Layman's talk : what is random variable

Random variables are functions

Introduce random variables via a simple example

Which one is easier to work with in a coin-flipping game?

- A_{1i} =$\{i$th throw gets a Head$\}$, A_{2i} =$\{i$th throw gets a Bottom$\}$;
- $X_i = 1$ if ith throw gets a Head, and $X_i = 0$ if ith throw gets a Bottom.

With the second notation, we can do more things in easier ways. E.g., think about what would happen if we let n increase in the following fomula

$$\frac{1}{n}\sum_{i=1}^{n} X_i,$$

this average is more likely to be closer and closer to $\frac{1}{2}$ as $n \to \infty$.

Now we can work with numbers!

Introduce random variables via a simple example

$$X_i(s) = \begin{cases} 1; & s \in A_{1i} = \{\ ith\ throw\ gets\ a\ Head\ \} \\ 0; & s \in A_{2i} = \{ith\ throw\ gets\ a\ Bottom\} \end{cases}$$

Random variables are functions!

Introduce random variables via a simple example

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- Let $\{X_i = 1\}$ denote the subset of the sampel space S such that for $s \in \{X_i = 1\}$, $X_i(s) = 1$.
- $P(X_i = 1) = P(A_{1i}) = 1/2$.

X **maps elements from** S **to numbers! Now we can work with functions/numbers!**

$$\begin{tikzcd}
s \in S \arrow[r, "X"] \arrow[dr, dashed, "P : A \subseteq S"'] & Numbers \in \mathcal{C} \arrow[d, dashed, "P : \{X \in C \subseteq \mathcal{C}\} \subseteq S"] \\
& {[0, 1]}
\end{tikzcd}$$

- X essentially is a function, and when we manipulate random variables, it would be similar to manipulating functions and there are so many transformations we can work with: summation, division, max, min...

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- We can now use $\{X \in C\}$ with $C \subseteq \mathcal{C}$ to represent different random events, and properties derived for random events would be inherited.
- Given the introduction of these additional structures, we have a larger space to explore: conditional probability/independence based on random variables, random variables with specific probability functions...

Random variables are functions, there are selected functions with nice properties, and so are some selected random variables.