Probability Theory for EOR

Expectation of discrete random variables

Expected values of r.v.'s (if exsist) are numbers.

Expectation of discrete r.v.'s

Definition (Expectation of discrete r.v.'s)

The expectation/expected value/first moment/mean/average of a **discrete** random variable X for $x_1, x_2, x_3, \ldots$ (if exists) is defined by

$$\mathrm{E}[X] = \sum_{i=1}^{\infty} x_i \mathbb{P}(X = x_i) \ \ (= \sum_{i=1}^{\infty} x_i \Delta F_X(x_i))$$

(we order all values from the support of X by the increasing order: $x_1 \leq x_2 \leq x_3, \cdots$.
We may choose $\Delta F_X(x_i) = F_X(x_i) - F_X(x_{i-1})$, let $x_0 < x_1, x_0 \notin supp(X)$ and thus $F(x_0) = 0$.
We may choose $\Delta F_X(x) = F(x) - \lim_{y \uparrow x} F(y)$, as another possible expression.
Essentially both expressions give: $\Delta F_X(x_i) = P(X = x_i)$.)

If $|\mathrm{supp}(X)|$ is finite with elements $x_1, x_2, x_3, \ldots, x_n$, then

$$\mathrm{E}[X] = \sum_{i=1}^{n} x_i \mathbb{P}(X = x_i) \ \ (= \sum_{i=1}^{n} x_i \Delta F_X(x_i))$$

- Expectation may not exist for some random variables, e.g., the summation $\sum_{i=1}^{\infty} x_i \mathbb{P}(X = x_i)$ may diverge.
- Same distributions, same expectations (if exist).

Example

A box of 10 apples has 3 bad apples. We choose 3 apples at random, without replacement. What is the number of bad apples that you expect?

Denote by X the number of bad apples. Then

$$P(X=0)=\frac{\binom{7}{3}\binom{3}{0}}{\binom{10}{3}}; P(X=1)=\frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}}$$
$$P(X=2)=\frac{\binom{7}{1}\binom{3}{2}}{\binom{10}{3}}; P(X=3)=\frac{\binom{7}{0}\binom{3}{3}}{\binom{10}{3}}$$

$\mathrm{E}[X]=P(X=0)\cdot 0+P(X=1)\cdot 1+P(X=2)\cdot 2+P(X=3)\cdot 3= 3\times\frac{3}{10}$.