# Layman's talk : what is expectation

# Expectation summarizes "average" value: a function mapping well-behaved random variables to numbers

- Distributions assign (probability values) numbers to random events associated with random variables.

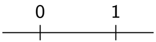
  *Many values associated with many events...*

- Sometimes, one would prefer to know what happens "on average".

  *E.g., if heads for 1 bottoms for 0, we know from our daily experience, the average would be 0.5.*

- **Expectation** formalizes the above idea and assigns a number (if exists) for one random variable.

**Example:**




$X \sim$Bern(p) with PMF:

$$P(X = 0) = 1 - p; P(X = 1) = p.$$

$$E(X) = 0 \times P(X = 0) + 1 \times P(X = 1) = p.$$

**Example:**


$X \sim$Bern(p), then $Y = 2 + 2X$ is also an r.v. with PMF:

$$P(Y = 2) = 1 - p; P(Y = 4) = p.$$

$$E(X) = 0 \times P(X = 0) + 1 \times P(X = 1) = p.$$

$$E(Y) = 2 \times P(Y = 2) + 4 \times P(Y = 4) = 2 + 2p.$$

**For a discrete r.v. $X$:**

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- A weighted average (with weights being the associated probability values, these weights sum up to 1) of possible values taken by $X$.
- For $E(X)$ to be well-defined, we need the condition that this summation $\sum_{x \in \mathrm{supp}(X)} xP(X = x)$ is well-defined.

- Expectation essentially maps a subset of the collection of all random variables to numbers. It is a function!

$$E : \{X : \textit{well-behaved r.v.'s}\} \mapsto \mathcal{C}$$

$E(X+Y) = E(X) + E(Y)$ and $E(aX) = aE(X)$ for real-valued r.v.'s $X, Y$ and $a \in \mathbb{R}$.

- **Expectation goes far beyond "average"** by working with different transformation of random variables.

E.g., it can recover probability values via the indicator random varaible $I_{\{X=x\}}$:

$$P(X = x) = EI_{\{X=x\}}.$$