The Goods Market (by Ling)

Overview	
The Goods Market I: Consumption & Saving	
Consumption & Saving	• Determinants of optimal consumption of a utility-maximizing consumer
Life-Cycle Model	• Pattern of income/consumption/saving over an individual's lifetime • $\underset{\text{low saving}}{[\text{early working life}]} \rightarrow \underset{\text{maximum saving}}{[\text{highest income}]} \rightarrow \underset{\text{retirement}}{[\text{dissaving}]}$
Intertemporal Choice	• Choices involving different periods of time • Assume forward-looking consumer who maximize lifetime satisfaction • Consumer's choices are subject to intertemporal budget constraint
The Consumer's Consumption-Saving Decision: A 2-Period Utility Maximization Problem	• Intuition/ Graph • Consumer utility maximization problem/ Budget constraint • First order derivative/ Lagraingan multiplier method
Government & Fiscal Policy	• Government decisions taken as exogenous • Government budget constraint • Fiscal policy
Ricardian Equivalence: Theory & Evidence	• Rationale consumer will save the increase in income from decrease tax cut, leaves government lump-sum tax cut with no-effect • However, in reality majority of consumers are short-sighted
The Goods Market II: Investment & Goods Market Equilibrium	
Investment & Desired Capital Stock	• Investment becomes capital with a lag • Desired capital stock is determined by MPK^f and user cost
The Firm's Investment Decision: A 2-Period Profit Maximization Problem	• Profit-maximising firms decides on labour and capital income to maximise profit
Goods Market Equilibrium	• Goods market equilibrium condition: • $Y = C^d + I^d + G \text{ or } S^d = I^d$ • Goods market equilibrium might not hold • Goods market equilibrium implies loanable funds market in equilibrium, vice versa
Loanable Funds Market & the Saving-Investment Diagram	• Loanable Funds Market reach equilibrium when savings = investment
Alternative: Keynesian Cross & the Income-Expenditure Diagram	•

	Definitions	***Formula***
Consumption & Saving		
Consumption	Goods and services purchased by consumers	Determined by optimal/desired consumption of a utility-maximizing consumer
Marginal propensity to consume	proportion of an aggregate raise in pay that a consumer spends on the consumption of goods and services, as opposed to saving it	$MPC = \frac{\partial c(y)}{\partial y}$ $0 < MPC < 1$
Consumer preferences	Consumer preferences are given by the consumer's utility function $U(c_1, c_2)$	Indifference curve shows all combinations of c_1 and c_2 of the same utility (a person's satisfaction) $U(c_1, c_2) = u(c_1) + \delta u(c_2)$
Saving	Current income not consumed	$\underset{S}{\text{saving}} = \underset{Y}{\text{income}} - \underset{C}{\text{consumption}}$
National saving	Private saving + public saving	$\overset{\text{private}}{S} = \overset{\text{disposable income}}{Y - T} - \overset{\text{consumption}}{C}$ $\overset{\text{public}}{S} = \overset{\text{revenue}}{T} - \overset{\text{expenditure}}{G}$ $\overset{\text{national}}{S} = \overset{\text{national}}{Y} - \overset{\text{consumer}}{C} - \overset{\text{government}}{G}$
Consumption-smoothing	The desire to have a relatively even pattern of consumption over time	$\frac{\partial c_1}{\partial y_1} = MPC\Delta y_1$; $\frac{\partial c_2}{\partial y_1} = (1 - MPC)\Delta y_1 \times (1 + r)$

