

# The Goods Market (by Ling)

## Overview

### The Goods Market I: Consumption & Saving

Consumption & Saving	<ul style="list-style-type: none"> <li>Determinants of optimal consumption of a utility-maximizing consumer</li> </ul>
Life-Cycle Model	<ul style="list-style-type: none"> <li>Pattern of income/consumption/saving over an individual's lifetime</li> <li>[early working life] → [highest income] → [dissaving] low saving                      maximum saving                      retirement</li> </ul>
Intertemporal Choice	<ul style="list-style-type: none"> <li>Choices involving different periods of time</li> <li>Assume forward-looking consumer who maximize lifetime satisfaction</li> <li>Consumer's choices are subject to intertemporal budget constraint</li> </ul>
The Consumer's Consumption-Saving Decision: A 2-Period Utility Maximization Problem	<ul style="list-style-type: none"> <li>Intuition/ Graph</li> <li>Consumer utility maximization problem/ Budget constraint</li> <li>First order derivative/ Lagrange multiplier method</li> </ul>
Government & Fiscal Policy	<ul style="list-style-type: none"> <li>Government decisions taken as exogenous</li> <li>Government budget constraint</li> <li>Fiscal policy</li> </ul>
Ricardian Equivalence: Theory & Evidence	<ul style="list-style-type: none"> <li>Rationale consumer will save the increase in income from decrease tax cut, leaves government lump-sum tax cut with no-effect</li> <li>However, in reality majority of consumers are short-sighted</li> </ul>

### The Goods Market II: Investment & Goods Market Equilibrium

Investment & Desired Capital Stock	<ul style="list-style-type: none"> <li>Investment becomes capital with a lag</li> <li>Desired capital stock is determined by <math>MPK^f</math> and user cost</li> </ul>
The Firm's Investment Decision: A 2-Period Profit Maximization Problem	<ul style="list-style-type: none"> <li>Profit-maximising firms decides on labour and capital income to maximise profit</li> </ul>
Goods Market Equilibrium	<ul style="list-style-type: none"> <li>Goods market equilibrium condition: <math>Y = C^d + I^d + G</math> or <math>S^d = I^d</math></li> <li>Goods market equilibrium might not hold</li> <li>Goods market equilibrium implies loanable funds market in equilibrium, vice versa</li> </ul>
Loanable Funds Market & the Saving-Investment Diagram	<ul style="list-style-type: none"> <li>Loanable Funds Market reach equilibrium when savings = investment</li> </ul>
Alternative: Keynesian Cross & the Income-Expenditure Diagram	<ul style="list-style-type: none"> <li></li> </ul>

## Definitions

## Formula

Consumption & Saving		
Consumption	Goods and services purchased by consumers	Determined by optimal/desired consumption of a utility-maximizing consumer
Marginal propensity to consume	proportion of an aggregate raise in pay that a consumer spends on the consumption of goods and services, as opposed to saving it	$MPC = \frac{\partial c(y)}{\partial y}$ $0 < MPC < 1$
Consumer preferences	Consumer preferences are given by the consumer's utility function $U(c_1, c_2)$	Indifference curve shows all combinations of $c_1$ and $c_2$ of the same utility (a person's satisfaction) $U(c_1, c_2) = u(c_1) + \delta u(c_2)$
Saving	Current income not consumed	$S = Y - C$
National saving	Private saving + public saving	$S^{\text{private}} = Y - T - C$ $S^{\text{public}} = T - G$ $S^{\text{national}} = Y - C - G$
Consumption-smoothing	The desire to have a relatively even pattern of consumption over time	$\frac{\partial c_1}{\partial y_1} = MPC \Delta y_1; \frac{\partial c_2}{\partial y_1} = (1 - MPC) \Delta y_1 \times (1 + r)$

