

# User Manual for KCC—a MATLAB package for K-means-based Consensus Clustering

Hao Lin<sup>1</sup>, Hongfu Liu<sup>2</sup>, Junjie Wu<sup>3</sup>, Hong Li<sup>4</sup>, Stephan Günnemann<sup>5</sup>

---

## 1. INTRODUCTION

This user manual systematically presents the usage of the MATLAB package on K-means-based Consensus Clustering (KCC) accompanying the following paper:

Hao Lin, Hongfu Liu, Junjie Wu, Hong Li, and Stephan Günnemann. 2022. Algorithm xxxx: KCC: A MATLAB Package for K-means-based Consensus Clustering. *ACM Trans. Math. Softw.*

The package was developed and tested in MATLAB R2022a under Linux. **To those without access to MATLAB and those who prefer to use free open source software, we also investigate the usage of KCC with OCTAVE, and find it is also compatible with OCTAVE without additional efforts. OCTAVE version?**

For package installation, you need to first unpack the compressed archive into your current directory. It consists of a *source code* folder `Matlab`, a folder `userManual` with this comprehensive *user manual*, and a *license* file `LICENSE` indicating that the package is distributed under GNU GENERAL PUBLIC LICENSE (Version 3). Then under the `Matlab` folder, you need to add one of its subfolder, i.e., the `Src` folder, to the MATLAB path. The directory structure of the `Matlab` folder is described as follows.

`Src` (core functions for conducting KCC)

- `BasicCluster_RFS.m` (function to generate BPs with RFS)
- `BasicCluster_RPS.m` (function to generate BPs with RPS)
- `Preprocess.m` (function to prepare for consensus clustering)
- `KCC.m` (consensus function)
- `exMeasure.m` (function to compute validity scores for clustering results)
- `load_sparse.m` (auxiliary function to load input text data as a sparse matrix)
- `hungarian.m` (auxiliary function for cluster label assignment)
- `BasicCluster_RPS_missing.m` (auxiliary function to generate IBPs with strategy-I)
- `addmissing.m` (auxiliary function to generate IBPs using strategy-II)
- `distance_*` (distance functions)
- `gClusterDistribution.m` (auxiliary function to calculate cluster distribution for BPs)
- `Ucompute.m`, `Ucompute_miss.m` (auxiliary function for utility calculation)
- `gCentroid.m`, `gCentroid_miss.m` (auxiliary function for centroid update)
- `sCentroid.m`, `sCentroid_miss.m` (auxiliary function for centroid initialization)

`Drivers` (illustrative examples)

- `data` (input data for illustration)
- `demo.m` (function for KCC with different utility functions)

---

<sup>1</sup>Department of Informatics, Technical University of Munich, Germany, [linh@in.tum.de](mailto:linh@in.tum.de).

<sup>2</sup>Mitchom School of Computer Science, Brandeis University, USA, [hongfuliu@brandeis.edu](mailto:hongfuliu@brandeis.edu).

<sup>3</sup>School of Economics and Management, Beihang University, China, [wujj@buaa.edu.cn](mailto:wujj@buaa.edu.cn).

<sup>4</sup>School of Economics and Management, Beihang University, China, [hong\\_lee@buaa.edu.cn](mailto:hong_lee@buaa.edu.cn).

<sup>5</sup>Department of Informatics, Technical University of Munich, Germany, [guennemann@in.tum.de](mailto:guennemann@in.tum.de).

