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A joint discrete-continuous model considering budget constraint for the continuous part: application in joint mode and departure time choice modelling

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This article presents an econometric modelling framework for a discrete-continuous choice structure. The proposed model ensures random utility maximisation (RUM) approach for both discrete and continuous decisions and explicit correlation between the two choices. The continuous choice is modelled as RUM under budget limitation. The model is applied for joint mode choice and departure time choice modelling. The empirical application considers continuous time scale for departure time under daily 24 h time budget constraint. The empirical model is estimated by using data collected in Toronto, Canada. The estimated parameters reveal many behavioural details of commuters' mode and departure time choices. The RUM-based discrete-continuous model with the incorporation of time budget constraints for the continuous part is a generic econometric model. This article considers modelling 'mode and departure time' choice bundle for empirical application of the model. However, the model can be applied for any other similar choice bundle situation, for example, in the case of activity-based travel demand and land use modelling.

Keywords: random utility maximisation; discrete-continuous model; mode choice; departure time

1. Introduction

Many of our choices related to transportation, land-use and daily activity engagements are joint discrete-continuous in nature. The importance of developing a joint discrete-continuous choice model in transportation, land-use and environmental impact assessment is now well recognised. Furthermore, several different types of discrete-continuous models have been outlined in the literature (Bhat *et al*. 2006, Munizaga *et al*. 2007, Ahn *et al*. 2008, Fang 2008, Habib *et al*. 2009, Pinjary *et al*. 2009). In the case of any discrete-continuous type choice, it is primarily the juncture between two types of choices (discrete and continuous) that generates endogeneity or self-selection issues. Joint discrete-continuous models can be referred to as a very 'tightly coupled' one if single functions are used to model pairs of discrete and continuous choices. In such cases, the juncture between a pair of discrete and continuous choices is seamlessly (implicitly) addressed and no extra measure is necessary to address the correlation. On the other hand, specifying separate

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functions for a pair of discrete and continuous choices with the recognition of endogeneity represents the 'loosely coupled' discrete-continuous choice situations. In this case, the correlation between the two types of decisions needs to be modelled explicitly. Although various types of discrete-continuous models are available in the literature, 'loosely coupled' discrete continuous model with random utility maximisation (RUM)-based approach for both discrete and continuous parts is not available in the literature. Since the RUM-based approach has proven to be a very powerful one with roots in behavioural realism (McFadden 2000, Wen 2010), development of such a RUM-based discrete-continuous model is important from a conceptual as well as from a practical policy analysis point of view.

The RUM-based econometric approach not only captures the behavioural realism of a specific choice process under consideration, but also assists in the establishment of a meaningful integration of different choice model components in order to simulate larger scenarios. Such models would include tour formation, activity scheduling and land-use–transportation interactions (Miller *et al*. 2005, Habib 2007) etc.

Most of the 'loosely coupled' discrete-continuous models presented in the literature are only RUM-based for the discrete choice part only. Such models are mostly sample selection models for continuous regression variables (see Habib *et al*. (2009) for a detailed literature review). On the other hand, all 'tightly coupled' discrete-continuous models presented in the literature are normally completely RUM-based (see Hanemann (1984) and Bhat (2005, 2008) for detailed descriptions of these model types). In such models, the attributes only enter through one utility index that simultaneously explains both the discrete and continuous choices, although with a clear interpretation. However, there are many situations, where discrete and continuous choices are apparently not very 'tightly coupled' but are endogenously related. For example, mode choice and departure time choice may be better represented by separate utility functions but decisions are highly correlated. This article presents a joint discrete-continuous model that considers RUM assumptions for both discrete and continuous choice components and also considers endogeneity between the two types of decisions. Hence, the proposed model is also a sample selection model, but is grounded in behavioural theory (RUM) for both discrete and continuous choice components. The model is applied for an empirical application in commuter's mode and departure time choice modelling.

The remainder of this article is organised as follows: Section 2 presents the literature review on discrete-continuous econometric models. Section 3 presents the details of econometric model formulations. Section 4 presents a description of the data source for the empirical investigation. Section 5 discusses the empirical model of joint mode and departure time choice. Section 6 presents a demonstration of application of the model. Finally, Section 7 concludes this article by summarising important findings and identifying directions for future research.

2. Literature review

Developing econometric models for discrete-continuous decision structures is not a new research area. However, considering the level of maturity of discrete choice models, discrete-continuous econometric models are still lagging as an area of research (Train 2003). Modelling discrete-continuous choice structures dates back to Heckman's (1979) sample selection model. The primary objective of Heckman-type models is to address the endogeneity issue between two types of decisions: discrete and continuous. Dubin and McFadden (1984) have extended Heckman's sequential two-step binomial model to a sequential multinomial model. In contrast to sequential estimation, Lee (1983) has proposed a full-information maximum likelihood estimation method for the joint discrete-continuous model. Lee's technique for deriving the closed-form likelihood function of the joint discrete-continuous case assists in devising a series of models in the following years. Recently, Munizaga *et al*. (2008) and Habib (2009) explained how Lee's technique can be used to model joint decisions of multinomial discrete choice with two or more continuous regression models. Bhat (1998a, 2001) has also explained how Lee's technique can be used to develop a joint hierarchical discrete choice model with multiple continuous regression model components. Ye and Pendyala (2009) have exploited Lee's technique to develop a discrete-continuous modelling framework where the discrete choice component is a probit-based model.

All the above-mentioned categories of discrete-continuous models use a non-utility maximisation-based regression approach for the continuous choice portion. These models use separate functions for discrete and continuous choice specifications and endogeneity is captured by considering correlation between the random elements. Hence, these types of models may also be referred to as 'loosely coupled' discrete-continuous models. Recently, Munizaga *et al*. (2007) have presented such a 'loosely coupled' model with apparently utility-based specifications for both discrete and continuous parts. In their specification the discrete choice portion uses indirect utility maximisation specifications, which involves a well-known RUM-based multinomial logit formulation. On the other hand, for the continuous portion, they derived an equation for optimum continuous choice using a deterministic Cobb–Douglas utility function. They later added a random term with the utility maximising optimum specification (Jara-Diaz and Guevara 2003). Such an approach of modelling continuous part is not consistent with RUM-based continuous allocation decision and is not significantly different from general non-linear regression approach.

Contrary to a 'loosely coupled' discrete-continuous specification, Hanemann (1984) has specified a completely RUM-based modelling framework with implicit discrete-continuous choice for two goods decisions. A similar trend in the research on discrete-continuous econometric models was encapsulated in the research studies of Chiang (1991), Chintagunta (1993) and Arora *et al*. (1998). Kim *et al*. (2002, 2007) have extended the discrete-continuous model for two goods to multiple-good cases, which is tantamount to continuous resource allocation to multiple alternatives under a budget constraint. Bhat (2005) has furthered Hanemann's approach for a closed-form joint likelihood function in a model referred to as multiple discrete continuous extreme value (MDCEV). Habib and Miller (2009) have presented a similar model, only with a variation in error term specification. The MDCEV model has been further elucidated by Bhat (2008), who has provided a generalised econometric structure for multiple discrete-continuous choice situations. All these models use single utility function to represent a pair of discrete and continuous choices. Hence, these types of models may also be referred to as 'tightly coupled' discrete-continuous model. Although these 'tightly coupled' joint models are referred to as discrete-continuous models, the discrete choice is captured implicitly in continuous choice situations with the possibility of allocating zero versus any positive amount to the continuous choice element. Bhat *et al*. (2006) and Pinjary *et al*. (2009) have further integrated multinomial decisions regarding specific continuous choice outcomes either through error nesting or through conditional probability assumptions. In fact, the 'tightly coupled' discrete-continuous models exploit the Kuhn–Tucker optimality conditions (Kuhn and Tucker 1951) to derive a resource allocation model with corner solution (possibility of zero allocation to any alternative). The capacity to accommodate corner solution is expressed as an implicit discrete choice embedded in RUM-based continuous choice resource allocation decisions.

All 'tightly coupled' discrete continuous models can be fully RUM-based. However, a 'loosely coupled' model with RUM-based multinomial discrete choice and corresponding RUM-based continuous choice is rare in the literature. Whereas, such joint choice situations are very common in many transportation and land-use-related decisions. One specific example to consider is the commuter's mode and departure time choice. Choosing one specific mode from a number of alternative modes may be endogenously related to the corresponding departure time choice (time expenditure on before-work at-home activities). For example, auto drivers may tend to depart late compared to transit users, who need to comply transit schedules. In such situations it is logical that both mode choice and departure time choice are modelled with an RUM-based assumption. Now, if we want to retain the basic continuous nature of time and corresponding departure time choice in a continuous choice model, the joint mode and departure time choice model becomes a discrete-continuous choice model. In such a case, modelling the continuous departure time choice under a RUM-based approach poses a challenge. The allocation of time to before-work at-home activities basically defines the departure time choice along a longitudinal time scale. Given this fact, the total day time budget limitation should also be considered. It is logical to assume that we maximise our utility of choosing a particular departure time given that we have limited time in a day to work and to complete other activities.

