

MATH412/COMPSCI434/MATH713

Fall 2025

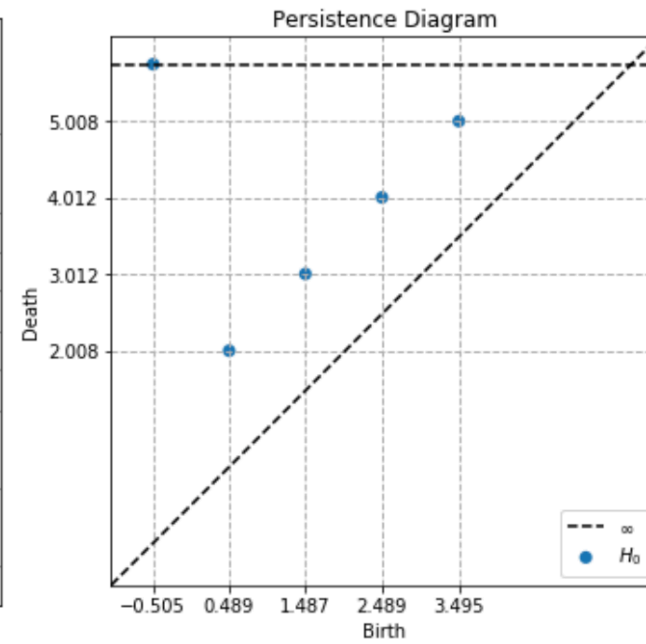
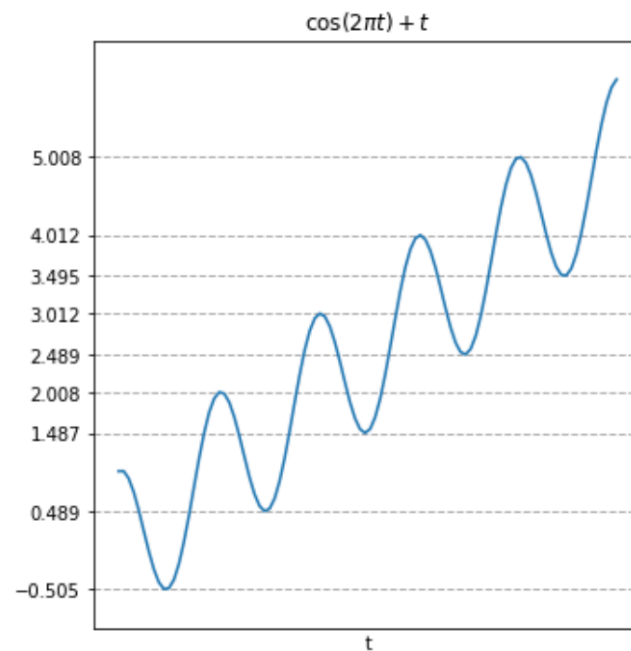
Topological Data Analysis

