Investigating the role of the cosmological constant in gravitational lensing using a numerical approach

Lingyi Hu

Introduction: motivation

Key question

- Does the cosmological constant directly affect gravitational lensing?

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Motivation

- Both the cosmological constant and gravitational lensing form important parts of our understanding of the universe
- Can become important for future precision cosmology measurements
- Most approaches in literature have been analytical rather than numerical

Introduction: overview of the literature

Key question

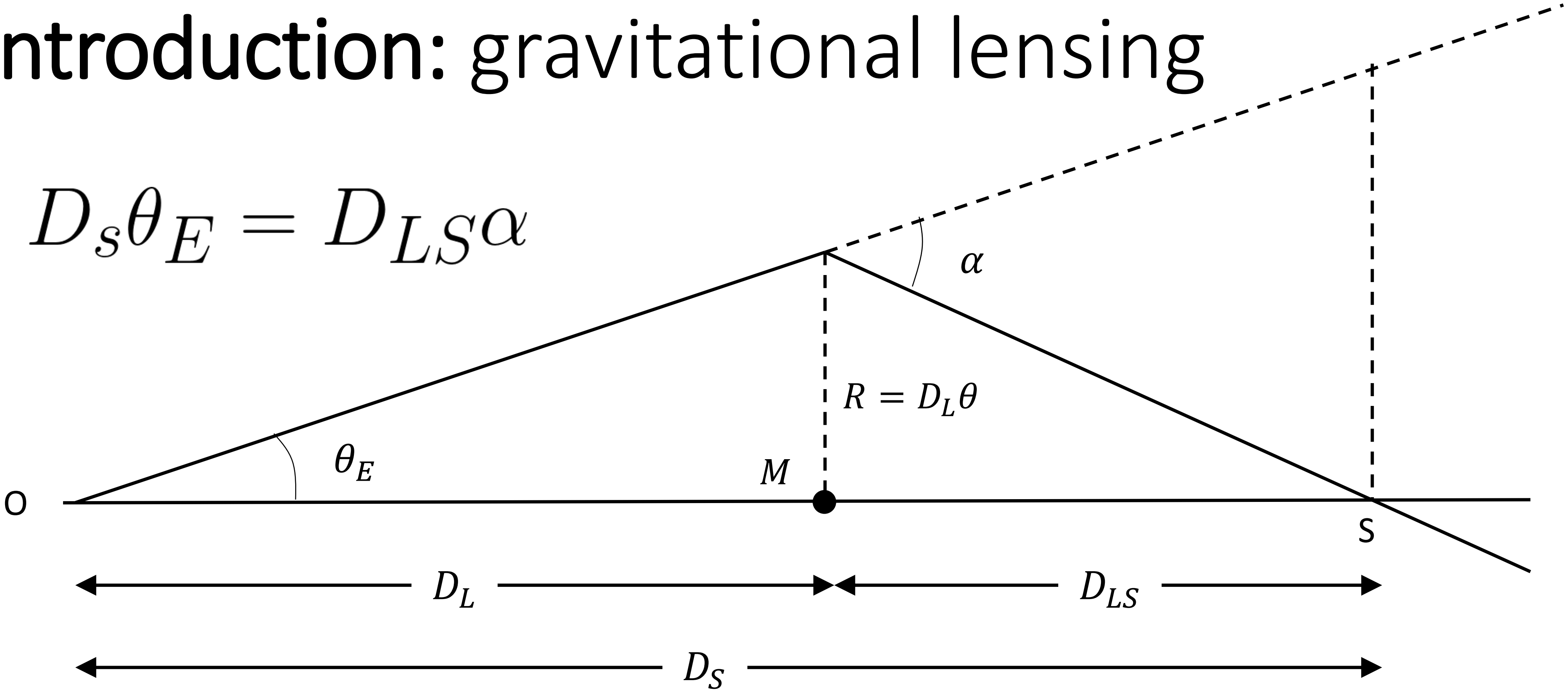
- Does the cosmological constant directly affect gravitational lensing?

Key papers

- Islam, 1983 (Conventional view)
- Rindler and Ishak, 2007 (Challenging the conventional view)

Introduction: gravitational lensing

$$D_s\theta_E = D_{LS}\alpha$$



Introduction: gravitational lensing

$$\alpha_{\mathrm{FRW}} = 4\frac{M}{R} + \frac{15\pi}{4}\left(\frac{M}{R}\right)^2 + \frac{401}{12}\left(\frac{M}{R}\right)^3$$

Introduction: gravitational lensing

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$$\begin{aligned}\alpha_{\mathrm{Kantowski}} = \left(\frac{r_s}{2r_0}\right)\cos\tilde{\phi}_1 & \left[-4\cos^2\tilde{\phi}_1 - 12\,\cos\tilde{\phi}_1\sin\tilde{\phi}_1\sqrt{\frac{\Lambda r_0^2}{3} + \frac{r_s}{r_0}\,\sin^3\tilde{\phi}_1} + \Lambda r_0^2\left(\frac{8}{3} - \frac{20}{3}\sin^2\tilde{\phi}_1\right)\right] \\ & + \left(\frac{r_s}{2r_0}\right)^2\left[\frac{15}{4}(2\tilde{\phi}_1 - \pi) + \cos\tilde{\phi}_1\left(4 + \frac{33}{2}\sin\tilde{\phi}_1 - 4\sin^2\tilde{\phi}_1 + 19\sin^3\tilde{\phi}_1 - 64\sin^5\tilde{\phi}_1\right)\right. \\ & \left. -12\log\left\{\tan\frac{\tilde{\phi}_1}{2}\right\}\sin^3\tilde{\phi}_1\right] + \mathcal{O}\left(\frac{r_s}{r_0} + \Lambda r_0^2\right)^{5/2}\end{aligned}$$

