Cryptography I

General concepts and some classical ciphers

- Basic concepts
- Attack models
- Classic ciphers: mono-alphabetic
- Vigenere cipher
- One-time-pad cipher

Security Goals

- Confidentiality (secrecy, privacy)
  - Assure that data is accessible to only one who are authorized to know
- Integrity
  - Assure that data is only modified by authorized parties and in authorized ways
- Availability
  - Assure that resource is available for authorized users

General tools

- Cryptography
- Software controls
- Hardware controls
- Policies and procedures
- Physical controls

What is Crypto?

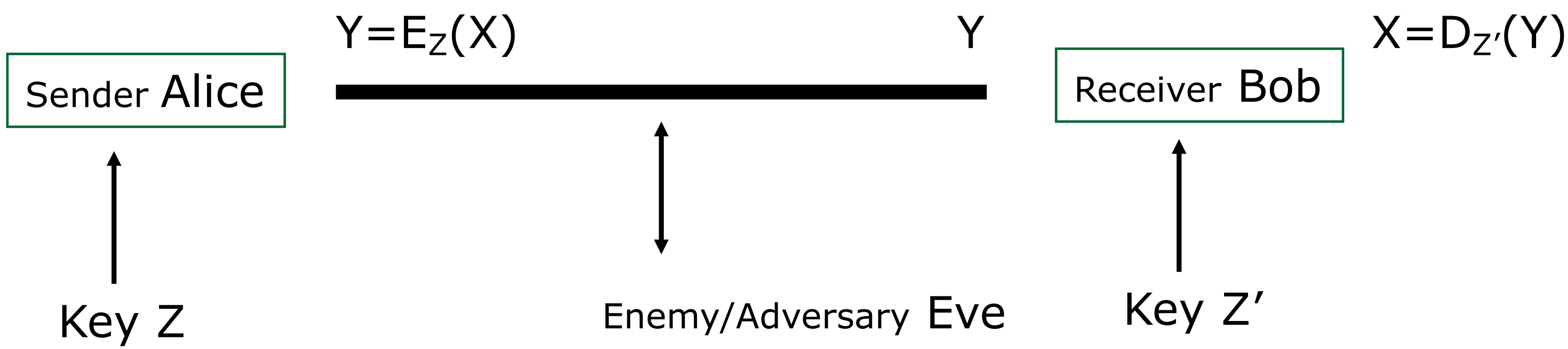
- Constructing and analyzing cryptographic protocols which enable parties to achieve security objectives
  - Under the present of adversaries.
- A protocol (or a scheme) is a suite of procedures that tell each party what to do
  - usually, computer algorithms
- Cryptographers devise and analyze protocols under Attack model
  - assumptions about the resources and actions available to the adversary
    - So, you need to think as an adversary

Terms

- **Cryptography:** the study of mathematical techniques for providing information security services.
- **Cryptanalysis:** the study of mathematical techniques for attempting to get security services breakdown.
- **Cryptology:** the study of cryptography and cryptanalysis.

Terms …

- plaintexts
- ciphertexts
- keys
- encryption
- decryption



Secret-key cryptography

- Also called: symmetric cryptography
- Use the same key for both encryption & decryption (Z=Z')
- Key must be kept secret
- Key distribution – how to share a secret between A and B very difficult

Public-key cryptography

- Also called: asymmetric cryptography
- Encryption key different from decryption key and
  - It is not possible to derive decryption key from encryption key
- Higher cost than symmetric cryptography

Is it a secure cipher system?

- **Why insecure**
  - **just break it under a certain reasonable attack model (show failures to assure security goals)**
- Why secure:
  - Evaluate/prove that under the considered attack model, security goals are assured
  - Provable security: Formally show that (with mathematical techniques) the system is as secure as a well-known secure one (usually simpler).

Breaking ciphers …

- There are different methods of breaking a cipher, depending on:
  - the type of information available to the attacker
  - the interaction with the cipher machine
  - the computational power available to the attacker

Breaking ciphers …

- **Ciphertext-only attack**:
  - The cryptanalyst knows **only the ciphertext**.
  - Goal: to find the plaintext and the key.
  - NOTE: such vulnerable is seen completely insecure
- **Known-plaintext attack**:
  - The cryptanalyst knows **one or several pairs of ciphertext and the corresponding plaintext.**
  - Goal: to find the key used to encrypt these messages
    - or a way to decrypt any new messages that use the same key (although may not know the key).

