Statistical Arbitrage Mining for Display Advertising

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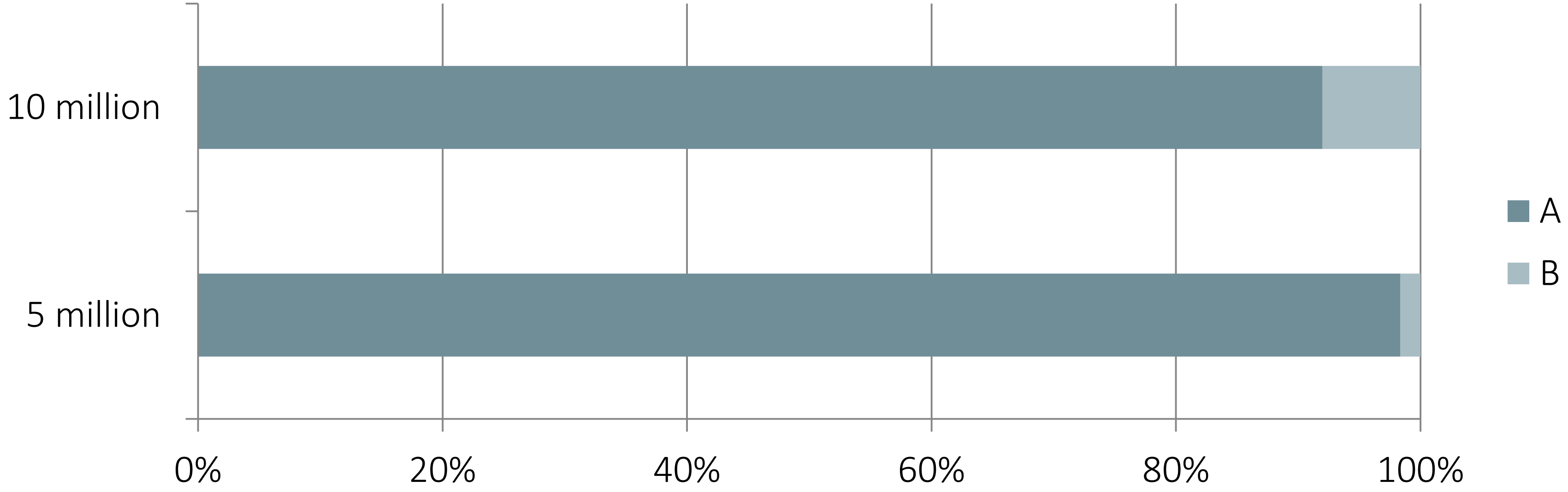
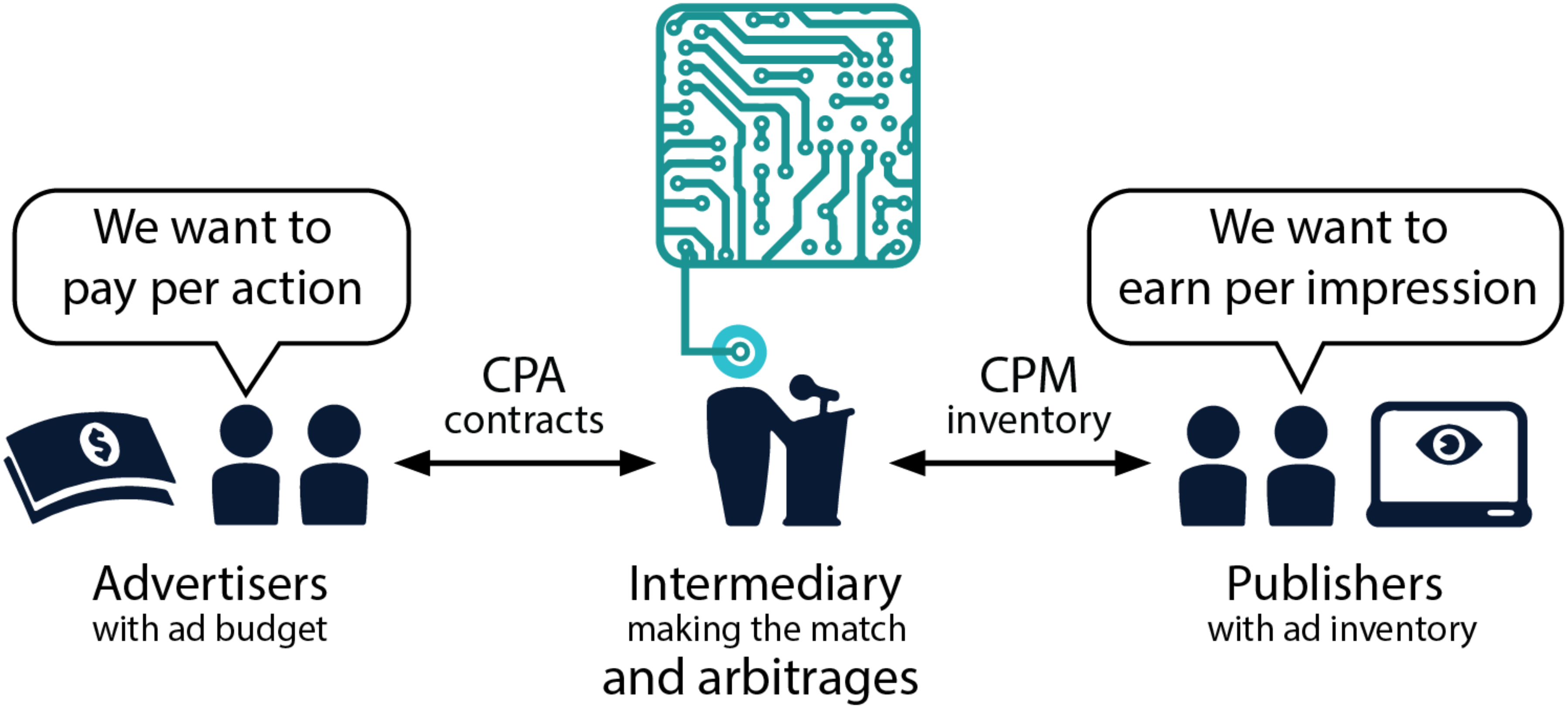
11 Aug 2015, KDD