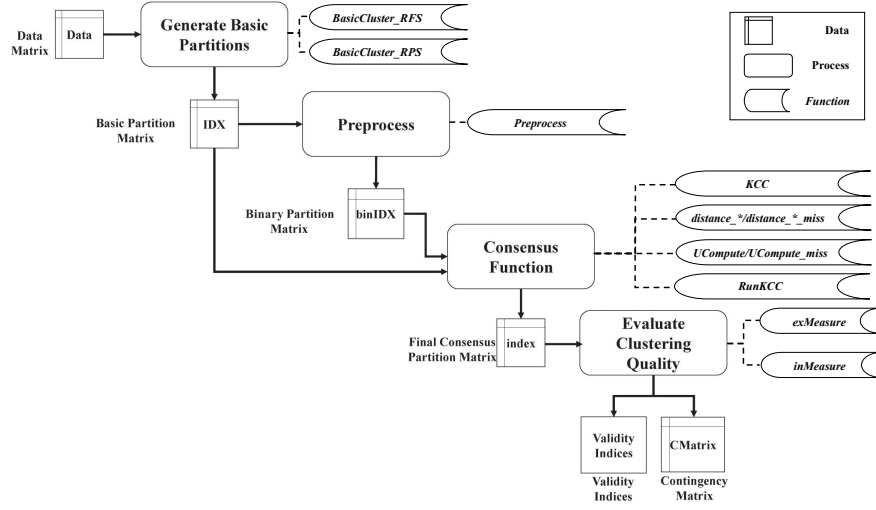


Fig. 1. Typical program flow of KCC package.

demolBPI.m (function for KCC with IBPs generated by strategy-I)  
 demolBPII.m (function for KCC with IBPs generated by strategy-II)  
 demoNumberBP.m (function for KCC with varying number of BPs)  
 demoStrategyBP.m (function for KCC with RFS strategy for BP generation)

A typical workflow of the package as shown in Figure 1 is first to input a real-world data matrix *Data* into the basic partition generation functions for generating a basic partition matrix *IDX*. The basic partition matrix is then input to a *Preprocess* function to produce a sparse representation of  $\mathcal{X}^{(b)}$ , i.e., the binary matrix *binIDX*. The binary matrix *binIDX* is then input to the final consensus clustering, i.e., the consensus function, which produces a consensus partition matrix *index*. Lastly, the clustering quality is evaluated with an *inMeasure*/*exMeasure* function, which outputs multiple internal/external validity indices.

## 2. GUIDES

### 2.1 BasicCluster\_RFS

This function generates basic partition results using *K*-means as a basic clustering algorithm with Random Feature Selection (RFS) strategy.

#### Syntax:

```
IDX = BasicCluster_RFS(Data, r, K, dist, nFeature)
```

#### Input parameters:

*Data* : input data matrix  
*r* : the predefined number of basic partitions in the cluster ensemble  
*K* : the predefined number of clusters in the basic partitions  
*dist* : the distance measure used in *K*-means clustering  
*nFeature* : the number of randomly selected partial features

#### Output parameters:

**IDX** : a matrix indicating the cluster labels for data points in basic partitions

#### Discussion:

The parameter **Data** is an  $n \times p$  matrix of data, whose rows correspond to  $n$  observations, and columns correspond to  $p$  features. The available options of the parameter **dist** can be found from the official documentation of the Matlab **kmeans** function, and the most widely used distance metric is '**sqEuclidean**', which denotes the squared Euclidean distance. For '**sqEuclidean**', the centroid for each cluster is calculated as the mean of the data points in that cluster. For selecting features, we first sample **nFeature** values uniformly at random without replacement from the integers 1 to  $p$ , where  $p$  is the dimension of all features in the input data. Then the sampled values are used as the indices of the selected features. The output **IDX** is a  $n \times r$  cluster labels matrix for  $n$  data points in  $r$  basic partitions. Note that this function is a non-deterministic function. Each call may yield a different output matrix **IDX**, due to the random feature sampling process, and the random initializations in the  $K$ -means algorithm.

## 2.2 BasicCluster\_RPS

This function generates basic partition results using  $K$ -means as a basic clustering algorithm with Random Parameter Selection (RPS) strategy.