Breaking ciphers …

- **Chosen-plaintext attack**
  - The cryptanalyst **can choose a number of messages and obtain the ciphertexts for them**
  - Goal: deduce the key used in the other encrypted messages or decrypt any new messages (using that key).
- **Chosen-ciphertext attack**
  - Similar to above, but the cryptanalyst **can choose a number of ciphertexts and obtain the plaintexts.**
- Both can be **adaptive**
  - The choice of ciphertext may depend on the plaintext received from previous requests.

Models for Evaluating Security

- **Unconditional (information-theoretic) security**
  - **Assumes that the adversary has unlimited computational resources**.
  - Plaintext and ciphertext modeled by their distribution
  - Analysis is made by using probability theory.
  - For encryption systems: **perfect secrecy**, observation of the ciphertext provides no information to an adversary.

Models for Evaluating Security

- **Provable security:**
  - Prove security properties based on assumptions that it is difficult to solve a well-known and supposedly difficult problem (NP-hard …)
    - E.g.: computation of discrete logarithms, factoring
- **Computational security (practical security)**
  - Measures the amount of computational effort required to defeat a system using the best-known attacks.
  - Sometimes related to the hard problems, but no proof of equivalence is known.

Models for Evaluating Security

- **Ad hoc security (heuristic security):**
  - Variety of convincing arguments that every successful attack requires more resources than the ones available to an attacker.
  - Unforeseen attacks remain a threat.
  - **THIS IS NOT A PROOF**

Classic ciphers

Shift cipher (additive cipher)

- Key Space: [1 .. 25]
- Encryption given a key K:
  - each letter in the plaintext P is replaced with the K'th letter following corresponding number (shift right):
  - Another way: $Y = X \oplus K$ ➔ additive cipher
- Decryption given K:
  - shift left

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
P = CRYPTOGRAPHYISFUN
K = 11
C = NCJAVZRCLASJTDQFY

Shift Cipher: Cryptanalysis

- Easy, just do exhaustive search
  - key space is small (<= 26 possible keys).
  - once K is found, very easy to decrypt

General Mono-alphabetical Substitution Cipher

- The key space: all permutations of $\Sigma = \{A, B, C, \ldots, Z\}$
- Encryption given a key π:
  - each letter X in the plaintext P is replaced with $\pi(X)$
- Decryption given a key π :
  - each letter Y in the cipherext P is replaced with $\pi^{-1}(Y)$

