Finding convergence.
① $\lim_{n\to\infty} a_n = L$
② M.I. $x_{n+1} > x_n$
③ Ratio test

Finding limits
① divide by highest powers
② L'H rule

Definitions

Sequence

A sequence $\{a_n\}$ is said to be
1. Increasing if $a_n \leq a_{n+1}$ for all n;
2. Decreasing if $a_n \geq a_{n+1}$ for all n;

Such a sequence is called a monotonic sequence

Arithmetic Sequence: $a_n = a + (n-1)d$ ⇒ always infinite

[MCT monotonic convergent theorem]

Sequence

① increasing + upper bound
② decreasing + lower bound

A number C is called a lower bound of $\{a_n\}$ if $a_n \geq C$ for all n, and C is called an upper bound of $\{a_n\}$ if $a_n \leq C$ for all n. A sequence $\{a_n\}$ is said to be bounded if it has an upper bound and lower bound

Infinite Series

If $\lim_{n\to\infty} \sum_{k=1}^{n} a_k = \sum_{k=1}^{\infty} a_k$ exist, infinite series is convergent. Else divergent

Geometric Series

express as $\sum A r^k$ to find convergent condition

$\sum_{n=1}^{\infty} ar^{n-1} = S_n = \frac{a(1-r^n)}{1-r}$, infinite series is convergent only if $|r|<1$, where r is the common ratio and a is a constant $1+r+r^2+r^3+\dots$

Telescopic Series

$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1-\frac{1}{n+1}\right) = 1$, making use of partial fraction

(eventually) non-negative series

A series $\sum_{k=1}^{\infty} a_k$ is called an (eventually) non-negative series if each $a_k \geq 0$ (eventually). $k \in \mathbb{N}$

Alternating Series

$\sum_{k=1}^{\infty} \underbrace{(-1)^{k+1}}_{\text{alternate sign}} a_k = a_1 - a_2 + a_3 - a_4$ or $\sum_{k=1}^{\infty} (-1)^k a_k = -a_1 + a_2 - a_3 + a_4$

Power Series

$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots c_n(x-a)^n + \dots$

Differentiation & Integration

to find constant ⇒ $\int_0^x f(x)\,dx$

$f'(x) = \sum_{n=0}^{\infty} n c_n (x-a)^{n-1}$; $\int_0^x f(x)\,dx = \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1} + C$

$f(x)$ always shares the same domain, regardless of diff/integration

Remember to investigate for Constant

Taylor Series (expand fn into series)

$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + \underbrace{f'(a)(x-a)}_{\text{order } 1} + \dots + \underbrace{\frac{f^{(n)}(a)}{n!}(x-a)^n}_{\text{order } n} + \dots$

Maclaurin Series

$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n \dots$

Taylor Polynomials

$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

Best polynomial approximation of degree n

Theorem

Sequence

An increasing sequence with an upper bound is convergent.
A decreasing sequence with a lower bound is convergent.

Squeeze Theorem

If $a_n \leq b_n \leq c_n$ for all n beyond some index n_0, and if $\lim_{n\to\infty}(a_n) = L = \lim_{n\to\infty}(c_n)$ then $\lim_{n\to\infty} b_n = L$

useful to identify
① perfect square
② power of n

Other indeterminate

$\frac{0}{0}$; $\frac{\infty}{\infty}$; 0^0 ; ∞^0 ; 1^∞ [solve by taking ln]

Infinite Series

If series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent, then the series $\sum_{n=1}^{\infty}(a_n+b_n)$ is also convergent and $\sum_{n=1}^{\infty}(a_n+b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$. If series $\sum_{n=1}^{\infty} a_n$ is convergent and $c \in \mathbb{R}$, then the series $\sum_{n=1}^{\infty} ca_n$ is also convergent and $\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$

n^{th} term test

[Test for divergence only] If $\{a_n\}$ does not converge to 0, then the infinite series $\sum_{k=1}^{\infty} a_k$ is divergent. A test for divergence ONLY, no conclusion if $\lim_{n\to\infty} a_n = 0$

Consider 2 (eventually) non-negative series $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ Test for convergence

p-series test

[Applicable to p-series only] The series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ is convergent if $p>1$ and is divergent if $p \leq 1$

Ratio Test [use for power series, find R]

$|x-a| < R$

① interval of convergence $x=a$
② radius of convergence

Suppose that the limit $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists

$\frac{a_{n+1}}{a_n}$ can be used to investigate if series is increasing/decreasing

1. If $\rho < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\rho > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
3. No conclusion if $\rho = 1$.