In general, the main tradeoff in a RUM-based 'tightly coupled' discrete-continuous model is that it yields models that are consistent with constrained utility maximisation, but that are also tightly 'coupled' due to a strong set of assumptions that might be inconsistent with actual behavioural decision-making. On the other extreme, such decisions might be modelled using the RUM approach and considering conditionally independent. However, in that case the interrelationship between the two choices would be very 'loose'. The contribution of this article is presenting a model that occupies a 'middle ground' of the two extreme options, which is grounded in RUM behavioural theory and also captures the interrelationship between the two decisions. The next section presents the econometric formulation of such a model.

3. Econometric modelling framework

Let us consider that an individual commuter, i, is choosing an alternative mode and corresponding departure time. For mode choice, she is selecting one from a number of available modes. Similarly, for departure time choice she is selecting an amount of time she will spend at home before departing to work. In both choice situations, an inherent assumption is that the commuter maximises her utility. The commuter chooses an alter mode that maximises her mode choice utility and she allocates the portion of the total 24 h time period to the before-work at-home activities that maximises her time expenditure utility. In the case of time expenditure, if we consider midnight-to-midnight as the 24 h daily time budget, the utility optimising the amount of time spent on before-work at-home activities defines the commuting departure time. The commuter trades off in selecting a specific mode and a specific amount of before-work at-home time that maximises the total utility of mode choice and departure time. It is conceivable that time expenditure on before-work at-home activities depends on the mode she is choosing for the commuting. The implication here is an endogeneity/self-selection issue between commuter's mode choice and departure time choice. Hence, the econometric model formulation should consider the utility maximisation approach while at the same time addressing probable endogeneity between these two decisions.

3.1. *Formulation of mode choice and time allocation utility function*

Let us assume that the utility function for the individual commuter (suppressing the subscript, $_i$, for the sake of simplicity) for the j alternative mode choices:

$$U_j = V_j + \varepsilon_j = \beta_j x_j + \varepsilon_j; \quad j = 1, 2, 3, \ldots, M, \tag{1}$$

where V_j is the systematic utility, which is a function of a set of explanatory variables, x_j, and corresponding parameters of weighting factors, β_j. The random variable, ε_j, is the unobserved random error component of the random utility function. Similarly, let us assume that the total utility function of the same individual for time expenditure, t_k, on before-leaving for work and after-leaving for work activities can be expressed as

$$U(t_k) = \sum_{k=1}^{2} \frac{1}{\alpha_k} \left[\exp(\psi_k z_k + \varepsilon_k^{/})\right](t_k^{\alpha_k} - 1). \tag{2}$$

Here $k = 1$ indicates before-work at-home activities and $k = 2$ indicates the rest of the day (the composite activity); z_k indicates a vector of explanatory variables; ψ_k indicates a vector of the coefficient corresponding to z_k; α_k is satiation parameter and $\varepsilon_k^{/}$ is the unobserved random error component of the random utility function. The composite activity encapsulates all the rest of the activities against which the before-work at-home activity time expenditure trade-offs are made. For departure time choice, this is a behaviourally plausible approach. The function of total utility of before-work at-home activities and the composite activity time allocation as specified in Equation (2) is well-known as a generalised variant of translated constant elastic of substitution (CES) direct utility function (Pollak and Wales 1992, Bhat 2005, 2008). This utility function specifies the direct utility derived from the time expenditure on before-work at-home activities and the composite activity. It is clear that for alternative modes, the time expenditure trade-off between before-work at-home activities and the composite activity may be different. If the total time budget is expressed as T, then the time expenditure decision becomes an optimisation problem under the time budget constraints:

$$t_j + t_c = T. \tag{3}$$

Here, for simplicity, the subscript, j, indicates the time expenditure on before-work at-home activities corresponding to chosen mode (j) and c indicates the time left over for the composite activity. To integrate the time budget constraints within the utility function of time allocation, we can use the Lagrangian function:

$$l = \sum_{k=1}^{2} \frac{1}{\alpha} \left[\exp(\psi_k z_k + \varepsilon_k^{/})\right](t_k^{\alpha_k} - 1) - \lambda \left[\sum_{k=1}^{2} t_k - T\right] \tag{4}$$

Here $k = 1, 2,$ refers to j and c, respectively.

In the above Lagrangian function, λ is the Lagrangian multiplier. According to the first-order Kuhn–Tucker optimality condition (Kuhn and Tucker 1951), the generalised expression can be written as

$$\left[\exp(\psi z + \varepsilon^{/})\right](t_k)^{\alpha_k - 1} - \lambda = 0, \quad \text{if } t_k > 0,\ k = 1, 2 \tag{5}$$

$$\left[\exp(\psi z + \varepsilon^{/})\right](t_k)^{\alpha_k - 1} - \lambda < 0, \quad \text{if } t_k = 0,\ k = 1, 2. \tag{6}$$

Equation (5) indicates the situation of spending time at-home before departure for work and Equation (6) indicates the situation of the earliest departure for work.[1] Considering the composite activity as the reference activity ($k = c$) and the circumstance that composite activity time allocation is non-zero (as obvious that individual will leave for work to spend time for the work), we can specify λ as a function of the composite activity function:

$$\lambda = \left[\exp(\psi_c z_c + \varepsilon_c^{/})\right](t_c)^{\alpha_c - 1}. \tag{7}$$

Substituting λ in the Kuhn–Tucker optimality conditions, we can specify that

$$\begin{aligned} &\left[\exp(\psi_j z_j + \varepsilon_j^{/})\right]\left(t_j\right)^{\alpha_j - 1} - \left[\exp(\psi_c z_c + \varepsilon_c^{/})\right](t_c)^{\alpha_c - 1} = 0 \\ &\left[\exp(\psi_j z_j + \varepsilon_j^{/})\right]\left(t_j\right)^{\alpha_j - 1} = \left[\exp(\psi_c z_c + \varepsilon_c^{/})\right](t_c)^{\alpha_c - 1}. \end{aligned}$$

Here the subscript j indicates time expenditure at-home before leaving for work. Taking the logarithms of both sides, we have

$$\begin{aligned} (\psi_j z_j + \varepsilon_j^{/}) + (\alpha_j - 1)\ln\left(t_j\right) &= (\psi_c z_c + \varepsilon_c^{/}) + (\alpha_c - 1)\ln(t_c) \\ \left[\psi_j z_j + (\alpha_j - 1)\ln\left(t_j\right)\right] + \varepsilon_j^{/} &= [\psi_c z_c + (\alpha_c - 1)\ln(t_c)] + \varepsilon_c^{/}. \end{aligned} \tag{8}$$

If we use the following expression:

$$V_k^{/} = [\psi_k z_k + (\alpha_k - 1)\ln(t_k)], \quad k = j, c$$

then Equation (8) can be written as

$$V_j^{/} + \varepsilon_j^{/} = V_c^{/} + \varepsilon_c^{/} \quad \text{for } t_j > 0.$$

Similarly, following Equation (6) we can write that

$$V_j^{/} + \varepsilon_j^{/} < V_c^{/} + \varepsilon_c^{/} \quad \text{for } t_j = 0.$$

In order to derive the probability function of allocating a specific amount of time, t_j, for before-work at-home activities (that defines the departure time), we can further write as follows:

$$\begin{aligned} \varepsilon_j^{/} - \varepsilon_c^{/} &= V_c^{/} - V_j^{/} \quad \text{for } t_j > 0 \\ \varepsilon_j^{/} - \varepsilon_c^{/} &< V_c^{/} - V_j^{/} \quad \text{for } t_j = 0. \end{aligned} \tag{9}$$

3.2. *Assumption of error term distributions and deriving probability distribution functions*

According to RUM theory (Train and McFadden 1978, Ben-Akiva and Lerman 1985), an alternative mode, *j*, will be chosen if the utility of that alternative mode is the maximum of all available alternatives.

$$U_j > \max_{n=1,2,3,\ldots,M,n\neq j} U_n$$

$$V_j > \left\{ \max_{n=1,2,3,\ldots,M,n\neq j} U_n \right\} - \varepsilon_j$$

So,

$$\begin{aligned}\Pr\left(U_j > \max_{n=1,2,3,\ldots,M,n\neq j} U_n\right) &= \Pr\left(V_j > \left\{\max_{n=1,2,3\ldots M,n\neq j} U_n\right\} - \varepsilon_j\right)\\ &= \Pr\big(V_j > (V_n + \varepsilon_n) - \varepsilon_j\big)\\ &= \Pr\big(V_n < V_j + (\varepsilon_j - \varepsilon_n)\big).\end{aligned} \tag{10}$$

