

An activity-based model for transit network design and activity location planning in a three-party game framework

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ARTICLE INFO

Keywords:

Transit network design
Activity location planning
Activity-based approach
Three-party game

ABSTRACT

Over the past few decades, the activity-based approach has received extensive attention in travel behaviour modelling in order to understand the underlying motivation of trip making. In this study, we optimize the transit network and activity location plan with explicit consideration of the correlations between passengers' activity choices and travel choices by using the activity-based approach. Considering the conflicting interests of three stakeholders (i.e., the government, the transit company and the passengers) involved in transit network design and activity location planning, a bi-level programming model is formulated to depict the three-party game, and multi-objective optimization is proposed. A Pareto genetic algorithm is adopted to solve the model, and a numerical example is provided to illustrate the application of the proposed model and solution algorithm. The results show that under different transit network designs and activity location plans, passengers' activity and travel choices, as well as the system-wide indicators including space-time accessibility of activity locations, social welfare and subsidy expenses vary significantly.

1. Introduction

In many densely populated areas around the world, increasing private vehicles cause severe traffic congestion, environmental pollution and road accidents. Public transport has been widely recognized as an effective way to improve the quality of people's daily life. In order to reduce the use of private cars while meeting the travel needs of people, it is necessary to design a practical, efficient and economical transit network (Guihaire and Hao, 2008).

As summarized by Kang et al. (2013), in transit network design problems, most studies usually consider the trip-based demand. While this consideration is sufficient in many applications, there is an increasing recognition that the emerging activity-based approach can better understand the motivation of trip making. Individuals' travel choices should be modelled integrating with their activity choices. That is, the consideration of individuals' daily activity-travel pattern (DATP) is needed, which includes activity sequence, activity type, travel choices, temporal decisions for each activity/trip in a day, etc. As individuals' activity and travel choice behaviours are dependent on transport network and activity locations, the interactions among individuals' activity/travel choices, transit network and activity location plan have gradually become a key research area for urban development particularly for congested

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areas.

Transit network design and activity location planning are interdependent and mutually restricted. On the one hand, the attractiveness of a new activity location to individuals depends largely on the design of the transit network around the activity location (such as transit line deployment); On the other hand, the design of the transit network depends on the demand that the activity location can attract (related to the type of activity, open hours of the activity location, etc.). If we plan activity locations on existing transit network, it will actually affect transit demand, and the transit network designed previously may not yield the best outcome. Considering transit network design and activity location planning separately will lead to misestimation of transit network performance. However, few literatures combine the problem of transit network design with the problem of activity location planning.

To comprehensively consider individuals' activity and travel choices, as well as activity location planning and transit network design, this study adopts the activity-based approach. In recent years, activity-based network equilibrium or design problem with space-time constraints has received much attention (Lam and Yin, 2001; Liao et al., 2013; Rasouli and Timmermans, 2014; Liu et al., 2015, 2016, 2020a; Liao, 2016, 2019, 2021; Li et al., 2010). Using activity-based approach in transit network design enables a better evaluation of the transit network and understanding of activity/travel choice behaviours. Unlike the trip-based approach, the travel demand in the activity-based model comes from activity behaviours. In this way, activity choice behaviours, travel choice behaviours, transit networks and activity location plans are connected. In other words, changes in transit network and activity location plans bring changes to individuals' DATPs. The activity-based approach allows an integrated investigation into individuals' daily scheduling and routing choice with space-time constraints. Individuals determine the activity sequence, activity location, activity duration, departure time and travel path to achieve the optimal DATP utility. With the activity-based approach, the utility/dis-utility of activity/travel can be depicted with consideration of crowding effects in vehicles or at activity locations (Fu et al., 2022a). Over the past decade, several activity-based network equilibrium or network design models were proposed (Kang et al., 2013; Chow and Djavadian, 2015; Fu and Lam, 2014, 2018; Li et al., 2018; Li and Liao, 2020; Vo et al., 2020, 2021; Wang et al., 2020; Nguyen et al., 2022). However, few studies jointly considered the transit network design and activity location planning (e.g., consider activity locations as decision variables) with the activity-based approach, referring to Table 1.

Generally, the transit network design and activity location planning involves different stakeholders such as the government (or transport authority), the transit company and the passengers. The different stakeholders have various or even conflicting interests and values. For instance, the government may focus on social benefits when design transit networks and making activity location plans, while the transit company focuses on the economy of operation (e.g., fleet size, operation cost, etc.) and passengers pursue fast trips and complete activities. Although some previous studies have taken different stakeholders into account (Gulhan et al., 2018; Nayeem et al., 2019), little attention has been paid to explicitly investigating the relationship of stakeholders in the decision-making process of transit network design and activity location planning.

Involving three stakeholders mentioned above, the determination of transit network and activity location plan is a three-party game, as yet largely unexplored in the literature (refer to Table 1). The government subcontracts the operation work to the transit company who provide and maintain transit services to passengers. The government also offers a certain amount of subsidy for the policy loss according to the operation effort made by the transit company. Hence, the relationship between government and transit company is usually described as a principal-agent game (Laffont and Martimort, 2002; Huang et al., 2016), in which the government makes decision for the planning level (such as transit network design and activity location planning), while the transit company makes decision for the operation level (such as service frequency, operation effort, etc.). This principal-agent game is rarely considered in a

Table 1
Comparison of studies on transit network design and activity location planning.

Publication	Demand modelling approach	Decision variables	Three-party game	Optimization objectives
Bielli et al. (2002)	Trip-based	Transit lines and service frequency	Not considered	Minimize fleet size, etc. (over 20 objectives)
Li et al. (2011)	Trip-based	Transit lines	Considered	Maximize the total social welfare
Huang et al. (2016)	Trip-based	Transit fares and service frequency	Considered	Maximize the total social welfare
Gulhan et al. (2018)	Trip-based	Transit lines	Not considered	Maximize accessibility; maximize network and operation efficiency; minimize fleet size
Liu et al. (2018)	Trip-based	Transit lines and service frequency	Not considered	Minimize user costs; minimize operation costs
Nayeem et al. (2019)	Trip-based	Transit lines	Not considered	Minimize in-vehicle travel time, etc. (7 objectives)
Bourbonnais et al. (2021)	Trip-based	Transit lines and service frequency	Not considered	Minimize system cost
Li et al. (2010)	Activity-based	Timetable	Not considered	Maximize total user net utility
Fu et al. (2022a)	Activity-based	Transit fares and service frequency	Not considered	Maximize space-time accessibility of activity locations
Kang et al. (2013)	Activity-based	Network link choice	Not considered	Minimize the sum of the construction cost and the operation cost
This study	Activity-based	Transit lines and activity locations	Considered	Maximize space-time accessibility of activity locations; Maximize social welfare; minimize subsidy expenditure

transit network design problem. Most studies formulate the problem into one leader-follower game using bi-level programming models (Bielli et al., 2002; Tom and Mohan, 2003; Liu et al., 2018; An et al., 2020; Di and Yang, 2020; Bourbonnais et al., 2021). In the bi-level model, the upper-level depicts the system-wide objectives of the leaders (i.e., the government or the transit company), while the lower-level reflects the passengers' reaction by passenger flow.

The government needs to take comprehensive consideration of a variety of factors when making decisions of transit network design and activity location planning, thus multiple optimization objectives arise. In the decision-making process, the government aims at proper allocation of resources and the maximization of social benefit, which is composed of two perspectives, namely, society and network. From the society perspective, many studies considered maximizing total social welfare (Li et al., 2011; Huang et al., 2016), or minimizing the system cost (Tom and Mohan, 2003; Bourbonnais et al., 2021), or minimizing the user and operation costs (Liu et al., 2018; Ma et al., 2021). From the network perspective, various accessibility indicators are widely used in transportation planning and facilities management (Chen et al., 2013, 2017, 2019; Wang et al., 2018; Volotskiy et al., 2018; Fu et al., 2022a; Gu et al., 2022). A Higher accessibility indicates a more convenient network, which means it is easier for passengers to conduct activities with the network. Gulhan et al. (2018) considered the utility-based accessibility and the potential accessibility in transit network design. Fu et al. (2022a) maximized space-time accessibility of activity locations in transit network design.

As summarized in Table 1, it can be found that most existing related studies simply design the transit network regardless of activity locations (only with transit network related decision variables such as transit lines, fares or service frequency). Most studies use a trip-based approach in demand modelling, which means that the correlation between activity choices and travel choices are ignored. In addition, only a few studies consider the three-party game and only consider a single objective.

To jointly consider the transit network design and the activity location planning, this paper uses an activity-based approach and model the three-party game. The major contributions of this study are summarized as follows. (1) The relationships between activity choices and travel choices, and between the transit network design and the activity location planning are explicitly investigated by using an activity-based approach, which are rarely studied in the existing studies; (2) As the transit network design and activity location planning involve the government, the transit company and the passengers, a bi-level framework is proposed to consider the three-party game; (3) To evaluate the solutions, a multi-objective analysis is carried out from the perspectives of the network, the society and the government to support the decision-making process of the government.

The structure of this paper is organized as follows. In Section 2, some basic concepts and assumptions are briefly introduced. Section 3 presents the activity-based three-party game framework. Section 4 describes the solution algorithm for solving the proposed model. A numerical example is provided in Section 5 for illustrating the application of the proposed model and solution algorithm. Conclusions are drawn in Section 6, together with suggestions for further research.

2. Problem statement

It is well recognized that the change of activity location plans has a significant impact on passengers' DATP choices. The conventional trip-based approach in transit network modelling ignores the underlying motivation of passengers' trip making and cannot reflect the linkages between activities and travels. The trip-based approach cannot capture the changes in passengers' DATP choices according to the changes occurring in the network (Fu et al., 2022b; Kang et al., 2013). The following simplified case demonstrates the impact of a change in activity location on the passenger's activity-travel choice, which would be unaccountable with trip-based approach.

Assume that each passenger has one or two activities to perform in a day, e.g., a compulsory Work activity and an optional Shop activity. In the following case, specifications of activity start/end time, duration and utility are shown in Table 2. Assume also that the objective of passengers is to maximize DATP utility.

Consider a grid network with four nodes. The link travel time and dis-utility are shown in Fig. 1(a). Suppose in this case that the link disutility is not influenced by the link flow. Fig. 1(b) shows the optimal DATP with the largest utility if no change is made on the current network. Passengers choose to spend the most time at home and the least time at work under the given link dis-utility and activity utility, and they do not pass Node 3 due to the large dis-utility of link 0–3.

If the activity location is changed and Node 3 is planned as a new shop location, passengers would adjust the DATP as shown in Fig. 1(c). In the adjusted DATP, passengers spend the most time at shop and at home, and spend the least time at work under the given link dis-utility and activity utility. The adjusted DATP indicates the new demand for the Shop activity at Node 3 and the derived new travel demand on link 0–3 and link 2–3. If we do not adopt the activity-based approach, we cannot observe the interaction among

Table 2
Settings of activities in the small case.