- **Example:**

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

π = B A D C Z H W Y G O Q X S V T R N M S K J I P F E U

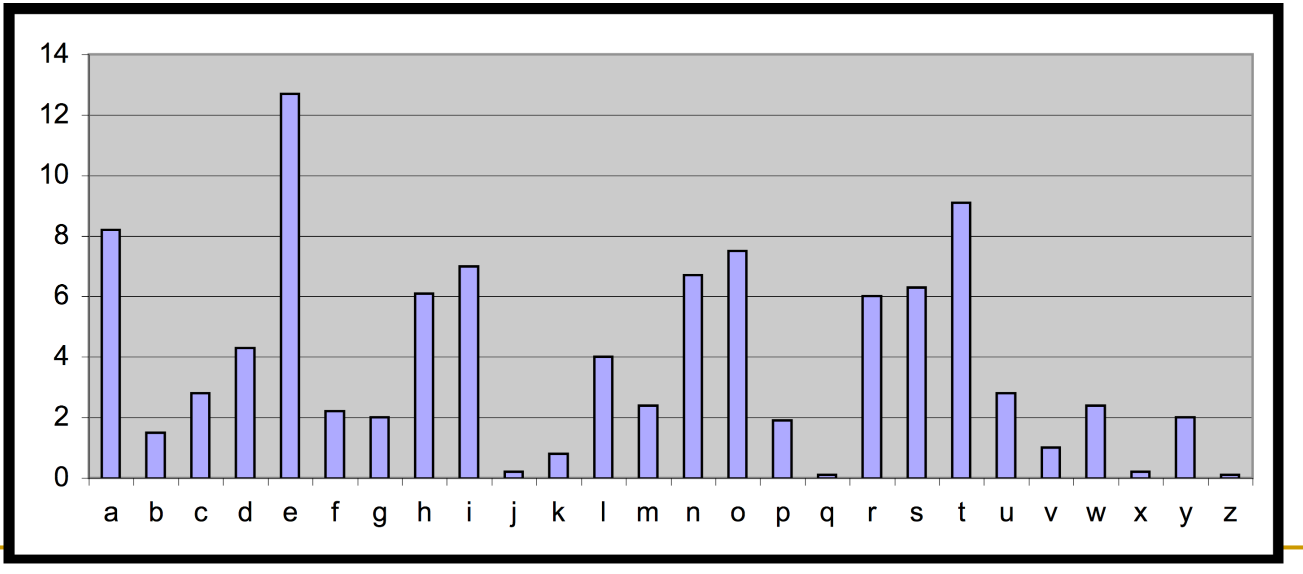
BECAUSE $\rightarrow$ AZDBJSZ

Looks secure, early days

- Exhaustive search is infeasible
  - key space size is $26! \approx 4*10^{26}$
- Dominates the art of secret writing throughout the first millennium A.D.
- Thought to be unbreakable by many back then

Cryptanalysis of Substitution Ciphers: Frequency Analysis

- Each language has certain features:
  - frequency of letters, or of groups of two or more letters.
- Substitution ciphers preserve the mentioned language features ➔ vulnerable to frequency analysis attacks



- Observations:
  - A cipher system should not allow statistical properties of plaintext to pass to the ciphertext.
  - The ciphertext ginerated by a "good" cipher systim should be satistically indistinguishable form random text.
- Idea for a stronger cipher (1460’s by Alberti)
  - use more than one cipher alphabet, and switch between them when encrypting different letters ➔ Polyalphabetic Substitution Ciphers
  - Developed into a practical cipher by Vigenère (published in 1586)

- **Definition**:
  - Given m, a positive integer, $P = C = (Z_{26})^n$, and $K = (k_1, k_2, \ldots, k_m)$ a key, we define:
- **Encryption**:

$e_k (p_1, p_2 \ldots p_m) = (p_1+k_1, p_2+k_2 \ldots p_m+k_m) \pmod{26}$

- **Decryption**:

$d_k (c_1, c_2 \ldots c_m) = (c_1-k_1, c_2-k_2 \ldots c_m- k_m) \pmod{26}$

- **Example:**

Plaintext: C R Y P T O G R A P H Y

Key: L U C K L U C K L U C K

Ciphertext: N L A Z E I I B L J J I

Vigenere Cipher: Cryptanalysis

- Find the length of the key.
- Divide the message into that many shift cipher encryptions.
- Use frequency analysis to solve the resulting shift ciphers.

One-Time Pad

Key is chosen randomly

Plaintext $X = (x_1\ x_2\ \ldots\ x_n)$

Key $K = (k_1\ k_2\ \ldots\ k_n)$

Ciphertext $Y = (y_1\ y_2\ \ldots\ y_n)$

$e_k(X) = (x_1+k_1\ \ x_2+k_2\ \ldots\ x_n+k_n) \bmod m$

$d_k(Y) = (x_1-k_1\ \ x_2-k_2\ \ldots\ x_n-k_n) \bmod m$

Example

Plaintext space = Ciphtertext space =

Keyspace = $\{0,1\}^n$

Key is chosen randomly

For example:

Plaintext is 10001011

Key is 00111001

Then ciphertext is 10110010

Main points in One-Time Pad

- The key is never to be reused
  - Thrown away after first and only use
  - If reused ➔ insecure!
- One-Time Pad uses a very long key, exactly the same length as of the plaintext
  - In old days, some suggest choose the key as texts from, e.g., a book ➔ i.e. not **randomly chosen**
    - Not One-Time Pad anymore ➔ this does not have perfect secrecy as in true One-Time-Pad and can be broken
  - Perfect secrecy means key length be at least message length
    - **Difficult in practice**!