In the above formulations, the systematic utility of a chosen alternative is a function of the difference between two random error terms: the error term of the chosen alternative, ε_j, and the error term of the second-best alternative, ε_n. Let us assume that the random variable, ε_j, has the identically and independently distributed (IID) Type I Extreme-Value (Gumbel) distribution with a mean value of 0 and a scale parameter of 1. According to the properties of IID Type I Extreme-Value distribution, the maximum over an IID Extreme-Value random variable is also extreme-value distributed. Furthermore, the difference between two IID Extreme-Value random terms is logistically distributed (Johnson *et al.* 1995, Pendyala and Bhat 2004). As advanced by Ben-Akiva and Lerman (1985) and Train (2003), the implied cumulative distribution of the random error term of the chosen alternative, $F(\varepsilon_A)$, can be written as

$$\begin{aligned}\Pr(\varepsilon_n < V_j - V_n + \varepsilon_j) &= \frac{\exp(V_j)}{\exp(V_j) + \sum_{n\neq j}\exp(V_n)} = \frac{\exp(\beta_j x_j)}{\exp(\beta_j x_j) + \sum_{n\neq j}\exp(\beta_n x_n)}\\ \therefore\quad \Pr(\varepsilon_n - \varepsilon_j < V_j - V_n) &= \frac{\exp(V_j)}{\exp(V_j) + \sum_{n\neq j}\exp(V_n)} = \frac{\exp(\beta_j x_j)}{\exp(\beta_j x_j) + \sum_{n\neq j}\exp(\beta_n x_n)}.\end{aligned} \tag{11}$$

For the time expenditure on before-work at-home activities to correspond to the chosen mode as shown in Equation (9), let us consider that $\varepsilon_k^{/}$ is of a Type I Extreme-Value (Gumbel) distribution with a mean value of 0 and a scale parameter, σ. Since the time expenditure corresponds to the alternative discrete mode choices, the time expenditure is concerned with two alternative options: (a) the chosen before-work at-home activities corresponding to the chosen mode and (b) the composite activity for the remainder of the day). According to RUM theory, considering the time budget constraints and the Kuhn–Tucker optimality condition, the time expenditure, t_j, on before-work at-home activities corresponding to the chosen mode depends on the condition shown in Equation (9). As mentioned above, the difference between two Type I Extreme-Value random terms is logistically distributed, such that the probability function can be expressed as

(Johnson *et al.* 1995)

$$\Pr\left(\varepsilon_j^{/} = (V_c^{/} - V_j^{/}) + \varepsilon_c^{/}\right) = \frac{1}{\sigma}\exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\left[1 + \exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\right]^{-2}$$
$$\Pr\left(\varepsilon_j^{/} - \varepsilon_c^{/} < (V_c^{/} - V_j^{/})\right) = \left[1 + \exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\right]^{-1}. \tag{12}$$

To ensure model identification, the specification of $V_{ji}^{/}$ and $V_{ci}^{/}$ can be further specified as (Bhat 2008)

$$V_j^{/} = \left[\psi_j z_j + (\alpha_j - 1)\ln\left(t_j\right)\right]$$
and
$$V_c^{/} = (\alpha_c - 1)\ln(t_c). \tag{13}$$

Here $V_c^{/}$ expresses the systematic utility of the time expenditure on the composite activity under time budget constraints. According to the transformation of variables theorem, the probability distribution function (PDF) of spending a specific amount of time, t_j, is expressed as (Kottegoda and Rosso 2008)

$$\Pr\left(t = t_j\right) = \left(\frac{\delta\left((V_c^{/} - V_j^{/})\right)}{\delta t_j}\right)\frac{1}{\sigma}\exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\left[1 + \exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\right]^{-2}$$
$$= \left(\frac{1 - \alpha_j}{t_j} + \frac{1 - \alpha_c}{t_c}\right)\frac{1}{\sigma}\exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\left[1 + \exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\right]^{-2},$$

where

$$\left(\frac{\delta\left((V_c^{/} - V_j^{/})\right)}{\delta t_j}\right) = \frac{\delta}{\delta t_j}\left((\alpha_c - 1)\ln(T - t_j) - \psi z - (\alpha_j - 1)\ln\left(t_j\right)\right)$$
$$= \frac{(1 - \alpha_j)}{t_j} + \frac{(1 - \alpha_c)}{t_c} \tag{14}$$

and

$$\Pr\left(\varepsilon_j^{/} - \varepsilon_c^{/} < V_c^{/} - V_j^{/}\right) = \left[1 + \exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\right]^{-1}. \tag{15}$$

3.3. *Joint probability of mode choice and departure time choice*

The joint estimation of RUM-based mode choice and time expenditure on before-work at-home activities requires the assumption that the random error terms of the two models have an unrestricted correlation. One means of specifying the unrestricted correlation between these two random variables is to transform both random variables into an equivalent standard normal variable and specify the joint distribution as an equivalent bivariate normal distribution (Lee 1983). In our case, both the random error components have completely specified the implied marginal distributions. Hence, the marginal distributions can be transformed into corresponding standard normal distributions. This implies that, transforming these marginal distributions into equivalent standard normal variables, it can be shown that (Lee 1983)

$$\begin{aligned} \varepsilon_j^* &= J_1(\varepsilon_j) = \Phi^{-1}[(\varepsilon_n - \varepsilon_j) < (V_j - V_n)] \\ \varepsilon_k^* &= J_2(\varepsilon_j^{/}) = \Phi^{-1}[(\varepsilon_j^{/} - \varepsilon_c^{/}) < (V_c^{/} - V_j^{/})]. \end{aligned} \tag{16}$$

Here, Φ^{-1} indicates an inverse of the cumulative standard normal variable. The transformed variables $J_1(\varepsilon_j)$ and $J_2(\varepsilon_j^{/})$ are transformed standard normal variables of the corresponding random variables, ε_j and $\varepsilon_j^{/}$, respectively. The joint decision process of mode choice and departure time can now be described by considering that the transformed standard normal variables are bivariate normal (BVN) distributed with correlation, ρ_{jt}: BVN$[J_1\varepsilon_{(j)}, J(\varepsilon_j^{/}), \rho_{jt}]$. Hence, the joint probability of observing that any individual (just ignoring the individual identifier, *i*) choosing an alternative mode, *j*, and a corresponding time allocation, t_j, can be expressed as

$$\begin{aligned} &\Pr(Time = t_j \cap Mode\ Type = j) = \Pr(Time = t_j \cap \varepsilon \le J_1(\varepsilon_j)) \\ &= \left(\frac{(1-\alpha_j)}{t_j} + \frac{(1-\alpha_c)}{t_c}\right)\frac{1}{\sigma}\exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right) \\ &\times \left[1 + \exp\left(\frac{-(V_c^{/} - V_j^{/})}{\sigma}\right)\right]^{-2} \Phi\left(\frac{J_1(\varepsilon_j) - \rho_{jt}J_2(\varepsilon_j^{/})]}{\sqrt{1-\rho_{jt}^2}}\right). \end{aligned} \tag{17}$$

Based on the above formulation, the likelihood function, L_i, of an individual observation, *i*, can be written as

$$L_i = \prod_{j=1}^{M} \left(\begin{array}{c} \left(\frac{(1-\alpha_j)}{t_j} + \frac{(1-\alpha_c)}{t_c}\right)\frac{1}{\sigma}\exp\left(\frac{-(V_{ci}^{/} - V_{ji}^{/})}{\sigma}\right)\left[1 + \exp\left(\frac{-(V_{ci}^{/} - V_{ji}^{/})}{\sigma}\right)\right]^{-2} \\ \Phi\left(\frac{J_1(\varepsilon_{ji}) - \rho_{jt}J_2(\varepsilon_{ji}^{/})]}{\sqrt{1-\rho_{jt}^2}}\right) \end{array} \right)^{D_{ji}}, \tag{18}$$

D_{ji} is a binary indicator variable for the chosen mode type.

Now, if we have a sample of observations with sample size, *N*, the joint likelihood function for the sample, *L*, becomes

$$L = \prod_{i=1}^{N} L_i. \tag{19}$$

This likelihood function is a likelihood function of a dynamic RUM discrete-continuous model. It is closed-form and can be estimated using classical maximum likelihood estimation (MLE) algorithms. In this article, the parameter estimates are obtained by maximising the log-likelihood function using a code written in GAUSS. This code applies the BFGS optimisation algorithm (Aptech Systems 2009). The goodness-of-fit measure of the mode is estimated using an adjusted ρ^2:

$$\text{Adjusted } \rho^2 = 1 - \frac{\text{Log likelihood of Full Model} - \text{Number of Parameters}}{\text{Log likelihood of the Constant Only Model}}.$$

Here the constant-only mode implies that the model has only two parameters: the generic baseline utility function constant and the generic constant for the satiation parameter. The null model considers all modes as equally likely. Finally, the number of parameters in the adjusted ρ^2 equation indicates the total number of parameters in the full model above that of the null model.

