

Layman's talk : what is random variable

Random variables are functions

Introduce random variables via a simple example

Which one is easier to work with in a coin-flipping game?

- ▶ $A_{1i} = \{i\text{th throw gets a Head}\}$, $A_{2i} = \{i\text{th throw gets a Bottom}\}$;
- ▶ $X_i = 1$ if $i\text{th throw gets a Head}$, and $X_i = 0$ if $i\text{th throw gets a Bottom}$.

With the second notation, we can do more things in easier ways. E.g., think about what would happen if we let n increase in the following fomula

$$\frac{1}{n} \sum_{i=1}^n X_i,$$

this average is more likely to be closer and closer to $\frac{1}{2}$ as $n \rightarrow \infty$.

Now we can work with numbers!

Introduce random variables via a simple example

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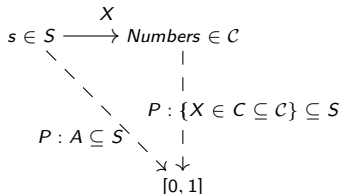
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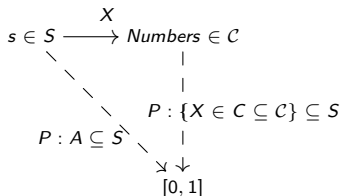
- ▶ Let $\{X_i = 1\}$ denote the subset of the sample space S such that for $s \in \{X_i = 1\}$, $X_i(s) = 1$.
- ▶ $P(X_i = 1) = P(A_{1i}) = 1/2$.

X maps elements from S to numbers! Now we can work with functions/numbers!



- ▶ X essentially is a function, and when we manipulate random variables, it would be similar to manipulating functions and there are so many transformations we can work with: summation, division, max, min...

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- ▶ We can now use $\{X \in \mathcal{C}\}$ with $\mathcal{C} \subseteq \mathcal{C}$ to represent different random events, and properties derived for random events would be inherited.
- ▶ Given the introduction of these additional structures, we have a larger space to explore: conditional probability/independence based on random variables, random variables with specific probability functions...

Random variables are functions, there are selected functions with nice properties, and so are some selected random variables.