Rexample

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Bollywood Box Office Data

Movie Budgets (X) Box Office Grosses (Y)

Residual vs. fitted

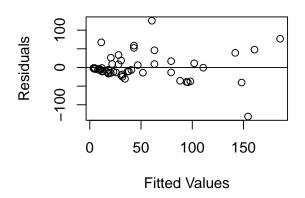
```
library(car) # for significance tests
bbo <- read.csv("https://raw.githubusercontent.com/lingxiaozhou/STA4210Rmaterial/main/data/bollywood_bo.
    header = T)
attach(bbo)
names(bbo)
## [1] "Movie" "Gross" "Budget"
bbo.reg1 <- lm(Gross ~ Budget)</pre>
summary(bbo.reg1)
##
## Call:
## lm(formula = Gross ~ Budget)
## Residuals:
        Min
                  1Q
                       Median
                                    3Q
## -131.349 -14.114
                       -6.371
                                 9.195 125.236
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.9549
                         7.2385 -0.270
                                              0.788
                 1.2510
## Budget
                            0.1359
                                    9.204 1.41e-12 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 36.51 on 53 degrees of freedom
## Multiple R-squared: 0.6151, Adjusted R-squared: 0.6079
## F-statistic: 84.71 on 1 and 53 DF, p-value: 1.411e-12
e1 <- residuals(bbo.reg1)</pre>
yhat1 <- predict(bbo.reg1)</pre>
par(mfrow = c(1, 2))
```

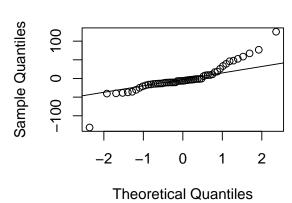
```
plot(yhat1, e1, main = "Residuals vs Fitted Values", xlab = "Fitted Values",
     ylab = "Residuals")
abline(h = 0)

# QQ plot
qqnorm(e1)
qqline(e1)
```

Residuals vs Fitted Values

Normal Q-Q Plot





```
shapiro.test(e1) # Shapiro-Wilk
```

```
##
## Shapiro-Wilk normality test
##
## data: e1
## W = 0.87003, p-value = 2.627e-05
```

ncvTest(bbo.reg1) # Breusch-Pagan

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 32.22785, Df = 1, p = 1.3711e-08
```

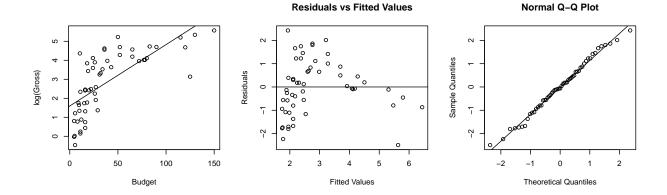
• Diagnostic plots and the significance tests indicate that there are violations to normality and constant variance assumption.

Log transformation of Y

```
bbo.reg2 <- lm(log(Gross) ~ Budget)
summary(bbo.reg2)</pre>
```

##

```
## Call:
## lm(formula = log(Gross) ~ Budget)
##
## Residuals:
##
                  1Q
                       Median
                                    3Q
## -2.49440 -0.79585 -0.06396 0.76221 2.42628
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
                           0.23114
                                      6.908 6.34e-09 ***
## (Intercept) 1.59676
## Budget
                0.03229
                           0.00434
                                     7.439 8.86e-10 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 1.166 on 53 degrees of freedom
## Multiple R-squared: 0.5108, Adjusted R-squared: 0.5016
## F-statistic: 55.34 on 1 and 53 DF, p-value: 8.859e-10
e2 <- residuals(bbo.reg2)</pre>
yhat2 <- predict(bbo.reg2)</pre>
par(mfrow = c(1, 3))
# Scatter plot
plot(Budget, log(Gross))
abline(bbo.reg2)
# Residual vs. fitted
plot(yhat2, e2, main = "Residuals vs Fitted Values", xlab = "Fitted Values",
    ylab = "Residuals")
abline(h = 0)
# QQ plot
qqnorm(e2)
qqline(e2)
```



