Implementation and Evaluation of a Compact Table Propagator in Gecode

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Contents

1 Introduction

In Constraint Programming (CP), every constraint is associated with a propagator algorithm. The propagator algorithm filters out impossible values for the variables related to the constraint. For the TABLE constraint, several propagator algorithms are known. In 2016, a new propagator algorithm for the TABLE constraint was published [1], called Compact Table (CT). Preliminary results indicate that CT outperforms the previously known algorithms. There has been no attempt to implement CT in the constraint solver Gecode [2], and consequently its performance in Gecode is unknown.

1.1 Goal

The goal of this thesis is to implement a CT propagator algorithm for the TABLE constraint in Gecode, and to evaluate its performance with respect to the existing propagators.

1.2 Contributions

State the contributions, perhaps as a bulleted list, referring to the different parts of the paper, as opposed to giving a traditional outline. (As suggested by Olle Gallmo.)

This thesis contributes with the following:

- The relevant preliminaries have been covered in Section 2.
- The algorithms presented in [1] have been modified to suit the target constraint solver Gecode, and are presented and explained in Section 3.
- The CT algorithm has been implemented in Gecode, see Section 4.
- The performance of the CT algorithm has been evaluated, see Section 5.
- ...

2 Background

This section provides a background that is relevant for the following sections. It is divided into five parts: Section 2.1 introduces Constraint Programming. Section 2.3 gives an overview of Gecode, a constraint solver. Section 2.4 introduces the TABLE constraint. Section 2.5 describes the main concepts of the Compact Table (CT) algorithm. Finally, Section 2.6 describes the main idea of Reversible? Sparse Bit-Sets, a data structure that is used in the CT algorithm.

2.1 Constraint Programming

This section introduces the concept of Constraint Programming (CP).

CP is a programming paradigm that is used for solving combinatorial problems. A problem is modelled as a set of *constraints* and a set of *variables* with possible values. The possible values of a variable is called the *domain* of the variable. All the variables are to be assigned a value from their domains, so that all the constraints of the problem are satisfied. Sometimes the solution should not only satisfy the set of constraints for the problem, but should also maximise or minimise some given function.

Perhaps add an example somewhere here.

A constraint solver (CP solver) is a software that solves constraint problems. The solving of a problem consists of generating a search tree by branching on possible values for the variables.

At each node in the search tree, the solver removes impossible values from the domains of the variables. This filtering process is called *propagation*. Each constraint is associated with at least one propagator algorithm, whose task is to detect values that would violate the constraint if the variables were to be assigned any of those values, and remove those values from the domain of the variables.

Definition 1. ***Constraint.*** *Consider a finite sequence of n variables $X = x_1, \dots, x_n$, and a corresponding sequence of* domains *$D = D_1, \dots, D_n$, that are possible values for the respective variable. For a variable $x_i \in X$, its domain D_i is denoted by $dom(x_i)$.*

- *A* constraint *c on X is a relation, denoted by $rel(c)$. The associated variables X are denoted $vars(c)$, and we call $|vars(c)|$ the* arity *of c. The relation $rel(c)$ contains the set of n-tuples that are allowed for X, we call those n-tuples* solutions *to the constraint c.*

- *For an n-tuple $\tau = \langle a_1, \dots, a_n \rangle$ associated with X, we denote the ith value of τ by $\tau[i]$ or $\tau[x_i]$. The tuple τ is* valid *for X if and only if each value of τ is in the domain of the corresponding variable: $\forall i \in 1 \dots n, \tau[i] \in dom(x_i)$, or equivalently, $\tau \in D_1 \times \dots \times D_n$.*

- *An n-tuple τ is a* support *on a n-ary constraint c if and only if τ is valid for $vars(c)$ and τ is a solution to c, that is, τ is contained in $rel(c)$.*

- *For an n-ary constraint c, involving a variable x such that the value $a \in dom(x)$, an n-tuple τ is a* support for (x, a) *on c if and only if τ is a support on c, and $\tau[x] = a$.*

Definition 2. ***CSP.*** *A Constraint Satisfaction Problem (CSP) is a triple $\langle V, D, C \rangle$, where: $V = v_1, \dots, v_n$ is a finite sequence of variables, $D = D_1, \dots, D_n$ is a finite sequence of domains for the respective variable, and $C = \{c_1, \dots, c_m\}$ is a set of constraints, each on a subsequence of V.*

Definition 3. ***Stores.*** *A* store *s is a function, mapping a finite set of variables $V = v_1, \dots, v_n$ to a finite set of domains. We denote the domain of a variable v_i under s by $s(v_i)$ or $dom(v_i)$.*

