Deterministic Models
Operations Research (OR 541)
Project Report

Super Chip Company Production And Distribution Operation Recommendations

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The optimization problem we are given is that the Super Chip computer chip company wants to minimize the computer chip production and distribution costs. We are provided with Super Chip company's production capacity, sales region demand, shipping costs, and production costs. Based on the problem description and data provided, we generated the prescriptive model components in Fig. 1, conducted optimization procedures, and generated the solution.

Decision Variables	S = { Alexandria, Richmond, Norfolk, Roanoke, Charlottesville } Djz = z chip's yearly demand in sales region j (thousand units) j = 1,…, 23 z = 1,…, 30 Ci = yearly chip production capacity in facility i (thousand units) i ∈ S Piz = production cost per z chip in facility i ($) i ∈ S z = 1,…, 30 Sijz = shipping cost per z chip from facility i to sales region j ($) i ∈ S j = 1,..., 23 z = 1,..., 30 Xijz = amount of z chips shipped from facility i to sales region j (unit) i ∈ S j = 1,..., 23 z = 1,..., 30 Yiz = amount of z chips produced in facility i (unit) j = 1,…, 23 z = 1,…, 30
Objective Function	Min. $Z = \sum_{i \epsilon S} \sum_{j=1}^{23} \sum_{z=1}^{30} SijzXijz + \sum_{i \epsilon S} \sum_{z=1}^{30} PizYiz$
Subjected To	$\sum_{z=1}^{30} Yiz \leqslant 1000Ci$ i ∈ S $\sum_{i \epsilon S} Xijz = 1000Djz$ j = 1 ,..., 23 z = 1 ,..., 30 $Yiz \geqslant \sum_{j=1}^{23} Xijz$ i ∈ S z = 1 ,..., 30 $Xijz \geqslant 0$ i ∈ S j = 1 ,..., 23 z = 1 ,..., 30 $Yiz \geqslant 0$ i ∈ S z = 1 ,..., 30

Fig. 1. Super Chip Company Cost Reduction Optimization Model

In the optimization model, S represents a set of facility names, i represents a facility in S, j represents a sales region j from 1 to 23, and z represents a type of computer chip from 1 to 30. The objective function is to minimize the total distribution and production costs. The first constraint restricts the summation of all z chips produced in facility i to be less than or equal to the yearly chip production capacity in facility i. The second constraint represents the summation of the amount of each type of z chip shipped from all facilities i to each sales region j needs to be equal to each z chip's yearly demand in sales region j. The third constraint represents the number of each type of z chip produced in each facility i needs to be greater than or equal to the summation of the number of each type of z chip shipped from each facility i to all sales regions j. The last two constraints set lower bounds of the number of z chips shipped from facility i to sales region j and the number of z chips produced in facility i to be 0. The optimal objective function value is shown in Fig 2.

```
Set parameter Username
Academic license - for non-commercial use only - expires 2025-03-25
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x756da770
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Cost =  49083430.39999999
```

Fig. 2. Super Chip Company Cost Reduction Model Optimal Objective Function Value

To solve question 1, we evaluated the current policy's production costs and the proposed alternative policy's production costs based on our model's optimal solution. Currently, a facility produces y% of every chip's total demand where y% is the facility's total production capacity across all facilities. First, we solved "y" and stored it in a "facilityPercentage" dictionary in Fig. 3. With this information, we confirmed the current policy's total production costs to be approximately $52,084,234 by calculating the current amount of chips produced in the facilities. Subsequently, we calculated the production cost of the alternative policy with the optimal solution. The total production costs of the alternative policy is approximately $47,309,680. The total production cost savings is approximately $4,774,553. We identified the percentage of the chip's total demand that a facility is producing, the result suggests that not every facility produces every computer chip, and facilities do not produce a certain percentage of every chip's total demand. For example, the Charlottesville facility only produces 6.46% of chip 1 and 100% of chip 14, Roanoke only produces chips 1, 8, 20, 25, and 26. In summary, Super Chip should

implement the alternative production policy based on our model's chip production distribution in facilities, because the alternative production policy yields greater production cost savings.

```
Facility Alexandria Production Percentage: 0.25327510917030566%
Facility Richmond Production Percentage: 0.22707423580786026%
Facility Norfolk Production Percentage: 0.21397379912663755%
Facility Roanoke Production Percentage: 0.1572052401746725%
Facility Charolottesville Production Percentage: 0.14847161572052403%
```

