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Abstract

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A joint between-day and within-day activity based travel demand with forward looking individuals

Oskar Blom Västberg and Anders Karlström

KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden

Including day-to-day planning to account for systematic variability in activity participation has the potential to further improve travel demand models. This paper introduce a dynamic discrete choice model of day-to-day and within-day planning in a joint framework. No model up to date jointly treats within-day and day-to-day planning with individuals that take future days into account.

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1. INTRODUCTION

During the last decades, activity based models has been the focus of travel demand research as the demand for travelling is considered derived from the demand of activity participation. A number of operational activity based travel demand models has now existed for some time, e.g., DaySim (Bowman and Ben-Akiva, 2001a), CEMDAP (Bhat et al., 2004b), Albatross Arentze and Timmermans (2004) and MATSim (Balmer et al., 2008). A common trait for these models is that they consider the choice of activities, destinations, departure times and modes for all trips during a single day. However, when making plans for what to do on a specific day, individuals are likely considering its impact on future days. Likewise, what they have done in the past likely influences what they need to do on that specific day. For example, although working is a mandatory activity for most people, flexible working schedules makes it possible to work more on certain days and less on others. Some activities are also more or less mandatory

Corresponding author: oskar.vastberg@abe.kth.se

to conduct with certain frequency, such as grocery shopping. The day-to-day variability in individuals' travel behaviour has been studied for a long time, both consisting of research quantifying such variability (see, e.g., Hanson and Huff, 1982; Huff and Hanson, 1986; Jones and Clarke, 1988; Pas, 1988; Pas and Koppelman, 1986) and research analysing whether variability is systematic (see, e.g., Hanson and Huff, 1988; Hirsh et al., 1986). Multiple day data has later been used to analyse the systematic variability of behavioural patterns using hazard models (Bhat et al., 2004a, 2005; Kim and Park, 1997), finding that especially for shopping there is an increasing probability for additional activity episodes with time. This means that the probability that an individual will perform certain activities on a specific day is dependent on the last time those activities were performed. Since day-to-day variability has been observed to have a systematic component, it is likely that travel demand models could benefit from extending the unit of analysis to a longer time period than a single day.

A multitude of models that connect choices of activity schedules for longer time frames (usually a week or more) exists in the literature. An early example is Hirsh et al. (1986) that developed a utility based model for weekly activity programs. The choice is modelled as a sequence of logit choices, where the utility of a day includes the expectation of future days in the week and the utility of a specific day is dependent on the activities executed earlier in the week.

Gärling et al. (1999) argues that time pressure arising from having too many activities planned in too short a time window increases the likelihood for different planning horizons for different activities. They specifically argue that activities might be subject to long-term or short-term planning. Activities which must be performed with certain frequency involves a long-term commitment. The short-term planning involves adjusting the daily schedule to alleviate time pressure arising from too many planned activities. They develop a model describing this process and argue that one important factor in long-term planning is expected future time pressure.

Bayarma et al. (2007) use a Markov chain to analyse the day-to-day variation in activity patterns. They divide the possible ways in which a day can be performed in five categories and estimate the probability that one type of day is followed by another type of day.

Another week long model is presented in Habib and Miller (2008), who uses Kuhn-Tucker optimality conditions to determine optimal activity schedules for all days in a week. Day-to-day dynamics is included by allowing activity episodes of the previous day to influence the utility function that determine the daily choices.

Another travel demand model which allows for the planning of seven consecutive days is presented in Kuhnimhof and Gringmuth (2009). They first assign each individual with an agenda, a set of activities that they wish to perform during the week, and then try to schedule each day to include as many of these activities as possible. Possible agendas are extracted from data, and individuals in the model are assigned agendas based on their similarity with the observed individuals. The within-day scheduling is performed using a greedy algorithm and performing a local search for the optimal way to schedule each day considering time-space constraints.

Arentze and Timmermans (2009) introduce a need-based model for activity scheduling on household level and later show how it can be estimated on single day data Arentze et al. (2011). The utility of performing an activity is said to be dependent on the need an individual has for the activity, which is dependent on the number of days since an activity was last performed and a random perturbation which is added each day. They then assume that the activity is performed on the first day when the utility of performing the activity is greater than a certain threshold. It does not take interactions of activities into account, as the threshold for each activity is treated separately. Individuals does not consider the future consequences of their actions, but Arentze and Timmermans (2009) argues that by updating the threshold during simulation, individuals will act close to optimal in the long run.

A combined day-to-day and within-day travel demand model was presented in Cirillo and Axhausen (2010). They use a mixed logit model to capture correlation in preferences between days. The utility of an activity is dependent on the number of days since it was last performed. Individuals are assumed not to be forward looking, so the choice only depends on the current state and the direct utility of the alternatives. The within-day model is a tour-based nested logit model.

Another combined day-to-day and within-day travel demand model was presented in Märki et al. (2014). They assume that individuals have behavioural targets related to their preferences and goals. Deviating from their behavioural targets cause discomfort and activities are conducted to avoid this discomfort. Since they have data available where individual answer question regarding such targets, they do not need any latent states to represent these variables. The utility of a specific plan within the day is then influenced by the discomfort, but individuals does not consider future days when making decisions.

We will here present a model that extends a within-day travel demand model to include day-to-day dynamics. The extension from within day to day-to-day planning is achieved using an infinite horizon dynamic discrete choice model. The idea of using a dynamic discrete choice model to jointly model day-to-day and within-day travelling was initially proposed in Karlström (2005). A small scale version of such a model was later used in Jonsson et al. (2014) to analyse the effect of time-space constraint on accessibility. An estimated single-day version of the same model was recently provided in Västberg et al. (2018), but the joint day-to-day and within-day model has never before been estimated, nor implemented in a larger scale. Resembling many of the above discussed models, a state is used to represent the relevant history that impacts the utility of performing an activity on a specific day. The state could for example include the number of days since an activity of a specific type was last performed. What is more, individuals are assumed to take the future into account when making decisions and thus consider the benefits of doing an activity today relative to postponing it the future. The proposed model has the following features:

- The utility of an activity is dependent on the history.
- Individuals are forward looking and explicitly considers the future impacts of their actions.
- The planning horizon is infinite.
- An estimatable discount factor determines the extent to which individuals are forward looking. As special cases the model can reproduce the myopic behaviour assumed in some of the models presented above but also individuals that act as if they maximize the utility of an average day.
- It extends a within-day travel demand model to multiple days, so that individuals within the day act as if they take infinite discounted future consequences of their actions into account. It thus consistently treats within day and day-to-day planning in a united framework.
- It is based on random utility theory and can thus be used for cost appraisal.

2. METHOD

The behavioural assumption underlying dynamic discrete choice models is that people act as if they maximize the expected future discounted utility of their actions. This implies seeking a decision rule $\mu(x_t, \epsilon_t)$ that, given the state consisting of x_t and ϵ_t, maximize the expected future discounted utility. The random state variable ϵ_t is only known in the current time period t and a decision d will lead to a transition in the observable state x to y with a probability $p_{xy}(d)$. They thus maximize the value function defined as:

$$V_\pi(x_0, \epsilon_0) = \lim_{N\to\infty} E_{x_t,\epsilon_t}\left[\sum_{t=0}^{N} \beta^t U(\mu_t(x_t, \epsilon_t), x_t, \epsilon_t) \middle| \pi, x_0, \epsilon_0\right].$$

If the utility is additively separable into u and ϵ this is known from Rust (1987) to be solvable recursively using Bellman's equation:

$$V(x, \epsilon) = \max_{d\in C(x)} \left(u(d, x) + \epsilon(d, x) + \beta \sum_{y\in S} p_{xy}(d)V(y) \right)$$

where $V(y)$ is the *expected* value function with respect to ϵ. The expected value function is therefore given by:

$$\begin{aligned} V(x) &= E_\epsilon\left[V(x,\epsilon)\right] \\ &= \int \max_{d\in C(i)} \left(u(d,x) + \epsilon(d,x) + \beta \sum_{y\in S} p_{xy}(d)V(y) \right) q(\,\mathrm{d}\epsilon|x). \end{aligned}$$

If ϵ has Gumbel distribution with mean zero, this becomes a log-sum:

$$V(x) = \log \sum_{d\in C(x)} e^{u(d,x)+\beta \sum\limits_{y\in S} p_{xy}(a)V(y)} \tag{2.1}$$

We will next show how the above framework can be used to construct a model both incorporating within-day and day-to-day planning of travel patterns. We start the development by describing how the within-day described in Västberg et al. (2018) can be extended to include day-to-day planning. This involves 1) having the utility of actions within a day being dependent of actions on previous days, and 2) taking into account that actions in a day will influence the utility of actions in future days. This therefore requires both extending the state space and changing the utility functions, as compared to Västberg et al. (2018).