Survey

- If you have two potential deals, which one will you choose?
  - Deal A: You will surely earn 2 million dollars.
  - Deal B: By 50% chance you will earn **5** million dollars, by 50% chance you will earn nothing.

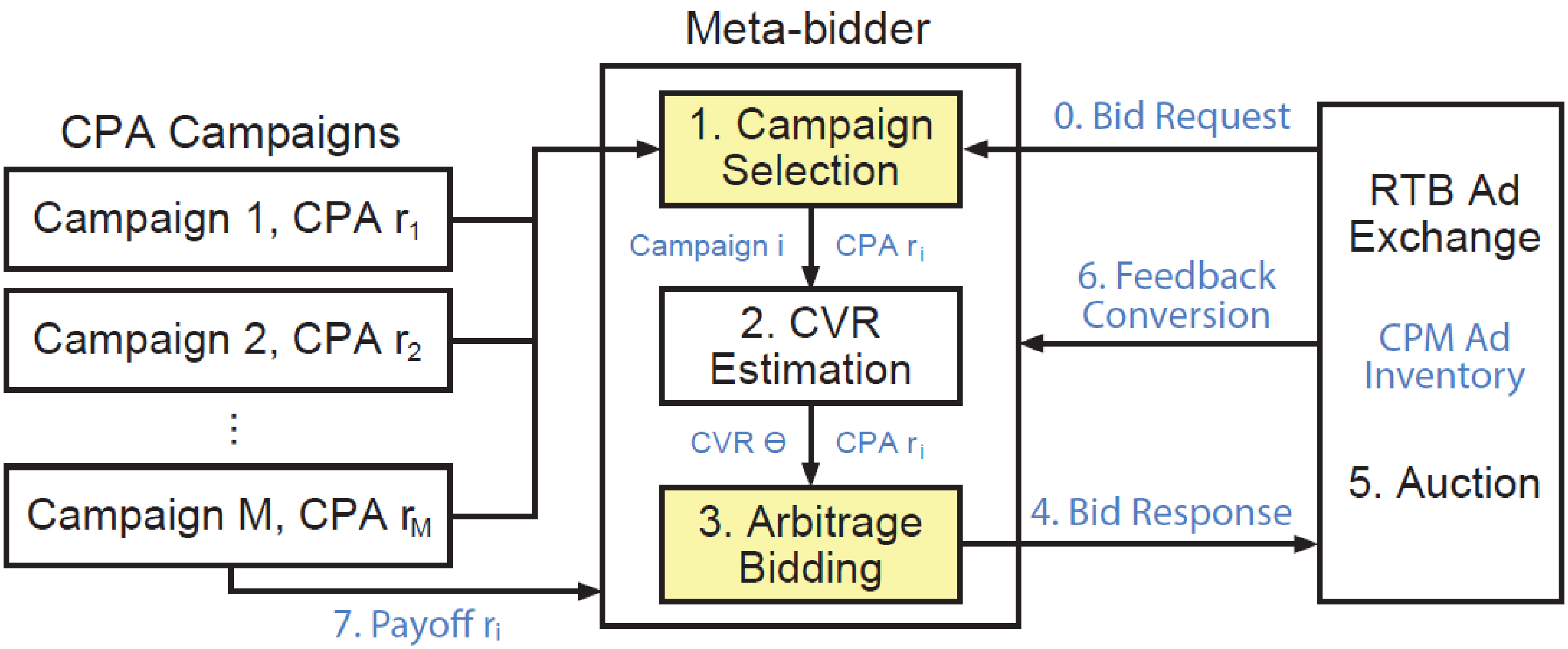


Display Advertising Intermediaries



This work: Intermediary arbitrage algorithms in RTB display advertising.

