5.2 The Definite Integral

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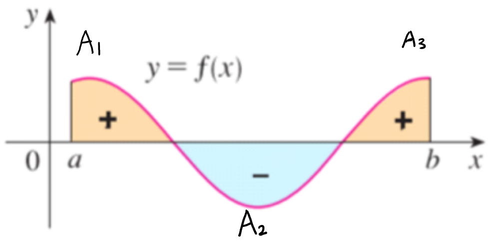
Let's keep our standing assumption of dealing with a continuous function on a closed internal, however, we'll remove non-negativity.

Definition: Let $f(x)$ be as above. Then the definite integral of $f(x)$ from a to b is defined to be

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n = \lim_{n\to\infty} M_n$$

Note1: Since $f(x)$ is not assumed to be non-negative, $\int_a^b f(x)\,\mathrm{d}x$ no longer represents the area beneath the graph of $f(x)$; instead, it represents the signed area between the graph of $f(x)$ and the x-axis

$$\int_a^b f(x)\,\mathrm{d}x = A_1 - A_2 + A_3$$



Note2: Although I define the definite integral as arising from midpoint rectangle or left/right end point rectangle, you can choose different set of point, not necessarily evenly spaced, so long as one and only one is chose from each sub-internal.

Note3: Although I define the integral for a continuous function, a function with finitely many jump discontinuation is also integrable.

Definitions & Theorems:

1. Theorem

   If f is continuous on $[a, b]$, or if f has only a finite number of jump discontinuities, then f is integrable on $[a, b]$; that is, the definite integral $\int_a^b f(x)\,dx$ exists.

Properties of definite integrals:

1. $\int_b^a f(x)\,\mathrm{d}x = -\int_a^b f(x)\,\mathrm{d}x$
2. $\int_a^a f(x)\,\mathrm{d}x = 0$
3. $\int_a^b c\,\mathrm{d}(x) = c(b-a), c \in R$
4. $\int_a^b [f(x) \pm g(\mathrm{x})]\,\mathrm{d}x = \int_a^b f(x) \pm \int_a^b g(x)$
5. $\int_a^b cf(x)\,\mathrm{d}x = c\int_a^b f(x), c \in R$
6. $\int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x$
7. If $f(x) = 0$ on [a, b], then $\int_a^b f(x)\,\mathrm{d}x = 0$
8. If $f(x) \leq \mathrm{g}(x)$ on [a, b], then $\int_a^b f(x)\,\mathrm{d}x \leq \int_a^b \mathrm{g}(x)\,\mathrm{d}x$
9. If $m \leq f(x) \leq M$ on [a, b] then $m(b-a) \leq \int_a^b f(x)\,\mathrm{d}x \leq M(b-a)$