Activity	Start time	End time	Duration (min)	Utility per 5 min (CNY¥)
Home (morning)	7:00	[7:00, 8:30]	—	12
Home (afternoon)	[17:00, 19:00]	19:00	—	12
Work	[7:00, 9:00]	[17:00, 18:00]	—	10
Shop	[17:00, 19:00]	[17:00, 19:00]	30, 60	18

Note: [a, b] refers to the time window.

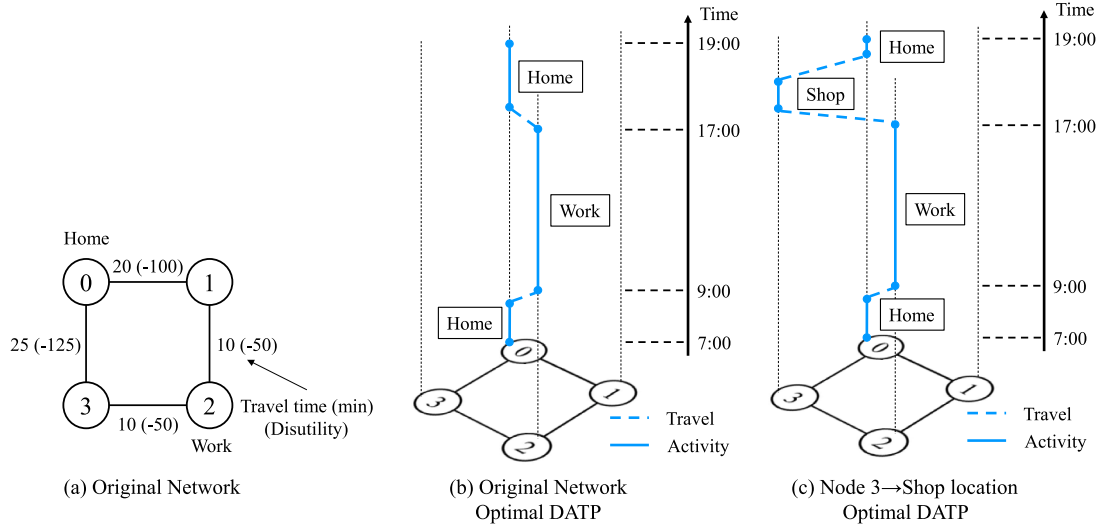


Fig. 1. Illustration of optimal DATP.

activity locations, transport network and passengers' choice behaviour.

From the above case, it can be found that the activity-based approach enables an integrated investigation into the DATP scheduling mechanism with time and space constraints, i.e., what activities are conducted, in what sequence, when and for how long, when each trip starts, which route is used, and how the activities and travels interrelate in the transit networks. Therefore, with the activity-based approach, we can jointly consider different stakeholders, i.e., the government, the transit company and the passengers, involved in transit network design and activity location planning. Individuals' activity choices (e.g., activity start time and duration, activity sequence and activity location) and travel choices (e.g., departure time, route and travel time) can be simultaneously investigated. Besides, considering various characteristics of public transportation, we adopt a multi-objective analysis in the transit network design and activity location planning problem, considering the space-time accessibility of activity locations, social welfare and subsidy expenses of the government.

The notations used in this paper are listed in Table 3.

2.1. Assumptions

In order to facilitate essential ideas without loss of generality, the following assumptions are adopted.

A1: The transit network design and activity location planning problem is a three-party game involving three stakeholders: the government (transport authority), the transit company, and passengers, each individually pursuing their various objectives (Schmöcker et al., 2009).

A2: The model can be formulated as a leader-follower game (Lam and Zhou, 2000; Farahani et al., 2013). The upper-level problem depicts the system-wide objectives of the leaders (who plan or operate the transit network), while the lower-level problem reflects the passenger flow pattern.

A3: In the upper level, the game between the government and the transit company is described as a principal-agent game, where the government is the principal and the transit company is the agent (Laffont and Martimort, 2002).

A4: The model is proposed for long-term planning at the strategic level. Therefore, it is assumed that passengers have perfect knowledge of traffic conditions throughout the whole network. All passengers schedule their DATPs to achieve the maximum DATP utility.

A5: The travel time of each in-vehicle link in the studied transit network is not influenced by the road congestion. The transit fare is flat and the frequency is given.

A6: In the study, home and work are considered as compulsory activities, while others are optional. Activity choices (including activity location, activity start time and duration) and travel choices (including departure time and route) are not fixed.

A7: The marginal utility of activities depends on the start time. The utility of each kind of activity is determined following a form of bell-shaped marginal utility function proposed by Joh et al. (2002), and has been widely adopted in related studies.

2.2. Transit network representation

Consider a connected transit network $G = (N, V)$, where N is the set of nodes and V is the set of links. N_a is the set of activity locations in the network, $N_a \subseteq N$. In the example network illustrated in Fig. 2, each node is featured by its land-use properties such as home (labelled H), shop (labelled O1) and work (labelled W), which are denoted by $n_H \in N_H$, $n_W \in N_W$ and $n_{O1} \in N_{O1}$ respectively. The candidate shop location (labelled C) in the network is denoted by $n_C \in N_C$. Let L be the set of all feasible transit lines. Every transit line

Table 3
Notations.

	Network presentation and demand
G	Transit network; $G = (N, V)$.
N	Set of nodes; $N = N_H \cup N_W \cup N_O \cup N_C$.
V	Set of physical links; $V = \{v\}$.
N_H	Set of home locations; $N_H = \{n_H\}$.
N_W	Set of work locations; $N_W = \{n_W\}$.
N_O	Set of optional activity locations; $N_O = \cup_{i \in \mathbb{N}} N_{O_i}$.
N_{O_i}	Set of i^{th} type of optional activity locations (e.g., Education, $i = 0$, Shop, $i = 1$, etc.); $N_{O_i} = \{n_{O_i}\}$.
N_a	Set of activity locations; $N_a = N_H \cup N_W \cup N_O$; $N_a \subseteq N$.
N_C	Set of candidate locations; $N_C = \{n_C\}$.
L	Set of feasible transit lines; $L = \{l\}$.
T	Equally spaced time intervals; $t \in T$.
t	Time of day; $t = 1, 2, \dots, T, T + 1$.
M	Set of nodes in super-network; $M = \{(n, l, t)\}$
A_a	Set of activity links; $A_a = \{a_a\}$.
A_d	Set of direct in-vehicle links; $A_d = \{a_d\}$.
$n(a_a)$	Physical location n of activity link a_a .
$t(a_a)$	Time period t of activity link a_a .
u_{a_a}	Utility of activity link a_a .
$\bar{u}_{a_a}(t)$	Marginal utility function of activity link a_a .
f_{a_a}	Passenger flow on activity link a_a .
$c_{n(a_a)}$	Capacity of activity location $n(a_a)$.
$\alpha_{a_a}, \beta_{a_a}$	Parameters by activity type in activity link utility function.
$\alpha_{a_a}, \beta_{a_a}, \gamma_{a_a}, u_{a_a}^{\max}, u_{a_a}^0$	Parameters by activity type in activity marginal utility function.
$t(a_d)$	Time period t of activity link a_d .
$\xi(a_d, v)$	Dummy variable indicating if physical link v is in a_d .
$a_{x, n}$	A direct in-vehicle link from node $x \in M$ to node $n \in M$.
$distu_{a_d}$	Dis-utility of direct in-vehicle link a_d .
vo_t	Value of time.
ψ	Duration of each time period.
f_{a_d}	Passenger flow on direct in-vehicle link a_d .
c_b	Capacity of bus.
α_v, β_v	Parameters in in-vehicle link dis-utility function.
K_0	The total number of travellers.
K_{n_H, n_W}	The number of travellers who live in n_H and work in n_W
P	Set of DATPs; $P = \{p\}$.
u_p	Utility of DATP p .
$\zeta(p, a_d), \zeta(p, a_a)$	Dummy variables in DATP utility function.
e	Optimization problem and result The operation effort level.
θ	Line frequency.
TR	Total revenue.
f_p	Passenger flow on the DATP p .
C_h	Unit cost of travel time.
$\delta(l)$	Dummy variable indicates whether transit line l is under operation.
RT_l	Roundtrip travel time of l .
S	Subsidy.
OC	Operation cost.
h_{a_d}	Transit fare function regarding the direct in-vehicle link a_d .
$\varphi(e)$	Profit of transit company under operation effort level e .
$\delta_C = (\dots, \delta(n_C^i), \dots)^T$	Decision vector for activity location.
$\delta_l = (\dots, \delta(l_i), \dots)^T$	Decision vector for transit network design.
Ω	Set of feasible transit network.
τ_{cr}	Regulation operation cost.
r	Regulation profit rate for transit companies.
$Acc(n)$	Space-time accessibility of node n .
STA	Network-wide space-time accessibility.
SW	Social welfare.
CS	Consumer surplus.
$D(e)$	Overall transit demand under operation effort level e .

$l_i \in L$ runs in both directions.

2.3. Activity-time-space super-network representation

To simultaneously consider activity choices and travel choices with an activity-based approach, the physical transit network is extended to an activity-time-space super-network following Fu and Lam (2014, 2018), with which the dynamic activity-travel

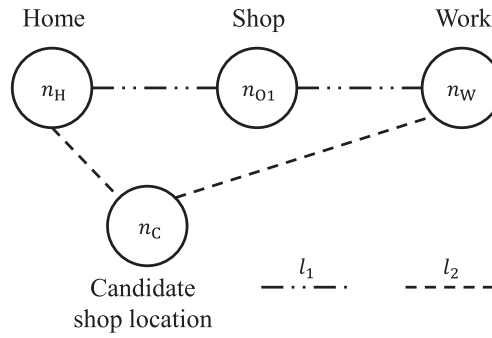


Fig. 2. Transit network presentation.

scheduling problem is converted into an approximately static problem by extending the transport network in the time dimension. Fig. 3 shows an example of the activity-time-space super-network.

The study horizon is divided into T equally spaced time intervals. Let $t = 1, 2, \dots, T, T+1$ be the start time of each link. Links in the super-network can be divided into direct in-vehicle links, activity links and transfer links. Nodes in the super-network can be regarded as the start of links. Each node in the super-network is described as $(n, l, t) \in M$, where n is the physical location of the node and l is the alight or aboard indicator. The value of l is equal to 1(0) indicating that the passenger is at the beginning (end) of an in-vehicle link. A path in the super-network represents an activity-travel pattern, including all activity behaviours (i.e., activity sequence, activity location, activity start time and duration) and all travel behaviours (i.e., departure time, path, and travel time) during the study period. Some possible activity-travel patterns can be found in Fig. 3. More details about the super-network can be found in Fu and Lam (2014, 2018).

Activity links. For each activity link $a_a \in A_a$, it is characterized by (n, t) where n denotes the physical location and t denotes the start time. $n(a_a)$ denotes the relationship of the physical location n and the activity link a_a ; $t(a_a)$ denotes the relationship of the start time t and the activity link a_a .

