ELEC 4700 Assignment 2

Finite Difference Method

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1. **Finite Difference Method**

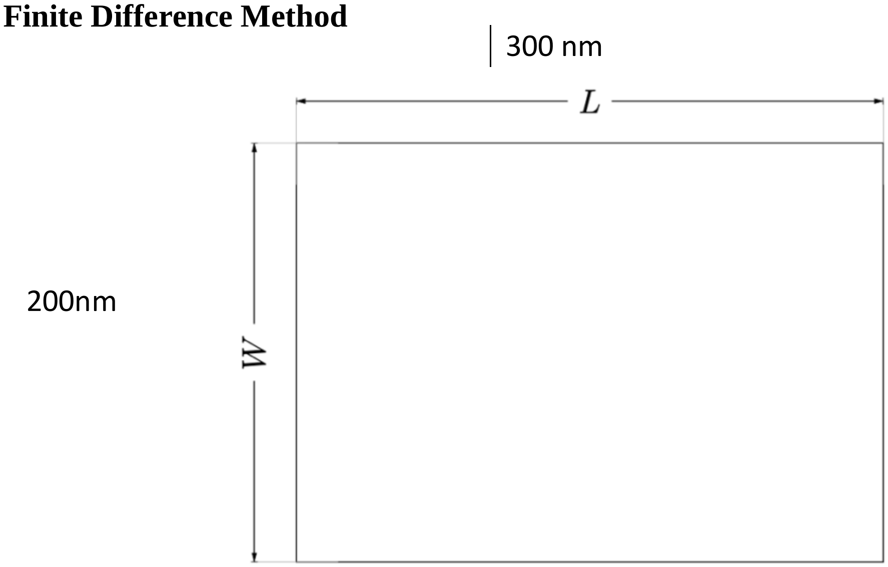

Figure 1. The rectangular region with sizes.

a. Simple case (1-D)

- Solving the electrostatic potential using finite difference method in the rectangular region as figure 1: GV=F => $V=G^{-1}F$

- The condition is set for V=V0 at x=0 and V=0 at x=L

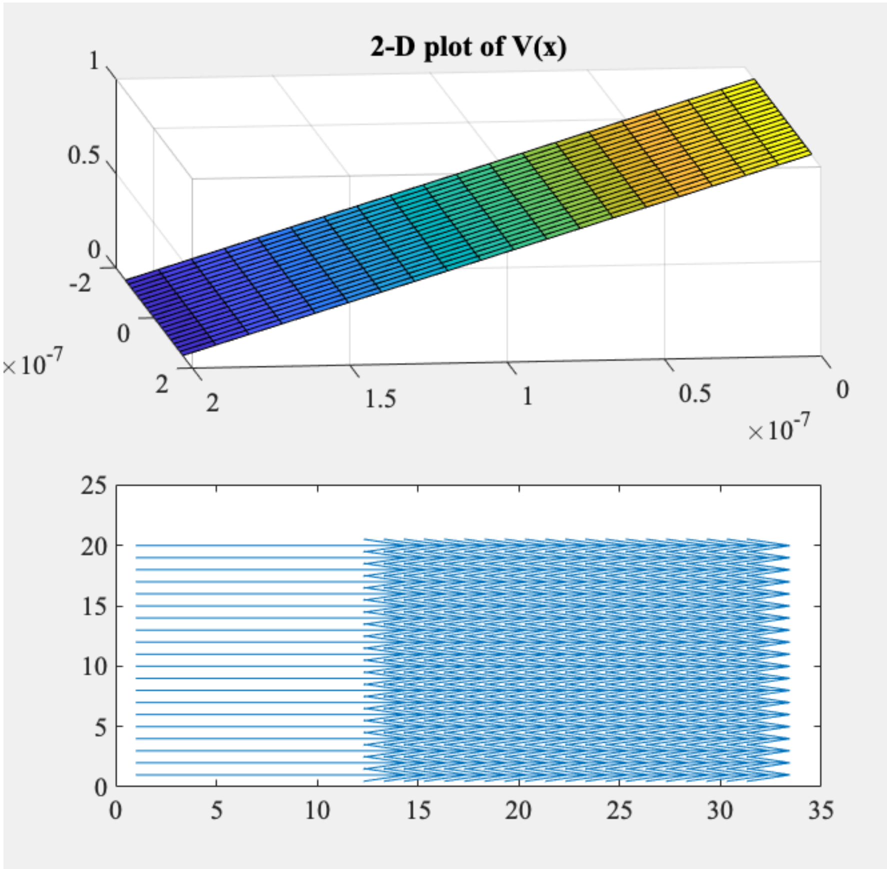

Figure 2. 1-D case of electrostatic potential using finite difference method.