In the joint probability distribution, the probability of choosing the discrete alternative is modified by the corresponding departure time choice probability. This modification is weighted by the correlation coefficient between random components influencing the two decisions. In our case the departure time is defined by the amount of time allocated to before-work at-home activities. Hence, as per Equation (16), the probability of choosing a particular mode is modified by the probability of not allocating time to the corresponding before-work at-home activities. The interpretation is that the probability of choosing a particular mode is modified by the probability of earliest departure time corresponding to that mode. As per Equation (17), a positive value of the correlation coefficient refers to a negative correlation and vice versa. In the context of application in the mode and departure time choice model presented in this article, a negative correlation coefficient value indicates a positive relationship between earliest departure and the corresponding mode choice. That means a negative correlation coefficient indicates a negative correlation between late departure and the corresponding mode choice or a positive correlation between early departure and the corresponding mode choice. However, it should be noted that here the correlation indicates a correlation between unobserved and random elements only. A higher value of this correlation coefficient is an indication of a poor deterministic specification of the utility functions of both model components. High statistical insignificance of this correlation coefficient will nullify the argument of joint estimation of mode and departure time.

4. Data preparation for empirical analysis

For the purpose of empirical modelling, a subset of the 2001 Transportation Tomorrow Survey (TTS) data collected in Toronto is used in this article. The TTS, which is a multimodal travel survey conducted in the Greater Toronto Area (GTA) every 5 years, was used as the primary dataset. The TTS survey records the detailed travel records of a random 5% sample of households within the GTA for a single day (Data Management Group (DMG) 2009). The survey collected all required individual-level trip data, including departure times, home and work locations, mode choices and household-/individual-level socioeconomic attributes. TTS identifies six major modes of transport: (1) auto driver; (2) auto passenger; (3) local transit; (4) park and ride with local transit; (5) park and ride with intercity transit: 'GO Transit' and (6) walking/biking. Although several modes in this classification have common elements (e.g. common transit element between park and ride-type modes, similar auto element between auto driver and auto passenger modes), because of the pattern of home–work location distribution, all of these modes act as more or less independent options (Day *et al.* 2008). Auto travel times are computed for the complete 24 h day, with one EMME/2 trip assignment conducted for each hour. For transit modes, peak-hour level-of-service variables are used. The mode availability to consider in the mode choice set is defined using general modal feasibility rules. Most of the mode choice set feasibility rules are fixed by applying practical feasibility constraints and thresholds in order to rule out unfeasible and highly unattractive trips. The thresholds and choice rules themselves were developed through previous experience with multinomial logit mode choice models in the GTA. To a large extent, these are based on the rules used in the University of Toronto's conventional four-stage transportation demand modelling system: GTA Model (Miller 2007). The most restrictive feasibility rules were used for the GO transit and local transit auto access modes. In particular, both the auto drive and passenger access modes – 'park and ride' and 'kiss and ride', respectively – are considered as one mode and users are assumed to access their closest (by straight-line distance) feasible station with on-site parking. This consideration is made in an effort to avoid unnecessary complexity in the mode choice model while still maintaining a practically large number of observations for econometric model estimation. Individual tour feasibility rules used in preparing data are as follows (Nick *et al.* 2010):

- Individuals must make a Home–Work and a Work–Home journey in the survey day.
- Tours that include intermediate stops along the Home–Work and/or Work–Home journeys are retained but not modelled explicitly. All mode-specific level-of-service variables are computed by summing their direct Home–Work and Work–Home journey values individually. However, the scheduling constraints posed by such indirect tours are captured indirectly through a variable denoting the total number of observed intermediate stops.
- Both the Home–Work and Work–Home journeys must use same the same mode. The mode choice model requires a 'primary' discrete mode choice for each individual since it is impractical to consider all of the possible multimodal combinations as distinct mode choices. Furthermore, such journeys do not constitute a large proportion of trip makers, and as such they do not constitute an area of focus in this article. The following exceptions are made for tours with intermediate stops where a clear 'primary' mode choice exists:
    - Local transit walk access used in combination with walk for intermediate stops (either on the way to or from work) is expressed as local transit walk access.
    - Park and ride used in combination with drive for intermediate stops (either on the way to or from work) is expressed as park and ride.
- The Home–Work and Work–Home journeys must use the same park and ride station (if this mode is used).

In this article we are interested in commuters' mode choice and home-to-office departure time choice. Hence, after searching out missing values and applying tour and mode feasibility rules, 103,278 individual commuting trip records are available for empirical investigation. The TTS survey classifies respondents into four major occupation groups: general office, manufacturing, professional and retail/service. Table 1 presents composition of samples in terms of occupation groups.

Table 1. Sample composition according to occupation group.

Occupation group	No. of observations	% of sample
General Office/Clerical	13,511	13
Manufacturing/Construction/Trades	22,431	22
Professional/Management/Technical	49,392	48
Retail/Service	17,944	17
Total	103,278	100

Table 2. Variable definitions for empirical model.

Variable	Round trip	Definition
Income	–	Median household income of home zone in thousands of dollars
Housing type	–	1 if individual's dwelling is a house, 0 otherwise
Household size	–	Number of individuals residing in household
Home location Urban density	–	Residents plus jobs per hectare of built area
Work location Urban density	–	Residents plus jobs per hectare of built area
Job status	–	1 if worker is employed full-time, 0 otherwise
Household vehicle	–	Total number of motorised vehicle at home
Age	–	Individual's age
Gender	–	Dummy variable taking 1 if individual is male, 0 otherwise
Free parking	–	Dummy variable 1 if individual has free parking at workplace, 0 otherwise
Distance	No	Straight line centroid-to-centroid distance from household to employment zone in Km
No. of stops	–	Total number of intermediate stops in Home–Work journey
AIVTT	Yes	Round trip auto in-vehicle time in minutes
Auto cost	Yes	AM peak period auto fuel cost in dollars
Parking cost	No	'All Day' average zonal parking cost in dollars, equal to zero if individual has free parking at workplace
TIVTT	Yes	Round trip transit in-vehicle travel time in minutes (peak period)
Transit wait time	Yes	Round trip transit wait time in minutes (peak period)
Transit walk time	Yes	Round trip transit walk time in minutes (peak period)
Transit cost	Yes	Round trip transit fare in dollars (peak period)
Transit fare/distance	Yes	Transit cost divided by distance
AIVTT park and ride	Yes	Round trip park and ride auto access time to closest station in minutes
Auto cost park and ride	Yes	Round trip park and ride auto access fuel cost to closest station in dollars
Total travel time	No	Total home–work local travel time in hours

In order to capture the specific heterogeneity of individual occupation groups, separate models are estimated for each occupational group. For the modelling of joint mode and departure time choice, modal attributes, socio-economic attributes and land-use attributes are considered for variables in the empirical specifications. Table 2 defines the complete set of explanatory variables used in the final specifications of the empirical models. Unfortunately, the data set does not include individual- or household-level income information. In order to obtain a surrogate measure, median zonal income is considered as a variable in the empirical models.

5. Empirical model of mode and departure time choice

The modelling of travellers' departure time choices is not a new topic in the literature. It dates back to Vickery's (1969) equilibrium scheduling theory for continuous time framework and Small's (1987) Ordered generalised extreme value (OGEV) model for discretised but sequentially-correlated time framework. However, researchers also began to concentrate on joint mode choice and departure time choice models. Bhat (1998b) has presented a joint nested structure with mode choice at the higher level and departure time choice at the lower level. In this model, the mode choice is modelled using a multinomial logit model (MNL) while the departure time choice is modelled using the OGEV model. The OGEV model considers sequential correlations between consecutive segments (discretised) of the day. De Jong *et al.* (2003) have presented a joint mode and departure time choice model based on a discrete choice modelling framework. They used an error component logit model to capture the juncture between discrete mode and departure time choices. Tringides and Pendyala (2004) have presented a joint model system for departure time and mode choice. They used a recursive bivariate probit model for joint mode and departure time choice modelling. Vrtic *et al.* (2007) have presented a joint model of mode and departure time choice with route choice, which is essentially an MNL model with nonlinear utility function. Hess *et al.* (2007) have also presented a mode choice and time period model which uses a discrete choice modelling framework. In a more recent paper, Bajwa *et al.* (2008) have presented a joint mode choice and departure time choice model using a cross-nested logit (CNL) approach. Their purpose is primarily to overcome problems of correlated discrete alternative departure times – problems which stem from the discretisation of time.

In all of these modelling exercises for mode and departure time choice, time is discretised into a number of mutually-exclusive alternatives that fit within a specific decision structure assumed by the researchers. A significant problem with discretising departure time is that the neighbouring time period choices are likely to be correlated, especially if the discrete time periods are short. Application of advanced econometric methods may overcome the limitations of correlations among discretised time alternatives, but an appropriate discretisation of time is always a significant challenge (Russo *et al.* 2009). Based on this argument, Habib *et al.* (2009) have presented a joint model of mode choice and departure time considering time as a continuous variable. They have used a discrete-continuous econometric structure, where the departure time choice component is modelled as a hazard model. Habib (2009) has further incorporated an endogenous work duration component within the joint mode and departure time choice model using a hazard model for both departure time and work duration choices. While the hazard model captures the behavioural dynamics of departure time (unlike the discrete choice model), it does not encompass any kind of RUM-based decision framework (Habib *et al.* 2008).

