EC2104 Tutorial 2 solution

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This week's tutorial is more mechanical (applied) than conceptual. I'll only highlight the more important concepts to understand. For full solution please refer to professor's uploaded solution.

Section 1

Question 1

Question 1

Key: understand the following relationship graphically

- $\frac{d}{dx} = 0 \Leftarrow$ stationary point
- $\frac{d}{dx}$ does not exist $\Leftrightarrow$ function not differentiable at the point
  - If $\frac{d}{dx}$ does not exist $\Rightarrow \frac{d^2}{dx^2}$ does not exist
- $\frac{d}{dx} < 0 \Rightarrow$ function is decreasing
- critical point:
  1. $\frac{d}{dx} = 0$
  2. Not differentiable
- $\frac{d^2}{dx^2} > 0 \Leftarrow$ convex function

Section 2

Question 2

Question 2

1. A continuous function is not always differentiable
   - Show using definition of differentiable

$$f'(0) = \lim_{\Delta x \to 0} \frac{f(\Delta x) + f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(\Delta x)}{\Delta x}$$

   - $\lim_{\Delta x \to 0^-} \frac{f(\Delta x)}{\Delta x} = \frac{\Delta x}{\Delta x} = 1; \lim_{\Delta x \to 0^+} \frac{f(\Delta x)}{\Delta x} = \frac{2\Delta x}{\Delta x} = 2$
   - $\lim_{\Delta x \to 0^-} \neq \lim_{\Delta x \to 0^+} \Rightarrow f'(0)$ does not exist
2. Non-continuous are not differentiable
3. Checking global minimum
   1. Check critical points
      - $f'(x) = 0$
      - non-differentiable points including boundary points
   2. Compare functional value to determine global extrema

Section 3

Question 3

Question 3

Key:

- Understand how to perform implicit differentiation
- Understand how to perform chain rule together with implicit differentiation (part c is slightly more troublesome to solve)

Section 4

Question 4

Question 4

Key:

- Understand how to do nested differentiation
- Understand how to perform first order and second order differentiation
- Understand how to check if a function is increasing/ decreasing
  - If $f'(a) > 0$, then function is increasing at point a
- Understand how to check if a function is concave or convex
  - A convex function has $f''(a) \geq 0$ at a
  - Remember to check if there is any inflection points

Section 5

Question 5

Question 5

Key:

- Able to perform differentiation to investigate the extrema by invoking first order necessary condition
- Understand that for convex function stationary points are minimum while for concave function stationary points are maximum
- Understand how to apply properties of elasticity

Question 5d

Key: using properties of elasticity

Given: $Q = \frac{10}{p+4p^2}$, WTS: ϵ_p

1. Using $\epsilon_x\left(\frac{f(x)}{g(x)}\right) = \epsilon_x(f(x)) - \epsilon_x(g(x))$
   - $f(p) := 10, g(p) := p + 4p^2$
   - $\Rightarrow \epsilon_p(\frac{10}{p+4p^2}) = \epsilon_p(10) - \epsilon_p(p + 4p^2)$
   - Note: $\epsilon_p(10) = 0$
2. Using $\epsilon_x(f \pm g) = \frac{f(x)\epsilon_x(f(x)) \pm g(x)\epsilon_x(g(x))}{f(x) \pm g(x)}$
   - $f(p) := p, g(p) := 4p^2$
   - $\Rightarrow \epsilon_p(p + 4p^2) = \frac{p\epsilon_p(p) + 4p^2\epsilon_p(4p^2)}{p+4p^2}$
3. Using $\epsilon_x(f(x)) = f'(x)\frac{x}{f(x)}$
   - $\epsilon_p(p) = \frac{p}{p} = 1$
   - $\epsilon_p(4p^2) = 8p\frac{p}{4p^2} = 2$

Section 6

Question 6

Question 6

Key:

- Understand how to find global extrema
- Techniques:
  1. Check interior points that are stationary points
  2. Compare functional value of non-differentiable points
     - Boundary points
     - non-differentiable points

Section 7

Question 7

Question 7

Key:

- Understand how to use Taylor expansion to approximate function
  - Linear approximation at point a:

$$f(x) \approx f(a) + f'(a)(x-a)$$

- Understand the implication of Taylor approximation in the context of convex and concave function
  - We know that linear approximation is just a line
    - in fact, think of linear approximation like moving along the tangent line
  - Now, we know the shape of convex and concave function
  - From there, it is easy to understand that linear approximation is likely to be smaller than convex function and larger than concave function
  - Note: this is different from the definition of convex and concave function

Section 8

Question 8

Question 8

Key:

- Understand the definition of and technique used to find inflection points
- Inflection points satisfy:
  1. $f''(x) = 0$
  2. $f''(x - \epsilon) \neq f''(x + \epsilon)$