EC2104 Lecture 1 - Functions & Limits

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Section 1

Functions

Functions

- Function: a procedure mapping domain onto range.
- Key Characteristics: one-to-one mapping between domain and range.
- Test invalid functions:
  1. Failed the vertical-line test
  2. Numeric example where function failed one-to-one mapping definition
- Composite Functions: nested function, or applying functions sequentially
  - Note: $g \circ f(x) = g(f(x)) \neq f(g(x)) = f \circ g(x)$ for most functions
- Inverse Functions: the reverse of the function, mapping from range to domain
  - Note: it is important to identify the domain and range correctly
  - Note: inverse function has to satisfy one-to-one mapping relationship as well

Subsection 1

Polynomial

Polynomial

Polynomial function

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

- non-zero coefficients $a_n, a_{n-1}, \cdots, a_0$
- x is the unknown, $a_n, a_{n-1}, \cdots, a_0$ are known
- Domain: $\mathbb{R}$, Range: $\mathbb{R}$
- Types of polynomial functions
  - Linear Functions: $P(x) = a_1 x + a_0$
  - Quadratic Functions: $P(x) = a_2 x^2 + a_1 x + a_0$
- Example of polynomial functions
  - Linear: solving equilibrium conditions (e.g. demand and supply)
  - Quadratic: Solving profit maximisation conditions (e.g. monopoly optimal output level to maximise profit)

Subsection 2

Power and Exponential functions

Power and Exponential functions

Power Functions

$f(x) = Ax^r$

- x is the unknown, r, A are known.
- Domain: $x > 0$ (in this module)
- Range: $f(x) > 0, A > 0$ or $f(x) < 0, A < 0$

Exponent Functions

$f(t) = Aa^t$

- t is the unknown, A, a are known.
- Domain: $\mathbb{R}$
- Range: $f(t) > 0$
- Note: $f(t+1) = f(t) \cdot a = Aa^{t+1}$

Subsection 3

Logarithmic function

Logarithmic function

Logarithmic Functions

$x := \log_a a^x = \log_a b$

- inverse of exponential function ($a^x = b$)
- commonly natural log is used ($\log_e x$ or $\ln x$)
- Note: $a^{log_a x} = x$
- Logarithmic is commonly used to simplify differentiation
  - Question: $\frac{d}{dx} exp(x-1)^2 = 0$
  - Solving Log transformed question: $\frac{d}{dx} log(exp(x-1)^2) = \frac{d}{dx}(x-1)^2 = 0 \Rightarrow x^* = 1$
  - Log transformed and original question differs in objective function value ($e^{(1-1)^2} = 1 \neq (1-1)^2 = 0$) but the optimal solution is the same ($x^* = 1$)

Subsection 4

Key economics Examples

Key economics Examples

- Solving equilibrium price and quantity
  - Linear demand: $Q_D = a - bP$
  - Linear supply: $Q_S = \alpha + \beta P$
- Solving optimal output Q^M and profit π^M
  - Cost function: $C = \alpha Q + \beta Q^2, Q \geq 0$
  - Demand function: $P = a - bQ$
  - Revenue function: $R = PQ$
  - Profit function: $\pi(Q) = R - C$

Section 2

Limits

Limits

Limits

$\lim_{x\to a} f(x) = A$

- Read: as x tends towards a, the limit of $f(x)$ tends towards A
- Commonly used when
  1. Functions does not exist at the point
  2. When value tends towards infinity
- For a limit to exist, the left limit and right limit must tends towards the same value
  - $\lim_{x\to a} f(x) = A \Leftrightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = A$