EC2104 Tutorial 3 solution

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Section 1

Question 3(a)

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Given: $\dot{S} = -aS(t)$, WTS: $S(t) = S_0 e^{at}$

1. We know $\dot{S} = \frac{dS}{dt}$, rearranging the given term we have $\frac{\left(\frac{dS}{dt}\right)}{S} = -a$
2. Now, consider $f(S) = \ln(S)$, we have $\frac{df}{dt} = \frac{1}{S} \cdot \frac{dS}{dt}$ by chain rule
3. Observe results from (1) and (2). We note that

$$\frac{d\ln(S)}{dt} = \frac{\left(\frac{dS}{dt}\right)}{S} = \frac{\dot{S}}{S} = -a$$

4. Now, solve for the differential (integrate both side wrt t)

$$\Rightarrow \ln(S) = -at + C$$
$$\Rightarrow S(t) = e^{-at+C} = e^C \cdot e^{-at}$$

5. When $t = 0, S(0) = S_0 \Rightarrow e^C \cdot e^0 = S_0 \Rightarrow e^C = S_0$
6. Therefore

$$\Rightarrow S(t) = S_0 e^{-at} \text{ (shown)}$$

Section 2

Question 4(a)

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Amount $A(0)$ is invested with $p\%$ yearly interest, ask $A(t) :=$ value of investment after t years

1. We know the rate of change over time (from the question)

$$\frac{dA(t)}{dt} = p \cdot A(t)$$

2. Expressing the equation as differential equation and integrate both side

$$\frac{1}{A(t)} dA(t) = pdt \Rightarrow \int \frac{1}{A(t)} dA(t) = \int pdt$$

3. Solution:
   - $\Rightarrow \ln(A(t)) = pt + C$
   - $A(t) = e^{pt+C} = e^C \cdot e^{pt}$
   - Since $A(0) = e^C \Rightarrow A(t) = A(0)e^{pt}$