MI3.22
Advanced Programming for HPC
Master ICT, USTH, 2nd year

Aveneau Lilian

lilian.aveneau@univ-poitiers.fr
XLIM/SIC/IG, CNRS, Computer Science Department
University of Poitiers

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Lecture 3 – Scan & other Patterns

- Scan
- Segmented Scan
- Others Parallel Patterns

Overview

- Scan
- Segmented Scan
- Others Parallel Patterns

SCAN

It consists to apply a binary, associative and commutative operator to all elements of a given input X of size n, to compute a list of new values:

$$\mathrm{SCAN}(X) = \{Y_i\}_{i=0}^{n-1} = \left\{ \bigoplus_{k=0}^{i} X_k \right\}_{i=0}^{n-1} \qquad \text{where } \bigoplus_{k=i}^{i} = X_i$$

$$X = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline X_0 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 \\ \hline \end{array}$$

?

$$Y = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline Y_0 & Y_1 & Y_2 & Y_3 & Y_4 & Y_5 & Y_6 & Y_7 & Y_8 \\ \hline \end{array}$$

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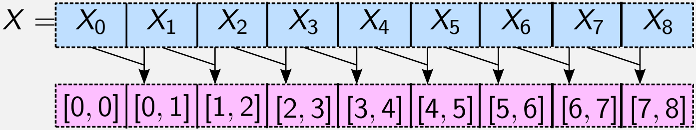
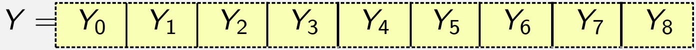


$$\text{with } [i,j] = X_i \oplus X_{i+1} \oplus \ldots \oplus X_j$$



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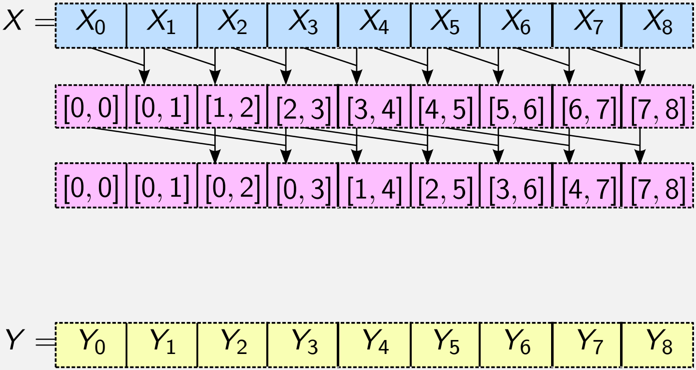

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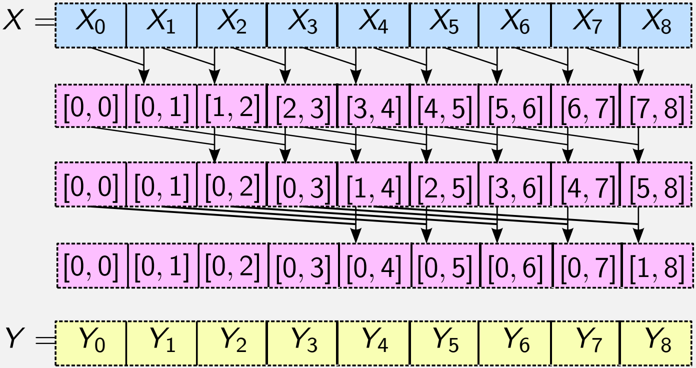

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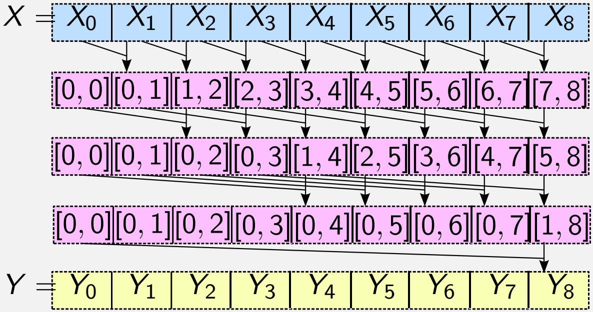

- The swiss-knife of any parallel programmer!
- In thrust: `thrust::inclusive_scan` with different possibilities

PREFIX-SCAN

It consists to apply a binary associative commutative operator to all elements of a given input X of size n, to compute a list of new values:

$$\text{PREFIX-SCAN}(X) = \left\{ \bigoplus_{k=0}^{\boxed{i-1}} X_k \right\}_{i=0}^{n-1}$$

where $\forall j < i$, $\bigoplus_i^j = 0$ or any other initialization value

- A *shifted* SCAN, in fact!
  Example with $\oplus = +$

X_i	1	1	1	1	1	1	1	1
SCAN	1	2	3	4	5	6	7	8
PRESCAN	0	1	2	3	4	5	6	7

- It is also the swiss-knife of any parallel programmer!
- In thrust: `thrust:exclusive_scan` with different possibilities

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SEGMENTED SCAN I

It is a SCAN or a PREFIX-SCAN done per segment of a given array ...