$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ for power series

Applicable to certain series which look like the geometric series, series with $n!$ and certain series defined recursively.

Root test

Suppose that the limit $\rho = \lim_{n\to\infty} a_n^{1/n}$ exists
1. If $\rho < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\rho > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
3. No conclusion if $\rho = 1$

Applicable to certain series where a_n involves a highe power such as n-th power.

Alternating Series Test

Let $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ be alternating series. Suppose that
1. $a_n \geq 0$ for all n
2. $\{a_n\}$ is decreasing (i.e. $a_n \geq a_{n+1}$ for all n), and
3. $\lim_{n\to\infty} a_n = 0$

If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then it converges. Every series is either absolutely convergent, conditionally convergent or divergent.

Comparison Test

Suppose that there exists $K \in \mathbb{N}$ such that $0 \leq a_k \leq b_k$ for all $k \geq K$
1. If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges.
2. If $\sum_{k=1}^{\infty} a_k$ diverges, then $\sum_{k=1}^{\infty} b_k$ diverges.

Usually compares the given series with a geometric series or a p-series. Easier to solve oscillating factor/term

Limit Comparison Test

Suppose that limit $\rho = \lim_{n\to\infty} \frac{a_n}{b_n}$ exists
1. If $\rho > 0$, then either the two series both converges or both diverge
2. If $\rho = 0$ and $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges

Usually compares the given series with a geometric series or a p-series.

	Usually easier to use than comparison test
Binomial expansion	$\Sigma_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots + x^n + \ldots = \frac{1}{1-x}$, $-1 < x < 1$ * Convert power series to Binomial expansion for simplification
Convergence of Power Series	Case i) $R=0$ Converges only at $x=a$ and diverges elsewhere
	Case ii) $R=h$ Converges for all x in interval $(a-h, a+h)$ Diverge elsewhere
	Case iii) $R=\infty$ Converges for all values of x
Standard Taylor Series	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$, $-\infty < x < \infty$ $\quad \sum_{n=0}^{\infty} \frac{x^n}{n!}$
	$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$, $-\infty < x < \infty$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
	$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$, $-\infty < x < \infty$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
	$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$, $-1 \le x \le 1$
	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, $-1 < x < 1$

$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$

$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ $\quad \lim_{x\to 0} \frac{\tan(x)}{x} = 1$

$\lim_{x\to\infty} \frac{\sin(x)}{x} = 0$ $\quad \sec(x) = \frac{1}{\cos(x)}$

$\lim_{x\to 0} \cos(x) = 1$

$\lim_{x\to\infty} f(a) = f(A)$
$= f(\lim_{x\to\infty} a)$

$\lim_{n\to\infty} a^{\frac{1}{n}} = 1$, for $a > 0$

$\lim_{n\to\infty} n^{\frac{1}{n}} = 1$; $\lim_{n\to\infty} \frac{1}{n}\ln(n) = 0$

$\lim_{n\to\infty} r^n = 0$ for $|r| < 1$

$\lim_{n\to\infty} (1 + \frac{a}{n})^n = e^a$, $a \in \mathbb{R}$

$\lim_{n\to\infty} x^x = e^0 = 1$

~~Different~~ Limit clue:
convert to $(1 + \frac{a}{n})^n$ form

Finding $\lim_{x\to a} f(x)^{g(x)}$

① consider
$\lim_{x\to a} \ln(f(x))\, g(x) = L$

② $\lim_{x\to a} f(x)^{g(x)} = e^L$

L'H rule:
only on
$\frac{0}{0}$ $\frac{\infty}{\infty}$

$(\tan(x))' = \sec^2(x)$
$(\sec(x))' = \sec(x)\tan(x)$
$(a^x)' = a^x \ln(a)$
$(\lg_a u)' = \frac{1}{\ln(a)} \frac{1}{u} \frac{du}{dx}$