- *A store s is* failed *if and only if $s(v_i) = \varnothing$ for some $v_i \in V$.*

- *A variable $v_i \in V$ is* fixed, *or* assigned, *by a store s if and only if $|s(v_i)| = 1$.*

- *A store s is an* assignment *store if all variables are fixed under s.*

- *Let c be an n-ary constraint on V. A store s is a* solution store *to c if and only if s is an assignment store and the corresponding n-tuple is a solution to c: $\forall i \in \{1, \dots, n\}, s(v_i) = \{a_i\}$, and $\langle a_1, \dots, a_n \rangle$ is a solution to c.*

- *A store s_1 is* stronger *than a store s_2, written $s_1 \preceq s_2$ if and only if $s_1(v) \subseteq s_2(v)$ for all $v \in V$.*

2.2 Propagation and propagators

Constraint propagation is the process of removing values from the domains of the variables in a CSP that can never appear in a solution store to the CSP. In a CP solver, each constraint that the solver implements is associated with one or more propagation algorithms, called propagators, whose task is to remove values that are in conflict with its respective constraint.

To have a well-defined behaviour of propagators, there are some properties that they must fulfill. Now follows a definition of a propagator and the obligations that they must meet, taken from Chapter 14 in handbook and MPG.

Definition 4. ***Propagators.*** *A* propagator *p is a function mapping stores to stores:*

$$p : store \mapsto store$$

In a CP-solver, a propagator is implemented as a procedure that also returns a status message*. A propagator must fulfill the following properties:*

- *A propagator p is a decreasing function: $p(s) \preceq s$ for any store s. This property guarantees that constraint propagation only removes values.*
- *A propagator p is a monotonic function: $s_1 \preceq s_2 \Rightarrow p(s_1) \preceq p(s_2)$ for any stores s_1 and s_2. This property guarantees that constraint propagation preserves the strength-ordering of stores.*
- *A propagator is correct for the constraint it implements. A propagator p is correct for a constraint c if and only if it does not remove values that are part of supports for c. This property guarantees that a propagator does not miss any potential solution store.*
- *A propagator is* checking*: for a given assignment store s, the propagator must decide whether s is a solution store or not for the constraint it implements. If s is a solution store, it must signal subsumption, otherwise it must signal failure.*
- *A propagator must be* honest*: it must be* fixpoint honest *and* subsumption honest*. A propagator p is fixpoint honest if and only if it does not signal fixpoint if does not return a fixpoint, and it is subsumption honest if and only if it does not signal subsumption if it is not subsumed by an input store s.*

  A propagator p is at fixpoint *on a store s if and only if applying p to to s gives no further propagation: $p(s) = s$ for a store s. If a propagator p always returns a fixpoint, that is, if $p(s) = p(p(s))$, p is* idempotent*.*

  A propagator is subsumed *by a store s if and only if all stronger stores are fixpoints: $\forall s' \preceq s, p(s') = s$.*

Note that the honest property of a propagator does not mean that a propagator is obliged to signal fixpoint or subsumption if it has computed a fixpoint or is subsumed, only that it may not claim that it is at fixpoint or is subsumed if it is not. Thus, it is always safe (though in many cases not so efficient for the CP-solver) for a propagator to signal 'not fixpoint', except for a solution store when it must signal either fail or subsumption. In fact, a propagator must not even prune values. An extreme case is the identity propagator i, with $i(s) = s$ for all input stores s, which would be correct for all constraints, as long at is it checking and honest.

To give a measure of how strong the constraint propagation of a propagator is, it is common to declare a *consistency level* of a propagator. There are three commonly used consistency levels, **value consistency, bounds consistency**, and **domain consistency**.

Definition 5. ***Domain consistency.*** *A constraint c is domain consistent on a store s if and only if for all variable-value pair (x, a) such that $x \in vars(c)$ and $a \in dom(x)$, there exists at least one support for (x, a) on c. A propagator p is domain consistent, iff c is domain consistent on $p(s)$ for all stores s such that $p(s)$ is not a failed store.*

Bounds consistency, Value consistency.

2.3 Gecode

Gecode [2] is a popular constraint programming solver written in C++.

Define the parts of the Gecode API that is used later (propagate(), status messages...)

2.4 The Table Constraint

The TABLE constraint, also called EXTENSIONAL , explicitly expresses the possible combinations of values for the variables as a sequence of n-tuples.

Definition 6. ***Table constraints.*** *A (positive*[1]*)* table constraint c *is a constraint such that* $rel(c)$ *is defined explicitly by listing all the tuples that are solutions to c.*

2.5 Compact-Table Propagator

2.6 Sparse Bit-Set

3 Algorithms

This chapter presents the algorithms that are used in the implementation of the CT propagator in Section 4. In what follows, when we refer to an array a, $a[0]$ denotes the first element (indexing starts from 0), a.length() the number of its cells and $a[i : j]$ all its cells in the closed interval $[i, j]$, where $0 \leq i \leq j \leq a.\text{length}() - 1$.

