

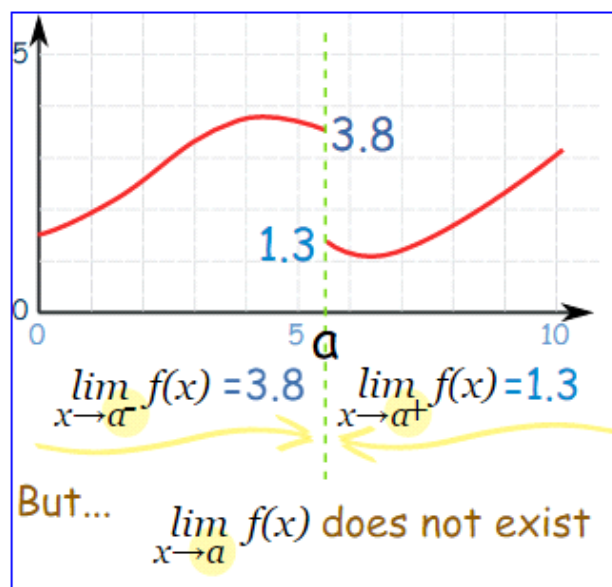
Limits

February 12, 2017 10:42

When we call the Limit "L", and the value that x gets close to "a" we can say

"f(x) gets close to L as x gets close to a"

$$f(x) \rightarrow L \text{ as } x \rightarrow a$$



Delta and Epsilon

But "small" is still English and not "Mathematical-ish".

Let's choose two values **to be smaller than**:

δ that $|x-a|$ must be smaller than

ϵ that $|f(x)-L|$ must be smaller than

(Note: Those two greek letters, δ is "delta" and ϵ is "epsilon", are often used for this, leading to the phrase "**delta-epsilon**")

And we have:

$$|f(x)-L| < \epsilon \text{ when } |x-a| < \delta$$

That actually says it! So if you understand that you understand limits ...

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... but to be **absolutely precise** we need to add these conditions:

1) it is true for any $\epsilon > 0$ 2) δ exists, and is > 0 3) x **not equal to** a means $0 < |x - a|$

And this is what we get:

"for any $\epsilon > 0$, there is a $\delta > 0$ so that $|f(x) - L| < \epsilon$ when $0 < |x - a| < \delta$ "

That is the formal definition. It actually looks pretty scary, doesn't it!

But in essence it still says something simple: when x *gets close to* a then $f(x)$ *gets close to* L .