b. The boundary is set to V=V0 at x=0 and x=L, V=0 at y=0 and y=W using 2 models: finite difference method and analytical method

- Finite difference method:

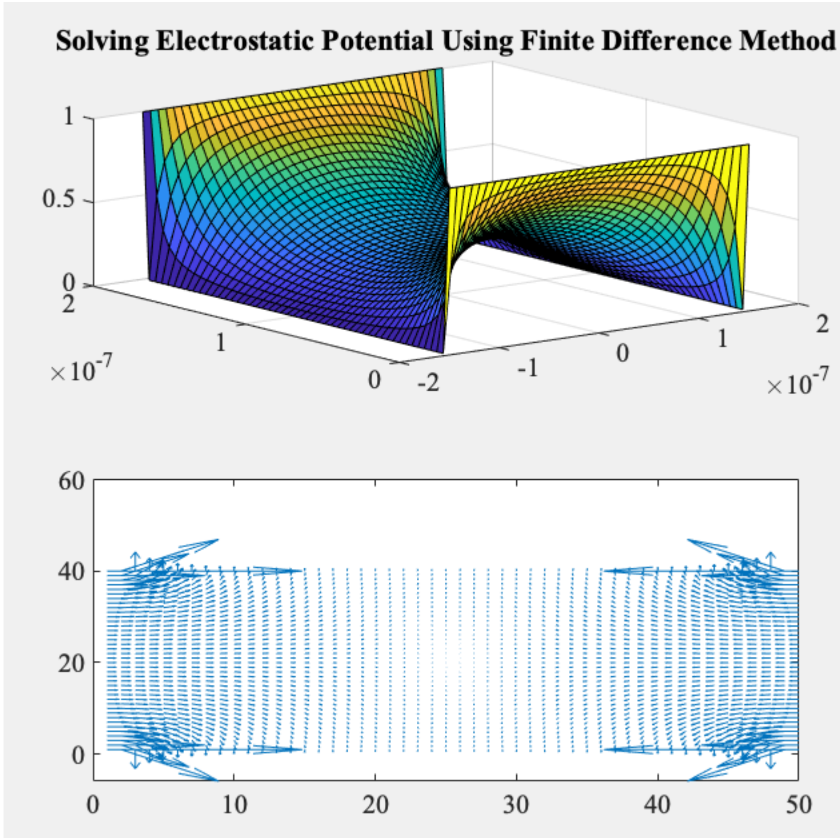

Figure 3. 2-D case of electrostatic potential using finite difference method.

- Analytical method:

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \frac{\cosh\left(\frac{n\pi x}{W}\right)}{\cosh\left(\frac{n\pi L}{2W}\right)} \sin\left(\frac{n\pi y}{W}\right)$$

