frequency counts	"Analyze" -> "Descriptive Statistic" -> "Crosstabs"
	- Bar chart	proc sgplot data=var; vbar col_name; #for two way frequency table panelby col_name; vbar col_name /group=group_name;	"Graphs" -> "Legacy Dialogs" -> "Bar"
Numerical data Test for location	Parametric test - One sample t-test (normal population)	proc univariate data=var mu0=value; var col_name;	"Analyze" -> "Compare Means" -> "One Sample t tests"
	Nonparametric test - Sign test		"Analyze" -> "Nonparametric Tests"
	- Sign rank test		-> "Legacy Dialogs" -> "2 Related Samples" *note: must duplicate sample mean in a new col first
Tests comparing two groups	Two independent sample - Parametric Test (t-test)	proc ttest data=var hu0=value sides=2/L/U; class col_name; #two groups var col_name; #values	"Analyze" -> "Compare Means" -> Independent Sample T tests"

Function; Variable	; Reserved Keywords	SAS	SPSS
	- Nonparametric tests (Wilcoxon rank sum test) (Mann-Whitney U-test)	proc nparlway data=var wilcoxon; class col_name; #two groups var col_name; #values exact wilcoxon;	#recode into numeric variable "Transform" -> "Recode" -> "Into different variable" #wilcoxon rank sum test "Analyze" -> "Nonparametric Tests" -> "Legacy Dialogs" -> "2 Independent Samples"
	Related samples - Paired t-test	proc ttest data=var mu0=value sides=2/L/U; paired var1*var2; #or create a difference and run 1 var t-test	"Analyze" -> "Compare Means" -> "paired Sample T test"
	Related samples: Nonparametric tests - Wilcoxon signed rank test	#create a difference at data import stage proc univariate data=var; var col_name;	"Analyze" -> "Nonparametric Tests" -> "Legacy Dialogs" -> "2 Related Samples"
Tests comparing three groups or more	Parametric Test - ANOVA table	proc glm data=var; class col_name; model y=x;	"Analyze" -> "Compare Means" -> "One-way ANOVA" "Options", "Descriptive"
	- LSD	means col_name/ lsd;	"Analyze" -> "Compare Means" -> "One-way ANOVA" "Post Hoc", "LSD"
	- Contrast	contrast "X vs. Y and Z" y_col -2 1 1; $C: -2\mu_x + \mu_y + \mu_z = 0$ contrast "Y vs. Z" y_col 0 1 -1; $C: \mu_y - \mu_z = 0$	"Analyze" -> "Compare Means" -> "One-way ANOVA" "Contrast"
	Testing for Equal Variances - Levene's test & Bartlett test	means col_name/ hovtest = levene hovtest = bartlett;	"Analyze" -> "Compare Means" -> "One-way ANOVA" "Options", "Homogeneity of variance test"

Function; Varia	ble; Reserved Keywords	SAS	SPSS
	Testing for Independence and Normality Assumptions	*obtain the residuals; proc glm data=var; class col_name; model y=x; output out=var2 p=yhat r=resid;  *test for normality;	"Analyze" ->"General Linear Model" -> "Univariate" "Dependent Variable", "Fixed Factor"
		proc univariate data=var; var resid; histogrm resid/ normal noplot; qqplot resid;  *residual plots; proc sgplot data=var; scatter x=yhat y=resid;	
	Nonparametric 1-way test - Kruskal-Wallis Test	proc npar1way data=var; class col_name; var col_name; exact wilcoxon;	"Analyze" -> "Nonparametric Tests" -> "Legacy Dialogs" -> "K Independent Samples" "Test Variable List", "Grouping Variable" "Define Range"
Regression Analysis	Pearson Correlation Coefficient	proc corr data=var nosimple; var col_name1 col_name2 col_name3;	"Analyze" -> "Correlate" -> "Bivariate" "Variables window", "Pearson"
	Simple regression	proc reg data=var; model y = x; output out=var p=yhat r=resid; quit;	"Analyze" _> "Regression" -> "Linear"
	Testing assumptions - Linear relationship: scatter plot	proc sgplot data=var; scatter y=col_name x=col_name;	"Graphs" -> "Legacy Dialogs" -> "Scatter/plot" "Simple Scagter" -> "Define"

Function; Variable;	Reserved Keyv	vords	SAS	SPSS
	- Normality and independence		proc sgplot data=var; scatter y=resid x=yhat; proc sgplot data=var; scatter y=resid x=height;	"Analyze" -> "Regression" -> "Linear" [and save residuals] "Analyze" -> "Descriptive Statistics" -> "QQ plots"
	- Prediction	Draw fitted lines	<pre>proc sgplot data=var; pbspline y=col_name x=col_name; *pbspline = penalized beta spline method;</pre>	
		Case1: value in dataset	Add statement output out=var p=yhat r=resid and print dataset to search	"Analyze" -> "Regresion" -> "Linear" "Save", "Unstandardized"
		Case2: value not in	Add data point into dataset with the missing values (using . for missing value)	Add missing data point
	- Transform	Quadratic	data var2; set var1; x2 = x**2; proc reg data=var; model y=x x2;	"Transform" -> "Compute Variable"
		Log	data var2; set var1; lx = log(x); proc reg data=var; model y=x lx;	

