EC2104 Lecture 4 - Integration

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Section 1

Integration

Integration

Indefinite integrals

If $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$. Given $A = F(0) + C \Rightarrow C = A - F(0)$

Definite integrals

When $F'(x) = f(x)$ for all $x \in (a, b)$, the definite integral of f over $[a, b]$ is $\int_a^b f(x)dx = F(b) - F(a)$

Integration as Area Under Curve

Let $A(t)$ be a function measures the area under the graph of f over the interval $[a, t] \Rightarrow A(t) = \int_a^b f(t)dt$

Subsection 1

Integration as Area Under Curve

Integration as Area Under Curve

Consider: Find area under the curve for $f(x) = cos(x)$ for $x \in [0, \pi]$

Note:

- $\int cos(x)dx = sin(x) + C$
- When finding area under a curve, one need to know which area are under the $x - axis$
- For area under the $x - axis$, take $-\int f(x)dx$ instead

Wrong method:

$$\int_0^{\pi} cos(x)dx = sin(\pi) - sin(0) = 0$$

Correct method:

$$\int_0^{\pi/2} cos(x)dx - \int_{\pi/2}^{\pi} cos(x)dx = sin(\pi/2) - sin(0) - sin(\pi) + sin(\pi/2) = 2$$

Section 2

Techniques in integration

Techniques in integration

1. Anti-differentiation
$$\int f(x)dx = F(x) + C$$
2. Integration by Substitution
$$\int f(g(x))\frac{dg(x)}{dx}dx = \int f(u)du = F(g(x)) + C$$
   - Differentials: $du = \frac{du}{dx}dx = g'(x)dx$, where $u = g(x)$
3. Integration by Parts
$$\int f(x)\frac{dg(x)}{dx} = f(x)g(x) - \int g(x)\frac{df(x)}{dx}dx$$
4. Leibniz Integral Rule (next slide)

Subsection 1

Leibniz Integral Rule

Leibniz Integral Rule

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(x,t)dx = \int_{a(t)}^{b(t)} \frac{\partial}{\partial t} f(x,t)dx + f(b(t),t)\frac{db(t)}{dt} - f(a(t),t)\frac{da(t)}{dt}$$

- If $a(t), b(t)$ are constants

$$\frac{d}{dt}\int_{a}^{b} f(x,t)dx = \int_{a}^{b} \frac{\partial}{\partial t} f(x,t)dx$$

- Variable limit form

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(x)dx = f(b(t))\frac{db(t)}{dt} - f(a(t))\frac{da(t)}{dt}$$

Subsection 2

Remarks: when to use Leibniz Integral Rule

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Now, there are only so few techniques we have learnt to solve analytical solutions in integration. If all else failed, try Leibniz integration.

Question ask to evaluate

$$\int_0^1 \frac{t^3 - 1}{\ln(t)} dt$$

1. Substitution?
   - Try sub $x = ln(t) \Rightarrow dt = tdx$. However, how to express x as a function of $t^3 - 1$?
   - Try sub $x = t^3 - 1 \Rightarrow dt = dx/3t$. However, how to express x as a function o $\ln(t)$?
2. By Parts? $uv' = uv - \int u'v$
   - Try let $u = 1/\ln(t), v' = t^3 - 1 \Rightarrow u' = -\frac{1}{t\ln(t)^2}, v = 3t^2$. However, how to solve $\int -\frac{3t^2}{t\ln(t)^2}$?
   - Try let $u = t^3 - 1, v' = 1/\ln(t) \Rightarrow u' = 3t^2$. However, what is $\int \frac{1}{\ln(t)}$?

Subsection 3

Remark: using Leibniz Integration Rule correctly

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Question ask to evaluate

$$\int_0^1 \frac{t^3 - 1}{\ln(t)} dt$$

1. Check if question is in the functional form we desire: $\frac{d}{dx} \int f(x, t) dt$
   - Note current expression is $\int f(t) dt$
2. If needed, fill in the missing ingredients
   - Want $f(x, t)$, have $f(t) \Rightarrow$ introduce x
   - Want $\frac{d}{dx}$, have only $\int f(x, t) dt \Rightarrow$ solve for the differentiated result instead
3. Choose the right ingredients to fill
   - What if we use $g(x) := \int_0^1 \frac{t^3 - x}{\ln(t)} dt \Rightarrow g'(x) = \int_0^1 \frac{\partial}{\partial x} \frac{t^3 - x}{\ln(t)} dt = \int_0^1 -\frac{1}{\ln(t)} dt = 0$
   - Then $g(x) = C$, $C = ?$

Subsection 4

Integrating infinite bounds

Integrating infinite bounds

To solve

$$\int_0^\infty f(x)dx$$

1. Express ∞ as s
   - i.e. $\int_0^s f(x)dx$
2. Solve the integration in terms of the symbol s
   - i.e. $\int_0^s f(x)dx = F(s) - F(0)$
3. Take limit to infinity
   - i.e. $\lim_{s\to\infty} F(s) - F(0)$

Note:

- $\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^0 f(x)dx + \int_0^\infty f(x)dx$