#### Syntax:

```
IDX = BasicCluster_RPS(Data, r, K, dist, randKi)
```

#### Input parameters:

**Data** : input data matrix  
**r** : the predefined number of basic partitions in the cluster ensemble  
**K** : the groundtruth number of clusters for the input data  
**dist** : the distance measure used in  $K$ -means clustering  
**randKi** : the parameter regarding the number of clusters in the basic partitions

#### Output parameters:

**IDX** : a  $n \times r$  cluster labels matrix for  $n$  data points in  $r$  basic partitions

#### Discussion:

The parameter **K** indicates the groundtruth number of clusters for the input data, which is used as the lower bound of the randomized number of cluster for each basic partition when **randKi** == 1. The parameter **randKi** has the following options. If **randKi** == 1, this function generates a random number of cluster ranging from  $K$  to  $\sqrt{n}$ , where  $n$  is the number of input data instances; if **randKi** is set to a  $r \times 1$  vector, this function produces  $r$  basic partitions of which the  $i$ -th BP's number of clusters is **RandKi(i)**; otherwise, this function produces  $r$  basic partitions with each partition having equal number (i.e.,  $K$ ) of clusters. Note that this function is a non-deterministic function. Each call may yield a different output matrix **IDX**, due to the random parameter selection process, and the random initializations in the  $K$ -means algorithm.

### 2.3 Preprocess

This function conduct some preprocessing on the input basic partition matrix `IDX` to produce the input for the final consensus clustering, as well as some auxiliary output variables that can help to save memory and accelerate computations.

**Syntax:**

```
Ki, sumKi, binIDX, missFlag, missMatrix, distance, Pvector, weight =
Preprocess(IDX, U, n, r, w, utilFlag)
```

**Input parameters:**

`IDX` : the input basic partition matrix  
`U` : the parameter regarding the chosen utility function  
`n` : the number of data instances  
`r` : the number of basic partitions in the cluster ensemble  
`w` : the weight vector for all basic partitions  
`utiFlag` : whether to calculate utility function

**Output parameters:**

`Ki` : a row vector indicating the number of clusters in all basic partitions  
`sumKi` : a matrix indicating the starting indexes for all basic partitions  
`binIDX` : a sparse representation of binarization of `IDX`  
`missFlag` : whether the input `IDX` matrix contains IBPs or not  
`missMatrix` : indices matrix of the non-zero entries in `IDX` if there exists IBPs  
`distance` : the distance function corresponding to a specific utility function  
`Pvector` : a row vector calculated from the contingency matrix  
`weight` : an adjusted weight vector adapted for convenient distance calculation

**Discussion:**

The parameter `U` is a  $1 \times 3$  cell array. Its first cell `U{1,1}` defines the chosen type of the KCC utility function. It currently supports four different types of utility functions which correspond to four different  $K$ -means point-to-centroid distance functions, i.e., 'U\_c' for Euclidean distance, 'U\_H' for Kullback-Leibler Divergence, 'U\_cos' for cosine similarity, and 'U\_Lp' for  $L_p$ -norm. It is worth noting that 'U\_Lp' corresponds the distance measure in  $L_p$  spaces, which are function spaces defined using a natural generalization of the  $p$ -norm for finite-dimensional vector spaces. The second cell `U{1,2}` is a parameter specifying the chosen form of the KCC utility function, i.e., 'std' for the Standard Form, and 'norm' for the Normalized Form. The third cell `U{1,3}` is only required to be set when 'U\_Lp' is chosen as the utility function. The settings of  $p = 1$ ,  $p = 2$ ,  $p \rightarrow \infty$  correspond to the Manhattan distance, the euclidean distance and the chebyshev distance, respectively. The parameter `w` is  $r \times 1$  weight vector, of which each entry indicates the weight value assigned to each basic partition in the optimization objective of consensus clustering. The parameter `utiFlag` is a variable indicating whether to calculate utility function during the iterative process of  $K$ -means clustering.