Intermediary's Statistical Arbitrage via RTB

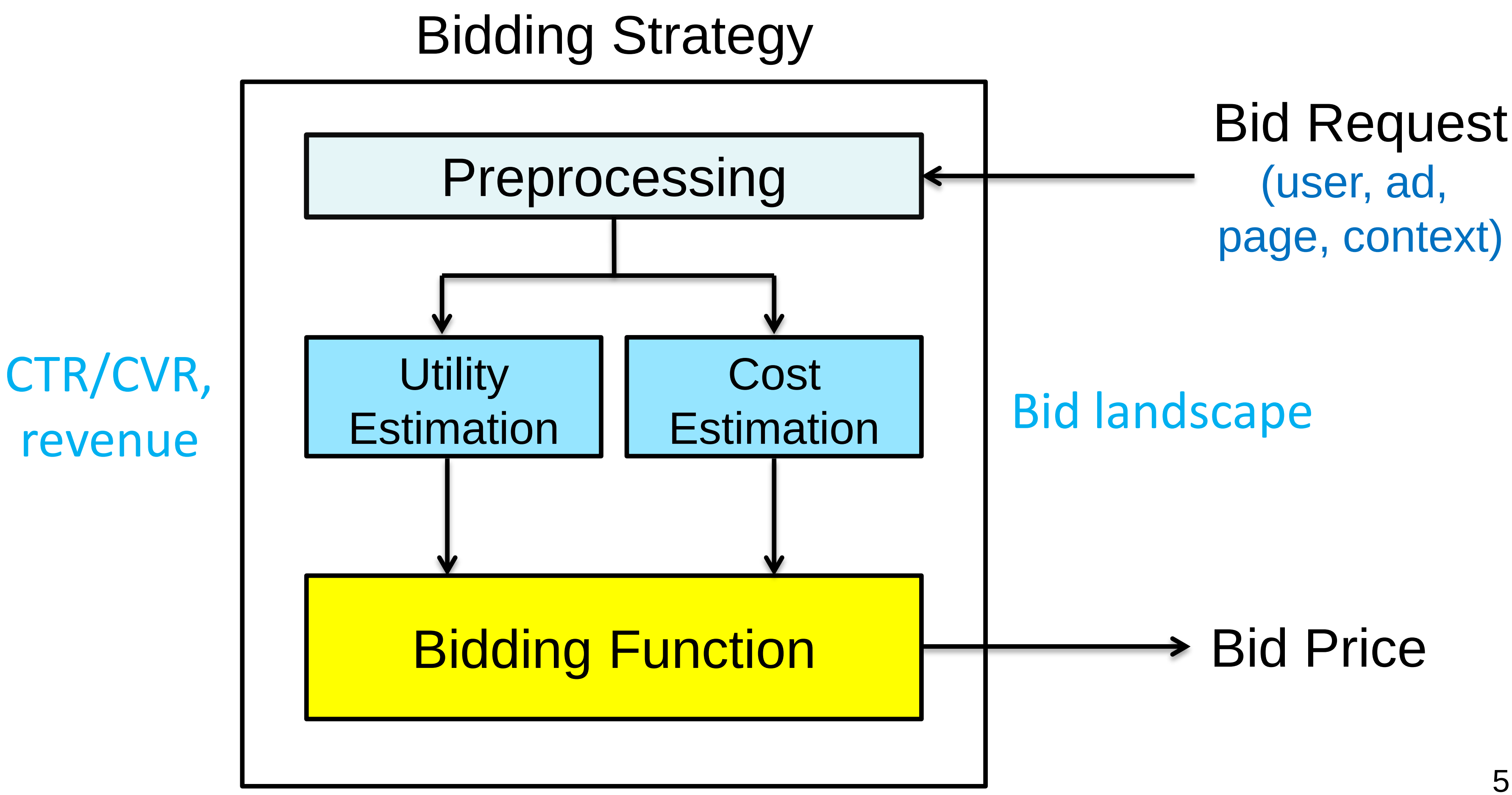


- Statistical arbitrage opportunity occurs, e.g., when

(CPM) cost per conversion < (CPA) payoff per conversion

1000 impressions * 5 cent < 8000 cent for 1 conversion

Bidding Strategy: Commoditising each ad display opportunity



Statistical Arbitrage Mining

- Expected utility (net profit) and cost on multiple campaigns

Bid request vol. Est. payoff winning function CVR estimation

$$\mathbb{E}[R(\boldsymbol{v}, b(\theta, r))] = T \sum_{i=1}^{M} v_i \int_{\theta} \Big(\theta r_i - b(\theta, r_i) \Big) w(b(\theta, r_i)) p_{\theta}^{i}(\theta) d\theta$$

bidding function

$$\mathbb{E}[C(\boldsymbol{v}, b(\theta, r))] = T \sum_{i=1}^{M} v_i \int_{\theta} b(\theta, r_i) w(b(\theta, r_i)) p_{\theta}^{i}(\theta) d\theta$$

Cost upper bound

Prob. of selecting Campaign i

Statistical Arbitrage Mining

- Optimising net profit by tuning bidding function and campaign volume allocation

$$b_{\mathrm{SAM}}(), \boldsymbol{v}^* = \underset{b(), \boldsymbol{v}}{\operatorname{argmax}} \quad \mathbb{E}[R]$$

Total arbitrage net profit

$$\text{s.t.} \quad \mathbb{E}[C] \le B$$

Total cost constraint

$$\mathrm{Var}[R] \le h$$

Risk control

$$\mathbf{0} \le \boldsymbol{v} \le \mathbf{1}$$