Rate of time preference	Also known as subjective discount rate, measures the discount of future utility, can be seen as a measure of consumer patient	$\rho > 0$									
		higher $\rho$ indicates higher discounting factor and lower patience, higher $\rho$ meaning prefer future consumption.									
		<table><tr><td>increasing consumption profile</td><td>perfect smoothing; constant profile</td><td>decreasing consumption profile</td></tr><tr><td><math>r &gt; \rho</math></td><td><math>r = \rho</math></td><td><math>r &lt; \rho</math></td></tr><tr><td><math>c_1 &lt; c_2</math></td><td><math>c_1 = c_2</math></td><td><math>c_1 &gt; c_2</math></td></tr></table>	increasing consumption profile	perfect smoothing; constant profile	decreasing consumption profile	$r > \rho$	$r = \rho$	$r < \rho$	$c_1 < c_2$	$c_1 = c_2$	$c_1 > c_2$
		increasing consumption profile	perfect smoothing; constant profile	decreasing consumption profile							
$r > \rho$	$r = \rho$	$r < \rho$									
$c_1 < c_2$	$c_1 = c_2$	$c_1 > c_2$									
Discounting factor	Measures the worth of utility today	$\delta = \frac{1}{1 + \rho}, 0 < \delta < 1$ Inverse relationship with $\rho$									
Marginal rate of substitution (MRS)	The amount of $c_2$ the consumer would be willing to substitute for one unit of $c_1$	On an indifference curve, utility is constant $U(c_1, c_2) = \bar{U}$ $\frac{\partial U}{\partial c_1} \partial c_1 + \frac{\partial U}{\partial c_2} \partial c_2 = d\bar{U} = 0$ $MRS = \frac{MU_{c_1}}{MU_{c_2}}$									
Interest rate	Opportunity cost of current consumption	$c_t^{\text{current}} = \frac{\text{opp cost}}{(1 + r)} \times c_{t+1}^{\text{future}}$									
The Government											
Government spending	Goods and services purchase by governments	Taken as exogenous									
Government bond	Bonds issued by government to borrow money	Budget Constraint: $\text{expenditure } G_1 = \text{revenue } T_1 + \text{bond } B_1$									
Lump-Sum Tax	A tax for which the individual's liability does not depend upon behaviour	non-distortionary tax that does not change behaviours									
Fiscal Policy	Consist of Government purchases and Taxes to stimulate economy/ control inflation	$\uparrow \text{current } G \text{ (resp. future) financed by } \text{current } T \text{ (resp. future) result in } \downarrow \text{current } Y \text{ (resp. future)}$ $\Rightarrow MPC \times \downarrow C \text{ in additional, consumption smoothing effect } \downarrow S$									
Fiscal Policy: Lump-sum Tax	A lowered tax rate today financed by increase future tax rate Assume $G_1$ and $G_2$ unchanged	Opposing effects: $\downarrow \frac{\text{current}}{T} \Rightarrow \uparrow \frac{\text{current}}{Y} \Rightarrow \uparrow \frac{\text{current}}{C_1}$ $\uparrow \frac{\text{future}}{T} \Rightarrow \downarrow [\text{expected}] \frac{\text{future}}{Y} \Rightarrow \downarrow \frac{\text{current}}{C_1}$									
Ricardian Equivalence	A cut in lump-sum tax today has no effect on consumers' consumption choices or on the equilibrium real interest rate	$G_1 + \frac{G_2}{1 + r} = T_1 - \Delta T + \frac{Y_2 + (1 + r)\Delta T}{1 + r} = T_1 + \frac{Y_2}{1 + r}$ Private households hold the entire tax cut + interest in anticipation of future tax rate increase									
Crowding Out Effect	The reduction in investment that results when expansionary fiscal policy raises the interest rate	Since $S^d = Y - C^d - G$ , $S$ decrease when $G$ increase, result in increased interest rate through loanable funds market and decrease $I$									
Investment											
Investment	Purchase of new capital goods by firms and new housings by consumer, and increases in firm's inventory holdings	$\text{investment } I = \text{desired } K^* - \text{current } K + \text{depreciation rate } d \times \text{current } K$ Inventory Investment = Production - Sales									
Desired capital stock	The amount of capital that allows firms to maximise expected profit	Investment becomes capital with a lag									
Profit-maximising	The firm maximises the present value of stream of profits by choosing how many workers to employ and how much to invest	$\text{lifetime } \Pi = \pi_1 + \frac{\pi_2}{1 + r}$									
Profit	Profit = Revenue - Cost	$\pi_t = Y_t - w_t \times N_t - I_t$									
Final profit	Firm's profit at shut down	$\pi_{t+1} = Y_{t+1} - w_{t+1} \times N_{t+1} - 0_I + (1 - d)K$									
Output	The total production by firm	$Y_t = A_t \times F(K_t, N_t)$									
Equation of motion for capital	Investment becomes capital with a one-period lag	$K_{t+1} = \frac{\text{net value, after depreciation}}{(1 - d)} \times \text{present } K_t + \text{present } I_t$									
User cost of capital	The expected real cost of using a unit of capital for a specified period of time	$uc = \frac{(r + d)P_k}{(1 + r)}$									