In this article we proposed an RUM-based framework from discrete-continuous decisions and applied it for modelling commuter's mode and departure time choice.

The framework considers RUM assumption separately for both discrete and continuous parts. As mentioned in Section 4, the sample population used is classified according to four occupation groups. Table 3 summarises the estimated model parameters for the four occupation groups. The presented model specifications are reached based on a systematic process of eliminating variables found to be statistically insignificant at the 5% level of significance and intuitive specification based on the results from earlier studies. However, some parameters with higher than 5% levels of significance are still retained in the models for discussion and comparison purposes. In terms of goodness-of-fit, the adjusted ρ^2 value varies from 0.12 to 0.19 depending on the occupation group. Given the complicated nature of the model, the goodness-of-fit seems to be reasonable. However, the goodness-of-fit of the empirical model is also investigated by applying the estimated model in order to predict observed behaviour. This is further discussed in Section 6 (application).

5.1. *Mode choice model component*

Numerous variables are accommodated in the mode choice components of all groups. All of these parameters had the expected sign. The highest number of variables were entered in the mode choice component of the professional occupation group (30 variables), followed by retail/service (29 variables), general office (28 variables) and manufacturing (26 variables). Other than the mode specific variables, the heterogeneity in mode choice behaviour across the population are mostly explained by socio-economic attributes. These attributes account for the difference in the number of variables in the mode choice model components. It becomes clear, then, that the mode choice behaviour of the professional occupation group varies more in terms of socio-economic attributes than other occupation groups. In addition to mode-specific and socio-economic variables, alternative mode-specific constants are integrated in the mode choice model specification. The purpose of this is to capture systematic mode choice utility that cannot be explained by the available variables. The signs of the alternative mode-specific constants should be interpreted carefully as these constants act as error baskets. Moreover, these constants should be considered in the context of the overall specifications.

The employment status of the commuter seems to play a major role in defining commuting mode choice behaviour. This is captured in the model through the full-time versus part-time job status specific dummy variable. It is clear that full-time workers in the professional, general office and manufacturing occupation groups are more likely to use park and ride than the auto drive mode and that they are the least likely commuters to use the walk/bike mode. This is an indication of study area land-use patterns: work and home location distributions, in particular. It captures the fact that full-time workers in all occupation groups who live far away from their respective workplaces view park and ride as the most attractive mode type. For commuters who live closer to the workplace, the auto drive mode is more attractive than local transit or walking/biking.

In order to investigate the effect of tour complexity on commuting mode choice behaviour, the number of stops in the home–work tour is considered as a variable affecting mode choice. Intuitively, this variable has a positive effect on auto drive mode choice utility compared to other modes, and this effect is consistent among all occupation groups. Naturally, the auto dive mode offers the flexibility of making stops at will during home–work tours. As a result, the need to make stops increases the attractiveness of this mode.

Table 3. Joint RUM-based mode and departure time choice model.

Variable	Professional		General office		Retail/service		Manufacturing	
Mode	Param	*t*-Stat	Param	*t*-Stat	Param	*t*-Stat	Param	*t*-Stat
Mode choice model component								
Constant								
Auto driver	−0.765	−3.17	−1.784	−5.04	−4.344	−16.76	−4.306	−14.35
Local transit	1.759	14.49	4.188	9.97	3.396	26.49	0.714	3.63
Local transit park and Ride	–	–	–	–	−4.102	−3.67	−1.005	−4.82
GO park and ride	–	–	–	–	–	–	–	–
Walk/bike	−1.521	−3.31	1.828	13.41	–	–	−1.098	−3.24
Full time employee								
Auto driver	0.382	6.08	0.229	3.02	0.326	5.70	0.149	1.62
Local transit park and ride	0.970	12.72	1.109	3.82	–	–	–	–
GO park and ride	0.819	3.89	0.500	1.42	–	–	1.393	5.53
Walk/bike	–	–	–	–	–	–	−0.263	−1.56
Total number of stops in home–work tour								
Auto drive	0.909	28.32	0.851	16.67	0.816	14.73	0.939	13.56
Total Cost: in vehicle + parking								
Auto driver, auto passenger	−0.048	−14.84	−0.073	−12.25	−0.070	−10.32	−0.053	−6.67
Fare per km distance								
Local transit, local transit park and ride, GO park and ride	−0.287	−7.27	−0.595	−8.72	−0.572	−10.27	−0.481	−7.00
Park land ride auto cost								
Local transit park and ride, GO park and ride	−0.319	−7.66	−0.535	−12.63	−0.370	−6.21	−0.352	−4.25
Auto in-vehicle travel time (minutes)								
Auto driver, auto passenger	−0.037	−33.06	−0.034	−16.42	−0.019	−7.68	−0.014	−5.29
Local transit park and Ride, GO park and ride	−0.014	−3.38	–	–	–	–	–	–
Transit in-vehicle travel time (minutes)								
Local transit, local transit park and ride, GO park and ride	−0.012	−17.77	−0.010	−9.24	−0.012	−9.36	−0.011	−8.25

(continued)

Table 3. Continued.

Variable Mode	Professional Param	Professional t-Stat	General office Param	General office t-Stat	Retail/service Param	Retail/service t-Stat	Manufacturing Param	Manufacturing t-Stat
Waiting time (minutes)								
Local transit, Local transit park and ride, GO park and ride	−0.149	−37.46	−0.134	−24.91	−0.085	−18.07	−0.061	−13.94
Walking time (minutes)								
Local transit, local transit park and ride, GO park and ride	−0.030	−21.14	−0.032	−15.26	−0.027	−13.98	−0.018	−9.84
Home–work place distance in km								
Walk/bike	−0.836	−37.83	−1.067	−19.58	−0.864	−23.89	−0.791	−19.47
More than 1vehicle in household								
Auto driver	0.820	20.50	0.845	13.12	0.612	9.88	0.706	13.33
Local transit	−0.977	−19.18	−0.889	−11.68	−1.125	−16.00	−1.045	−14.56
Local transit park and Ride	–	–	0.281	2.02	0.366	1.74	–	–
GO park and ride	0.304	3.76	–	–	–	–	–	–
Walk/bike	–	–	–	–	−0.669	−6.73	–	–
Logarithm of age in years								
Auto driver	0.778	13.56	0.830	8.68	1.585	21.67	1.407	17.61
Local transit	–	–	−0.189	−1.69	–	–	–	–
Local transit park and Ride	–	–	−0.166	−1.95	0.997	3.35	–	–
GO park and ride	0.229	3.83	0.287	2.75	0.568	9.86	–	–
Walk/bike	0.252	2.15	–	–	0.559	16.22	–	–
Gender: male								
Auto driver	0.664	19.09	1.175	11.67	1.030	16.35	1.511	26.96
Local transit	0.171	4.00	0.585	5.41	0.136	1.92	0.449	6.94
Local transit park and Ride	–	–	0.540	3.05	0.814	4.02	–	–
GO park and ride	–	–	0.329	1.70	–	–	–	–
Walk/bike	–	–	1.237	7.38	0.853	8.68	1.060	8.66
Median zonal income in thousands								
Auto driver	−0.001	−2.78	–	–	–	–	–	–
Local transit	−0.003	−5.35	−0.002	−1.94	−0.002	−2.99	−0.001	−1.67
Walk/bike	−0.005	−5.95	–	–	−0.004	−3.82	−0.005	−2.82