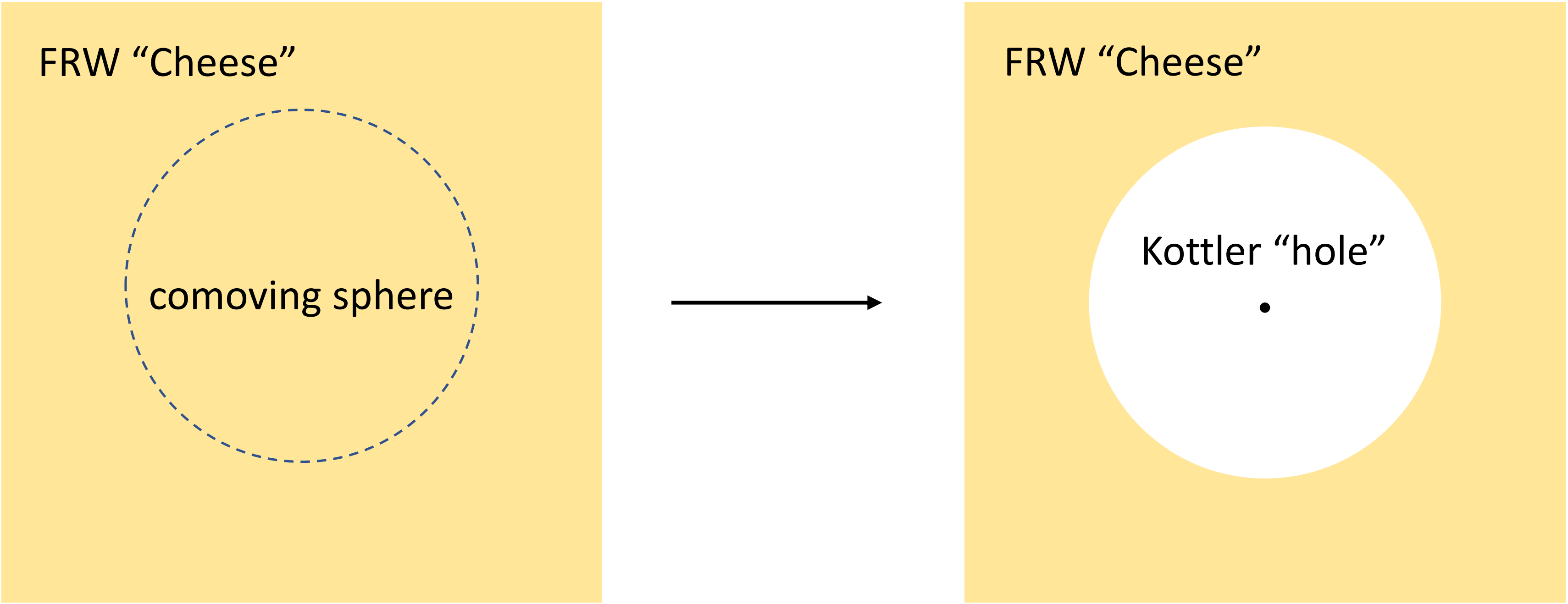
Method: our approach

Numerical approach in a **Swiss-Cheese model**

Previous similar research:

- Schücker (2009): partially numerical approach in a Swiss-Cheese model
- Kantowski et al (2010) analytical estimation of the effect in a Swiss Cheese universe
- Aghili et al (2017) investigated numerical approach in a McVittie metric

