AP Calculus AB Sample Practice Homework

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Description

This is a sample homework to demonstrate the curriculum of our AP Calculus AB tutoring. If you decide to pay for a real homework packet, you will receive a fixed amount of problems and an answer key with detailed explanations.

Instructions

Answer the following questions. Show complete work. You may use a graphing calculator. There is an answer key on page 2.

Questions

1. Evaluate the limit:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

2. If $f(x) = \sin(\ln(2x))$, $f'(x) = ?$

3. Evaluate the definite integral:

$$\int_0^1 e^{3x}\, dx$$

4. What point on the graph of the function $y = \sqrt{x}$ is closest to the point $(\pi, 0)$?

5. Let

$$f(x) = \frac{x - 4}{(x + 4)(x - 2)}.$$

   Find the intervals where $f(x)$ is increasing.

Answer Key

1. One way of solving this limit is by factoring the numerator, recognizing it as the difference of two squares.
$$\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2}$$
For $x \neq 2$, we can cancel the common factor:
$$= x+2$$
Now take the limit:
$$\lim_{x\to 2}(x+2) = 4$$
Alternatively, you may use L'Hôpital's rule. Differentiating both the numerator and the denominator produces the fraction
$$\frac{2x}{1} = 2x$$
and L'Hôpital's rule states that
$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2} 2x$$
Plugging in our value for x, we obtain
$$2x = 2(2) = 4$$

2. Use the chain rule:
$$\frac{d}{dx}\left[f\left(u\right)\right] = \frac{d}{du}\left[f\left(u\right)\right]\frac{du}{dx}$$
In this case, your function $f = \sin u$ and your function $u = \ln(2x)$. First find the derivative of $\ln(2x)$ with respect to x.
$$\frac{d}{dx}\left[\ln(2x)\right]$$
Using the chain rule again:
$$= \frac{1}{2x}\cdot\frac{d}{dx}(2x)$$
$$= \frac{1}{2x}\cdot 2 = \frac{1}{x} = \frac{du}{dx}$$

Now, return to the original application of the chain rule.

$$[f\,(u)]\,\frac{1}{x}$$

The derivative of $\sin u$ with respect to u is $\cos u$ (as per an identity). Substituting u back in, we obtain:

$$\frac{\cos\left(\ln(2x)\right)}{x}$$

3. First, find the antiderivative using u-substitution ($u = 3x \Rightarrow du = 3dx$):

$$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$$

Now evaluate the definite integral:

$$\int_0^1 e^{3x}\,dx = \left[\frac{1}{3}e^{3x}\right]_0^1 = \frac{1}{3}e^3 - \frac{1}{3}e^0$$

$$= \frac{1}{3}(e^3 - 1)$$

4. Let the point on the curve be $(x, \sqrt{x})$. The distance from this point to $(\pi, 0)$ is given by the distance formula:

$$D = \sqrt{(x-\pi)^2 + (\sqrt{x} - 0)^2} = \sqrt{(x-\pi)^2 + x}$$

Since square root is increasing, minimizing D is equivalent to minimizing D^2:

$$D^2 = (x-\pi)^2 + x$$

$$= x^2 - 2\pi x + \pi^2 + x = x^2 - (2\pi - 1)x + \pi^2$$

Now take the derivative and find the critical point:

$$\frac{d}{dx}D^2 = 2x - (2\pi - 1)$$

$$= 2x - 2\pi + 1$$

Set the derivative equal to zero:

$$2x - 2\pi + 1 = 0 \Rightarrow x = \pi - \frac{1}{2}$$

So the closest point on the curve is:

$$\left(\pi - \frac{1}{2}, \sqrt{\pi - \frac{1}{2}}\right)$$

5. We begin by taking the derivative using the quotient rule.

   Let

$$f(x) = \frac{u}{v}, \quad \text{where } u = x - 4, \quad v = (x+4)(x-2).$$

   First, compute v and v':

$$v = (x+4)(x-2) = x^2 + 2x - 8$$

$$v' = \frac{d}{dx}(x^2 + 2x - 8) = 2x + 2$$

$$u' = \frac{d}{dx}(x - 4) = 1$$

   Now apply the quotient rule:

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{(1)(x^2 + 2x - 8) - (x-4)(2x+2)}{(x^2 + 2x - 8)^2}$$

   Simplify the numerator:

   First, expand both parts:

$$(1)(x^2 + 2x - 8) = x^2 + 2x - 8$$

$$(x-4)(2x+2) = 2x^2 + 2x - 8x - 8 = 2x^2 - 6x - 8$$

   Now subtract:

$$x^2 + 2x - 8 - (2x^2 - 6x - 8) = x^2 + 2x - 8 - 2x^2 + 6x + 8 = -x^2 + 8x$$

   So,

$$f'(x) = \frac{-x^2 + 8x}{(x^2 + 2x - 8)^2}$$

Now find where $f'(x) > 0$. This occurs when the numerator is positive (denominator is always positive except where undefined).

Set the numerator $-x^2 + 8x > 0$:

$$-x^2 + 8x > 0 \Rightarrow x(-x + 8) > 0$$

$$x(8 - x) > 0$$

This inequality is true when $0 < x < 8$

Now exclude points where $f(x)$ is undefined. The original function is undefined at $x = -4$ and $x = 2$, so we must remove those from the interval.

Therefore, the function is increasing on the intervals:

$$(0, 2) \cup (2, 8)$$