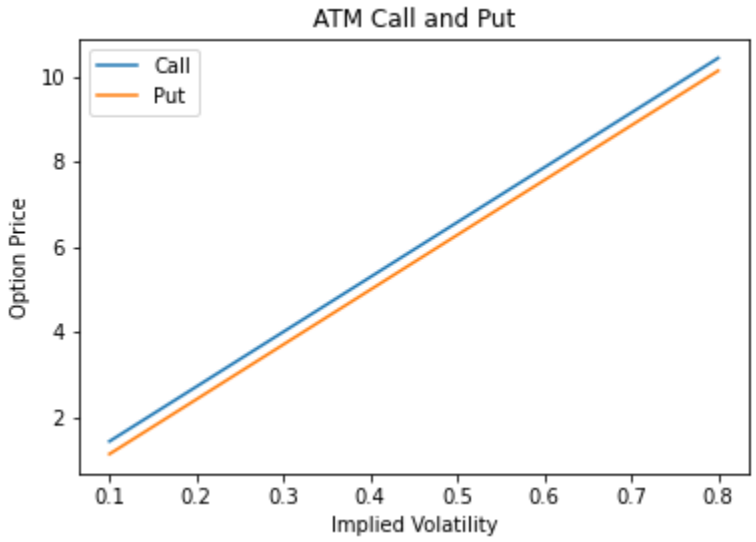
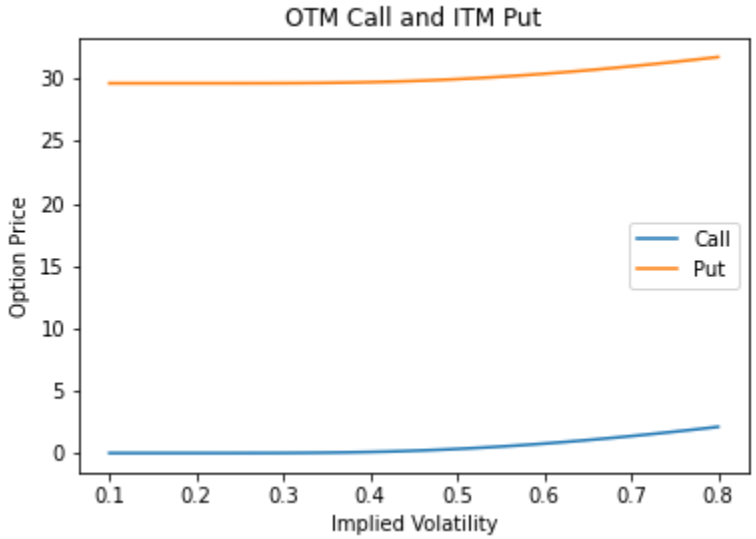
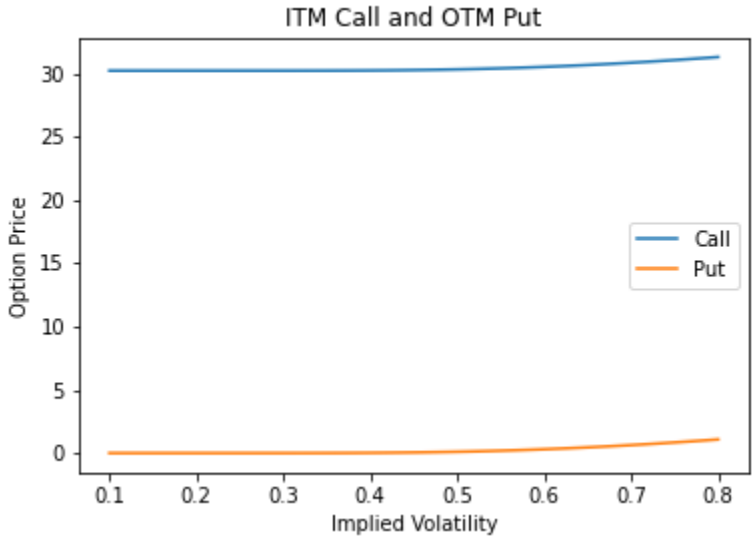
Problem 1

In this problem, we are asked to calculate the time to maturity and plot the value of a call and put option over a range of implied volatility (10%-80%). Both options have the underlying price of 165$, current date 03/03/2023 and expiration date 03/17/2023, risk free rate of 5.25%, and continuous compounding coupon of 0.53%. The time to maturity is 14 days. The following graphs show the values of the call and put options over the implied volatility from 10% to 80%, for at-the-money, in-the-money, and out-of-the-money.