Home zone urban density								
Log of density-auto driver	−0.085	−4.03	–	–	–	–	–	–
Work zone urban density								
Log of density-local transit	0.362	23.61	–	–	–	–	0.362	11.58
Log of density-walk/bike	0.702	26.10	–	–	–	–	0.572	9.97
Local transit	–	–	–	–	0.001	12.52	–	–
Walk/bike	–	–	–	–	0.001	8.84	–	–
Departure time model component								
Constant								
All modes	63.791	218.67	61.999	109.13	32.736	104.98	34.006	123.66
Full time employee								
All modes	−0.701	−19.59	−0.923	−19.21	−1.229	−32.51	−0.834	−15.73
Work duration in hours								
All modes	−0.161	−10.67	−0.120	−3.68	−0.111	−4.43	−0.047	−3.10
Total travel time (home–work) in hours								
Auto driver, auto passenger	−1.624	−46.69	−1.742	−23.47	−1.320	−20.20	−0.854	−15.22
Local transit	−1.409	−21.37	−1.379	−14.17	−0.869	−9.21	−0.983	−11.08
Local transit park and Ride	−0.930	−7.64	−0.405	−0.295	−1.598	−2.67	−0.295	−0.46
GO park and ride	−0.930	−7.64	−0.757	−3.02	0.837	1.66	−0.295	−0.46
Home–work place distance in km								
walk/bike	−0.165	−4.90	−0.307	−3.78	−0.125	−2.18	−0.136	−2.05
Household size								
All modes	0.010	1.62	0.061	5.65	0.064	6.92	0.049	6.41
Gender: male								
All modes	−0.056	−3.59	0.082	2.34	0.171	6.37	−0.170	−6.12
Satiation parameter								
Constant								
Auto driver, auto passenger	−2.400	−586.92	−2.364	−293.90	−1.785	−232.26	−1.847	−286.95
Local transit	−2.390	−575.39	−2.357	−289.82	−1.775	−222.43	−1.837	−271.65
Local transit park and Ride	−2.398	−379.91	−2.336	−220.15	−1.878	−121.96	−1.872	−73.04
GO park and ride	−2.391	−457.58	−2.384	−244.70	−1.894	−95.10	−1.785	−55.01
Walk/bike	−2.394	−568.03	−2.356	−281.75	−1.765	−214.30	−1.836	−244.98
Composite activity								
Work duration in hours	0.009	3.56	0.0269	3.89	0.012	2.78	0.020	6.96
Correlation coefficient								
Auto driver, auto passenger	−0.125	−12.56	−0.166	−9.95	−0.228	−14.60	−0.218	−14.17
Local transit	−0.063	−2.83	−0.021	−0.52	−0.072	−2.17	−0.196	−5.37

(*continued*)

Table 3. Continued.

Variable	Professional		General office		Retail/service		Manufacturing	
Mode	Param	t-Stat	Param	t-Stat	Param	t-Stat	Param	t-Stat
Local transit park and Ride	0.162	1.57	0.855	24.70	−0.921	−30.34	−0.145	−0.71
GO park and ride	0.632	15.51	−0.395	−3.02	−0.778	−11.11	0.869	11.82
Walk/bike	−0.165	−3.93	−0.190	−2.55	−0.008	−0.14	−0.141	−1.93
No. of observations	49392		13511		17944		22431	
Log likelihood value	−307544		−86576		−123806		−149655	
Adjusted ρ^2 Value		0.19		0.13		0.12		0.13
VOTS – Auto users		45.62		27.63		16.26		16.25

The respective costs of auto drive and auto passenger modes are each specified as the total cost of fuel and parking. This variable has a consistent negative sign across the occupation groups, with lower values for general office, retail/service and manufacturing than for the professional occupation group. Similarly, the auto in-vehicle travel time variable has a consistent negative coefficient for all occupation groups. The value of travel time savings (VOTS) for auto driver and auto passengers is calculated using the in-vehicle travel time and travel cost. The resulting VOTS are \$45.62, \$27.63, \$16.26 and \$16.25 for professional, general office, retail/service and manufacturing occupation groups, respectively. It is important to mention that considering an RUM-based departure time choice model affects the mode choice model parameters also. As presented in Habib *et al.* (2009), a similar empirical application was explored, but the departure time choice model was a non-RUM-based hazard model. Comparing the VOTS calculated in these studies, it is clear that the RUM approach improves the VOTS for some occupational groups significantly. For example, the VOTS for the general office occupational group was found to be \$32.32 in Habib *et al.* (2009) as compared to \$27.63 found in this study. Similarly, the VOTS for the manufacturing occupational group was found to be \$12.99 in Habib *et al.* (2009) as compared to \$16.25 found in this fully RUM-based model. There are no standard values available in the literature for comparing the VOTS for different occupational groups in the study area. However, there is one report presented by Transport Canada (2006), where the VOTS is suggested (not based on any specific mode choice model-based calculation though) as around \$30 for all occupational group together for Toronto. Although it is difficult to compare against such aggregate suggestive VOTS, it is understandable that the non-RUM-based departure time model seems to overestimate the VOTS for general office occupational group and underestimates the VOTS for manufacturing occupational group.

Because of the flat fare system of both local and GO transit (the inter-city transit system) in the study area, it is found to be very difficult to capture the effect of transit cost on transit mode choice probability. Considering the total cost as a variable in the model does not help us capture the behaviour since, for short-distance trips, total cost makes the local transit mode too costly. To overcome this problem, the transit cost of local transit- and park and ride-type modes is expressed as a cost per km of travel. Consistently, this variable has a negative coefficient for all occupation groups with similar magnitudes. In the case of park and ride-type modes, auto cost is considered as a variable and it has similar negative values for all occupational groups.

However, auto travel time for park and ride-type modes enters as a variable in the mode choice utility function only with respect to the professional occupation group, with an expected negative coefficient. In the cases of the other occupation groups this variable does not have any significant effect on park and ride-type mode choice utility. This is most likely an indication of the home–work location distribution of these occupation groups within the study area. Unlike professional occupation groups, general office, retail/service and manufacturing groups represent mostly homogenous worker groups. Commuters belonging to these groups who tend to consider park and ride modes as attractive ones usually live relatively close to a specific transit station with a park and ride facility. These commuters probably have very few alternative park and ride options to choose from and, hence, auto travel time to get to the station is always more or less fixed and minimal.

Transit in-vehicle travel time becomes relevant with regard to local transit, local transit park and ride and GO transit park and ride mode choice utility functions with the expected negative sign. Unlike auto in-vehicle travel time, the transit in-vehicle travel time variable has closely similar parameter values across the occupation groups. This illustrates the fact that occupation type has little effect on transit in-vehicle travel time perception. Wait time is an important variable for transit modes. This variable enters into local transit, local transit park and ride and GO transit park and ride mode choice utility functions with the expected negative sign. However, it is clear that commuters from professional and general office occupation groups are more sensitive to wait time than are members of the other two occupation groups. This is consistent with the fact that professional and general office occupation groups earn comparatively higher wages than the other two groups. Similarly, walking time also becomes a significant variable defining the utility of local transit, local transit park and ride and GO transit park and ride modes with the expected negative sign across all occupation groups. For the walk/bike mode, home-to-work distance becomes a significant variable with the expected negative sign for all occupation groups as well.

Among socio-economic variables, automobile ownership, age and gender are found to be significant in defining variations in mode choice utility function. Furthermore, socio-economic variables in the mode choice utility model capture variations in mode choice behaviour across the population. It is clear that a higher level of household automobile ownership primarily increases the utility of the auto driver mode. However, a higher automobile ownership level also increases the likelihood of choosing park and ride-type modes, only to a lesser degree. Intuitively, this variable reduces the utility of local transit and walk/bike modes. The effect of this variable is consistent across all occupation groups.

Age also plays a significant role in defining mode choice utility. It is clear that older commuters from the professional occupation group who live far away from the workplace are more likely to choose the auto driver and GO park & ride modes. Older professionals who live within walking/biking distance of the workplace are more likely to choose walk/bike than local transit or auto passenger modes. For older commuters of the general occupation group, it is clear that the auto driver and GO park and ride modes are the most attractive modes. On the other hand, auto passenger and walking modes are more attractive than local transit and local transit park and ride modes to older commuters from the general office group. Among older commuters from the retail/service occupation group, the auto driver mode is the most attractive while the auto passenger mode is the least likely to be used. Interestingly, age only has a significant effect on auto driving mode choice utility for the manufacturing occupation group. In the case of this group, older commuters are more likely to choose the auto driving mode than are younger commuters. Males across all occupation groups are more likely to choose auto driving, local transit and walk/bike modes than are females. Males from the general office and retail/service occupation groups who live far from the workplace are more likely to choose park and ride-type modes than are females.

In order to capture the spatial variation of mode choice behaviour, some land-use variables are considered in the mode choice utility functions. However, these variables are aggregated at the zonal level and hence are found to be difficult to include in many cases. The possible explanation of failing to accommodate aggregate variable in disaggregate choice model is that the individual heterogeneities are concealed in aggregate variables. It is clear that the commuters from the professional group who live in higher-income areas are more likely to choose the auto passenger and park and ride-type modes. In fact, higher-income professional commuting professionals live primarily in suburban areas with better park and ride facilities, such that park and ride-type modes are the most attractive.

Commuters living in higher-income neighbourhoods whose respective workplaces are close to home and are more likely to choose auto passenger and auto driving modes than transit and walk/bike modes. For all occupation groups, it is clear that commuters living in higher-income neighbourhoods do not find transit and non-motorised modes attractive. In terms of the effects of urban density, it seems that the auto driving mode is most unattractive to the commuting professionals who live in densely populated neighbourhoods. A plausible explanation is that commuters who live in densely populated neighbourhoods have lower-incomes and lower levels of household automobiles. Intuitively, workplace urban density increases the positive utility of local transit and walk/bike modes. In fact, this trend is consistent across all occupation groups. There is no doubt that densely populated urban areas are more conducive to transit service and walking/biking facilities, and this fact is captured by this model.