<table>
<tr><td>Rate of time preference</td><td>Also known as subjective discount rate, measures the discount of future utility, can be seen as a measure of consumer patient</td><td>$$\rho > 0$$ higher ρ indicates higher discounting factor and lower patience, higher ρ meaning prefer future consumption.
<table>
<tr><td>increasing consumption profile</td><td>perfect smoothing; constant profile</td><td>decreasing consumption profile</td></tr>
<tr><td>$r > \rho$</td><td>$r = \rho$</td><td>$r < \rho$</td></tr>
<tr><td>$c_1 < c_2$</td><td>$c_1 = c_2$</td><td>$c_1 > c_2$</td></tr>
</table></td></tr>
<tr><td>Discounting factor</td><td>Measures the worth of utility today</td><td>$$\delta = \frac{1}{1+\rho}, 0 < \delta < 1$$ Inverse relationship with ρ</td></tr>
<tr><td>Marginal rate of substitution (MRS)</td><td>The amount of c_2 the consumer would be willing to substitute for one unit of c_1</td><td>On an indifference curve, utility is constant $$U(c_1, c_2) = \bar{U}$$ $$\frac{\partial U}{\partial c_1}\partial c_1 + \frac{\partial U}{\partial c_2}\partial c_2 = d\bar{U} = 0$$ $$MRS = \frac{MU_{c_1}}{MU_{c_1}}$$</td></tr>
<tr><td>Interest rate</td><td>Opportunity cost of current consumption</td><td>$$\underset{c_t}{\text{current}} = \underset{(1+r)}{\text{opp cost}} \times \underset{c_{t+1}}{\text{future}}$$</td></tr>
<tr><td colspan="3">The Government</td></tr>
<tr><td>Government spending</td><td>Goods and services purchase by governments</td><td>Taken as exogenous</td></tr>
<tr><td>Government bond</td><td>Bonds issued by government to borrow money</td><td>Budget Constraint: $\underset{G_1}{\text{expenditure}} = \underset{T_1}{\text{revenue}} + \underset{B_1}{\text{bond}}$</td></tr>
<tr><td>Lump-Sum Tax</td><td>A tax for which the individual's liability does not depend upon behaviour</td><td>non-distortionary tax that does not change behaviours</td></tr>
<tr><td>Fiscal Policy</td><td>Consist of Government purchases and Taxes to stimulate economy/ control inflation</td><td>$\uparrow \underset{G}{\text{current(resp.future)}}$ financed by $\underset{T}{\text{current(resp.future)}}$ result in $\downarrow \underset{Y}{\text{current(resp.future)}}$
$\Rightarrow MPC \times\downarrow \underset{C}{\text{current(resp.future)}}$ in additional, consumption smoothing effect $\downarrow \underset{S}{\text{current}}$</td></tr>
<tr><td>Fiscal Policy: Lump-sum Tax</td><td>A lowered tax rate today financed by increase future tax rate
Assume G_1 and G_2 unchanged</td><td>Opposing effects:
$\downarrow \underset{T}{current} \Rightarrow\uparrow \underset{Y}{current} \Rightarrow\uparrow \underset{C_1}{current}$
$\uparrow \underset{T}{future} \Rightarrow\downarrow \underset{Y}{[expected]\ future} \Rightarrow\downarrow \underset{C_1}{current}$</td></tr>
<tr><td>Ricardian Equivalence</td><td>A cut in lump-sum tax today has no effect on consumers' consumption choices or on the equilibrium real interest rate</td><td>$$G_1 + \frac{G_2}{1+r} = T_1 - \Delta T + \frac{Y_2 + (1+r)\Delta T}{1+r} = T_1 + \frac{Y_2}{1+r}$$ Private households hold the entire tax cut + interest in anticipation of future tax rate increase</td></tr>
<tr><td>Crowding Out Effect</td><td>The reduction in investment that results when expansionary fiscal policy raises the interest rate</td><td>Since $S^d = Y - C^d - G$, S decrease when G increase, result in increased interest rate through loanable funds market and decrease I</td></tr>
<tr><td colspan="3">Investment</td></tr>
<tr><td>Investment</td><td>Purchase of new capital goods by firms and new housings by consumer, and increases in firm's inventory holdings</td><td>$\underset{I}{\text{investment}} = \underset{K^*}{\text{desired}} - \underset{K}{\text{current}} + \underset{d}{\text{depreciation rate}} \times \underset{K}{\text{current}}$
Inventory Investment = Production - Sales</td></tr>
<tr><td>Desired capital stock</td><td>The amount of capital that allows firms to maximise expected profit</td><td>Investment becomes capital with a lag</td></tr>
<tr><td>Profit-maximising</td><td>The firm maximises the present value of stream of profits by choosing how many workers to employ and how much to invest</td><td>$\underset{\Pi}{\text{lifetime}} = \pi_1 + \frac{\pi_2}{1+r}$</td></tr>
<tr><td>Profit</td><td>Profit = Revenue - Cost</td><td>$\underset{\pi_t}{\text{profit}} = \underset{Y_t}{\text{output}} - \underset{w_t}{\text{real wage}} \times \underset{N_t}{\text{amount of labour}} - \underset{I_t}{\text{investment}}$</td></tr>
<tr><td>Final profit</td><td>Firm's profit at shut down</td><td>$\underset{\pi_{t+1}}{\text{profit}} = \underset{Y_{t+1}}{\text{output}} - \underset{w_{t+1}}{\text{real wage}} \times \underset{N_{t+1}}{\text{amount of labour}} - \underset{0_I}{\text{investment}} + \underset{(1-d)K}{\text{capital}}$</td></tr>
<tr><td>Output</td><td>The total production by firm</td><td>$\underset{Y_t}{\text{real output}} = \underset{A_t}{\text{total factor productivity}} \times \underset{F(K_t, N_t)}{\text{production function}}$</td></tr>
<tr><td>Equation of motion for capital</td><td>Investment becomes capital with a one-period lag</td><td>$\underset{K_{t+1}}{\text{future}} = \underset{(1-d)}{\text{net value, after depreciation}} \times \underset{K_t}{\text{present}} + \underset{I_t}{present}$</td></tr>
<tr><td>User cost of capital</td><td>The expected real cost of using a unit of capital for a specified period of time</td><td>$$uc = \frac{(r+d)P_k}{(1+\tau)}$$</td></tr>
</table>

