Polynomials

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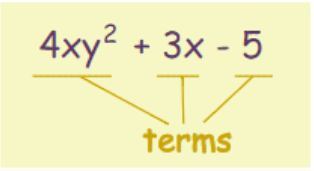
1. Polynomials

A polynomial looks like this:



example of a polynomial
this one has 3 terms

Polynomial comes from *poly-* (meaning "many") and *-nomial* (in this case meaning "term") ... so it says "many terms"

A polynomial can have:

constants (like **3**, **-20**, or **½**)

variables (like ***x*** and ***y***)

exponents (like the 2 in y^2), but only **0, 1, 2, 3, ...** etc are allowed

that can be combined using **addition, subtraction, multiplication and division** ...

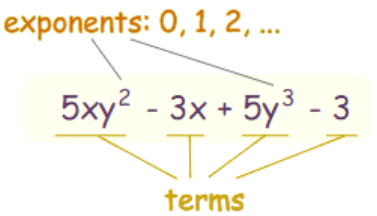
... except ...

... **not** division by a variable (so something like **2/x** is right out)

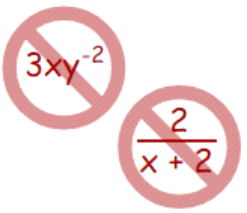
So:

A polynomial can have constants, variables and exponents,
but never division by a variable.

Polynomial or Not?



A Polynomial



Not Polynomials

Monomial, Binomial, Trinomial

There are special names for polynomials with 1, 2 or 3 terms:

$3xy^2$	$5x - 1$	$3x + 5y^2 - 3$
Monomial (1 term)	Binomial (2 terms)	Trinomial (3 terms)

Degree

The **degree** of a polynomial with only one variable is the **largest exponent** of that variable.

Example:

$4x^3 - x + 3$ The Degree is **3** (the largest exponent of x)

For more complicated cases, read Degree (of an Expression).

2. Special Products Formulas
   1) $(A+B)(A-B) = A^2 - B^2$
   2) $(A+B)^2 = A^2 + 2AB + B^2$
   3) $(A-B)^2 = A^2 - 2AB + B^2$
   4) $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$
   5) $(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$
3. Special Factoring Formulas
   1) Difference of Squares
      $x^2 - y^2 = (x - y)(x + y)$
   2) Square of Sum
      $x^2 + 2xy + y^2 = (x + y)^2$
   3) Square of Difference
      $x^2 - 2xy + y^2 = (x - y)^2$
   4) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
   5) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

4. Linear Equations
   A linear equation is a polynomial of degree 1.
   A linear equation is an equation for a straight line.

5. Quadratic Functions

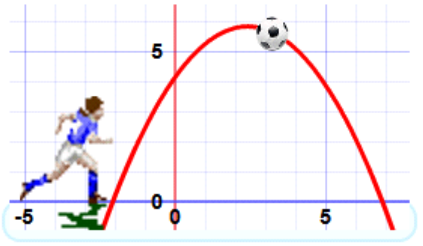
Quadratic Equations

An example of a **Quadratic Equation**:

this makes it Quadratic

$$5x^2 + 3x + 3$$

Quadratic Equations make nice curves, like this one:



Name

The name **Quadratic** comes from "quad" meaning square, because the variable gets squared (like $\mathbf{x^2}$).

It is also called an "Equation of Degree 2" (because of the "2" on the **x**)

Standard Form

The **Standard Form** of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0$$

- **a**, **b** and **c** are known values. **a** can't be 0.
- "**x**" is the **variable** or unknown (we don't know it yet).

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6. Cubic Function

   A cubic function is a polynomial of degree 3.

   $f(x) = ax^3 + bx^2 + c, \quad a \neq 0$

7. Power Functions

   $f(x) = ax^n$

8. Rational Functions

   $f(x) = \frac{P(x)}{Q(x)}, \quad P(x) and\ Q(x) are\ polynomials$

9. Algebraic Functions

   An algebraic function is a function that can be constructed using algebraic operations (such as addition, subtraction, multiplication, division, and taking roots) starting with polynomials.

10. Exponential Functions

$f(x) = b^x, \quad$ *the base b is a positive constant*

11. Logarithmic Functions

$f(x) = \log_b x\,, \quad$ *the base b is a positive constant*