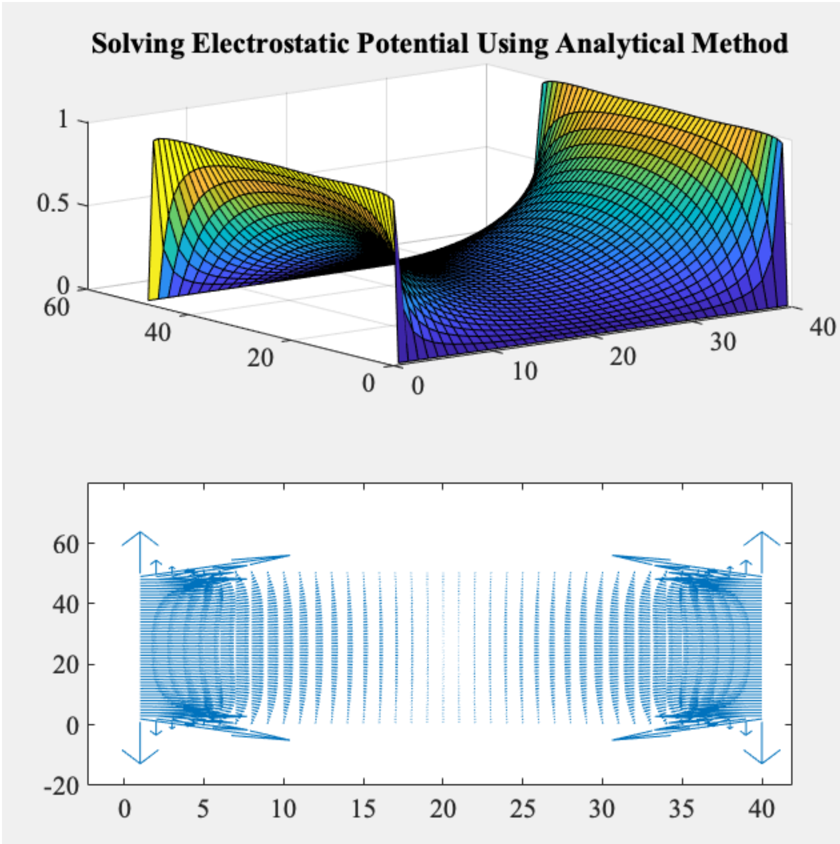

Figure 4. 2-D case of electrostatic potential using analytical method in 300 iterations

Both methods all have error, the finite difference method is done with discretization, so it is limited to uniform meshing only. Meanwhile, the analytical method cuts off after a certain number of sums in iteration. The analytical series solution breaks after 300 iterations. After this point, the analytical series does not give the saddle structure anymore, so this is when the analytical series needs to be stopped. The problem could be the number n only is non-zero when it is odd and is 0 if n is even. Increasing iteration increases the missing of V in case of even n and results in the greater difference between two methods. The finite difference method is simpler with transparent formulation, meshing, programming, and debugging are simpler and easier than the analytical method to keep track on.

2. Using the finite difference method to solve for current flow

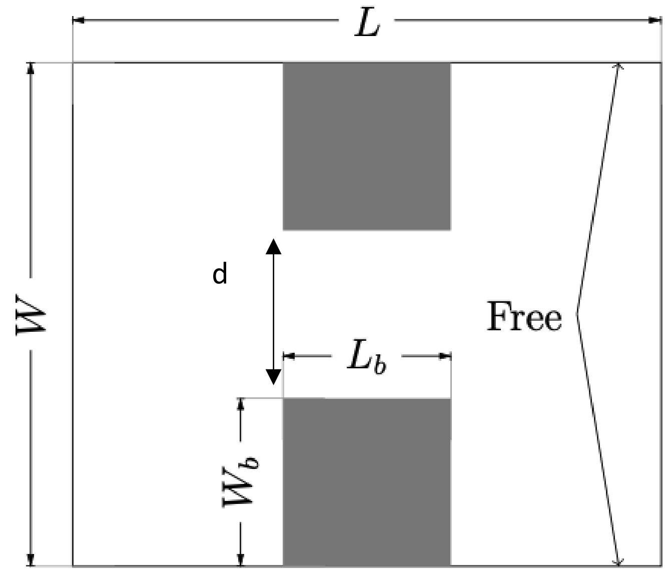

Figure 5. The rectangular region with isolated conducting sides and "bottle-neck" with d is the size of the bottle neck

a.

