Layman's talk : what is conditional probability

Conditional probability is also a probability (function)

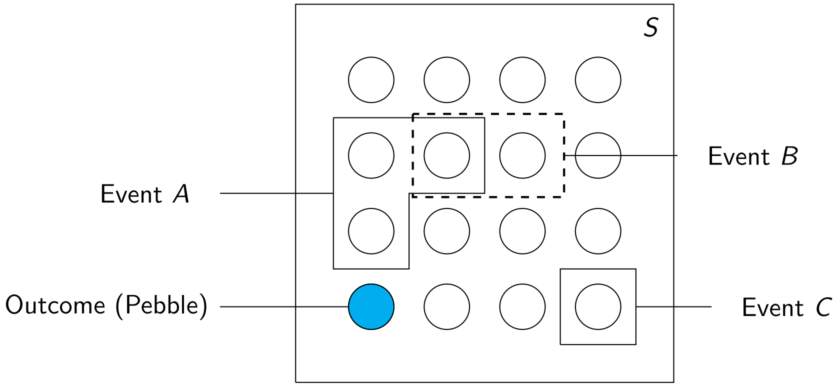
Suppose we have already had a probability function maps a collection of events to numbers within $[0, 1]$.

How do we quantify the uncertainty if we know an event B ($P(B) > 0$) already happen?

Conditional probability (also a probability) maps again events to numbers within $[0, 1]$.

Example

Each pebble is equally likely to be chosen.



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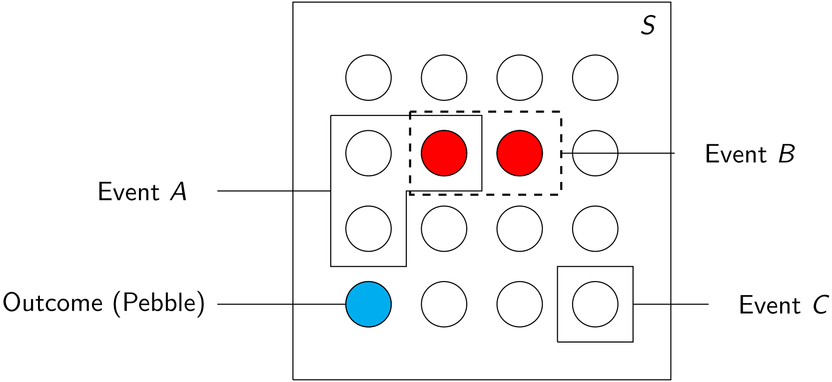

If we know B occured, what can you say about event A and C?
$P(A|B) = 1/2$, $P(C|B) = 0$, $P(B|B) = 1$ (B is like our new sample space.)

$$\mathbb{P}(\cdot|B): \quad \{A_i \subseteq S, i \in I\} \mapsto [0,1]$$
$$A \mapsto P(A \cap B)/P(B)$$

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Measure how likely A occurred given the B already occurred ($P(B) > 0$).
(Informally, the percentage of outcomes in B that also in A.)

How the conditional probability/probability are linked to each other?

$$P(A|B) = P(A \cap B)/P(B) \ (*)$$

- Bayes' rule:
- $$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$
- LOTP:
- $$P(A) = \sum_i P(A|B_i)P(B_i); \ \cup_i B_i = S, B_i \cap B_j = \emptyset, (i \neq j).$$

Independence means products.

Think about throwing two fair coins separately, knowing the results of the first coin does not provide any information on the second one.

With $P(B) > 0$:

$$P(A|B) = P(A).$$

Eqivalently,

$$P(A \cap B) = P(A)P(B).$$

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