Implementation and Evaluation of a Compact-Table Propagator in Gecode

Linnea Ingmar
<linnea.ingmar.3244@student.uu.se>

The ASTRA Group
on Combinatorial Optimisation

Uppsala University

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Supervisor: Mats Carlsson (SICS)
Reviewer: Pierre Flener

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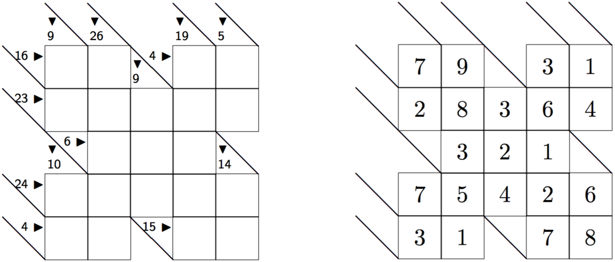
Outline

Outline

Kakuro puzzle



Assign the cells digits from 1 to 9 such that for each row and column:

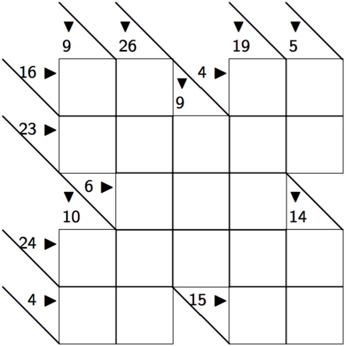
- digits are distinct, and
- the sum of the digits is equal to the *clue*

Kakuro puzzle as a constraint problem (1)

Variables One per cell.

Domains $\{1 \ldots 9\}$ for all variables.

Constraints For each row and column: *distinct* digits, and the *sum* of the digits is equal to the clue.

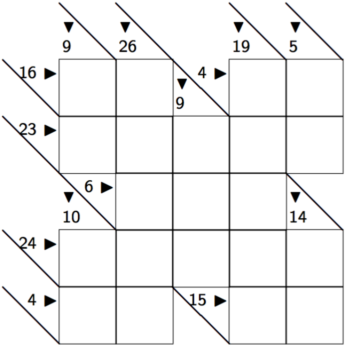


Kakuro puzzle as a constraint problem (2)

Variables One per cell.

Domains $\{1 ... 9\}$ for all variables.

Constraints For each row and column: state the *possible combinations of values* that the variables can take.



For an entry of size 2 and clue 4: $\langle 1, 3\rangle$ and $\langle 3, 1\rangle$ are the only combinations.

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Constraint problems (definition)

Definition (Constraint problem)

A **constraint satisfaction problem (CSP)** is a triple

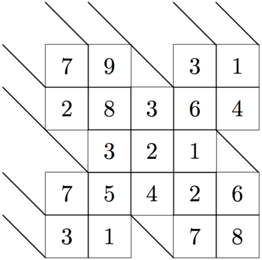
$$\langle V, D, C \rangle$$

where:

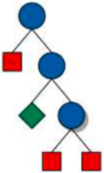
- $V = v_1, \ldots, v_n$ is a finite sequence of variables,
- $D = D_1, \ldots, D_n$ is a finite sequence of domains, that are possible values for the respective variable,
- $C = \{c_1, \ldots, c_m\}$ is a finite set of constraints, each on a subset of V. Express relations among the variables that have to be true.

Solving Constraint Problems

Solution A complete variable-value assignment that satisfy all the constraints.

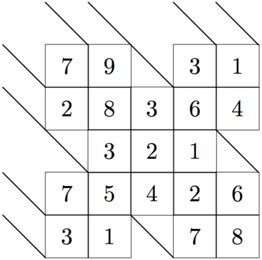


- Solutions are found by **search**
  - Propagation
  - Branching

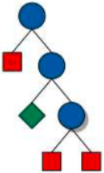


Solving Constraint Problems

Solution A complete variable-value assignment that satisfy all the constraints.