(Strike price 195)



(Strike price 135)

It's expected that as the implied volatility increases, the value of options will increase. For at-the-money options, the reason why the value of the call option is higher than the put options is probably that the owner of a call option has the right to buy the stock at the expiration but potentially lose the underlying's value where its dividend is distributed.

It's easy to observe that when the strike price is 195, which is in-the-money for put and out-of-the-money for call, the value of call option is closer to zero because the payoff of the call is zero and the value of put option is much more positive, and vice versa for when the strike price is 135.

Implied volatility reflects the market's expected volatility for the underlying asset, as implied by the price of the option. When the demand is high for options on a particular underlying asset, the price of those options increase, leading to an increase in implied volatility.

Furthermore,supply and demand for the underlying asset can also affect implied volatility in the sense that changes in demand for the underlying asset may lead to changes in the price of the asset and thus corresponding changes in the implied volatility of its options.

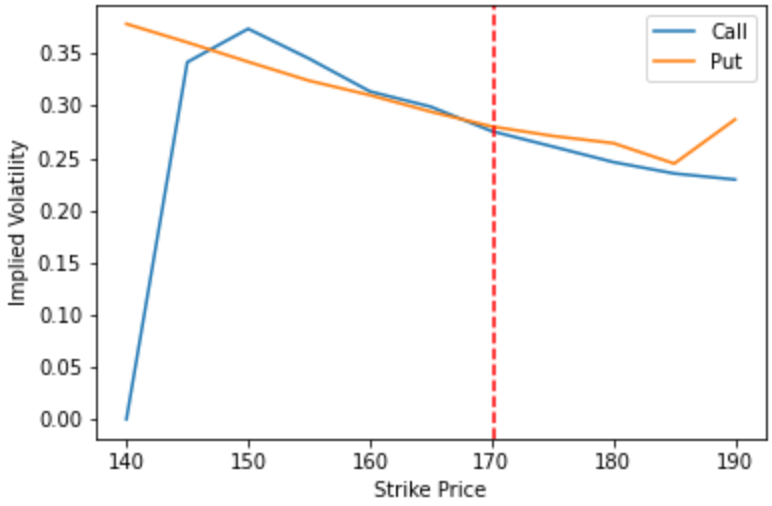
Problem 2

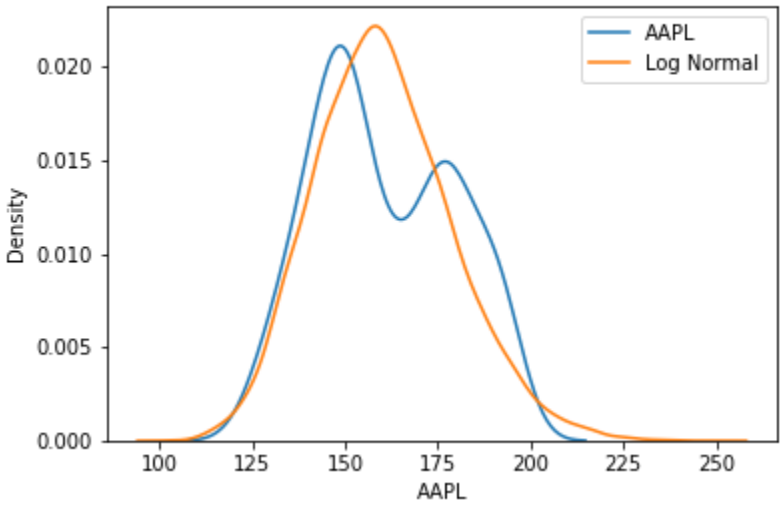
In this problem we are given a set of AAPL options, given the current price of AAPL stock is 170.15, current date as 10/30/2023, risk free rate as 5.25%, the dividend rate is 0.57%, and calculate the implied volatility for each option.

A graph is plotted as shown below. These are not typical volatility smile curves. The minimum value of implied volatility does not occur at which the strike price equals the current underlying price. This may reflect a market expectation that the underlying stock price of AAPL may increase. On the other hand , the implied volatility is high for in-the-money call options and low for out-of-the-money call options, with one exception. This may also because the market is expecting an increase in the AAPL stock price so the in-the-money call is having a higher implied volatility.

