Probability Theory for EOR

Random variables and their distributions

Independence/Conditional Independence (two random events)

Independence means products

- Sometimes, 'additional' information B does *not* change the probability of an event A.
- In cases with $P(B) > 0$, $P(A|B) = P(A)$.
  What does this imply? If $P(A) > 0$, $P(B|A) = P(B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A)P(B)$$

$$(\mathit{if}\ P(A) > 0) \Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B)$$

- We say that the events A and B are independent.

A and B are independent if

$$P(A \cap B) = P(A)P(B).$$

Quiz: P({fist coin head} and {second coin head}) =1/4 =
P({fist coin head}) P({second coin head}).

Conditional independence means products

- A and B are independent if

$$P(A \cap B) = P(A)P(B).$$

- Conditional independence!: A and B are independent **conditional on/given** C if

$$P(A \cap B|C) = P(A|C)P(B|C)$$

- Knowing that B (or A) also occured does not provide any extra information on the probability of A (or B) once we already learn that C occured:

$$P(A|C,B) = \frac{P(A \cap B|C)}{P(B|C)} = \frac{P(A|C)P(B|C)}{P(B|C)} = P(A|C)$$
$$P(B|C,A) = \frac{P(A \cap B|C)}{P(A|C)} = \frac{P(A|C)P(B|C)}{P(A|C)} = P(B|C)$$

$P(A|C,B) := P(A|C \cap B) = \frac{P(A\cap(C\cap B))}{P(C\cap B)} = \frac{P((A\cap B)\cap C))}{P(B\cap C)} = \frac{P((A\cap B)\cap C))/P(C)}{P(B\cap C)/P(C)} = \frac{P(A\cap B|C))}{P(B|C)}$.
When we write $P(A|B)$ down, we always assume $P(B) > 0$. So here $P(B \cap C) > 0$ as well.