Activity links are associated with positive utility. Previous studies have indicated that the crowding discomfort has a significant impact on individuals' choice of transit service for long-term planning. Thus, the crowding effects occurring at activity locations and within transit vehicles should be explicitly considered in the proposed activity-based model (Fu et al., 2022a). With this consideration,

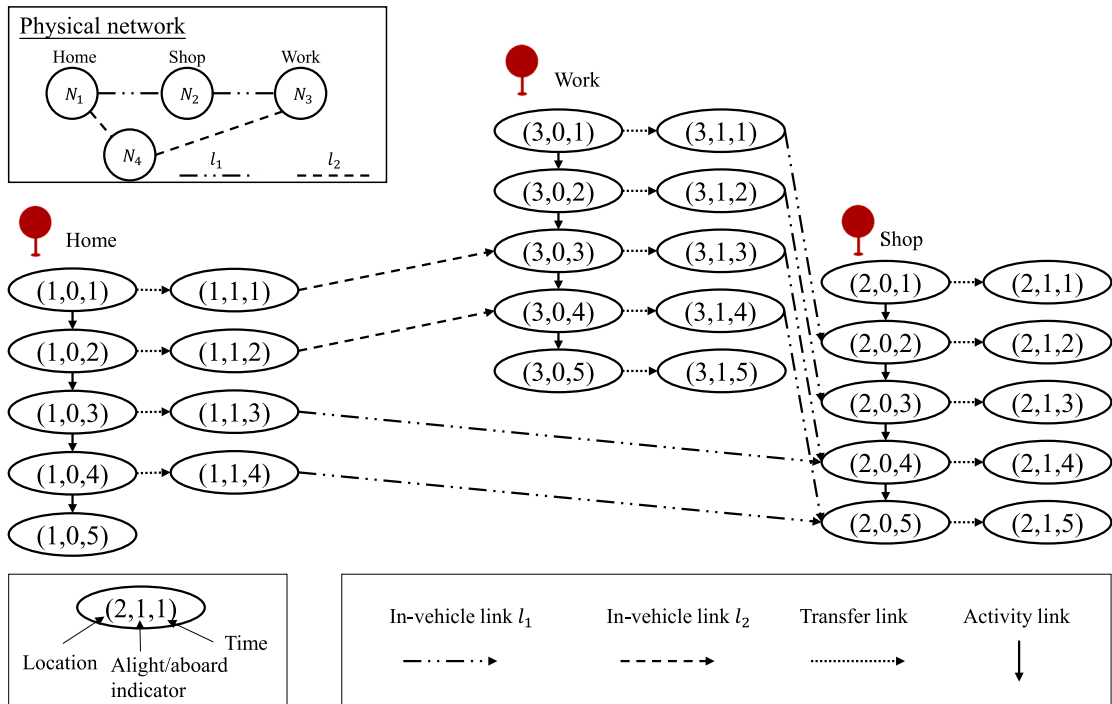


Fig. 3. An illustrative example of the activity-time-space super-network.

passengers may choose different activity locations, durations and departure times, etc. when the network is crowded.

The concept of activity utility is widely used in the activity-based models. Considering the crowding discomfort at activity locations, the utility of activity link a_a (from start time t for one interval) can be expressed as a Bureau of Public Roads (BPR) type of function.

$$u_{a_a} = \left(1 + \alpha_{a_a} \left(\frac{f_{a_a}}{c_{n(a_a)}} \right)^{\beta_{a_a}} \right) \int_t^{t+1} \bar{u}_{a_a}(\omega) d\omega, \forall a_a \in A_a \quad (1)$$

In Eq. (1), f_{a_a} denotes the flow of the activity link; $c_{n(a_a)}$ is the capacity of the activity location; α_{a_a} and β_{a_a} are model parameters by activity type. The utility of compulsory activities (i.e., Home and Work) is not affected by the crowding at activity locations, that is, $\alpha_{a_a} = 0$ for compulsory activities. $\int_t^{t+1} \bar{u}_{a_a}(\omega) d\omega$ expresses the utility of performing activity link a_a from start time t to end time $t + 1$; $\bar{u}_{a_a}(t)$ is the marginal utility function of activity link a_a at the time interval t , which can be defined as Eq. (2) (Joh et al., 2002):

$$\bar{u}_{a_a} = u_{a_a}^0 + \frac{\gamma_{a_a} \beta'_{a_a} u_{a_a}^{\max}}{\exp \left[\beta'_{a_a} (t(a_a) - \alpha'_{a_a}) \right] \left\{ 1 + \exp \left[-\beta'_{a_a} (t(a_a) - \alpha'_{a_a}) \right] \right\}^{\gamma_{a_a} + 1}} \quad (2)$$

where $u_{a_a}^{\max}$ represents the maximum accumulated utility of the activity; α'_{a_a} , β'_{a_a} , γ_{a_a} and $u_{a_a}^0$ are parameters. This is a bell-shaped marginal utility function. Fig. 4 shows the marginal utility of the compulsory activities (Home and Work) and an example of optional activity (Shop).

In-vehicle links. $A_d = \{a_d\}$ is the set of in-vehicle links. Each in-vehicle link is made up of physical links. In-vehicle link a_d can also be denoted as $a_{x,n}$ to represent a direct in-vehicle movement from node $x \in M$ to node $n \in M$. Let $t(a_d)$ denote the start time of the in-vehicle link a_d ; $\xi(a_d, v)$ equals to 1 if physical link v is in a_d , 0 otherwise.

In-vehicle links are associated with dis-utility of travel. Similar to the activity links, the dis-utility of in-vehicle links is influenced by in-vehicle crowding effects. The dis-utility of physical link v with start time interval t (denoted as $disu_v(t)$) is expressed using a BPR type of function to quantify the increasing discomfort when the number of passengers increases (Lo et al., 2003; Nielsen, 2000):

$$disu_v(t) = -vot \cdot \psi \left(1 + \alpha_v \left(\frac{f_{a_d}}{c_b} \right)^{\beta_v} \right), \forall v \in V \quad (3)$$

where vot denotes the value of time; ψ is the duration of each time period; f_{a_d} is the passenger flow on a_d ; c_b denotes the bus capacity; α_v and β_v are parameters.

The in-vehicle link dis-utility can be obtained by the summation of related physical link dis-utilities and the transit fare:

$$disu_{a_d} = \sum_{v \in V} disu_v(t) \cdot \xi(a_d, v) + h_{a_d} \quad (4)$$

where h_{a_d} denotes the transit fare with respect to the direct in-vehicle link a_d ; $\xi(a_d, v)$ is equal to 1 if physical link v is in direct in-vehicle link a_d , 0 otherwise. To facilitate the essential ideas of the proposed model, we focus only on the activity utility and travel dis-utility. The dis-utility of transfer is neglected. For considering transfer behaviour in the activity-time-space super-network, readers can refer to Fu and Lam (2014).

Number of Travellers. In this study, Home and Work are considered as compulsory activities, and the locations of the activities are

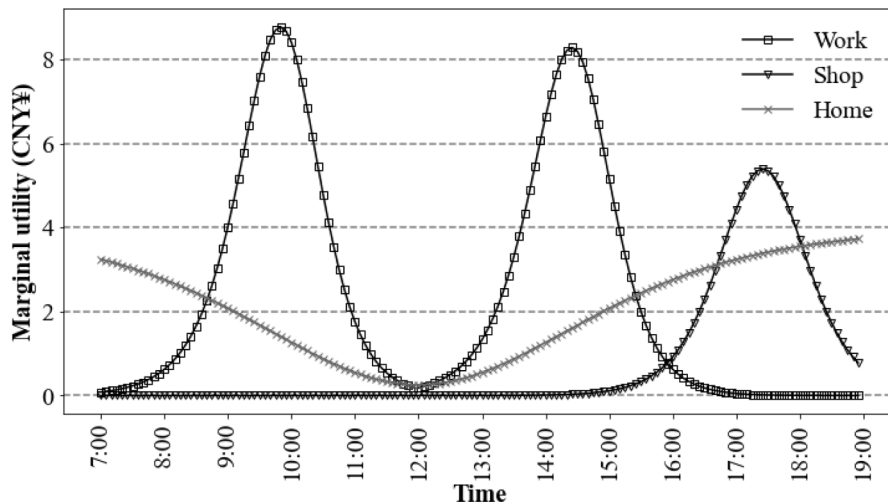


Fig. 4. Examples of marginal utility functions for different activities.

fixed (we can also extend the model to other situations with different kinds of compulsory activities); in contrast, other activities are optional, and passengers can choose any location at any time after work to perform the optional activity. The total population is given as

$$K_0 = \sum_{n_H \in N_H} \sum_{n_W \in N_W} K_{n_H, n_W} \quad (5)$$

where K_{n_H, n_W} denotes the number of travellers live in n_H and work in n_W .

3. Model formulation

In this section, we develop a bi-level programming model to solve the transit network design and activity location planning problem with an activity-based approach. The concept of DATP, the formations of the lower level and the upper level, objective functions and constraints are elaborated.

3.1. Daily activity-travel pattern

The DATP, denoted by $p \in P$, is an ordered sequence of activity links and in-vehicle links. Therefore, the utility of DATP u_p can be obtained by the sum of total utility of activity links and dis-utility of in-vehicle links:

$$u_p = \sum_{a_a \in A_a} u_{a_a} \zeta(p, a_a) + \sum_{a_d \in A_d} \text{dis} u_{a_d} \zeta(p, a_d), \forall p \in P \quad (6)$$

where $\zeta(p, a_a)$ is a dummy variable which equals to 1 if the DATP p contains the activity link a_a , 0 otherwise; $\zeta(p, a_d)$ is a dummy variable which equals to 1 if the DATP p contains the in-vehicle link a_d , 0 otherwise.

Assume that it takes 1 time interval to travel between nodes in the physical transit network. In 10 intervals, four DATP examples of the network are illustrated in Fig. 5. The horizontal axis represents the activity location dimension; the vertical axis represents the time period dimension; each path in the network represents a DATP. Among the four examples of DATP, there are only two types of activity sequences (“Home-Work-Home”, “Home-Work-Shop-Home”). Even with the same activity sequence, the activity-travel choices can be different. Associating an activity sequence with different departure times, activity durations and activity locations can result in various DATPs. In an activity-based approach for network equilibrium or network design, the key is the modelling of individuals’ DATP choice (i.e., path choice in the activity-time-space super-network).