$$\boldsymbol{v}^T \mathbf{1} = 1$$

M-Step

E-Step

- Solve it in an EM fashion

M-Step: Bidding function optimisation

- Fix v and tune b()

$$\max_{b()} \; T \sum_{i=1}^{M} v_i \int_{\theta} \Big(\theta r_i - b(\theta, r_i)\Big) w(b(\theta, r_i)) p_{\theta}^{i}(\theta) d\theta$$

$$\text{s.t.} \;\; T \sum_{i=1}^{M} v_i \int_{\theta} b(\theta, r_i) w(b(\theta, r_i)) p_{\theta}^{i}(\theta) d\theta \leq B.$$

$$\frac{\mathcal{L}(b(), \boldsymbol{v})}{b()} = 0 \Rightarrow \Big(\frac{\theta r_i}{1+\lambda} - b(\theta, r_i)\Big) \frac{\partial w(b(\theta, r_i))}{\partial b(\theta, r_i)} = w(b(\theta, r_i))$$

$$\Downarrow$$

$$w(b(\theta, r)) = \frac{b(\theta, r)}{l} \Rightarrow b_{\text{sam1}}(\theta, r) = \frac{r\theta}{2(1+\lambda)}$$

$$w(b(\theta, r)) = \frac{b(\theta, r)}{b(\theta, r) + l} \Rightarrow b_{\text{sam2}}(\theta, r) = \sqrt{\frac{rl\theta}{1+\lambda} + l^2} - l$$

E-Step: Campaign volume allocation

- Multi-campaign portfolio optimisation

Portfolio margin mean

Portfolio margin variance

$$\max_{\boldsymbol{v}} \quad \boxed{\boldsymbol{v}^T \boldsymbol{\mu}(b)} - \alpha \boxed{\boldsymbol{v}^T \boldsymbol{\Sigma}(b) \boldsymbol{v}},$$

$$\text{s.t.} \quad \boldsymbol{v}^T \mathbf{1} = 1, \quad \mathbf{0} \leq \boldsymbol{v} \leq \mathbf{1}$$

