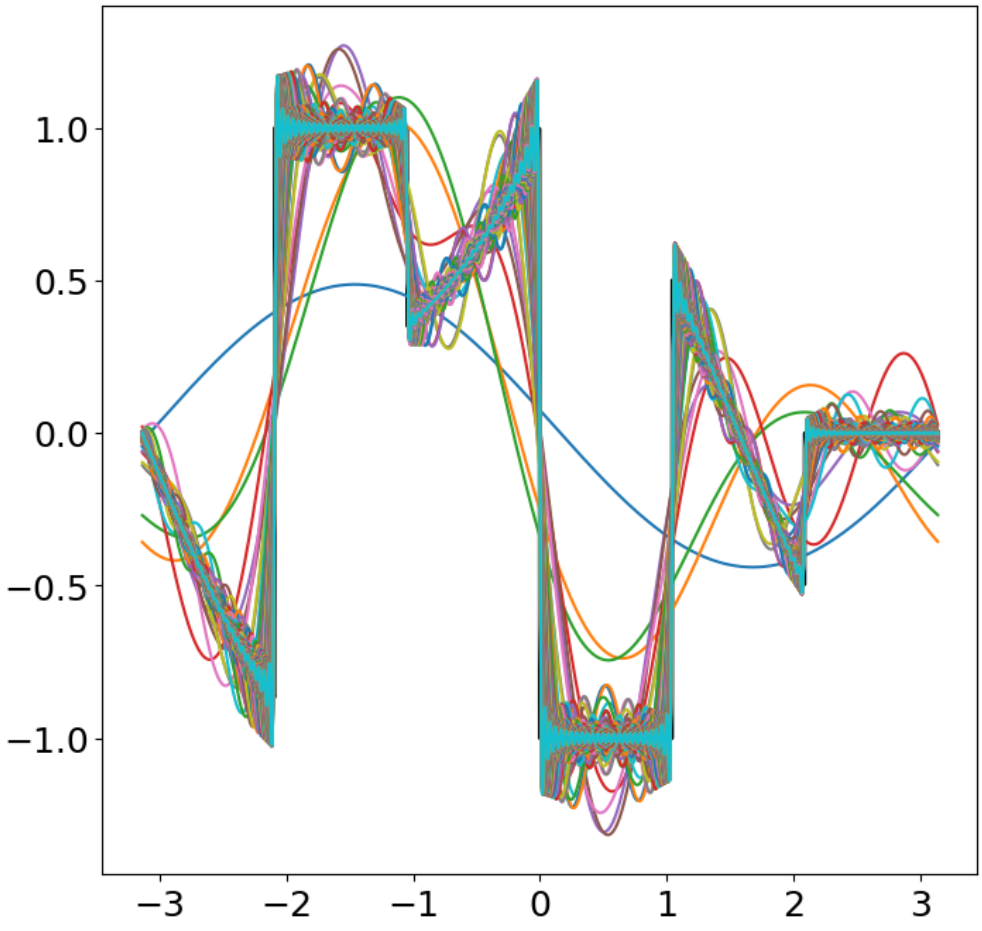
Exam2

1. (30%)Given a piecewise function as follow:

$$f(x)=\begin{cases}\sin(x) & \text{for } -\pi \le x < -\frac{2\pi}{3} \\ 1 & \text{for } -\frac{2\pi}{3} \le x < -\frac{\pi}{3} \\ e^x & \text{for } -\frac{\pi}{3} \le x < 0 \\ -1 & \text{for } 0 \le x < \frac{\pi}{3} \\ \cos(x) & \text{for } \frac{\pi}{3} \le x < \frac{2\pi}{3} \\ 0 & \text{for } \frac{2\pi}{3} \le x \le \pi \end{cases}.$$

Use Fourier series with 150 cosine and sine functions to approximate the function and the result should look like the following picture



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2. Perform denoising on a grayscale `dog.png`. The image is loaded, converted to grayscale, and then Gaussian noise is added. The code should then display the following:

- (10%)The original noisy image given by:

$$B_{noise} = B + 200 \times N(0, 1)$$

- (5%)The Fourier transform of the noisy image on a log scale.
- (5%)The Fourier transform of the image after being filtered, also on a log scale.
- (5%)The denoised image.

3. (15%)Compress `dog.png` using various thresholds to keep 10%, 3%, and 0.1% of the largest wavelet coefficients.

4. You are given a dataset `data.csv` that includes 50 samples of x and y values. However, there are few outliers in the dataset. You are asked to use L1 optimization to reject outliers and find the best line that fits the data points.

- (5%)Load the dataset `data.csv` and visualize the scatter plot of the data points.
- (15%)Implement the L1 optimization to find the best line that fits the data points.
- (5%)Plot the data points and the line that is affected by outliers in the dataset. Make sure to label the axes and the title of the plot.
- (5%)Plot the data points and the best line that fits the remaining data points after rejecting the outlier. Make sure to label the axes and the title of the plot.