5.2. *Departure time choice component*

In this article, commuter departure time choice is modelled as an outcome of time expenditure decisions. We take the RUM approach of modelling continuous time expenditures on before-work at-home activities from a 24 h time budget. It should be clarified that the increase in before-work at-home time expenditure entails a late departure to work, and vice versa. Separate models are developed for all occupation groups.

5.2.1. *Baseline utility component*

The baseline utility of before-work at-home time expenditure refers to the baseline preference by commuters to consider commuting departure time. The baseline utility is expressed as an exponential of a linear in the parameter utility function, thus ensuring a positive value. The variable in this utility function increases the baseline preference if the corresponding parameter has a positive sign, and vice versa. In terms of covariates, only level-of-service variable (total travel time) is considered to have alternative mode-specific coefficients in the model. All other socio-economic and constant terms are considered to be generic across the mode specific departure time model components. The constant term is added in order to capture the systematic variations in departure time that cannot be captured by the available variables. It is clear that the constant has high positive coefficients and most of the other variables have negative coefficients. This indicates that in general people want to depart late, but various factors (as represented by socio-economic variables in the model) influence the commuters to start early. However, in terms of magnitude of the constant term, it is very clear that the values are almost double for professional and general office occupation groups, compared to the values of other two occupation groups. This captures the reference departure time for retail/service and manufacturing occupation groups, which are earlier than the reference departure time for professional and general office occupation groups.

In terms of commuter employment status, it seems that the full-time employees are more likely to depart early than are the part-time employees. In terms of magnitude of this coefficient, it seems that retail/service occupation groups is most sensitive to this factor followed by general office, manufacturing and professional occupation groups. Work duration, which is relevant with respect to medium- to long-term decisions, is considered as a variable in baseline utility function. A worker's work duration creates time pressure and thus significantly influences the home–work departure time decision, which is captured in the models. For all occupation groups it is clear that commuters who work longer hours are more likely to depart early in the morning, which is intuitive. However, in terms of the magnitude of the effects of work duration, it is clear that commuters of manufacturing occupation group are less sensitive than all other occupation groups do. The total travel time required in the home–work trip is considered as a variable at play in capturing the influence of transportation system performance in the departure time choice model. As expected, this variable has a negative sign, indicating that a longer travel time influences earlier departure, and vice versa. Among members of the professional and general office occupation groups, auto users are the most sensitive to travel time followed by local transit users. Among members of the retail/service occupation group, local transit park and ride users are the most sensitive to travel time. Among members of the manufacturing occupation group, auto users and local transit users are the most sensitive to travel time. Home–work trip distance enters into the baseline utility function of walk/biker users with the expected negative sign. This indicates that a longer distance leads to an early departure, and vice versa.

It has been found to be problematic to consider zonal aggregate land-use variables in the utility-maximising departure time choice model component. This is most likely due to the fact that departure time choice is less influenced by aggregate neighbourhood attributes, as we have seen in the case of mode choice. Among the socio-economic variables, only household size and gender enter into the baseline utility function of the departure time choice model component. It is clear that having a larger household influences late departure time for all commuters. Also, males are more likely to depart early than are females.

5.2.2. *Satiation parameter*

As per the model specification, the satiation parameter, α, must be less than 1. In order to satisfy these restrictions and in accordance with Bhat (2008), the following specifications are considered: $\alpha = 1 - \exp(-\tau z)$. Here τ is a vector of parameters corresponding to variable z. The estimated τ parameters are presented in Table 3 as the satiation parameters. In the case of mode specific constant values, all of the estimated values are negative across all occupation groups. A negative value of this parameter indicates quick satiation, meaning that there is no basic impetus to delay departure to work other than the various factors influencing time expenditure utility. On the other hand, for the time-of-day variable in composite activity satiation, the coefficient is positive. This captures the fact that with increasing time-of-day (delay in departure), psychological time pressure increases significantly.

5.3. *Correlation coefficients*

The justification for joint estimation of mode and departure choice is substantiated by the fact that most of the correlation coefficients are highly significant. Estimated correlation coefficients are consistently negative for auto driver, auto passenger, local transit and walk/bike modes across the occupation groups. This indicates that the unobserved factors influencing the choices of these modes also influence corresponding early departure time choice. In the case of both park and ride-type modes for the professional occupation group, local transit park and ride mode for the general office occupation group and GO park and ride mode for the manufacturing occupation group, the correlation coefficients are positive. It indicates that the unobserved factors influencing park and ride-type mode choice for the professional occupation groups, local transit park and ride mode for the general office occupation group and GO park and ride mode for the manufacturing occupation group also influences corresponding late departure time choice. On the contrary, both park and ride-type modes for the retail/services occupation group, local transit park and ride mode for the manufacturing occupation group and GO park and ride mode for the general office occupation group, the correlation coefficients are negative. However, it should be noted that these correlation coefficients must be interpreted very carefully. The values and signs of this parameter may change with different specifications of the two choice model components. Hence, the estimated parameter sign should be considered in the context of the presented specification only. In terms of the absolute values of the estimated correlation coefficients, the values vary from 0.02 to 0.92. The lower values are indications of better specification of individual mode choice and departure time choice model components and vice versa. Correlation coefficient values are higher for the park and ride-type modes. This indicates the lack of enough information to model park and ride-type mode and departure time choice model in the data set.

However, accommodation of correlations among unobserved factors influencing mode choice and departure time choice decisions is not only to improve model fit or having good '*t*'-statistics of the estimated parameters but also to recognise the fact that the survey data used to model such decisions may not be enough to capture all influential factors affecting mode and departure time choices, and the relationship between such unobserved factors affecting these two choice is critical to capture the endogeneity effects. Neglecting endogeneity between mode and departure time choice decisions may have serious consequences in terms of over or under estimation of the influences of observed factors (variables). For example, neglecting endogeneity between mode and departure time choice may cause larger constant terms in the mode choice model as well as the overestimation of VOTS (readers can compare the mode choice model component of this article with the mode choice model presented in Nick *et al.* (2010), which uses the same data set).

6. Demonstration of model application

In order to demonstrate the potential of this model in explaining travel behaviour, it has been applied in order to replicate the observed behaviour. It is clear that the proposed model can capture the observed behaviour very well. Figure 1 presents the graphical comparisons of observed versus model predicted mode choice for different occupation groups. The auto driver mode is slightly under-estimated for the professional occupation group and slightly over-estimated for the retail/service and manufacturing occupation groups. The auto passenger mode is under-estimated in all cases because of an insufficient number of variables available to characterise the model. Local transit is slightly over-estimated for all occupation groups except manufacturing. Park and ride-type modes are chosen by a very small number of commuters in the data set, but the mode predictions for this mode are very close to the observed values for all occupation groups. Essentially, the same can be said of the walk/bike mode.

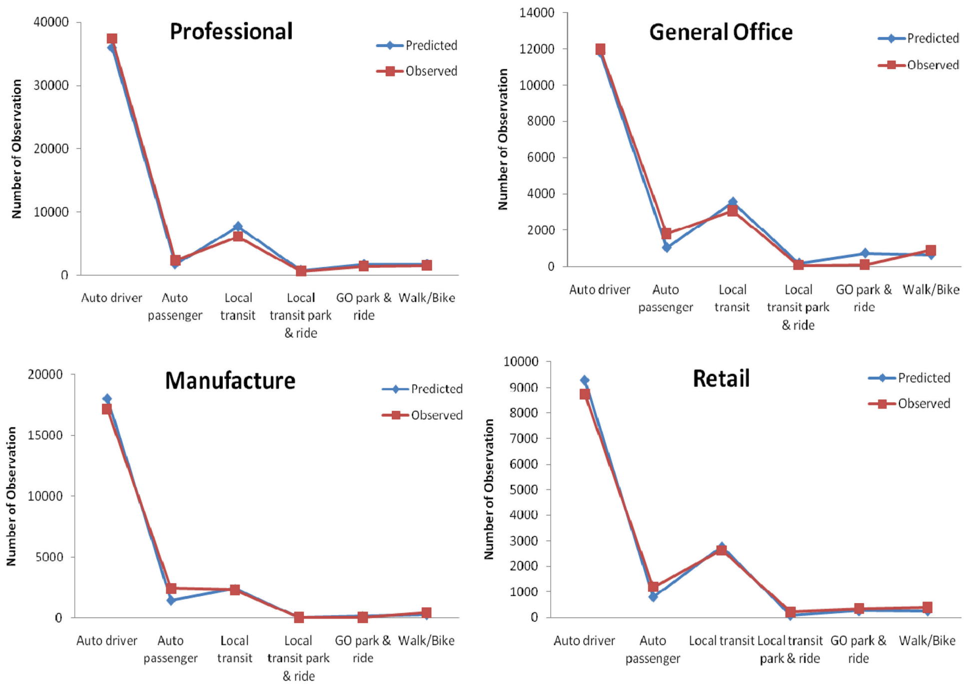

Figure 1. Validation of mode choice model component.

