Layman's talk : what is moment generating function

A real-valued r.v. X and its nth moment $\mathbb{E}X^n$.

Can we find something to store the information of a sequence of moments?

	$\mathbb{E}X^0$	$\mathbb{E}X$	$\mathbb{E}X^2$	$\mathbb{E}X^3$	$\mathbb{E}X^4$	$\cdots$
	$t^0/0!$	$t^1/1!$	$t^2/2!$	$t^3/3!$	$t^4/4!$	$\cdots$

We use polynomials to store these moment values, label each moment with a unique polynomial!

$$M_X(t) = \sum_{i=0}^{\infty} E(X^i) t^i / i! = E \sum_{i=0}^{\infty} (X^i) t^i / i! = E e^{tX}.$$

Moment generating function (MGF) $M_X(t)$ is a function.

We say it is well-defined, if we can find one $a > 0$ such that
$M_X(t) : (-a, a) \mapsto \mathbb{R}$.

► **Moments via the derivative of the MGF**:

$$EX^n = M_X^{(n)}(0)$$

► **MGF (if exists) determines the distribution type**:

If two r.v.s have the same MGF, they have the same distribution!