1 Phan 5.2.3

1.1 Chung minh bat dang thuc Minkowski

Ta co:

$$\begin{aligned}
\Sigma_{k=1}^{n}|x_k+y_k|^p &= \Sigma_{k=1}^{n}|x_k+y_k||x_k+y_k|^{p-1} \\
&\leq \Sigma_{k=1}^{n}(|x_k|+|y_k|)|x_k+y_k|^{p-1} \\
&= \Sigma_{k=1}^{n}|x_k||x_k+y_k|^{p-1}+\Sigma_{k=1}^{n}|y_k||x_k+y_k|^{p-1} \\
&\leq (\Sigma_{k=1}^{n}|x_k|^p)^{\frac{1}{p}}(\Sigma_{k=1}^{n}|x_k+y_k|^{(p-1)*q})^{\frac{1}{q}}+(\Sigma_{k=1}^{n}|y_k|^p)^{\frac{1}{p}}(\Sigma_{k=1}^{n}|x_k+y_k|^{(p-1)*q})^{\frac{1}{q}} \\
&= ((\Sigma_{k=1}^{n}|x_k|^p)^{\frac{1}{p}}+(\Sigma_{k=1}^{n}|y_k|^p)^{\frac{1}{p}}))*(\Sigma_{k=1}^{n}|x_k+y_k|^{(p-1)*q})^{\frac{1}{q}} \\
&= ((\Sigma_{k=1}^{n}|x_k|^p)^{\frac{1}{p}}+(\Sigma_{k=1}^{n}|y_k|^p)^{\frac{1}{p}}))*(\Sigma_{k=1}^{n}|x_k+y_k|^p)^{\frac{1}{q}} \qquad (1)
\end{aligned}$$

Suy ra:

$$\begin{aligned}
(\Sigma_{k=1}^{n}|x_k+y_k|^p)^{1-1/q} &= (\Sigma_{k=1}^{n}|x_k+y_k|^p)^{1/p} \\
&\leq (\Sigma_{k=1}^{n}|x_k|^p)^{\frac{1}{p}}+(\Sigma_{k=1}^{n}|y_k|^p)^{\frac{1}{p}})(dpcm)
\end{aligned}$$

Trong bat dang thuc so (1):

- Dong thu 1 sang dong thu 2 la do $|x+y| \leq |x|+|y|$.
- Dong thu 3 sang dong thu 4 la do ap dung bat dang thuc Hoelder.
- Dong thu 5 sang dong thu 6 la do $\frac{1}{p}+\frac{1}{q}=1$ nen $(p-1)*q=p$.