3.1 Sparse Bit-Set

This section describes Class `SparseBitSet`, which is the main data-structure in the CT algorithm for maintaining the supports. Algorithm 1 shows the pseudo code for Class `SparseBitSet`. The rest of this section describes its fields and methods in detail.

[1]There are also negative table constraints, that list the forbidden tuples instead of the allowed tuples.

```
Class SparseBitSet

words: array of long                                         // words.length() = $p$
index: array of int                                          // index.length() = $p$
limit: int
mask: array of long                                          // mask.length() = $p$

Method initSparseBitSet($nbits$: int)
  $p \leftarrow \lceil \frac{nbits}{64} \rceil$
  words $\leftarrow$ array of long of length $p$, first $nbits$ set to 1
  mask $\leftarrow$ array of long of length$p$, all bits set to 0
  index $\leftarrow [0, \ldots, p-1]$
  limit $\leftarrow p - 1$

Method isEmpty() : Boolean
  return limit $= -1$

Method clearMask()
  for $i \leftarrow 0$ to limit do
    $offset \leftarrow$ index[$i$]
    mask[$offset$] $\leftarrow 0^{64}$

Method addToMask($m$: array of long)
  for $i \leftarrow 0$ to limit do
    $offset \leftarrow$ index[$i$]
    mask[$offset$] $\leftarrow$ mask[$offset$] | $m$[$offset$]          // bitwise OR

Method intersectWithMask()
  for $i \leftarrow$ limit downto 0 do
    $offset \leftarrow$ index[$i$]
    $w \leftarrow$ words[$offset$] & mask[$offset$]                 // bitwise AND
    if $w \neq$ words[$offset$] then
      words[$offset$] $\leftarrow w$
      if $w = 0^{64}$ then
        index[$i$] $\leftarrow$ index[limit]
        index[limit] $\leftarrow offset$
        limit $\leftarrow$ limit $- 1$

Method intersectIndex($m$: array of long) : int
  for $i \leftarrow 0$ to limit do
    offset $\leftarrow$ index[$i$]
    if words[offset] & $m$[offset] $\neq 0^{64}$ then
      return offset
  return $-1$
```

Algorithm 1: Pseudo code for Class SparseBitSet.

3.1.1 Fields

Todo: Add examples.

Lines 2-5 of Algorithm 1 shows the fields of Class `SparseBitSet` and their types. Now follows a more detailed description of them.

- `words` is an array of p 64-bit words which defines the current value of the bit-set: the ith bit of the jth word is 1 if and only if the $(j-1)\cdot 64+i$th element of the set is present. Initially, all words in this array have all their bits set to 1, except the last word that may have a suffix of bits set to 0. Example.
- `index` is an array that manages the indices of the words in `words`, making it possible to performing operations to non-zero words only. In `index`, the indices of all the non-zero words are at positions less than or equal to the value of the field `limit`, and the indices of the zero-words are at indices strictly greater than `limit`.
- `limit` is the index of `index` corresponding to the last non-zero word in `words`. Thus it is one smaller than the number of non-zero words in `words`.
- `mask` is a local temporary array that is used to modify the bits in `words`.

The class invariant describing the state of the class is as follows:

$$\begin{aligned}&\texttt{index}\text{ is a permutation of }[0,\dots,p-1],\text{ and}\\&\forall i\in\{0,\dots,p-1\}: i\le\texttt{limit}\Leftrightarrow\texttt{words}[\texttt{index}[i]]\neq 0^{64}\end{aligned}\tag{3.1}$$

3.1.2 Methods

We now describe the methods in Class `SparseBitSet` in Algorithm 1.

- initSparseBitSet() in lines 6-11 initialises a sparse bit-set-object. It takes the number of bits as an argument and initialises the fields described in 3.1.1 in a straightforward way.
- isEmpty() in lines 12-13 checks if the number of non-zero words is different from zero. If the limit is set to -1, that means that all words are zero-words and the bit-set is empty.
- clearMask() in lines 14-17 clears the temporary mask. This means setting to 0 all words of `mask` corresponding to non-zero words of `words`.
- addToMask() in lines 18-21 collects elements to the temporary mask by applying a word-by-word logical bit-wise *or* operation with a given bit-set (array of long). Once again, this operation is only applied to indices corresponding to non-zero words in `words`.
- intersectWithMask() in lines 22-31 considers each non-zero word of `words` in turn and replaces it by its intersection with the corresponding word of `mask`. In case the resulting new word is 0, it (its index) is switched by (the index of) the last non-zero word, and `limit` is decreased by one.