Although we here derive the between-day model as an extension of a specific within-day model, it is worth noting that the within-day model could be exchanged for any model capable of producing the expected utility of a day conditional on that day including a set of activities. This could especially be the case for any MEV based demand model, such as the once presented in Bowman and Ben-Akiva (2001b) and Cirillo and Axhausen (2010).

2.1. *Within day model*

We will here give a brief description of the model developed in Västberg et al. (2018) and show how it can be extended to include day-to-day dynamics. For a full explanation of the within-day model, readers are referred to Västberg et al. (2018). Individuals are assumed to make a sequence of choices throughout the day, starting at home in the morning and ending at home in the evening, as illustrated in Figure 1. This is modelled as a sequence of decisions d between states x. Defining the model requires defining: 1) the state space, 2) available decisions conditional on the states, 3) the transition to a new state from a a specific decision, and 4) the one-stage utility associated with a state-decision pair.

State space

A state x consists of:

Time $t \in [5\,\text{am}, 11\,\text{pm}]$	Continuous variable for time of day. A day starts at 5 am and ends at 11 pm
Location $l \in [0, 1239]$	Curnt location. One of 1240 zones in the region of Stockholm.
Activity a	Current activity, one out of {social, recreational, shop, home, work escorting children}. The activity must be included in the state since the individual can choose to continue with the same activity for yet another time-period, but have to travel (possibly within the zone) to change activity.
Errand indicator $e \in [0, N_e]$	A state keeping track of the number of finished mandatory activities. The number of mandatory activities varies from 0 to N_e depending on the individual. Mandatory activities are work, picking up children or dropping of children.
Main mode $m \in \{car, bike, other\}$	Mode used on first trip of a tour. Used to control available modes.

To allow for multiple day planning, we will add new states to keep track off the number of days since an activity was last performed as well as the number of times an activity has been performed the current day. It will also be necessary to keep track of the day-of-week.

$n_a \in [0, N_a]$:	Number of times activity a has been performed during the current day, where N_a is the maximum number of times that are recorded for activity a.
$\tau_a \in [0, N_{\tau_a}]$:	Number of days since activity a was last performed, where N_{τ_a} is the length of the memory, so that if $\tau_a = N_{\tau_a}$ it was N_{τ_a} days or more since the activity a was last performed. Allows the modelling of an increased need for activity participation.
dow:	Current day-of-week. Allows the utility of different activities to vary over the week. Travel times and working schedules can also be day-of-week dependent.

Available actions

In each time-step t an individual chooses between: 1) continuing with the same activity for another time step; or 2) travel with a mode $m \in M(x)$ to a location

$l \in L(x)$ and start a new activity $a \in A(x)$. Here $M(x)$, $L(x)$ and $A(x)$ are used to denote the set of feasible modes, destinations and activities conditional on the current state x. The set of feasible decisions in a state x is denoted $C(x)$. If a new trips is performed the choice sets are defined by:

$L(x)$: All locations are possible for social, recreational, other and shop activities, but only an individual specific location is available for home, work and child errands.

$A(x)$: Social, recreational, other, shop and home can potentially be started at any time. The possibility of work and child errands depend on the errand indicator e, which defines the next mandatory activity to be performed in the following way: If an individual should 1) drop of a child in the morning, 2) go to work and 3) pick up a child after work, $N_e = 3$ and $E = 0$ means that the individual can drop of the child (but not got to work or pick up the child), $e = 1$ that they can go to work and $e = 2$ that they can pick up the child. When $e = 3$ there are no more mandatory activities to perform. These activities are further bound by time constraints and can only be started within time windows that are individual specific.

$M(x)$: The available modes depends on the current main mode (car or not) and activity (home or not) in the following way:

$$M(x) = \begin{cases} \{\text{Car}, \text{Bike}, \text{Walk}, \text{PT}\} & \text{if } x_a = \text{Home} \\ \{\text{Bike}, \text{Walk}, \text{PT}\} & \text{if } x_a \neq \text{Home}, x_m \neq \text{Car} \\ \{\text{Car}\} & \text{if } x_a \neq \text{Home}, x_m = \text{Car} \end{cases}.$$

This means that all modes are always available at home. It also guarantees that car is used for all trips on a tour. Car ownership determines the availability of car and is given exogenously.

Conditional state transition within day

The new state x_{k+1} reached when making decisions d_k in state x_k is easily defined with the actions and states above. Transitions in the state variables $N_{\tau,a}$ and dow occurs between days, and will be discussed later.

Time: $t_{k+1} = (t_k + \text{travel time} + \text{activity time})$ where travel time and activity time are dependent on origin, destination, time-of-day and mode. Activity time varies with the activity and the decision.

Location: l_{k+1} is the destination of the decision

Activity: a_{k+1} is the activity of the decision

Errand indicator: $e_{k+1} = e_k + 1$ if the decision involves the next mandatory activity, and E_k else.

Main mode: $m_{k+1} = m_k$ if neither origin nor destination is home. At home there are no main mode. If the origin of a trip is home, the main mode will be car if car is used and bike if bike is used.

$n_{a,k+1}$: Increases with one if activity a is started:

$$n_{a,k+1} = \begin{cases} \min\left(n_{a,k} + 1, N_{\tau_a}\right) & \text{if start activity } a \\ n_{a,k} & \text{else} \end{cases}$$

Distribution of error component

The most common assumption in DDCM's is that the error component is i.i.d. Gumble distributed, so that the choice in each stage is given by a logit model and the value function is given from (2.1). We will make a slightly different assumption here that will produce a Nested-Logit (NL) model over day-paths.

We assume that two random parts enters the within-day utility:

$$\begin{aligned} \epsilon &\sim \text{Gumble (with zero mean)} \\ \nu &\sim \mathcal{V}(\lambda) \end{aligned}$$

where $\mathcal{V}(\lambda)$ is such that $\nu + \lambda \cdot \epsilon$ is a random variable with a marginal distribution identical to that of ϵ, i.e., a Gumble with zero mean. The existence, uniqueness and functional form of the p.d.f for such a distribution $\mathcal{V}(\lambda)$ with $0 \leq \lambda \leq 1$ is given in Cardell (1997).

The random term ν is assumed to be known by the individual in the beginning of each day and is dependent on the initial and final state of each day. However, as we defined the state in the previous section, each feasible state in the end of the day can only be reached from a single feasible state in the beginning of the day, so the random term will be uniquely defined by the final state. We will therefore use $\nu(y)$ to denote the random state variable associated with a final state y. The random part $\epsilon(a)$ will on the other hand be observed by the individual for all actions $a \in C(x)$ only when reaching a specific state x. Both ν and ϵ_a are assumed to be i.i.d., but the inclusion of ν into all day-paths between a final and initial states creates a correlation structure.

It is worth noting that Mai et al. (2015) and Mai (2016) previously showed how the Recursive Logit framework can allow for Nested Logit and MEV formulations, but the correlation is then assumed to occur on link basis in specific choice situations rather than over paths sharing common attributes that the error component ν may allow here.

One-stage utility of state-action pair

When making a decision $d = (l', a', m')$ in a state $x = (t, l, a, e, m, n_a, n_\tau, \text{dow})$, the observable instantaneous utility $u_n(d, x; \theta_u)$ obtained by an individual n is the sum of the (dis)utility of travelling and the utility of activity participation. The full set of parameters is given by $\theta = (\theta_u, \beta, \lambda)$ including both parameters related to the instantaneous utility (θ_u), the discount factor β (introduced later) and the scale parameter λ. The utility of travelling is dependent on the origin, destination, main mode, mode of transport and day-of-week, and can be written as: $u_{n,m}(t, l, l', m, m', \text{dow}; \theta)$. The utility of activity participation is dependent on the destination (the location where the activity is being performed), the activity, the time of day, the number of times the activity has been performed as well as the last time the activity was performed: $u_{n,a}(t, a', n_a, n_\tau, l'; \theta)$. For a detailed description of the utility functions of the within day model, see Västberg et al. (2018).