Desired Capital Stock	The optimal level of capital to achieve profit maximising output	$\frac{\partial \Pi}{\partial k_2} = MPF^f \frac{(r+d)P_k}{(1+\tau)}$, taking k_1 as given, only investigate k_2
	Calculation	
Present Value	The amount today that is equivalent to an amount to be received in the future, taking into account the interest that could be earned over the interval of time	PV = $\frac{x}{(1+r)^T} = \frac{\text{amount}}{(\text{discounting factor})^{\text{time}}}$
Future Value	Value in terms of dollars or goods in the future The factor (1+r) is the price of c_1 measured in terms of c_2	FV = x$(1+r)^T = \text{amount} \times \text{compounding factor}^{\text{time}}$
Intertemporal Budget Constraint (simple)	Total resources available for present and future consumption Incentives for future planning	Period 1: $\overset{\text{period 1}}{y_1} = \overset{\text{period 1}}{c_1} + \overset{\text{period 1}}{s_1}$ Period 2: $\overset{\text{period 2}}{c_2} = \overset{\text{period 2}}{y_2} + \overset{\text{period 1}}{(1+r) \times s_1}$ $\underset{\text{PVLC}}{\text{present value of lifetime consumption}} = \underset{\text{PVLR}}{\text{present value of lifetime resources}}$ $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$
Intertemporal Budget Constraint	Advanced version of the previous budget constraint, taking into consideration tax and dividends	$c_1 + \frac{c_2}{1+r} = y_1 + \pi_1 - t_1 + \frac{y_2 + \pi_2 - t_2}{1+r}$
Intertemporal Budget Constraint with Income-Leisure Decision	Combining the utility maximising objective of consumer and leisure requirement of consumer	$c_1 + \frac{c_2}{1+r} = (h - l_1)w_1 + \pi_1 - t_1 + \frac{(h - l_2)w_2 + \pi_2 - t_2}{1+r}$
Lagrangian multiplier	A strategy to find local maxima and minima of a function subject to a constraint	Single constraint $\Lambda(x, y, \lambda) = f(x, y) - \lambda g(x, y)$ where f(x,y) is the function and g(x,y) is the constraint. Solve for $\frac{\partial \Lambda}{\partial x}; \frac{\partial \Lambda}{\partial y}; \frac{\partial \Lambda}{\partial \lambda}$=0
Euler Equation	An differential equation representing an intertemporal first order condition for a dynamic choice problem	$\frac{\partial U}{\partial c_1} = \frac{\delta}{1+r} \times \frac{\partial U}{\partial c_2}$ where δ is the discount factor and r is interest rate
Hicks Substitution Effect	Investigate substitution effect by keeping the utility constant and change relative price	At the intermediate basket, $\overline{U}$ and $(1+r_2)$

Graphs & max/min problems

Consumption & Saving

Solve for

- Determinants of optimal consumption of a utility-maximizing consumer

Parameters

- Intertemporal Budget Constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

- Consumer preference

$$U(c_1, c_2) = u(c_1) \; + \delta u(c_2)$$

Intuition

- A rational consumer wants to maximise one's utility while subject to intertemporal budget constraint