Fig. 3. Percentage of Facility's Total Production Capacity Across All Facilities

In response to question 2, we evaluated the total operation costs in every facility assuming the facility received the capital investment. We identified the facility that will benefit the most from expansion by calculating the cost savings in total operation costs after expansion in comparison to the optimal model's total operation costs. Two scenarios are considered: a modest expansion of 50,000 units in facility capacity, and a more substantial expansion of 100,000 units in facility capacity. A facility's production capacity is modified in each evaluation, the model is optimized based on the new production capacity, and optimal production and distribution costs are generated. Expanding the Alexandria facility production capacity does not affect total operation costs and optimal objective function value as shown in Fig. 4 and Fig. 5. As shown in Fig. 6, if expanding the production capacity of the Richmond facility by 50,000 units, we noticed a reduction in total shipping costs in the Alexandria facility by $49,645, in the Roanoke facility by $26,913, and in the Charlottesville facility by $29. Moreover, we observed total production costs reduction of $1,642,686 in the Alexandria facility, $606,457 in the Roanoke facility, and $488 in the Charlottesville facility. The objective function value of the total operation costs was reduced by $23,044, indicating a more optimal outcome. In a more substantial expansion of 100,000 units in Richmond in Fig. 7, we observed a reduction of $23,794 in total operation costs reflected by the objective function value, which is $750 more in cost reduction compared to 50,000 units expansion. Fig. 8 and Fig. 9 show expanding the Norfolk facility does not affect total operation costs and optimal objective function value. Fig. 10, Fig. 11, Fig. 12, and Fig. 13 show expanding Roanoke and Charlottesville facilities does not affect total operation cost and optimal objective function value. In conclusion, the Richmond facility should receive the investment capital.

```
Production Capacitiy After 50,000 Expansion:  {1: 398, 2: 312, 3: 294, 4: 216, 5: 204}
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x95ca7e48
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 4e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Total Shipping Cost Change in Alexandria Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost change in Alexandria Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value with 50,000 Expansion in Alexandria Facility:  0.0
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)
```

Fig. 4. Results for Alexandria Facility Production Capacity Expansion of 50,000 (units)

```
CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0xe857918d
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 4e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Production Capacitiy After 100,000 Expansion:  {1: 448, 2: 312, 3: 294, 4: 216, 5: 204}
Total Shipping Cost Change in Alexandria Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Alexandria Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value with 50,000 Expansion in Alexandria Facility:  0.0
```

Fig. 5. Results for Alexandria Facility Production Capacity Expansion of 100,000 (units)

```
Production Capacitiy after 50,000 Expansion:  {1: 348, 2: 362, 3: 294, 4: 216, 5: 204}
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x4c8231ec
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 4e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.01s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   1.487500e+03   0.000000e+00      0s
       8    4.9060386e+07   0.000000e+00   0.000000e+00      0s

Solved in 8 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.906038600e+07
Optimal Production and Distribution Costs =  49060386.0
Total Shipping Cost Change in Richmond Facility with 50,000 Expansion:  {'Alexandria': -49645.0, 'Richmond': 76656.70000000007, 'Norfolk': 0.0, 'Roanoke': -26913.50000000003, 'Charolottesville': -29.0}
Total Production Cost Change in Richmond Facility with 50,000 Expansion:  {'Alexandria': -1642686.7999999989, 'Richmond': 2226518.5999999996, 'Norfolk': 0.0, 'Roanoke': -606457.0000000005, 'Charolottesville': -488.39999999990687}
Change in Objective Function Value in Richmond Facility with 50,000 Expansion:  -23044.39999999106
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)
```

Fig. 6. Results for Richmond Facility Production Capacity Expansion of 50,000 (units)

```
CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x5dd032cb
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 4e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   0.000000e+00   0.000000e+00      0s
       0    4.9059636e+07   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.90596360e+07
Optimal Production and Distribution Costs =  49059636.199999996
Production Capacitiy after 50,000 Expansion:  {1: 348, 2: 412, 3: 294, 4: 216, 5: 204}
Total Shipping Cost Change in Richmond Facility with 100,000 Expansion:  {'Alexandria': -58539.20000000001, 'Richmond': 94258.00000000023, 'Norfolk': -13020.799999999988, 'Roanoke': -26913.50000000003, 'Charolottesville': -2200.8000000000003}
Total Production Cost Change in Richmond Facility with 100,000 Expansion:  {'Alexandria': -1920463.0, 'Richmond': 2779738.5, 'Norfolk': -233566.40000000224, 'Roanoke': -606457.0000000005, 'Charolottesville': -36630.0}
Change in Objective Function Value in Richmond Facility with 100,000 Expansion:  -23794.19999999553
```

