Antisymmetric and Symmetric relations

June 16, 2016 11:08 PM

Here's a way to think about symmetry and antisymmetry that some people find helpful. A relation R on a set A has a *directed graph* (or *digraph*) G_R: the vertices of G_R are the elements of A, and for any $a, b \in A$ there is an edge in G_R from a to b if and only if $\langle a, b \rangle \in R$. Think of the edges of G_R as streets. The properties of symmetry, antisymmetry, and reflexivity have very simple interpretations in these terms:

- R is reflexive if and only if there is a loop at every vertex. (A loop is an edge from some vertex to itself.)
- R is symmetric if and only if every edge in G_R is a two-way street or a loop. Equivalently, G_R has no one-way streets between distinct vertices.
- R is antisymmetric if and only every edge of G_R is either a one-way street or a loop. Equivalently, G_R has no two-way streets between distinct vertices.

This makes it clear that if G_R has only loops, R is both symmetric and antisymmetric: R is symmetric because G_R has no one-way streets between distinct vertices, and R is antisymmetric because G_R has no two-way streets between distinct vertices.

To make a relation that is neither symmetric nor antisymmetric, just find a digraph that has both a one-way street and a two-way street, like this one:

$$0 \longrightarrow 1 \longleftrightarrow 2$$

It corresponds to the relation $R = \{\langle 0,1 \rangle, \langle 1,2 \rangle, \langle 2,1 \rangle\}$. on $A = \{0,1,2\}$.

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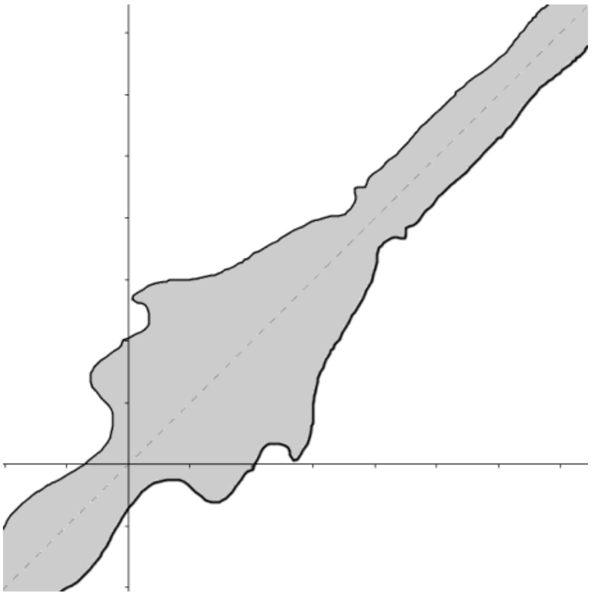
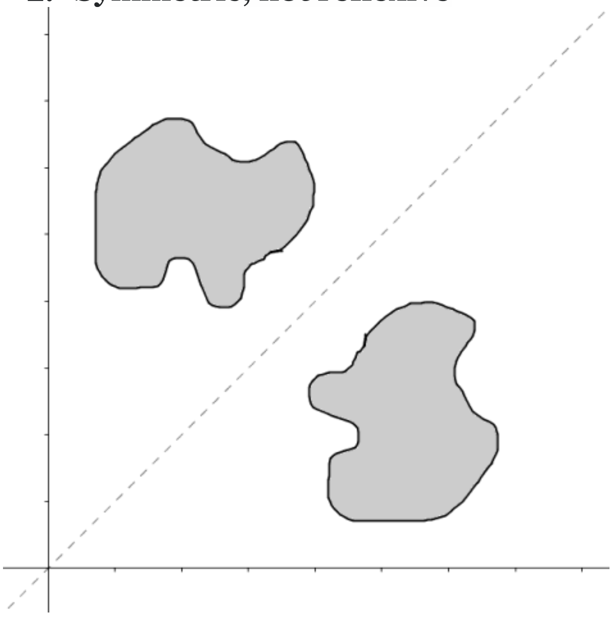
http://math.stackexchange.com/questions/225808/is-my-understanding-of-antisymmetric-and-symmetric-relations-correct

Here are a few relations on subsets of $\mathbb{R}$, represented as subsets of $\mathbb{R}^2$. The dotted line represents $\{(x, y) \in \mathbb{R}^2 \mid y = x\}$.