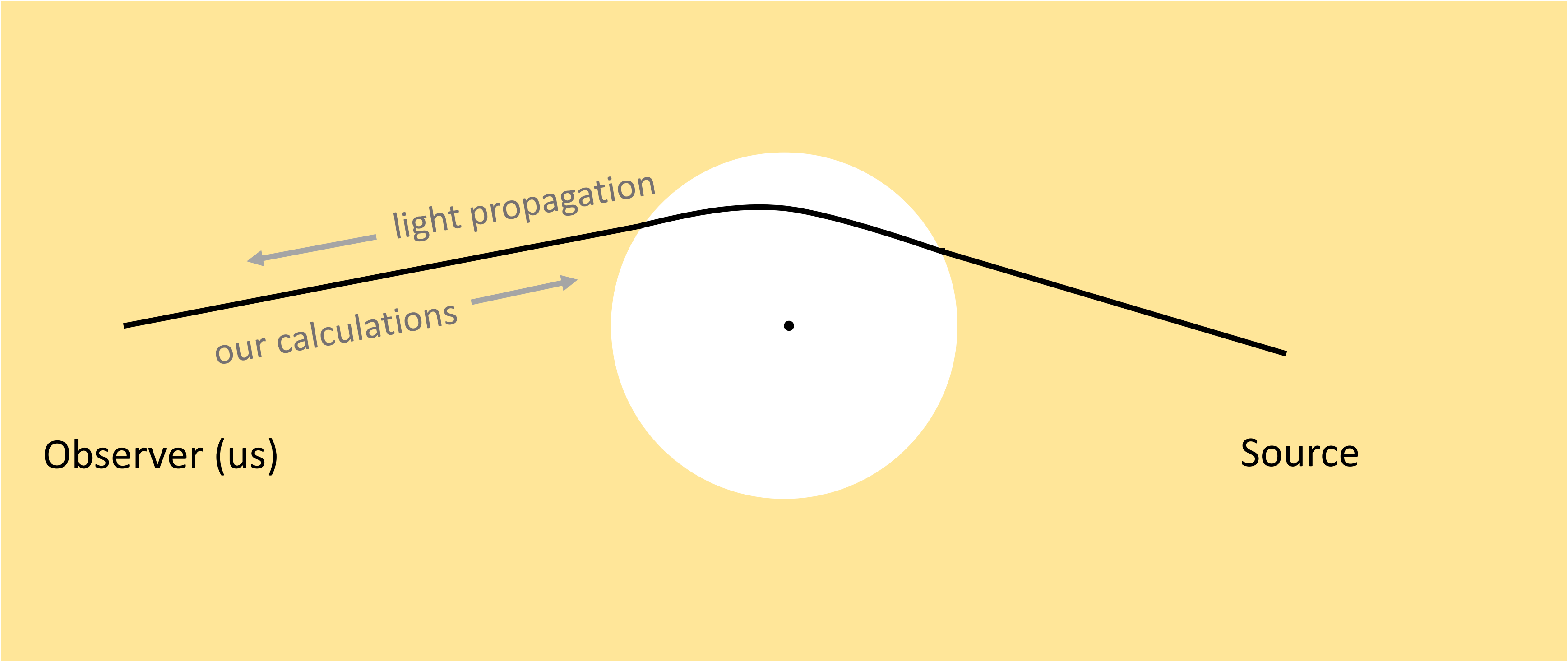
The Swiss Cheese Model



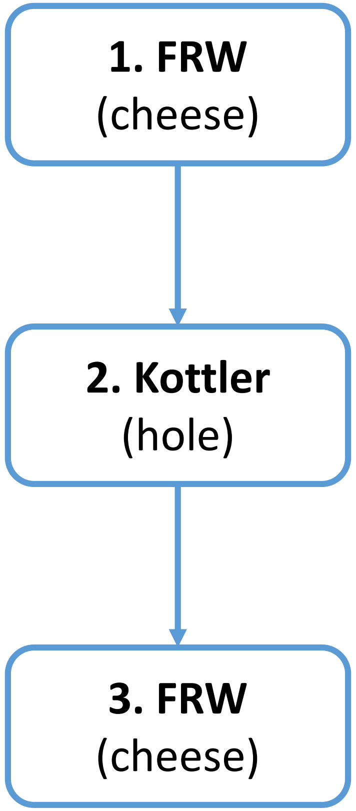
Cheese:
- Homogeneous and expanding
- Filled with pressureless matter (dust)

Hole:
- Point mass at the centre
- Vacuum everywhere else with a cosmological constant

Propagation of light



Propagation of light: an overview



General method

To find light path in each region,

1. Write down the metric
2. Calculate the Christoffel symbols and hence the equations of motion using

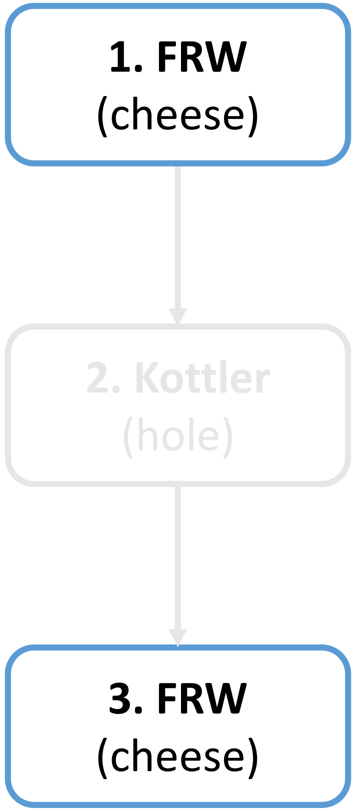
$$\ddot{x}^{\mu} + \Gamma^{\mu}_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta} = 0$$

   and the null condition

$$ds^2 = 0$$

3. Solve the resulting differential equations (numerically or analytically)

Propagation of light



Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

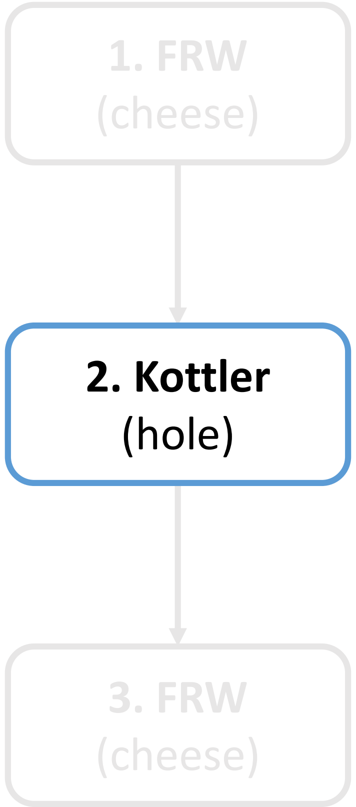
Null geodesics

$$\ddot{r} = (1 - kr^2) r \dot{\phi}^2 - \frac{k\dot{r}^2}{1 - kr^2} - \frac{2a_{,t}}{a} \dot{r}\dot{t}$$

$$\dot{\phi} = \frac{L}{a^2 r^2}$$

$$a_{,t} = aH_0 \sqrt{\Omega_M / a^3 + \Omega_k / a^2 + \Omega_\Lambda}$$

Propagation of light



Kottler metric

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

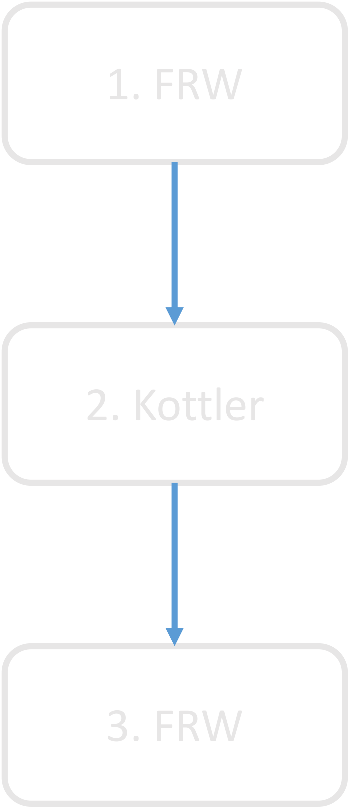
where $f(R) = 1 - \dfrac{2M}{R} - \dfrac{\Lambda R^2}{3}$