Function; Variable; F	Reserved Keywords		R
	•	Describing numerical da	
Numerical descriptive measures			
- Location	- Mean	mean(var)	summary(var)
	- Median	median(var)	Min/1stQ/Median/Mean/3rdQ/Max
	- Mode	var[order(var)[1	L:n]]#smallest 5
- Variability	- Variance	var(var)	
	- Standard deviation	sd(var)	
	- Range	range(var)	
	- Interquartile range	IQR(var)	
	- Coefficient of	cv = function(x)	sd(x)/mean(x)*100
	Variation		
- Other measures	- Min and Max	min(var); max(va	
	- First & third	quantile(var, le	evel)
	quartile		
	- Percentiles		
	_	mn = mean((x-mean))	
	- Skewness [measure the	skew = function sk = $m3/m2^{(3/2)}$	(x) {   *sqrt (n* (n-1))/(n-2)
	[measure the symmetric]	return(sk)}	5410(II (II 1))/(II 2)
	- Kurtosis [measure	kurt = function	(x) {
	the tail]		$(n-3)$ (n-3) (n+1) *m4/m2^2-3* (n-
		1))	
		return(ku)}	
		Graphical methods	
- Histogram	_	1)) #2 graphs in one col	
			<pre>freq=TRUE, col='color',</pre>
	main=paste("Tit	<del>-</del>	"sub-title"), xlab ="x", ylab
	#normal curve impose		
	xpt = seq(n, m,		
			name),sd(col name))
	aypt=ypt*length	$n(col_name)*10 \#sca$	aling the line to match area of histogram, 10 is
	the width of bin		
	lines(xpt,aypt)		
		1)) #1 graphs in one col	
- QQplot	_	e); qqline(col_na	
- Boxplot		$me\sim col\_name2) \#\sim me$	eans split by group
- Stem and leaf plot	stem(col_name)	2 tr-col nama **1:	m=c(lower,upper),
- Scatter plot for bivariate data		e,y=coi_name, xii oper), axes=FALSE	
orvariate data	_	plot new graph and old g	
	1 = ( = : = : = : = : = : = : = : = : = :	Robust Location estimate	* *
- Trimmed mean		mean(var, trim=0	
- Winsorized mean			
- Huber's M-estimato	ors		
- Tukey's bisquare estimator			
- Hampel's M-estima			
T		ust measures of scale par	
- Interquartile Range (IQR)		IQR (var) /1.34898	
- Median Absolute Deviation (MAD)		<pre>median(abs(x-med mad(x)</pre>	llan(X)))
- Gini's mean difference		mau (x)	
- Onn s mean unitele		Categorical data	
- Frequency tables		table(var)	
- Contingency tables		table(var, var2)	
- Fisher test		fisher.test(tabl	Le(var),
			e="less/greater/two.sided")
•			

Function; Variable; R	eserved Keywords	R
- Chi-square tests	teserved fley words	chisq.test(table(var), correct=F) #applied w/o
1		continuity correction
- Charts		Continuity Correction
Paired data	- McNemar's test	<pre>mcnemar.test(table1, correct=T)</pre>
Tanoa aata	- Bar chart	<pre>barplot(height= c(table1),</pre>
	Bur onart	<pre>names.arg=c("Nam1","Name2"))</pre>
		#two-way frequency table
		<pre>barplot(tablecount, beside=T, col=c(1,8))</pre>
		<pre>legend("top-left", c("Name1","Name2"),</pre>
		fill=c(1,8))
		Numerical data
Tests for location	Parametric test	t.test(var, mu=value,
	- One sample t-test	alterntive="two.sided/less/greater")
	Nonparametric test	testvalue = var[var!=mu0]
	- Sign test	<pre>binom.test(sum(testvalue&gt;mu0),</pre>
		length(testvalue))
	- Signed Rank test	<pre>wilcox.test(testvalue, mu=value,</pre>
		alternative="two.sided")
Two independent	- t-test	#first create different groups
samples		#test for variances
		var.test(var1, var2)
		#t-test
		t.test(var1, var2, mu=0, var.equal=T/F,
	NT 4 '	alternative="two.sided")
	- Nonparametric	#first create different groups
	(Wilcoxon rank	wilcox.test(groupa, groupb)
T 1-4-1	sum test)	or wilcox.test(y~x)
Two related	- Paired t-test	<pre>t.test(var1, var2, mu=0, paired=T) #create a diff first</pre>
amples	Nonparametric test	diff = varA-varB
	- Signed test	ncount = sum(diff>0)
		binom.test(ncount, length(diff), 0.5)
	- Signed rank test	wilcox.test(diff)
Three or more	Parametric test	$model1 = anov(y\sim x)$
independent	- ANOVA	summary (model1)
samples	111.00.111	<pre>tapply(x,y,mean); mean(x); tapply(x,y,mean)-</pre>
<b>-</b>		mean(x); boxplot(y~x)
	- LSD	group.1 = $y[x=="1"]$ ; group.2 = $y[x=="1"]$ ;
		group.3 = $y[x=="3"];$
		<pre>group.mean = tapply(x,y,mean)</pre>
		<pre>treat.group = cbind(group.1, group.2, group.3)</pre>
		#d=(N-k) degree of freedom, where k is the no. of groups
		$mse = sum(model1$res^2)/d$
		lsd = qt(0.975,d) * sqrt(mse* (1/n1 + 1/n2))
		<pre>diff = mean(treat.group[,i]) - maan(treat.group[,i])</pre>
		<pre>mean(treat.group[,j]) if(abs(diff)&gt;lsd) {cat("There is significant")</pre>
		difference between groups") }
	- Contrast	contrasts(x) = $matrix(c(2,-1,-1,0),$
	Commast	-1,1), nrow=3)
		modelc = lm(y~x)
		summary (modelc)
	Testing for Equal	<pre>group.means = tapply(y,x,mean)</pre>
		<pre>gmean = group.means[x]</pre>
İ	Variances	gmodii grodp imodiio [ii]
	- Levene's test	#Absolute deviation from group meams is used
		#Absolute deviation from group meams is used
		#Absolute deviation from group meams is used words.abs.dev = abs(y-gmean)

Function; Variable; Reserved Keywords		R
	Test for	#Test for normaity (Kolmogorove-Smirnov test)
	independence and	resid=model1\$res
	Normality	<pre>ks.test(resid, "pnorm", mean(resid), sd(resid))</pre>
	assumptions	#Model checking: residual plots
	•	<pre>fv = model1\$fitted</pre>
		par(mfrow=c(2,1))
		<pre>plot(fv,resid); abline(h=0,lty=2)</pre>
		<pre>qqnorm(resid); qqline(resid, lty=2)</pre>
		par(mfrow=c(1,1))
	Nonparametric 1-	kruskal.test(y,x)
	way Test	or
	- Kruskal-Wallis	kruskal.test(y~x)
	Test	
Simulation Study		