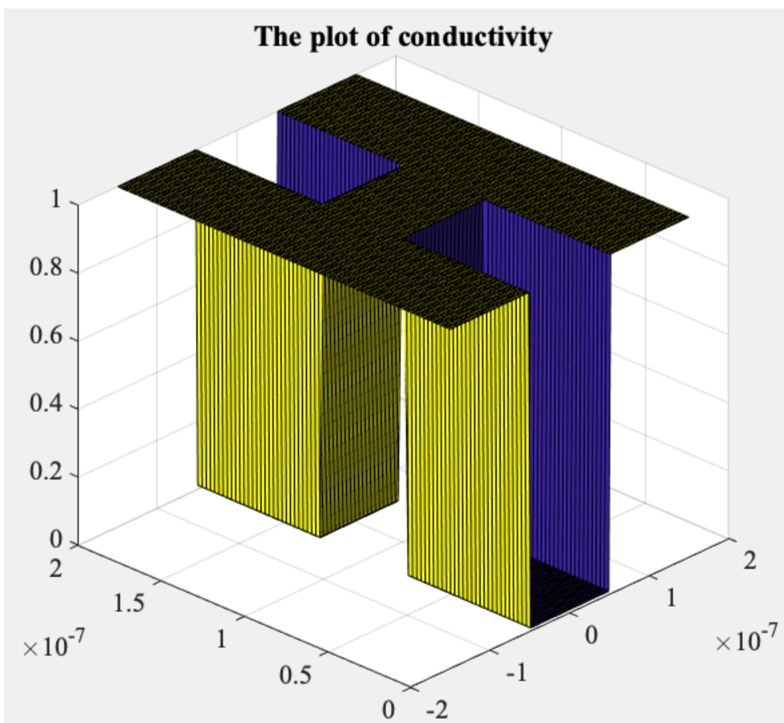

Figure 6. The plots of $\sigma(x, y)$

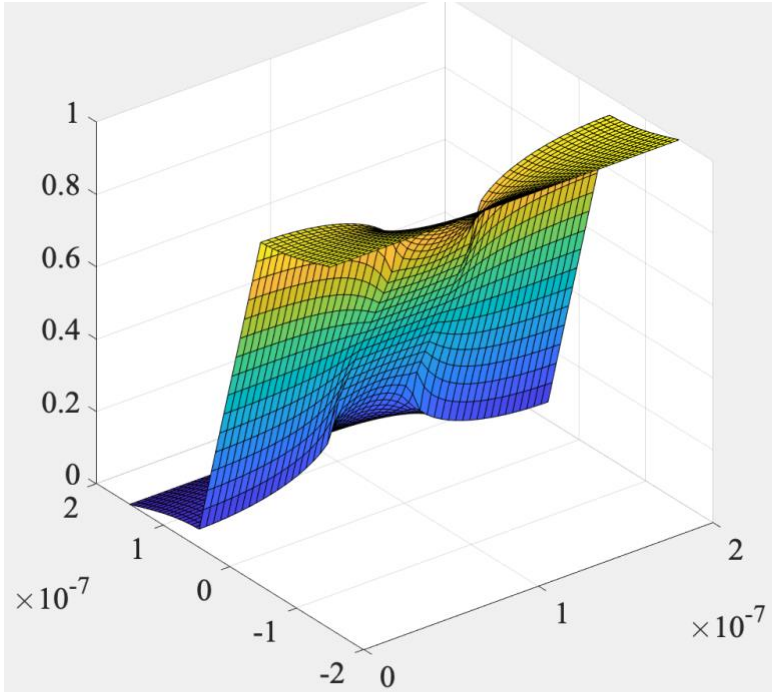

Figure 7. The voltage plot V(x,y)

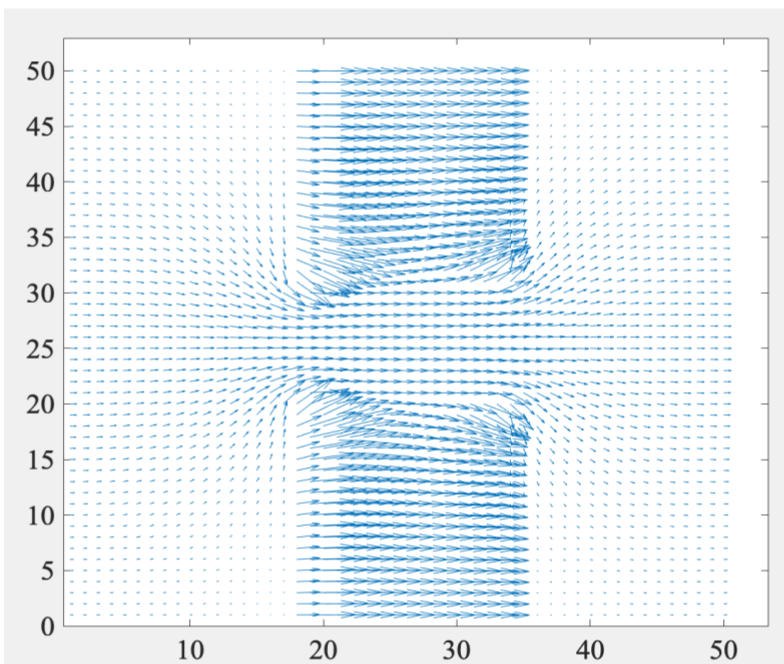

Figure 8. The plot of vector E(x,y).

