Ex 2: $\sum_{n=1}^{N} a+[n-1]d = a + (a+d) + (a+2d) + (a+3d) + (a+4d) + \ldots. (a+[N-1]d) = \frac{N}{2}(2a+[N-1]d), \quad n \in Z+$

$\rightarrow \sum_{n=1}^{4} 2+3n = 5 + 8 + 11 + 14 = \frac{4}{2}(2[5] + [4-1][3]) = 38$

P(n): $a + (a+d) + (a+2d) + \ldots\ldots\ldots (a+[n-1]d) = \frac{n}{2}(2a + [n-1]d)\ \forall n \in Z+$

Proof: (Mathematical Induction)

Step 1 Prove: P(1)

LHS = a — 1st term of LHS

RHS = $\frac{1}{2}(2a + [1-1]d = a$ — Subst. n=1

Concl: P(1) = T — LHS = RHS

Step 2 Assume P(**k**): $a+(a+d)+(a+2d)+\ldots +(a+[k-1]d) = \frac{k}{2}(2a+[k-1]d)$, for any one $k \in Z+$

Step 3 From Step 2 assumption Prove P(**k+1**): $a+(a+d)+(a+2d)+\ldots +(a+kd) = \frac{k+1}{2}(2a+kd)$

Step 4 P(**k**): $a+(a+d)+(a+2d)+\ldots +(a+[k-1]d) = \frac{k}{2}(2a+[k-1]d), \quad k \in Z+$

P(**k+1**): $a+(a+d)+(a+2d)+\ldots +(a+[k-1]d)+ (a+kd) = \frac{k}{2}(2a+[k-1]d)+(a+kd)$

$= \frac{1}{2}(2ak+k[k-1]d+2a+2kd)$ — Factor out $\frac{1}{2}$

$= \frac{1}{2}(2a[k+1]+d[k^2-k+2k])$ — Common factor

$= \frac{1}{2}(2a[k+1]+d[k^2+k])$ — Cancel

$= \frac{1}{2}(2a[k+1]+d[(k+1)k])$ — Common factor

$= \frac{k+1}{2}(2a+kd)$

Concl: P(k) $\rightarrow$ P(k+1) QED