## Conditon of interest

- Compare 3 estimators for location  $\mu$  through a simulation study 3 estimators are: sample mean, sample median, sample 10% trimmed mean Underlying condition: N(1,1)
- Sample size: 15 Simulation size: 1000

- Simulation size: 100	00	
Comparing the	Environment set up	ns = 1000; n = 15; mu = 1; sd = 1;
estimators		<pre>meax = numeric(ns); medx = numeric(ns);</pre>
		<pre>trmx = numeric(ns); stdx = numeric(ns);</pre>
		set.seed(12345)
	Compute the	for (i in 1:ns) {
	numeric value of	x = rnorm(n, mu, sd)
	the estimators	meax[i] = mean(x)
		medx[i] = median(x)
		trmx[i] = mean(x, trim=0.1)
		$stdx[i] = sd(x)$ }
	Compute the mean	<pre>simumean = apply(cbind(meax,medx,trmx),2,mean)</pre>
	of estimators	
	Compute the sd of	<pre>simustd = apply(cbind(meax, medx, trmx), 2, sd)</pre>
	estimators	
	Compute bias	simubias = simumean - rep(mu,3)
	Compute MSE	<pre>simumse = simubias^2 + simustd^2</pre>
	Presentation	<pre>sumdata = rbind(c(mu, mu, mu), ns, simumean,</pre>
		simustd, simubias, simumse)
		col.name = c("mean","median","10% tmean")
		row.name = c("True value", "No. of simu", "MC
		Mean", "MC sd", "MC Bias", "MC MSE")
		<pre>dimnames(sumdata) = list(row.name,col.name)</pre>
		round(sumdata,4)
Checking coverage	getting $t_{0.025,n-1}$	t05 = qt(0.975, n-1)
probability of	Compute CI	<pre>lower = meax-t05*stdx/sqrt(n)</pre>
confidence interval		upper = meax+t05*stdx /sqrt(n)
	Test logic	coverage = sum(lower <= mu & upper >= mu)
	Compute coverage	coverage = coverage/ns
Checking size of t-	Calculate t stat	<pre>ttests = (meanx-mu) / (stdx/sqrt(n))</pre>
test	Compute rejection	reject = abs(ttests) > t05
	Calculate Size	size = sum(reject)/ns
Checking power of	Generate data under	ns = 1000; n = 15; mu0 = 1; sigma = 1;
t-test when	$H_1$	mu1 = mu0 + 0.5*sigma
$\mu_1 = \mu_0 + \frac{1}{2}\sigma$		<pre>meanx = numeric(ns); stdx = numeric(ns);</pre>
2		set.seed(12345)
		for (i in 1:ns) {
		x = rnorm(n, mu1, sigma)
		meanx[i] = mean(x)
		$stdx[i] = sd(x)$ }

Function; Variable; R	Reserved Keywords	R
1 33133131, 1 331313131, 1	Calculate t stat	t05 = qt(0.975, n-1)
		ttests = (meanx-mu0)/(stdx/sqrt(n))
	Compute rejection	#alternate method of calculating rejection
	Compare rejection	reject = ttests >= t05   ttests <= -t05
	Calculate power	<pre>power = sum(reject)/ns</pre>
	Carcarate power	Bootstrap method
Generate bootstrap	Compute true value	options (digit=4)
samples	Compute true varue	theta.hat = corr(var1, var2)
Samples	Generate bootstrap	B = 1000; n = nrow(data);
	samples	theta.b = numeric(B);
		for (b in 1:B) {
	& compute stat of interest	<pre>index = sample(1:n, size=n, replace=TRUE)</pre>
		bsample1 = var1[index]
	(corr in this case)	bsample1 = var1[index] bsample2 = var2[index]
		_
D	G 1	theta.b[b] = corr(bsample1, bsample2)
Bootstrap	Compute sd	sd(theta.b)
Estimation of		
Standard Error		
Bootstrap	Compute bias	<pre>bias = mean(theta.b) - theta.hat</pre>
Estimation of Bias		
Alternate method	Using Library boot	bcor = function (data, b) {
for SE and Bias		<pre>return(cor(data[b,1],data[b,2])) }</pre>
		library(boot)
		<pre>boot.cor = boot(data, statistic=bcor, R=5000)</pre>
		#gives original value from sample, bias and standard error
Basic bootstrap	Compute lower and	alfa = 0.05
Confidence interval	upper quantile	<pre>low = quantile(theta.b,alfa/2)</pre>
		high = quantile(theta.b,1-alfa/2)
	Compute CI	<pre>lower = 2*theta.hat-high</pre>
	•	upper = 2*theta.hat-low
Percentile bootstrap	Compute lower and	alfa = 0.05
confidence interval	upper quantile	<pre>low = quantile(theta.b,alfa/2)</pre>
		high = quantile(theta.b,1-alfa/2)
Alternate method	Using Library boot	<pre>bcor = function (data, b) {</pre>
for CI		<pre>return(cor(data[b,1],data[b,2])) }</pre>
		library(boot)
		<pre>boot.cor = boot(data, statistic=bcor, R=5000)</pre>
		<pre>boot.ci(boot.cor, type=c("basic", "norm", "perc"))</pre>
		#gives normal, basic and percentile CI
		#must generate boot.cor object first
	•	Numerical methods in R
Finding Root	One-dimension	a = 0.5; n = 20;
<i>3</i>	*uniroot only finds	$fy = function(y) \{a^2+y^2+2*a*y/(n-1)-(n-2)\}$
	1 root in region	rt = uniroot(fy, lower=0, upper=100)
	*function must	root = rt\$root #to get root
	have opposite signs	value = rt\$f.root #to get the value of fn
	in two end points	5 , 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	Finding roots	#input the coefficient of polynomial
	*polyroot finds	a0 = $a^2 - (n-2)$ ; a1 = $2*a/(n-1)$ ; a2 = 1;
	multiple roots,	polyroot(c(a0,a1,a2))
	including complex	$\#a_i$ is the coefficient of $x^i$
	roots	mal is the coefficient of x
Integration	One variable	<pre>fcn = function(y, mu, sigma) {</pre>
micgianon		abs(y)*exp(-(y-mu)^2/(2*sigma^2)) /
	integral	sqrt(2*pi)/sigma }
		integrate(fcn,lower=-Inf, upper=Inf, mu=0,
Ontinui ti	O 1:	sigma=1)
Optimiation	One-dimentional	$f = function(x) \{log(x+log(x))/log(1+x)\}$
		<pre>optimize(f,lower=4,upper=8,maximum=TRUE)</pre>

```
Function; Variable; Reserved Keywords
                  Two-dimensional
                                    LL = function(theta, sx, slogx, n)
                                      r = theta[1]; lambda = theta[2]
                  *"optim" performs
                                       loglik = n*r*log(lambda) + (r-1)*slogx-
                  minimization by
                                                        lambda*sx-n*log(gamma(r))
                  default
                                       return(loglik) }
                                    #or return(-loglik) if lazy to use fnscale
                                    n = 200; r = 5; lambda = 2;
                                    theta.start = c(1,1)
                                    x = rgamma(n, shape=r, rate=lambda)
                                    out = optim(par=theta.start, fn=LL, sx=sum(x),
                                                        sloqx=sum(loq(x)),
                                                        n=n,control=list(fnscale=-1))
                                    r = out$par[1]; lambda = out$par[2]
                                    result = out$value
                                      Regression Analysis
                                    cor(dataframe, method="person")
Pearson Correlation Coefficient
                                    #test if true correlation is 0
                                    cor.test(var1, var2)
Simple regression
                                    model1 = lm(y~x)
                                    #to see sum of squared regression and error
                                    anova (model1)
                                    #to see R-squared and linear model, F-test
                                    summary(model1)
                                    model1 = lm(y\sim x)
Testing assumptions
- Linear relationship: scatter plot
                                    par(mfrow=c(2,2))
                                    #scatter plot
                                    plot(y~x); abline(model1);
                                    title("Scatter plot and Regression Line")
- Normality and independence
                                    #residual plot
                                    rs = model1$resid; fv = model1$fitted;
      observe if there's any trend in
                                    plot(rs~x);abline(h=0)
residuals, it shouldn't have
                                    #normal QQ plot
                                    qqnorm(rs); qqline(rs)
                                    par(mfrow=c(1,1))
- Prediction
                                    model1$coeff[1]+value*model1$coeff[2]
                  case1: data in set
                                    predict(model1, data.frame(x=value))
                  case2: data not in
                                    #or the previous method
- Transformation
                  Investigate rs
                                    plot(x,y); lines(smooth.spline(x,y))
                                    x2 = x^2;
                  quadratic
                                    model2 = lm(y\sim x+x2)
                                    anova (model2); summary (model2)
                                    lx = log(x);
                  log
                                    model3 = lm(y\sim lx)
                                    anova (model3); summary (model3)
```

