# Probability Theory for EOR

## Some special discrete random variables

Some **discrete random variables** are associated very special/ubiquitous **distributions (PMFs)**, they get their own names!

## Definition

The **probability mass function (PMF)** of a **discrete** r.v., $X : S \mapsto \{x_1, x_2, \cdots\}$, is the function:

$$p_X(x) = P(X = x).$$

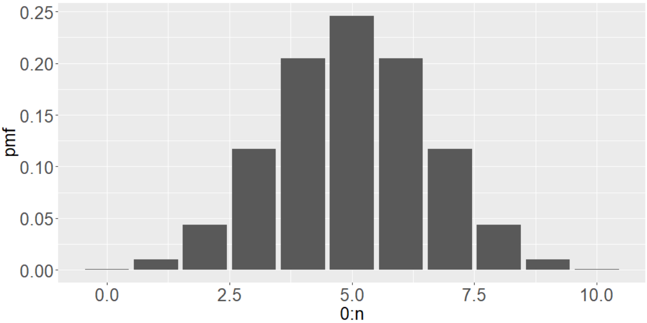
# Binomial: Bin$(n, p)$

- Throw a special coin $n$ times (with probability $p$ getting head in each flip), let $X$ denotes the number of heads in $n$ throws.
- $X \sim$Bin$(n, p)$.
- The PMF of $X$ is

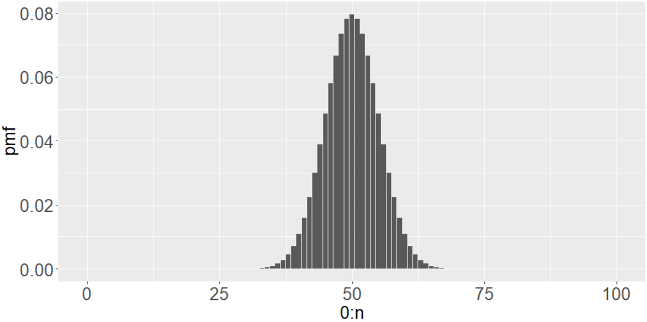
$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}; k = 0, 1, \cdots, n.$$

- Bin$(1, p)$ is also called Bernolli distribution Bern$(p)$.

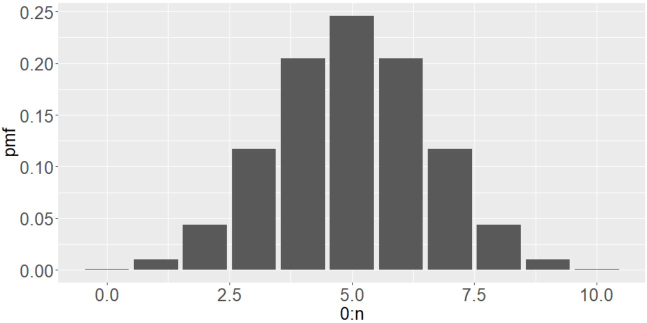
## Some Binomial PMFs in R ($n = 10$, $p = 0.5$)



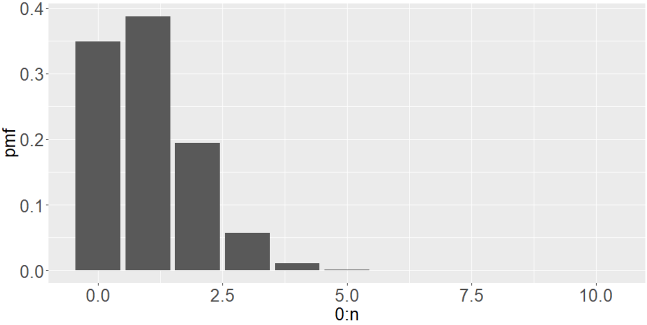
## Some Binomial PMFs in R ($n = 100$, $p = 0.5$)



## Some Binomial PMFs in R ($n = 10$, $p = 0.5$)



## Some Binomial PMFs in R ($n = 10$, $p = 0.1$)



# Discrete uniform: Dunif($C$)

- $C$ contains $n$ different numbers ($|C| = n$), Throw a n-sided fair die and each side is marked with a different number in $C$. Denote $X$ the number of one die roll, and $\{X = x\}$ for any $x \in C$ are equally likely to occur.
- $X \sim$Dunif($C$).
- The PMF of $X$ is

$$P(X = x) = 1/n; x \in C.$$

- What is $P(X \in \{x_1, x_2\}), x_1 \neq x_2, x_1, x_2 \in C$?

  $2/n$.
- Bin$(1, 0.5)$ also a Dunif$(\{0, 1\})$.

# Hypogeometric: HGeom$(w, b, n)$

- Mark $n$ different balls with $*$'s from a urn of $w$ white balls and $b$ **black** balls (each ball is equally likely to be marked), and $X$ is the number of balls being both white and marked with $*$'s.
- $X \sim$HGeom$(w, b, n)$.
- The PMF of $X$ is

$$P(X = k) = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}; \max\{n-b, 0\} \leq k \leq \min\{n, w\}.$$

- $X \sim$HGeom$(w, b, n)$ and $Y \sim$HGeom$(n, w+b-n, w)$ have the identical distribution:

$$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}} = \frac{\binom{n}{k}\binom{w+b-n}{w-k}}{\binom{w+b}{w}}$$

Mark $w$ different balls with white from a urn of $n$ balls with $*$'s and $w+b-n$ balls without $*$'s (each ball is equally likely to be marked), and $Y$ is the number of balls being both white and marked with $*$'s.