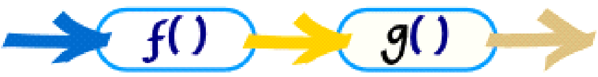
# Composition of Functions

February 11, 2017 22:38

"Function Composition" is applying one function to the results of another:




The result of f() is sent through g()

It is written: $(g \circ f)(x)$

Which means: $g(f(x))$

Example: $f(x) = 2x+3$ and $g(x) = x^2$

**"x" is just a placeholder**, and to avoid confusion let's just call it "input":

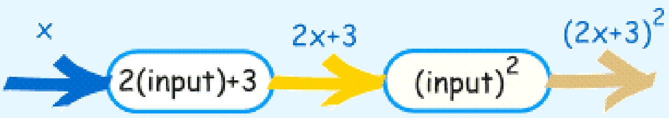
$$f(\text{input}) = 2(\text{input})+3$$

$$g(\text{input}) = (\text{input})^2$$

So, let's start:

$$(g \circ f)(x) = g(f(x))$$

First we apply f, then apply g to that result:

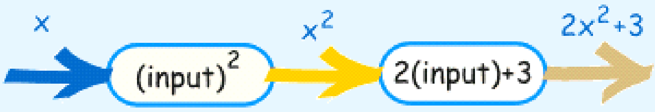



$$(g \circ f)(x) = (2x+3)^2$$

What if we reverse the order of f and g?

$$(f \circ g)(x) = f(g(x))$$

First we apply g, then apply f to that result:




$$(f \circ g)(x) = 2x^2+3$$

**We got a different result!**

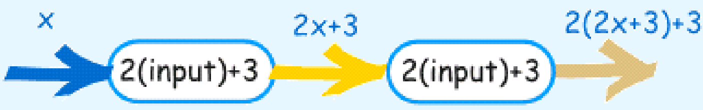
## Composed With Itself

You can even compose a function with itself!

**Example: $f(x) = 2x+3$**

$$(f \circ f)(x) = f(f(x))$$

First we apply $f$, then apply $f$ to that result:



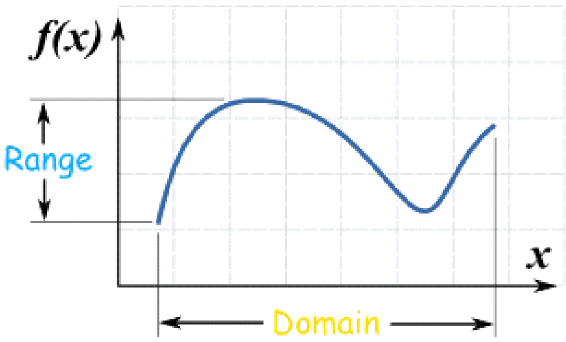

$$(f \circ f)(x) = 2(2x+3)+3 = 4x + 9$$

You should be able to do this without the pretty diagram:

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f(2x+3) \\ &= 2(2x+3)+3 \\ &= 4x + 9\end{aligned}$$

# Domains

It has been easy so far, but now you must consider the **Domains** of the functions.




The domain is **the set of all the values** that go into a function.

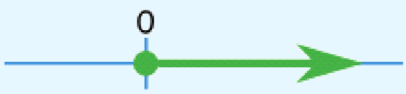
The function must work for all values you give it, so it is **up to you** to make sure you get the domain correct!

### Example: the domain for √x (the square root of x)

You cannot have the square root of a negative number (unless you use imaginary numbers, but we aren't), so we must **exclude** negative numbers:

The Domain of √x is all non-negative Real Numbers

On the Number Line it looks like:




Using set-builder notation it is written:

$$\{ x \in \mathbb{R} \mid x \geq 0 \}$$

Or using interval notation it is:

$$[0, +\infty)$$

## Domain of Composite Function

You must get **both Domains** right (the composed function **and** the first function used).

When doing, for example, $(g \circ f)(x) = g(f(x))$:

- Make sure you get the Domain for **f(x)** right,
- Then also make sure that **g(x)** gets the correct Domain

Example: $\mathbf{f(x) = \sqrt{x}}$ and $\mathbf{g(x) = x^2}$

The Domain of $\mathbf{f(x) = \sqrt{x}}$ is all non-negative Real Numbers

The Domain of $\mathbf{g(x) = x^2}$ is all the Real Numbers

The composed function is:

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= (\sqrt{x})^2 \\ &= x\end{aligned}$$

Now, "x" normally has the Domain of all Real Numbers ...

... but because it is a **composed function** you must **also consider f(x)**,

So the Domain is all non-negative Real Numbers