Topic 6: TDA + time series

Instructor: Ling Zhou

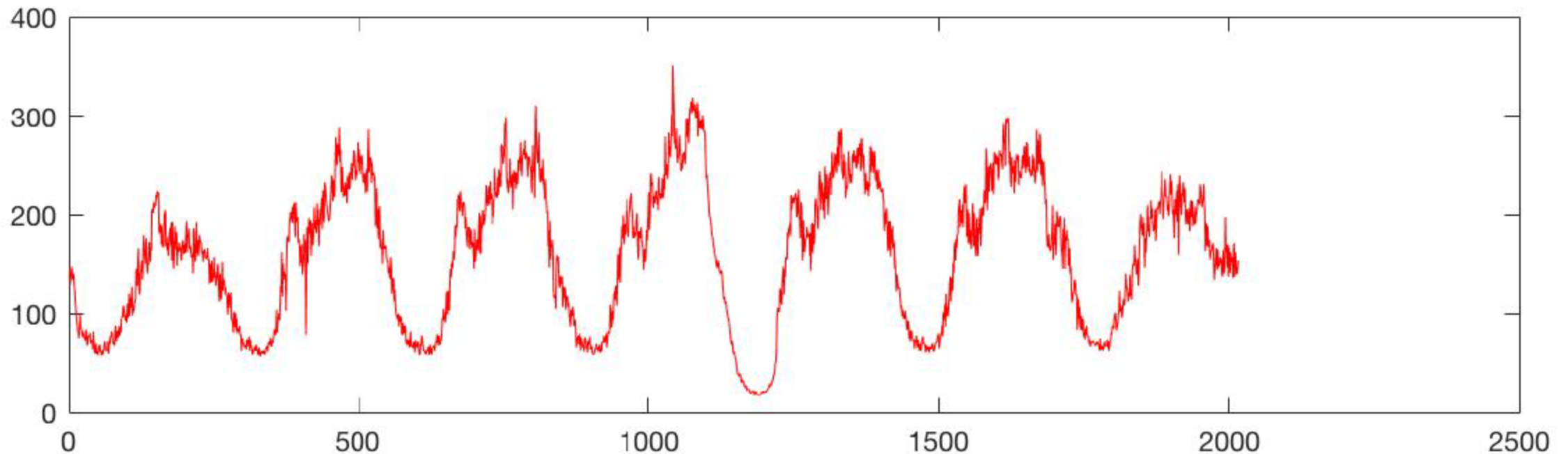
Time series data

- ▶ How do we create a filtration on top of a time series data?
- ▶ Idea 1: Use sub level set filtration



Time series data

- ▶ How do we create a filtration on top of a time series data?
- ▶ Idea 2: Use some sort of “sliding window” embedding
 - ▶ Often one aims to capture quasi-periodic features



Sliding Window Embedding

Given a time series $f(t)$, one can extract a ***sequence of vectors*** of the form

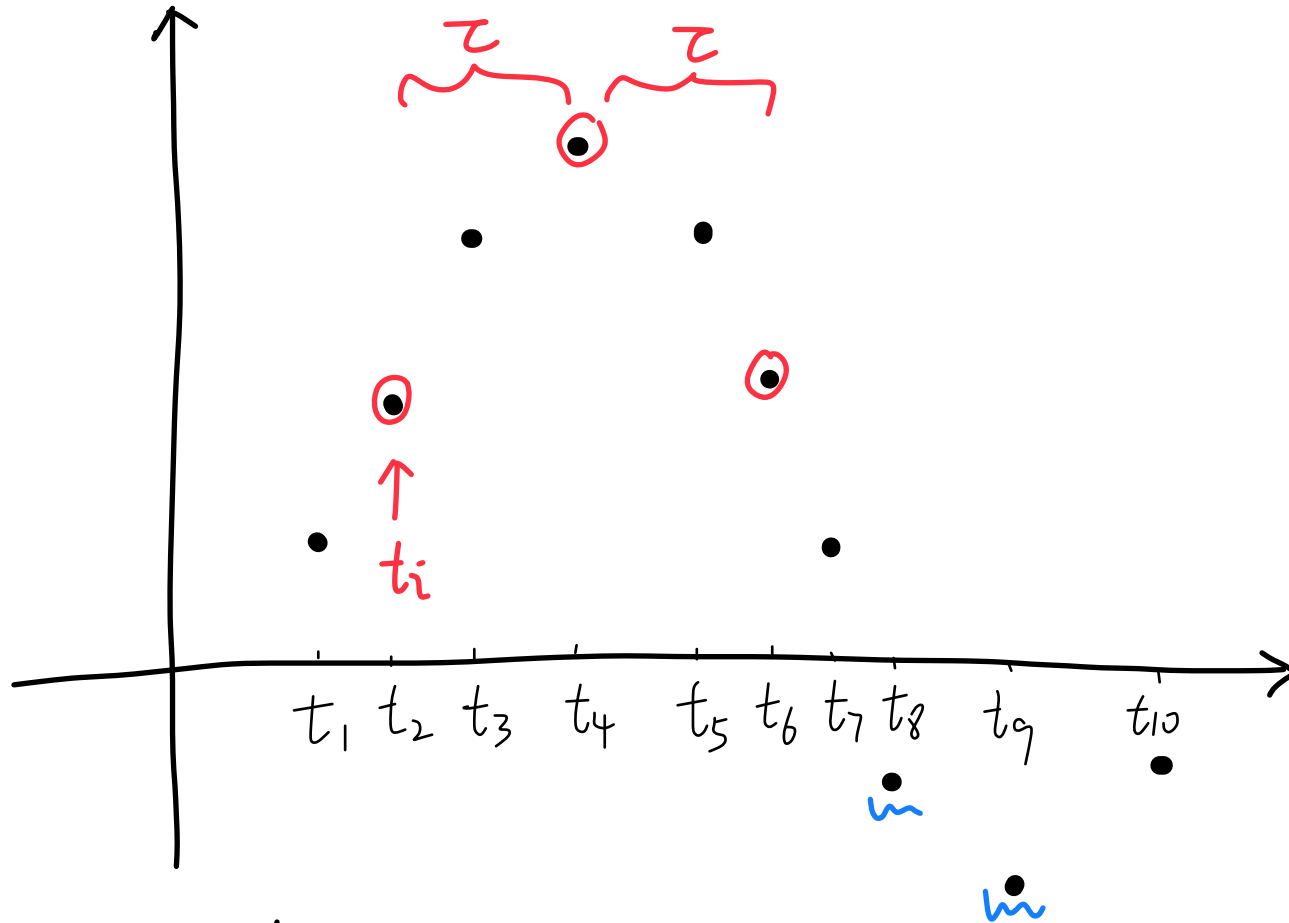
$$f_i = [f(t_i), f(t_i + \tau), f(t_i + 2\tau), \dots, f(t_i + (d - 1)\tau)] \in \mathbb{R}^d,$$