where

Net profit margin on each campaign

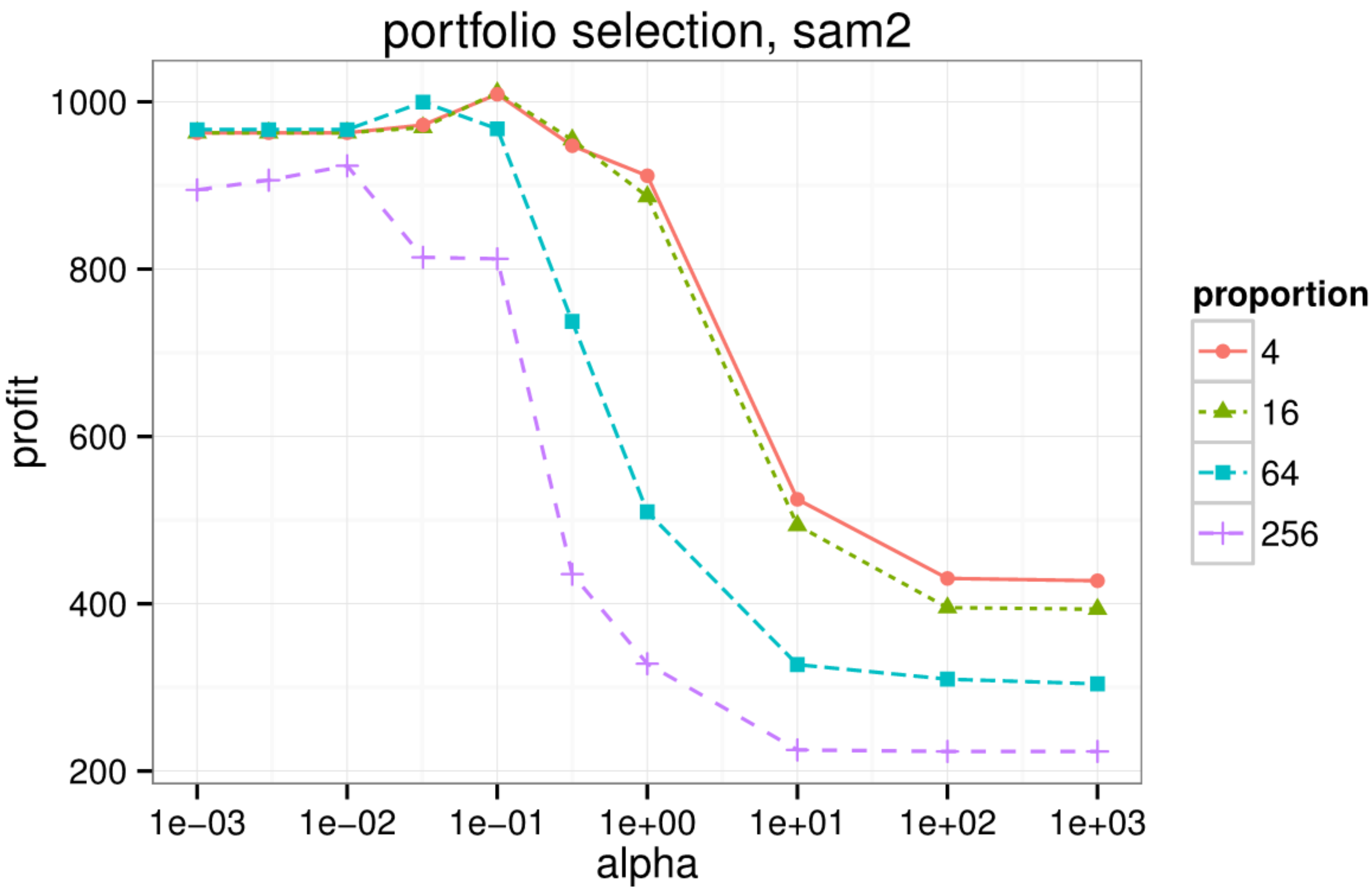
$$\boldsymbol{\mu}(b) = (\mu_1(b), \mu_2(b), \ldots, \mu_M(b))^T$$

$$\boldsymbol{\Sigma}(b) = \{\sigma_{i,j}(b)\}_{i=1\ldots M, j=1\ldots M}$$

$$\mu_i(b) = \mathbb{E}[\gamma_i] = \mathbb{E}\left[\frac{R_i(\boldsymbol{v}_{i=1}, b)}{C_i(\boldsymbol{v}_{i=1}, b)}\right], \ \sigma_i^2(b) = \mathbb{E}\left[\frac{R_i(\boldsymbol{v}_{i=1}, b)^2}{C_i(\boldsymbol{v}_{i=1}, b)^2}\right] - \mathbb{E}\left[\frac{R_i(\boldsymbol{v}_{i=1}, b)}{C_i(\boldsymbol{v}_{i=1}, b)}\right]^2$$