  In Section 4 we will see that the implementation actually can skip line 30 because it is unneccesary to save the index of a zero-word in a copy-based solver such as Gecode. We keep this line here though, because otherwise the invariant in (3.1) would not hold.
- intersectIndex() in lines 32-37 checks whether the intersection of `words` and a given bit-set (array of long) is empty or not. For all non-zero words in `words`, we perform a logical bit-wise *and* operation in line 35 and return the index of the word if the intersection is non-empty. If the intersection is empty for all words, -1 is returned.

3.2 Compact-Table (CT) Algorithm

This section describes the CT algorithm, a domain consistent propagation algorithm for any Table constraint c. Algorithm 2 shows the interface for Class CT-Propagator, which implements the CT algorithm. The rest of this section will describe its fields and methods in detail.

```
1:  Class CT-Propagator

2:  vars: array of variables
3:  validTuples: SparseBitSet                                   // Current valid tuples
4:  supports: array of bit-sets        // supports[$x, a$] is the bit-set of supports for $(x, a)$
5:  residues: array of int    // residues[$x, a$] is the last found index for a support for $(x, a)$

6:  Method initCT($variables$: array of variables, $tuples$: list of tuples)
7:      Algorithm 3

8:  Method updateTable($x$: variable)
9:      Algorithm 4

10: Method filterDomains()
11:     Algorithm 5

12: Method propagate()
13:     Algorithm 6
```

Algorithm 2: Interface for CT propagator class.

3.2.1 Fields

Add examples with figures for describing the fields.

Lines 2-5 of Algorithm 2 shows the fields of Class `CT-Propagator` and their types. Now follows a more detailed description of them. In what follows, we let the *initial domain* of a variable $x \in \texttt{vars}(c)$, denoted $\underline{\text{dom}}(x)$, be the domain that x has before CT has performed any propagation, in contrast to $\text{dom}(x)$ which is the current domain of x. The *initial table* for a table constraint c is the list of tuples $T_0 = \langle \tau_0, \tau_1, \ldots, \tau_{p_0 - 1} \rangle$ of length p_o that are given as input to initCT(), and the *initial valid table* for c is the subset $T \subseteq T_0$ of size $p \leq p_0$ such that $\forall i \in \{1, \ldots, p_0\} : \tau_i \in T$ iff τ_i is a support on c for the initial domains of the variables.

- `vars` represents $vars(c)$, the variables associated with c.
- `validTuples` represents the current table, that is, the current valid supports for c. If the initial valid table for c is $\langle \tau_0, \tau_1, \ldots, \tau_{p-1} \rangle$, then `validTuples` is a `SparseBitSet` object of initial size p, such that value i is contained (is set to 1) if and only if the ith tuple is valid:

$$i \in \texttt{validTuples} \Leftrightarrow \forall x \in vars(c) : \tau_i[x] \in \text{dom}(x) \tag{3.2}$$

- `supports` represents the supports for each variable-value pair (x, a), where $x \in vars(c) \land a \in \underline{\text{dom}}(x)$. It is a static array of words $\texttt{supports}[x, a]$, seen as bit-sets. The bit-set $\texttt{supports}[x, a]$ is such that the bit at position i is set to 1 if and only if the tuple τ_i in the initial valid table of c is initially a support for (x, a):

$$\forall x \in vars(c) : \forall a \in \text{dom}(x) : \texttt{supports}[x,a][i] = 1 \Leftrightarrow \tau_i[x] = a \wedge \forall y \in vars(c) : \tau_i[y] \in \underline{\text{dom}}(y)$$

`supports` is computed once during the initialisation of CT and then remains unchanged.

- `residues` is an array such that for each variable-value pair (x, a), $\texttt{residues}[x, a]$ denotes the index of the word in `validTuples` where a support was found for (x, a) the last time it was sought for.

3.2.2 Methods

We now describe the methods of Class `CT-Propagator`.

