# EC2104 Tutorial 6 solution

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Section 1

# Question 4

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Key matrix transpose properties

- $(\mathbf{A} + \mathbf{B})' = \mathbf{A}' + \mathbf{B}'$
- $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$
- Symmetric matrix $\Leftrightarrow \mathbf{A} = \mathbf{A}'$

Section 2

## Question 9b

# Question 9b

Given $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Given $\mathbf{A}^2 = (a+d)\mathbf{A} - (ad - bc)\mathbf{I}$

WTF: $\mathbf{A}$ such that $\mathbf{A}^2 = 0, \mathbf{A} \neq 0$

- Require: $a + d = 0$ and $ad - bc = 0$
  - $\Rightarrow a = -d, ad = bc = -d^2$
  - $d = \sqrt{-bc}$, $\{b, c | (b < 0, c > 0) \cup (b > 0, c < 0)\}$
- Therefore, we require any $\mathbf{A}$ such that

$$\mathbf{A} = \begin{pmatrix} -\sqrt{-st} & s \\ t & \sqrt{-st} \end{pmatrix}$$

$\{s, t | st < 0\} - \{s, t | s = 0, t = 0\}$

Section 3

# Question 9c

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Given $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Given $\mathbf{A}^2 = (a+d)\mathbf{A} - (ad-bc)\mathbf{I}$

WTS: $\mathbf{A}^3 = 0 \Rightarrow \mathbf{A}^2 = 0$

- $\mathbf{A}^2 = (a+d)\mathbf{A} - (ad-bc)\mathbf{I}$
  - $\Rightarrow \mathbf{A}^3 = (a+d)\mathbf{A}^2 - (ad-bc)\mathbf{A} = 0$
  - (1) $(a+d)\mathbf{A}^2 = (ad-bc)\mathbf{A}$
  - we also have $(a+d)\mathbf{A}^3 = (ad-bc)\mathbf{A}^2 = 0$
  - (2) $\Rightarrow (ad-bc)\mathbf{A}^2 = 0$

Consider cases:

1. $\mathbf{A}^2 = 0$
2. $(ad-bc) = 0 \Rightarrow (a+d)\mathbf{A}^2 = 0$
   1. $\mathbf{A}^2 = 0$
   2. $a+d = 0 \Rightarrow \mathbf{A}^2 = (a+d)\mathbf{A} - (ad-bc)\mathbf{I} = 0$

Therefore, in all possible cases $\mathbf{A}^2 = 0$