Desired Capital Stock	The optimal level of capital to achieve profit maximising output	$\frac{\partial \pi}{\partial k_2} = MPF^f \frac{(r+d)P_k}{(1+r)}$ , taking $k_1$ as given, only investigate $k_2$
Calculation		
Present Value	The amount today that is equivalent to an amount to be received in the future, taking into account the interest that could be earned over the interval of time	$PV = \frac{x}{(1+r)^T} = \frac{\text{amount}}{(\text{discounting factor})^{\text{time}}}$
Future Value	Value in terms of dollars or goods in the future The factor $(1+r)$ is the price of $c_1$ measured in terms of $c_2$	$FV = x(1+r)^T = \text{amount} \times \text{compounding factor}^{\text{time}}$
Intertemporal Budget Constraint (simple)	Total resources available for present and future consumption Incentives for future planning	Period 1: $y_1 = c_1 + s_1$ Period 2: $c_2 = y_2 + (1+r) \times s_1$ present value of lifetime consumption = present value of lifetime resources PVLC PVLR $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$
Intertemporal Budget Constraint	Advanced version of the previous budget constraint, taking into consideration tax and dividends	$c_1 + \frac{c_2}{1+r} = y_1 + \pi_1 - t_1 + \frac{y_2 + \pi_2 - t_2}{1+r}$
Intertemporal Budget Constraint with Income-Leisure Decision	Combining the utility maximising objective of consumer and leisure requirement of consumer	$c_1 + \frac{c_2}{1+r} = (h - l_1)w_1 + \pi_1 - t_1 + \frac{(h - l_2)w_2 + \pi_2 - t_2}{1+r}$
Lagrangian multiplier	A strategy to find local maxima and minima of a function subject to a constraint	Single constraint $\Lambda(x, y, \lambda) = f(x, y) - \lambda g(x, y)$ where $f(x, y)$ is the function and $g(x, y)$ is the constraint. Solve for $\frac{\partial \Lambda}{\partial x}; \frac{\partial \Lambda}{\partial y}; \frac{\partial \Lambda}{\partial \lambda} = 0$
Euler Equation	An differential equation representing an intertemporal first order condition for a dynamic choice problem	$\frac{\partial U}{\partial c_1} = \frac{\delta}{1+r} \times \frac{\partial U}{\partial c_2}$ where $\delta$ is the discount factor and $r$ is interest rate
Hicks Substitution Effect	Investigate substitution effect by keeping the utility constant and change relative price	At the intermediate basket, $\bar{U}$ and $(1 + r_2)$

## Graphs & max/min problems

### Consumption & Saving

Solve for

- Determinants of optimal consumption of a utility-maximizing consumer

- Intertemporal Budget Constraint

Parameters

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

- Consumer preference

$$U(c_1, c_2) = u(c_1) + \delta u(c_2)$$

Intuition

- A rational consumer wants to maximise one's utility while subject to intertemporal budget constraint

Math

$$\max_{c_1, c_2} U(c_1, c_2)$$

$$s. t. PVLC = PVLR$$

Solve through first order derivatives or Lagrangian multiplier

$$U(c_1, c_2) = u(c_1) + \delta u(c_2)$$

$$c_2 = (1+r)(y_1 - c_1) + y_2$$

$$dU = \frac{\partial U}{\partial c_1} + \frac{\partial U}{\partial c_2} \frac{dc_2}{dc_1} = 0$$

	slope	y-intercept	x-intercept
Budget Line	$-(1+r)$	$(1+r)y_1 + y_2$	$y_1 + \frac{y_2}{1+r}$
Indifference Curve	$MRS = \frac{MU_{c_1}}{MU_{c_2}}$	-	-