Function; Variable	Function; Variable; Reserved Keywords done by: Ling 1 Dec 19			
		SAS	R	
		Simulation		
Generate random numbers	Linear congruential generators	<pre>data try1;   seed = 1234;   do i = 1 to 10;     x = ranuni(seed);   output;   end; keep x;</pre>	<pre>ran = NULL for (i in 1:n) {     x1 = (a*x0+c)%%m     x0 = x1     ran = c(ran, x1/m) }</pre>	
	Uniform random numbers	data unif; seed = 1234; call streaminit(seed); n=100; a=0; b=10; do i = 1 to n; x = a + (b-a) * rand("UNIFORM"); output; end; keep x;	<pre>#uniform with (0,1) x = runif(1000) #uniform with (a,b) set.seed(1234) x = runif(n,a,b)</pre>	
	Normal $(\mu, \sigma^2)$	data norm; seed = 1234; call streaminit(seed); n=100; mu=0; sigma=1; do i = 1 to n; x = rand("NORMAL", mu, sigma); output; end; keep x;	x = rnorm(n, mean=mu, sd=sigma)	
	Exponential( $\lambda$ )	data expno; seed = 1234; call streaminit(seed); n=100; lambda=5; do i = 1 to n; x = lambda * rand('EXPONENTIAL'); output; end; keep x;	x = rexp(n, mean=lambda)	
	Gamma $(\alpha, \beta)$	data gammano; seed = 1234; call streaminit(seed); n=100; alpha=1; beta=2; do i = 1 to n; x = beta * rand('GAMMA', alpha); output; end; keep x;	x = rgamma(n, shape=alpha, scale=beta)	
	Chi-square(p)	data chisqno; seed = 1234; call streaminit(seed); n=100; df=10; do i = 1 to n; x = rand('CHISQUARE', df); output; end; keep x;	x = rchisq(n, df=p)	

$Beta(\alpha, \beta)$	data betano; seed = 1234; call streaminit(seed); n=100; alpha=2; beta=3;	x = rbeta(n, shape1=a, shape2=b)
	do i = 1 to n; x = rand('BETA', alpha, beta); output; end; keep x;	
t(k)	data tno; seed = 1234; call streaminit(seed); n=100; df=5; do i = 1 to n; x = rand('T', df); output; end; keep x;	x = rt(n, df=k)
F(m,n)	data fno; seed = 1234; call streaminit(seed); n=100; df1=5; df2=10; do i = 1 to n; x = rand('F', df1, df2); output; end; keep x;	x = rf(n, df1 = n1, df2 = n2)
Binomial(n,p)	data binomno; seed = 1234; call streaminit(seed); ns=100; n=10; p=0.3; do i = 1 to ns; x = rand('BINOMIAL', p, n); output; end; keep x;	x = rbinom(n, size, prob = p)
$Poisson(\lambda)$	data poisno; seed = 1234; call streaminit(seed); n=100; lambda=3; do i = 1 to n; x = rand('POISSON', lambda); output; end; keep x;	x = rpois(n, lamba)
Hypergeo(n,N,S)	data hypergeono; seed = 1234; call streaminit(seed); ns=100; popnsize=3; numbsucc=5; samplesize=3; do i = 1 to ns; x = rand('HYPERGEOMETRIC', popnsize, numbsucc, samplesize); output; end; keep x;	x = rhyper(nn, m=S, n=N, k=n)

