Layman's talk : what is probability

Probability is a function

In our daily life, we sometimes would see:

- this stock price is more likely to increase
- the risk of default is getting higher
- chances are that tomorrow is going to rain

The aforementioned are predictions, and there are many other relevant words such as coincidence, lucky or randomness. How do we formally discuss these terms?

Probability!

A measure of uncertainty and random events.

Quantify the uncertainty

A natural thought would be assigning numbers to those random events, the larger the number the more likely it is going to happen.

-
- This is similar to measuring one's height:
  (a) pick one people from a group of people
  (b) use a ruler to measure the height
  (c) You assign each people a number to represent their heights; the larger number the taller

  The above procedure is a function!
- Similarly, probability is also a function, relates random events to real numbers within [0,1].

Probability is a function

$$\begin{array}{rccc} \mathbb{P}: & A & \mapsto & x \\ & \subseteq & & \in \\ & S & & [0,1] \end{array}$$

where S denotes a collection of all possible outcomes, or a sample space, and A denotes an arbitrary **(but properly chosen)** subset of S.

For example, if we return to the first sentence in the slides we could denote $S = A \cup B$ where A represents the event where the stock price increases while B represents the event it doesn't increase, and if we have $\mathbb{P}(A) > 1/2$ then we can say it is more likely to increase.

Probability is a special function

$$\mathbb{P}(\emptyset) = 0;$$
$$\mathbb{P}(S) = 1;$$
$$\mathbb{P}\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}\left(A_i\right).$$

where $A_i \cap A_j = \emptyset, i \neq j$. The right-hand side of the third equation (countable additivity) involves a summable/convergent infinite series.

Return to the example, $S = A \cup B$ where A represents the event where the stock price increases while B represents the event it doesn't increase. Here, apparently we would have

$$\mathbb{P}\left(A \cup B\right) = \mathbb{P}\left(A\right) + \mathbb{P}\left(B\right)$$

Probability is a function with special properties.