Example:

$S =$	0	0	0	1	1	2	2	2	2
$X =$	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8

$\downarrow$

$Y =$	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8

where S defines the segments, and so:

- $Y_0 = X_0$, $Y_1 = X_0 \oplus X_1$, and $Y_2 = X_0 \oplus X_1 \oplus X_2$
- $Y_3 = X_3$, $Y_4 = X_3 \oplus X_4$
- $Y_5 = X_5$, $Y_6 = X_5 \oplus X_6$, $Y_7 = X_5 \oplus X_6 \oplus X_7$ and $Y_8 = X_5 \oplus X_6 \oplus X_7 \oplus X_8$

SEGMENTED SCAN II

Obviously, for PREFIX-SEG-SCAN, we have

- $Y_0 = 0$, $Y_1 = X_0$, and $Y_2 = X_0 \oplus X_1$
- $Y_3 = 0$, $Y_4 = X_3$
- $Y_5 = 0$, $Y_6 = X_5$, $Y_7 = X_5 \oplus X_6$ and $Y_8 = X_5 \oplus X_6 \oplus X_7$

Notice that it is a SCAN (or PREFIX-SCAN) using a particular operator:

$$\otimes : \{\mathbb{N} \times T\}^2 \to T$$
$$\otimes\left(\{S_i, X_i\}, \{S_j, X_j\}\right) = \begin{cases} X_i \oplus X_j & \text{if } S_i = S_j, \\ X_j & \text{otherwise.} \end{cases}$$

In practice, computation cost is a little higher:

- Need to read more data (the segment id)
- Avoid it when possible (prefer SCAN)

With Thrust, implemented in `thrust::inclusive_scan_by_key` and `thrust::exclusive_scan_by_key`

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SPLIT or PARTITION

Allows to separate an array into two segments using a given predicate:

$P =$	1	0	1	0	1	0	0	0	1
$X =$	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8

$\downarrow$

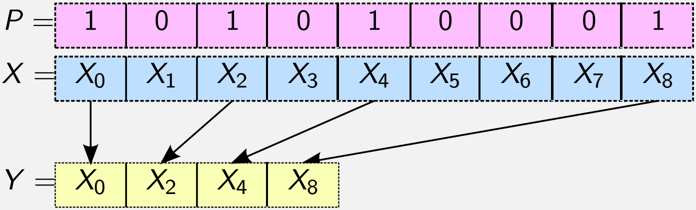
$Y =$	X_0	X_2	X_4	X_8	X_1	X_3	X_5	X_6	X_7

Using Thrust, you can use `thrust::partition` and variants ...
Notice that it can be written using others patterns:

```
PARTITION(X, P)
    { X and P are arrays of size N, X being data and P_i = {0,1} a boolean array }
    Y, Z: arrays of size N
    FOR each PE i in parallel
        Y_i ← P_i
        Z_i ← P_i − 1 { so contains 0 or −1 }
    Y ← PREFIX−SCAN(Y)
    Z ← REVERSE−SCAN(Z) { scan in reverse order }
    FOR each PE i in parallel
        IF NOT P_i THEN Y_i ← N + Z_i
    return SCATTER(X, Y)
```

COMPACT

COMPACT is similar to PARTITION, but forgetting the second part (for which predicate $P_i = 0$) ...



- Here, destination is not completely modified ...
- Result has variable length!

With Thrust, take a look at *stream compaction* at
`http://thrust.github.io/doc/group__stream__compaction.html`

SORT

SORT is a generic pattern for sorting data, given an order

- Many algorithms exist, for parallel computers
- An old one: RICHARD COLE's sorting machine 1986, "Parallel Merge Sort"
- Bitonic sort
- Parallel Quicksort
- Probably the most efficient: radix sort (see labwork)

In thrust, you can use `thrust::sort` which implements a fast radix-sort with bitonic sort into blocks to accelerate the whole process ...