	NBinom(r,p)	data negbinomno;	x = rnbinom(n, size=r, prob=p)
	(1,p)	seed = 1234; call streaminit(seed);	11 21 21 1, F-12 F,
		n=100; p=0.3; k=5;	
		do i = 1 to n;	
		x = rand('NEGBINOMIAL', p, k); output;	
		end; keep x;	
	_IL	Simulation Study	
Conditon of interes	est.		
	nators for location $\mu$ through a	simulation study	
	: sample mean, sample median		
- Underlying cond			
- Sample size: 15	,		
- Simulation size:	1000		
	Generate sample set	data simu;	
		seed = 123; ns=1000; n=15; mu=0; sigma=1;	
		call streaminit(seed);	
		do mcrep = 1 to ns;	
		do i = 1 to n;	
		value = rand("NORMAL",mu,sigma); output;	
		end;	
		keep mcrep value;	
		end;	
	Calculate t test	proc sort data=simu;	
		by mcrep;	
Size of t-test		proc univariate data=simu noprint mu0=0;	
α level		by mcrep;	
		var value;	
		<pre>output out=outtest probt = p;</pre>	
	Calculate rejection	data outtest1;	
		set outtest;	
		reject = $(p<0.05)$ ;	
	Calculate rejection rate	proc means data=outtest1 noprint;	
		var reject;	
		output out = results mean=rejrate;	
		proc print data=results;	
		var freq rejrate;	

		done by: Ling 1 Dec 19		
NT : 1		Describing numerical data		
Numerical	- Location	- Mean		
descriptive		- Median		
measures		- Mode		
	- Variability	- Variance or standard deviation		
		- Range		
		- Interquartile range		
		- Coefficient of Variation		
	- Other measures	- Minimum and Maximum	unbiased estimator of skewness:	
	unbiased estimator of	- First quartile and third	$\frac{\sqrt{n(n-1)}}{n-2}(\frac{m_3}{m_2^{3/2}})$	
	<u>kurtosis:</u>	quartile	$\frac{1}{n-2} (\frac{1}{m_2^{3/2}})$	
	$\frac{(n-1)}{(n-2)(n-3)} \left(\frac{(n+1)m_4}{m_2^2}\right)$	- Percentiles	2	
	$(n-2)(n-3)$ $m_2^2$	- Skewness [measure the		
	-3(n - 1)	symmetric]		
	- 1))	- Kurtosis [measure the tail]		
Graphical	- Histogram			
methods	- Boxplot, Stem and leaf			
	plot			
	- Scatter plot for bivariate			
	data			
		Robust Location estimator		
Robust location	- Trimmed mean (top-	mean: $\psi(x) = x$	6.2.2.3 Graph of Ψ(x) Trimmed Main Mober	
estimators	left)	median: $\psi(x) = sign(x)$		
$\sum_{i} \psi(y_i - \mu) = 0$	- Winsorized mean	trimmed: $\psi(x) = x  x  < c$ , els		
	- Huber's M-estimators	winsorized: $\psi(x) = -c$ , $x < -c$	c e. g	
	(top-right)	turkey:	Tokey Hampel	
	- Tukey's bisquare	$\psi(x) = x[1 - (\frac{x}{R})^2]^2 \text{ for }  X  < R$		
	estimator (bottom-left)	$\int_{0}^{\infty} \frac{\varphi(x) - x[1 - (R)]}{R} \int_{0}^{\infty} \frac{1}{1} \int_{0}^{\infty} $	4 4 4 9 2 4 9	
	- Hampel's M-estimator		X N = 4 805 CVM = + 2, b × 4, e = 5	
	(bottom-right)			
Robust	- Interquartile Range	Normal: σ̂	= IQR/1.34898	
measures of	(IQR)			
scale parameter	- Median Absolute		$_{i}( y_{i}-median_{j}(y_{j}) )$	
	Deviation (MAD)	Normal: $\hat{\sigma} =$	$= 1.4826 \times MAD$	
	(median of the diff in			
	median)			
	- Gini's mean difference	$G = \frac{1}{\cdots}$	$\sum  y_i - y_i $	
		$\binom{n}{2}$	<u></u>	
			$\sum_{i < j}  y_i - y_j  \\ \approx \frac{G}{2} \sqrt{\pi}$	
		σε	$\approx \frac{1}{2} \sqrt{\pi}$	
		e.g. $G = (a1 + + an)/n$		
		Categorical data		
	- Frequency tables			
	- Contingency tables			
	- Chi-square tests (single	$H_0$ : $\mu_1$ and $\mu_2$	are independent	
	tail test)	$H_1$ : $\mu_1$ and $\mu_2$ ar	re not independent	
	- Charts			
Paired Data	Same subjects under two di	fferent conditions		
	- McNemar's test	$\overline{H_0}$ : before	ore = after	
		H <sub>1</sub> : befo	ore ≠ after	
	- Bar Chart			
		Numerical data		
T		mple Tests (Hypothesis testing)		
Location	Parametric test		$a_0 = 0$	
	- One sample t-test	$\mu$	$t_0 \neq 0$	
	(normal population)			

