Probability Theory for EOR

Conditional probability (conditional on a non-zero probability event)

Definition and Intuition

Definition

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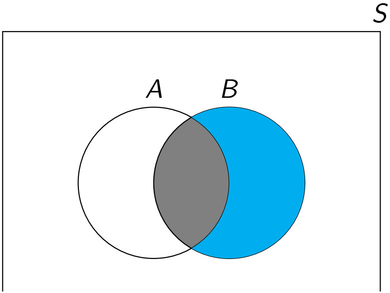
Conditional probability of A given B ($P(B) > 0$):
if A and B are events with $P(B) > 0$, then the conditional probability of A given B, denoted by $P(A|B)$, is defined as $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

$$\begin{array}{llll} \mathbb{P}(\cdot|B): & \{A_i \subseteq S, i \in I\} & \mapsto & [0,1] \\ & A & \mapsto & P(A \cap B)/P(B)\ (*) \end{array}$$



$P(B|B) =?$ $P(A|B) =?$

Conditional probability is a probability

We have a new probability!

- $P(S|B) = 1$;
- $P(\emptyset|B) = 0$;
- $P(\cup_i A_i|B) = \sum_i P(A_i|B)$, $A_i \cap A_j = \emptyset$ $(\forall, i \neq j)$.

In Latin

Some terminology

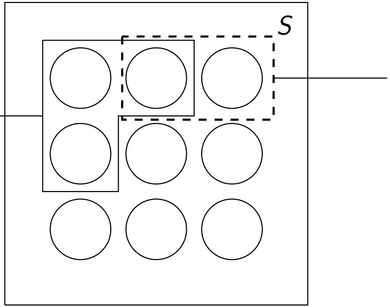
- $P(A)$ is sometimes called the *prior* of A (or unconditional/marginal probability of event A).
- When information B enters, we *update* the prior.
- The updated probability $P(A|B)$ is called the *posterior* of A given B (or conditional probability of event A conditional on event B).

Example

Example: visualizing conditional probabilities

If we know that B happens, what is the probability that A happens?

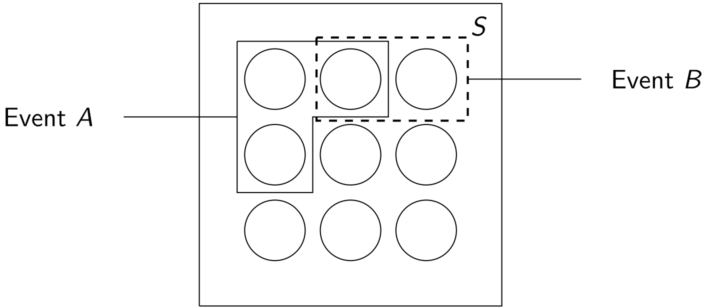
Event A



Event B

Example: visualizing conditional probabilities

If we know that A happens, what is the probability that B happens?



Important lesson

(In general)

$$P(A|B) \neq P(B|A)$$

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