Simulation-based population synthesis using Gibbs sampling

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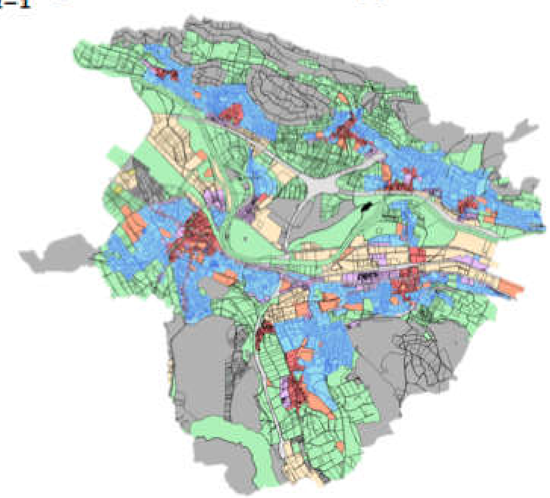
Outline

Modelling and Micosimulation

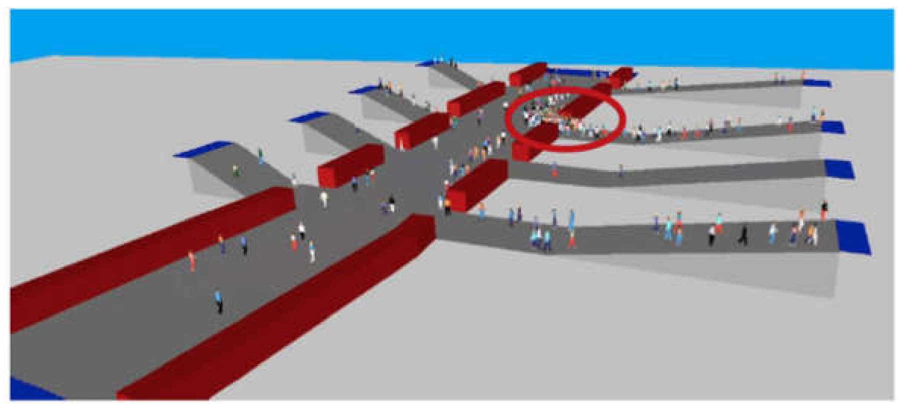
Urban area

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (f^r(X_i^r) - f^c(X_i^c))^{\vartheta} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + f^z(z)$$



Mobility Hub

$$Q(\xi,\tau) = \begin{cases} n(\xi,\tau) \left\{ 1 - \exp\left[-\gamma_\xi A_\xi \left(\frac{1}{n(\xi,\tau)} - \frac{1}{N_\xi} \right) \right] \right\} & \text{if } 0 < n(\xi,\tau) < N_\xi \\ 0 & \text{otherwise} \end{cases}$$



TRANSP-OR

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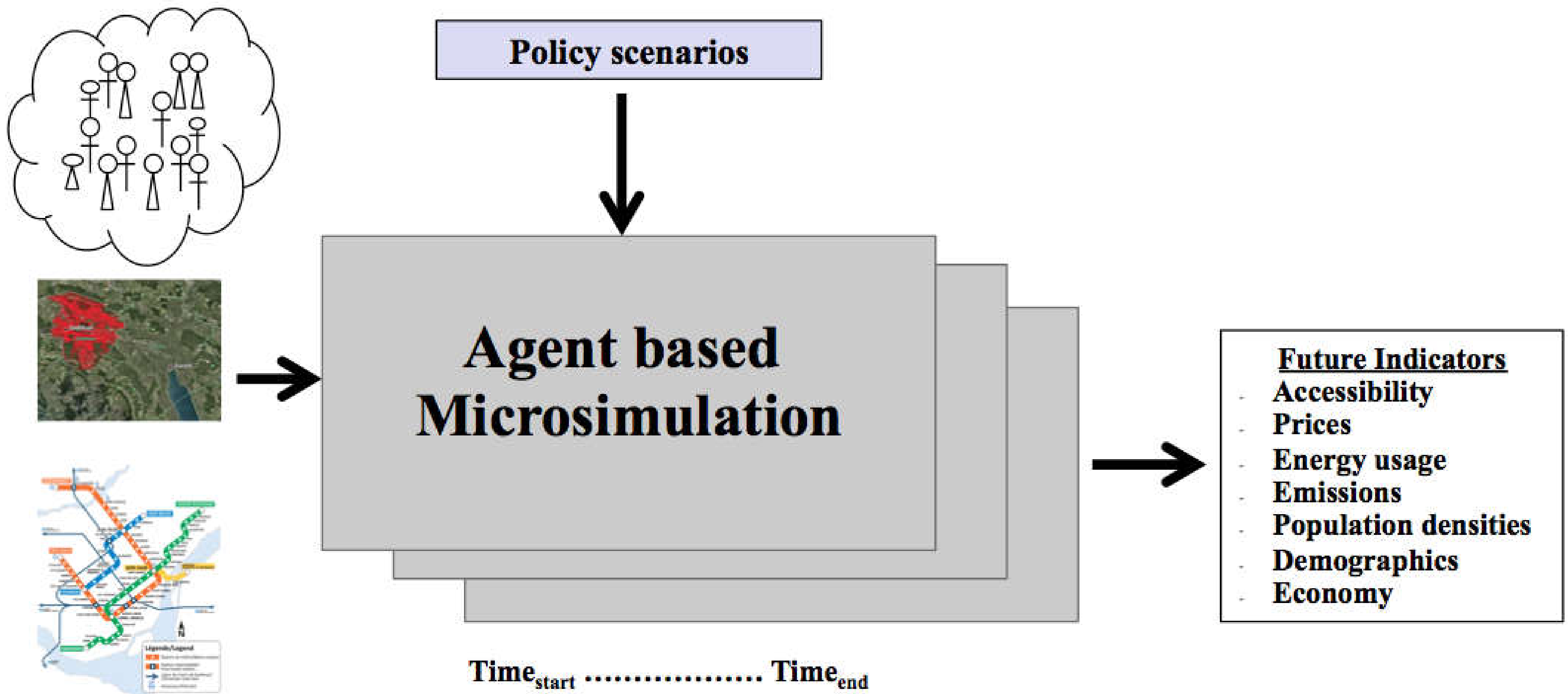
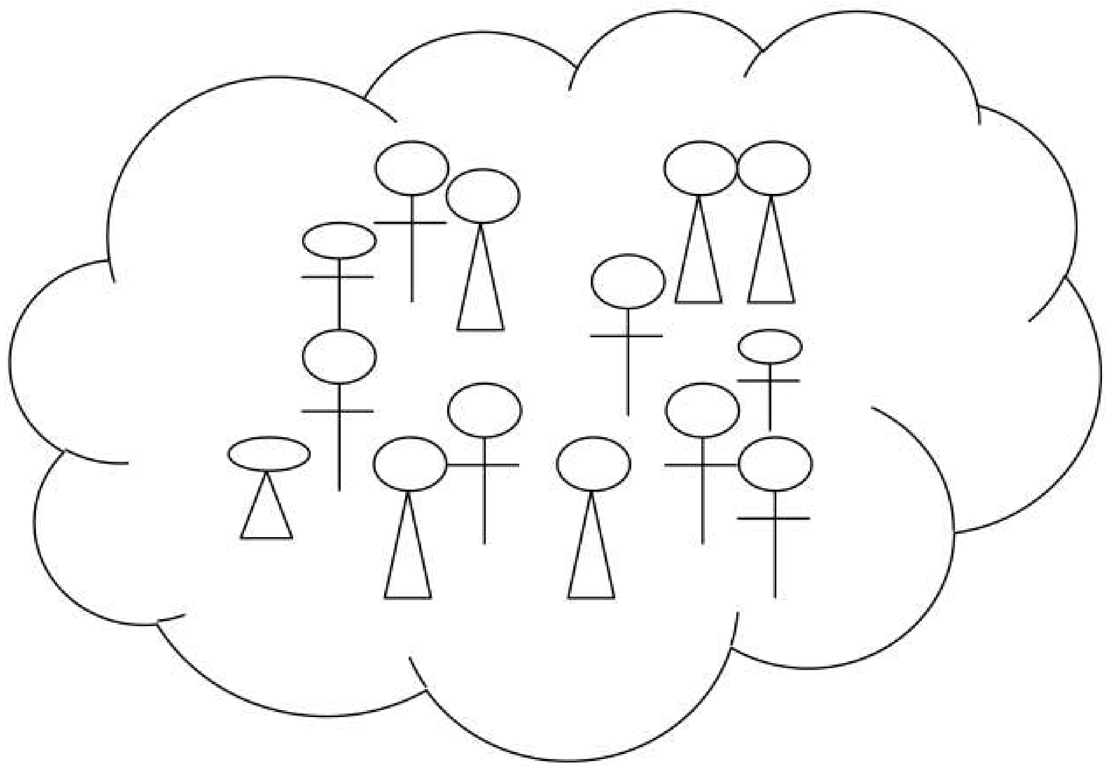
Agent based Microsimulation



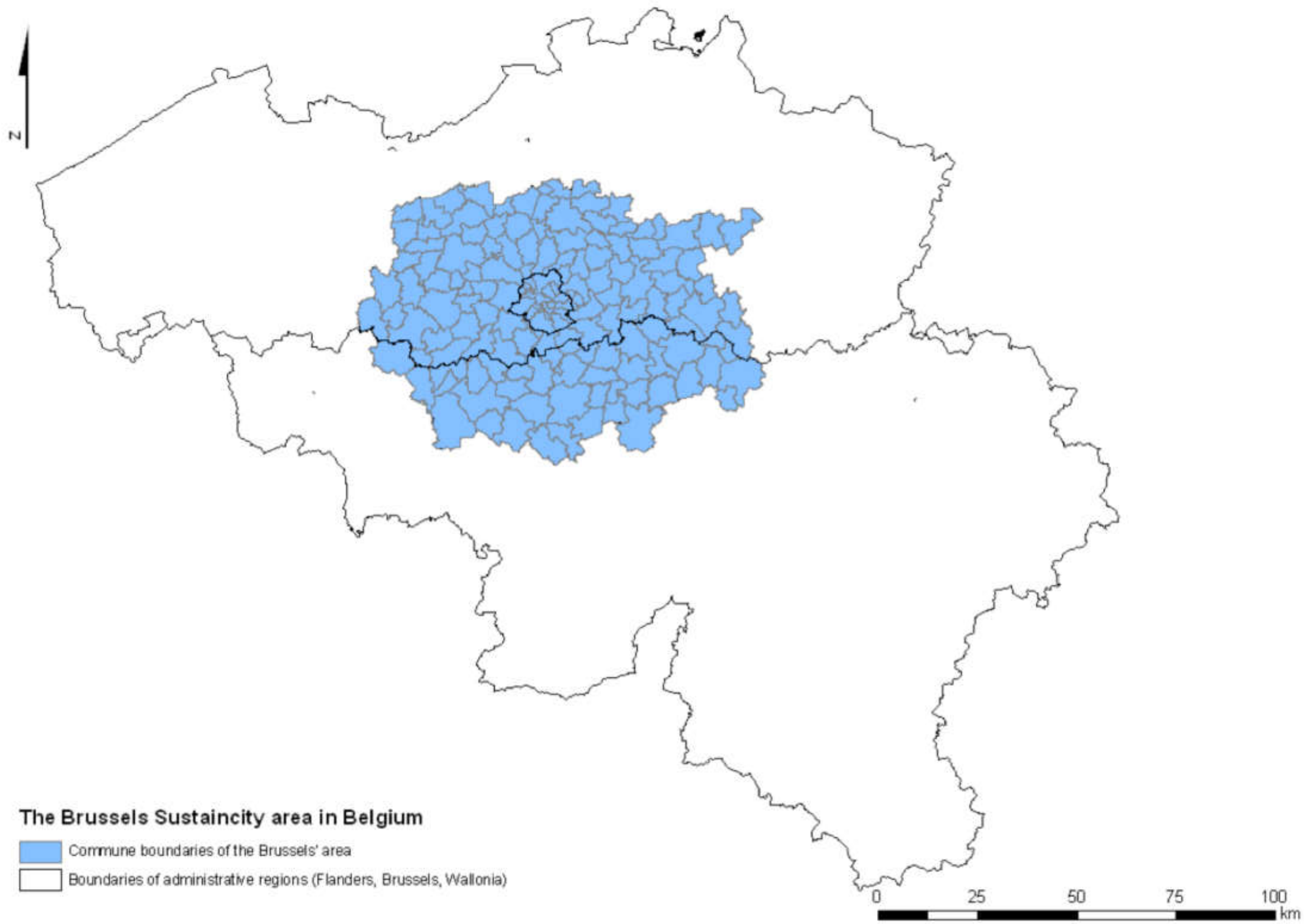
Population Synthesis



SustainCity project

- European Union funded mega research project
- More than 10 major European universities involved
- Aims:
  - Integrated land use and transportation modelling framework
  - Demographics, environment, and multi-scale issues
- Case studies
  - Paris
  - Zurich
  - Brussels

SustainCity: Brussels case study [Farooq et al., 2015]



Brussels case study

- Data sources (extremely limited)
  - Incomplete conditionals of households and persons (Census 2001)
  - Travel survey of households and individuals (MOBEL 1999)
    - **3063 observations (0.2%)**
- Synthetic household attributes
  - Size, children, workers, cars, income, university education, dwelling type, sector

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- *Conventional synthesis procedures were not usable*

Evolution of Synthesis Methods in Transport

Initial efforts

- From *Four-Stage* to *Activity based Integrated* modelling
- Forecasting behaviour using individual level models
- Synthesis for TRansportation ANalysis SIMulation System (TRANSIMS) [Beckman et al., 1996]

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Existing approach

- **Fitting based approach**
  - Iterative proportional fitting
    - By far the most commonly used approach
  - Combinatorial optimization
- Adjusting sample weights to fit the aggregate statistics

Iterative Proportional Fitting (IPF) [Beckman et al., 1996]

- Contingency Table (CT) from sample
  - Categorization of variables of interest
  - Totals for each cell of the resulting multi-way table
- Fitting: Multi-constraint gravity model sort of formulation
  - Sample used to initialize the contingency table
  - Use marginal as dimensional totals
  - Adjust the cell proportions to fit dimension totals
  - Iterate while the error is large
  - Odd-ratio is maintained
- Generation of agents based on fitted weights
  - Monte Carlo simulation for fractions

Combinatorial Optimization (CO) [Williamson et al., 1998]

- Zone-by-zone
- 0-1 weights for each row in the sample
- Optimizing the weights to fit zonal marginals
- Use of hill-climbing, simulated annealing, and genetic algorithm to estimate the best set of obs. weights for each zone

Key issues

- Optimization resulting in one synthetic population
  - Data are incomplete and purposely tampered with sophisticated anonymizing techniques
  - There can be any number of solutions
- Cloning of data rather than creation of a heterogeneous representative population
- Focus on fitting marginals
  - Generation of correct correlation structure is more important, as that is what the behavioural models are operating on

Key issues

- Over reliance on the accuracy of the microdata, without serious consideration to the sampling process and assumptions
- Large enough sample size
- Inefficient use of the available data
- Discrete agent attributes only
- Scalability issues

Problem statement

- True population: Individual agents defined as a set of attributes $X = (X^1, X^2, ..., X^n)$
  - Discrete (e.g. marital status) or continuous (e.g. income)
  - Unique joint distribution represented by $\pi_X(x)$
- No direct access to $\pi_X(x)$ and hard to draw from
- Instead, only partial views of $\pi_X(x)$
  - Marginals, conditional-marginals, and samples

Problem statement

- Develop a synthesis procedure that lets us use these views to draw a synthetic population as if we were drawing from $\pi_X(x)$
    - At the same time, ensuring that the empirical distribution $\pi_{\hat{X}}(\hat{x})$ of $\hat{X}$ resulting from the realized synthetic population is as close to $\pi_X(x)$ as possible

Simulation based approach [Farooq et al., 2013]

- Propose to use **Gibbs sampler** for drawing synthetic population
- MCMC method that uses $\pi(X^i | X^j = x^j$, for $j = 1 \ldots n$ & $i \neq j) = \pi(X^i | X^{-i})$ for $i = 1, \ldots, n$ to simulate drawing from $\pi_X(x)$ [Geman and Geman, 1984]
- Key challenge: Preparation of the conditional distributions for attributes from available data sources

Incomplete conditionals

- Full-conditionals rarely available

Completing conditionals by assumptions

- If in $\pi(X^1|X^{-1}) = \pi(X^1|X^{(2\ldots k)}, X^{((k+1)\ldots n)})$ only $\pi(X^1|X^{(2\ldots k)})$ is available
  - In case of no other information, $\pi(X^1|X^{-1}) = \pi(X^1|X^{(2\ldots k)}), \forall X^{((k+1)\ldots n)}$
  - Worst case, we can use $\pi(X^1|X^{-1}) = \pi(X^1)$

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- For $(Age|Sex, Income)$
  - From data only $(Age|Income)$ available
  - Assume that for all values of Sex, $(Age|Sex, Income) = (Age|Income)$
  - No matter the Sex of a person is, Age is only dependent on $Income$

Completing conditionals by domain knowledge

- In case of domain knowledge
  $\pi(X^1|X^{(2\ldots k)}, X^{((k+1)\ldots n)} = a) = \pi^a(X^1|X^{(2\ldots k)}),$
  $\pi(X^1|X^{(2\ldots k)}, X^{((k+1)\ldots n)} = b) = \pi^b(X^1|X^{(2\ldots k)}),$
  $\ldots$

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  $\pi(X^1|X^{(2\ldots k)}, X^{((k+1)\ldots n)} = b) = \pi^b(X^1|X^{(2\ldots k)})$,
  ...

- For $(Income|Sex, Age)$
  - From data only $(Income|Sex)$ available
  - Known: Infants do not have income, students have low income
    - $(Income|Sex, Age) = \alpha(Income|Sex)$ for $Age = 1 \ldots 12$
    - $(Income|Sex, Age) = \beta(Income|Sex)$ for $Age = 13 \ldots 18$
    - $(Income|Sex, Age) = \gamma(Income|Sex)$ for $Age > 18$
    - $\alpha + \beta + \gamma = 1$ and $\alpha < \beta < \gamma$

Completing conditionals by parametric models

- For instance, Logit model $\pi\left(X_l^1|X_m^{-1}\right) = \frac{e^{\left(V_{X_l^1|X_m^{-1}}\right)}}{\sum_{p=1}^{L}\left(e^{\left(V_{X_p^1|X_m^{-1}}\right)}\right)}$

Completing conditionals by parametric models

- For instance, Logit model $\pi(X_l^1|X_m^{-1}) = \frac{e^{(V_{X_l^1}|X_m^{-1})}}{\sum_{p=1}^{L}\left(e^{(V_{X_p^1}|X_m^{-1})}\right)}$

- For $(Dwelling|Income, Sex, Age)$
  - In sample $(Dwelling, Age, Sex)_p$ for a person are available
  - In zone (z) where person is living
    - Average income by dwelling type (av_inc)
    - ...
  - Dwelling choice model can be estimated for person:
    $dwel\_typ = (attached, semidetached, detached, apartment)$
    and $V^i_{(p,z)} = ASC^i + \beta^i_{age_p} \times Age + \beta^i_{av\_inc_z} \times av\_inc_z + interactions + \ldots$

Population from Swiss Census

- Access to Swiss Census for 2000
  - Person and household attributes (Except for Income)
- Selected area: postal code in Lausanne
  - CH-1004
  - 28,533 persons
- Four Person attributes (384 combinations)
  - Age (<15, 15-24, 25-34, 35-44, 45-54, 55-64, 65-74, >74)
  - Sex (Female, Male)
  - Household size (1, 2, 3, 4, 5, 6 or more)
  - Education level (none, primary, secondary, university/college)

Comparison between IPF and Simulation

- Criteria: how well the joint distribution is reproduced?

Data preparation

- Prepared same type of datasets as commonly available
  - Individual level microsample
    - Drawing from Census: Uniformly, without replacement
    - No sampling-zero
  - Zonal level conditionals (with various level of completion)
    - By counting from Census

List of available sample sizes

No.	Sample Size
1	20%
2	10%
3	5%
4	3%
5	1%

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1	20%
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- In practice the sample size is 5% or less
- Larger sizes used to investigate representativeness

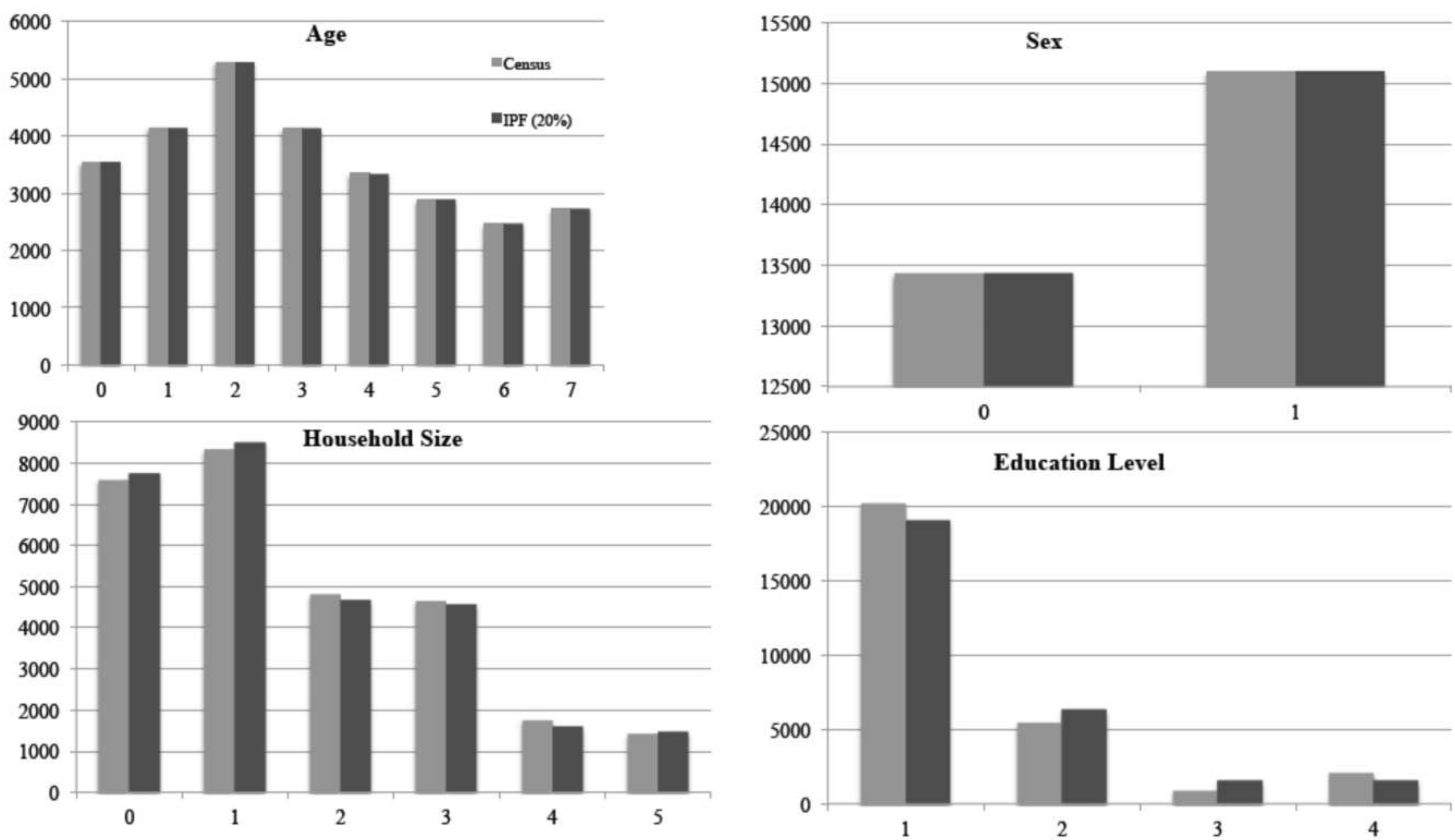
List of available conditionals

No.	ID	Conditionals
1	*FullCond*	$\pi(age \mid sex, hhld\_size, edu\_level)$
		$\pi(sex \mid age, hhld\_size, edu\_level)$
		$\pi(hhld\_size \mid age, sex, edu\_level)$
		$\pi(edu\_level \mid age, sex, hhld\_size)$
2	*Partial_1*	$\pi(age \mid \cancel{sex}, hhld\_size, edu\_level)$
		$\pi(sex \mid age, hhld\_size, edu\_level)$
		$\pi(hhld\_size \mid age, sex, edu\_level)$
		$\pi(edu\_level \mid age, sex, hhld\_size)$
3	*Partial_2*	$\pi(age \mid \cancel{sex}, hhld\_size, edu\_level)$
		$\pi(sex \mid age, hhld\_size, edu\_level)$
		$\pi(hhld\_size \mid age, \cancel{sex}, edu\_level)$
		$\pi(edu\_level \mid age, sex, hhld\_size)$
4	*Partial_3*	$\pi(age \mid \cancel{sex}, hhld\_size, edu\_level)$
		$\pi(sex \mid age, hhld\_size, edu\_level)$
		$\pi(hhld\_size \mid age, \cancel{sex}, edu\_level)$
		$\pi(edu\_level \mid age, \cancel{sex}, hhld\_size)$

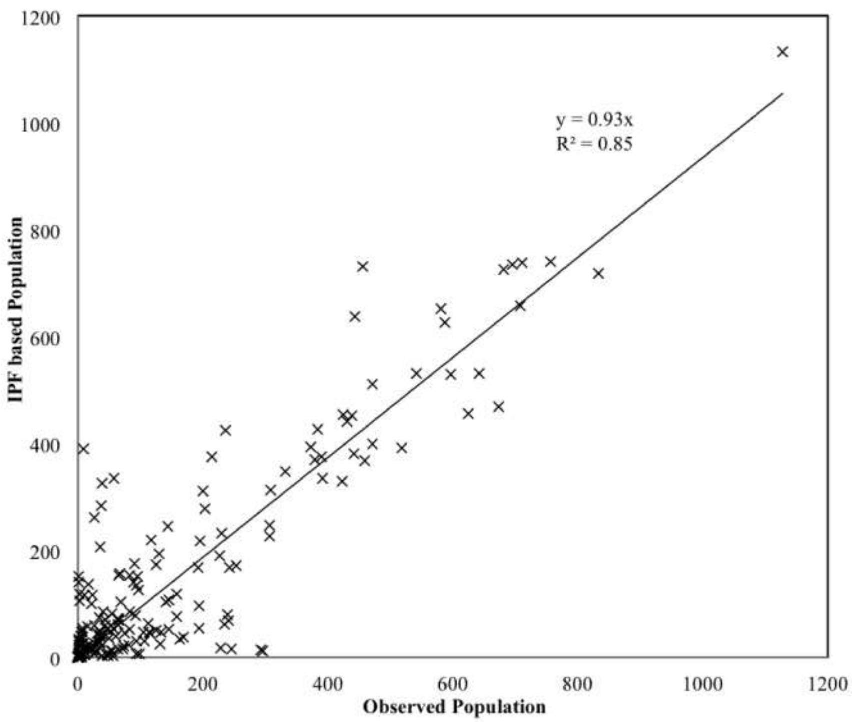
Data preparation

- Based on sample-conditional combinations
  - 20 possibilities
- IPF can use marginals only
  - Number of experiments collapses to 5
- Simulation based synthesis
  - Used conditionals only (used lesser information)
  - Number of experiments collapses to 4

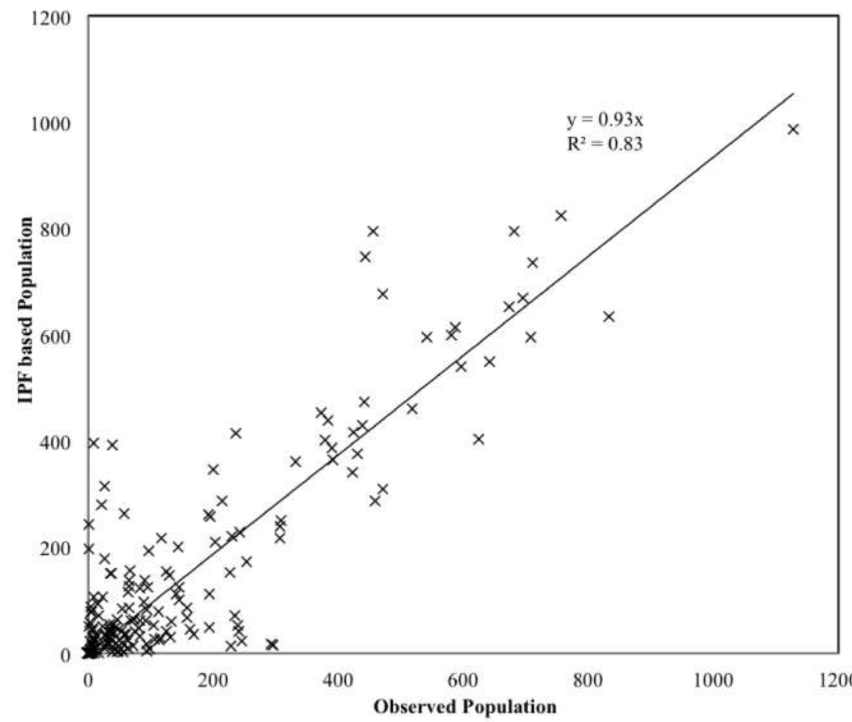
Results: IPF and Census marginals



Results: Fit of IPF with Census joint distribution

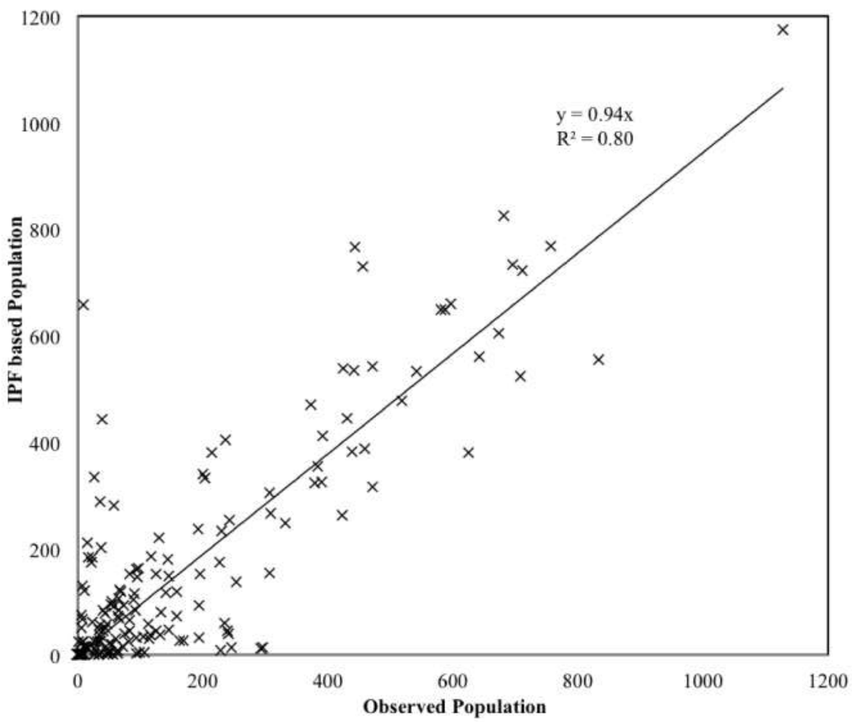


IPF with 20% sample

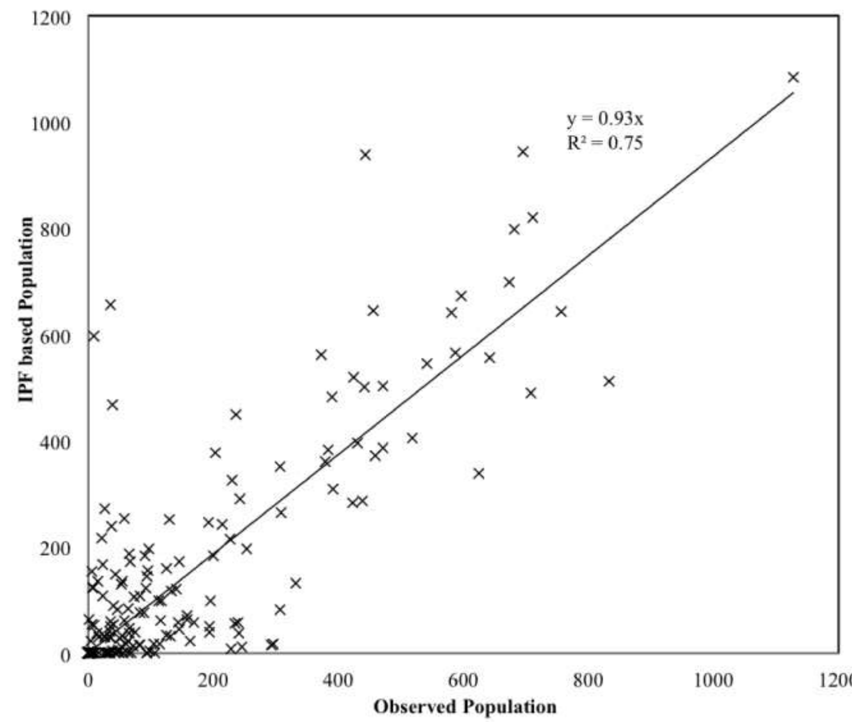


IPF with 10% sample

Results: Fit of IPF with Census joint distribution

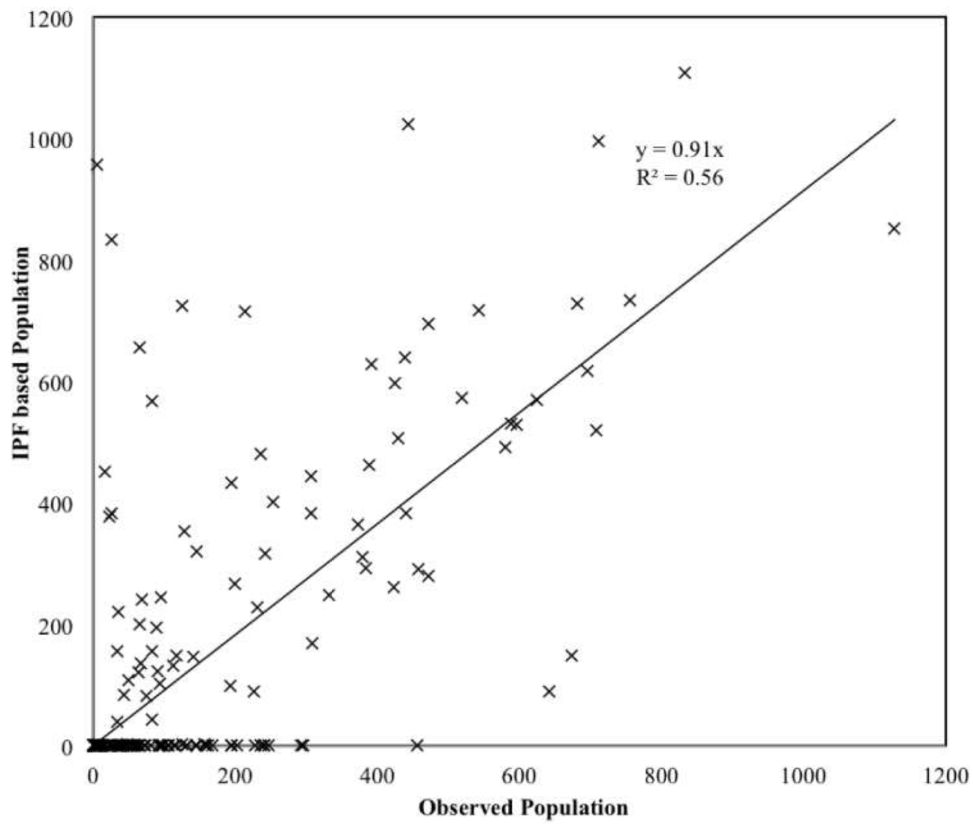


IPF with 5% sample



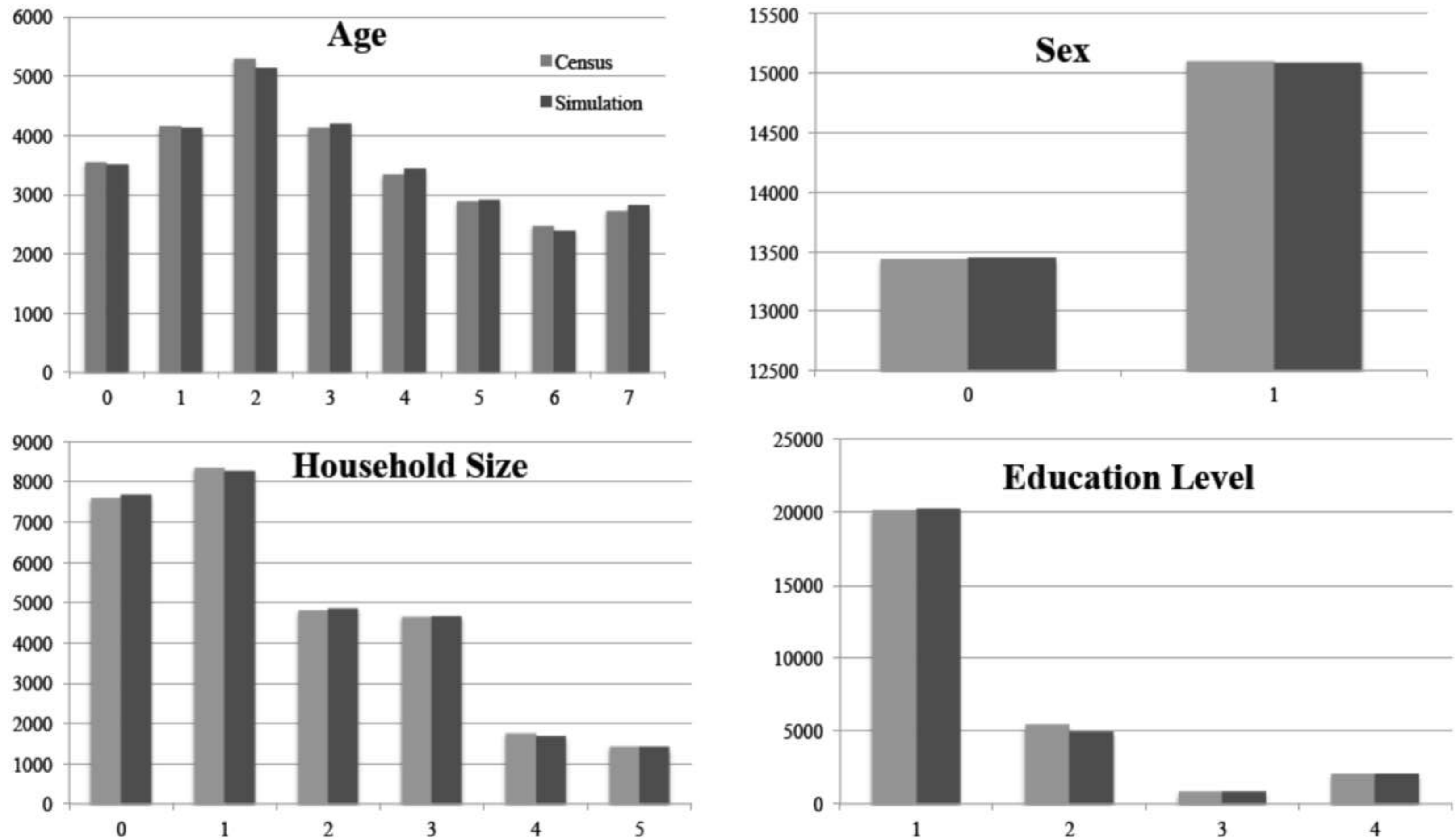
IPF with 3% sample

Results: Fit of IPF with Census joint distribution



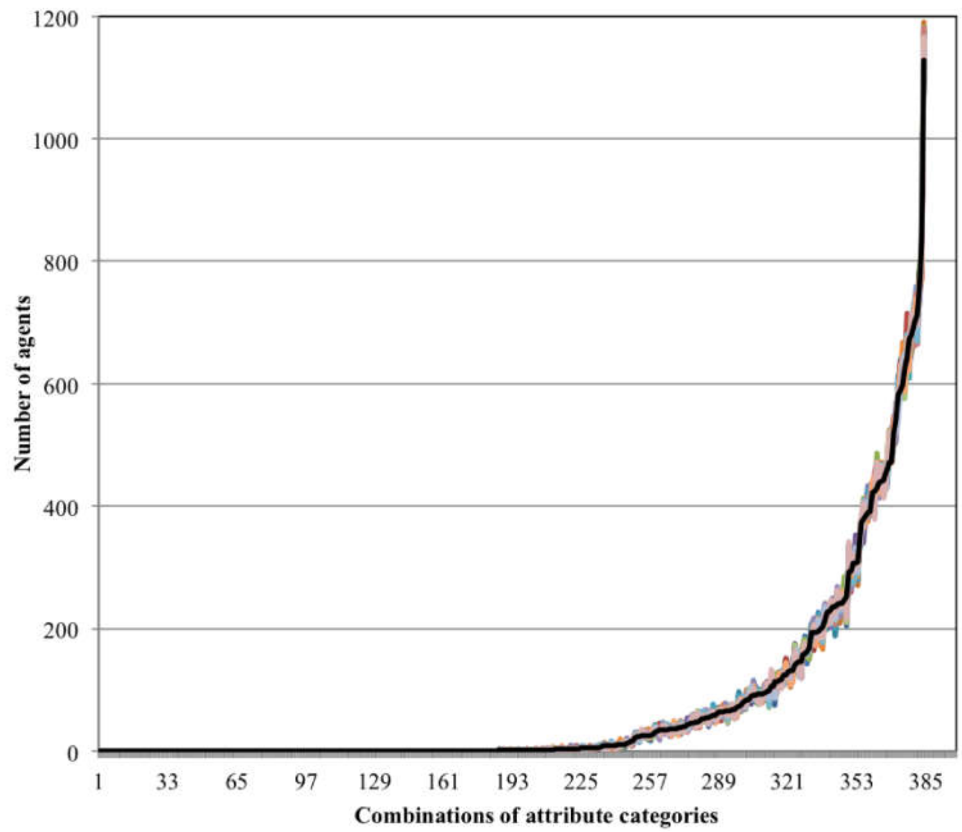
IPF with 1% sample

Results: Simulation and Census marginals



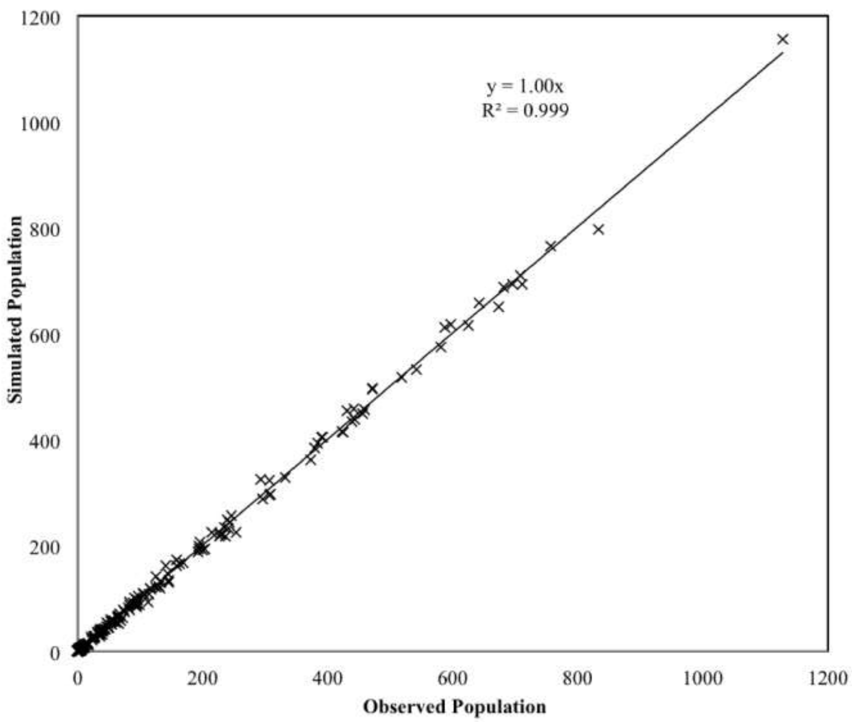
Using full-conditionals (*FullCond*)

Results: Simulation and Census joint dist.

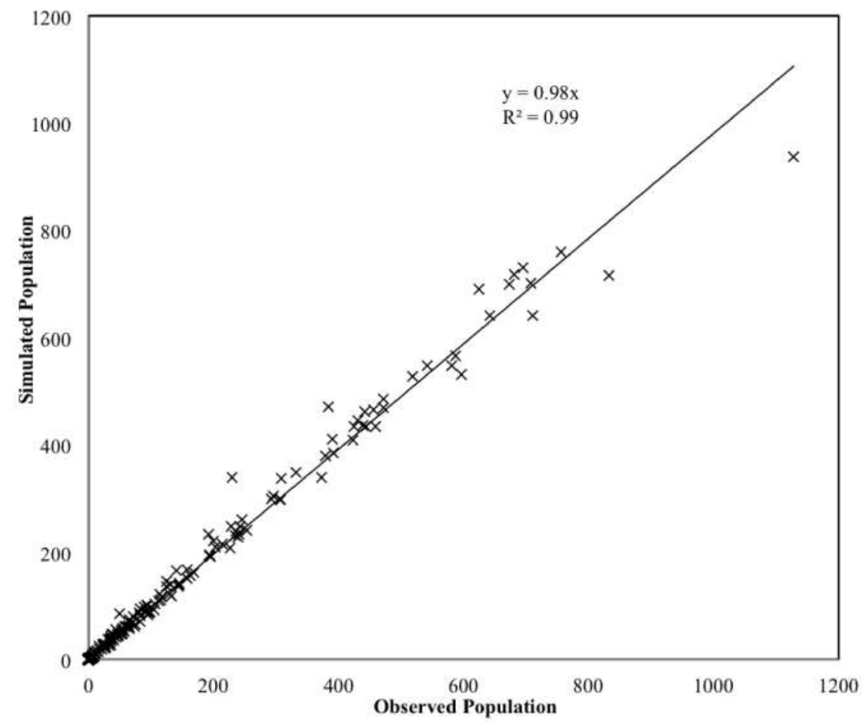


20 runs based on *FullCond* with real population superimposed

Results: Fit of Simulation with Census joint dist.

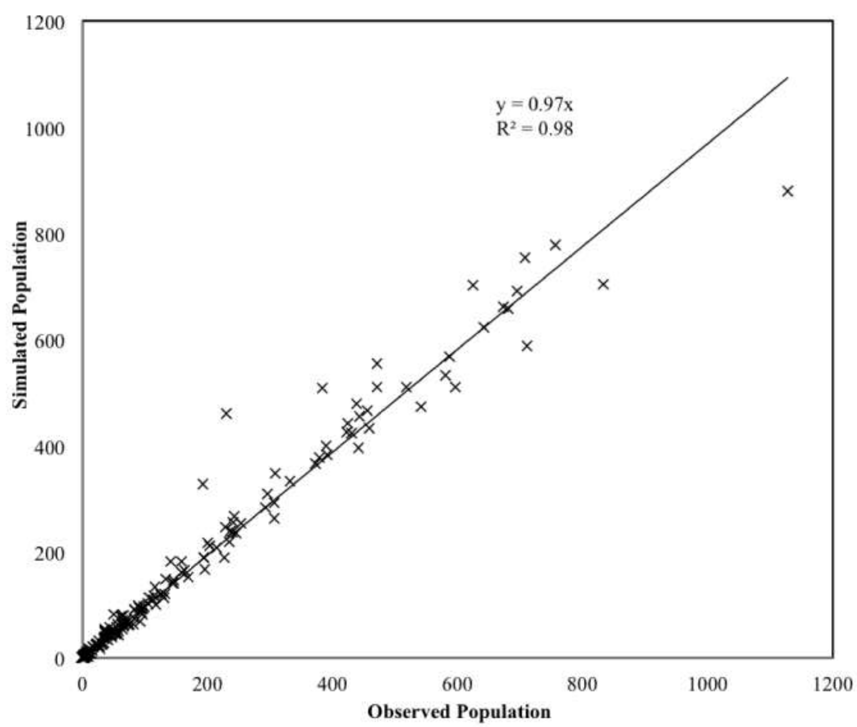


FullCond

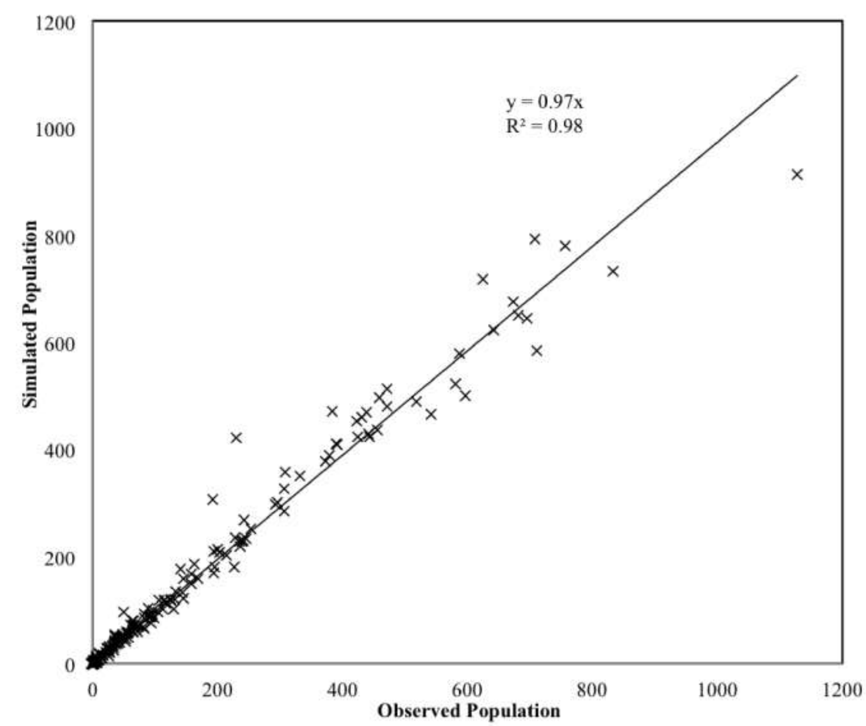


Partial_1 (Sex missing in 1 conditional)

Results: Fit of Simulation with Census joint dist.



Partial_2 (Sex missing in 2 conditionals)



Partial_3 (Sex missing in all conditional)

Comparison: Standard Root Mean Square Error

$$SRSME = \frac{\left[\sum_{i=1}^{m} \cdots \sum_{j=1}^{n} (R_{i\ldots j} - T_{i\ldots j})^2 / N\right]^{1/2}}{\sum_{i=1}^{m} \cdots \sum_{j=1}^{n} (T_{i\ldots j}) / N}$$

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Input	IPF	Simulation
20%*Sample*	0.853	-
10%*Sample*	0.928	-
5%*Sample*	1.020	-
3%*Sample*	1.160	-
1%*Sample*	1.730	-
FullCond	-	0.130
*Partial*_1	-	0.240
*Partial*_2	-	0.340
*Partial*_3	-	0.350

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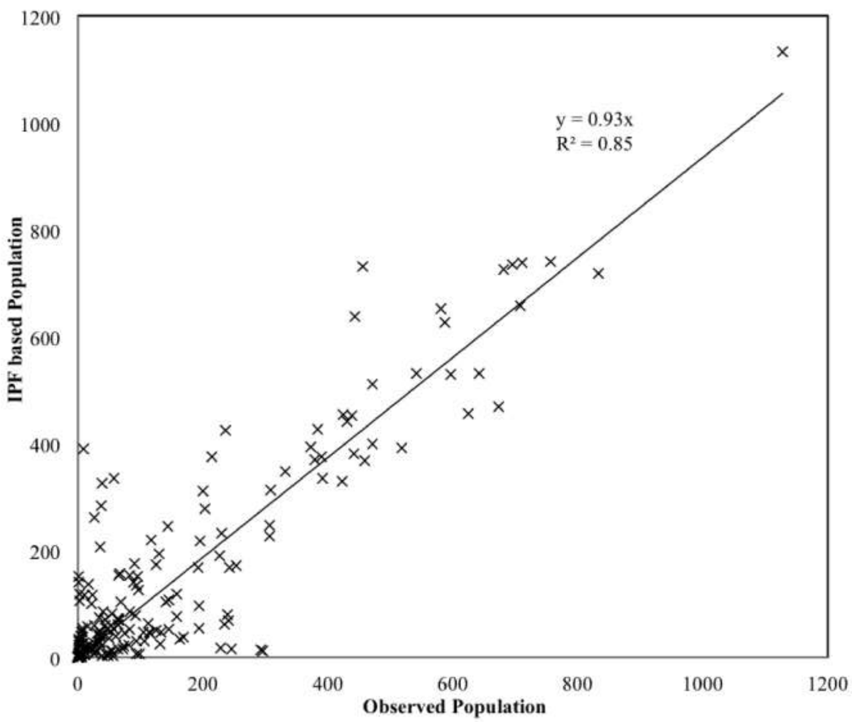
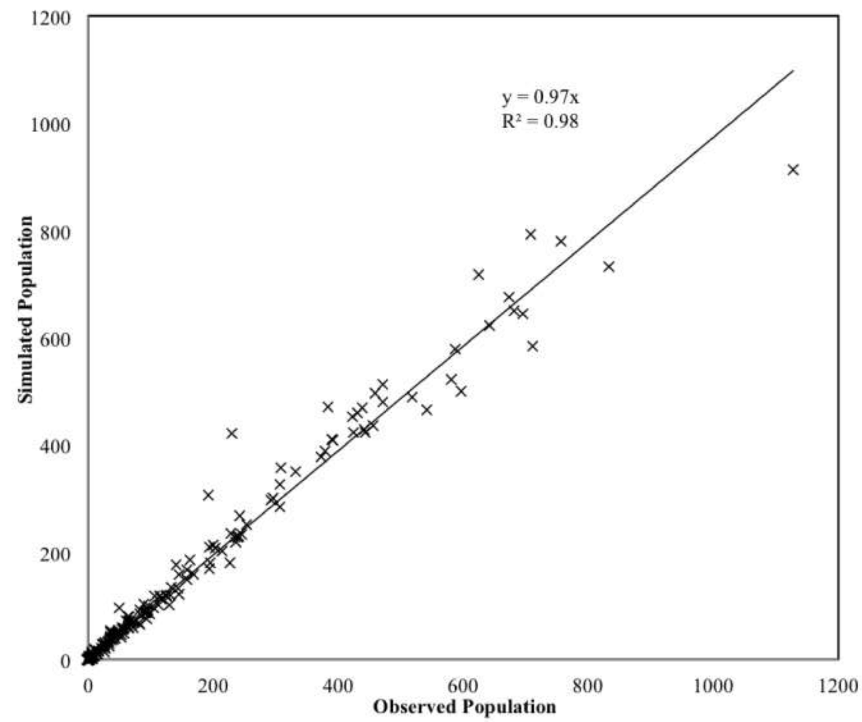
- For Marginals only, both methods give the same fit

Best case IPF and worst case Simulation

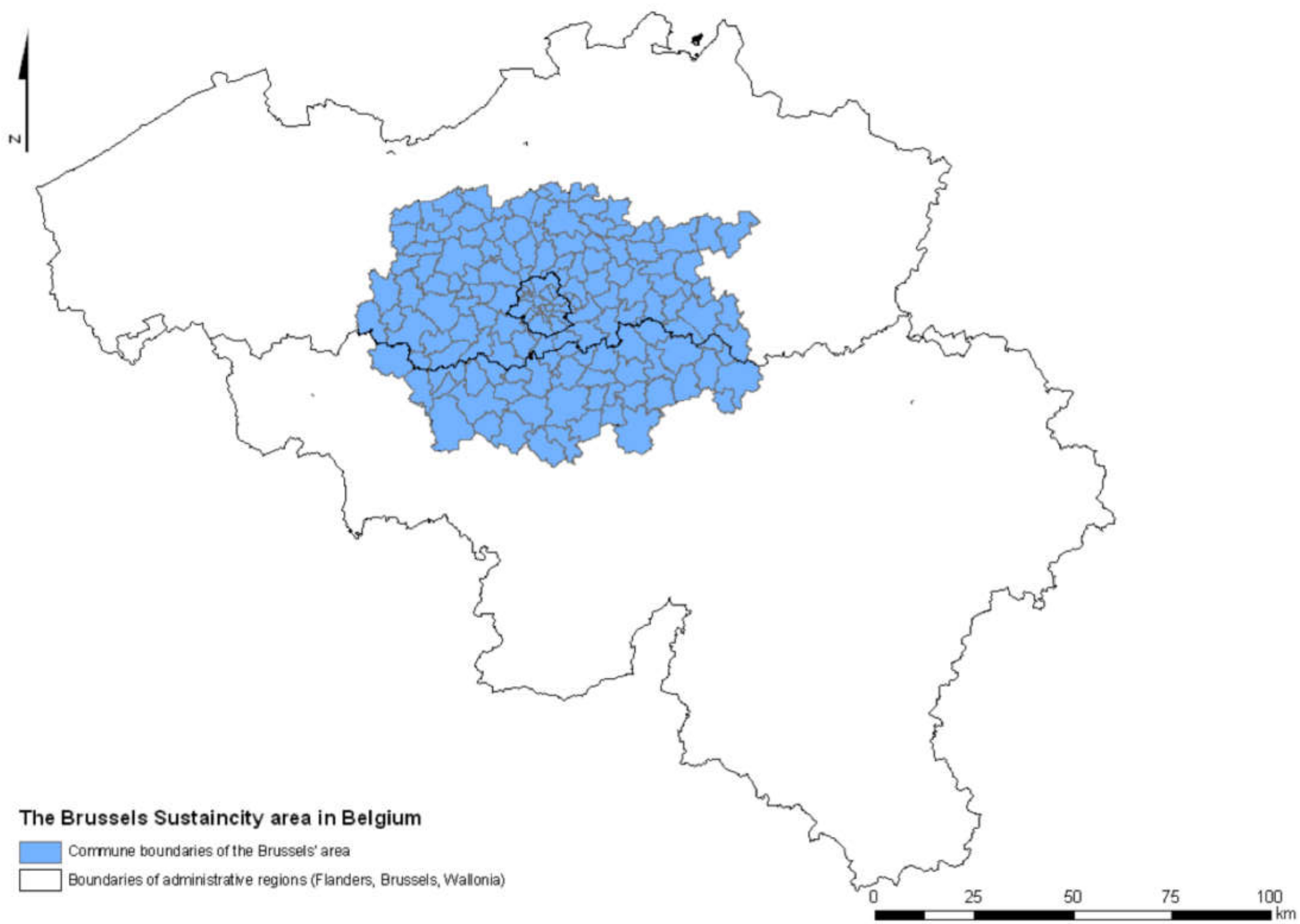


IPF with 20% sample



Partial_4 (Sex missing from all the conditionals)

Back to Brussels case study



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- Data Preparation
  - Aggregation
    - Spatial
    - Categorical
  - Model based conditionals (Logit)
    - Income, univ edu, cars, and dwelling type

Income level model (5 levels)

$$V^1_{(hh,z)} = 0$$

$$V^2_{(hh,z)} = ASC^2 + \beta^2_{zonal\_inc_z} \times zonal\_inc_z + \beta^2_{cars_{hh}} \times cars_{hh} + \beta^2_{workers_{hh}} \times workers_{hh}$$

$$V^3_{(hh,z)} = ASC^3 + \beta^3_{educ_{hh}} \times educ_{hh} + \beta^3_{zonal\_inc_z} \times zonal\_inc_z + \beta^3_{cars_{hh}} \times cars_{hh} + \beta^3_{house_{hh}} \times house_{hh} + \beta^3_{workers_{hh}} \times workers_{hh}$$