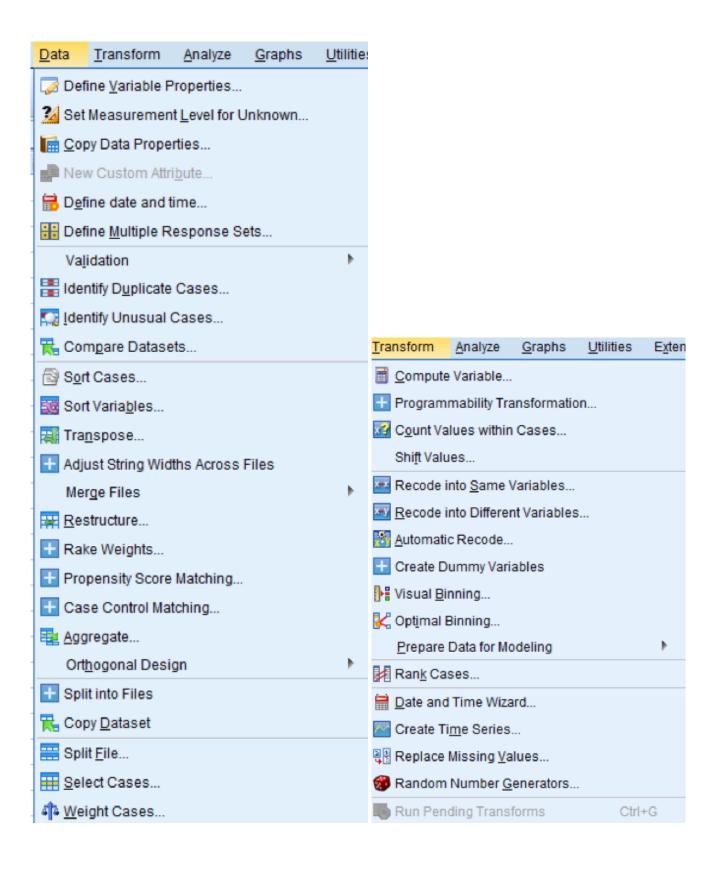
	Nonparametric tests	$\mu_0 = 0$			
	- Sign test	$\mu_0 \neq 0$			
	- Sign rank test				
	Two comparing two groups (Hypothesis testing)				
Independent	Two-sample t-test	$\mu_0 = \mu_1$			
samples	(normal population)	$\mu_0 \neq \mu_1$			
	Wilcoxon rank-sum test				
	(non-parametric)				
Related	Paired t-test (normal	$\mu_{before} = \mu_{after}$			
samples	population)	$\mu_{before} \neq \mu_{after}$			
	Sign Test or Wilcoxon				
	signed rank test(non-				
	parametric)	dent groups (One way englysis of verience)			
Domono otnio	1	dent groups (One-way analysis of variance)			
Parametric method	One-way analysis of variance	$\mu_x = \mu_y = \mu_z$			
memod	- ANOVA	$\mu_x \neq \mu_y \neq \mu_z$			
	Assumptions of ANOVA	1. Random sample			
	(based on F-test)	Equal variances for all the groups			
	& Model Checking	a. Bartlett test or Levene's test			
		3. Independence of errors			
		a. Residual plots			
		i. QQplot on residuals			
		ii. Plot residuals against the groups			
		4. Normal distribution of errors			
		a. Normality test on residuals			
		5. Additivitiy of treatment effects			
	Multiple comparison tests	LSD for $\mu_i - \mu_j$			
	- Least significant				
	difference (LSD)	$t_{\frac{\alpha}{2}.N-k} MSE(\frac{1}{n_1} + \frac{1}{n_2})$			
		$\overline{2}^{.N-k}\sqrt{\qquad n_1  n_2}$			
		We conclude $\mu_i$ is different from $\mu_j$ if			
		$ \overline{X}_{\iota} - \overline{X}_{J}  > LSD$			
	- Contrast method	$\left  \overline{X}_{\iota} - \overline{X}_{J} \right  > LSD$ A contrast of means is a linear combination of the means such that			
		the coefficients of the means such that the coefficients of the			
		means should sum up to zero			
		e.g. $C_1 = \mu_2 - \mu_3$ , $C_2 = 2\mu_1 - \mu_2 - \mu_3$			
		$H_0: C_1 = 0 \text{ and } C_2 = 0$			
	Others				
	- Duncan's multiple-range				
	- Student-Newman-Keul's	1			
Nonnananatai	- Scheffe's multiple-compa	rison procedure			
Nonparametric	Kruskal-Wallis test				
method		Simulation			
Usage	- Checking distribution	Theory:			
Usage	theory	Take a sample of size 4 from normal distribution with mean 0			
		$T = \frac{\bar{X}}{S/\sqrt{4}} \sim t(3)$			
		Simulation:			
		Generate 1000 random samples with size 4. Compute statistic T and			
		construct a historygram.			
		Superimpose the t(3) curve and compare			

	- Comparing estimators  - Buffon's needle experiment	Question: Comparing robustness of estimators in different conditions (underling distributions) Consider $MSE = Bias^2 + Variance$ Simulation: Generate random samples. Compute MSE for each estimators.  Question: Compute a question of interest through a function. Consider the probability that needle intersects a line $2/l$
		$\frac{2/l}{\pi d}$ Simulation: Simulate the experiement by N times and count the chances of success n/N give the estimate of the probability of success
		Generate numbers
Random number	- Congruential generators	$def: X_i = (aX_{i-1} + c) \bmod M$ Uniform random numbers: $U_i = X_i/M$
Uniform numbers	- Uniform random numbers	
Non-uniform random numbers	- Inversion method	If X has a continuous distribution function $F(x)$ , where $F(x) = \Pr(X \le x)$ , then $F(x) \sim Uniform(0,1)$ Generate U from Uniform(0,1) and set $X = F^{-1}(U)$
	- Exponential distribution	Generate U from Uniform(0,1) and set $X = F^{-1}(U)$ Let $U = F(X) = 1 - e^{-x/\lambda}$ Then $X = F^{-1}(U) = -\lambda \log (1 - U)$
	- Weibull distribution	Then $X = F^{-1}(U) = -\lambda \log (1 - U)$ $F(x) = 1 - \exp(-x^{\beta}), D = (0, \infty)$ $X = (-\log(1 - U))^{1/\beta}$
	- Cauchy distribution	$X = (-\log(1 - U))^{1/\beta}$ $F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} \left(\frac{x - \mu}{\sigma}\right)$ $X = \sigma \tan[\pi(U - 0.5)] + \mu$
Generate Normal random variable	- Box-Muller Algorithm	Generate $U_1$ and $U_2$ from uniform(0,1) $set \ \theta = 2\pi U_1; R = (-2log U_2)^{1/2}$ $set \ X = R \cos(\theta) \ and \ Y = R \sin(\theta)$ Now we have X and Y, two independent standard normal variables
Random variable from other random variables	- Cauchy distributions	$Y, Z \ independent \ and \ \sim N(0,1)$ $X = \frac{Y}{Z} \sim Cauchy(0,1)$ $If \ Y \sim N(\mu, \sigma^2) \ and \ Z \sim N(0,1)$ $X \sim Cauchy(\mu, \sigma^2)$ $If \ Y \sim N(0,1)$
	- Chi-square distribution	$If Y \sim N(0,1)$ $X = Y^2 \sim \chi^2(1)$ $If Y_1 \dots Y_n \text{ are } i.i.d \text{ and } X = Y_1^2 + \dots + Y_n^2$ $X \sim \chi^2(n)$ $If Y \sim N(0,1) \text{ and } Z \sim \chi^2(p), \text{ then}$
	- Student's t-distribution	$X = \frac{Y}{\sqrt{\left(\frac{Z}{p}\right)}} \sim t(p)$
	- F distribution	$Y \sim \chi^{2}(m) \text{ and } Z \sim \chi^{2}(n)$ $X = \frac{Y/m}{Z/n} \sim F(m, n)$
Functions	Other functions to generate the random no.	https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Distributions.html http://support.sas.com/documentation/cdl/en/lrdict/64316/HTML/default/viewer.htm#a001466748.htm
Generate random	Uniform(a,b)	$f(x) = \frac{1}{b-a}, a < x < b$
numbers	Normal( $\mu$ , $\sigma^2$ )	$f(x) = \frac{1}{b-a}, a < x < b$ $f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), -\infty < x < \infty$