```
shapiro.test(e2) # Shapiro-Wilk
```

```
## Shapiro-Wilk normality test
##
## data: e2
## W = 0.98861, p-value = 0.8803

ncvTest(bbo.reg2) # Breusch-Pagan

## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 0.6531736, Df = 1, p = 0.41898
```

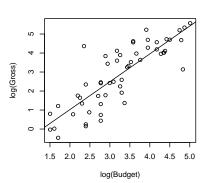
• Based on the Shapiro-Wilk test (Normality) and the Breusch-Pagan test (Constant Variance), the new model appears to be better. However, the linearity assumption is violated based on the residuals vs. fitted plot.

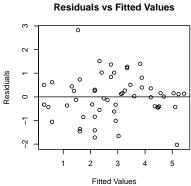
Log transformation of x and Y

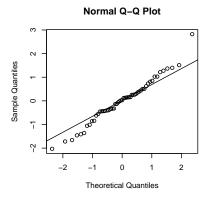
```
bbo.reg3 <- lm(log(Gross) ~ log(Budget))</pre>
summary(bbo.reg3)
##
## Call:
## lm(formula = log(Gross) ~ log(Budget))
##
## Residuals:
       Min
                1Q Median
                                3Q
                                       Max
## -2.0288 -0.4361 0.0317 0.4726 2.8222
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -1.9038
                           0.4489 -4.241 8.95e-05 ***
## log(Budget)
                            0.1327 11.034 2.41e-15 ***
                 1.4645
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 0.9181 on 53 degrees of freedom
## Multiple R-squared: 0.6967, Adjusted R-squared: 0.691
## F-statistic: 121.7 on 1 and 53 DF, p-value: 2.411e-15
e3 <- residuals(bbo.reg3)
yhat3 <- predict(bbo.reg3)</pre>
par(mfrow = c(1, 3))
# Scatter plot
plot(log(Budget), log(Gross))
abline(bbo.reg3)
# Residual vs. fitted
plot(yhat3, e3, main = "Residuals vs Fitted Values", xlab = "Fitted Values",
   ylab = "Residuals")
```

```
abline(h = 0)

# QQ plot
qqnorm(e3)
qqline(e3)
```







shapiro.test(e3) # Shapiro-Wilk

```
##
## Shapiro-Wilk normality test
##
## data: e3
## W = 0.97997, p-value = 0.4866
```

ncvTest(bbo.reg3) # Breusch-Pagan

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 1.105583, Df = 1, p = 0.29304
```

Box-Cox transformation

$$\bullet \ \ W = \begin{cases} \frac{Y^{\lambda}-1}{\lambda} & \lambda \neq 0 \\ \ln(Y) & \lambda = 0 \end{cases}$$

```
library(MASS) # for boxcox
bbo.reg4 <- lm(Gross ~ Budget)
bc <- boxcox(bbo.reg4, plotit = T)</pre>
```

```
lambda <- bc$x[which.max(bc$y)]
lambda</pre>
```

[1] 0.222222

```
# fit new linear regression model using the Box-Cox
# transformation
bbo.reg.bc <- lm(((Gross^lambda - 1)/lambda) ~ Budget)</pre>
```

- The optimal λ is 0.22
- The procedure chooses a "quarter root" transformation for Y. We will not pursue that here, as we have seen that log transformations of Y and X work quite well.

Lowess

- Nonparametric method of obtaining a smooth plot of the regression relation between Y and X
- Fits regression in small neighborhoods around points along the regression line on the X axis
- Weights observations closer to the specific point higher than more distant points
- Re-weights after fitting, putting lower weights on larger residuals (in absolute value)
- Obtains fitted value for each point after "final" regression is fit
- Model is plotted along with linear fit, and confidence bands, linear fit is good if lowess lies within bands

```
x_seq = seq(20, 120, by = 1)
fit1 = loess(workhrs ~ lotsize, span = 0.5, data = toluca) # span controls the size of the neighborhoo
predl = predict(fit1, x_seq, se = TRUE) # Get predicted y values for x_seq based on lowess model

plot(toluca)
lines(x_seq, predl$fit, lty = 1, col = "darkred")
lines(x_seq, predl$fit - 1.96 * predl$se.fit, lty = 2, col = "blue",
    lwd = 1)
lines(x_seq, predl$fit + 1.96 * predl$se.fit, lty = 2, col = "blue",
    lwd = 1)

polygon(c(x_seq, rev(x_seq)), c(predl$fit + 1.96 * predl$se.fit,
    rev(predl$fit - 1.96 * predl$se.fit)), col = "#00009933",
    border = "NA")

abline(toluca.reg, col = "darkgreen")
legend("bottomright", legend = c("Loess", "95% CB", "SLR"), col = c("darkred",
    "blue", "darkgreen"), lty = c(1, 2, 1), bty = "n")
```