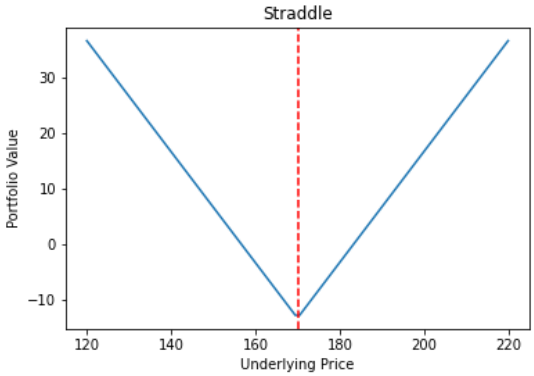
One exception from the volatility smile in this graph is that the implied volatility for some deep in-the-money call options are lower than the ATM and OTM call options. This may be due to several reasons. One of them is that deep ITM call options have a delta close to 1, meaning that the option price moves almost unit-by-unit with the underlying asset. Because of this, there's less uncertainty or volatility about the intrinsic value of the option as the extrinsic value is minimal. The option price is less sensitive to changes in volatility since it's primarily composed of intrinsic value.

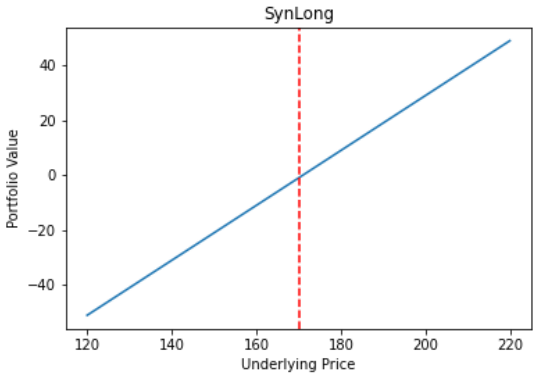
In addition, the black-scholes valuation model assumes that the volatility remains the same and the prices of the underlying follows a log-normal distribution. Both assumptions are hard to be satisfied. The first one is not necessarily true, and from the second graph the prices of AAPL does not strictly follow the log-normal distribution.

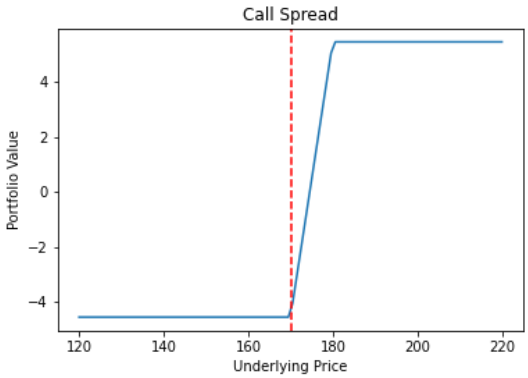

Call
Put
Implied Volatility
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
140
150
160
170
180
190
Strike Price

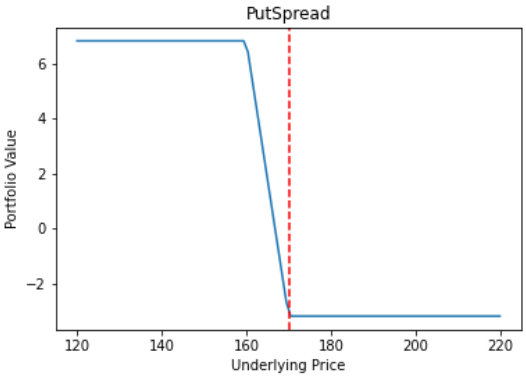

AAPL
Log Normal
Density
0.020
0.015
0.010
0.005
0.000
100
125
150
175
200
225
250
AAPL

