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1)

(a)

$$\sigma(x) = \frac{1}{1+e^{-x}} \qquad -\frac{\partial e^{-x}}{\partial x} = e^{-x}$$

$$\frac{\partial \sigma(x)}{\partial x} = \frac{0\cdot(1+e^{-x}) - 1(0-e^{-x})}{(1+e^{-x})^2} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1}{(1+e^{-x})}\frac{e^{-x}}{(1+e^{-x})} = \sigma(x)(1-\sigma(x))$$

(b)

$$\sigma(-x) = \frac{1}{1+e^{x}} = \frac{\frac{1}{e^{x}}}{\frac{1}{e^{x}}+\frac{e^{x}}{e^{x}}} = \frac{e^{-x}}{1+e^{-x}}$$

$$= 1-\sigma(x)$$

(c)

$$\sigma(x) = \frac{1}{1+e^{-x}} = y$$

$$\Rightarrow \frac{1}{y} = 1+e^{-x}$$

$$\Rightarrow e^{-x} = \frac{1-y}{y}$$

$$\Rightarrow -x = \ln\left(\frac{1-y}{y}\right)$$

$$\Rightarrow x = \ln\left(\frac{y}{1-y}\right)$$

2)

$\hat{y}_n = \sigma(w^T \phi_n)$

要求gradient，先求一個變數導數

$$\frac{\partial L}{\partial w_i} = \left(\frac{\partial L}{\partial \hat{y}_n}\right) \cdot \left(\frac{\partial \hat{y}_n}{\partial w_i}\right)$$

分開處理 ① $\frac{\partial L}{\partial \hat{y}_n} = -\frac{y_n}{\hat{y}_n} + \frac{(1-y_n)}{(1-\hat{y}_n)}$

from problem 1

② $$\frac{\partial \hat{y}_n}{\partial w_i} = \sigma(w^T\phi_n)(1-\sigma(w^T\phi_n)) \frac{\partial w^T\phi_n}{\partial w_i}$$

$$= \hat{y}_n(1-\hat{y}_n)\phi_n$$

合併

$$\frac{\partial L}{\partial w_i} = \left(\frac{\hat{y}_n - y_n}{\hat{y}_n(1-\hat{y}_n)}\right) \cdot \left(\hat{y}_n(1-\hat{y}_n)\frac{\partial w^T\phi_n}{\partial w_i}\right)$$

$$= (\hat{y}_n - y_n)\phi_n \qquad i = 0, 1, \dots n$$

所以 $\nabla L(x) = \sum_{n=1}^{N} (\hat{y}_n - y_n)\phi_n$ #

3)

(a)

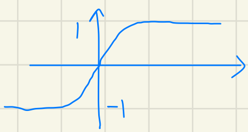
已知 output $-1 \le \hat{y}_n \le 1$ 且 label $\{-1,1\}$
,如果要計算 loss function, 且值在 $(-1, 1)$,
最適合的應該 Mean square error,

$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i-\hat{y}_i)^2$,因為對 MSE
$\hat{y}$ 可以在任意範圍
如果用 Cross-entropy loss, 因為有 log
存在, 會導致 log(負數) ,不符合數學式.
∴ 選擇 MSE.

(b)

要限定 $-1 \le \hat{y}_n \le 1$ 之間, 選擇 tanh 做為 activation function

$$\tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$



$$\lim_{x\to\infty}\tanh(x) = \frac{e^{\infty}-e^{-\infty}}{e^{\infty}+e^{-\infty}} = 1$$

$$\lim_{x\to-\infty}\tanh(x) = \frac{e^{-\infty}-e^{\infty}}{e^{-\infty}+e^{\infty}} = -1$$

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