Similarly, the empirical model is also validated in order to compare departure time choices for all mode users corresponding to individual occupation groups. Figure 2 presents a sample comparison of observed versus predicted departure time choice distribution for profession occupation group (which is the largest group in the data set) and for three major modes (auto driver, auto passenger and local transit). In general, the model's prediction of departure time choice shows a similar degree of fit to the observed data for all occupation groups. The models clearly capture departure time choice trends across the mode types and occupation groups. In particular, early departure time trends of two park and ride-type mode users are well predicted by the model. For non-park and ride-type modes, early departures are better predicted than are late departures. The models capture departure time choice trends of transit users better than that of auto users in general. Possible explanations are the strict departure time schedule and the relatively narrow window of time available for transit users compared to auto users. Each of these factors leads to utility-maximising behaviour.

A full application of the joint models requires the use of an iterative procedure. Iteration is necessary due to the use of travel time as a variable in both the mode choice and the departure time choice model components. Habib *et al*. (2009) have presented an iterative algorithm for application of such joint models. However, considering level-of-service variables, e.g. travel time by mode, is crucial to capture the dynamics of travel behaviour in response to transportation system performance. This is also a critical issue for any dynamic activity-based travel demand modelling (Roorda *et al*. 2008). For the sake of brevity, a full forecasting application of the joint mode choice and departure time for the

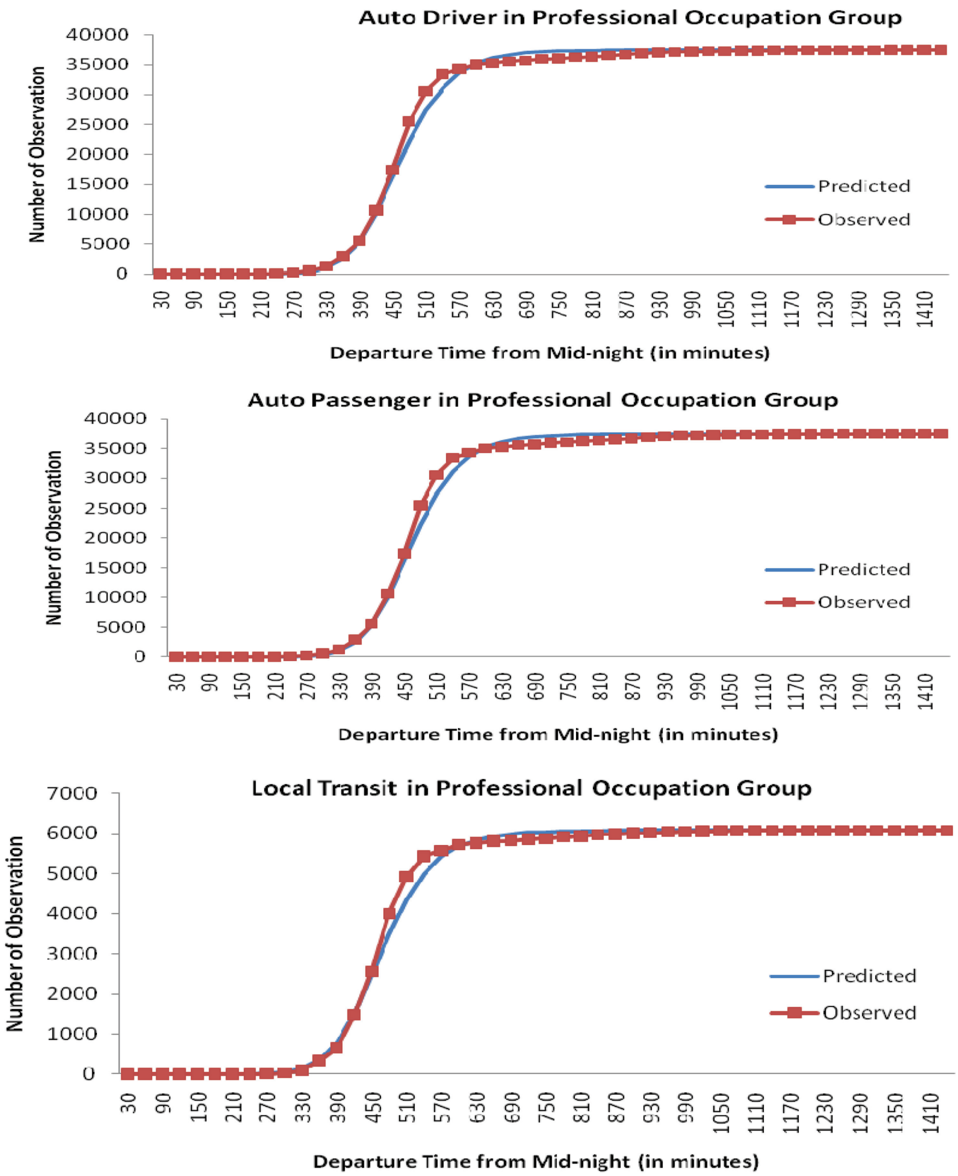

Figure 2. Validation of departure time choice component for three major modes of professional group.

purpose of analysing policies or forecasting future demand is not presented herein. However, the model is applied for alternative scenario analyses. Alternative scenarios for changing travel time and travel costs are evaluated. For example, for the professional occupation group (the largest occupation groups in the study area), increasing traffic congestion and resulting increases in auto in-vehicle travel time make park and ride-type modes very attractive. This is probably because the majority of commuters in this group live in suburban communities with long commuting distances. However, changes in travel time do not lead to drastic changes in departure time distributions as in the case of modal shares. Commuters who still choose the auto driver mode even as auto in-vehicle travel time increases tend to depart early to overcome the effects of increasing travel time. This stems from the fact that departure time is strongly influenced by planned work start time and flexibility of work start time. It seems that as auto in-vehicle time increases – *ceteris paribus* – a change of travel mode becomes preferable to a change in departure time.

7. Conclusions and future research

This study contributes to the travel demand modelling literature by presenting a joint discrete continuous using RUM-based assumption. It then applies the model for the purpose of modelling commuter mode and departure time choice. The model demonstrates the method of modelling temporal decisions under continuous time and RUM assumptions. The commuter's time pressure in deciding departure time jointly with mode choice is recognised by considering the time budget pressure embedded in the econometric modelling framework. For empirical application, the model is estimated using a data set collected in Toronto, Canada. The joint model is estimated on a sample of 103,278 commuters from four general occupation groups. In order to capture variations across the occupation groups, separate models are estimated for each group. The joint model is designed to capture the correlation between unobserved factors influencing mode choice and departure time choice. These correlations are found to be statistically significant for all occupation groups. The empirical models are validated in order to replicate observed behaviour and, in turn, prove that the model provides a reasonable goodness-of-fit. The joint model proves its merit by confirming the expected sign and relative coefficient values of all estimated parameters.

The model results offer several insights regarding commuters' mode and departure time behaviour in the study area. It replicates the fact that full-time employees are less likely to use non-motorised modes and more likely to depart early in the morning. Commuters' activity scheduling complexity in terms of increasing number of stops along the home–work tour increases the attractiveness of auto modes. A higher level of household auto ownership increases auto driver mode choice. Higher-income commuters usually live in the outskirts of the city and so park & ride-type modes are more attractive to them. Increasing work zone urban density increases the attractiveness of transit usage. Furthermore, empirical models can capture the effects of time pressure from work duration and travel time requirements on departure time choices. A longer duration of work and longer travel time requirements influence early departure, which is intuitive. The models also capture the effects of travel time on departure time decision. The models are validated against the observed data set and the results show a high degree of fit. The models clearly capture the trends of mode choice and departure time behaviour. The empirical models also demonstrate their capacity in predicted alternative scenarios with respect to changes in level-of-service variables.

The research presented in this article can be built upon in several ways. A few possible directions for the next stage of the research can be (1) considering endogenous work duration modelling within the joint mode and departure time choice model; (2) extending the analysis to include heterogeneous behaviour; (3) applying the model to richer data sets with attitudinal variables that may further enhance our understanding of the relationship between mode choice and departure time choice modelling and (4) incorporating work and home location choice model components, etc. Furthermore, the econometric modelling framework is generalised sufficiently to apply any other joint discrete and continuous choice situation under budget constraints. Examples of this might be dynamic activity scheduling, household vehicle and task allocation, etc. One interesting issue is left unresolved in this empirical application of the model, which is the overall 24 h time pressure effect versus smaller scale time pressure effect in the departure time choice model. In order to capture both, we derived departure time choice model that recognises 24 h time budget constraint as well as consider 'number of stops in the home–work–home tour' as a variable in the model. However, we agree that jointly developing model for mode choice, departure time and number of stop choice would be interesting to explore for future investigation.

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Note

1. For the empirical application we used the TTS dataset, where the day starts at 4 am. So, for the empirical application of the model presented in this article, the earliest departure time refers to not allocating time at-home after 4 am.

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