The output variable `missFlag`  $\in \{0, 1\}$  indicates whether the input `IDX` matrix contains incomplete basic partitions (IBPs) or not. The output `missMatrix` is a mask matrix which

represents the indices of the non-zero entries in `IDX` if there exists IBPs. The output variable `distance` determines the corresponding distance function to deal with the specific utility function defined by  $U\{1,1\}$ . The output vector `Pvector` is a  $1 \times r$  row vector calculated from the contingency matrix, i.e.,  $P_k^{(i)}$ , which can later be used in calculating distance and utility functions. The output vector `weight` is a  $r \times 1$  adjusted weight vector adapted for convenient distance calculation in later  $K$ -means heuristic.

## 2.4 KCC

This function employs the  $K$ -means heuristic to generate a final consensus partition.

### Syntax:

```
sumbest, index, converge, utility = KCC(IDX, K, U, w, weight, distance,
maxIter, minThres, utilFlag, missFlag, missMatrix, n, r, Ki, sumKi, binIDX,
Pvector)
```

### Input parameters:

`IDX` : the input basic partition matrix  
`K` : the number of clusters in the consensus partition  
`U` : the parameter regarding the chosen utility function  
`w` : the weight vector for all basic partitions  
`weight` : an adjusted weight vector adapted for convenient distance calculation  
`distance` : the distance function corresponding to a specific utility function  
`maxIter` : the maximum number of iterations for the convergence  
`minThres` : the minimum reduced threshold of objective function  
`utiFlag` : whether to calculate utility function  
`missFlag` : whether there exist IBPs in `IDX`  
`missMatrix` : the indices matrix of the non-zero entries in `IDX` if there exists IBPs  
`n` : the number of data points  
`r` : the number of basic partitions in the cluster ensemble  
`Ki` : a row vector indicating the number of clusters in all basic partitions  
`sumKi` : a matrix indicating the starting indexes for all basic partitions  
`binIDX` : a sparse representation of binarization of `IDX`  
`Pvector` : a row vector calculated from the contingency matrix

### Output parameters:

`sumbest` : the optimal value of the consensus clustering's objective function  
`index` : the label vector for the final consensus partition  
`converge` : the iterative values of objective function  
`utility` : the final utility function value

### Discussion:

For convenience of computation, `KCC` function achieves the consensus clustering under four different conditions, e.g., (a) utility calculation is enabled and there are missing

values (`utilFlag==1 && missFlag==1`); (b) utility calculation is enabled and there are no missing values (`utilFlag==1 && missFlag==0`); (c) utility calculation is disabled and there are missing values (`utilFlag==0 && missFlag==1`); (d) utility calculation is disabled and there are no missing values (`utilFlag==0 && missFlag==0`). Notably, we introduce a parameter called `utilFlag` in the `KCC` function to control whether the K-means heuristic computes the value of the KCC utility function, since the value of the KCC utility function might be of interest to some users in using the package. The users can choose to enable or disable the computation of the value of the KCC utility function since it does not affect the process of K-means heuristic. Disabling the computation of the value of the KCC utility function does not affect the clustering performance and can accelerate the computation of KCC. We have added more elaborations on the usage of the parameter `utilFlag` in the function `KCC` in the user manual.

## 2.5 distance\_euc

This function performs point-to-centroid distance calculation using euclidean distance measure.

### Syntax:

```
D = distance_euc(U, C, weight, n, r, K, sumKi, binIDX)
```

### Input parameters:

U : the  $1 \times 3$  utility parameter cell array  
C : the centroid matrix  
weight : an  $r \times 1$  adjusted weight vector  
n : the number of data points  
r : the predefined number of basic partitions in the cluster ensemble  
K : the predefined number of clusters in the consensus partition  
sumKi : a matrix indicating the starting indexes for all basic partitions  
binIDX : a sparse representation of binarization of IDX

### Output parameters:

D : an  $n \times K$  point-to-centroid distance matrix

## 2.6 distance\_cos

This function performs point-to-centroid distance calculation using cosine distance measure.