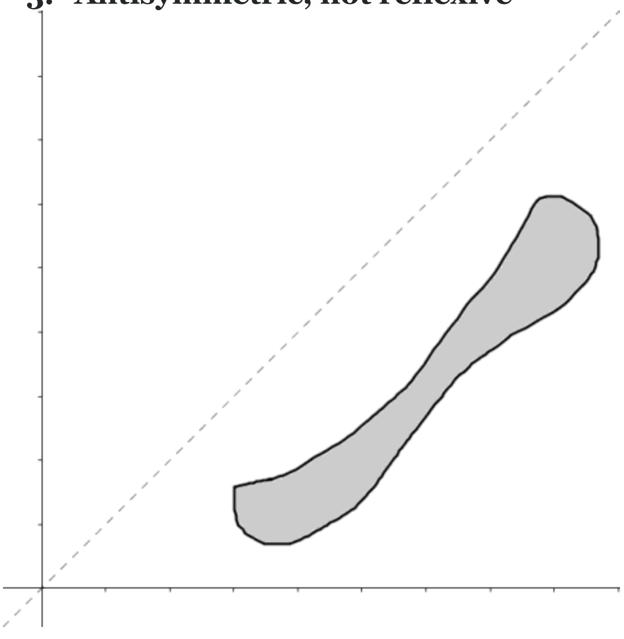
1. **Symmetric, reflexive**



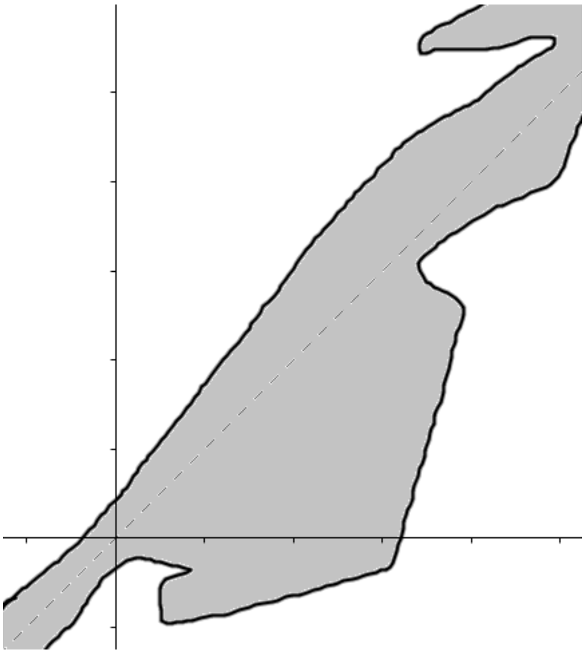
2. **Symmetric, not reflexive**



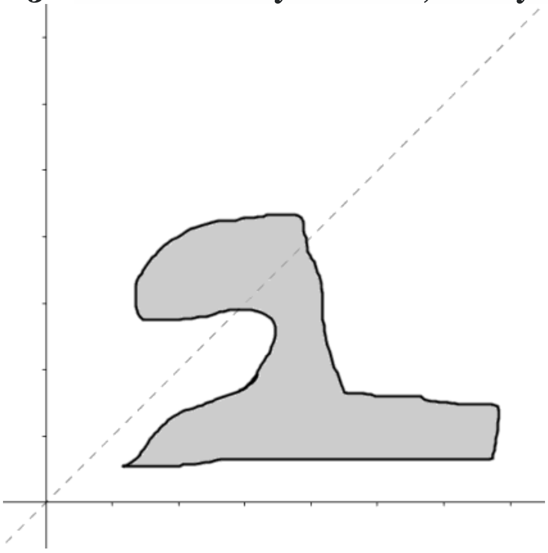
3. **Antisymmetric, not reflexive**



4. **Neither antisymmetric, nor symmetric, but reflexive**



5. **Neither antisymmetric, nor symmetric, nor reflexive**



http://math.stackexchange.com/questions/1254572/whats-the-difference-between-antisymmetric-and-reflexive-set-theory-discrete-m