Math

$$\max_{c_1, c_2} U(c_1, c_2)$$

$$s.t. PVLC \; = \; PVLR$$

Solve through first order derivatives or Lagrangian multiplier

$$U(c_1, c_2) = u(c_1) \; + \delta u(c_2)$$

$$c_2 = (1+r)(y_1 - c_1) + y_2$$

$$dU = \frac{\partial U}{\partial c_1} + \frac{\partial U}{\partial c_2}\frac{dc_2}{dc_1} = 0$$

	slope	y-intercept	x-intercept
Budget Line	-(1+r)	$(1+r)y_1 + y_2$	$y_1 + \frac{y_2}{1+r}$
Indifference Curve	$MRS = \frac{MU_{c_1}}{MU_{c_2}}$	-	-

	Increase	Decrease
Income y_1	BL, c_1, c_2 ↑ Saving ↑	BL, c_1, c_2 ↓ Saving ↓
Income y_2	BL, c_1, c_2 ↑ Saving ↓	BL, c_1, c_2 ↓ Saving ↑
Wealth ω_0	BL, c_1, c_2 ↑ Saving ↓	BL, c_1, c_2 ↓ Saving ↑

c_1 vs c_2 Graphs

↑ r for a saver

	c_1	c_2
income effect	↑	↑
substitution effect	↓	↑
overall	ambiguous	↑

↑ r for a borrower

	c_1	c_2
income effect	↑	↑
substitution effect	↑	↓
overall	↑	ambiguous

$$\frac{\partial U}{\partial r}$$

Graphs & max/min problems

Investment & Desired Capital

Solve for	• Determinants of desired input (including capital) of a profit-maximizing firm
Parameters	• Profit maximising condition $$\max_{N_1,N_2,K_2} \Pi_1 = \pi_1 + \frac{\pi_2}{1+r}$$
Intuition	• A rational firm chooses the optimal level of input to maximise profit, considering the cost involved
Math	considering $\frac{\partial \Pi}{\partial N_1} = 0; \frac{\partial \Pi}{\partial N_2} = 0; \frac{\partial \Pi}{\partial K_2} = 0$ Solve through first order derivatives common identity: $\frac{\partial \Pi}{\partial N_1} = MPN_1 - w_1; \frac{\partial \Pi}{\partial N_2} = MPN_2 - w_2;$ $\frac{\partial \Pi}{\partial K_2} = MPK^f - \frac{(r+d)P_k}{(1+\tau)}$

	slope	y-intercept	x-intercept
User Cost	$\frac{(r+d)P_k}{(1+\tau)}$	$\frac{(r+d)P_k}{(1+\tau)}$	–
MPK^f	$MPK^{f\prime}$	$\frac{\partial Y}{\partial K_2}, K_2 = 0$	$\frac{\partial Y}{\partial K_2}, K_2 = k$

	Increase	Decrease
Interest rate r	user cost ↑ K* ↓	user cost ↓ K* ↑
Depreciation rate d	user cost ↑ K* ↓	user cost ↓ K* ↑
Price of capital P_K	user cost ↑ K* ↓	user cost ↓ K* ↑
Technology affects MPK^f	MPK^f ↑ K* ↑	MPK^f ↓ K* ↓

MPK^f vs K

Graphs

Graphs & max/min problems

Loanable Fund Framework: Saving-Investment Diagram

Solve for

- Determinants of optimal saving/investment quantity through loanable fund framework

Parameters

- market equilibrium condition

$$S^d = I^d$$

Intuition

- In a market equilibrium, the output equals to total desired demand. Which implies a equilibrium condition in loanable funds market
- Note as assume government expenditure as exogenous

Math

Solve by investigating the equilibrium quantity of saving/investment and equilibrium price (interest rate)

$$S^d = Y - C^d - G$$
$$I^d = K^* - K - dK$$

	Increase	Decrease
Output Y	$S^d \uparrow$ r $\downarrow$	$S^d \downarrow$ r $\uparrow$
Wealth ω	$S^d \downarrow$ r $\uparrow$	$S^d \uparrow$ r $\downarrow$
Effective tax rate τ	$I^d \downarrow$ r $\downarrow$	$I^d \uparrow$ r $\uparrow$
Technology affects MPK^f	$I^d \uparrow$ r $\uparrow$	$I^d \downarrow$ r $\downarrow$

Saving vs Investment Graphs