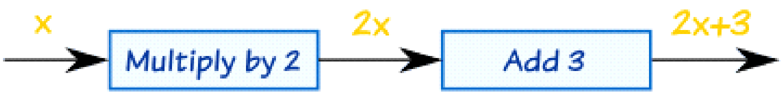
Inverse Functions

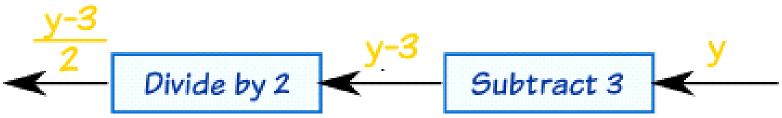
February 11, 2017 22:45

Let us start with an example:

Here we have the function **f(x) = 2x+3**, written as a flow diagram:



The **Inverse Function** just goes the other way:



So the inverse of: 2x+3 is: (y-3)/2

The inverse is usually shown by putting a little "-1" after the function name, like this:

$$f^{-1}(y)$$

*We say "**f inverse** of y"*

So, the inverse of $f(x) = 2x+3$ is written:

$$f^{-1}(y) = (y-3)/2$$

(I also used **y** instead of **x** to show that we are using a different value.)

So applying a function f and then its inverse f^{-1} gives us the original value back again:

$$f^{-1}(\ f(x)\ ) = x$$

We could also have put the functions in the other order and it still works:

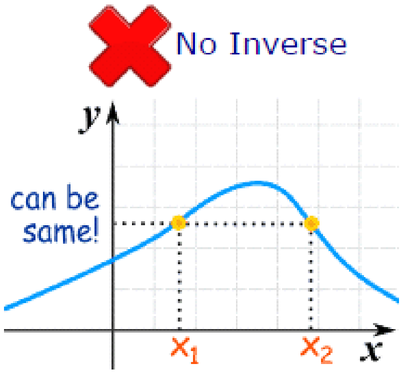
$$f(\ f^{-1}(x)\ ) = x$$

No Inverse?

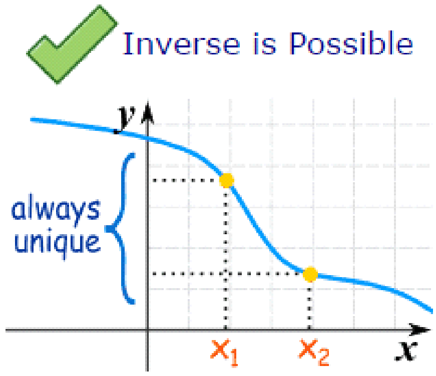
Let us see graphically what is going on here:

To be able to have an inverse we need **unique values**.

Just think ... if there are two or more **x-values** for one **y-value**, how do we know which one to choose when going back?.



When a y-value has more than one x-value, how do we know which x-value to go back to?



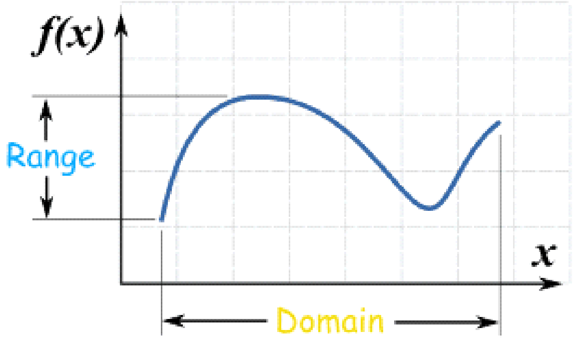
When there is a unique y-value for every x-value we can always "go back" from y to x.

So we have this idea of "a unique y-value for every x-value", and it actually has a name. It is called "**Injective**" or "**One-to-one**":

When a function is "One-to-one" (Injective) it has an inverse.

Domain and Range

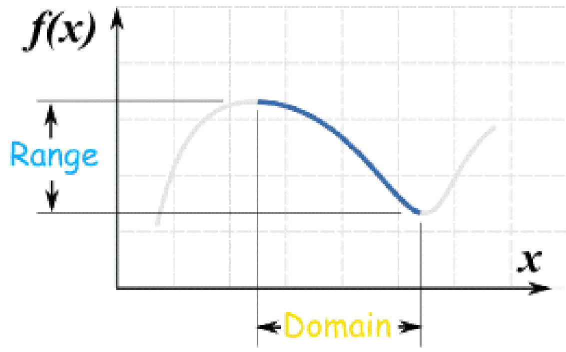
So what is all this talk about "**Restricting the Domain**"?



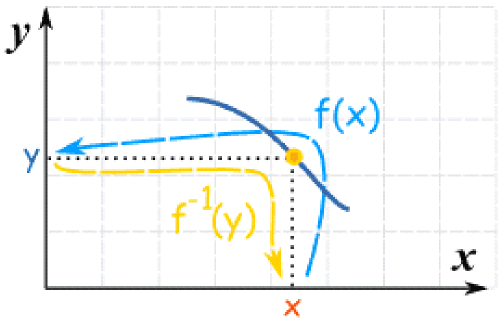
In its simplest form the **domain** is all the values that go into a function (and the **range** is all the values that come out).

As it stands the function above does **not** have an inverse.

But we could restrict the domain so there is a **unique y for every x** ...



... and **now** we can have an inverse:

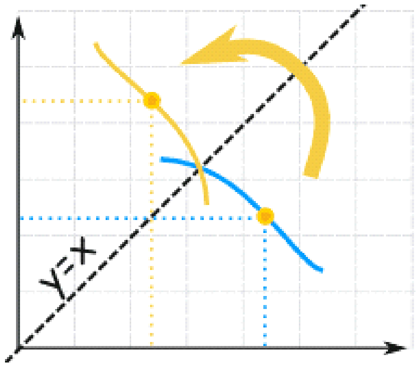


Note also:

- The function **f(x)** goes from the domain to the range,
- The inverse function **f^{-1}(y)** goes from the range back to the domain.

Or...

We could plot them both in terms of **x** ... so it is now $\mathbf{f^{-1}(x)}$, not $\mathbf{f^{-1}(y)}$:



$\mathbf{f(x)}$ and $\mathbf{f^{-1}(x)}$ are like mirror images
(flipped about the diagonal).

In other words:

> The graph of $\mathbf{f(x)}$ and $\mathbf{f^{-1}(x)}$ are symmetric across the line $\mathbf{y=x}$