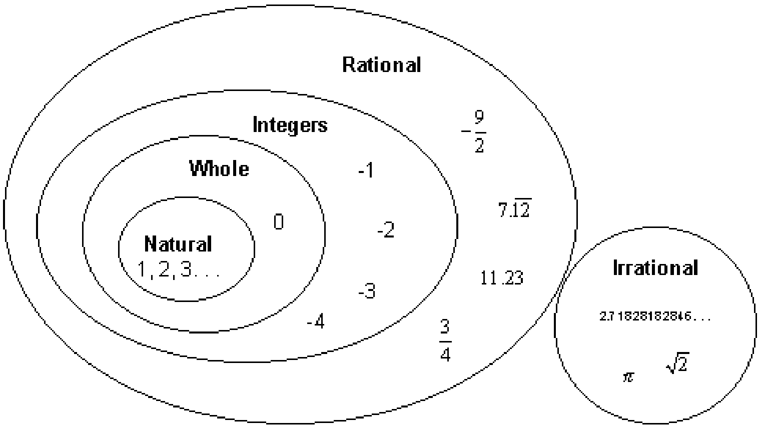
The real number system

February 11, 2017 21:32

1. The Real Number System



Why are they called "Real" Numbers?

Because they are not [Imaginary Numbers](#).

2. Fraction

$$\frac{1}{2} = 0.50000 \ldots = 0.5\overline{0}$$

$$\frac{a}{b} = \frac{Numerator}{Denominator}$$

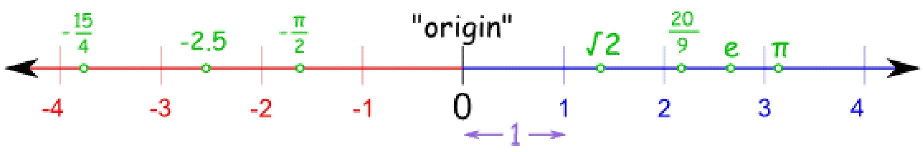
3. Properties of Real Numbers

	Property (*a*, *b* and *c* are real numbers, variables or algebraic expressions)	Examples	Verbal hints
1.	Distributive Property $a \bullet (b + c) = a \bullet b + a \bullet c$	$3 \bullet (4 + 5) = 3 \bullet 4 + 3 \bullet 5$	"multiplication distributes across addition"
2.	Commutative Property of Addition $a + b = b + a$	$3 + 4 = 4 + 3$	"commute = to get up and move to a new location : switch places"
3.	Commutative Property of Multiplication $a \bullet b = b \bullet a$	$3 \bullet 4 = 4 \bullet 3$	"commute = to get up and move to a new location: switch places"
4.	Associative Property of Addition $a + (b + c) = (a + b) + c$	$3 + (4 + 5) = (3 + 4) + 5$	"regroup - elements do not physically move, they simply group with a new friend."
5.	Associative Property of Multiplication $a \bullet (b \bullet c) = (a \bullet b) \bullet c$	$3 \bullet (4 \bullet 5) = (3 \bullet 4) \bullet 5$	"regroup - elements do not physically move, they simply group with a new friend."
6.	Additive Identity Property $a + 0 = a$	$4 + 0 = 4$	"the value that returns the input unchanged"
7.	Multiplicative Identity Property $a \bullet 1 = a$	$4 \bullet 1 = 4$	"the value that returns the input unchanged"
8.	Additive Inverse Property $a + (-a) = 0$	$4 + (-4) = 0$	"the value that brings you back to the identity element under addition"
9.	Multiplicative Inverse Property $a \bullet \left(\frac{1}{a}\right) = 1$ where $a \neq 0$	$4 \bullet \left(\frac{1}{4}\right) = 1$	"the value that brings you back to the identity element under multiplication"
10.	Zero Property of Multiplication $a \bullet 0 = 0$	$4 \bullet 0 = 0$	"zero times any value is 0"