Besides the observable part of the one-stage utility, an individual considers the random utility ϵ. As discussed earlier, this random utility enters the one-stage utility additively so that $v_n(d, x; \theta, \epsilon) = u_n(d, x; \theta) + \lambda \cdot \epsilon(d)$.

Solving the within-day value function

All randomness are assumed to be captured in the random terms ϵ and ν so there are no transition probabilities q as in (2.1). We will use $V_n(x; \theta, \nu, \epsilon)$ to denote the value function in a state x when ν and ϵ are both known. When taking an action within a day, the value of ϵ is unknown for any future stages, but ν is known throughout the day. The value function for a specific ν and ϵ therefore becomes:

$$V_n(x; \theta, \nu, \epsilon) = \max_{d \in \mathbb{C}_n(x)} u_n(d, x; \theta_u) + \lambda \cdot \epsilon(d) + E\big[V_n(f(d, x); \theta, \nu, \epsilon)\big|\nu\big]$$

and to simplify notations we will in the future denote the expected value function with respect to ϵ as $V_n(x; \theta, \nu) = E\left[V_n(x; \theta, \nu, \epsilon)|\nu\right]$. The expected value function in a state x is for individual n then given by:

$$V_n(x; \theta, \nu) = \lambda \cdot \log \sum_{d \in \mathbb{C}_n(x)} e^{\frac{1}{\lambda} \cdot (u_n(d, x; \theta_u) + V_n(f(d, x); \theta, \nu))} \tag{2.2}$$

where $f(d, x)$ is the state reached when making decision d in state x. The probability of a decision d is given by:

$$P_n(d|x; \theta, \nu) = \frac{e^{\frac{1}{\lambda} \cdot (u_n(d, x; \theta_u) + V_n(f(d, x); \theta, \nu))}}{\sum_{d' \in \mathbb{C}_n(x)} e^{\frac{1}{\lambda} \cdot (u_n(d', x; \theta_u) + V_n(f(d', x); \theta, \nu))}}. \tag{2.3}$$

The value function can be calculated in all states x by using backward induction, i.e., starting in the end of the day where the value functions is given by $J_n^T(y;\theta)+\nu(y)$ and iterating backwards until the start of the day. The probability of a day-path can then be calculated using the probabilities in (2.3). This, however, requires the value function in the end of the day to be known. In the within-day model as presented in Västberg et al. (2018), there is a single feasible end-state of each day where the value function can trivially be set to zero. Here, we want to create a connection between days. This means that $J_n^T(y;\theta)$, the expected value function in the end of the day, is the expected value of all future days as seen from a terminal state y, when both ν and ϵ of future days are unknown. If we let all decisions be defined by the day-paths individual chooses, there are no decisions made between days that affect the transition to the state in the beginning of the next day. We will allow for the possibility that individuals value utilities in future days less than utilities in the current day by introducing a discount factor β when going from terminal state $J_n^T(x)$ to the initial state in the subsequent day, where the expected value function is given by $J_n^0(g(x);\theta)$, where g denotes the transition from a state $x \in X(T, \text{dow})$ to a state $x \in X(0, \text{dow}+1)$. Here $X(T, \text{dow})$ denotes the available states at the last time step of a day T during day-of-week dowand $X(0, \text{dow}+1)$ the feasible states at time 0 in the following day-of-week. This means that:

$$J^T(x;\theta) = \beta J^0(g(x);\theta). \tag{2.4}$$

The connection of states between days means that the planning horizon becomes infinite, although the value of the discount factor β will impact the relative importance of future utilities. The infinite horizon means that the value function cannot be calculated by starting backward induction from a terminal state, as there is no final state from which to start. This also means that (2.2) will be an implicit rather than explicit function, as the network eventually will returns to the state x. This fixed point problem can still be solved by value iteration as long as $\beta < 1$ (and in some special cases for some values of $\beta \geq 1$, see: Västberg and Karlström, 2018). This is done by starting in an arbitrary state and iterating backwards, just as in the terminal state problem, until convergence is reached. To speed up convergence, Newton iterations can be used, as outlined in Rust (1987). Once the value functions are obtained, conditional choice probabilities can once again be calculated using (2.3). The standard method of estimating dynamic discrete choice models, namely the nested fixed point algorithm (NFXP) (Rust, 1988), is based on solving this fixed-point equation in each step of a standard maximum-likelihood estimation.

The problem with implementing this method directly, or any other estimation method we know off in the dynamic discrete choice literature, is that computation time would be immensely long.

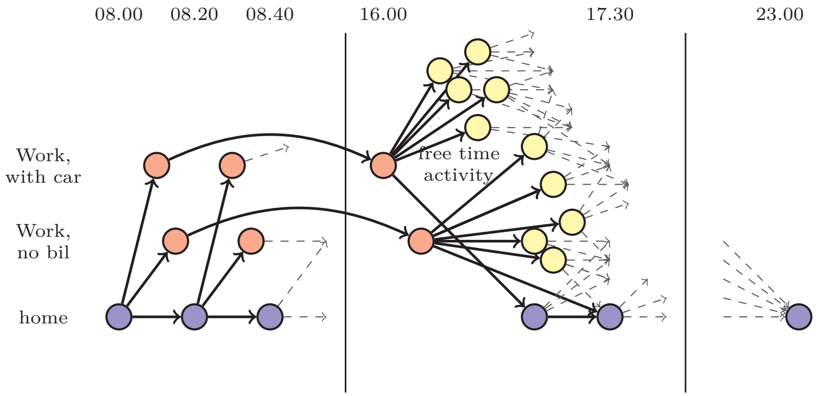


Figure 1: Example of how decisions yields new states with different future opportunities. Individuals are affected by the past through their current state and are assumed to act as if they take the expected future utility of their actions into account when making a decision.

Dimensionality of problem and Computation details

Observe that the possibility to go from-all-to-all destinations with any mode in any time step means that the number of links in this network is very large, in the order of 10^9. This makes calculating the value function and choice probabilities a computational challenge, even in the within-day setting presented in Västberg et al. (2018). With the current implementation which simply uses backward induction it takes around 10 s per individual on for a single workday on a Intel(R) Core(TM) i7-6820HQ 2.7Ghz processor when using a single core. This computation time further increases linearly with the number of performed-activity-states: $\prod_a N_{\tau_a}$. The code is written in C# but uses Intel MKL when applicable (e.g., exp and matrix operations) and C++ functions for other computationally expensive operations. Observe that simply taking *exp* on a 10^9 vector on the same machine would take 6 s in MATLAB.

Since evaluating the choice probabilities is computationally expensive, some approximative methods must be used. The method we have used here involves sequential estimation by 1) using sampling of alternatives to solve the within day model, and 2) based on these estimates estimate the remaining parameters in the between day model.

2.2. *Day-to-day model*

As we explained above, the extension of the within-day model to day-to-day planning is obtained by connecting the daily network above between numerous

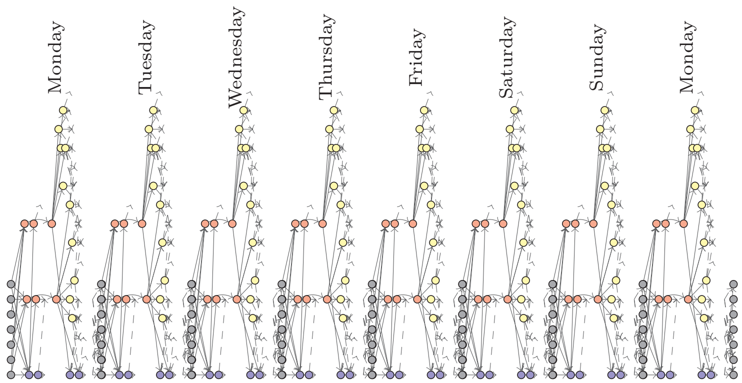


Figure 2: An extension to day-to-day planning is obtained by connecting states in the end of each day to states in the beginning of the subsequent day. The number of states connecting days are constrained by the assumption that individuals start and end their day at home.

days. This means connecting the value function in the final state of a day with the value function in the first state of the subsequent day through (2.4). Observe that although the number of states in the middle of the day is huge, the number of feasible states in the end and beginning of the day is typically very small since we require individuals to start and end their day at home. This is illustrated in Figure 2. This will be utilized in the following to allow estimation of the combined within and between-day model.

Transitions and states between days

The part of the history which will be remembered between days are 1) the number of days since a trip with a specific activity was last performed (τ_a) and 2) the current day-of-week dow. The part of the state necessary to give transitions between days is n_a, i.e., the number of times an activity has been performed during the last day. The relevant length of the history (M_{τ_a}) is not always straight forward to determine and will depend on the data at hand as well as the one-stage utility functions. We have used histories varying from 1 to 14 days during the development of this model.

With this specification, it is straight forward to specify the first state in day $k+1$ given the last state in day k. If $x_T^k = \{n_a^k, \tau_a^k, \mathrm{dow}^k, \dots\}$ is the last state reached in day k, the first state in the following day will have $n_a^{k+1} = 0$ (as the number of activities finished in the beginning of the day is always zero), dow simply increased by one day and:

$$\tau_a^{k+1} = \begin{cases} 0 & \text{if } N_a > 0 \\ \min(\tau_a^k + 1, N_{\tau_a}) & \text{else.} \end{cases}$$

If activity a is performed in day k, then $\tau_a^{k+1} = 0$. Otherwise it will increase by 1 until the maximum length of the memory is reached. The total number of states in the day-to-day model is thus $\prod_a (N_{\tau_a} + 1) \cdot 7$ since there are 7 possible days of the week.

2.3. *Estimation*

Conditional choice probabilities

Following Fosgerau et al. (2013), Västberg et al. (2018) shows that the probability of a day-path, i.e., a sequence of choices in accordance with (2.3) starting at time $t = 0$ and ending in $t = T$, is given by:

$$P(\sigma) = \frac{e^{u(\sigma)}}{\sum_{\sigma}^{\prime} e^{u(\sigma')}}.$$

The model is thus equivalent to an MNL model over all day-paths. In the within-day model, there is a single state in the end of the day, so the value function in that state can be trivially set to zero, which is why it is not included in the equation above. However, in the day-to-day extension presented here there are multiple possible states in the end of the day as well as in the beginning of the day. Let d_t denote the decision at time t of day-path σ and $x_{t+1} = f(d_t, x_t)$, i.e., the corresponding state reached when making decision d_t in state x_t. The probability of a day-path then becomes

$$\begin{aligned} P(\sigma| \quad x_0, \nu) &= \frac{e^{\frac{1}{\lambda}\left(u(\sigma,x_0)+J^T(x_T)+\nu(x_T)\right)}}{\sum_{\sigma}^{\prime} e^{\frac{1}{\lambda}\left(u(\sigma',x_0)+J^T(x'_T)+\nu(x'_T)\right)}} \\ &= P(\sigma = \underset{\sigma \in C(x_0)}{\arg\max} \left\{u(\sigma, x_0) + \underbrace{\lambda \cdot \epsilon(\sigma) + \nu(x_T)}_{\xi(\sigma)} + J^T(x_T)\right\} | x_0, \nu) \end{aligned}$$

where $\epsilon(\sigma)$ is Gumbel distributed. If both ϵ and ν are unknown, the joint error term becomes $\xi(\sigma) = \lambda \cdot \epsilon(\sigma) + \nu(x_T)$ which by definition of $\mathcal{V}$ also has a so that ξ over all day-paths σ generate a nested logit model. The probability of a day-path

σ when both ϵ and ν is then given by:

$$P(\sigma|x) = \underbrace{\frac{e^{\frac{1}{\lambda}\cdot u(\sigma)}}{\sum\limits_{\sigma' \in C(x,y)} e^{\frac{1}{\lambda}\cdot u(\sigma')}}}_{P(\sigma|x,y)} \cdot \underbrace{\frac{e^{\lambda \cdot L(x,y) + J^T(y)}}{\sum\limits_{y' \in T(x)} e^{\lambda \cdot L(x,y') + J^T(y')}}}_{P(y|x)} \tag{2.5}$$

where $C(x, y)$ denote all the paths that given an initial state state x reach state y in the end of the day and:

$$L(x, y) = \log \sum_{\sigma' \in C(x,y)} e^{\frac{1}{\lambda}\cdot u(\sigma')}. \tag{2.6}$$

From (2.5) it is clear that the likelihood conditional on the initial state can be divided into two parts. Firstly,

$$P(\sigma|x, y) = \frac{e^{\frac{1}{\lambda}\cdot u(\sigma)}}{\sum\limits_{\sigma' \in C(x,y)} e^{\frac{1}{\lambda}\cdot u(\sigma')}} \tag{2.7}$$

which is the likelihood of a day-path σ conditional on the initial choice x and the final state y. This is exactly the same likelihood as the one estimated in Västberg et al. (2018), with the exception of the restriction on the choice-set to $C(x, y)$. Secondly, the probability conditional on the initial state x to choose one of the day-paths ends in state y, given by:

$$\begin{aligned} P(y|x) &= \frac{e^{\lambda \cdot L(x,y) + J^T(y)}}{\sum\limits_{y' \in T(x)} e^{\lambda \cdot L(x,y') + J^T(y')}} \\ &= \frac{e^{\lambda \cdot L(x,y) + \beta J^0(g(y))}}{\sum\limits_{y' \in T(x)} e^{\lambda \cdot L(x,y') + \beta J^0(g(y'))}} \end{aligned} \tag{2.8}$$

Sequential estimation, which gives consistent but inefficient estimates, could be used to estimate this model, by first using the choice probabilities in (2.7) to estimate the parameters with attributes that varies within the same choice-set $C(x, y)$, and secondly fix these and estimate the remaining parameters. However, this requires that the initial state x is known. This is typically the case in DDCMs, and conditional choice probabilities are therefore usually used for estimation, as in Rust (1987). However, when modelling between-day planning with states representing the number of days since an activity was last performed, the state in at least the first response day will typically be unobserved. When the initial state x is latent and the utility of a day-path is dependent on the initial state, additional conditions on the one-stage utility functions are needed for sequential estimation to be possible.

2.3.1. *Expected value function in the beginning of a day*

The expected value function in a state x in the beginning of a day is given by:

$$
\begin{aligned}
J^0(x) &= \log \sum_{y \in T(x)} \left(\sum_{\sigma' \in C(x,y)} e^{\frac{1}{\lambda} \cdot \left(u(\sigma') + J^T(y) \right)} \right)^{\lambda} \\
&= \log \sum_{y \in T(x)} e^{\lambda \cdot L(x,y) + J^T(y)} \\
&= \log \sum_{y \in T(x)} e^{\lambda \cdot L(x,y) + \beta J^0(g(y))}
\end{aligned}
\tag{2.9}
$$

This gives us a first idea on how we can speed up the value-iteration calculation. The naive way, to move through the entire activity network, would require the repeated calculation of the value function in each state in the activity network, probably in the hundreds per day before convergence. However, if we can first calculate the log-sums in (2.6), we can then use these to solve the much smaller fixed-point equation defined by (2.9). Solving this problem can be done in negligible time in comparison. The calculation of the log-sums can be expected to take the same amount of time as the iteration would take over a single day, so the speed up is in the hundreds already. However, if we want to estimate the joint model, we would still need to calculate the log-sums and the gradients of the log-sums for each iteration of the optimization problem, and this is as computationally infeasible here as in the within-day model. We will therefore look for a way that allows us to use the sample-of-alternatives estimation method proposed in Västberg et al. (2018) to estimate the within-day part of the problem, and use these estimates to sequentially estimate the between-day part of the problem. To see when this is possible, we need to look further at the likelihood functions when the initial state is latent.

Log-likelihood function with unobserved initial state

We will next derive the log-likelihood function for an individual n when the initial state x is assumed to be partially unknown, i.e., it is only known to be a member of the set I_n^0. The set I_n^0 will contain all possible initial states x for a specific value of the dow, which is typically observed. Also remember that an individual can be observed on many subsequent days, making a sequence of day-paths $\bar{\sigma}^n = \{\sigma_0^n, \sigma_2^n, \ldots, \sigma_{D_n-1}^n\}$ where D_n is the number of days for which the individual has been observed.

After a day-path σ is performed in a state x_k, we have previously denoted the state reached in the end of the day by $y(\sigma, x)$, and the state reached in the beginning of the next day by $x_{k+1} = g(y(\sigma, x_k))$. When taking a day-path σ from the initial state x_{k+1}, an individual will thus start the next day in $x_{k+1} = g(f(\sigma, x_k))$. To simplify notations we use $\{x_k\}_{k=0}^{D_n}$ to denote the initial states reached when performing a number of day-paths $\{\sigma_k\}_{k=0}^{D_n-1}$ starting in state x_0, so that $x_k = g(y(\sigma_{k-1}, x_{k-1}))$. The corresponding set of terminal states are denoted $\{y_k\}_{k=0}^{D_n-1}$ so that $y_k = y(\sigma_k, x_k)$ and $x_{k+1} = g(y_k)$.

If the initial state of the first observation is known to be contained in the set I_n^0, the likelihood function for an individual n that has been observed to choose day-paths $\bar{\sigma}_n$ can with the above notations be written as:

$$P_n(\bar{\sigma}^n) = \sum_{x_0 \in I_n^0} \pi_n(x_0) \prod_{k=0}^{D_n-1} P_n(y_k|x_k) P_n(\sigma_k^n|y_k, x_k) \tag{2.10}$$

where $\pi(x)$ is the probability to be in the (partially latent) initial state x. As noted before, the state x between-days consist of the day of week, which is observed, and the number of days since an activity was last performed n_a, which the researcher will generally be unaware of for the first observed day. However, the transition probabilities between initial states and terminal states within a day are given by $P(y|x)$ and can be obtained by (2.8). The transition probability between initial states in subsequent days are then given by:

$$Pr_n(x_{k+1}|x_k) = \sum_{y_k \in T \text{ s.t.} g(y_k) = x_{k+1}} P_n(y_k|x_k)$$

and this is an irreducible and positive recurrent Markov chain which has a unique stationary distribution π. It is worth noting that the Markov process is not aperiodic, since each state can only be returned to after 7 days. The stationary distribution is thus not a limiting distribution. To find the stationary distribution, we solve the linear equation system defined by:

$$\pi_n(x') = \sum_{x \in I} Pr_n(x'|x) \pi_n(x) \quad \forall x' \in I. \tag{2.11}$$

The probability that an individual starts a day in specific state x is thus given endogenously by (2.11).

We will next introduce an that $P(\sigma|x, y(\sigma, x))$ to be independent on the initial state x. This means that the part of the utility of all day-paths $\sigma \in C(x, y)$ that depend upon the initial state x must be the same for all alternatives in $C(x, y)$.

Assumption 1 *Assume that for all initial states $x \in I$ and all final states $y \in T$, the utility of a day-path $\sigma \in C(x, y)$ is additively separable into*

$$u(\sigma, x; \theta_u) = u(\sigma; \theta_{u,1}) + u(y(x, \sigma), x; \theta_{u,2})$$

where $\theta_u = (\theta_{u,1}, \theta_{u,2})$ *are the parameters of the one stage utility functions. The utility of a day-path thus depends on two parts: one that is only dependent on the set of actions performed, but not on the initial state of the day* x*; and a second which is only dependent on the transition taking place throughout the day, but not on the sequence of actions that led to it.*

With Assumption 1, the utility of, e.g., a shopping trip cannot be dependent on a non-additively functional combination of the initial state x and the mode used for the shopping trip or the time of day at which it was performed. We think it is a reasonable restriction, even though such combinations possibly could be useful. The utility of a shopping trip can still be dependent on the initial state, so that a need for shopping increases over days, as long as the combination is only dependent on the fact that a shopping trip is performed during the day and not on details about how the shopping is performed.

ASSUMPTION 2 *Let* $\Delta n(x, y)$ *denote the change in* n_a *between a state* $x \in I$ *in* $y \in T$*. Then, if for some* $x' \in I$ *and* $y' \in T$*:*

$$\Delta n(x, y) = \Delta n(x', y')$$

and $\sigma \in C(x, y)$ *then* $\sigma \in C(x', y')$*, i.e., if two other states have the same transition* Δn*, all day-paths that connect* x *with* y *also connects* x' *and* y'*.*

With Assumption 2, the choice set $C(x, y)$ is uniquely defined by the transition $\Delta n(x, y)$. We will often use $C(\Delta n)$ instead of $C(x, y)$ to denote the set of all day-paths connecting x and y. This means that $P(\sigma|x, y)$ in (2.7) becomes:

$$\begin{aligned} P_n(\sigma|x, y; \theta) &= \frac{e^{\frac{1}{\lambda}\cdot u(\sigma,\theta_u)}}{\sum\limits_{\sigma' \in C(x,y)} e^{\frac{1}{\lambda}\cdot u(\sigma';\theta_u)}} \\ &= \frac{e^{\frac{1}{\lambda}\cdot(u(\sigma;\theta_{u,1})+u(y,x;\theta_{u,2}))}}{\sum\limits_{\sigma' \in C(x,y)} e^{\frac{1}{\lambda}\cdot(u(\sigma';\theta_{u,1})+u(y,x;\theta_{u,2}))}} \\ &= \frac{e^{\frac{1}{\lambda}\cdot u(\sigma;\theta_{u,1})}}{\sum\limits_{\sigma' \in C(\Delta n)} e^{\frac{1}{\lambda}\cdot u(\sigma';\theta_{u,1})}} \end{aligned} \tag{2.12}$$

so that $P_n(\sigma|x, y; \theta) = P_n(\sigma|\Delta n; \theta_{u,1}, \lambda)$. The likelihood in (2.10) for individual n then becomes:

$$P_n(\bar{\sigma}^n; \theta) = \prod_{k=0}^{D_n-1} P_n(\sigma_k^n|\Delta n_k^n, \theta_{u,1}, \lambda) \cdot \sum_{x_0 \in I_n^0} \pi_n(x_0; \theta) \prod_{k=0}^{D_n-1} P_n(y_k|x_k; \theta)$$

and the log-likelihood for $\mathscr{L}\mathscr{L}_n$ for individual n becomes:

$$
\begin{aligned}
\mathscr{L}\mathscr{L}_n(\bar{\sigma}^n;\theta) = &\sum_{k=0}^{D_n-1} \log\left(P_n(\sigma_k^n|\Delta n_k^n;\theta_{u,1})\right) \\
&+ \log\left(\sum_{x_0\in I_n^0} \pi_n(x_0;\theta) \prod_{k=0}^{D_n-1} P_n(y_k|x_k;\theta)\right)
\end{aligned}
\tag{2.13}
$$

The likelihood conditional on the observed transitions Δn_k^n is thus simply given by MNL models over day paths from the choice-set $C(\Delta n_k^n)$.

Calculating within-day lsm

If $\theta_{u,1}$ is known and $\theta_{u,2}$ is set so that $u(y,x;\theta_{u,2}) = 0\ \forall x, y$, the log-sums in (2.6) can be calculated once for each possible transition Δn_a and reused for all initial and final state pairs, instead of calculating it for each pair x and y separately.

We have previously defined the state dimension n_a to be the number of times activity a has been performed throughout the day. To reduce the size of the state-space, a maximum of N_a times were remembered for each activity, so that $n_a = N_a$ means that activity a has been performed at least N_a times throughout the day. This means that activity a can be performed independently of the value of n_a, that the only feasible value at $t = 0$ is $n_a = 0\ \forall a$ and that $n_a = 0$ at time T implies that the activity has not been performed throughout the day. This definition of the state is straight forward to implement when using backward induction if J_n^T is known. However, when calculating the log-sums conditional Δn_a, a transformation of the state dimension will be practical from a computational perspective, as it will allow us to calculate the log-sums conditional on a transition and simulate day-paths conditional on a transition using the same value functions. We will therefore use an alternative definition of the state n_a which will be denoted as $\tilde{n}_a$. In this transformed state space, an activity a can be performed whenever $\tilde{n}_a < N_a$, but not when $\tilde{n}_a = N_a$. However, if $\tilde{n}_a = 0$, there are two different ways in which activity a can be performed; 1) by increasing $\tilde{n}_a$ by one; 2) without increasing the value of $\tilde{n}_a$. If $\tilde{n}_a = k$, then activity a has been performed at least a times, so $\tilde{n}_a \geq n_a$.

With the new transformed state space, we set the terminal states to:

$$
J_n^T(\ldots,\tilde{n}_a) = \begin{cases} 0 & \text{if } \tilde{n}_a = N_a\ \forall a \\ -\infty & \text{else,} \end{cases}
\tag{2.14}
$$

i.e., the terminal state is feasible only if $\tilde{n}_a = N_a$. If $\tilde{n}_{a,t} = k \leq N_a$ in a state x_t, it means that any path including that state with a non-zero probability must include exactly $N_a - k$ additional performances of activity a if $k > 0$ and at least N_a performances of activity a if $k = 0$. This especially means that at $t = 0$, the value function $J^0(\ldots, \tilde{n}_a = k)$ gives the log-sum of all day-paths that includes exactly k performances of activity a if $k > 0$; and $k = 0$ gives the log-sum for all day-paths that include at least N_a performances of the activity. The value of $\tilde{n}_a$ can thus be interpreted as the least amount of times activity a must be performed throughout the day, whereas n_a was defined as the number of times activity a has been performed.

Without this transformation, setting the value function $J_n^T(\ldots, N_a)$ as in (2.14) would mean that any path leading from a state with $n_a = k$ would include *at least* $N_a - k$ additional performances of the activity. Obtaining the required log-sums would therefore be possible although it would include solving an equation system. However, if we later wanted to simulate a day with exactly k trips, it would require the recalculation of the value functions in each state starting the backward induction using $J_n^T(\ldots, N_a) = 0$ only when $n_a = k$.

Constructing choice set for within-day model

As in Västberg et al. (2018), we have used sampling of alternatives to estimate the within-day model. The method consisted of first finding a set of parameters that reproduced realistic behaviour from the model. With these parameters the value function can be calculated in all states in the network through backward induction using (2.2). Conditional on the value function, day-paths can subsequently be simulated using (2.3). For each individual, $N_{ch} = 1000$ alternatives is be sampled with replacement.

To get consistent estimates when sampling a choice set, a correction term must be added to the utility of each alternative. The correction term to add to alternative j should equal $\log(\bar{q}_n(\tilde{\mathbb{C}}_n|j))$, i.e., the probability that the choice set $\tilde{\mathbb{C}}_n$ is sampled conditional on the alternative σ being included in the choice-set (McFadden, 1978). For consistent estimates, the probability $\bar{q}$ must further satisfy the positive conditioning property, i.e., if $j \in \tilde{\mathbb{C}}_n$ and $\bar{q}_n(\tilde{\mathbb{C}}_n|i) > 0$ for some i, then $\bar{q}_n(\tilde{\mathbb{C}}_n|j) > 0$. This obviously holds if sampling is performed from the universal choice set so that all alternatives have a non-zero probability of being added, as in Västberg et al. (2018).

Here, we will sample from a restricted choice-set $\mathbb{C}_n(\Delta n)$, as we want to estimate the likelihood function given by (2.7). To sample from this choice-set, we follow the procedure to calculate the value function in all states described above. If we want to sample conditional on a specific Δn_a, we start simulation in $M_a - \Delta n_a$ for each a. As $\mathbb{C}_n(\Delta n)$ is the universal choice set for the problem we want to estimate, this is exactly the same case as in Västberg et al. (2018).

Let the probability to sample an alternative j be denoted q_j and the number of times it was sampled be denoted k_j. With the proposed sampling protocol, consistent estimates are obtained using the correction term $\log(\tilde{k}_j/q_j)$ (Frejinger et al., 2009) where:

$$\tilde{k}_j = \begin{cases} k_j + 1 & \text{if } j \text{ is the observed alternative} \\ k_j & \text{else.} \end{cases} \tag{2.15}$$

The likelihood function in (2.3) finally becomes:

$$P_n(i|\tilde{\mathbb{C}}_n;\theta_1) = \frac{e^{\frac{1}{\lambda}\cdot u(i;\theta_1)+\log(\tilde{k}_i/q_i)}}{\sum\limits_{j\in\tilde{\mathbb{C}}_n} e^{\frac{1}{\lambda}\cdot u(j;\theta_1)+\log(\tilde{k}_j/q_j)}} \tag{2.16}$$

Calculating between-day value function and likelihood

Once $\theta_{u,1}$ is estimated using the likelihood function in (2.16), $\theta_{u,2}$, μ and β are estimated using the NFXP. To be able to obtain estimates for the long-term average cost case ($\beta = 1$) we use the differential value function as outlined in Västberg and Karlström (2018). The differential value function is a normalized version of the value function which can have a solution for $\beta \geq 1$. The value function for the between day model was defined in (2.9). Define the log-sum operator as T, i.e.,

$$T(J_n))(i) = \log \sum_{j\in I_n} e^{\lambda\cdot L_n(i,j,\theta_1)+\beta J_n(j;\theta)} \quad \forall i \in I_n$$

The relative value function h is then defined as the solution to:

$$\begin{aligned} h_n(i;\theta) &= T(h_n)(i) - \lambda_n \quad \forall i \in I_n \\ h_n(0;\theta) &= 0 \end{aligned} \tag{2.17}$$

where λ_n is a constant that guarantees the normalization of $h_n(0;\theta)$ to zero. Since $h_n(i;\theta) - h_n(j;\theta) = J_n(i;\theta) - J_n(j;\theta)\ \forall\beta < 1$ it can be used to calculate the choice-probabilities instead of J.

The NFXP algorithm iterates between two steps. Firstly, the value function in (2.17) (and its gradients) are calculated. This again involves two steps: 1) use value iteration to get sufficiently close to the solution, and 2) use newton iterations to directly solve the non-linear equation system. Value iterations to obtain the differential value function are performed as:

$$h_n^{k+1}(i) = T(h_n^k)(i) - T(h_n^k)(0) \quad \forall i \in I_n$$

which is iterated until $\|h_n^{k+1} - h_n^k\|$ is sufficiently small.

Secondly, the conditional choice probabilities $P_n(y|x;\theta)$ are calculated using (2.8). Using these, the equation system in (2.11) is solved to obtain the stationary probabilities $\pi_n(x;\theta)$. With these obtained, the log-likelihood for individual n becomes:

$$\mathscr{L}\mathscr{L}_n(\bar{\sigma}_n;\theta) = K_n(\theta_{1,u}) \quad + \log\left(\sum_{x\in I_n} \pi_n(x,\theta) P_n(\Delta n_n|x;\theta)\right)$$

where $K_n(\theta_1, u)$ is the part of the utility that only depends on θ_1.

3. RESULTS

We have implemented and estimated a version of the day-to-day model described in the previous section only including day-to-day planning of grocery shopping. The data used for estimation is a combination of a selected subset of 4153 individuals from a travel survey conducted 2005/2006 in Stockholm (see Västberg et al., 2018, for a discussion of how individuals are selected from the data set) and an anonymized sample of 161 individuals with a total of 606 days obtained during a trial of MEILI (Prelipcean et al., 2017), a recently developed smart phone application for collection of travel surveys. The within-day model needs level-of-service variables for all OD pairs both for estimation and for generation of log-sums. To obtain a consistent treatment of observed and unobserved OD pairs, simulation generated values are used instead of reported travel times. Observe that different travel times are used dependent on if it is a weekend or a workday. Different parameters in the within-day model are also estimated depending on if is a workday or not, so that different log-sums are used depending on if it is a free day or a workday. The data set obtained from MEILI does not contain any socio-demographic variables, so no parameters related to socio-demographics could be estimated.

The following section includes estimation result of the between-day model followed by results from a policy experiment.

3.1. *Estimation result*

We have estimated four different models:

Static	including the log-sum term but removing all parameters that change with the state
Myopic	including the log-sum term but fixing $\beta = 0$.
Long-term	including the log-sum term but fixing $\beta = 1$.
Free β	including the log-sum term L and estimating the discount factor β.

We have first estimated the within-day model as described in Västberg et al. (2018), and the resulting estimates for that model can be found in Appendix A.

In Tabel 1, the estimation result for the static model as well as three alternative dynamic models are presented. The resulting loglikelihood tell us a number of things. Firstly, introducing dynamics has a big impact on the model fit. The loglikelihood difference between the static model and the best dynamic model is 21.87 at the cost of three additional parameters. Comparing the loglikelihood between the static model with the myopic model, which performs the worst, the difference is still 18.5, so there is a large benefit in terms of model fit from introducing dynamics.

This lead us to the second conclusion, namely that introducing day-to-day dynamics without incorporating forward looking behaviour will give most of the model fit benefits. The difference in log-likelihood between the myopic model and the free-beta model is 3.5 so although β is significantly greater than 0, the difference in model fit might not be enough to motivate the additional model difficulties needed to consistently considering the future. It is also quite possible that a method which approximates the value of the future, like the one suggested in Arentze and Timmermans (2009), would suffice to obtain most of the additional benefits observed here.

For the model presented here, the small difference in model fit might be because there is a single attribute which varies across individuals, namely the within-day log-sum. If the difference between the expected value function between different individuals is small, constants related to when to shop will pick up most of the difference. However, these constants will not pick up how the value function changes due to policies. It is therefore possible that the myopic model will produce unrealistic forecasts.

Thirdly, even though the change in model fit is not huge when introducing forward looking behaviour, the parameter values obtained changes qualitatively. For example, in the myopic model, shopping between day $1-3$ is preferable to shopping between day $4-5$, but the relationship is reverse in the dynamic models. This is needed in the myopic model to reproduce the correct rates of shopping, but give counter-intuitive result. In the forward-looking model, the utility to shop grows with the number of days since the last shopping trip was performed.

The log-sum parameters for free days λ_{freeday} where not significant in any of the models. There may be several explanations for this. Firstly, it is possible that accessibility to shopping facilities has less impact on the decisions to perform shopping trips on a free day. Secondly, we have fewer observations on weekends, so the uncertainty about the estimates will naturally be larger. Thirdly, it is likely that the need to shop on free days are determined by whether shopping trips have been performed on working days, so that the probability to shop on a weekend will be negatively correlated to the log-sum on workdays. As the

Table 1: Estimation of four different models using 4314 individuals. The best model is the model where β is allowed to take any value, but the discount factor is not significantly different from 1. The largest improvement in model fit comes from introducing dynamics. The additional improvement of having forward looking agents is relatively small, but significant.

Variable	Static	Myopic	Long-term	free
Shop on workday	-1.245	-1.654	0.488	0.709
	(-11.5)	(-7.0)	(1.0)	(1.2)
Shop on freeday	-0.191	-0.618	1.281	1.586
	(-2.1)	(-2.8)	(2.8)	(2.8)
Shop when $x_{shop} \leq 3$		0.188	-1.092	-1.628
		(0.8)	(-2.6)	(-3.9)
Shop when $x_{shop} \geq 6$		2.118	1.6	2.935
		(4.9)	(3.9)	(4.0)
λ_{workday}	0.154	0.195	0.150	0.187
	(2.4)	(2.6)	(2.5)	(2.6)
λ_{freeday}	0.014	0.014	0.042	0.033
	(0.19)	(0.18)	(0.48)	(0.41)
β	-	0	1	0.772
				(8.8)
$\mathscr{LL}$	-2442.1	-2423.7	-2422.1	-2420.2
$\mathscr{LL} - \max \mathscr{LL}$	-21.87	-3.496	-1.853	0

log-sum on workdays and free days are likely to be correlated as well, this may influence the value of λ_{freeday}. The last explanation is possibly strengthened by the fact that the value of λ_{freeday} increases with β.

3.2. *Policy test: grocery shopping unavailable on Sundays*

To test how the myopic model and the long-term model behave we implement a scenario where grocery stores are closed on Sundays. This is compared to the base case where things are left unchanged. In the base case, both models will perform very similar as they are fitted to the same data, so the change will be from the same initial levels. Observe that a purely static model would not predict changes on any other day than the day at which the policy has an effect.

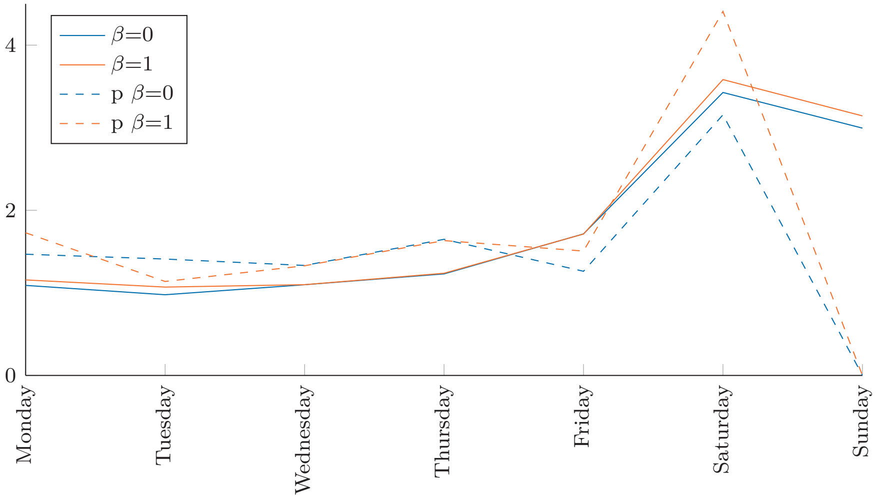
The resulting change in average shopping probabilities can be observed in Figure 3. Observe that both myopic and forward-looking agents perform more shopping on the days following the Sunday (Monday-Thursday). However, myopic agents actually shops less on both Fridays and Saturdays. From Figure 4 and 5 it seems as if the reason for this is that since they are forced to shop in the beginning of the week, agents in general have a higher state in the end of the week and therefore have a lower need for shopping then. This produce the counter-intuitive result that they shop less on Saturdays. Forward looking agents behave more as one would expect. Since they are aware that shopping wont be possible on Sundays they compensate by shopping more on Saturdays. They also perform more shopping on Mondays-Thursdays and marginally less on Fridays. Both models further predict a decrease in shopping trips over the week but the change is almost twice as high for the myopic model. We see the fact that the two models behaved so differently and that the myopic model performed so counter intuitively as a strong argument for the inclusion of forward looking behaviour in day-to-day models, even though the model fit within the data set was very similar.

4. CONCLUSIONS

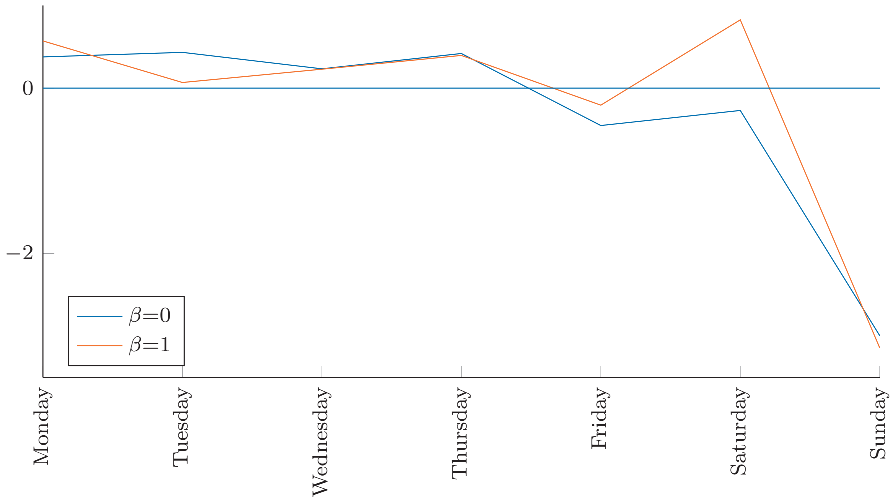
Modelling day-to-day dynamics has the potential to further improve travel demand models and is a natural extension to the activity based approach. There are multiple approaches suggested in the literature, but no combined within-day and day-to-day model up to date allows for forward looking agents. Even in models only treating day-to-day choices, individuals are often assumed to either consider a limited time span (such as a week) or to be myopic (i.e., they only consider the past but not the future). In this model we extend a within day travel demand model previously developed in Västberg et al. (2018) to include day-to-day dynamics. This is done using a dynamic discrete choice model, where the expected utility of the future enters the utility function of choices within the day. The day-to-day model is developed with the specific within-day model in mind, but any model that can produce the expected utility of a day conditional on the performed activities could be used.

Estimation of three types of models are compared, static models, myopic models and forward looking models. Introducing dynamics gives a large increase in model fit. Forward looking behaviour further improves the model but not nearly as much as the initial inclusion of dynamics. A policy experiment was conducted where grocery stores were closed on Sundays. Counter-intuitively, the myopic model predicted that the amount of shopping trips would decrease on Saturdays and increase on Mondays-Thursdays. A model allowing for forward-looking agents instead predicted that more shopping would be performed mainly during Saturdays but also during weekdays. Failing to account for forward-looking behaviour in day-to-day models can thus cause unrealistic responses to policy changes.

Future work includes introducing heterogeneity of preferences both by including socio demographic and with latent constructs, e.g., allowing for random and routine shoppers as has been common in hazard models (Bhat et al., 2004a; Kim and Park, 1997). One potential way of modelling this would be through classes of forward-looking and myopic agents respectively. More data on longer time

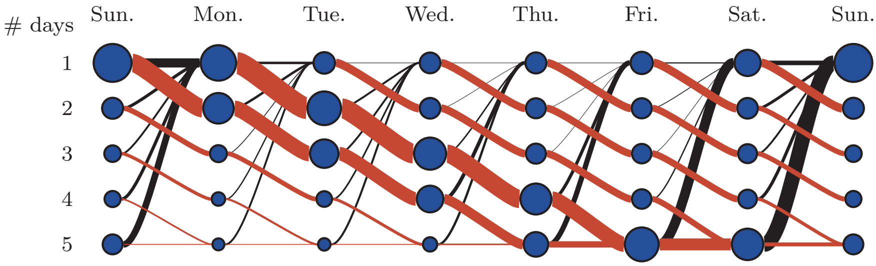


(a) Average probability of shopping trip before (solid) and after (dashed) policy change due to policy

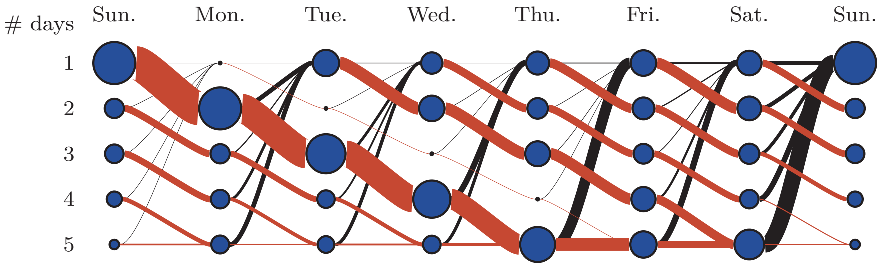


(b) Change in probability to perform shopping trip

Figure 3: Figures shows the average probability that a shopping trips is performed on specific day of the week in the base case and when shopping can no longer be performed on Sundays (dashed). Myopic agents (blue, $\beta = 0$) only react to the policy by shopping the days following the change (Monday-Thursday) but forward looking agents (red, $\beta = 1$) react by shopping both before and after the change.

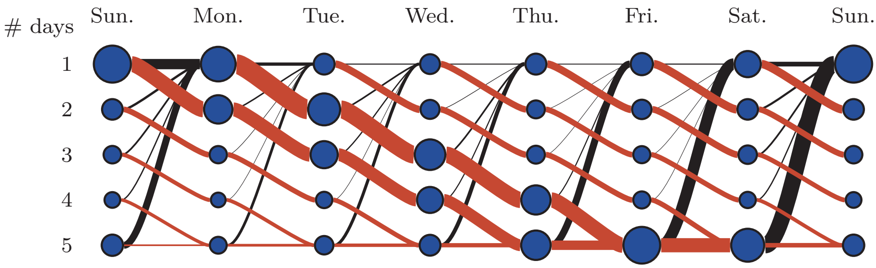


(a) $\beta = 1$, base case

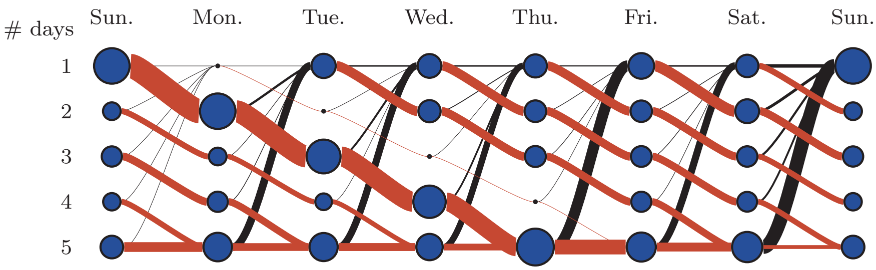


(b) $\beta = 1$, no shopping on sundays

Figure 4: Stationary probabilities and transition probabilities for base and policy scenario with $\beta = 1$. The size of the nodes are proportional to the stationary probabilities $\pi(i)$ and the size of the links are proportional to $\pi(i) \cdot P(a|i)$.



(a) $\beta = 0$, base case



(b) $\beta = 0$, no shopping on sundays

Figure 5: Stationary probabilities and transition probabilities for base and policy scenario with $\beta = 0$. The size of the nodes are proportional to the stationary probabilities $\pi(i)$ and the size of the links are proportional to $\pi(i) \cdot P(a|i)$.

horizons would help improve the type of behaviour the model could account for, and especially data that would allow to estimation of how people plan their flexible working hours would be interesting. Currently, only day-to-day planning of grocery shopping is modelled but more activities could jointly be treated in an identical manner.

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APPENDIX A: ESTIMATES

Parameter		Estimate	Rob. t-test
Car	Time	-0.081	-9.8
	ASC	-2.8	-14
PT	Time	-0.043	-3.1
	ASC	-3.7	-20
	Total Wait	0.0065	0.38
	ASC Bike main mode	-2.7	-4.1
Walk	Time	-0.049	-12
	same zone	-0.63	-2.7
	ASC	-1.8	-9.3
	ASC Bike main mode	-2	-3.7
Bike	Time	-0.05	-7.2
	ASC	-7.1	-11
	Main Mode	-1.7	-3.1
	ASC Bike main mode	4.3	6.7
Cost		-0.011	-3.7

Table 2: Estimates of workday version of within-day model.

Parameter		Estimate	Rob. t-test
Home	6AM Time	0.045	4.1
	7AM Time	0.037	5.6
	8AM Time	0.019	3
	9AM Time	0.014	1.5
	1PM Time	-0.01	-4.9
	5PM Time	0.0042	2.4
	7PM Time	0.0022	1.3
	9PM Time	0.019	6.9
Work	ASC 6AM	0.91	1.2
	ASC 7AM	0.37	1
	ASC 9AM	-1.2	-3.8
	ASC 10AM	-4.9	-6.3
Social	Time	-9.9e-05	-0.041
	ASC	-8.8	-22
	LSM Size	0.014	0.8
Shop	Time	-0.021	-7
	Small ASC	-6.6	-15
	LSM Size	0.55	5.3
	No Employed	3.6	6.7
Other	Time	-0.0094	-3.2
	ASC	-6.8	-22
	LSM Size	0.3	2.8
	No Employed Rest.	-29	-12
Rec.	ASC	-7.4	-25
	LSM Size	0.064	1
	No Employed Rest.	5.9	4.8
	No Emplyoyed OE	-1.1e+02	-20

Table 3: Estimates of workday version of within-day model.

Parameter		Estimate	Rob. t-test
Car	Time	-0.098	-11
	ASC	-1.7	-7.7
		-0.19	-1.4
PT	Time	-0.041	-8.8
	ASC	-3.4	-15
Walk	Time	-0.062	-12
	same zone	-0.23	-1.1
	ASC	-1.1	-5
	ASC Bike main mode	-3.4	-2
Bike	Time	-0.087	-5.4
	ASC	-6.2	-4.3
	ASC Main Mode	-0.68	-0.41
	ASC Bike main mode	3.9	2.4
Cost		0.0014	0.42

Table 4: Estimates of free-day version of within-day model.

Parameter		Estimate	Rob. t-test
Home	6AM Time W	0.053	1.3
	8AM Time W	0.01	1.6
	10AM Time W	-0.016	-6.4
	1PM Time W	-0.023	-11
	4PM Time W	-0.023	-7.7
	5PM Time W	-0.016	-2.8
	6PM Time W	-0.012	-1.8
	7PM Time W	-0.014	-1.9
	8PM Time W	-0.011	-1.5
	9PM Time W	-0.0085	-0.8
	10PM Time W	0.042	2.2
Social	Time	-0.021	-16
	ASC	-8.3	-25
	LSM Size	0.029	0.88
Shop	Time	-0.033	-17
	Small ASC	-6.7	-19
	LSM Size	0.0039	0.93
Other	Time	-0.027	-9.7
	ASC	-7.7	-20
	LSM Size	0.32	2.4
Rec.	ASC	-7.8	-26
	LSM Size	0.019	1
	No Employed Rest.	7.5	3.3

Table 5: Estimates of free-day version of within-day model.