$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)\left(1 - \frac{1}{n+2}\right) \cdots \left(\frac{2n-2}{2n-1}\right)\left(\frac{2n-1}{2n}\right)$
$= \lim_{n\to\infty} \left(\frac{1}{2}\right)$
$= \frac{1}{2}$

$\sqrt{a} - \sqrt{b} \Rightarrow \frac{a^2 - b^2}{\sqrt{a} + \sqrt{b}}$

$1 - \frac{1}{a^2} \Rightarrow \frac{(a+1)(a-1)}{a^2}$

$\left|\frac{\sin(f(x))}{a} + \frac{\cos(g(x))}{b}\right| \le \frac{1}{a} + \frac{1}{b}$

$\ln(f(x)!) \le \ln(f(x))^{f(x)}$

$\ln(ax^2 + bx + c) \le \ln(ax^2 + bx^2 + cx^2) = \ln((a+b+c)x^2)$

$\ln(ab) = \ln(a) + \ln(b)$

[Shortcut to get summation] → Taylor series
can sub x as $f(x)$

$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n$: G.S. $-1 < x < 1$
$= \sum_{n=0}^{\infty} x^n$

$d(\ln(1+x)) = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-1)^n x^n$

$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$

$d(\tan^{-1} x) = \frac{1}{1+x^2}$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

MA2311 Cheatsheet

Three Dimensional Space

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$\|v_1\| = \sqrt{x_1^2+y_1^2+z_1^2}$ (magnitude)

$\|v_1 - v_2\|^2 = \|v_1\|^2 + \|v_2\|^2 - 2\|v_1\|\|v_2\|\cos\theta$ (law of cosine)

$\cos\theta = \dfrac{v_1 \cdot v_2}{\|v_1\|\|v_2\|}$ (angle)

$v_1 \cdot v_2 = x_1x_2 + y_1y_2 + z_1z_2$ (dot product)

[if perpendicular] $v_1 \cdot v_2 = 0$

[same vector] $v_1 \cdot v_1 = \|v_1\|^2 \geq 0$

[length of projection] $\|\text{proj}_a b\| = \|b\|\cos\theta = \dfrac{a \cdot b}{\|a\|}$ ← aka scaler projection

[unit vector of p] $\dfrac{a \cdot b}{\|a\|^2} a$ ← aka vector projection

[Vector Product] $v_1 \times v_2 = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

$v_1 \times v_2 = -v_2 \times v_1$

$v_1 \times (v_2 + v_3) = v_1 \times v_2 + v_1 \times v_3$

$v_1 \times v_2$ is $\perp$ to the plane

$\|v_1 \times v_2\| = \|v_1\|\|v_2\|\sin\theta$

Lines in 3D space

Vector equation:

$(x_0 i + y_0 j + z_0 k) + t(ai + bj + ck), \ \forall t \in \mathbb{R}$

$= r_0 + t\vec{v} = \vec{r}$

Parametric equations:

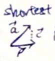
$\vec{r} = xi + yj + zk$

$\Rightarrow x = \cdots ; y = \cdots ; z = \cdots$ ← parametric form

solve for:

① points of intersection (equate $r_1 = r_2$)

② shortest distance (~~projection~~ perpendicular length)



find projection then: $c^2 = a^2 - p^2$

planes in 3D space

Cartesian equation of a plane

$ax + by + cz = d$

where $d = ax_0 + by_0 + cz_0$

[dot product of vector & normal)

Distance from a point to a plane

$\dfrac{\text{point} \cdot \vec{n}}{\|\vec{n}\|} = \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2+b^2+c^2}}$

if finding vector = intersection of line & plane

where point $S = (x_0, y_0, z_0)$

plane : $ax + by + cz = d$

Vector functions of one variable

$r(t) = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix} = f(t)i + g(t)j + h(t)k$

