6.1 Areas Between Curves

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Definitions & Theorems:

1. Formula:

   The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the line $x = a$, $x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, is

   $$A = \int_a^b [f(x) - g(x)]\,dx$$

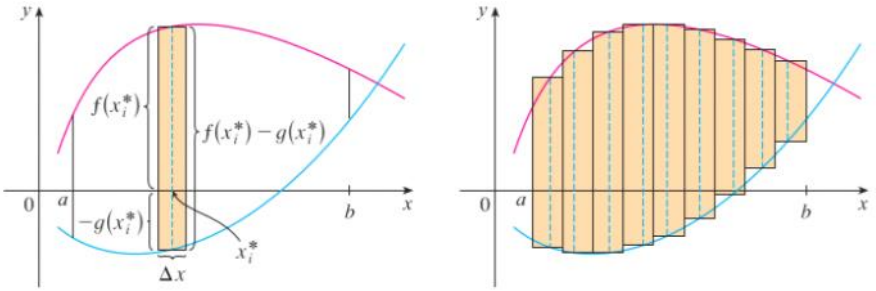
2. Formula:

   The area A between the curves $f = f(x)$ and $f = g(x)$ and between $x = a$ and $x = b$ is

   $$A = \int_a^b |f(x) - g(x)|\,dx$$

Proofs or Explanations:

1. Formula1

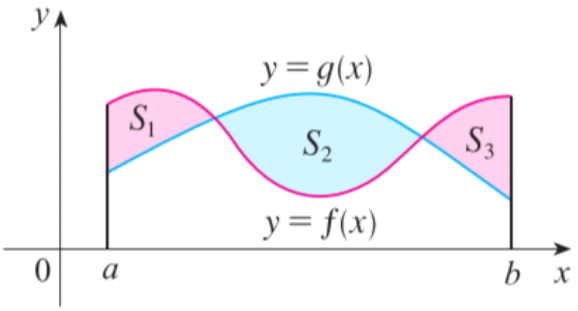


(a) Typical rectangle (b) Approximating rectangles

Consider the region S that lies between two curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a. b]$

$$A = \lim_{\lambda \to x} \sum_{i-\lambda}^{n} [f(x_i^*) - g(x_i^*)]\Delta x = \int_a^b [f(x) - g(x)]\,dx$$

2. Formula2



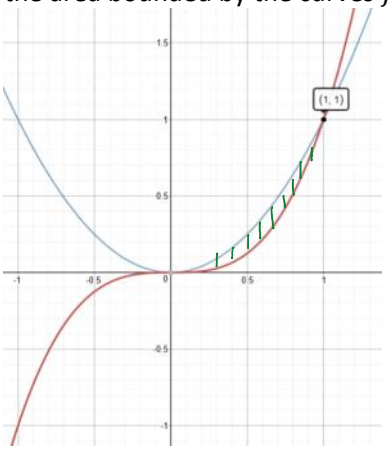
$$|f(x) - g(x)| = \begin{cases} f(x) - g(x), where\ f(x) \geq g(x) \\ g(x) - f(x), where\ g(x) \geq f(x) \end{cases}$$

Extra topics:

1. Steps for solving for the area between curves:
   a. Find the intersection
   b. Sketch the situation
   c. Determine "top" and "bottom"
   d. Compute the definite integral of "top" - "bottom" for each partition.

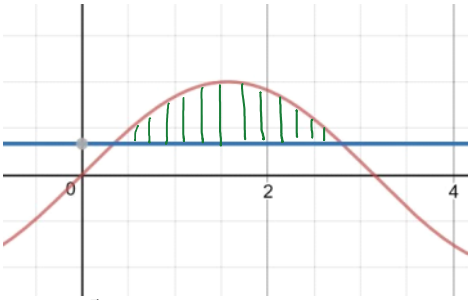
Examples:

1. Find the area bounded by the curves $f = x^3$ and $f = x^2$.



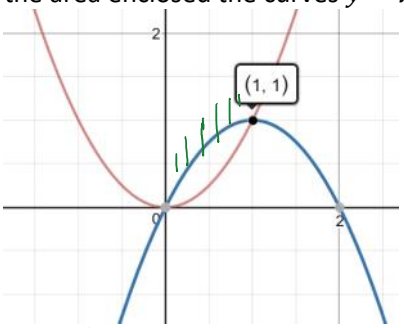
$$A = \int_0^1 (x^2 - x^3)\,dx = \frac{1}{12}$$

2. Find the area between the curves $y = \sin x$ and $y = \frac{1}{3}$ on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$



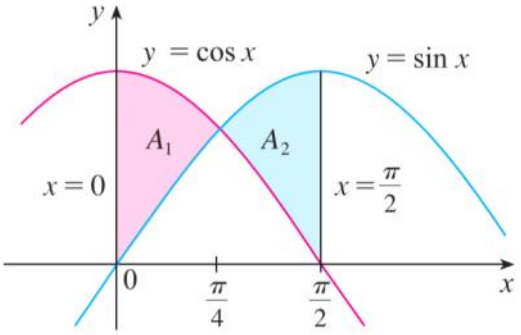
$$A = \int_{\frac{\pi}{6}}^{\frac{5}{6}\pi} \left[\sin x - \frac{1}{3}\right] dx = \sqrt{3} - \frac{2}{9}\pi$$

3. Find the area enclosed the curves $y = x^2$ and $y = 2x - x^2$



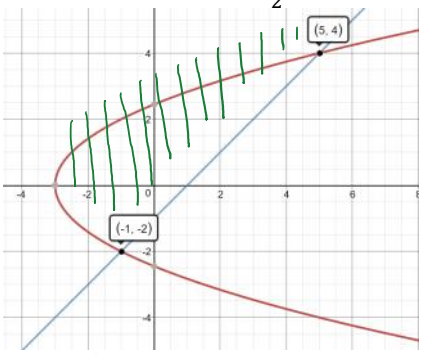
$$A = \int_0^1 \left[(2x - x^2) - x^2\right] dx$$

4. Find the area between the curves $y = \sin x$ and $y = \cos x$ on $[0, \frac{\pi}{2}]$



$$A = \int_0^{\frac{\pi}{4}} [\cos x - \sin x] dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [\sin x - \cos x]\, dx$$

5. Find the area between $x = \frac{1}{2}y^2 - 3$ and $x = y + 1$



$$A = \int_{-2}^{4} \left[(y+1) - \left(\frac{1}{2}y^2 - 3\right)\right] dy$$