Null geodesics

$$\ddot{R} = \frac{L_{\mathrm{k}}^2(R-3M)}{R^4}$$

$$\dot{\phi} = \frac{L_k}{R^2}$$

Propagation of light



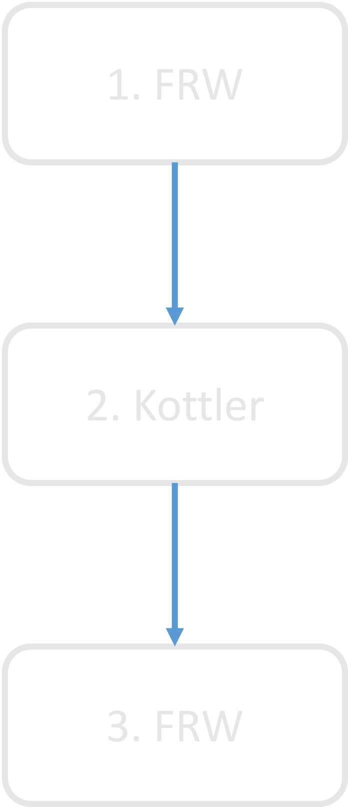
Gluing the two metrics together

A spacetime obtained by gluing two different geometries via a hypersurface Σ is well defined if it satisfies the Israel junction conditions (Israel, 1966):

Both geometries must induce, on Σ,

1. the same 3-metric, and
2. the same extrinsic curvature

Propagation of light



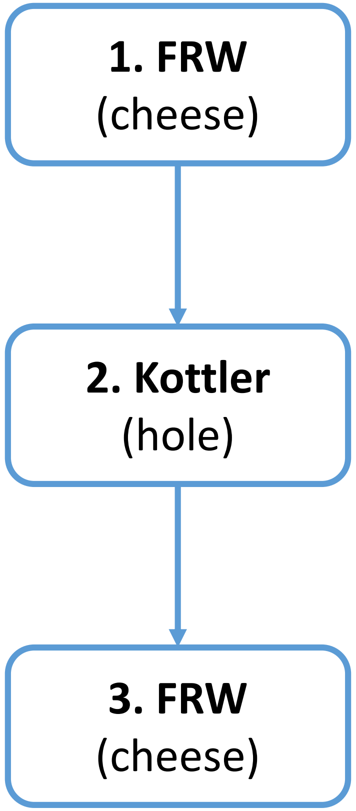
Gluing the two metrics together

A spacetime obtained by gluing two different geometries via a hypersurface Σ is well defined if it satisfies the Israel junction conditions (Israel, 1966):

These junction conditions tell us:

1. The size of the hole is related to the mass by
$$M = \frac{4\pi}{3}\rho a^3 r^3$$
2. The rate of expansion of the hole in Kottler coordinates:
$$R_{h,t} = \left(1 - \frac{2M}{R_h} - \frac{\Lambda R_h^2}{3}\right)\sqrt{\frac{2M}{R_h} + \frac{\Lambda R_h^2}{3}}$$
3. the Jacobian for transforming velocities from one coordinate to the other

Propagation of light: the full picture



- Start off light ray with a fixed θ
- Propagate light rays until it reaches the boundary of the hole
- Convert from FRW coordinates to Kottler coordinates using the Jacobian obtained from matching conditions

- Propagate light rays in hole using null geodesic equations
- At the same time, the boundary of the hole is also changing
- Stop when it has reached the boundary of the hole
- Convert from Kottler coordinates back to FRW coordinates

- Continue propagating light rays until it crosses the axis
- Record the coordinate at which it crosses the axis

Propagation of light: the full picture

1. FRW
(cheese)

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2. Kottler
(hole)