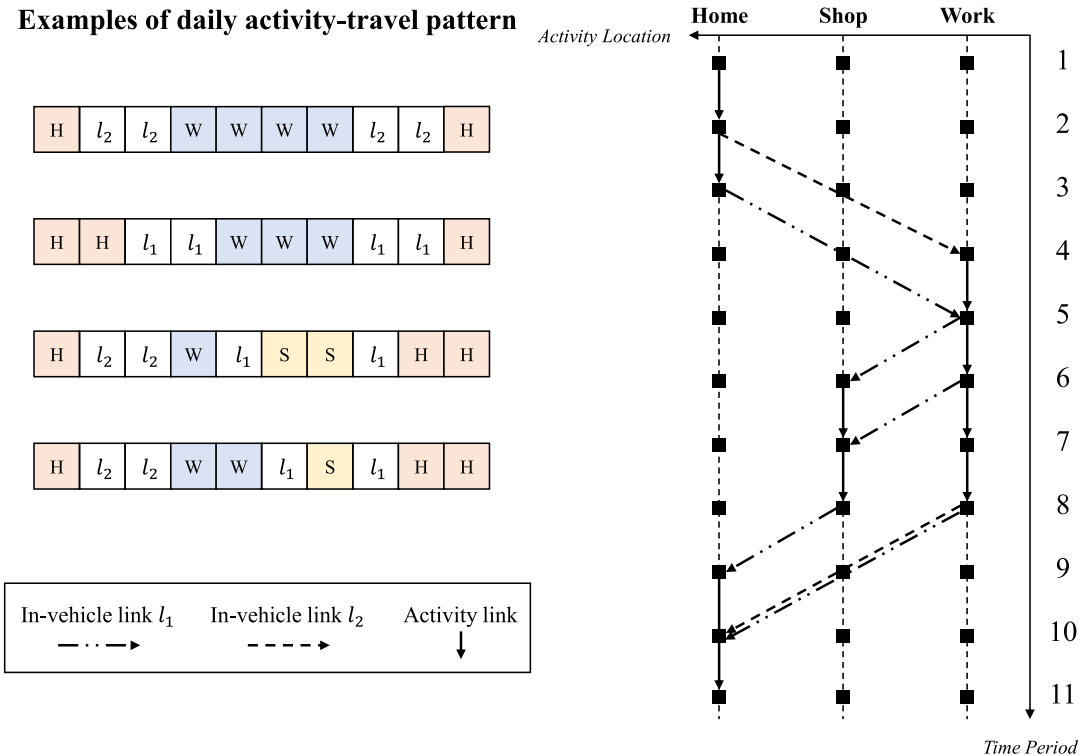


Fig. 5. DATPs representation.

3.2. The upper level: The principal-agent game

The government, the transit company and the passengers are three main stakeholders involved in a transit network design problem. The government determines the transit network and activity location plans with the aim of seeking benefit for the public. The objective of the government can be measured from three perspectives: (1) Space-time accessibility of activity locations. This measure jointly considers the transit network design and the activity location planning. It reflects the convenience of travellers to perform various activities in a given transit network; (2) Social welfare. It measures the social benefit of the decision; (3) Subsidy expenses. The government aims to lower the subsidy expenses under similar space-time accessibility and social welfare. The transit company conducts the operation and aims to maximize its profit, which is the sum of the total revenue and the subsidy minus the operation cost. The passengers respond to the decision of the government and the transit company by choosing the DATP to achieve the optimal utility. There are three games in this problem and form a three-party game, shown in Fig. 6. One is between the government and transit companies, which is described as a principal-agent game. The other two are leader-follower games relating to the interaction between the passenger and the government, as well as between the passenger and transit company. In this study, the three-party game relationship is formulated by a bi-level model: the government and the transit company represent the upper level and the passengers represent the lower level.

In the upper level, we consider the principal-agent game between the government and the transit company. The government acts as the principal, and the transit company acts as the agent.

The government makes decisions for transit network design as well as activity location planning. The government subcontracts the operation work to the transit company. For sustainable development of transit company, the government also compensates for the losses in operation according to the effort made by the transit company, which leads to subsidy. However, the government cannot observe the effort made by the transit company in operation, namely the operation effort level, so there exists information asymmetry. Specifically, compared with the government, the transit company has a more accurate picture about the operation of the transit networks while the government cannot directly observe the actual effort that the transit company takes. However, the government can still observe the operation status through the external information that are determined by the company's effort level (Tscharaktschiew and Hirte, 2012), e.g., total passenger demand.

Accordingly, the core aspect of this principal-agent game is how the principal decides the amount of subsidy required to incite the agents and induce them to make decisions that comply with the principal's desired benefit. However, the decisions made by the agents cannot be completely observed by the principal because of the asymmetric nature of the information between them. If the benefit resulting from a higher operation effort level outweighs that obtained in other ways, the transit company may choose a higher effort level by improving the level of service in the whole transit system (e.g. repairing the vehicles), in order to obtain additional subsidy from the government (Pilar Socorro and de Rus, 2010).

3.2.1. The transit company side

In the model, the transit company provides the transit service and determines its operation effort level. Generally, the operation effort level is related to a specific type of transit company's activities called support activities. For a transit company, the support

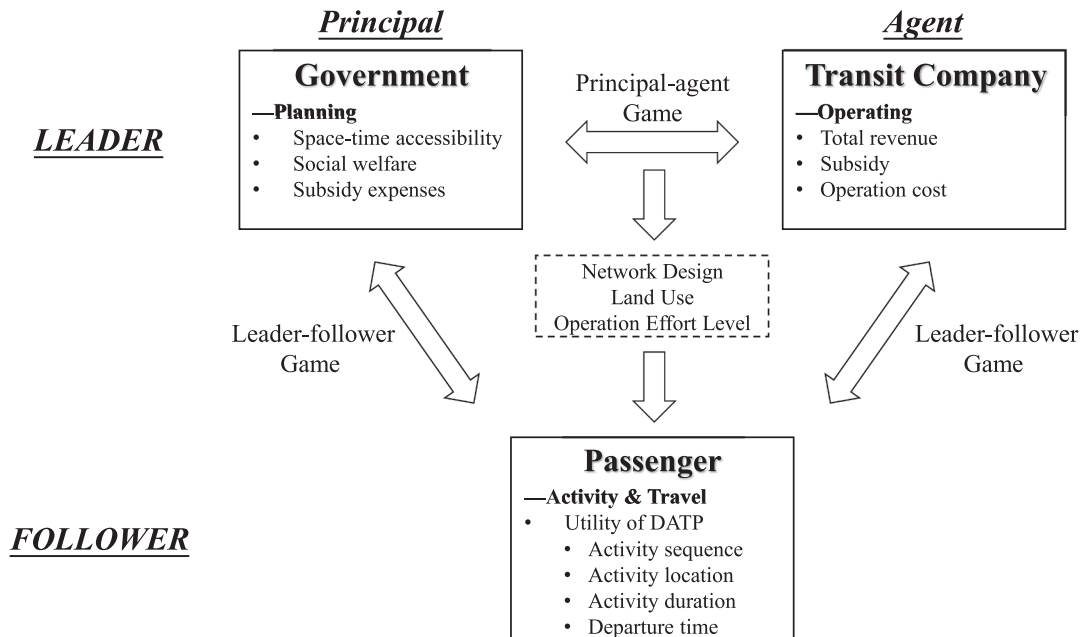


Fig. 6. Three-party game framework in the activity-based model.

activities can take many forms such as equipment maintenance, vehicle repair, management activities, team meetings, etc. For ease of assessment, researchers often rate the effort on a scale of 0 to 1. A higher effort level chosen by a transit company results in a higher level of service being provided on the transit network (Hensher and Stanley, 2008). Thus, in this study, we assume the operation effort level is a one-dimensional and continuous variable, denoted as $e \in [0, 1]$.

Profit of the transit company. The profit of the transit company is defined as the total revenue TR plus subsidy S from the government minus the operation cost OC . The total revenue is generated from transit fares, expressed as

$$TR = \sum_{p \in P} \sum_{\zeta(p, a_d)=1} f_p \cdot h_{a_d} \quad (7)$$

where f_p denotes the passenger flow on DATP p ; h_{a_d} denotes the transit fare regarding the direct in-vehicle link a_d .

The operation cost (denoted as OC) is following a quadratic form which is widely adopted in the literature considering the operation effort level e and the transit line frequency ϑ (Huang et al., 2016; Pilar Socorro and de Rus, 2010):

$$OC = \frac{e^2}{2} \cdot C_h \cdot \sum_{l \in L} \delta(l) RT_l \cdot \vartheta \quad (8)$$

where C_h is the unit cost of travel time; $\delta(l)$ is a dummy variable which equals to 1 if transit line l is under operation, 0 otherwise; RT_l is the roundtrip travel time of line l .

Therefore, the profit of the transit company under operation effort level e is:

$$\varphi(e) = \sum_{p \in P} \sum_{\zeta(p, a_d)=1} f_p \cdot h_{a_d} - \frac{e^2}{2} \cdot C_h \cdot \sum_{l \in L} \delta(l) RT_l \cdot \vartheta + S \quad (9)$$

Incentive-compatibility constraint. The incentive-compatibility constraint is intended to persuade the agent to choose the action that the principal expects the agent to do under the incentive mechanism. e is the decision variable of the transit company. The transit company is always pursuing interests. Therefore, the decision-making of transit companies in the game is subject to the incentive-compatibility constraint, which indicates that under the incentive mechanism, the effort level chosen by the transit company yields the largest profit (Huang et al., 2016). This constraint reflects the aim of the transit company (i.e., the maximization of its own profit).

$$\varphi(e) \geq \varphi(e') \quad (10)$$

where e is the effort level chosen by the transit company; e' represents any other possible choices of the transit company.

3.2.2. The government side

In this study, the government is the decision maker for transit network design and activity location planning. Therefore, at the planning level, the decision variables are formed with two vectors δ_C and δ_l . $\delta_C = (\dots, \delta(n_C^i), \dots)^T$ is a decision vector of activity location plan which indicates whether candidate location n_C^i is planned to be a new activity location. This can influence the activity location choices of passengers, thus changing their DATP and resulting in a totally different space-time flow distribution. $\delta_l = (\dots, \delta(l_i), \dots)^T$ is a decision vector of transit network design which indicates whether transit line l_i is used in the planned transit network. The constrain for δ_l is expressed as $\delta_l \in \Omega$, where Ω denotes the set of feasible transit networks which requires all nodes in the transit network should be accessible and all transit lines in the transit network should be feasible (Gulhan et al., 2018).

As an important guarantee of individuals' daily life, the social benefits of the transit network and the activity locations far outweigh their economic benefits. Therefore, the government needs to take many factors into consideration when making decisions. In this study, we consider three system-wide indicators from the perspective of the government, the network and the society (i.e., subsidy expenses, space-time accessibility of activity locations and social welfare, respectively). The three indicators are explained in detail below.

Subsidy. As the information asymmetry exists, the government can only observe the operation effort level of the transit company indirectly from the total travel demand. Therefore, when the transit company makes more effort in operation and attract more travel demand in the transit system, the government offer more subsidy. The subsidy S is defined in Eq. (11):

$$S = \sum_{p \in P} \sum_{\zeta(p, a_d)=1} [\tau_{cr}(1+r) - h_{a_d}] \cdot f_p \quad (11)$$

where τ_{cr} is the regulation operation cost per passenger, determined by the transit company based on the previous year's financial results; r is the regulation profit rate of the transit company for its sustainable development. Note that if $h_{a_d} \geq \tau_{cr}(1+r)$, the subsidy is treated as 0. In other words, the income from the transit fare is sufficient to ensure the company's sustainable development.

Space-time accessibility of activity locations. In the activity-based model, the purpose of travel is getting involved in some activities at activity locations, so the space-time accessibility of activity locations is one of the important evaluation indicators of the transit network design and activity location planning. To combine travels and activities into a unified measure of accessibility, following the space-time utility accessibility approach (Miller, 1999).