$r'(t) = \begin{bmatrix} f'(t) \\ g'(t) \\ h'(t) \end{bmatrix}$

space curve: $\vec{OP}$ lies on the image of the vector function

$\Rightarrow \vec{OP} = r(t_0) \ \forall t_0 \in \mathbb{R}$

smooth curves:

$r'(t)$ is continuous

$r'(t)$ is never zero

Tangent vector

unit tangent: $\dfrac{r'(t_0)}{\|r'(t_0)\|}$

Arc length of a space curve

$L = \int_a^b \|r'(t)\| \, dt$

Lagrange Multipliers

$F(x,y,\lambda) = f(x,y) - \lambda g(x,y)$

solve for $\dfrac{\partial F}{\partial x} ; \dfrac{\partial F}{\partial y} ; \dfrac{\partial F}{\partial \lambda} = 0$

Functions of several variables

$f(x,y) = \sqrt{1-x^2-y^2}$

$D = \{(x,y) : x^2+y^2 \leq 1\}$ domain

Partial Derivatives

similiar def for higher ~~for~~ order derivatives

First order partial derivatives

$\dfrac{d}{dx} f(x,b)\Big|_{x=a} = \lim_{h\to 0} \dfrac{f(a+h,b) - f(a,b)}{h}$

$\dfrac{\partial f}{\partial x}\Big|_{(a,b)}$ or $f_x(a,b)$

Chain rule

$z(t) = f(x(t), y(t))$

$\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

for two independent var

$z(s,t) = f(x(s,t), y(s,t))$

$\dfrac{\partial z}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial s}$

$\dfrac{\partial z}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial t}$

Directional Derivatives

rate of Δ in any direction

directional derivatives of $f(a,b)$

in direction of unit vector $\vec{u} = (u_1 i + u_2 j)$

$D_u f(a,b) = \lim_{h\to 0} \dfrac{f(a+hu_1, b+hu_2) - f(a,b)}{h}$

note:

$D_i f(a,b) = f_x(a,b)$

$D_j f(a,b) = f_y(a,b)$

$D_u f(a,b) = f_x(a,b) \cdot u_1 + f_y(a,b) \cdot u_2$

Gradient Vector

$\nabla f = f_x i + f_y j$

$\Rightarrow D_u f(a,b) = \nabla f(a,b) \cdot \vec{u}$

where $\vec{u}$ is a unit vector

physical meaning:

$df = D_u f(a,b) \cdot dt$

Critical points

$f_x(a,b) = 0$ or does not exist

Saddle points

neither max/min

max from one direction & min from another

Second order derivative test

$f_x(a,b) = 0$ & $f_y(a,b) = 0$

$D = f_{xx}(a,b) f_{yy}(a,b) - f_{xy}(a,b)^2$

Decision for max/min / saddle

critical points	f_{xx}	f_{xy}	f_{yy}	$D(x,y)$	conclusion
	>0			>0	min
	<0			>0	max
				<0	saddle
				$=0$	no conclusion

Mixed Derivatives

for most function

$f_{xy}(a,b) = f_{yx}(a,b)$

Multiple Integrals

double integral

$\iint_R f(x,y)\,dA = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i, y_i)\Delta A_i$

= vol under surface $z = f(x,y)$
above x-y plane
over the region R

properties

if $f(x,y) \geq g(x,y)\ \forall (x,y) \in R$
$\Rightarrow \iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$

$\iint_R dA = A(R)$, area of R

$m \leq f(x,y) \leq M\ \forall (x,y) \in R$
$\Rightarrow mA(R) \leq \iint_R f(x,y)\,dA \leq MA(R)$

Rectangular Regions

$a \leq x \leq b \quad c \leq y \leq d$

$\iint_R f(x,y)\,dA = \int_c^d \left[\int_a^b f(x,y)\,dx\right] dy$
$= \int_a^b \left[\int_c^d f(x,y)\,dy\right] dx$