- Solutions are found by **search**
  - **Propagation**
  - Branching



Constraint Propagation

Important concepts:

- Constraint store
- Propagator

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Constraint Stores

Definition (Constraint store)

A **constraint store** *s* is a function mapping variables to domains:

$$s : variables \mapsto domains$$

s_0

$$\{x \mapsto \{1,2,3\}, \\ y \mapsto \{1,2,3\}\}$$

Propagators

Definition (Propagator)

A **propagator** p is a function mapping stores to stores:

$$p : store \mapsto store$$

- Implement constraints

$$\underset{s_0}{\{x \mapsto \{1,2,3\},\ y \mapsto \{1,2,3\}\}} \xrightarrow{p_{x<y}} \underset{s_1}{\{x \mapsto \{1,2,\cancel{3}\},\ y \mapsto \{1,2,3\}\}}$$

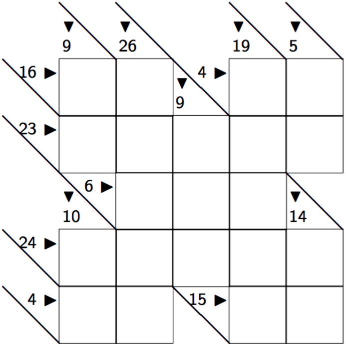
Constraint Propagation

$x_0 \in \{1,2,3,4,5,6,7,8,9\}$
$x_1 \in \{1,2,3,4,5,6,7,8,9\}$
$\dots$
$x_4 \in \{1,2,3,4,5,6,7,8,9\}$
$\dots$

x_0	x_1
7	9
9	7

x_0	x_4
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1

$\dots$



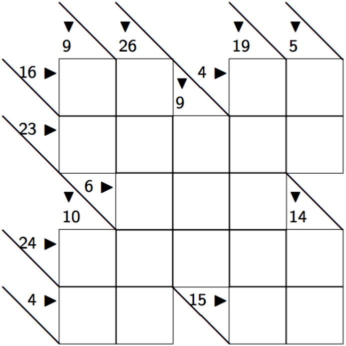
Constraint Propagation

$$x_0 \in \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},7,\cancel{8},9\}$$
$$x_1 \in \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},7,\cancel{8},9\}$$
$$\ldots$$
$$x_4 \in \{1,2,3,4,5,6,7,8,9\}$$
$$\ldots$$

x_0	x_1
7	9
9	7

x_0	x_4
~~1~~	~~8~~
~~2~~	~~7~~
~~3~~	~~6~~
~~4~~	~~5~~
~~5~~	~~4~~
~~6~~	~~3~~
7	2
~~8~~	~~1~~

...



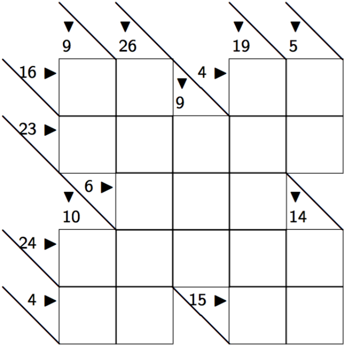
Constraint Propagation

$$x_0 \in \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},\mathbf{7},\cancel{8},\cancel{9}\}$$
$$x_1 \in \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},7,\cancel{8},9\}$$
$$\ldots$$
$$x_4 \in \{\cancel{1},\mathbf{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},\cancel{7},\cancel{8},\cancel{9}\}$$
$$\ldots$$

x_0	x_1
7	9
9	7

x_0	x_4
~~1~~	~~8~~
~~2~~	~~7~~
~~3~~	~~6~~
~~4~~	~~5~~
~~5~~	~~4~~
~~6~~	~~3~~
7	2
~~8~~	~~1~~

$\ldots$



Constraint Propagation

$$x_0 \in \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},\mathbf{7},\cancel{8},\cancel{9}\}$$
$$x_1 \in \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},\cancel{7},\cancel{8},\mathbf{9}\}$$
$$\ldots$$
$$x_4 \in \{\cancel{1},\mathbf{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},\cancel{7},\cancel{8},\cancel{9}\}$$
$$\ldots$$

x_0	x_1
7	9
~~9~~	~~7~~

x_0	x_4
~~1~~	~~8~~
~~2~~	~~7~~
~~3~~	~~6~~
~~4~~	~~5~~
~~5~~	~~4~~
~~6~~	~~3~~
7	2
~~8~~	~~1~~

$\ldots$

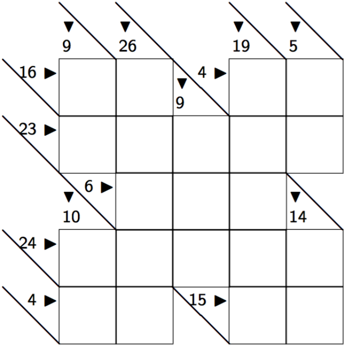


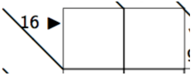
TABLE constraints

Definition (TABLE constraint)

A TABLE constraint lists the possible combinations of values that the variables can take as a sequence of n-tuples.

$$\text{TABLE}(\{x_0, x_1\}, [\langle 7,9 \rangle, \langle 9,7 \rangle])$$

x_0	x_1
7	9
9	7



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Gecode

Gecode (Generic Constraint Development Environment) is...

