Problem 1

For this problem, the normal distribution and generalized T distribution are fitted on the given data, the PDFs are shown and VaRs and Expected Shortfalls are calculated.

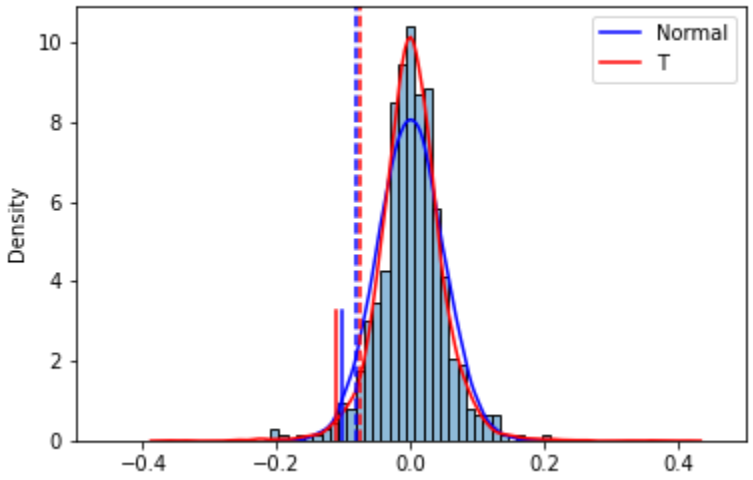

Figure 1: PDFs, VaRs, and ESs for Normal and Generalized T Distribution

By calculation, the VaR and ES for fitted normal distribution are roughly 0.0817 and 0.1023 respectively. Those values for fitted generalized T distribution are approximately 0.0747, and 0.1111. There are 10,000 used for the simulation. Thus, even though the generalized T simulation looks similar to the normal simulation, it still has a higher kurtosis (from the PDFs). As a result, it is reasonable that the 5% VaR level of the T distribution is lower than that of the normal distribution because it has a thinner tail, and therefore a higher ES.

Problem 2

All functions perform as expected.

Problem 3

In this problem, I first fit the data of stock returns in a t-distribution to get the CDF and parameters. Then, the spearman correlation for the fitted CDFs and PCA simulation method are used to simulate a series of CDFs. FInally, simulated returns are drawn from the simulated CDFs. From there, the portfolio values and thus the VaR and ES are calculated.

Portfolio	VaR	ES
A	18242.4365	29137.4060
B	11337.0634	15782.4641
C	23752.3975	37556.2955
Total	50682.2674	78666.7443

Table 1: VaR and ES for Portfolios

Since VaR represents the minimum value of loss on an adverse event and ES represents the expected loss, it makes sense that ES is larger than the VaR. Comparing to the VaRs calculated using Delta Normal VaR method (Portfolio A: $15526.87, Portfolio B: $8090.29, Portfolio C: $18726.90, Total: $42344.06), the VaRs using generalized T model are higher. This is probably because that t-distribution fit the data of returns better than normal distribution.