Income

$y_1$

Income

$y_2$

Wealth

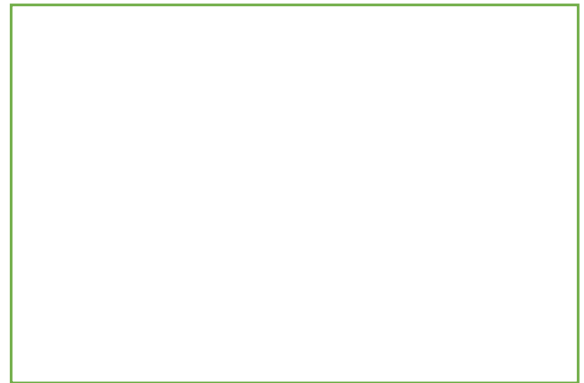
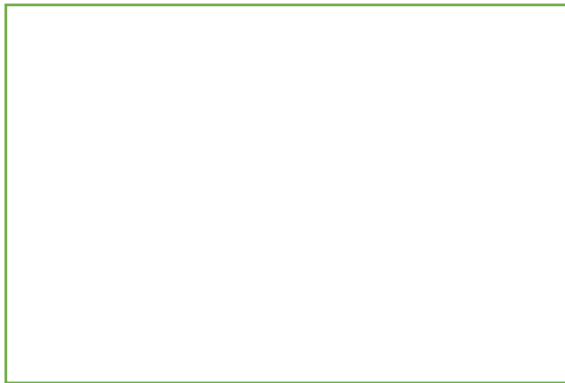
$\omega_0$

Increase

Decrease

BL, $c_1, c_2 \uparrow$	BL, $c_1, c_2 \downarrow$
Saving $\uparrow$	Saving $\downarrow$
BL, $c_1, c_2 \uparrow$	BL, $c_1, c_2 \downarrow$
Saving $\downarrow$	Saving $\uparrow$
BL, $c_1, c_2 \uparrow$	BL, $c_1, c_2 \downarrow$
Saving $\downarrow$	Saving $\uparrow$

$c_1$  vs  $c_2$   
Graphs



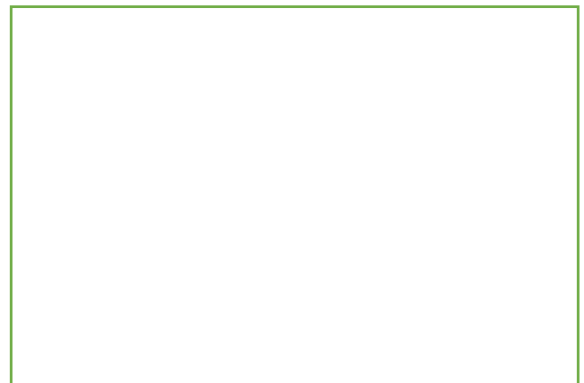
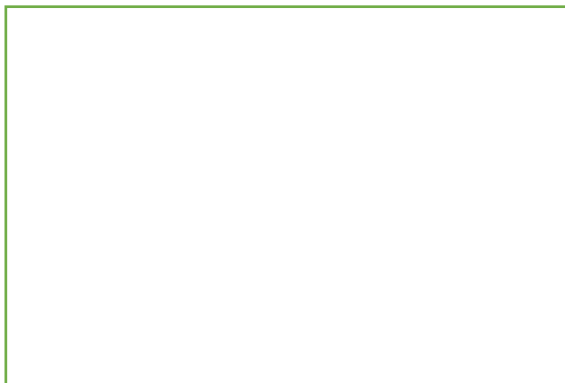
$\uparrow r$  for a saver

	$c_1$	$c_2$
income effect	$\uparrow$	$\uparrow$
substitution effect	$\downarrow$	$\uparrow$
overall	ambiguous	$\uparrow$

$\uparrow r$  for a borrower

	$c_1$	$c_2$
income effect	$\uparrow$	$\uparrow$
substitution effect	$\uparrow$	$\downarrow$
overall	$\uparrow$	ambiguous

$\frac{\partial U}{\partial r}$



## Graphs & max/min problems

### Investment & Desired Capital

Solve for • Determinants of desired input (including capital) of a profit-maximizing firm

Parameters • Profit maximising condition

$$\max_{N_1, N_2, K_2} \Pi_1 = \pi_1 + \frac{\pi_2}{1+r}$$

Intuition • A rational firm chooses the optimal level of input to maximise profit, considering the cost involved

