6.5 Average value of a function

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Definitions & Theorems:

1. Definition:

   We define the average value of f on the interval $[a, b]$ as

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x$$

Proofs or Explanations:

1. Definition1

$$f_{ave} = \frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{n}$$

$$\Delta x = \frac{b-a}{n} \to n = \frac{b-a}{\Delta x}$$

$$\to f_{ave} = \frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{\frac{b-a}{\Delta x}}$$

$$\to f_{ave} = \frac{1}{b-a}\left[f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)\right]\Delta x$$

$$\to f_{ave} = \frac{1}{b-a}\sum_{i=1}^{n} f(x_i^*)\Delta x = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x$$

Examples:

1. Find the average value of $f(x) = \sin(2x)\, e^{1-\cos(2x)}$ on $[-\pi, \pi]$

$$f_{ave} = \frac{1}{2\pi}\int_{\pi}^{\pi} \sin(2x)\, e^{1-\cos(2x)}\,\mathrm{d}x$$

Let $u = 1 - \cos(2x) \to du = 2\sin(2x)\,dx$

$$f_{ave} = \frac{1}{2\pi}\int_0^0 \frac{1}{2}\mathrm{e}^{\mathrm{u}}\,\mathrm{d}u = 0$$