Fig. 7. Results for Richmond Facility Production Capacity Expansion of 100,000 (units)

```
Production Capacitiy after 50,000 Expansion:  {1: 348, 2: 312, 3: 344, 4: 216, 5: 204}
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x4e774d3b
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Total Shipping Cost Change in Norfolk Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Norfolk Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value in Norfolk Facility with 50,000 Expansion:  0.0
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)
```

Fig. 8. Results for Norfolk Facility Production Capacity Expansion of 50,000 (units)

```
CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x56c6181d
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 4e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Production Capacitiy after 100,000 Expansion:  {1: 348, 2: 312, 3: 394, 4: 216, 5: 204}
Total Shipping Cost Change in Norfolk Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Norfolk Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value in Norfolk Facility with 100,000 Expansion:  0.0
```

Fig. 9. Results for Norfolk Facility Production Capacity Expansion of 100,000 (units)

```
Production Capacitiy after 50,000 Expansion:  {1: 348, 2: 312, 3: 294, 4: 266, 5: 204}
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0xad5ba122
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Total Shipping Cost Change in Roanoke Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Roanoke Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value in Roanoke Facility:  0.0
```

Fig. 10. Results for Roanoke Facility Production Capacity Expansion of 50,000 (units)

```
CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x0b0e8e2e
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Production Capacitiy after 100,000 Expansion:  {1: 348, 2: 312, 3: 294, 4: 316, 5: 204}
Total Shipping Cost Change in Roanoke Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Roanoke Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value in Roanoke Facility:  0.0
```

Fig. 11. Results for Roanoke Facility Production Capacity Expansion of 100,000 (units)

```
Production Capacitiy after 50,000 Expansion: {1: 348, 2: 312, 3: 294, 4: 216, 5: 254}
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0xfac570f6
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Total Shipping Cost Change in Charolottesville Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Charolottesville Facility with 50,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value in Charolottesville Facility:  0.0
```

Fig. 12. Results for Charlottesville Facility Production Capacity Expansion of 50,000 (units)

```
CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x477f0174
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.9059636e+07   7.737500e+03   0.000000e+00      0s
      45    4.9083430e+07   0.000000e+00   0.000000e+00      0s

Solved in 45 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.908343040e+07
Optimal Production and Distribution Costs =  49083430.39999999
Production Capacitiy after 100,000 Expansion:  {1: 348, 2: 312, 3: 294, 4: 216, 5: 304}
Total Shipping Cost Change in Charolottesville Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Total Production Cost Change in Charolottesville Facility with 100,000 Expansion:  {'Alexandria': 0.0, 'Richmond': 0.0, 'Norfolk': 0.0, 'Roanoke': 0.0, 'Charolottesville': 0.0}
Change in Objective Function Value in Charolottesville Facility:  0.0
```

Fig. 13. Results for Charlottesville Facility Production Capacity Expansion of 100,000 (units)

Regarding question 3, with the estimated 10% demand increase for every chip type in every region, the second constraint in the model should be adjusted to Fig. 14. With no change in other sections of the model, we conducted model optimization and confirmed the feasibility of our solution in Fig. 15, indicating Super Chip has sufficient capacity to handle the estimated increase in demand. The new optimal objective function value regarding total operation costs is approximately $54,024,420. In comparison to the optimal objective function value with current demand, the total operation costs increased by approximately $4,940,990.

$$\sum_{i \epsilon S} Xijz = 1000((0.1 * Djz) + Djz) \qquad j = 1,..., 23 \quad z = 1,..., 30$$

Fig. 14. Constraint Change of 10% Demand Increase for Every Chip Type in Every Region

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x3a380a35
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    5.3965600e+07   1.241125e+04   0.000000e+00      0s
      81    5.4024420e+07   0.000000e+00   0.000000e+00      0s