	Exponential( $\lambda$ )	$f(x) = \frac{1}{2} \exp\left(-\frac{x}{2}\right), x > 0$
	$Gamma(\alpha, \beta)$	$f(x) = \frac{1}{\beta^{\alpha} \Gamma(\alpha)} x^{\alpha - 1} \exp\left(-\frac{x}{\beta}\right), x > 0$
	Chi-square(p)	$f(x) = \frac{1}{\frac{p}{2^2}\Gamma(\frac{p}{2})} x^{\frac{p}{2}-1} \exp\left(-\frac{x}{2}\right), x > 0$
	$Beta(\alpha, \beta)$	$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}, \text{ for } 0 < x < 1$
	t(k)	$f(x) = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right), x > 0$ $f(x) = \frac{1}{\beta^{\alpha} \Gamma(\alpha)} x^{\alpha - 1} \exp\left(-\frac{x}{\beta}\right), x > 0$ $f(x) = \frac{1}{2^{\frac{p}{2}} \Gamma(\frac{p}{2})} x^{\frac{p}{2} - 1} \exp\left(-\frac{x}{2}\right), x > 0$ $f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}, for \ 0 < x < 1$ $f(x) = \frac{\Gamma(\frac{k+1}{2})}{\Gamma(\frac{k}{2})} \frac{1}{\sqrt{k\pi}} \frac{1}{\left(1 + \frac{x^2}{k}\right)^{\frac{k+1}{2}}}, for \ -\infty < x < \infty$ $f(x) = \frac{\Gamma\left(\frac{n_1 + n_2}{2}\right)}{\Gamma\left(\frac{n_1}{2}\right) \Gamma\left(\frac{n_2}{2}\right)} \left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}} \frac{x^{\frac{n_1}{2} - 1}}{\left(1 + \frac{n_1}{n_2}x\right)^{\frac{n_1 + n_2}{2}}}, 0 < x < \infty$
	F(m,n)	$f(x) = \frac{\Gamma\left(\frac{n_1 + n_2}{2}\right)}{\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)} \left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}} \frac{x^{\frac{n_1}{2} - 1}}{\left(1 + \frac{n_1}{n_2}x\right)^{\frac{n_1 + n_2}{2}}}, 0 < x < \infty$
	Binomial(n,p)	$f(x) = {n \choose k} p^{x} (1-p)^{n-x}, for \ x = 0, 1,, n$
	$Poisson(\lambda)$	$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0,1,2,$
	Hypergeo(n,N,S)	$f(x) = \binom{n}{k} p^x (1-p)^{n-x}, for \ x = 0,1,,n$ $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0,1,2,$ $f(x) = \frac{\binom{S}{x} \binom{N-S}{n-x}}{\binom{N}{n}}, for \ x = 0,1,, \min(n,S)$ $f(x) = \binom{r+x-1}{x} p^r (1-p)^x, for \ x = 0,1,2,$ Simulation Studies in Stats
	NBinom(r,p)	$f(x) = {r + x - 1 \choose x} p^{r} (1 - p)^{x}, for x = 0,1,2,$
	S	imulation Studies in Stats
Rationale (in statistics)	Properties of estimators	<ul> <li>Bias</li> <li>Consistent</li> <li>Sampling variance</li> <li>Comparison with competing estimators on bias, precision etc</li> <li>Does confidence interval achieve the advertised nominal level of coverage (e.g.95%)?</li> </ul>
	Properties of hypothesis testing procedures	<ul> <li>Does hypothesis testing procedure attain the advertised level of significance or size (e.g. α = 0.05)?</li> <li>What power is possible against different alternatives to the null hypothesis? Do different test procedures deliver different power?</li> </ul>
Properties of estimators	Monte Carlo simulation approximation	<ul> <li>Generate S independent data sets under the conditions of interest</li> <li>Compute the numerical value of the estimators</li> <li>If S large enough, summart statistics should a good approximations to the true properties of the estimator <u>under the conditions of interest</u></li> </ul>
	Checking the coverage probability of confidence interval	$(\bar{Y} - t_{0.025,n-1} \frac{S}{\sqrt{n}}, \bar{Y} + t_{0.025,n-1} \frac{S}{\sqrt{n}})$
Properties of hypothesis tests	Testing size/ level of test	Generate data under $H_0$ : $\mu = \mu_0$ Calculate the proportion of rejections of $H_0$ Approximate the true probability of rejecting $H_0$ when it is true. Proportion should be $\approx \alpha$
	Evaluate power	Generate data under $H_1: \mu \neq \mu_0$ Calculate the proportion of rejections of $H_0$ Approximate the true probability of rejecting $H_0$ when it is false.
		Bootstrap Method
Bootstrap distribution	Empiritcal distribution	$f_n(x) = \begin{cases} 1/n, & for \ x = x_1 \dots x_n \\ 0, & otherwise \end{cases}$
	Empirical cumulative distribution	$f_n(x) = \begin{cases} 1/n, & for \ x = x_1 \dots x_n \\ 0, & otherwise \end{cases}$ $F_n(t) = \frac{\# \ of \ x's < t}{n}$