Campaign Portfolio Optimisation Results

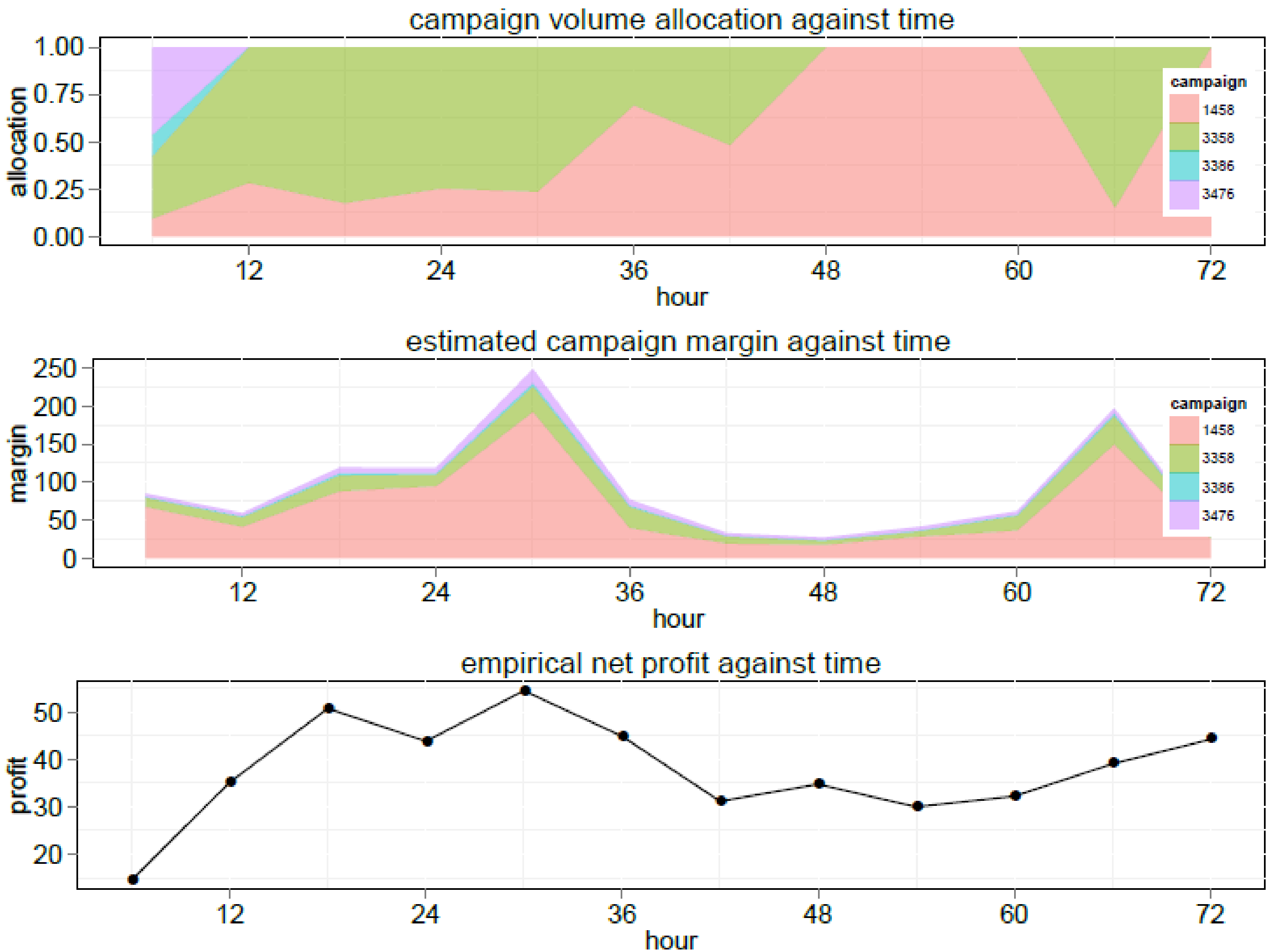
strategies		easy payoff		hard payoff	
bid. algo.	cam. select.	profit (CNY)	margin	profit (CNY)	margin
lin	greedy	501.12	6.63	68.59	0.91
lin	portfolio	925.45	13.11	181.54	2.50
lin	uniform	747.00	9.53	127.14	1.62
ortb	greedy	517.02	6.65	70.96	0.91
ortb	portfolio	802.15	10.32	146.13	1.88
ortb	uniform	765.12	9.89	133.16	1.72
sam1	greedy	966.02	20.81	230.38	11.13
sam1	portfolio	**1,037.98**	15.84	240.63	7.96
sam1	uniform	768.38	9.78	172.43	7.57
sam2	greedy	961.68	28.73	235.31	24.00
sam2	portfolio	983.01	17.21	**248.65**	13.61
sam2	uniform	774.09	10.32	168.15	5.16
truth	greedy	787.10	14.69	227.86	29.05
truth	portfolio	787.10	14.69	242.07	18.34
truth	uniform	326.57	4.14	101.12	5.36