### Syntax:

```
D = distance_cos(U, C, weight, n, r, K, sumKi, binIDX)
```

### Input parameters:

U : the  $1 \times 3$  utility parameter cell array  
C : the centroid matrix  
weight : an  $r \times 1$  adjusted weight vector

$n$  : the number of data points  
 $r$  : the predefined number of basic partitions in the cluster ensemble  
 $K$  : the predefined number of clusters in the consensus partition  
 $\text{sumKi}$  : a matrix indicating the starting indexes for all basic partitions  
 $\text{binIDX}$  : a sparse representation of binarization of  $\text{IDX}$

**Output parameters:**

$D$  : an  $n \times K$  point-to-centroid distance matrix

## 2.7 distance\_kl

This function performs point-to-centroid distance calculation using KL-divergence measure.

**Syntax:**

$D = \text{distance\_kl}(U, C, \text{weight}, n, r, K, \text{sumKi}, \text{binIDX})$

**Input parameters:**

$U$  : the  $1 \times 3$  utility parameter cell array  
 $C$  : the centroid matrix  
 $\text{weight}$  : an  $r \times 1$  adjusted weight vector  
 $n$  : the number of data points  
 $r$  : the predefined number of basic partitions in the cluster ensemble  
 $K$  : the predefined number of clusters in the consensus partition  
 $\text{sumKi}$  : a matrix indicating the starting indexes for all basic partitions  
 $\text{binIDX}$  : a sparse representation of binarization of  $\text{IDX}$

**Output parameters:**

$D$  : an  $n \times K$  point-to-centroid distance matrix

## 2.8 distance\_lp

This function performs point-to-centroid distance calculation using  $L_p$  norm measure.

**Syntax:**

$D = \text{distance\_lp}(U, C, \text{weight}, n, r, K, \text{sumKi}, \text{binIDX})$

**Input parameters:**

$U$  : the  $1 \times 3$  utility parameter cell array  
 $C$  : the centroid matrix  
 $\text{weight}$  : an  $r \times 1$  adjusted weight vector  
 $n$  : the number of data points  
 $r$  : the predefined number of basic partitions in the cluster ensemble  
 $K$  : the predefined number of clusters in the consensus partition  
 $\text{sumKi}$  : a matrix indicating the starting indexes for all basic partitions  
 $\text{binIDX}$  : a sparse representation of binarization of  $\text{IDX}$

**Output parameters:**

$D$  : an  $n \times K$  point-to-centroid distance matrix

**2.9 distance\_euc\_miss**

This function performs point-to-centroid distance calculation over data with IBPs using euclidean distance measure.

**Syntax:**

```
D = distance_euc_miss(U, C, weight, n, r, K, sumKi, binIDX, missMatrix)
```

**Input parameters:**

$U$  : the  $1 \times 3$  utility parameter cell array  
 $C$  : the centroid matrix  
 $weight$  : an  $r \times 1$  adjusted weight vector  
 $n$  : the number of data points  
 $r$  : the predefined number of basic partitions in the cluster ensemble  
 $K$  : the predefined number of clusters in the consensus partition  
 $sumKi$  : a matrix indicating the starting indexes for all basic partitions  
 $binIDX$  : a sparse representation of binarization of  $IDX$   
 $missMatrix$  : the indices matrix of the non-zero entries in  $IDX$  if there exists IBPs

**Output parameters:**

$D$  : an  $n \times K$  point-to-centroid distance matrix

**2.10 distance\_cos\_miss**

This function performs point-to-centroid distance calculation over data with IBPs using cosine distance measure.

