EC2104 Tutorial 7 solution

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Section 1

Question 7: Finding eigenvalues and eigenvectors

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Given $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$

- Find eigenvectors and eigenvalues
  - $\mathbf{A} - \lambda \mathbf{I} = \begin{pmatrix} -1-\lambda & 3 \\ 2 & -\lambda \end{pmatrix}$
  - Solving $det(\mathbf{A} - \lambda \mathbf{I}) = 0 = (1+\lambda)\lambda - 6 \Rightarrow \lambda_1 = 2, \lambda_2 = -3$
  - Solving $(\mathbf{A} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}$
    - $\lambda = 2 : \left[\begin{array}{cc|c} -3 & 3 & 0 \\ 2 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \mathbf{v} = s \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
    - $\lambda = -3 : \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 2 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & \frac{3}{2} & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \mathbf{v} = s \begin{pmatrix} -\frac{3}{2} \\ 1 \end{pmatrix}$

Section 2

Question 7: Eigendecomposition

Question 7: Eigendecomposition

Given $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$ $\mathbf{P} = \begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}$ $\mathbf{P}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$

- Since $\mathbf{Av} = \lambda\mathbf{v}$, we have $\mathbf{A} = \mathbf{PDP}^{-1}$
  - $\mathbf{D} := [\lambda_1\ \lambda_2]$, $\mathbf{P} := [\mathbf{v}_1\ \mathbf{v}_2]$
  - $\mathbf{AP} = \mathbf{PD} \Rightarrow \mathbf{APP}^{-1} = \mathbf{PDP}^{-1}$
    - $\mathbf{AP} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 9 \\ 2 & -6 \end{pmatrix}$
    - $\mathbf{PD} = \begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 9 \\ 2 & -6 \end{pmatrix}$
    - Note: $\mathbf{DP} = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ -3 & -6 \end{pmatrix}$

Eigendecomposition (cont)

Given $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$ $\mathbf{P} = \begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}$ $\mathbf{P}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$

Eigendecomposition: $\mathbf{A} = \mathbf{PDP}^{-1}$

- In general, power of diagonal matrix is the power of the entries
  - i.e. $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}^n = \begin{pmatrix} 2^n & 0 \\ 0 & (-3)^n \end{pmatrix}$
- Using eigendecomposition we have $\mathbf{A}^n = \mathbf{PD}^n\mathbf{P}^{-1}$
  - $\mathbf{A}^n = \mathbf{AA}\cdots\mathbf{A} = (\mathbf{PDP}^{-1})(\mathbf{PDP}^{-1})\cdots(\mathbf{PDP}^{-1})$
  - Note that $\mathbf{PP}^{-1} = \mathbf{I}$
  - Simplifying: $\mathbf{A}^n = \mathbf{PDD}\cdots\mathbf{DP}^{-1} = \mathbf{PD}^n\mathbf{P}^{-1}$