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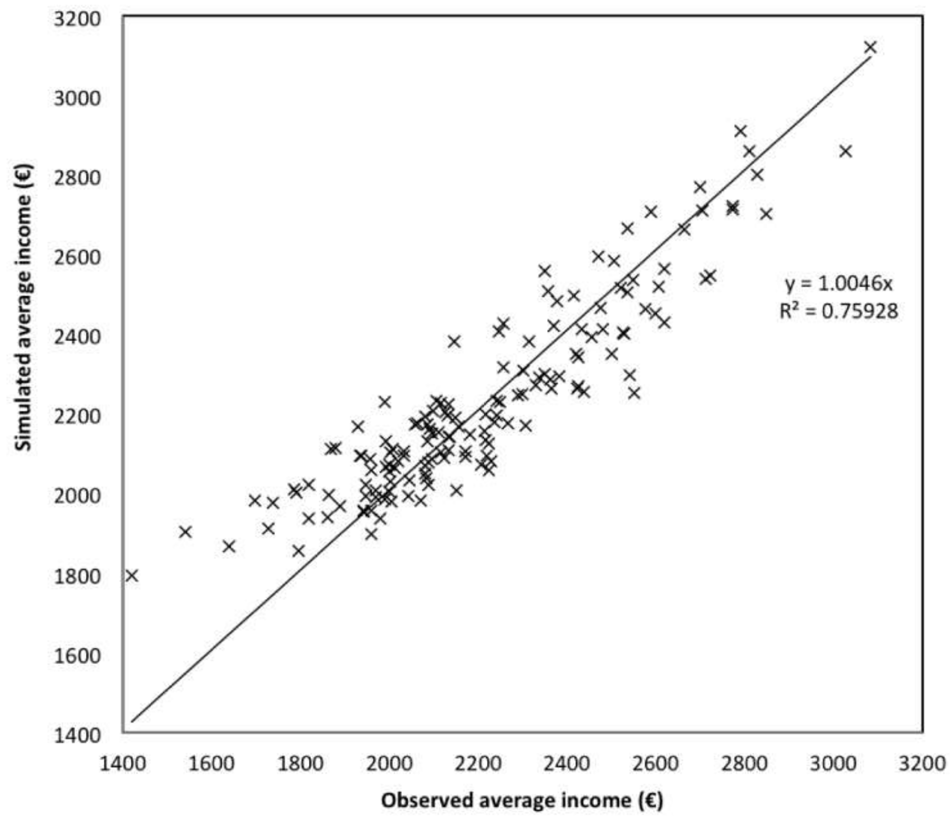
$$V^5_{(hh,z)} = ASC^5 + \beta^5_{educ_{hh}} \times educ_{hh} + \beta^5_{zonal\_inc_z} \times zonal\_inc_z + \beta^5_{cars_{hh}} \times cars_{hh} + \beta^5_{house_{hh}} \times house_{hh} + \beta^5_{workers_{hh}} \times workers_{hh}$$

Back to original problem

Income level model

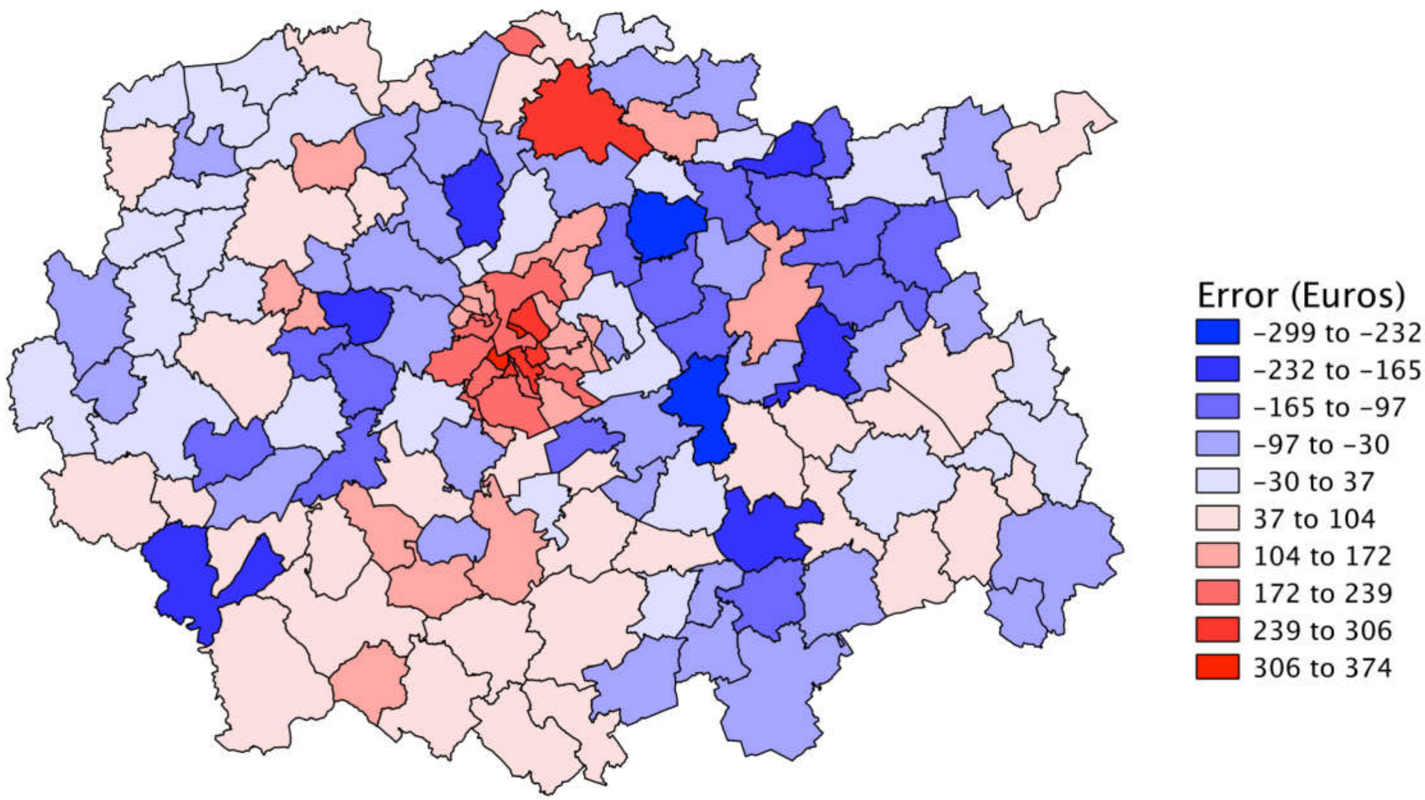
Parameter	Variable	Value	Std err	t-test
ASC^2	constant for income level 2	-0.86	0.789	-1.09
ASC^3	constant for income level 3	-4.64	0.901	-5.14
ASC^4	constant for income level 4	-8.31	1.12	-7.39
ASC^5	constant for income level 5	-10.6	1.55	-6.82
β^3_{educ}	dummy for presence of people with higher educ in the hh	0.831	0.177	4.69
β^4_{educ}	dummy for presence of people with higher educ in the hh	1.72	0.314	5.49
β^5_{educ}	dummy for presence of people with higher educ in the hh	1.92	0.656	2.93
$\beta^2_{\text{zonal\_inc}}$	average zonal income	0.0008	0.0004	1.84
$\beta^3_{\text{zonal\_inc}}$	average zonal income	0.0012	0.0005	2.55
$\beta^4_{\text{zonal\_inc}}$	average zonal income	0.0016	0.0005	3.09
$\beta^5_{\text{zonal\_inc}}$	average zonal income	0.0016	0.0006	2.47
β^2_{cars}	number of cars in the household	1.16	0.265	4.39
β^3_{cars}	number of cars in the household	1.92	0.299	6.41
β^4_{cars}	number of cars in the household	2.33	0.341	6.83
β^5_{cars}	number of cars in the household	3.2	0.466	6.87
β^3_{house}	dummy for dwelling being a house	0.45	0.193	2.34
β^4_{house}	dummy for dwelling being a house	0.485	0.294	1.65
β^5_{house}	dummy for dwelling being a house	0.485	0.294	1.65
β^2_{workers}	number of workers in the household	1.14	0.277	4.11
β^3_{workers}	number of workers in the household	2.22	0.295	7.53
β^4_{workers}	number of workers in the household	2.46	0.345	7.13
β^5_{workers}	number of workers in the household	1.74	0.428	4.07

Results: Brussels case study



Fit between simulation based and observed average commune-level income

Results: Brussels case study



Spatial distribution of error in average income

- More zonal level demographic statistics are required to further decrease the error

Concluding remarks

- From single solution optimization problem to sampling from joint distribution
  - Output of microsimulation models

$$O = \int_{p_{syn}} microsim(p_{syn})\, dp_{syn}.$$

- Focus on reproducing not just marginals, but the whole joint distribution
- Heterogeneous not cloned population
- Population synthesis as part of microsimulation
  - Sensitivity analysis in a coherent way
- Separation of data preparation from agent generation
  - Data, models, assumptions

Concluding remarks

- Mix of sampling process can be utilized based on the situation
- Works both for continuous and discrete or mixture of conditionals
- Computationally efficient and scalable
  - Clean and simple
- Issue of inconsistency
  - Open research question [Buuren, 2007][Chen et al., 2011]
- Use of new and unconventional data
  - WiFi network (Pedestrian movement)
  - Online check-in / social media
- Resource and Agents association
  - from bi-partite to k-partite graph [Anderson et al., 2014]

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