Initialisation. The initialisation of the fields of Class `CT-Propagator` is described in Algorithm 3. The method initialiseCT() takes two parameters: *variables*, that are the variables associated to the constraint *c*, and *tuples*, that is a list of tuples that define the initial table for *c*.

```
Method initCT(variables: array of variables, tuples: list of tuples)
  foreach $x \in$ variables do
    $\text{dom}(x) \leftarrow \text{dom}(x) \setminus \{a \in \text{dom}(x) : a > tuples.\text{max}() \text{ or } a < tuples.\text{min}()\}$
    if $\text{dom}(x) = \varnothing$ then
      return Failed
  npairs ← sum $\{|\text{dom}(x)| : x \in variables\}$                // Number of variable-value pairs
  ntuples ← tuples.size()                                            // Number of tuples
  nsupports ← 0                                                      // Number of found supports
  vars ← variables
  residues ← array of length npairs
  supports ← array of length npairs with bit-sets of size ntuples
  foreach $t \in$ tuples do
    supported ← true
    foreach $x \in$ vars do
      if $t[x] \notin \text{dom}(x)$ then
        supported ← false
        break                                                        // Exit loop
    if supported then
      foreach $x \in$ vars do
        supports$[x, t[x]][nsupports]$ ← 1
        residues$[x, t[x]]$ ← $\lfloor \frac{nsupports}{64} \rfloor$          // Index for the support in validTuples
        nsupports ← nsupports + 1
  foreach $x \in$ vars do
    $\text{dom}(x) \leftarrow \text{dom}(x) \setminus \{a \in \text{dom}(x) : \texttt{supports}[x, a] \text{ is empty}\}$
    if $\text{dom}(x) = \varnothing$ then
      return Failed
  validTuples ← SparseBitSet with nsupports bits
  return Fixpoint
```

Algorithm 3: Pseudo code for initialising the CT-propagator.

Lines 2-5 performs simple bounds propagation to limit the domain sizes of the variables, which limit the sizes of the incremental data structures. It removes from the domain of each variable x all values that are either greater than the largest element or smaller than the smallest element in the initial table. If a variable has a domain wipe-out, *Failed* is returned.

Lines 6-8 initialise local variables that will be used later.

Lines 9-11 initialise the fields `vars`, `residues` and `supports`. The field `supports` is initialised as an array of bit-sets, with one bit-set for each variable-value pair, and the size of each bit-set being the number of tuples in *tuples*. Each bit-set is assumed to be initially filled with zeros.

Lines 12-22 set the correct bits to 1 in `supports`. For each tuple t, we check if t is a valid support for c. Recall that t is a valid support for c if and only if $t[x] \in \mathrm{dom}(x)$ for all $x \in scp(c)$. We keep a counter, *nsupports*, for the number of valid supports for c. This is used for indexing the tuples in `supports` (we only index the tuples that are valid supports). If t is a valid support, all elements in `supports` corresponding to t are set to 1 in line 20. We also take the opportunity to store the word index of the found support in `residues`$[x, t[x]]$ in line 21.

Lines 23-24 removes values that are not supported by any tuple in the initial valid table. In case a variable has a wipe-out of its domain, *Failed* is returned.

Line 27 initialises `validTuples` as a `SparseBitSet` object with *nsupports* bits, initially with all bits set to 1 since *nsupports* number of tuples are initially valid supports for c. *nsupports* > 0, otherwise we would have returned *Failed* as no variable-value pair would be supported.

Performing propagation. When the propagator is invoked for propagation, the method propagate() in Algorithm 2 is called. Before defining this function, we need to define the help functions updateTable() and filterDomains(). Performing propagation consists of two steps: updating the current table and filtering out inconsistent values from the domains of the variables. We now describe these processes in more detail.

1. *Updating the current table.*

```
1: Method updateTable($x$: variable)
2:   validTuples.clearMask()
3:   foreach $a \in \mathrm{dom}(x)$ do
4:     validTuples.addToMask(supports[$x, a$])
5:   validTuples.intersectWithMask()
6:   if validTuples.isEmpty() then
7:     return Failed                                   // No valid tuples left
```

Algorithm 4: Method updateTable() in Class CT-Propagator. The infrastructure is such that this method is called for each variable whose domain is modified since the previous call to propagate().

The method updateTable() in Algorithm 4 filters out (indices of) tuples that have ceased to be supports for the input variable x. Lines 3-4 stores the union of the set of valid tuples for each value $a \in \mathrm{dom}(x)$ in the temporary mask and Line 5 intersects `validTuples` with the mask, so that the indices that correspond to tuples that are no longer valid are set to 0 in the bit-set. Line 6 checks whether the current table is empty, in which case we return *Failed* in line 7 because there are no valid tuples left.

The algorithm is assumed to be run on an infrastructure that runs updateTable() before every call to propagate(), for each variable $x \in vars(c)$ whose domain has changed since the previous call to propagate().