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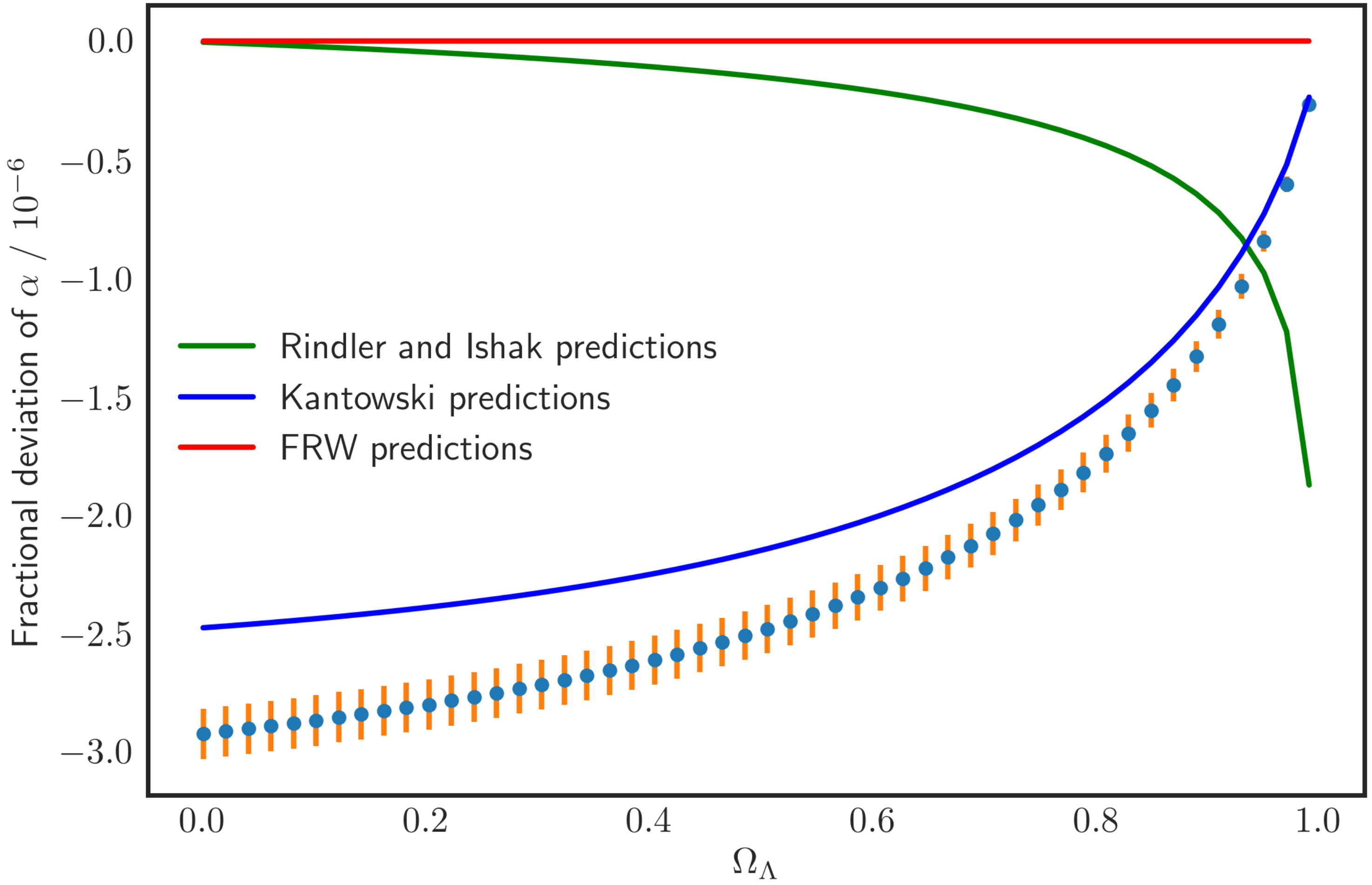
Results

Results: Keeping the mass fixed



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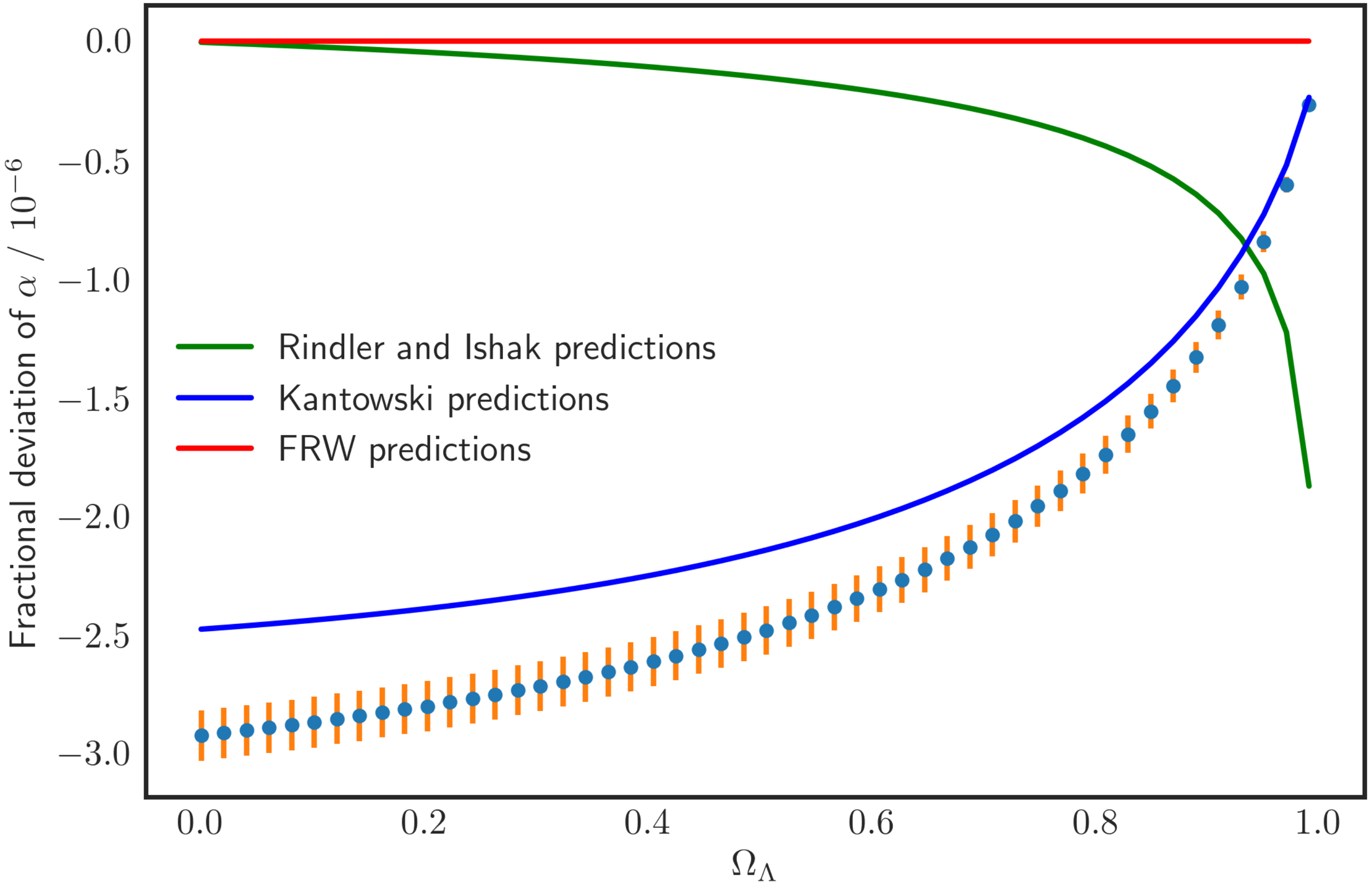