**Syntax:**

```
D = distance_cos_miss(U, C, weight, n, r, K, sumKi, binIDX, missMatrix)
```

**Input parameters:**

$U$  : the  $1 \times 3$  utility parameter cell array  
 $C$  : the centroid matrix  
 $weight$  : an  $r \times 1$  adjusted weight vector  
 $n$  : the number of data points  
 $r$  : the predefined number of basic partitions in the cluster ensemble  
 $K$  : the predefined number of clusters in the consensus partition  
 $sumKi$  : a matrix indicating the starting indexes for all basic partitions  
 $binIDX$  : a sparse representation of binarization of  $IDX$   
 $missMatrix$  : the indices matrix of the non-zero entries in  $IDX$  if there exists IBPs

**Output parameters:**

$D$  : an  $n \times K$  point-to-centroid distance matrix



### 2.11 distance\_kl\_miss

This function performs point-to-centroid distance calculation over data with IBPs using KL-divergence measure.

**Syntax:**

```
D = distance_kl_miss(U, C, weight, n, r, K, sumKi, binIDX, missMatrix)
```

**Input parameters:**

U : the  $1 \times 3$  utility parameter cell array  
 C : the centroid matrix  
 weight : an  $r \times 1$  adjusted weight vector  
 n : the number of data points  
 r : the predefined number of basic partitions in the cluster ensemble  
 K : the predefined number of clusters in the consensus partition  
 sumKi : a matrix indicating the starting indexes for all basic partitions  
 binIDX : a sparse representation of binarization of IDX  
 missMatrix : the indices matrix of the non-zero entries in IDX if there exists IBPs

**Output parameters:**

D : an  $n \times K$  point-to-centroid distance matrix

### 2.12 distance\_lp\_miss

This function performs point-to-centroid distance calculation over data with IBPs using  $L_p$  norm measure.

**Syntax:**

```
D = distance_lp_miss(U, C, weight, n, r, K, sumKi, binIDX, missMatrix)
```

**Input parameters:**

U : the  $1 \times 3$  utility parameter cell array  
 C : the centroid matrix  
 weight : a  $r \times 1$  adjusted weight vector  
 n : the number of data points  
 r : the number of basic partitions in the cluster ensemble  
 K : the predefined number of clusters  
 sumKi : a matrix indicating the starting indexes for all basic partitions  
 binIDX : the sparse representation matrix  
 missMatrix : the indices matrix of the non-zero entries in IDX if there exists IBPs

**Output parameters:**

D : an  $n \times K$  point-to-centroid distance matrix

### 2.13 UCompute

This function performs utility calculation on data sets without missing values.

**Syntax:**

```
util = UCompute(index, U, w, C, n, r, K, sumKi, Pvector)
```

**Input parameters:**

`index`: an  $n \times 1$  cluster assignment matrix  
`U` : the  $1 \times 3$  utility parameter cell array  
`w` : the  $r \times 1$  adjusted weight parameter vector  
`C` : the centroid matrix  
`n` : the number of data points  
`r` : the number of basic partitions in the cluster ensemble  
`K` : the predefined number of clusters in the consensus partition  
`sumKi`: a matrix indicating the starting indexes for all basic partitions  
`Pvector` : a  $1 \times r$  row vector calculated from the contingency matrix

**Output parameters:**

`util` : the utility values calculated from the optimization objective of consensus clustering

**Discussion:**

The output `util` is a  $2 \times 1$  cell array including a utility gain and an adjusted utility value.

### 2.14 UCompute\_miss

This function performs utility calculation on data sets with missing values.

**Syntax:**

```
util = UCompute_miss(index, U, w, C, n, r, K, sumKi, Pvector, M)
```

**Input parameters:**

`index`: an  $n \times 1$  cluster assignment matrix  
`U` : the  $1 \times 3$  utility parameter cell array  
`w` : the  $r \times 1$  adjusted weight parameter vector  
`C` : the centroid matrix  
`n` : the number of data points  
`r` : the number of basic partitions in the cluster ensemble  
`K` : the predefined number of clusters in the consensus partition  
`sumKi`: a matrix indicating the starting indexes for all basic partitions  
`Pvector` : a  $1 \times r$  row vector calculated from the contingency matrix  
`M` : a mask matrix indicating the indices of the non-zero entries in `IDX`

**Output parameters:**

`util` : the utility values calculated from the optimization objective of consensus clustering

### 2.15 RunKCC

This function combines the two functions, i.e., `Preprocess` and `KCC`, for finding the consensus partition in one step.