2. *Filtering of domains.* After the current table has been updated, inconsistent values must be removed from the domains of the variables. It follows from the definition of the bitsets `validTuples` and `supports`$[x, a]$ that (x, a) has a valid support if and only if

$$(\texttt{validTuples} \cap \texttt{supports}[x, a]) \neq \emptyset \tag{3.3}$$

Therefore, we must check this condition for every variable-value pair (x, a) and remove a from the domain of x if the condition is not satisfied any more. This is implemented in the method filterDomains() in Algorithm 5.

```
1: Method filterDomains()
2:   $count\_unassigned \leftarrow 0$
3:   foreach $x \in$ vars such that $|\text{dom}(x)| > 1$ do
4:     foreach $a \in \text{dom}(x)$ do
5:       $index \leftarrow$ residues$[x, a]$
6:       if validTuples$[index]$ & supports$[x, a][index] = 0$ then
7:         $index \leftarrow$ validTuples.intersectIndex(supports$[x, a]$)
8:         if $index \neq -1$ then
9:           residues$[x, a] \leftarrow index$
10:        else
11:          $\text{dom}(x) \leftarrow \text{dom}(x) \setminus \{a\}$
12:    if $|\text{dom}(x)| > 1$ then
13:      $count\_unassigned \leftarrow count\_unassigned + 1$
14:  if $count\_unassigned \leq 1$ then
15:    return $Subsumed$
16:  else
17:    return $Fixpoint$
```

Algorithm 5: Method filterDomains() in Class CT-Propagator.

Line 2 initialises a counter for the number of unassigned variables.

Lines 3-13 performs the actual filtering of the domains. We note that it is only necessary to consider a variable $x \in$ `vars` whose domain size is larger than 1, because we will never filter out values from the domain of an assigned variable. To see this, assume we removed the last value for a variable x, causing a wipe-out for x. Then by the definition in equation (3.2) `validTuples` must be empty, which it will not be upon invocation of filterDomains(). Hence, we need only consider $x \in$ `vars` such that $|\text{dom}(x) > 1|$.

In Lines 5-6 we see if the cached word index still has a support for (x, a). It it has not, we we search for an index in line 7in `validTuples` where a valid support for the variable-value pair (x, a) is found, thereby checking the condition in (3.3). If such an index exists, we cache it in `residues`$[x, a]$, and if it does not, we remove a from $\text{dom}(x)$ if (x, a) in line 11 since there is no support left for (x, a).

Lines 12-13 increments the counter of unassigned variables if $|\text{dom}(x)| > 1$.

Lines 14-15 return the correct propagator status message. If the number of unassigned variables is at most one, the propagator is subsumed. Otherwise, the propagator is at fixpoint.

After defining updateTable() and filterDomains(), we are now ready to define the method propagate() in Class CT-Propagator, shown in Algorithm 6.

It should be unneccessary to check if validTuples is empty as that is done in updateTable already. However, when I try to remove the check in the c++ code it crashes, maybe because of synchronisation issues between advise() and propagate().

```
1: Method propagate()
2:   if validTuples.isEmpty() then
3:     return Failed
4:   return filterDomains()
```

Algorithm 6: Method propagate() in Class CT-Propagator. updateTable() (Algorithm 4) is called, and if the current table is empty, we are in a failed node. Otherwise, filterDomains() (Algorithm 5) is called, and the return value of that method is returned.

Optimisation of propagate(). If x is the only variable that has been modified since the last invocation of CT, it is not necessary to attempt to filter out values from x, because every value of of x will have a support in `validTuples`. Show the implementation of this explicitly?

3.2.3 Proof of properties for CT

This section proves that the CT Propagator is indeed a well-defined propagator implementing the Table constraint. We formulate the following theorem, which we will prove by a number of lemmas.

Theorem 3.1. *CT is an idempotent, domain consistent propagator implementing the* Table *constraint, fulfilling the properties in Definition 4.*

To prove Theorem 3.1, we formulate and prove the following lemmas. In what follows, we denote $CT(s)$ the resulting store of running either `initCT()` or `propagate()` on an input store s, depending on if it is the first time or not that the propagator is called. Or skip initCT() and only count calls to propagate()?

Lemma 3.2. *CT is a decreasing function.*

Proof of Lemma 3.2. Since CT only removes values from the domains of the variables, we have $CT(s) \preceq s$ for any store s. Thus, CT is a decreasing function. □

Lemma 3.3. *CT is idempotent.*

Proof of Lemma 3.3. To prove that CT is idempotent, we shall show that CT always reaches fixpoint for any input store s, that is, $CT(CT(s)) = CT(s)$ for any store s.

Suppose $CT(CT(s)) \neq CT(s)$ for a store s. Since CT is monotonic and decreasing, we must have $CT(CT(s)) \prec CT(s)$, that is, CT must prune at least one value a from a variable x from the store $CT(s)$.