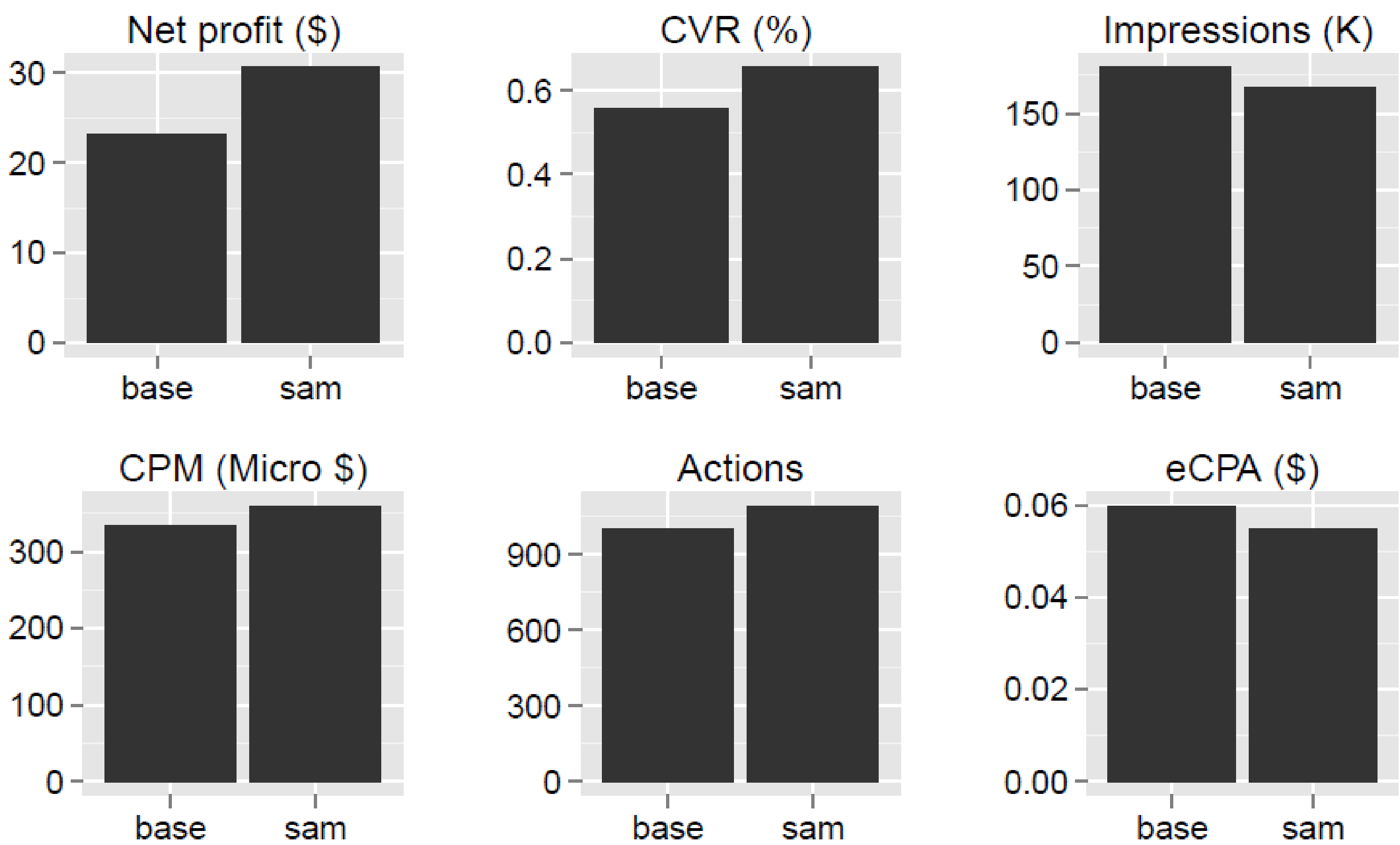
$$\max_{\boldsymbol{v}} \quad \boldsymbol{v}^T \boldsymbol{\mu}(b) - \alpha \boldsymbol{v}^T \boldsymbol{\Sigma}(b) \boldsymbol{v},$$

$$\text{s.t.} \quad \boldsymbol{v}^T \mathbf{1} = 1, \quad \mathbf{0} \leq \boldsymbol{v} \leq \mathbf{1}$$

