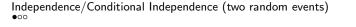
Probability Theory for EOR Random variables and their distributions

Probability Theory for EOR 1 of 4



Independence means products

- ► Sometimes, 'additional' information *B* does *not* change the probability of an event *A*.
- ► In cases with P(B) > 0, P(A|B) = P(A). What does this imply? If P(A) > 0, P(B|A) = P(B).

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A)P(B)$$

$$(if P(A) > 0) \Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B)$$

► We say that the events *A* and *B* are independent.

A and B are independent if

$$P(A \cap B) = P(A)P(B)$$
.

Quiz: $P(\{\text{fist coin head}\}) = 1/4 = P(\{\text{fist coin head}\}) P(\{\text{second coin head}\})$.

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Conditional independence means products

► A and B are independent if

$$P(A \cap B) = P(A)P(B).$$

► Conditional independence!: A and B are independent conditional on/given C if

$$P(A \cap B|C) = P(A|C)P(B|C)$$

► Knowing that *B* (or *A*) also occured does not provide any extra information on the probability of *A* (or *B*) once we already learn that *C* occured:

$$P(A|C,B) = \frac{P(A \cap B|C)}{P(B|C)} = \frac{P(A|C)P(B|C)}{P(B|C)} = P(A|C)$$

$$P(B|C,A) = \frac{P(A \cap B|C)}{P(A|C)} = \frac{P(A|C)P(B|C)}{P(A|C)} = P(B|C)$$

 $P(A|C,B) := P(A|C\cap B) = \frac{P(A\cap(C\cap B))}{P(C\cap B)} = \frac{P((A\cap B)\cap C))}{P(B\cap C)} = \frac{P((A\cap B)\cap C))}{P(B\cap C)} = \frac{P(A\cap B)\cap C)}{P(B\cap C)} = \frac{P(A\cap B)\cap C)}{P(B\cap C)}.$ When we write P(A|B) down, we always assume P(B) > 0. So here $P(B\cap C) > 0$ as well.

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