* without Δ values, only for rectangular
if $f(x,y) = g(x)h(y)$,
$\iint_R g(x)h(y)\,dA = \left(\int_a^b g(x)\,dx\right)\left(\int_c^d h(y)\,dy\right)$

General region – Type A

$R: g_1(x) \leq y \leq g_2(x) \quad a \leq x \leq b$

$\iint_R f(x,y)\,dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\right] dx$

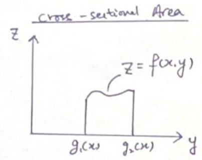
General region – Type B

$R: h_1(y) \leq x \leq h_2(y), \quad c \leq y \leq d$

$\iint_R f(x,y)\,dA = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\right] dy$

Note:
think of left/right top/bottom bounds
the pair with easier straight line will be preffered to use as the bounds

Cross-sectional Area



$C(x) = \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy$

Volume:
$\int_a^b C(x)\,dx$

polar coordinates

→ useful for circle
e.g. $R: -\sqrt{1-x^2} \leq y \leq \sqrt{1+x^2}, -1 \leq x \leq 1$
convert $R: 0 \leq r \leq 1 \quad 0 \leq \theta \leq 2\pi$
e.g. ~~big~~ ring: $1 \leq r \leq 2, 0 \leq \theta \leq 2\pi$
~~plo~~ polar triangles:

$R: a \leq r \leq b$
$\alpha \leq \theta \leq \beta$

change of variables

$x = r\cos\theta$
$y = r\sin\theta$
$dA = r\,dr\,d\theta$

Application of double integrals

Volume

$\iint_R f(x,y)\,dA$
bounded by $z = f(x,y)$ and R

Surface Area

$S = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\ dA$

Average value of a function

$\frac{1}{\text{Area of } R} \iint_R f(x,y)\,dA$

Triple integral

$\iiint_D f(x,y,z)\,dV = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i, y_i, z_i)\Delta V_i$

= D solid region in xyz space
divide into sub solids
ΔV_i & (x_i, y_i, z_i) in D_i

Rectangular region (only, in this mod)

$D: a \leq x \leq b$, ~~eg~~ $c \leq y \leq d$, $r \leq z \leq s$

$\iiint_D f(x,y,z)\,dV = \int_a^b \int_c^d \int_r^s f(x,y,z)\,dz\,dy\,dx$

Useful identities

$\sin(2x) = 2\sin x\cos x$ $\qquad \cos^2(\theta) + \sin^2(\theta) = 1$

$\cos(2x) = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$[\sin^{-1}(x)]' = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

$[\tan^{-1}(x)]' = \frac{1}{1+x^2} \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

$[\sec^{-1}(x)]' = \frac{1}{x\sqrt{x^2-1}} \qquad \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$

integration by parts:
$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$
Note: $f(x) \longrightarrow f'(x)$
$g'(x) \longrightarrow g(x)$

differenciation:
$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$
$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

special remarks

If Q given in polar form
1. plot graph by considering $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$
2. if area: $\iint_R r\,dr\,d\theta$

Finding centre of mass
1. $\iint_R f(x,y)\,dA \Rightarrow$ total mass
2. $\iint_R y\,f(x,y)\,dA \Rightarrow$ mass in y-axis
3. $\iint_R x\,f(x,y)\,dA \Rightarrow$ mass in x-axis
4. center $= \left(\frac{③}{①}, \frac{②}{①}\right)$

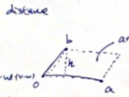
Finding $b = x + y$, where x is $\parallel a$ and $y \perp a$
use $b = \text{proj}_a b + (b - \text{proj}_a b)$

note: $n = b - \text{proj}_a b$

Area of parallagram $= \|\vec{a} \times \vec{b}\|$
first prove two sides are $\parallel$

to get the shortest distance
$\|\vec{n}\| = \frac{\|\vec{a} \times \vec{b}\|}{\|\vec{a}\|}$

Consider $\|v-w\|^2 = (v-w)\cdot(v-w)$
or $\|v+w\|^2 = (v+w)\cdot(v+w)$



$h = \frac{\text{area}}{\|a\|}$