Method	Estimate distribution function $F_{\overline{\theta}}(\cdot)$	Generate bootstrap replicate $x^{*(b)} = (x_1^*,, x_n^*)$ into	
	$\Gamma_{\theta}(f)$	$\widehat{\theta}^{*(b)} = (\widehat{\theta}_1^{\ *}, \dots, \widehat{\theta}_n^{\ *})$ Calculate bootstrap estimate of $F_{\overline{\theta}}(\cdot)$	
Statistics	Standard Error	$\widehat{se}\big(\widehat{\theta}^*\big) = \sqrt{\frac{1}{B-1}\sum_{b=1}^B(\widehat{\theta}^{*(b)} - \overline{\widehat{\theta}^*})^2}$ where	
	Bias	$ \frac{\overline{\hat{\theta}^*} = \frac{1}{B} \sum_{b=1}^{B} \hat{\theta}^{*(b)}}{bias(\hat{\theta}) = E(\hat{\theta} - \theta) = E(\hat{\theta}) - \theta} $	
	Confidence interval	$\widehat{bias}(\widehat{\theta}) = \overline{\widehat{\theta}^*} - \widehat{\theta}$ Basic $(2\widehat{\theta} - \widehat{\theta}^*_{1-\alpha/2}, 2\widehat{\theta} - \widehat{\theta}^*_{\alpha/2})$	
		Percentile $(\hat{\theta}^*{}_{\alpha/2},\hat{\theta}^*{}_{1-\alpha/2})$ where $\hat{\theta}^*{}_{\alpha}$ is the $\alpha$ sample quantitle from the empirical cdf of the	
		replicates $\hat{\theta}^*$	
Root-finding	One-dimension	Numerical methods in R e.g. solve for y in equation	
Root Inding	one dimension	$a^2 + y^2 + \frac{2ay}{n-1} = n - 2, a = 0.5, n = 20$	
Integration	One-variable	e.g. solve for the integral	
		$\int \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{(y-\mu)^2}{2\sigma^2})$	
Maximum Likelihood Method	Concept	A method to estimate the parametes of a distribution, the parameter should maximizes the likelihood function $\max L(\theta) = f(x_1, x_2,, x_n   \theta)$ Assuming independent and identically distributed random values $f(x_1, x_2,, x_n   \theta) = \prod_{i=1}^n f(x_i   \theta)$	
		Since population parameter $\theta$ is unknown, we estimate with $\hat{\theta}$ In reality the product function is difficult to differentiate, so we take log of the function	
	Method1: finding $\theta$ which maximizes $L(\theta)$ or $l(\theta)$	Newton's method to find the value which maximises the function	
	Method2: finding the first order condition	$\frac{d}{d\theta}L(\theta) = 0 \text{ or } \frac{d}{d\theta}l(\theta) = 0$	
Optimization	One-dimension	e.g. Maximize the fuction $f(x) = \frac{\log(1 + \log(x))}{\log(1 + x)}, wrt \ x \ and \ x > 0$	
	Two-dimension	e.g. Find the maximum likelihood estimator of $r$ and $\lambda$	
		$\max L(r,\lambda) = \frac{(\lambda)^{nr}}{\Gamma(r)^n} \prod_{i=1}^n x_i^{r-1} \exp\left(-\lambda \sum_{i=1}^n x_i\right), x_i \ge 0$	
		or consider the log-likelihood function $\max \ l(r,\lambda) = nr \ log\lambda - nlog\Gamma(r) + (r-1)\sum_{i=1}^n logx_i - \lambda\sum_{i=1}^n x_i$	
		Regression Analysis	
Correlation	Pearson Correlation Coefficient - Indicates the strength of linear relationship	$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$ $\rho = \frac{E[(X - \mu_x)(Y - \mu_y)]}{\sqrt{E[(X - \mu_x)^2\sqrt{E[(Y - \mu_y)^2]}}}$	
	between two variabels	$\sqrt{E[(X-\mu_x)^2\sqrt{E[(Y-\mu_y)^2]}]}$	

Sample regression	Linear model establish a relationship between two	$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ where $\varepsilon_i \sim N(0, \sigma^2)$ independently
regression	variables	$E(Y_i x_i) = \beta_0 + \beta_1 x_i$
		$Var(Y_i x_i) = Var(\varepsilon_i) = \sigma^2$
	Assumptions	- $Y_i$ 's are independent
		- $Y_i$ 's follow normal distribution with given parameters
		Study residuals
		- $Y_i$ 's have the same variances
		Study residuals
		- Linear relationship exist between Y and X
		Test using scatter plot
	Prediction	Case1: values in the dataset
		Case2: values not within the dataset
	Transformation	- Quadratic term to the regression line
		- Log transformation
		Bonus
Machine	Extra	https://www.analyticsvidhya.com/blog/2017/09/common-
Learning		machine-learning-algorithms/



<u>A</u> nalyze	<u>G</u> raphs	<u>U</u> tilities	E <u>x</u> tensions
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Co <u>m</u>	pare Means	3	<b>&gt;</b>
<u>G</u> ene	eral Linear I	Model	<b>&gt;</b>
Gene	erali <u>z</u> ed Line	ear Models	<b>&gt;</b>
Mi <u>x</u> e	d Models		<b>&gt;</b>
Corre	elate		<b>&gt;</b>
<u>R</u> egr	ession		<b>&gt;</b>
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Neur	al Net <u>w</u> orks	3	<b>&gt;</b>
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Fore	casting		•
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