By (3.3), there must exists at least one tuple τ_i that is a support for (x, a) under the store $CT(s)$: $\exists i : i \in$ `validTuples` $\wedge\ \tau_i[x] = a$. After `updateTable()` is perfomed on $CT(s)$, we still have $i \in$ `validTuples`, because τ_i is still valid in $CT(s)$. Since `filterDomains()` only removes values that have no supports, it is impossible that a is pruned from x, since τ_i is a support for (x, a). Hence, we must have $CT(CT(s)) = CT(s)$. □

Lemma 3.4. *CT is correct for the* Table *constraint.*

Proof of Lemma 3.4. CT does not remove values that participate in tuples that are supports on a Table constratint c, since `filterDomains()` and `initCT()` only removes values that have no supports on c. Thus, CT is correct for Table . □

Lemma 3.5. *CT is checking.*

Proof of Lemma 3.5. For an input store s that is an assignment store, we shall show that CT signals failure if s is not a solution store, and signals subsumption if s is a solution store.

First, assume that s is not a solution store. That means that the tuple $\tau = \langle s(x_1), \ldots, s(s_n) \rangle \notin c$.

There are two cases, either it is the first time CT is applied or it has been applied before. If it is the first time, then `initCT()` is called. Since τ is not a solution to c, there is at least one variable-value pair $(x_i, s(x_i))$ which is not supported, so $s(x_i)$ will be pruned from x in `initCT()`, which reports failure in line which line.

If it is not the first time that CT is called, `propagate()` is called. Since there are no valid tuples left, `validTuples` will be empty after the call to `updateTable()` and CT reports failure.

Now assume that s is a solution store. CT signals failure in `filterDomains()` because all variables are assigned. Initialisation? □

Lemma 3.6. *CT is honest.*

Proof of Lemma 3.6. Since CT is idempotent, CT is fixpoint honest. It remains to show that CT is subsumption honest. CT signals subsumption on input store s if there is at most one unassigned variable x in `filterDomains()`. After this point, no values will ever be pruned from x by CT, because there will always be a support for (x, a) for each value $a \in dom(x)$. Hence, CT is indeed subsumed by s when it signals subsumption.

□

Lemma 3.7. *CT is domain consistent.*

Proof of Lemma 3.7. There are two cases; either it is the first time CT is called, or it is not. Both after a call to `initCT()` and `filterDomains()`, for each variable-value pair (x, a) there exists at least one support, because we filter out those values that have no support. □

Lemma 3.8. *CT is a monotonic function.*

Proof of Lemma 3.8. Consider two stores s_1 and s_2 such that $s_1 \preceq s_2$. Since CT is domain consistent, each variable-value pair (x, a) that is part of $CT(s_1)$, must also be part of $CT(s_2)$, so $CT(s_1) \preceq CT(s_2)$. □

After proving Lemmas 3.2-3.8, proving Theorem 3.1 is trivial.

Proof of Theorem 3.1. The result follows by Lemmas 3.2- 3.8. □

4 Implementation

Copy-function.

This section describes an implementation of the CT propagator using the algorithms presented in Section 3. The implementation was made in the C++ programming language in the Gecode library.

The bit-set matrix `supports` is static and could be shared between all solution spaces.

The bit-set `validTuples` changes dynamically during propagation and must therefore be copied for every new space. Can save memory by only copying the non-zero words.

No need to save the `tuples` as a field in the propagator class as all the necessary information is encoded in `validTuples` and `supports`.

How is the index mapping done in supports and residues?

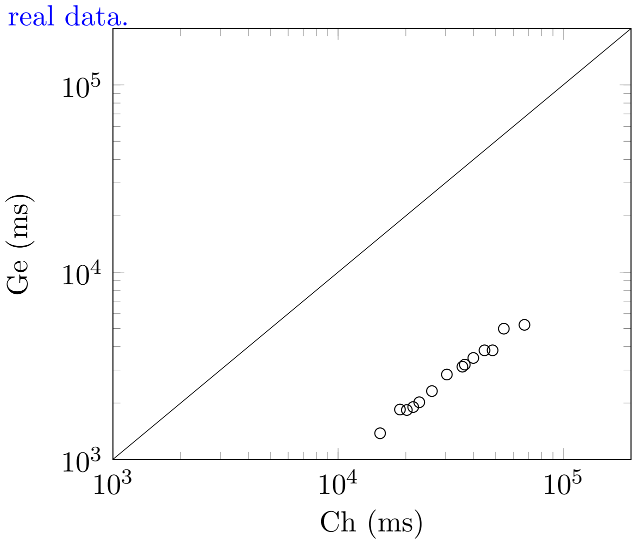
5 Evaluation

This chapter presents the evaluation of the implementation of the CT propagator presented in Section 4. In Section 5.1, the evaluation setup is described. In Section 5.2 presents the results of the evaluation. The results are discussed in Section 5.3.