**Syntax:**

```
[pi_sumbest, pi_index, pi_converge, pi_utility, t] = RunKCC(IDX, K, U, w,
rep, maxIter, minThres, utilFlag)
```

**Input parameters:**

`IDX` : the basic partition matrix  
`K` : the predefined number of clusters in the consensus partition  
`U` : the  $1 \times 3$  utility parameter cell array  
`w` : the  $r \times 1$  adjusted weight parameter vector  
`rep` : the number of repeated KCC experiments for selecting the best result  
`maxIter` : the maximum number of iterations for the convergence  
`minThres` : the minimum reduced threshold of objective function  
`utiFlag` : whether to calculate utility function

**Output parameters:**

`pi_sumbest` : the value of objective function for the best KCC experiment  
`pi_index` : the cluster assignment matrix in the consensus partition for the best KCC experiment  
`pi_converge`: the iterative values of objective function for the best KCC experiment  
`pi_utility` : the utility values for the best KCC experiment  
`t` : the execution time cost of the whole process for this function

### 2.16 exMeasure

This function assesses the clustering quality of the results obtained by a clustering algorithm.

**Syntax:**

```
[Acc, Rn, NMI, VIn, VDn, labelnum, ncluster, cmatrix] = exMeasure(cluster,
true_label)
```

**Input parameters:**

`cluster` : a  $n \times 1$  cluster assignment matrix returned by a clustering algorithm  
`true_label` : the true class labels for the data points

**Output parameters:**

`Acc` : the classification accuracy  
`Rn` : the normalized Rand statistic  
`NMI` : the normalized mutual information  
`VIn` : the normalized Variation of Information  
`VDn` : the normalized van Dongen criterion  
`labelnum` : the number of unique labels in the groundtruth data

**ncluster** : the number of clusters returned by the algorithm  
**cmatrix** : a **ncluster**  $\times$  **labelnum** contingency matrix

### Discussion:

This function implements five external validity indices, including classification accuracy [Nguyen and Caruana 2007] ( $CA$ ), the normalized mutual information [Cover and Thomas 2012] ( $NMI$ ), the normalized Rand statistic [Rand 1971] ( $R_n$ ), the normalized van Dongen criterion [Dongen 2000] ( $VD_n$ ), and the normalized Variation of Information [Cover and Thomas 2012] ( $VI_n$ ). In implementing the **exMeasure** function, we utilize a function called **bestMap** written by [Cai et al. 2005], in which the Hungarian method [Carpaneto and Toth 1980] is employed to resolve the label assignment issue in clustering.

### REFERENCES

- D. Cai, X. He, and J. Han. 2005. Document Clustering Using Locality Preserving Indexing. *IEEE Trans. Knowl. Data Eng.* 17, 12 (Dec 2005), 1624–1637.
- Giorgio Carpaneto and Paolo Toth. 1980. Algorithm 548: Solution of the assignment problem [H]. *ACM Transactions on Mathematical Software (TOMS)* 6, 1 (1980), 104–111.
- Thomas M Cover and Joy A Thomas. 2012. *Elements of Information Theory*. John Wiley & Sons.
- Stijn Dongen. 2000. *Performance Criteria for Graph Clustering and Markov Cluster Experiments*. Technical Report. Amsterdam, The Netherlands, The Netherlands.
- Nam Nguyen and Rich Caruana. 2007. Consensus Clusterings. In *Proceedings of the 2007 Seventh IEEE International Conference on Data Mining (ICDM '07)*. IEEE Computer Society, Washington, DC, USA, 607–612. <https://doi.org/10.1109/ICDM.2007.73>
- William M Rand. 1971. Objective Criteria for the Evaluation of Clustering Methods. *J. Am. Stat. Assoc.* 66, 336 (1971), 846–850.