where

- d is the ***embedding dimension***
- τ is the ***time delay***
- $(d - 1)\tau$ is known as the **window size**
- the difference between t_{i+1} and t_i is called the ***stride***.

$$f_i = [f(t_i), f(t_i + \tau), f(t_i + 2\tau), \dots, f(t_i + (d-1)\tau)] \in \mathbb{R}^d,$$

Sliding Window Embedding



Assume $t_j - t_{j-1} = 1$

For example,

Let $i=2, \tau=2$ and $d=3$

$$\Rightarrow f_2 = [f(t_2), f(t_4), f(t_6)]$$

$$\in \mathbb{R}^3$$

Let $i=8, \tau=1$ and $d=2$

$$\Rightarrow f_8 = [f(t_8), f(t_9)]$$

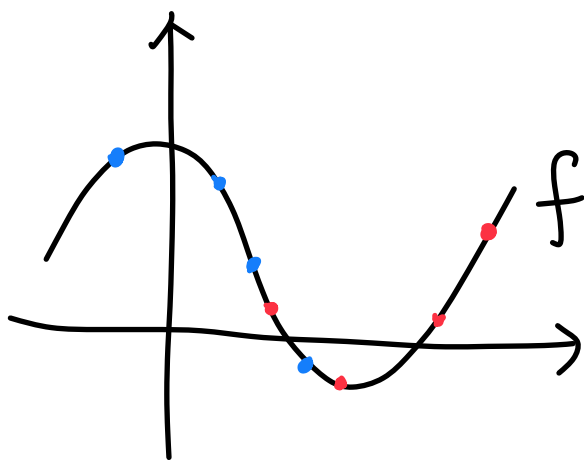
Sliding Window Embedding

If the time-series is discrete time, then this will give a point cloud data.

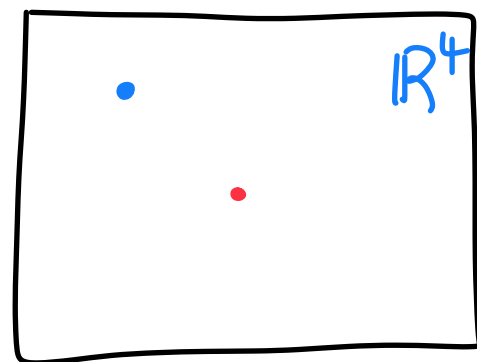
- ▶ Map a time-series data $f: \mathbb{R} \rightarrow \mathbb{R}$ to a point cloud data