5.1 Evaluation Setup

5.2 Results

Not real data.



5.3 Discussion

6 Conclusions and Future Work

References

[1] J. Demeulenaere, R. Hartert, C. Lecoutre, G. Perez, L. Perron, J. Régin, and P. Schaus. Compact-Table: Efficiently filtering table constraints with reversible sparse bit-sets. In M. Rueher, editor, *CP*, volume 9892 of *Lecture Notes in Computer Science*, pages 207–223. Springer, 2016.

[2] Gecode Team. Gecode: A generic constraint development environment, 2016.

A Source Code

This appendix presents the source code for the implementation described in Section 4.

n	$runtime_g$	$fail_g$	$nprops_g$	$runtime_c$	$fail_c$	$nprops_c$
0	0.119	7	4679	0.366	6	4474
1	0.085	2	1780	0.166	11	5119
2	0.073	1	1684	0.197	0	1241
3	0.052	6	2065	0.137	6	2266
4	0.033	0	845	0.064	0	850
5	0.087	0	1275	0.169	0	1260
6	0.082	7	3952	0.136	6	3967
7	0.077	3	2304	0.102	4	2460
8	0.051	2	1281	0.103	2	1306
9	0.034	0	825	0.072	0	875
10	1.227	16	171162	1.112	46	298509
11	1.399	41	410228	1.063	21	230481
12	1.067	3	95731	1.217	71	405968
13	2.342	381	1700606	600.00	147028	603399614
14	0.978	25	191179	0.873	10	124704
15	3.154	245	1146764	2.368	178	879023
16	2.910	185	992421	1.710	152	734316
17	1.006	11	136377	1.119	16	128781
18	1.052	7	108977	0.958	11	122557
19	3.012	166	1083832	2.131	219	1381581
20	2.690	32	319505	1.858	21	332253
21	2.818	61	766182	5.233	1091	8287668
22	600.00	28601	171635604	600.00	69906	464113700
23	2.475	53	616660	600.00	61878	414187753
24	3.513	177	841650	1.681	13	324284
25	2.441	18	301053	1.694	19	339487
26	2.415	25	374594	1.711	16	326141
27	2.215	7	264087	2.238	5	270104
28	1.919	14	272577	1.804	13	273908
29	2.228	8	212822	1.936	6	221706
30	600.00	18058	119444044	600.00	47925	497445022
31	600.00	23283	178289620	2.670	30	532937
32	4.870	63	913320	600.00	166635	858563943
33	600.00	15181	147948821	600.00	67148	763053956
34	600.00	39057	303289935	9.809	1726	14322164
35	4.376	52	904626	600.00	117958	1084029060
36	4.396	114	1628267	4.487	197	1934868
37	600.00	21286	132575730	3.146	35	542375
38	4.577	79	1045936	3.162	51	856676
39	600.00	9040	99188489	600.00	39272	399951117
40	2.930	1	34787	2.915	1	69542
41	5.303	40	790942	3.325	44	834269
42	600.00	12605	148700057	600.00	42626	406830329
43	9.819	361	5325834	6.875	373	5490353
44	5.528	116	1336494	4.065	37	789395
45	600.00	8415	111542169	600.00	27352	334646024
46	6.539	248	2498691	4.149	80	1240475
47	600.00	22179	282707631	600.00	34120	451342876
48	600.00	19778	164139781	600.00	51875	649868285
49	600.00	15047	208250589	600.00	32003	508583392
50	0.021	0	24	0.021	0	29
51	0.076	0	192	0.054	0	209
52	0.055	0	521	0.062	0	525
53	0.366	6	2065	0.235	6	2266
54	0.144	0	1030	0.416	0	988
55	0.155	2	1699	0.333	2	1766
56	0.212	0	2499	0.284	0	2477
57	0.279	0	4201	0.226	0	4440
58	0.359	0	3124	0.303	0	3259
59	0.248	0	4518	0.257	0	4559
60	0.246	2	4170	0.301	0	3489
61	0.229	0	7555	0.223	1	8870
62	0.303	2	12954	0.416	3	13698
63	0.378	1	10738	0.603	1	10691
64	0.459	2	28017	0.638	12	51083
65	0.721	7	57191	0.888	6	54937
66	1.134	12	127865	1.490	6	107939
67	1.014	10	124827	0.877	13	117966
68	1.499	8	111993	1.572	6	114614
69	0.351	0	6613	0.310	0	6631
70	0.693	20	82813	0.882	16	87958
71	0.588	12	92658	0.932	12	98391

Table 1: