Problem 1

- Current Stock Price $151.03
- Strike Price $165
- Current Date 03/13/2022
- Options Expiration Date 04/15/2022
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Implement the closed form greeks for GBSM. Implement a finite difference derivative calculation.
Compare the values between the two methods for both a call and a put.
Implement the binomial tree valuation for American options with and without discrete dividends.
Assume the stock above:

- Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put. Calculate the Greeks of each.
What is the sensitivity of the put and call to a change in the dividend amount?

Using the Black-Scholes model to calculate the greeks for call and put option, the results are shown below.

	Call		Put	
	Close form	Finite difference derivative	Closed form	Finite difference derivative
Delta	0.083	0.0829	-0.9169	-0.9165
Gamma	0.01683	0.01682	0.01683	0.01682
Vega	6.9420	6.9386	6.9420	6.9386
Theta	-8.1265	-8.1263	-1.9409	-1.9407
Rho	-0.03035	-0.03035	-1.2427	-1.2427
Carry rho	1.13295	1.13295	-12.51527	-12.51527

It can be seen from the above results that the closed form calculation and the finite difference derivative calculator for Greeks are roughly the same. Some numbers shown in the above table are the same because they are in five decimal places but are actually not exactly the same. Nevertheless, it still indicates that the values are very close. In addition, in this problem I replaced the formula for rho in the notes with the formula where b = r-q.

Using the binomial tree valuation for American options with and without discrete dividend payout to calculate the prices, greeks, and sensitivity to dividend amount, the results are shown below.

	Call	Put
Price without div	0.34204	14.02023
Price with div	0.29816	14.55911
Delta	0.06940	-0.93843
Gamma	0.01887	0.01769
Vega	6.14319	5.66412
Theta	-7.27652	-0.46564
Rho	0.94268	-12.40758
Sensitivity to div amount	-0.025	0.941

Here we see that with higher dividend the call option has a lower price and the put option has a higher price. This is because when there's a dividend paid, the underlying stock loses its intrinsic value and thus harms the benefit of the owner of the call option, and vice versa for the owner of the put option.

Problem 2

Using the options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming :

- American Options
- Current Date 03/03/2023
- Current AAPL price is 165
- Risk Free Rate of 4.25%
- Dividend Payment of $1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return.
Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.
Calculate VaR and ES using Delta-Normal.
Present all VaR and ES values a $ loss, not percentages.
Compare these results to last week's results.

The below graph shows the Mean, VaR, and ES using the normal distribution to simulate the 10-day ahead returns. The first group of results are from last week, the second group are from this week, and the last table shows the results obtained from the delta-normal method.

I think there are some serious problems with my simulation functions and the pd.grouby.sum methods since my results from last week are not that correct, and the results from the normal simulation this week are even more absurd.

However, in theory I think the results generated from the AR model (last week) should be a more accurate simulation than the normal simulation, especially with only 100 simulations and 25 depth of the binomial tree. Moreover, if the delta-neutral strategy is used, the risk should be reduced.

```
Straddle          0.552506
SynLong          -0.250180
CallSpread        3.790313
PutSpread         0.234849
Stock             0.095414
Call              0.151163
Put               0.401343
CoveredCall      -1.860412
ProtectedPut      0.378989
dtype: float64
```

```
Straddle          10.697175
SynLong            0.577707
CallSpread         1.008630
PutSpread          2.487403
Stock             15.273265
Call               5.586934
Put                5.110241
CoveredCall       29.949773
ProtectedPut       7.891118
dtype: float64
```

```
Straddle          11.580706
SynLong            0.690100
CallSpread         1.131866
PutSpread          2.726893
Stock             18.795021
Call               6.093218
Put                5.487488
CoveredCall       36.991291
ProtectedPut       8.643038
dtype: float64
```

```
Straddle           7.353008
SynLong            7.362989
CallSpread     -9382.310680
PutSpread         -0.071006
Stock             12.638441
Call               7.357998
Put               -0.004991
CoveredCall        7.015403
ProtectedPut      12.622878
dtype: float64
```

```
Straddle           0.663364
SynLong           10.625890
CallSpread     55854.213491
PutSpread          1.855309
Stock              1.802739
Call               5.795571
Put                3.302657
CoveredCall       -2.247261
ProtectedPut      -1.764296
dtype: float64
```

```
Straddle           0.904176
SynLong           13.874528
CallSpread     75458.951341
PutSpread          2.182845
Stock              6.297929
Call               6.636909
Put                3.783170
CoveredCall        2.247929
ProtectedPut       0.662133
dtype: float64
```

Portfolio		Mean	VaR	ES
Call	0		12.693933	15.918705
CallSpread	0		4.305794	5.399639
CoveredCall	0		4.755691	5.963828
ProtectedPut	0		12.649392	15.862847
Put	0		2.898182	3.634437
PutSpread	0		1.411042	1.769504
Stock	0		15.490022	19.425112
Straddle	0		9.795751	12.284267
SynLong	0		15.592115	19.553142

Problem 3

Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the following stocks.
AAPL FB UNH MA
MSFT NVDA HD PFE
AMZN BRK-B PG XOM
TSLA JPM V DIS
GOOGL JNJ BAC CSCO
Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.
Based on the past 10 years of factor returns, find the expected annual return of each stock.
Construct an annual covariance matrix for the 10 stocks.
Assume the risk free rate is 0.0425. Find the super efficient portfolio.

Using the Fama-French four factor model and the data of the past 10 years, the expected annual returns of these 20 stocks are as follows。

Symbol	Return	Symbol	Return	Symbol	Return	Symbol	Return
AAPL	0.263543	META	0.238417	UNH	0.492107	MA	0.15057
MSFT	0.228763	NVDA	-0.056238	HD	-0.00817	PFE	0.253431
AMZN	0.041766	BRK-B	0.154862	PG	0.017089	XOM	0.632155
TSLA	0.433966	JPM	-0.128065	V	0.147108	DIS	-0.382507
GOOGL	0.006573	JNJ	0.290983	BAC	-0.254746	CSCO	0.026509

The weights of the stocks in the super efficient portfolio are as follows.

Symbol	Weight %	Symbol	Weight %	Symbol	Weight %	Symbol	Weight %
AAPL	0.0	META	0.0	UNH	34.82	MA	0.0
MSFT	0.0	NVDA	0.0	HD	0.0	PFE	0.0
AMZN	0.0	BRK-B	0.0	PG	0.0	XOM	31.17

TSLA	0.31	JPM	0.0	V	0.0	DIS	0.0
GOOGL	0.0	JNJ	33.77	BAC	0.0	CSCO	0.0

The Sharpe ratio of the portfolio is 2.26.