- ...a constraint solver (a software that solves constraint problems).
- ...written in C++, modular, extensible, and has state-of-the-art performance.
- ...supports the programming of new propagators.

Two existing propagators for the TABLE constraint

Compact Table

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Compact-Table

- A new propagation algorithm for the TABLE constraint.
- Published in a 2016 paper
- No attempt to implement it in Gecode (until now).

Linnea Ingmar | *linnea.ingmar.3244@student.uu.se*

Outline

The Compact-Table Algorithm

- **Initialisation**
- Variable modifications
- Filtering
- Putting it all together

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The Compact-Table Algorithm
Initialisation

$$\text{dom}(x_0) = \text{dom}(x_1) = \text{dom}(x_2) = \{1, 2, 3, 4\}$$

x_0	1	2	1	2	6	7	4	1	7	8	2	0	2	5	4
x_1	5	1	3	4	5	7	2	1	8	9	2	0	3	8	3
x_2	8	4	2	2	9	8	1	1	9	6	3	0	1	5	1

The Compact-Table Algorithm
Initialisation

$$\text{dom}(x_0) = \text{dom}(x_1) = \text{dom}(x_2) = \{1, 2, 3, 4\}$$

x_0	1	**2**	**1**	**2**	6	7	**4**	**1**	7	8	**2**	0	**2**	5	**4**
x_1	5	**1**	**3**	**4**	5	7	**2**	**1**	8	9	**2**	0	**3**	8	**3**
x_2	8	**4**	**2**	**2**	9	8	**1**	**1**	9	6	**3**	0	**1**	5	**1**

The Compact-Table Algorithm

Initialisation

$$\mathrm{dom}(x_0) = \mathrm{dom}(x_1) = \mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

	0	1	2	3	4	5	6	7
x_0	**2**	**1**	**2**	**4**	**1**	**2**	**2**	**4**
x_1	**1**	**3**	**4**	**2**	**1**	**2**	**3**	**3**
x_2	**4**	**2**	**2**	**1**	**1**	**3**	**1**	**1**

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The Compact-Table Algorithm
Initialisation

$$\mathrm{dom}(x_0) = \mathrm{dom}(x_1) = \mathrm{dom}(x_2) = \{1,2,3,4\}$$

$$\left.\boxed{\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}}\right\}\texttt{validTuples}$$

x_0	**2**	**1**	**2**	**4**	**1**	**2**	**2**	**4**
x_1	**1**	**3**	**4**	**2**	**1**	**2**	**3**	**3**
x_2	**4**	**2**	**2**	**1**	**1**	**3**	**1**	**1**
	0	1	2	3	4	5	6	7

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Background

The Compact-Table Algorithm

Evaluation

Conclusions

The Compact-Table Algorithm
Initialisation

$$\mathrm{dom}(x_0) = \mathrm{dom}(x_1) = \mathrm{dom}(x_2) = \{1,2,3,4\}$$

$$\left.\boxed{\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}}\right\}\texttt{validTuples}$$

$$\left.\begin{array}{c|cccccccc|} \langle x_0,1\rangle & 0 & \mathbf{1} & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\ \langle x_0,2\rangle & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \langle x_0,3\rangle & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & & & & & & & & \\ \langle x_2,4\rangle & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right\}\texttt{supports}$$

	0	1	2	3	4	5	6	7
x_0	**2**	**1**	**2**	**4**	**1**	**2**	**2**	**4**
x_1	**1**	**3**	**4**	**2**	**1**	**2**	**3**	**3**
x_2	**4**	**2**	**2**	**1**	**1**	**3**	**1**	**1**

Linnea Ingmar | *linnea.ingmar.3244@student.uu.se*

The Compact-Table Algorithm
Initialisation

$$\mathrm{dom}(x_0) = \{1, 2, \cancel{3}, 4\}$$
$$\mathrm{dom}(x_1) = \mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

validTuples	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

supports:

$\langle x_0, 1\rangle$	0	**1**	0	0	**1**	0	0	0
$\langle x_0, 2\rangle$	**1**	0	**1**	0	0	**1**	**1**	0
$\langle x_0, 3\rangle$	0	0	0	0	0	0	0	0
$\dots$								
$\langle x_2, 4\rangle$	**1**	0	0	0	0	0	0	0

x_0	**2**	**1**	**2**	**4**	**1**	**2**	**2**	**4**
x_1	**1**	**3**	**4**	**2**	**1**	**2**	**3**	**3**
x_2	**4**	**2**	**2**	**1**	**1**	**3**	**1**	**1**
	0	1	2	3	4	5	6	7

The Compact-Table Algorithm

- Initialisation
- **Variable modifications**
- Filtering
- Putting it all together

The Compact-Table Algorithm

When a variable is modified

Background

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$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

The Compact-Table Algorithm
When a variable is modified

$$\begin{array}{r|cccccccc|}
\texttt{words} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\texttt{mask} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \Bigg\} \texttt{validTuples}$$

$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

The Compact-Table Algorithm

When a variable is modified

`words`	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	`validTuples`
`mask`	0	0	0	0	0	0	0	0	

`supports`$[x_1,3]$	0	**1**	0	0	0	0	**1**	**1**
`supports`$[x_1,4]$	0	0	**1**	0	0	0	0	0

$$\mathrm{dom}(x_0) = \{1,2,4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1},\cancel{2},3,4\}$$
$$\mathrm{dom}(x_2) = \{1,2,3,4\}$$

The Compact-Table Algorithm

When a variable is modified

`words`	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	$\Big\}$ `validTuples`
`mask`	0	0	0	0	0	0	0	0	

$\texttt{supports}[x_1, 3]$	0	**1**	0	0	0	0	**1**	**1**
$\texttt{supports}[x_1, 4]$	0	0	**1**	0	0	0	0	0

$$\texttt{mask} = \texttt{supports}[x_1, 3] \mid \texttt{supports}[x_1, 4]$$

$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

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The Compact-Table Algorithm

When a variable is modified

`words`	1	1	1	1	1	1	1	1	validTuples
`mask`	0	1	1	0	0	0	1	1	

`supports`$[x_1, 3]$	0	1	0	0	0	0	1	1
`supports`$[x_1, 4]$	0	0	1	0	0	0	0	0

$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

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The Compact-Table Algorithm

When a variable is modified

`words`	1	1	1	1	1	1	1	1
`mask`	0	1	1	0	0	0	1	1

`validTuples`

`supports`$[x_1, 3]$	0	1	0	0	0	0	1	1
`supports`$[x_1, 4]$	0	0	1	0	0	0	0	0

`words = words & mask`

$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

The Compact-Table Algorithm

When a variable is modified

`words`	0	1	1	0	0	0	1	1	`validTuples`
`mask`	0	1	1	0	0	0	1	1	

$\texttt{supports}[x_1, 3]$	0	1	0	0	0	0	1	1
$\texttt{supports}[x_1, 4]$	0	0	1	0	0	0	0	0

$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

The Compact-Table Algorithm

When a variable is modified

Background

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`words`	0	1	1	0	0	0	1	1
`mask`	0	1	1	0	0	0	1	1

$\}$ `validTuples`

`supports`$[x_1, 3]$	0	1	0	0	0	0	1	1
`supports`$[x_1, 4]$	0	0	1	0	0	0	0	0

x_0	2	1	2	4	1	2	2	4
x_1	1	3	4	2	1	2	3	3
x_2	4	2	2	1	1	3	1	1

$$\mathrm{dom}(x_0) = \{1, 2, 4\}$$
$$\mathrm{dom}(x_1) = \{\cancel{1}, \cancel{2}, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

The Compact-Table Algorithm

When a variable is modified

PROCEDURE UPDATETABLE(s: store, x: variable)

1: `validTuples`.clearMask()
2: **foreach** $a \in s(x)$ **do**
3:     `validTuples`.addToMask(`supports`$[x, a]$)
4: `validTuples`.intersectWithMask()

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The Compact-Table Algorithm

- Initialisation
- Variable modifications
- **Filtering**
- Putting it all together

Compact-Table Algorithm
Filtering

- Intersect every support entry with `validTuples`
- Remove value if intersection is empty

`validTuples`:

`words`	0	**1**	**1**	0	0	0	**1**	**1**

&

$\langle x_2, 1\rangle$	0	0	0	**1**	**1**	0	**1**	**1**
$\langle x_2, 2\rangle$	0	**1**	**1**	0	0	0	0	0
$\langle x_2, 3\rangle$	0	0	0	0	0	**1**	0	0
$\langle x_2, 4\rangle$	**1**	0	0	0	0	0	0	0