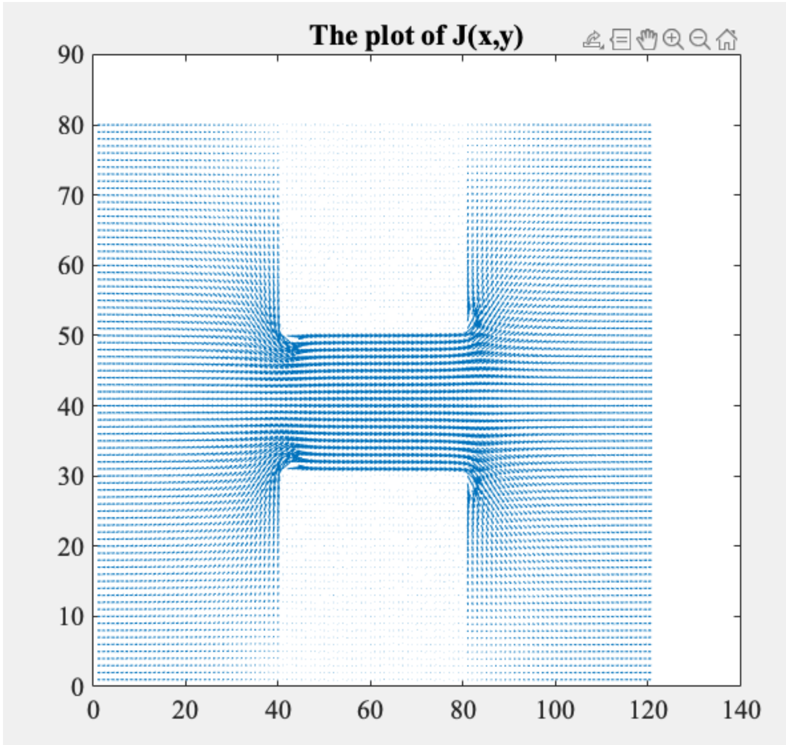

Figure 9. The plot of vector J(x,y).

The current is found to be -1.5 mA
b)

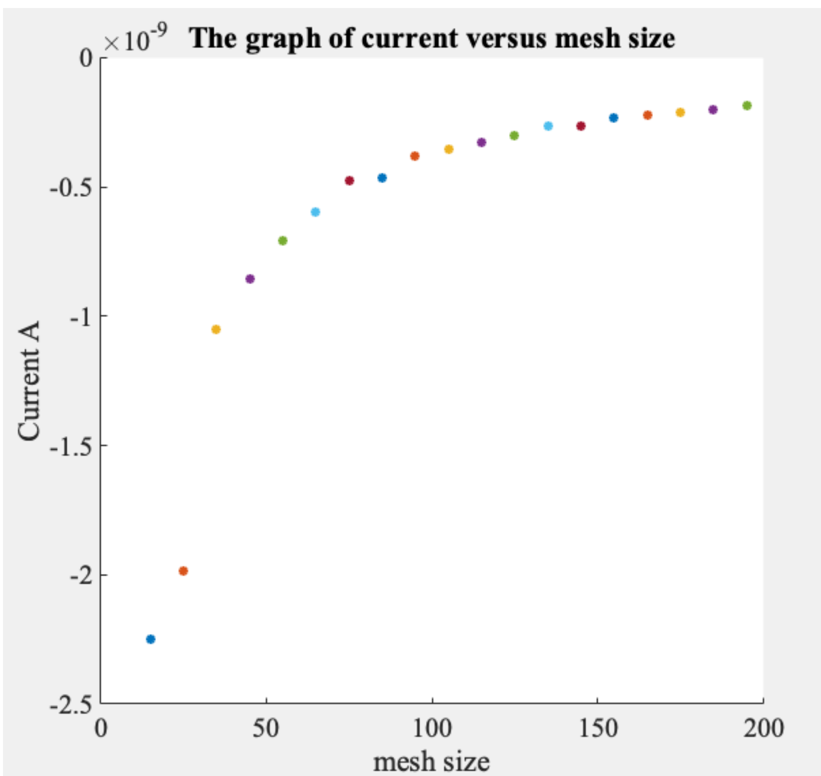

Figure 10. The plot of current versus different mesh size.

c)

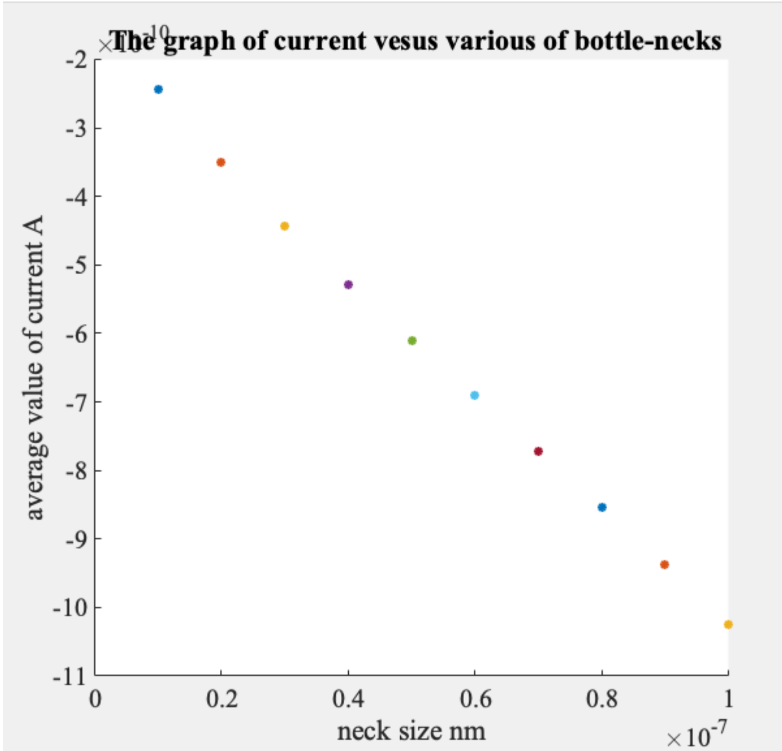

Figure 11. The plot of current with different bottle-neck sizes d.

d)

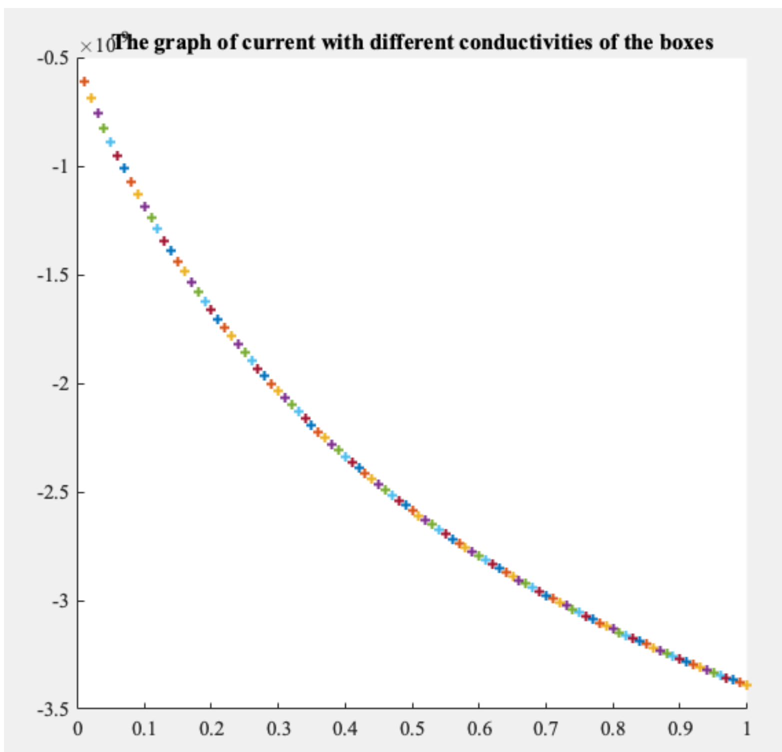

Figure 12. The plot of current versus various conductivities of the boxes.