Dynamic Portfolio Optimisation

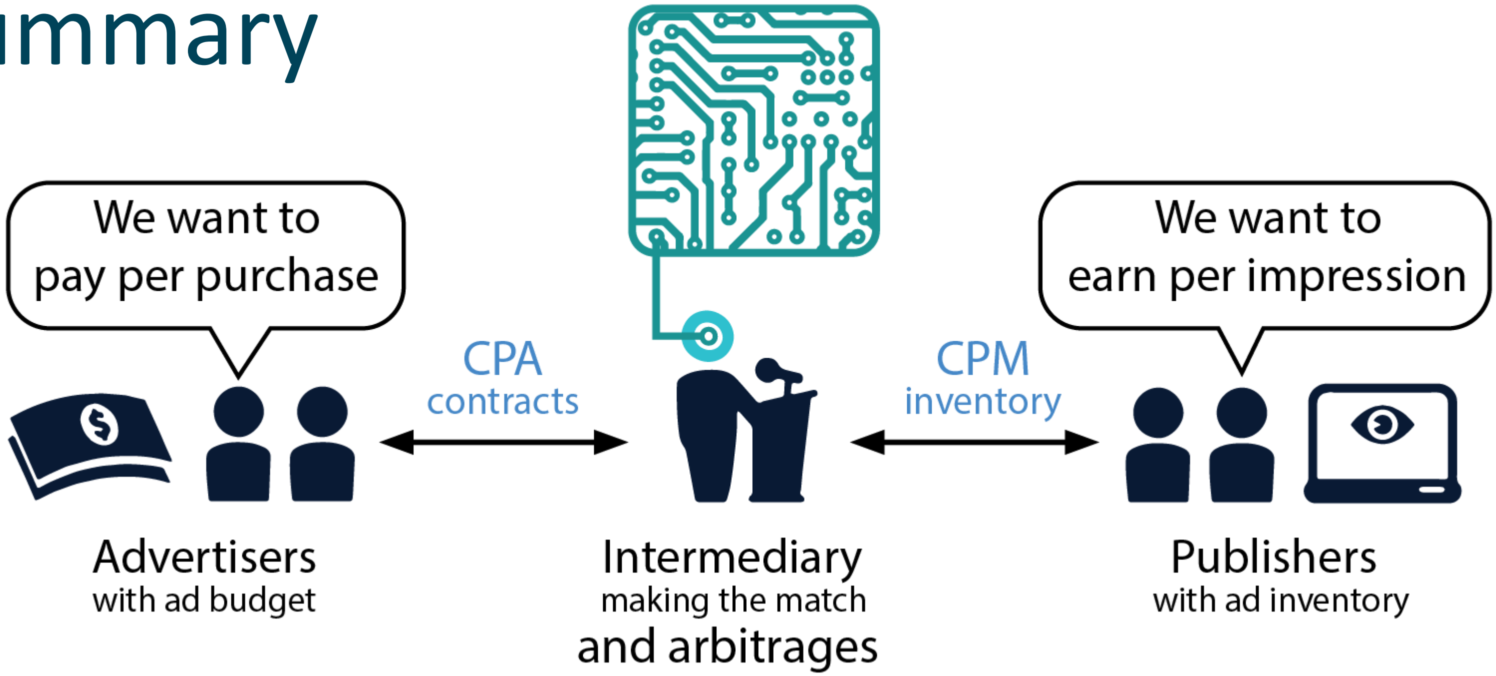


Online A/B Test on BigTree™ DSP



- 23 hours, 13-14 Feb. 2015, with $60 budget each

Summary



- Statistical arbitrage mining algorithm
  - E-step: campaign portfolio optimisation
  - M-step: bidding function optimisation
- Dynamic arbitrage is more effective in practice

Thank You!
Questions?

OpenBidder

OpenBidder Project: www.openbidder.com