$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

Compact-Table Algorithm
Filtering

- Intersect every support entry with `validTuples`
- Remove value if intersection is empty

`validTuples`:

`words`	0	1	1	0	0	0	1	1

&

$\langle x_2, 1 \rangle$	0	0	0	1	1	0	1	1
$\langle x_2, 2 \rangle$	0	1	1	0	0	0	0	0
$\langle x_2, 3 \rangle$	0	0	0	0	0	1	0	0
$\langle x_2, 4 \rangle$	1	0	0	0	0	0	0	0

$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

Compact-Table Algorithm
Filtering

- Intersect every support entry with `validTuples`
- Remove value if intersection is empty

`validTuples`:

`words`	0	1	1	0	0	0	1	1

&

$\langle x_2, 1\rangle$	0	0	0	1	1	0	1	1
$\langle x_2, 2\rangle$	0	1	1	0	0	0	0	0
$\langle x_2, 3\rangle$	0	0	0	0	0	1	0	0
$\langle x_2, 4\rangle$	1	0	0	0	0	0	0	0

$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

Compact-Table Algorithm
Filtering

- Intersect every support entry with `validTuples`
- Remove value if intersection is empty

`validTuples:`

`words`	0	1	1	0	0	0	1	1

&

$\langle x_2, 1\rangle$	0	0	0	1	1	0	1	1
$\langle x_2, 2\rangle$	0	1	1	0	0	0	0	0
$\langle x_2, 3\rangle$	0	0	0	0	0	1	0	0
$\langle x_2, 4\rangle$	1	0	0	0	0	0	0	0

$$\mathrm{dom}(x_2) = \{1,2,3,4\}$$

Compact-Table Algorithm
Filtering

- Intersect every support entry with `validTuples`
- Remove value if intersection is empty

`validTuples`:

`words`	0	1	1	0	0	0	1	1

&

$\langle x_2, 1\rangle$	0	0	0	1	1	0	1	1
$\langle x_2, 2\rangle$	0	1	1	0	0	0	0	0
$\langle x_2, 3\rangle$	0	0	0	0	0	1	0	0
$\langle x_2, 4\rangle$	1	0	0	0	0	0	0	0

$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$

Compact-Table Algorithm

Filtering

- Intersect every support entry with `validTuples`
- Remove value if intersection is empty

`validTuples:`

`words`	0	1	1	0	0	0	1	1

&

$\langle x_2, 1\rangle$	0	0	0	1	1	0	1	1
$\langle x_2, 2\rangle$	0	1	1	0	0	0	0	0
$\langle x_2, 3\rangle$	0	0	0	0	0	1	0	0
$\langle x_2, 4\rangle$	1	0	0	0	0	0	0	0

$$\mathrm{dom}(x_2) = \{1, 2, \cancel{3}, \cancel{4}\}$$

Compact-Table Algorithm

Filtering out values

PROCEDURE $\textsc{FilterDomains}(s)$: store

foreach $x \in s$ such that $|s(x)| > 1$ **do**
  foreach $a \in s(x)$ **do**
    $index \leftarrow \mathtt{residues}[x, a]$ // remembered last index
    if $\mathtt{validTuples}[index]$ & $\mathtt{supports}[x, a][index] = 0$ **then**
      $index \leftarrow \mathtt{validTuples}.\text{intersectIndex}(\mathtt{supports}[x, a])$
      if $index \neq -1$ **then**
        $\mathtt{residues}[x, a] \leftarrow index$
      else
        $s \leftarrow s[x \mapsto s(x) \setminus \{a\}]$
return s

The Compact-Table Algorithm

- Initialisation
- Variable modifications
- Filtering
- **Putting it all together**

The Compact-Table Algorithm

Putting it all together

PROCEDURE COMPACTTABLE(s : store) : $\langle$*StatusMsg*, store$\rangle$

```
if the propagator is being posted then
    s ← INITIALISECT(s, T_0)
    if s = ∅ then
        return ⟨FAIL, ∅⟩
else
    foreach variable x ∈ s whose domain has changed since last time do
        UPDATETABLE(s, x)
        if validTuples.isEmpty() then
            return ⟨FAIL, ∅⟩
    if validTuples has changed since last time then
        s ← FILTERDOMAINS(s)
if there is at most one unassigned variable left then
    return ⟨SUBSUMED, s⟩
else
    return ⟨FIX, s⟩
```

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Outline

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