Math considering  $\frac{\partial \Pi}{\partial N_1} = 0; \frac{\partial \Pi}{\partial N_2} = 0; \frac{\partial \Pi}{\partial K_2} = 0$   
Solve through first order derivatives

common identity:

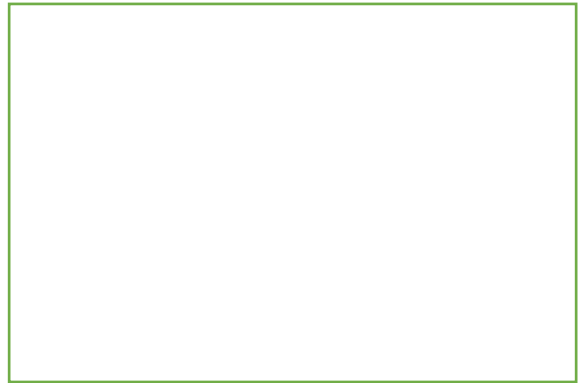
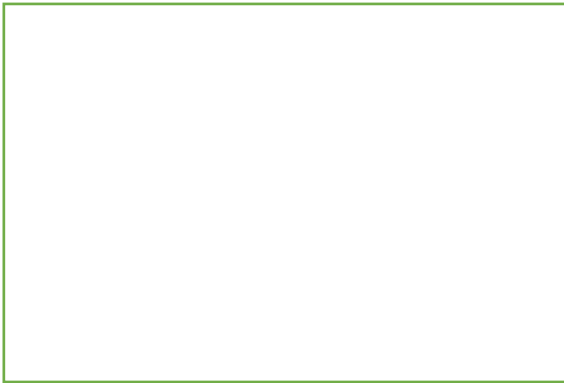
$$\frac{\partial \Pi}{\partial N_1} = MPN_1 - w_1; \frac{\partial \Pi}{\partial N_2} = MPN_2 - w_2;$$

$$\frac{\partial \Pi}{\partial K_2} = MPK^f - \frac{(r+d)P_k}{(1+r)}$$

	slope	y-intercept	x-intercept
User Cost	$\frac{(r+d)P_k}{(1+r)}$	$\frac{(r+d)P_k}{(1+r)}$	—
MPK <sup>f</sup>	$MPK^{f'}$	$\frac{\partial Y}{\partial K_2}, K_2 = 0$	$\frac{\partial Y}{\partial K_2}, K_2 = k$

	Increase	Decrease
Interest rate r	user cost ↑ K* ↓	user cost ↓ K* ↑
Depreciation rate d	user cost ↑ K* ↓	user cost ↓ K* ↑
Price of capital P <sub>k</sub>	user cost ↑ K* ↓	user cost ↓ K* ↑
Technology affects MPK <sup>f</sup>	MPK <sup>f</sup> ↑ K* ↑	MPK <sup>f</sup> ↓ K* ↓

MPK<sup>f</sup> vs K  
Graphs



### Graphs & max/min problems

#### Loanable Fund Framework: Saving-Investment Diagram

Solve for	<ul style="list-style-type: none"> <li>Determinants of optimal saving/investment quantity through loanable fund framework</li> </ul>
Parameters	<ul style="list-style-type: none"> <li>market equilibrium condition</li> </ul> $S^d = I^d$
Intuition	<ul style="list-style-type: none"> <li>In a market equilibrium, the output equals to total desired demand. Which implies a equilibrium condition in loanable funds market</li> <li>Note as assume government expenditure as exogenous</li> </ul>
Math	<p>Solve by investigating the equilibrium quantity of saving/investment and equilibrium price (interest rate)</p> $S^d = Y - C^d - G$ $I^d = K^* - K - dK$

	Increase	Decrease
Output	$S^d \uparrow$	$S^d \downarrow$
Y	$r \downarrow$	$r \uparrow$
Wealth	$S^d \downarrow$	$S^d \uparrow$
$\omega$	$r \uparrow$	$r \downarrow$
Effective tax rate	$I^d \downarrow$	$I^d \uparrow$
$\tau$	$r \downarrow$	$r \uparrow$
Technology affects MPK <sup>f</sup>	$I^d \uparrow$	$I^d \downarrow$
	$r \uparrow$	$r \downarrow$

Saving vs  
Investment  
Graphs