The utility of individuals at node x going to node n in the physical network can be modified using exponential function as

$$\bar{U}_n(x) = \exp(\chi \cdot \text{disu}_{a_x, n}), x \in N, n \in N, a_x, n \in A_d \quad (12)$$

where χ is the utility increasing parameter.

Following the definition in Fu et al. (2022a), the space-time accessibility of node n (denoted as $Acc(n)$) is measured by the sum of the utility from node n to all activity locations in the research area:

$$Acc(n) = \sum_{x \in N_a} \bar{U}_n(x) \quad (13)$$

Thus, the network-wide activity-based space-time accessibility (denoted as STA) of all activity locations for time period t' to t' is

$$STA = \sum_{n \in N_a} \sum_{t' \leq t' \leq t'} Acc(n), t' \in T, t' \in T \quad (14)$$

Social welfare. Previous studies have shown that social welfare is one of the commonly used optimization indicators in transit network related problems. Social welfare consists of the consumer surplus CS and the producer benefit. In this model, the producer benefit can be described as the transit company's profit $u(e)$ minus the subsidy expenses of the government S . Therefore, social welfare in the model can be expressed as

$$SW = CS + \varphi(e) - S \quad (15)$$

Consumer surplus was originally defined as the difference between consumers' willingness to pay and market price (Cohen et al., 2022). Following Huang et al. (2016), the consumer surplus is derived from the total social benefit minus the total cost. From the land use and transit network system's perspective, total social benefit results from the utility from activity, and describes the passengers' willingness to pay for the transit service. The total cost comes from the in-vehicle dis-utility, which is a negative value which represents the generalized cost that the passengers actually pay. Therefore, the consumer surplus is obtained as

$$CS = \sum_{p \in P} f_p \cdot u_p \quad (16)$$

where u_p is the utility of DATP p .

Therefore, social welfare SW is the sum of consumer surplus and transit company's profit:

$$SW = \sum_{p \in P} f_p \cdot u_p + \sum_{p \in P} \sum_{\xi(p,a_d)=1} f_p \cdot h_{a_d} - \frac{e^2}{2} \cdot C_h \cdot \sum_{l \in L} \delta(l) RT_l \cdot \vartheta \quad (17)$$

3.3. The lower level: The leader-follower game

There are two leader-follower games between the passenger and the government, and between the passenger and the transit company. Passengers pursue the largest DATP utility. The activity location planning and the transit network design at the planning level, and the operation effort at the operation level both affect the passenger flow distribution.

Overall transit demand. The higher effort level brings the higher service level (Hensher and Stanley, 2008). Some research shows that the service level significantly influences the overall demand between any OD pair for public transit (Fan and Machemehl, 2008), leading to a variety of transit assignments. It affects the crowding effect in activity and travel behaviour and changes the utility of activity-travel patterns, thus leads to a different network equilibrium result. In this study, a higher operation effort level leads to a larger transit demand. The overall transit demand is expressed as (Lam and Zhou, 2000; Huang et al., 2016):

$$D(e) = \sum_{n_H \in N_H} \sum_{n_W \in N_W} e \cdot K_{n_H, n_W} \quad (18)$$

where e is the operation effort level.

Passenger flow distribution. In this study, user equilibrium condition defined in Eqs. (19)–(22) is used to capture the activity-travel choice behaviour of passengers in the activity-time-space super-network. The passenger flow in the super-network is allocated to the optimal DATP (rather than the optimal route in the conventional trip-based model). The utility of all DATPs assigned with passenger flow is the largest and equal, while the utility of all unused DATPs is smaller.

$$f_p(u_\pi - u_p) = 0 \quad (19)$$

$$D(e) = \sum_{p \in P} f_p \quad (20)$$

$$u_\pi - u_p \geq 0 \quad (21)$$

$$f_p \geq 0 \quad (22)$$

where f_p is the flow of an activity-travel pattern in the activity-time-space super-network; π is the optimal DATP.

The flow conservation constraints in the model are expressed in Eqs. (23)–(25). Eqs. (23)–(24) express the flow identity between DATPs and activity/in-vehicle links. Eq. (25) expresses the flow conservation at each time period t in the activity-time-space super-network.

$$f_{a_d} = \sum_{p \in P} f_p \cdot \zeta(p, a_d), \forall a_d \in A_d, \forall p \in P \quad (23)$$

$$f_{a_a} = \sum_{p \in P} f_p \cdot \zeta(p, a_a), \forall a_a \in A_a, \forall p \in P \quad (24)$$

$$D(e) = \sum_{t(a_a)=t} f_{a_a} + \sum_{t(a_d)=t} f_{a_d}, \forall t \in T, a_a \in A_a, a_d \in A_d \quad (25)$$

where f_{a_d} denotes the passenger flow on the in-vehicle link a_d ; f_{a_a} denotes the passenger flow on the activity link a_a .

3.4. The overall model

To summarize, this model jointly designs the transit network and plans the activity locations in an activity-based approach. Passengers' responses to the transit network and the activity locations are reflected by their DATP choices (including activity type, activity start time and duration, departure time, etc.). In this way, the relationship between travel and activity, and the relationship between the transit network and the activity locations are jointly considered.

Considering the conflicting interests of the government, the transit company and the passengers, we employ a bi-level framework to depict the three-party game. The principal-agent game between the government and the transit company is included in the upper level. The government is the decision-maker of the transit network design and activity location planning, and the aims of optimization are maximizing the network-wide space-time accessibility of activity locations, maximizing the social welfare, and minimizing the subsidy expenses. As the government plays a leading role in this problem, we consider the optimization goals of the government in the objective function.

For ease of presentation, we define the objective vector as $Z(S, -STA, -SW)$ and form the problem as a minimization problem. The overall model for the studied activity-based model for transit network design and activity location planning problem involving a three-party game is shown in Eq. (26). The constraints ensure the feasibility of the transit network and flow conservation, and meanwhile constrain the transit company to choose the operation effort level that leads to the optimal profit and constrain the passengers to choose the DATP that yields the largest utility.

$$\begin{aligned} & \min_{\delta \in \mathcal{D}, \epsilon} Z(S, -STA, -SW) \\ & \text{s.t. Eqs. (10), (19) - (25)} \end{aligned} \quad (26)$$

In the following section, we propose a solution method to solve this problem efficiently.

4. The solution method

As discussed in Section 3, the model forms a principal-agent game and two leader-follower games into a bi-level framework. In

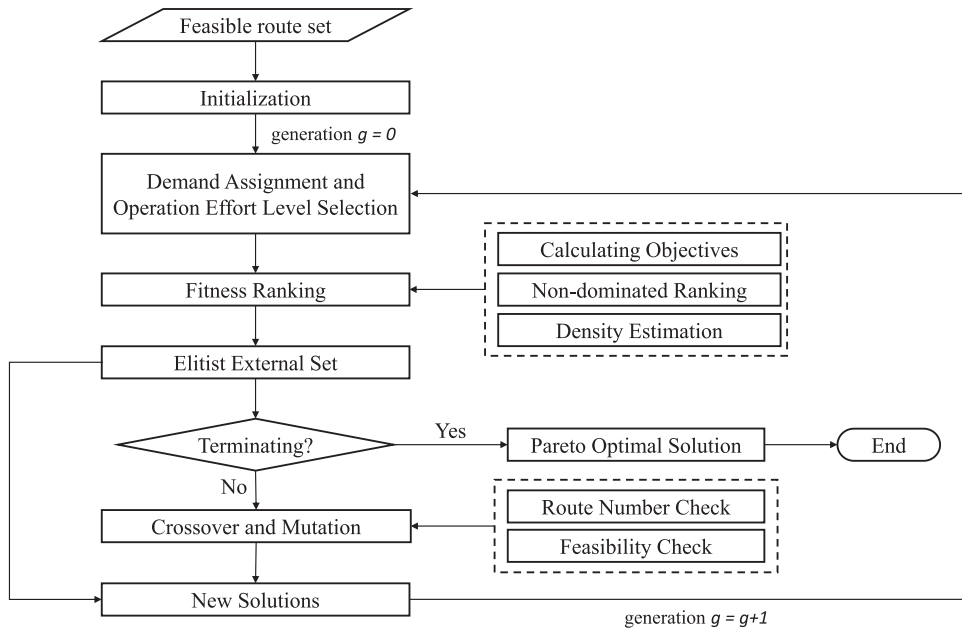


Fig. 7. Solution algorithm.

transport network design problems, the bi-level framework is widely adopted and is acknowledged as an non-deterministic polynomial hard (NP-hard) problem (Brands et al., 2014; Pattnaik et al., 1998). Considering the computational efficiency in practice, heuristic algorithms are more suitable than exact algorithms in this study. In the literature, many heuristic solution algorithms have been developed for solving such transit network design problem (Bielli et al., 2002; Bourbonnais et al., 2021; Liu et al., 2020b). In this study, the Pareto genetic algorithm (PGA) is adapted to generate the compromise solutions between the conflicting objectives in the proposed model (Liu et al., 2018). In the genetic algorithm, each solution represents a set of transit lines and new activity locations. In each generation, the solutions are evaluated by the objectives and ranked according to non-dominated ranking and density estimation. During the whole evolution process, there is an elitist external set created to store elitist solutions for each generation (Van Veldhuizen and Lamont, 2000). We generate new feasible solutions based on the previous elitist external set, and combine the new feasible solutions and the previous elitist external set to form the next generation. This process is repeated until the stopping criteria is satisfied. The process of the solution algorithm for solving the bi-level programming model is displayed in Fig. 7 and outlined as follows.

Step 1. Initialization. The PGA starts with the initial feasible solution R_0 (also generation 0) of size $\nu_p + \nu_o$, where ν_p is the size of parent solutions and ν_o is the size of offspring solutions in the algorithm. The feasible transit lines are generated following Ceder (2007). Each feasible transit line corresponds to a 0–1 variable in the decision vector. It is the same with each candidate location. Fig. 8 shows how each chromosome represents the solution of transit network design and the activity location planning. The chromosome is composed with two decision vectors $\delta_c = (\dots, \delta(n_c^i), \dots)^T$ and $\delta_l = (\dots, \delta(l_i), \dots)^T$, which are series of 0–1 variables.

Step 2. Demand assignment and operation effort level selection. When it comes to a determined plan of transit network and activity locations, the transit company chooses the optimal operation effort level to achieve the largest profit. In this sub-problem, the decision variable is operation effort level. Different operation effort levels lead to different transit demand, and thus passengers may re-decide their DATPs. This problem is also a NP-hard problem, and heuristic algorithm is an efficient method to solve this kind of problem. In this study, we adopt an iterative framework with the artificial bee colony algorithm.

Algorithm 1. (Pseudocode for solving the three-party game)

Data: a given transit network and initial parameters
while stopping criterion is not satisfied **do**
 Upper Level
 • Choose a new operation effort level
 • Obtain the subsidy, operation cost, revenue and total profit of the transit company
 Lower Level
 • Compute the public transit demand in the network
 • Solve the transit assignment on the current solution
End
Report the operation effort level and network equilibrium result

Step 3. Fitness Evaluation. Once the demands have been assigned and operation effort level computed, three objectives for each solution are then calculated following Eqs. (11), (14), (17). In order to identify the best compromise solutions, solution i is evaluated with non-domination rank i_{rank} and crowding distance i_{distance} according to the non-dominated sorting and the density estimation approach given in Algorithm 2 and Algorithm 3. In the study, the fitness of solution i is represented by i_{rank} and i_{distance} . The partial order \prec_n used in the study follows Deb et al. (2002). As is shown in Eq. (27), the solution with the lower rank is preferred; And if both solutions belong to the same rank, we prefer the solution located in a less crowded region considering the diversity of the solutions.

$$i \prec_n j \quad \text{if } (i_{\text{rank}} < j_{\text{rank}}) \quad \text{or} \quad ((i_{\text{rank}} = j_{\text{rank}}) \quad \text{and} \quad (i_{\text{distance}} > j_{\text{distance}})) \quad (27)$$

Step 4. Reserve the elitist solutions. In generation g , we select ν_o elitist solutions to form P_{g+1} for the next generation, which is also called the offspring solution of generation g . The selection is also according to the non-dominated sorting and the density estimation approach. If g exceeds the maximum generation, stop and output the Pareto optimal solutions; otherwise, go to Step 5.

Step 5. New generation. In generation $g + 1$, the parent solution Q_{g+1} of size ν_p is generated with the crossover operator and the mutation operator from P_{g+1} . The parent and the offspring solution are combined to form a new solution set, denoted as $R_{g+1} = P_{g+1} \cup Q_{g+1}$. Then, increase the iteration counter by one and go to Step 2.

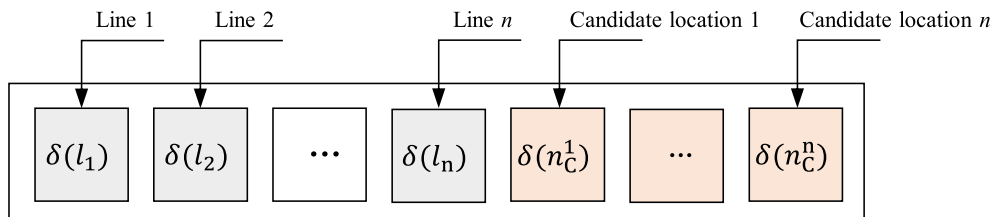


Fig. 8. Chromosome representation.

Algorithm 2. (Pseudocode for the non-dominated sorting)

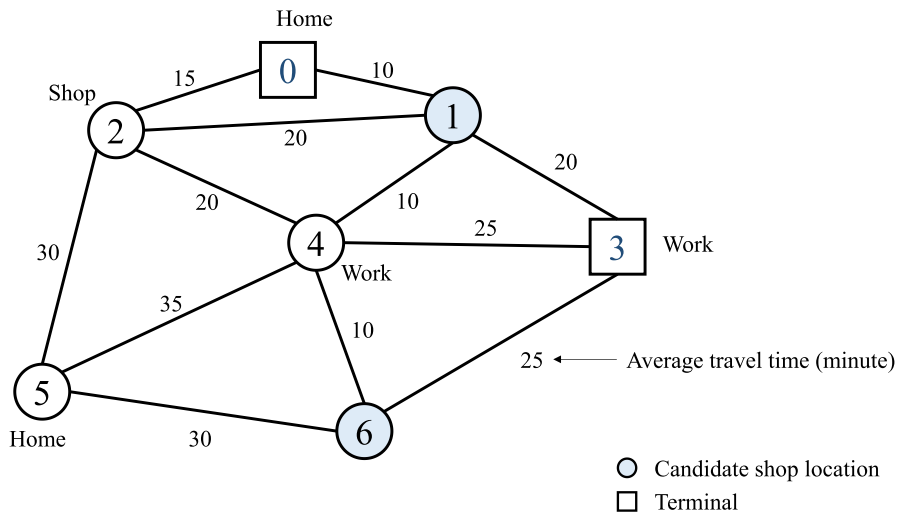
Data: solution set R
Define the domination count x_r and a set of solutions that the solution dominates Y_r .
for each r in R
 $Y_r = \emptyset, x_r = 0$
for each r' in R
 if r dominates r' **then** add r' to Y_r
 if r' dominates r **then** increase the domination count x_r by 1
if $x_r = 0$ **then** add r to the first front counter F_1
 $i = 1$
while $F_i \neq \emptyset$ **do**
 $F_{i+1} = \emptyset$
for each r in F_i
 for each r' in Y_r
 Decrease the domination count $x_{r'}$ by 1
 if $x_{r'} = 0$ **then** add r' to the next front counter F_{i+1}
 $i = i + 1$
End
Report the front counters

Algorithm 3. (Pseudocode for the density estimation)

Data: non-dominated set I
Define the m^{th} objective function as o_m
Initialize the distance of each solution as zero.
for each objective m
 Sort the solutions by their objective values
 Set the distance of two boundary points as ∞
 for all other points
 Calculate the individual distance value as the absolute normalized difference
 in the objective values of two adjacent solutions
Calculate the overall crowding-distance value as the sum of individual distance values
corresponding to each objective
Report the distance value for the solution in the non-dominated set I

5. Numerical example

The numerical example is carried out to present the merits of the proposed model and influence of activity-based considerations in decision making. The optimized transit lines and new activity locations can be generated by the proposed model. The solution algorithm is executed in Python 3.7 and run on a personal computer with Intel Core TM i7-7500CPU of 2.7 GHz and 8 GB RAM.

**Fig. 9.** The example network.

We use a network following the network used by Ceder (2007) and Gulhan et al. (2018) to carry out the numerical example, as shown in Fig. 9. This example aims to compare the single-objective model and the multi-objective model, and analyse the interaction between transit network design and activity location planning. In the example, a new shop location is expected to be located at Node 1 or Node 6 in the studied network, considered as Scenario 1 and Scenario 2 respectively in this numerical example for simplification. The new activity location affects the activity choice of passengers (including activity location choice, activity sequence and duration choices, etc.) and thus the travel choice (including transit line choice, departure time choice, etc.). In this way, the interactions between transit network design and activity location planning can be explored.

For a better illustration of commuters' DATP choice behaviour, the study period is 7 am-7 pm, divided into 144 periods with 5-minute duration. In the study, three types of activity are considered, which are Home (H), Work (W) and Shop (S). The first two are considered as compulsory activities and the latter one is optional. The home-based flow of passengers is assigned to DATPs covering two kinds of activity sequence which are "H-W-H" and "H-W-S-H" (Kang et al., 2013). The time windows of start and end time and activity durations are given in Table 4.

For each activity, a marginal utility expresses the utility gained from a time unit of participation. Different functions can be used to describe the marginal utility. In this study, the bell-shaped marginal utility function (described in Section 2.3) is used and the parameters are given in Table 5.

We firstly consider the single-objective optimization problem under one scenario (with shop located at Node 6). In single-objective optimization, the results and independent objective values are obtained as Table 6 shows. As shown in Table 6, when social welfare is optimized (i.e., 69,612), the subsidy indicator 25,500 is much higher than that in the other two sets of results; when the subsidy is optimized, the social welfare is much lower (i.e., 25,471) than that in the other two sets of results; when the space-time accessibility is optimized, the performance of the other two indicators is just acceptable, which are 43,002 and 15,127 respectively. These results prove that: (1) trade-offs between different objectives exist in this problem; (2) in the single-objective optimization, while the objective is optimized, the performance of other objectives reflected by other indicators is not always averagely good or even acceptable. The results show the necessity of multi-objective optimization in this problem. By normalization of objective values, Fig. 10 illustrates this phenomenon more intuitively. In this study, for each objective function f , value f_i is normalized following Eq. (28) for maximization problem; value f_i is normalized following Eq. (29) for minimization problem. The larger the value, the better the result.

$$f_n^i = (f_i - f_{\min}) / (f_{\max} - f_{\min}) \quad (28)$$

$$f_n^i = 1 - (f_i - f_{\min}) / (f_{\max} - f_{\min}) \quad (29)$$

where f_{\max} represents the maximum value of the objective function f ; f_{\min} represents the minimum value of the objective function f ; f_n^i represents the normalized value of f_i , $f_n^i \in [0, 1]$.

From Fig. 10, we can see the conflicting relationship among the three objectives. If we optimize the transit network only for a single objective, some results are relatively unacceptable, for example, the 0.19 (subsidy) in Fig. 10(a) and the 0.29 (social welfare) in Fig. 10(b).

Different from single-objective optimization problems, in multi-objective optimization problems, it is usually impossible to obtain an optimal solution that optimizes all objectives due to conflicts among various objectives. Decision makers can only choose a more satisfactory compromise solution based on preferences. In the multi-objective optimization problem, a set of non-dominated solutions of the objective function is called the Pareto non-dominated solution set, and the curve or surface formed in the objective function space is called the Pareto front.

In the study, we set the size of the parent solution $\nu_p = 20$, and the offspring solution $\nu_o = 20$. Suppose that for each a_H and a_W , $K_{a_H, a_W} = 300$. Other parameters were set as $c_b = 100$, $\tau_{cr} = 7$ CNY, $r = 0.5$, $\theta = 0.3$.

The optimal values of each objective in each generation are presented to illustrate the evolution process, shown in Fig. 11. It can be observed that most updates of optimal values occur before the 50th generation, and then remain almost constant until the 100th generation. Although we aim for the non-dominated solutions instead of optimal solutions of each single objective, the updates of the optimal values ensure the diversity of solutions in the elitist set. Therefore, in this case, we set the iteration limit as 50 in PGA.

In the study, we generate 20 elitist solutions by PGA (see Fig. 12). The compromise solutions are found near the boundary in Fig. 12(a), and two-dimensional representation is given in Fig. 12(b), Fig. 12(c) and Fig. 12(d). The figure also shows the nonlinearity of the problem. Generally, the social welfare grows when the subsidy increases. However, the relationship between space-time accessibility and social welfare, as well as the relationship between the space-time accessibility and subsidy are not obvious. According to Fig. 12(b) and Fig. 12(d), given the amount of subsidy, the values of space-time accessibility and social welfare vary under different solutions. As the government aims at optimizing the space-time accessibility and the social welfare, and at the same time reduce the subsidy

Table 4
Settings of activities.

Activity	Start time	End time	Duration (min)
Home (morning)	7:00	[7:00, 8:30]	—
Home (afternoon)	[17:00, 19:00]	19:00	—
Work	[7:00, 9:00]	[17:00, 18:00]	—
Shop	[17:00, 19:00]	[17:00, 19:00]	30, 60

Note: [a, b] refers to the time window.

Table 5

Given parameters in the marginal utility function.

	Work (0:00–12:00)	Work (12:00–24:00)	Home (0:00–24:00)	Shop (0:00–24:00)
$u_{a_i}^{\max}$	180	170	120	120
α_{a_i}	35	90	60	125
$\beta_{a_i}^j$	0.21	0.21	0.048	0.18
γ_{a_i}	0.8	0.8	−2.6	1
$u_{a_i}^0$	0	0	4	0

expenses under similar space-time accessibility and social welfare, this multi-objective consideration is necessary in this three-party problem.

By this approach, the result can not only provide the previously mentioned individual optimal solutions, but also other solutions which include solutions that perform well on each single objective, defined as satisfactory solutions. Each single objective of satisfactory solutions performs well and is above average. In this study, we define the normalized performance of each objective in satisfactory solutions above 0.6. Although these satisfactory solutions may not be the non-dominated solution in all possible solutions, they are better than many other possible solutions.

In the multi-objective optimization problems, the radar charts of the satisfactory solutions appear to be more balanced comparing the radar charts in Fig. 10 and Fig. 13. In fact, the larger size of the elitist set or iteration times, the higher probability we generate satisfactory solutions. However, when the size of the elitist set is larger than the number of solutions in the Pareto front, the elitist set can include some solutions that in the second front and we need to select dominant ones. In the set of elitist solutions we generated, transit network $[[0, 1, 4, 5], [3, 1, 2, 5], [3, 4, 6, 5]]$ with new shop location Node 6 is the only one dominant satisfactory solution, shown in Fig. 13.

Due to the complexity of the problem, the computation time is longer than most network design problems. This is because the original network is extended to a large activity-time-space super-network with much more nodes and links. Larger network leads to much longer computation time since one additional node can bring hundreds of additional combination of transit lines (even with line number constraint) and DATP choices. With the aim of efficiency, we use a small network to prove the model can have a reasonable solution and the computation can complete within 3 h. This result indicates that the proposed algorithm can solve the bi-level programming problem for this typical network efficiently. Although the example network is relatively small, this example considers the compulsory activities, optional activities, flexible activity sequences, flexible activity start/end time, etc. Therefore, this is a representative case, and the proposed method can be applied in a larger network with more nodes and links, different types of compulsory activities and optional activities, and activity sequences constraints. To further illustrate the proposed model, the optimal solutions of transit network design associate with activity location plans will be presented in the following discussions.

The optimal solution of the proposed multi-objective model and the performance of the three stakeholders can be found in Table 6. The optimal transit network is $[[0, 1, 4, 5], [3, 1, 2, 5], [3, 4, 6, 5]]$, and the optimal new location for shop activity is Node 6. By observing the results of single-objective model and multi-objective model, it is demonstrated that the transit company responds differently under different transit network designs. The profit of the transit company relies on subsidy from the government, and the profit is often less than the subsidy. This means the total revenue cannot always cover the operational cost. Therefore, to ensure the sustainable development of the transit company, the subsidy is necessary. The responds of the transit company influence the total demand of passengers. A higher operation effort level encourages individuals to travel by the transit network.

Table 6 also shows some results regarding individuals' activity and travel choice behaviours. With the proposed model, we can obtain results of average in-vehicle duration and average activity duration under different transit network and shop location solutions. This indicates that the passengers may reschedule their DATPs to achieve the maximum utility when the transit network and shop location changes, and leads to totally different objective values. To investigate individuals' choice behaviour in different activity location plans, we conduct a comparison experiment (as the last row in Table 6 shows), that is, using the optimal transit network design generated from the multi-objective model but changing the location of the new shop to Node 1. The comparison shows that the indicators of the same transit network under different activity location plans perform differently. For the given transit network $[[0, 1, 4, 5], [3, 1, 2, 5], [3, 4, 6, 5]]$, when the new shop location changes from Node 6 (Scenario 2) to Node 1 (Scenario 1), although social welfare increases from 44,725 (normalized value 0.60) to 61,864 (normalized value 0.87), the subsidy expenditure for the government increases from 14,383 (normalized value 0.65) to 26,412 (normalized value 0.15), which may not be acceptable for the government. Therefore, the optimal solution of transit network under one scenario is often not the optimal solution under another scenario. This result indicates the activity location plan has significant influence on transit network design, and individuals' activity choice behaviour needs to be taken into account when planning the transit network. Separately considering the transit network design from activity location planning can misunderstand the performance of the transit network.

Fig. 14 shows the temporal flow distributions of travels and different activities, and the link flows for the two scenarios. It can be seen that individuals choose different DATPs under different activity location plans even with the same transit network design, which cannot be explored by conventional trip-based approach. When the new shop is located at another location, the temporal flow distributions are different due to that individuals may choose different DATPs. Some individuals may reduce/increase the durations for activities and travel (as Fig. 15 shows), and the link flows in transit networks change accordingly.

Thus, with the proposed bi-level programming model in a three-party game framework, transit network and activity location plan can be jointly optimized with consideration of maximizing the network-wide space-time accessibility of activity locations, maximizing

Table 6

Optimal solution and the performance of the three stakeholders.

	The government					The transit company		The passengers				
	Transit line set	Shop location	Social welfare (CNY¥)▲	Subsidy (CNY¥)▼	Space-time accessibility of activity locations (CNY¥)▲	Operation effort level	Profit (CNY¥)	Total demand of passengers	Average in-vehicle duration (min)	Average activity duration (min)		
										Home	Work	Shop
Single-objective model	[[0, 1, 4, 6], [3, 6, 5], [3, 1, 0, 2]]	6	69,612	25,500	2,182	1	19,500	1,200	68	130	500	22
	[[3, 1, 2, 5], [3, 6, 4, 5], [3, 4, 1, 0, 2]]	6	25,471	6,120	2,277	0.12	7,268	360	68	161	491	0
	[[3, 6, 4, 2], [3, 1, 0, 2, 5], [3, 1, 4, 6, 5]]	6	43,002	15,127	2,349	0.6	11,744	720	68	136	494	23
Multi-objective model	[[0, 1, 4, 5], [3, 1, 2, 5], [3, 4, 6, 5]]	6	44,725	14,383	2,217	0.62	10,614	744	69	140	496	15
Comparison experiment for the multi-objective model	[[0, 1, 4, 5], [3, 1, 2, 5], [3, 4, 6, 5]]	1	61,864	26,412	2,209	0.99	14,572	1,188	68	124	501	27

Note: the bold number represents the optimization result of the objective in single-objective model. ▲ means the higher the value, the better; ▼ means the lower the value, the better.

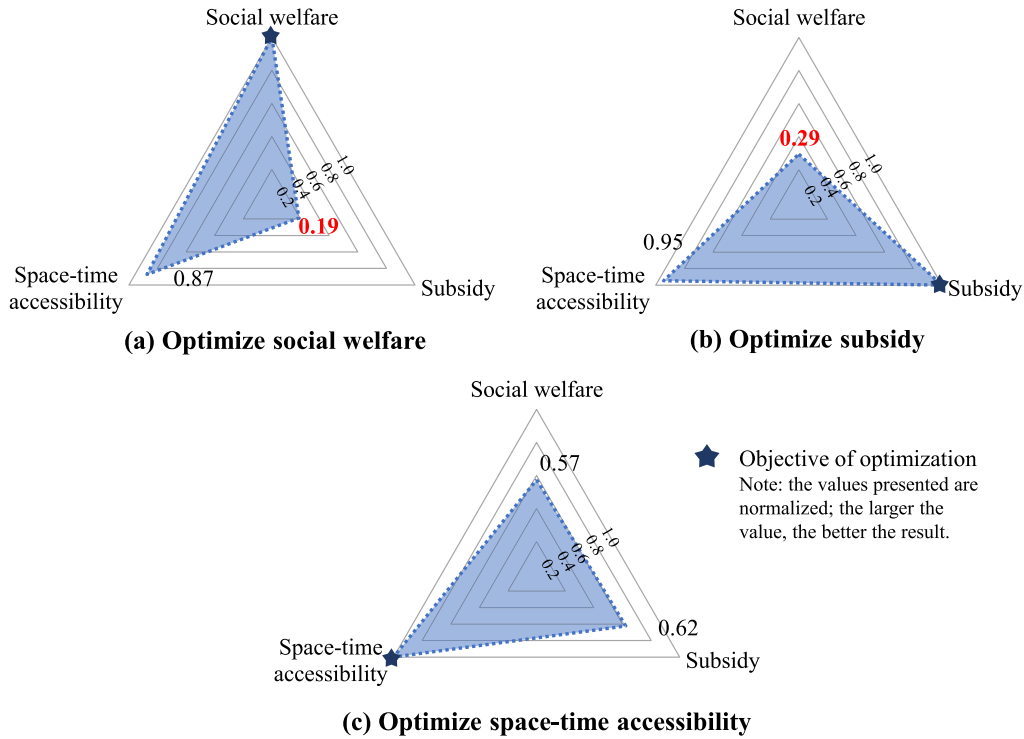


Fig. 10. Evaluation for the optimal solutions with different objectives.

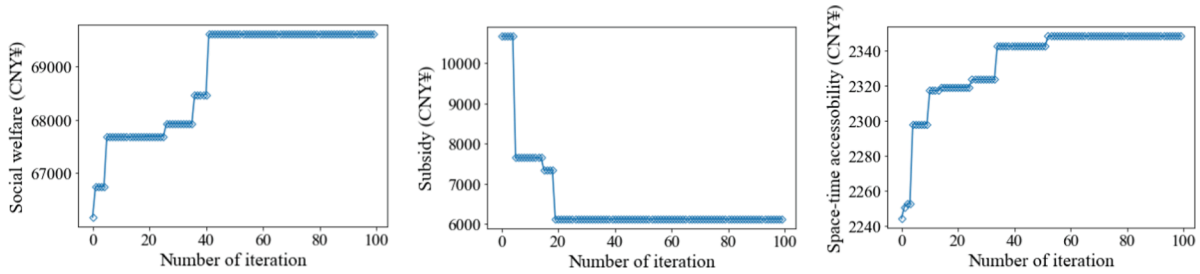


Fig. 11. Evolution process of objective values.

the social welfare, and minimizing the subsidy expenses. The resultant changes of passengers' activity and travel choice behaviours (including activity sequence, activity start time and duration, departure time of each trip, transit line, etc.) can be explicitly investigated.

6. Conclusions

Current works of transit network design in the literature seldom consider activity location plans, and lack consideration of individuals' activity choice behaviour. This paper jointly designs the transit network and the activity locations in an activity-based approach. Considering the conflicting interests of the three stakeholders involved (i.e., the government, the transit company and the passengers), we employ a bi-level framework to depict the three-party game. To evaluate the transit network design and the activity location planning, we carry out a multi-objective analysis from the perspective of the network, the society and the government with three objectives to support the decision-making process of the government.