11.	Closure Property of Addition $a + b$ is a real number	$10 + 5 = 15$ (a real number)	"the sum of any two real numbers is another real number"
12.	Closure Property of Multiplication $a \cdot b$ is a real number	$10 \cdot 5 = 50$ (a real number)	"the product of any two real numbers is another real number"
13.	Addition Property of Equality If $a = b$, then $a + c = b + c$.	If $x = 10$, then $x + 3 = 10 + 3$	"adding the same value to both sides of an equation will not change the truth value of the equation."
14.	Subtraction Property of Equality If $a = b$, then $a - c = b - c$.	If $x = 10$, then $x - 3 = 10 - 3$	"subtracting the same value from both sides of an equation will not change the truth value of the equation."
15.	Multiplication Property of Equality If $a = b$, then $a \cdot c = b \cdot c$.	If $x = 10$, then $x \cdot 3 = 10 \cdot 3$	"multiplying both sides of an equation by the same value will not change the truth value of the equation."
16.	Division Property of Equality If $a = b$, then $a / c = b / c$, assuming $c \neq 0$.	If $x = 10$, then $x / 3 = 10 / 3$	"dividing both sides of an equation by the same non-zero value will not change truth value of the equation."
17.	Substitution Property If $a = b$, then a may be substituted for b, or conversely.	If $x = 5$, and $x + y = z$, then $5 + y = z$.	"a value may be substituted for its equal."
18.	Reflexive (or Identity) Property of Equality $a = a$	$12 = 12$	"a real number is always equal to itself"
19.	Symmetric Property of Equality If $a = b$, then $b = a$.	If $3.5 = 3\frac{1}{2}$, then $3\frac{1}{2} = 3.5$.	"quantities that are equal can be read forward or backward"
20.	Transitive Property of Equality If $a = b$ and $b = c$, then $a = c$.	If $2a = 10$ and $10 = 4b$, then $2a = 4b$.	"if two numbers are equal to the same number, then the two numbers are equal to each other"
21.	Law of Trichotomy Exactly ONE of the following holds: $a < b$, $a = b$, $a > b$	If $8 > 6$, then $8 \neq 6$ and 8 is not < 6.	"for two real numbers a and b, a is either equal to b, greater than b, or less than b." (common sense)

4. The Real Number Line

A point is chosen on the line to be the **"origin"**. Points to the right are positive, and points to the left are negative.



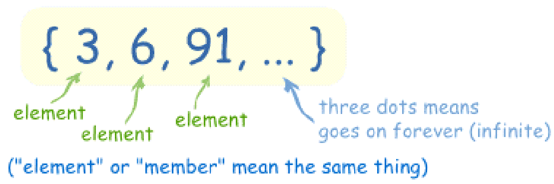
A distance is chosen to be "1", then whole numbers are marked off: {1,2,3,...}, and also in the negative direction: {...,−3,−2,−1}

Any point on the line is a Real Number:

- The numbers could be whole (like 7)
- or rational (like 20/9)
- or irrational (like π)

But we won't find Infinity, or an Imaginary Number.

5. Sets



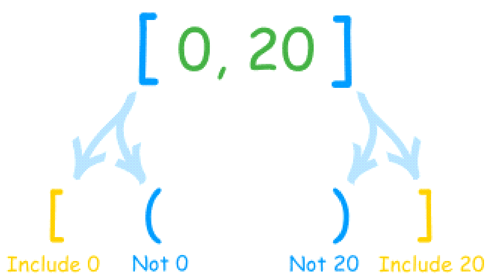
The curly brackets { } are sometimes called "set brackets" or "braces".

Empty set: Ø

Order: In sets it does not mater what order the elements are in.

Example: {1,2,3,4} is the same set as {3,1,4,2}

6. Intervals

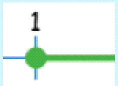
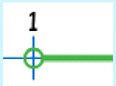
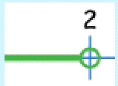
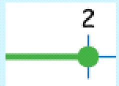


Example: (5, 12]

Means from 5 to 12, do **not** include 5, but **do** include 12

All Three Methods Together

Here is a handy table showing all 3 methods (the interval is 1 to 2):

	From 1			To 2	
	Including 1	**Not** Including 1		**Not** Including 2	Including 2
Inequality:	$x \geq 1$ "greater than or equal to"	$x > 1$ "greater than"		$x < 2$ "less than"	$x \leq 2$ "less than or equal to"
Number line:					
Interval notation:	[1	(1		2)	2]