The time delay embedding of f with parameters (d, τ) is the function
(or sliding window embedding)
(or Takens embedding)
 $SW_{d,\tau} f: \mathbb{R} \rightarrow \mathbb{R}^d$,

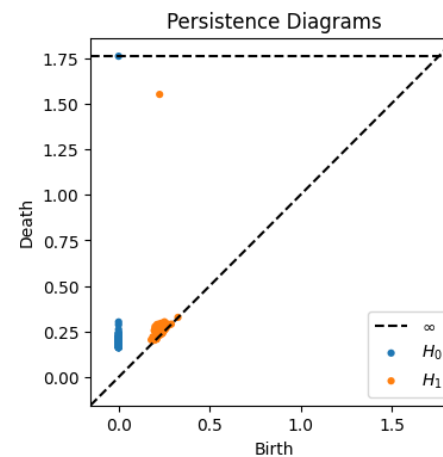
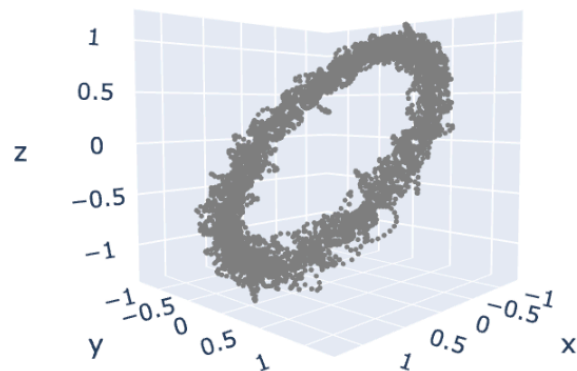
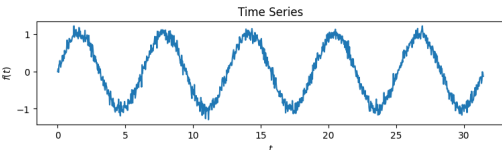
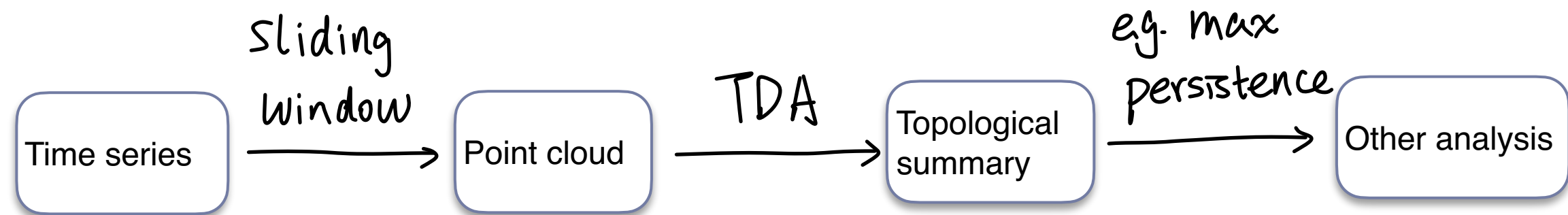
$$t \rightarrow \begin{bmatrix} f(t) \\ f(t + \tau) \\ f(t + 2\tau) \\ \vdots \\ f(t + (d - 1)\tau) \end{bmatrix}$$



$SW_{d,\tau}$



Sliding Window + TDA



The TDA ranking is
[10 4 1 2 16 6 3 12 17 8 15 20 14 13 9 19 5 7 18 11]
The true ranking is
[10 4 1 2 14 6 3 18 17 8 15 20 13 12 9 19 5 7 16 11]
The agreement between the rankings is 0.9157894736842106

Sliding Window + TDA

- ▶ Continue with the Google colab notebook below

[https://colab.research.google.com/drive/1ZNK8RuDJ5dnekOEZvfBuJtoIT0xZHljY?
usp=sharing](https://colab.research.google.com/drive/1ZNK8RuDJ5dnekOEZvfBuJtoIT0xZHljY?usp=sharing)