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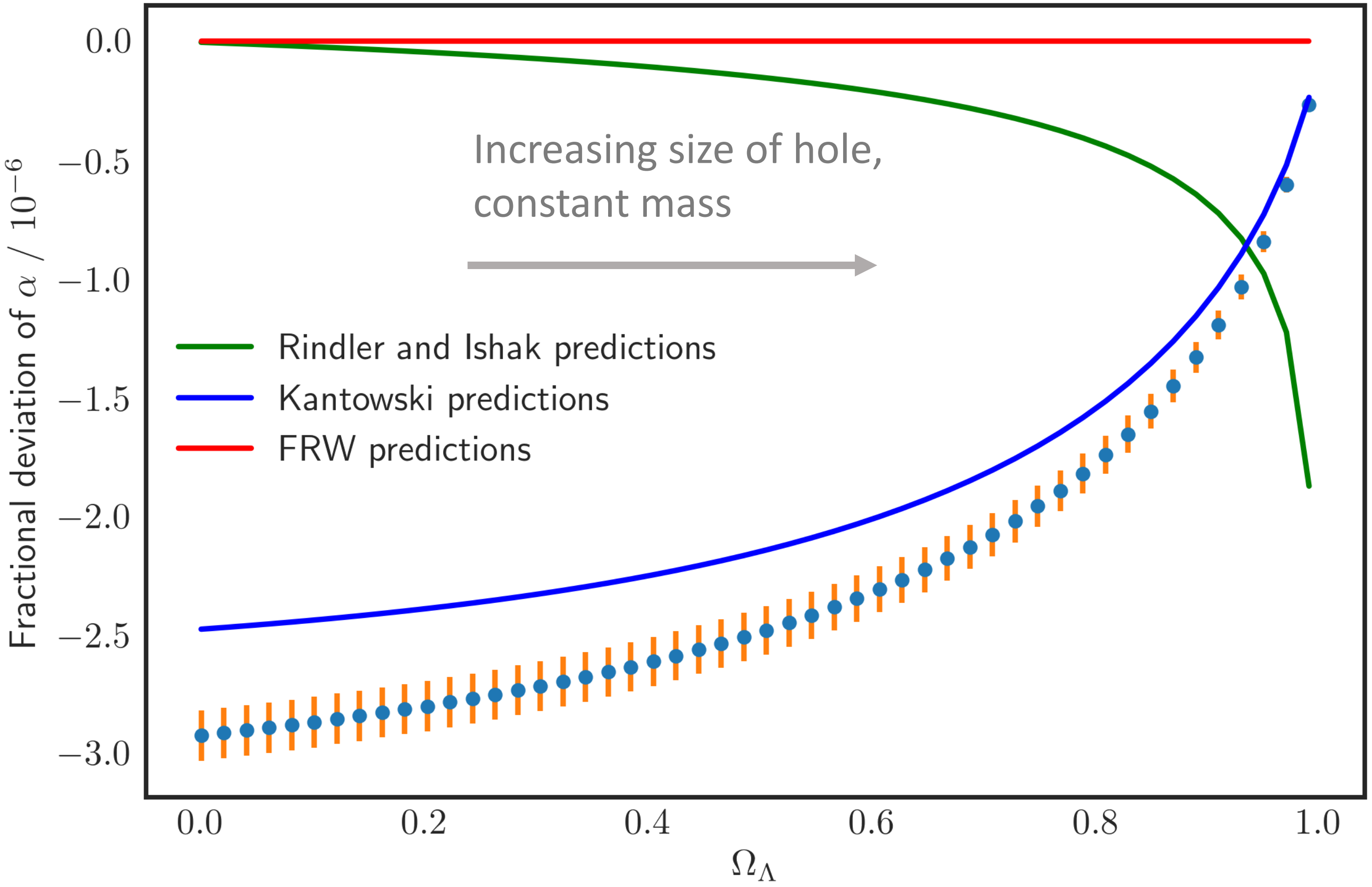

A few factors at play:

- Size of the hole is increased as Λ increases

$$M = \frac{4\pi}{3}\rho a^3 r^3$$

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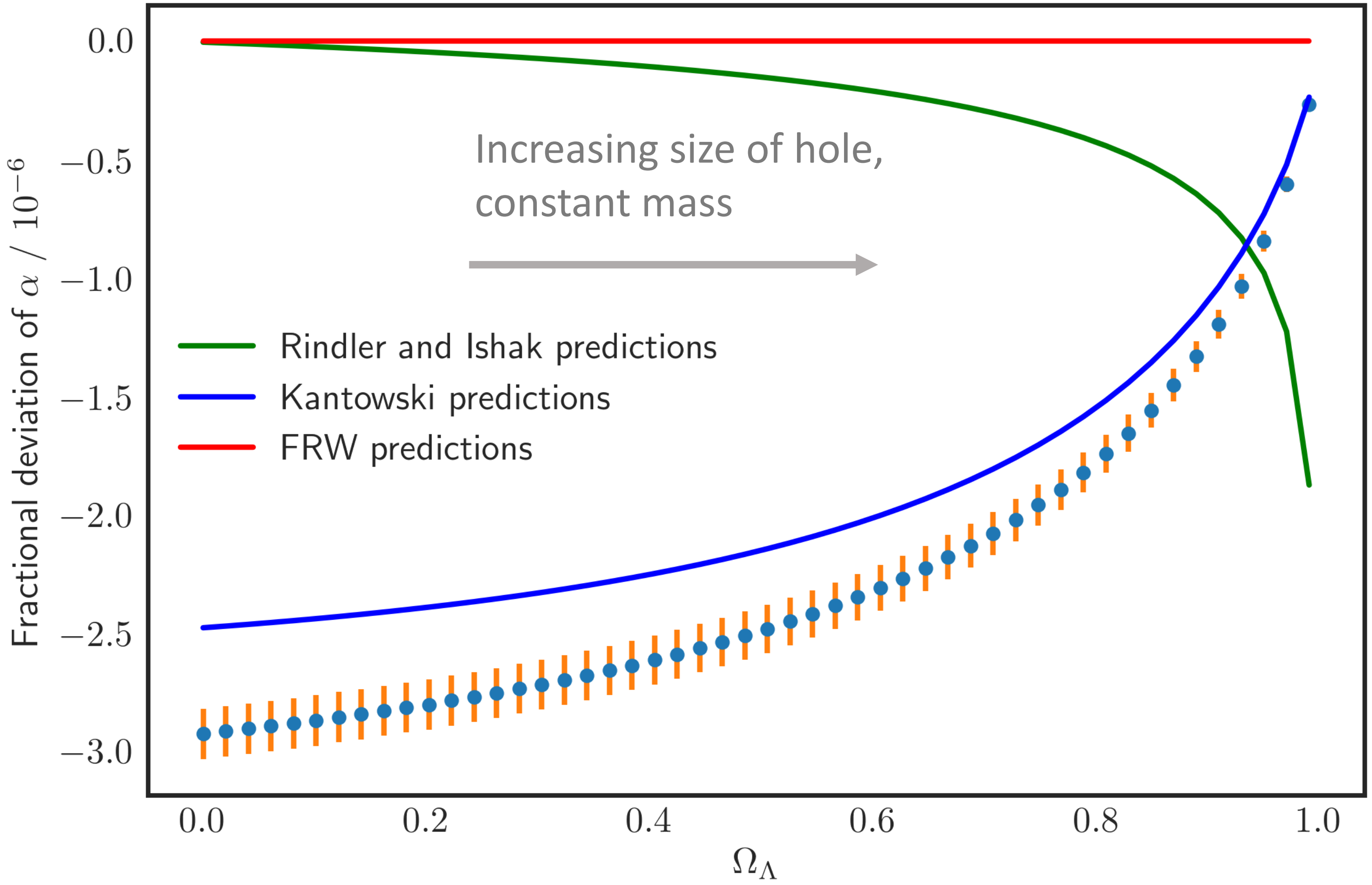

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$$M = \frac{4\pi}{3}\rho a^3 r^3$$

- Expansion rate of the hole boundary depends on Λ

$$R_{h,t} = \left(1 - \frac{2M}{R_h} - \frac{\Lambda R_h^2}{3}\right)\sqrt{\frac{2M}{R_h} + \frac{\Lambda R_h^2}{3}}$$

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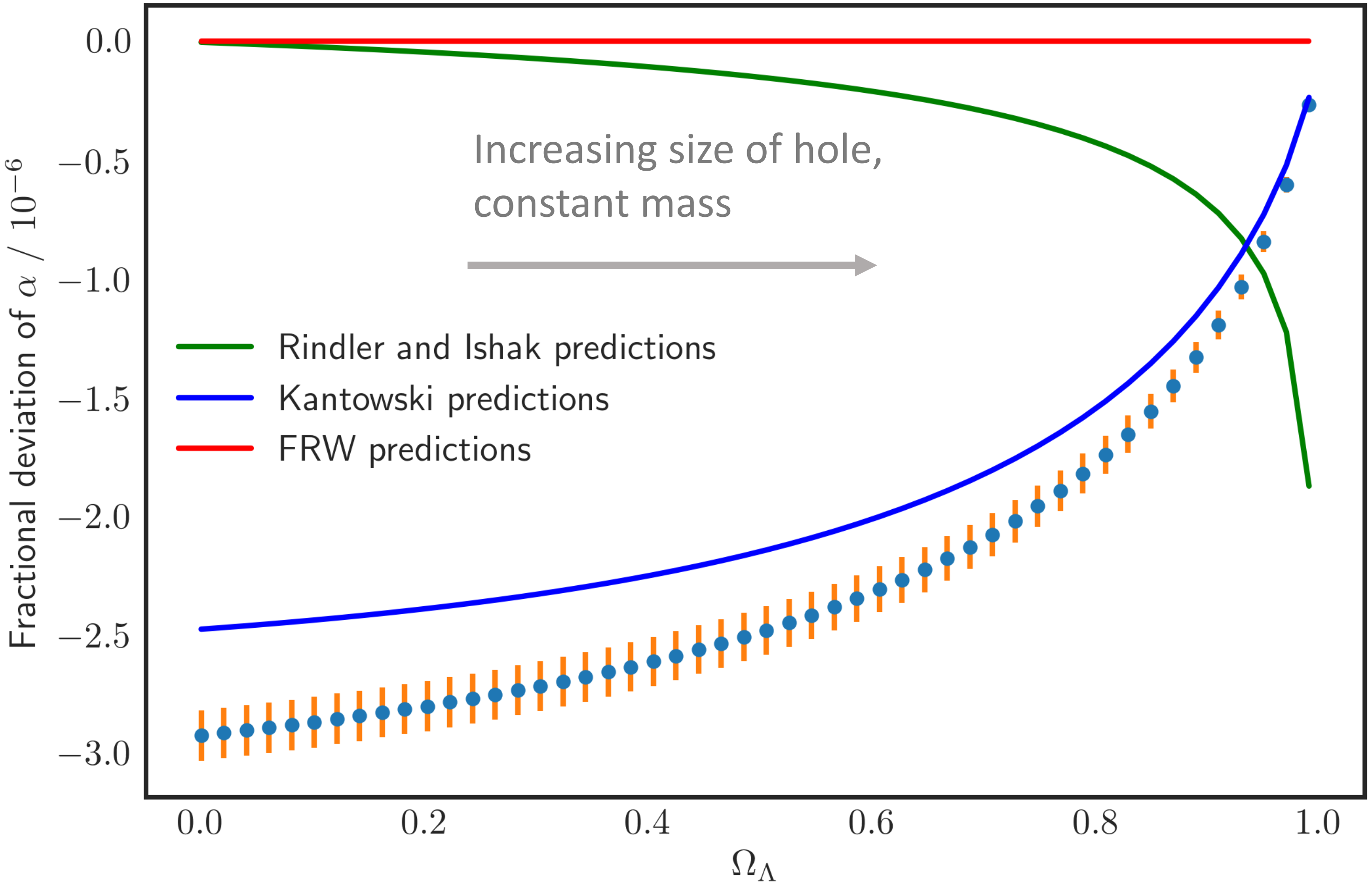

