The role of the cosmological constant in gravitational lensing (Outline)

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1 Introduction

Description of the problem: whether the current gravitational lensing formula require a modification.

Give an overview of the literature, starting with the conventional view (Islam's paper) and then Rindler and Ishak's paper. Outline the main points of disagreement (comoving observers as opposed to static, observable quantities, whether the effect is already accounted for).

Point out other work done on the Swiss Cheese such as Schucker 2009 (numerical), Kantowski 2010 (analytical) and also other numerical work that used a different metric (Aghili 2017, Effect of Accelerated Global Expansion on Bending of Light, used a McVittie metric).

Introduce some formulas and terms used in General Relativity:

- The metric and Einstein field equations
- The geodesic equation
- The light path follows a null geodesic

Introduce the structure of the rest of the report.

2 Gravitational lensing formalism

State the formula for Schwarzschild lensing angle

Derivation of Einstein angle (using a diagram)

Make a note that angular diameter distances already depend on Lambda, the question is whether there is an additional dependence on Lambda that we need to take care of (i.e. do we need to modify the existing formula for lensing).

3 Description of the Swiss Cheese Model

Describe the Swiss Cheese model. It involves embedding one metric in another based on a set of boundary conditions. The mass also has limited influence, i.e. the bending stops after exiting the hole, back into the cheese.

Reasons for using this model:

- It is an exact solution of Einstein's equations
- Influence of the central mass is confined to the size of the hole
- tackles the problem of observations by comoving observers (instead of static)

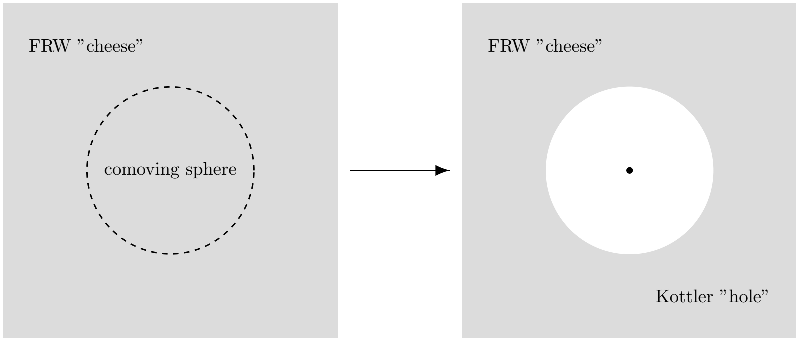

Figure 1: Illustration of a Swiss Cheese model

3.1 Spacetime patches

Write down the FRW and Kottler metrics and explain what the terms mean

3.2 Matching conditions

Matching of induced metric and extrinsic curvature. Deriving the Jacobian for conversions of the velocities between the two coordinate systems at the boundary.

3.3 Light propagation

Light is propagated backwards assuming a certain Einstein angle.

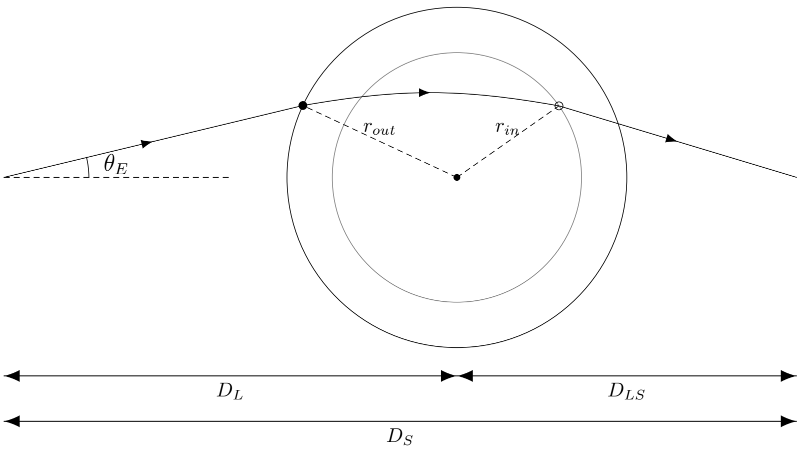

Figure 2: Light propagation in a Swiss Cheese model

3.3.1 In FRW

Friedmann equation, null condition, geodesic equations

Write down the steps for calculating the intersection point with the hole analytically in flat space.

3.3.2 In Kottler

Null condition, geodesic equations

Make a note that the geodesic equations in Kottler are exactly the same as in Schwarzschild (this was why Islam (1983) first wrote that Λ has no effect), but the conversion of velocities at the boundary and the rate of expansion of the boundary of the hole, which moves with the FRW universe, depend on Λ.

3.4 Calculation of the Einstein angle

- Obtaining D_{LS} and D_S from the raw r coordinate used
- Calculate the expected D_S using the conventional lensing formula and using the formula including correction suggested by Rindler and Ishak
- Compare them

3.5 Hole with a generalised mass distribution

Introduce LTB metric. It is a more realistic model as lenses are not point masses but extended objects. It has similar matching conditions at the cheese-hole boundary.

3.5.1 Density profile

State the NFW mass profile and the free parameters. Include plots of pressure, $\rho(r)$ and $M(r)$.

Lensing mass is a function of the distance from centre at the turning point, calculated from thin screen approximation.

4 Results and discussion

4.1 Kottler swiss cheese

Robustness is checked by comparing with the $\Lambda = 0$ case where the lensing formula is known. Integration was repeated for different lens redshifts, and the ratio $r_{\text{numerical}}/r_{\text{expected}}$ was calculated and averaged for each value of Λ.

As varying Λ but keeping a constant M changes the size of the hole, M was varied with Λ to keep the radius of the hole the same, so that the sphere of influence remains constant.

Talk about lensed and unlensed distances (from Luke Butcher's paper) and give an estimate of its magnitude.

4.2 Static LTB swiss cheese

Integration checked against the Schwarzschild / Kottler case when the mass becomes a point mass at the centre, or when the light ray passes far enough such that it does not go through the mass.

5 Conclusion

Summarize what is written above, suggest further work, for example:

- Extend to a non-static LTB in the hole