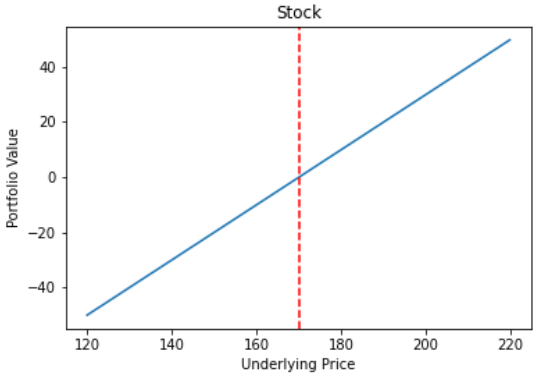
Problem 3

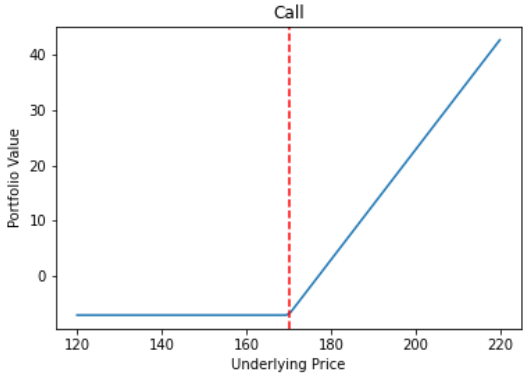

Straddle
Portfolio Value
Underlying Price
30
20
10
0
−10
120
140
160
180
200
220

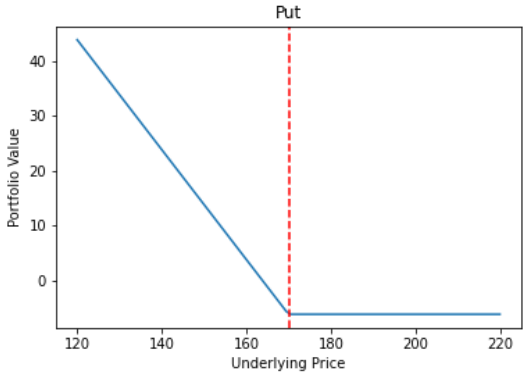

SynLong
Portfolio Value
Underlying Price
40
20
0
−20
−40
120
140
160
180
200
220

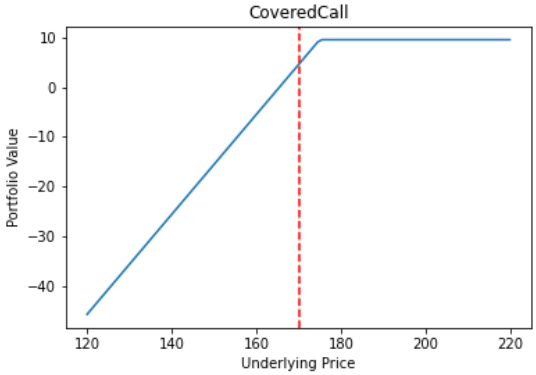

Call Spread
Portfolio Value
Underlying Price
4
2
0
−2
−4
120
140
160
180
200
220

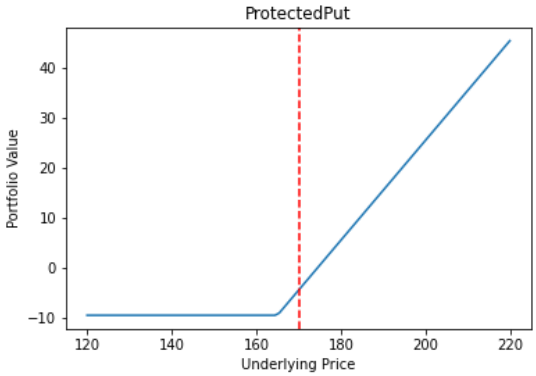

PutSpread
Portfolio Value
Underlying Price
6
4
2
0
−2
120
140
160
180
200
220


Stock
Portfolio Value
Underlying Price
40
20
0
−20
−40
120
140
160
180
200
220


Call
Portfolio Value
Underlying Price
40
30
20
10
0
120
140
160
180
200
220








```
Straddle         0.552506
SynLong         -0.250180
CallSpread       3.790313
PutSpread        0.234849
Stock            0.095414
Call             0.151163
Put              0.401343
CoveredCall     -1.860412
ProtectedPut     0.378989
dtype: float64
```

```
Straddle        10.697175
SynLong          0.577707
CallSpread       1.008630
PutSpread        2.487403
Stock           15.273265
Call             5.586934
Put              5.110241
CoveredCall     29.949773
ProtectedPut     7.891118
dtype: float64
```

```
Straddle        11.580706
SynLong          0.690100
CallSpread       1.131866
PutSpread        2.726893
Stock           18.795021
Call             6.093218
Put              5.487488
CoveredCall     36.991291
ProtectedPut     8.643038
dtype: float64
```

From the above graph show the value of the portfolio against the underlying price. The **straddle** strategy makes money when the underlying price goes away from the current price. The **synlong** strategy replicates the performance of longing the underlying asset but with less capital used. The **call spread** strategy makes money when the price goes up by more than the difference of the two options, with limited profit and loss and thus limited risk. The **put spread** is the mirror reflection of the call spread. The **long stock** strategy makes money when the stock price goes up with the risk of your whole capital. The **long call** strategy makes money when the price of the underlying goes up by more than the call price with limited loss. The **long put** strategy makes money when the price of the underlying goes down by more than the put price with limited loss and profit. The **covered call** is a neutral strategy and is used when you believe the price of the underlying asset does not fluctuate much, and in this case you can hold the

underlying long and short the call option on the underlying to generate income via option premium. The **protective put** strategy is to protect the loss of an investor's long position in an underlying in the case when the price of the underlying declines by capping the loss.

The three columns in the last set of pictures show the mean, VaR, and ES respectively. The results closely correspond to the previous discussion.