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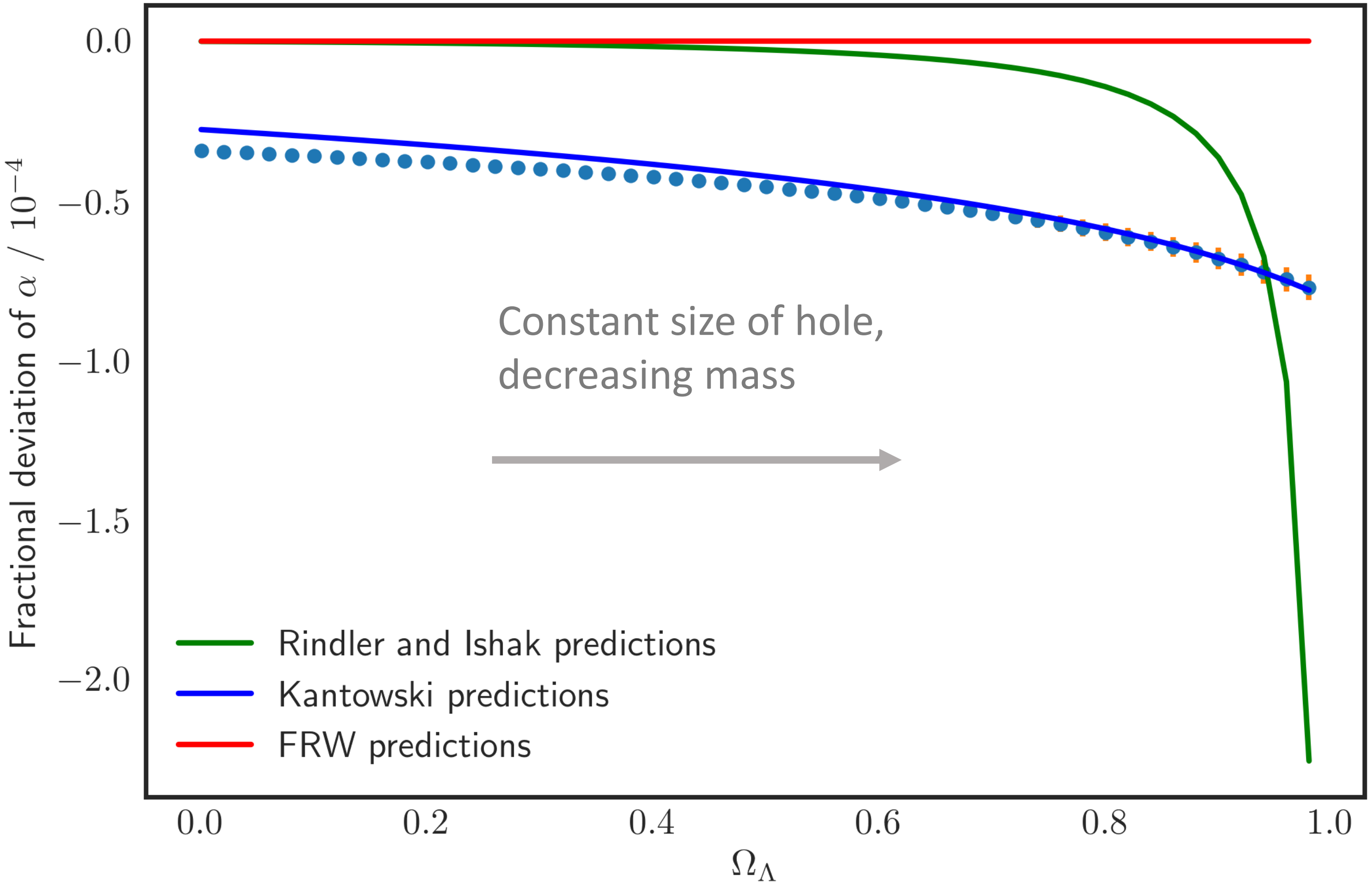
- Jacobian at the boundary depends on Λ

Results: Keeping size of the hole fixed



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- Jacobian at the boundary depends on Λ

Currently working on / future work

- Curved space
- Better estimation of numerical errors
- Extend the model to a general mass distribution instead of a point mass

References

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Thank you!