At the lower level of the proposed bi-level model, passengers' DATP choices are investigated using an activity-based network equilibrium approach. Passengers' various choice behaviour in their DATPs (including activity type, activity start time and duration, departure time, transit line, etc.) are reflected using an activity-time-space super-network. Activity choices are generated endogenously by specifying time-dependent utilities of activity participation. Passengers' activity and travel choices are not fixed and do not need to be pre-defined at the lower level. The upper level involves the principal-agent game between the government and the transit company. At the upper level, the social welfare, the subsidy and the space-time accessibility of activity locations are optimized by

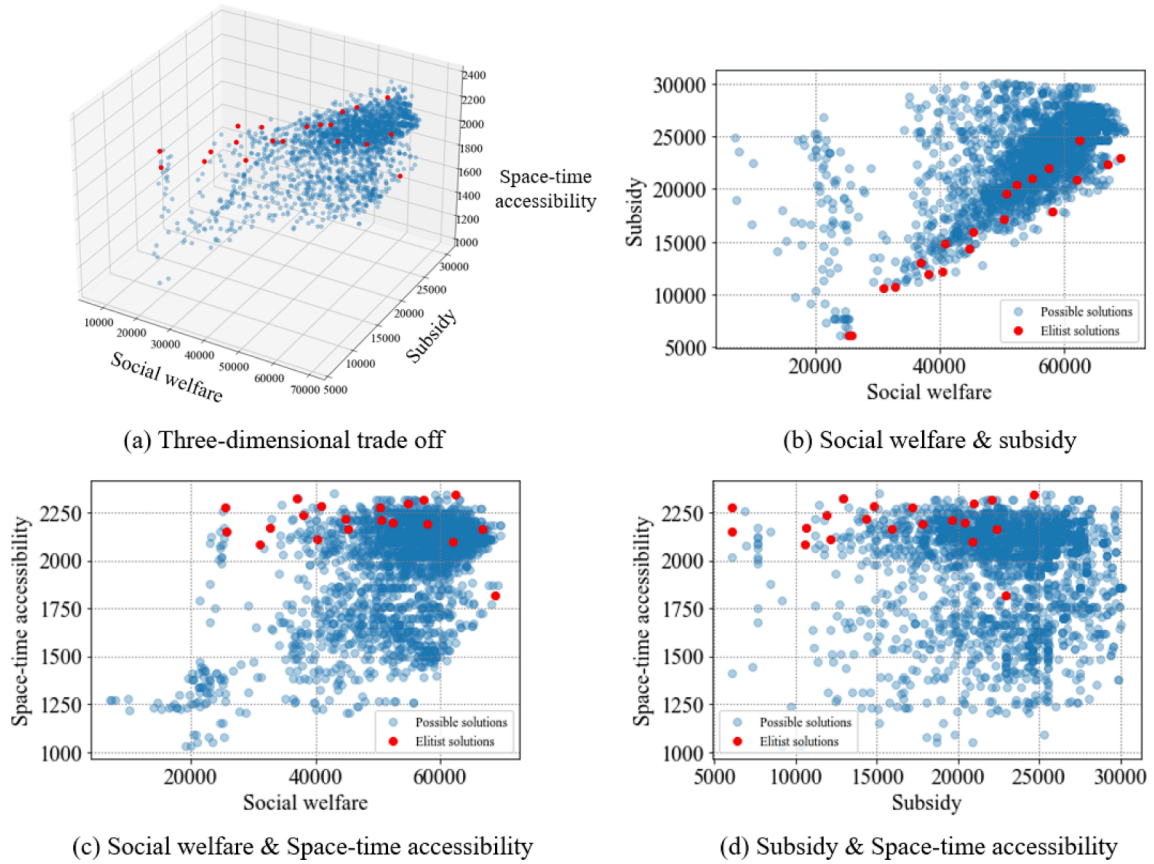


Fig. 12. Solutions and the trade-offs.

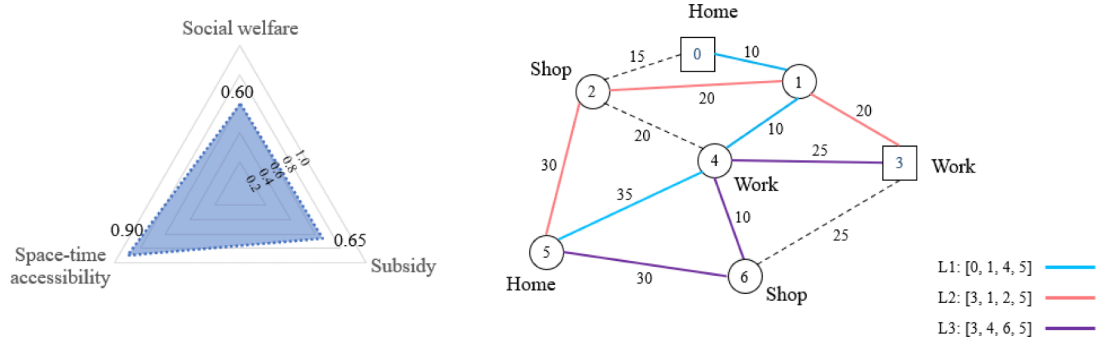
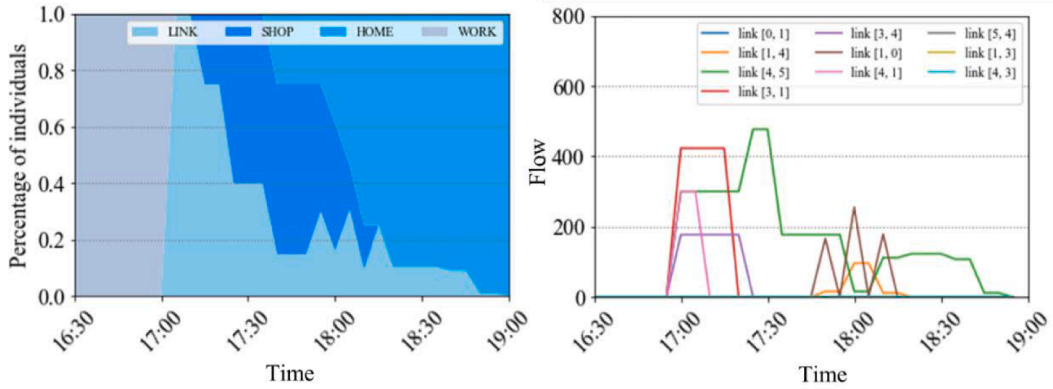


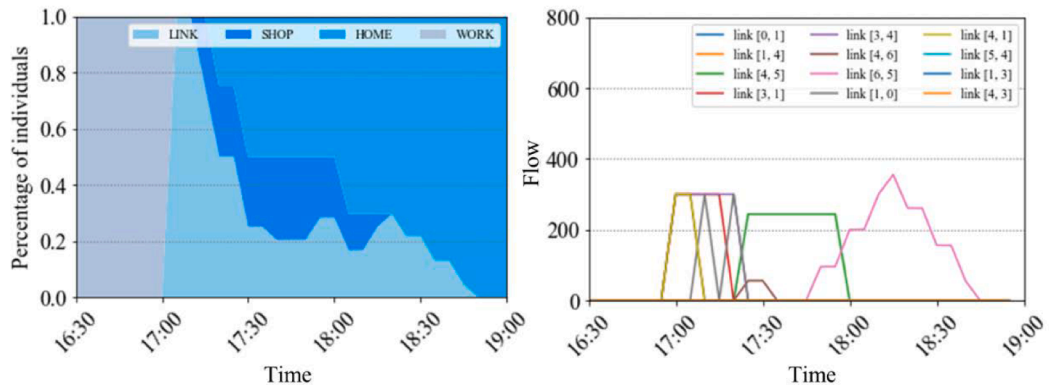
Fig. 13. Evaluation for the optimal solution (transit network: [[0, 1, 4, 5], [3, 1, 2, 5], [3, 4, 6, 5]], new shop location: Node 6).

multi-objective analysis to support the decision-making process of the government. By the activity-based approach, utility gain from activities and loss from travels for different time periods at different locations are reflected. In this way, the relationship between activity and travel, and the relationship between the transit network design and the activity location planning are jointly considered. A Pareto genetic algorithm is used to solve the proposed bi-level model and generate the solutions on the Pareto front. The proposed model can be used by governments to design transit networks and plan activity locations from the perspective of fully exploring individuals' activity and travel choice behaviour.

A case study is carried out to show the merits of the proposed model. We can attain the space-time flow distribution from the activity-based model. It is also found that in the single-objective optimization problem, while the objective is optimized, the performance of other objectives reflected by other indicators is not always averagely good or even barely acceptable. Therefore, the multi-objective optimization is an essential part of the three-party game. Considering the conflicting interests of the three parties, the results show that under different transit network designs and activity location plans, passengers' activity and travel choices, as well as the



(a) Scenario 1



(a) Scenario 2

Fig. 14. Temporal flow distributions under different activity location plans (16:30–19:00).

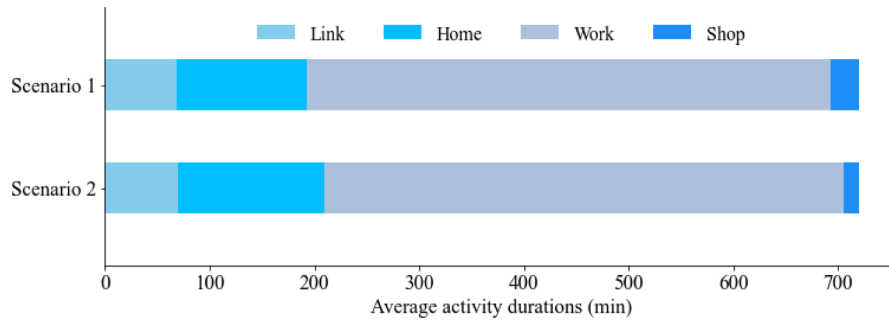


Fig. 15. Average activity time durations under different activity location plans.

system-wide indicators including space-time accessibility of activity locations, social welfare and subsidy expenses vary significantly. Separately considering the transit network design from activity location planning can overestimate or underestimate the activity/travel demand and lead to unreliable evaluation results.

In this paper, we focus on activity and travel choice behaviour, so we simplify the problem of land use planning to the problem of activity location planning. However, the actual land use planning problem is very complex, and a land use planning model considering more factors is needed in further study. Besides, the idea of jointly modelling the transit network and activity location plans can be adopted in multi-modal transport network design problems. We can take more transport modes into consideration including private cars, metro, etc. For transit network design, we need to consider more variables such as transit service frequency, and it would also be valuable to model the relationship between service frequency and operation effort level. The current work also needs to be extended by

accounting for other practical considerations, e.g., travel time uncertainty in the network, more activity types and other system-wide indicators (such as social equity). Relevant parameters need to be further calibrated to improve the accuracy of the model.

CRedit authorship contribution statement

Xiao Fu: Conceptualization, Methodology, Data curation, Visualization, Writing – original draft. **Youqi Wu:** Data curation, Visualization, Writing – review & editing. **Di Huang:** Methodology, Writing – review & editing. **Jianjun Wu:** Methodology, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgement

This work was supported by Humanities and Social Science Fund of Ministry of Education of the People's Republic of China [No. 21YJC790030], National Natural Science Foundation of China [71890972/71890970], “Zhishan” Scholars Programs of Southeast University [No. 2242021R41162], and the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University [Contract No. RCS2021K002].

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