Solved in 81 iterations and 0.01 seconds (0.01 work units)
Optimal objective  5.402442027e+07
Optimal Production and Distribution Costs After 10% Demand Increase =  54024420.27
Optimal Production and Distribution Costs Diference =  4940989.870000012
```

Fig. 15. Question 3 Model Result

To address question 4, we first obtained the total production costs for each facility. Subsequently, we identified that the Alexandria facility has the highest total production costs of approximately $14,837,101. To further investigate the cost savings generated by manufacturing technologies in each facility, we implemented a reduction of 15% in production costs for all chips in each facility per evaluation and generated the respective optimization model with the new production costs. The cost reductions are observed in these models. Fig. 16 shows reducing production costs of all chips by 15% in the Alexandria facility generated a total operation costs reduction of approximately $2,401,007. Fig. 17 shows the reduction in production costs in the Richmond facility generated a total operation costs reduction of approximately $2,088,422. Fig. 18 indicates the reduction in production costs in the Norfolk facility reduced total operation costs by approximately $2,031,229. In Roanoke, the production cost reduction reduced total operation costs by approximately $1,347,040 as shown in Fig. 19. In Fig. 20, it is apparent that production cost reduction in Charlottesville produced a total operation cost reduction of approximately $902,509. Based on the production cost observations and the above optimization model results, implementing the new manufacturing technologies in the Alexandria facility will produce the most optimal cost savings.

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0xd16c3c90
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.5692423e+07   3.876500e+04   0.000000e+00      0s
     221    4.6682423e+07   0.000000e+00   0.000000e+00      0s

Solved in 221 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.668242343e+07
Optimal Production and Distribution Costs =  46682423.43
Total Production Costs Change After 15% Reduction in Alexandria:  -2390405.1700000023
Change in Objective Function Value:  -2401006.9699999914
```

Fig. 16. Reduction of Production Costs of All Chips By 15% in Alexandria Facility

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x24d34231
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.4981606e+07   6.311625e+04   0.000000e+00      0s
     345    4.6995008e+07   0.000000e+00   0.000000e+00      0s

Solved in 345 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.699500848e+07
Optimal Production and Distribution Costs =  46995008.47500001
Total Production Costs Change After 15% Reduction in Richmond:  -2094274.325000002
Change in Objective Function Value:  -2088421.9249999821
```

Fig. 17. Reduction of Production Costs of All Chips By 15% in Richmond Facility

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x4214e589
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.6053253e+07   4.427625e+04   0.000000e+00      0s
     282    4.7052202e+07   0.000000e+00   0.000000e+00      0s

Solved in 282 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.705220176e+07
Optimal Production and Distribution Costs =  47052201.76
Total Production Cost Change After 15% Production Costs Reduction in Norfolk:  -2021960.0399999982
Change in Objective Function Value:  -2031228.6399999931
```

Fig. 18. Reduction of Production Costs of All Chips By 15% in Norfolk Facility

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x723f508d
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 7e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.7050270e+07   4.033375e+04   0.000000e+00      0s
     219    4.7736391e+07   0.000000e+00   0.000000e+00      0s

Solved in 219 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.773639082e+07
Optimal Production and Distribution Costs =  47736390.82000001
Total Production Cost Change After 15% Production Costs Reduction in Roanoke:  -1190336.8800000018
Change in Objective Function Value:  -1347039.5799999833
```

Fig. 19. Reduction of Production Costs of All Chips By 15% in Roanoke Facility

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (mac64[arm] - Darwin 22.3.0 22D49)

CPU model: Apple M1 Pro
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Optimize a model with 845 rows, 3600 columns and 7200 nonzeros
Model fingerprint: 0x7ecd516f
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 6e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 3e+05]
Presolve removed 151 rows and 155 columns
Presolve time: 0.00s
Presolved: 694 rows, 3445 columns, 6890 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.8062449e+07   1.510375e+04   0.000000e+00      0s
     102    4.8180921e+07   0.000000e+00   0.000000e+00      0s

Solved in 102 iterations and 0.01 seconds (0.01 work units)
Optimal objective  4.818092134e+07
Optimal Production and Distribution Costs =  48180921.33999999
Total Production Cost Change After 15% Production Costs Reduction in Charolottesville:  -757675.3599999994
Change in Objective Function Value:  -902509.0600000024
```

Fig. 20. Reduction of Production Costs of All Chips By 15% in Charlottesville Facility