EC2104 Tutorial 5 solution

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Section 1

Question 1

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- MR: $\frac{d\pi}{dQ} = P(Q) + Q\frac{dP}{dQ}$
- $\epsilon_D = \frac{dQ}{dP}\left(\frac{P}{Q}\right)$
- Relationship: $Q\frac{dP}{dQ} = \frac{P}{\epsilon_D}$
- Result: $MR = P + Q\frac{dP}{dQ} = P + \frac{P}{\epsilon_D}$ (ans: a)

Section 2

Question 2

Question 2

Given equation system, find $\frac{du}{dv}, \frac{du}{dy}, \frac{dv}{dx}, \frac{dv}{dy}$

$$\begin{aligned} \ln(x+u) + uv - y^2 e^y + y &= 0 \\ u^2 - x^v &= v \end{aligned}$$

- What do I need? (1) Solve differentials (2) Substitute values at the point (3) Solve system of linear equations (4) Find coefficients

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- Differentials:

$$\frac{dx + du}{x + u} + udv + vdu - 2ye^v dy + y^2 e^v dv + dy = 0$$
$$2udu - vx^{v-1}dx - x^v \ln(x)dv = dv$$

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$$dx + du - dv - 2dy - dv + dy = 0 \Rightarrow du - 2dv = dy - dx$$
$$-2du - \ln(2)dv = dv \Rightarrow dv = -\frac{2}{1+\ln(2)}du$$
$$\Rightarrow du = -\frac{1+\ln(2)}{2}dv$$

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- Solve: $du - 2dv = dy - dx$

$$du + \frac{4}{1+\ln(2)}du = dy - dx \Rightarrow du = \frac{1+\ln(2)}{5+\ln(2)}(dy - dx)$$

$$-\frac{1+\ln(2)}{2}dv + 2dv = dy - dx \Rightarrow dv = -\frac{2}{5+\ln(2)}(dy - dx)$$

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- Find coefficients:
$\frac{du}{dy} = \frac{1+\ln(2)}{5+\ln(2)}, \frac{du}{dx} = -\frac{1+\ln(2)}{5+\ln(2)}, \frac{dv}{dy} = -\frac{2}{5+\ln(2)}, \frac{dv}{dx} = \frac{2}{5+\ln(2)}$ (ans: a)

Section 3

Question 3, 4, 5

Question 3, 4, 5

(True/False) If $g(x) = x^5, \lim_{x\to 2} \frac{g(x)-g(2)}{x-2} = 80$

- Applying L'H rule: $\Rightarrow \lim_{x\to 2} g'(x) = 5(2)^4 = 80$ (True)

(True/False) If f is differentiable, then $\frac{d}{dx} f(\sqrt{x}) = \frac{f'(x)}{2\sqrt{x}}$

Suppose f is continuous on $[a, b]$ and differentiable on (a, b). If $f'(x) > 0$ on (a, b). Which is necessarily true?

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   - False, $f'(x) > 0$. Constant function has $f'(x) = 0$
4. f is convex on (a, b)
   - False, not necessarily true. Require knowledge on $f''(x)$

Section 4

Question 6 (a)

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Given pdf: $f(x) = \frac{\lambda a \exp(-\lambda x)}{(exp(-\lambda x)+a)^2}, x \in \mathbf{R}, a, \lambda > 0$

- Show that $F(x) = \frac{a}{exp(-\lambda x)+a}$, given $C = 0$

Question 6 (a)

Given pdf: $f(x) = \frac{\lambda a \exp(-\lambda x)}{(exp(-\lambda x)+a)^2}, x \in \mathbf{R}, a, \lambda > 0$

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  - Note $F(x) = \int f(x)dx$, $F(x)$ is called Cumulative distribution function

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    1. Substitution method: $u = exp(-\lambda x) + a$

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    1. Substitution method: $u = exp(-\lambda x) + a$
    2. Observation: $\frac{d}{dx} \frac{1}{exp(-\lambda x)+a} = \frac{\lambda exp(-\lambda x)}{(exp(-\lambda x)+a)^2}$

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  - Note that for all CDF, $\lim_{x \to +\infty} F(x) = 1$
  - Formally: $\lim_{x \to +\infty} \frac{a}{exp(-\lambda x)+a} = \frac{a}{a} = 1 \because \lim_{x \to +\infty} exp(-\lambda x) = 0$

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  - Note that for all CDF, $\lim_{x \to -\infty} F(x) = 0$
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Question 6b

Given pdf: $f(x) = \frac{\lambda a \exp(-\lambda x)}{(exp(-\lambda x)+a)^2}, F(x) = \frac{a}{exp(-\lambda x)+a}, x \in \mathbf{R}, a, \lambda > 0$

- Show that $\int_{-\infty}^{x} f(t)dt = F(x)$

Question 6b

Given pdf: $f(x) = \frac{\lambda a \exp(-\lambda x)}{(exp(-\lambda x)+a)^2}, F(x) = \frac{a}{exp(-\lambda x)+a}, x \in \mathbf{R}, a, \lambda > 0$

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- Show that $\int_{-\infty}^{x} f(t)dt = F(x)$
  - Note: integrating pdf = cdf
  - Formally: $\int_{-\infty}^{x} f(t)dt = \lim_{s \to -\infty} F(x) - F(s) = F(x)$

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  - Formally: $f(x) > 0 \because exp(\cdot) > 0$

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  - Note: in general cdf is an increasing function, this question asks a narrower definition of strictly increasing function
  - Formally: $f(x) > 0 \because exp(\cdot) > 0$
  - Can you think of why this cdf is strictly increasing? Look at the support of the pdf

Question 6c

Given pdf: $f(x) = \frac{\lambda a \exp(-\lambda x)}{(exp(-\lambda x)+a)^2}, F(x) = \frac{a}{exp(-\lambda x)+a}, x \in \mathbf{R}, a, \lambda > 0$

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Section 5

Question 7

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  - E.g. $f(x, y) = xy + exp(xy)$

Question 7 c

Given $H(Q) = Q^2, Q(K, L) = AK^\alpha L^\beta, K > 0, L > 0$

- Find marginal rate of technical substitution for the isoquants of H

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  - MRTS:

$$\frac{H'_K}{H'_L} = \frac{2(AK^\alpha L^\beta)\alpha K^{\alpha-1}AL^\beta}{2(AK^\alpha L^\beta)\beta L^{\beta-1}AK^\alpha} = \frac{\alpha L}{\beta K}$$