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A dynamic discrete choice activity based travel demand model

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This paper presents a dynamic discrete choice model (DDCM) for daily activity-travel planning. A daily activity-travel pattern is constructed from a sequence of decisions of when, where, why, and how to travel. Individuals' preferences for activity-travel patterns are described by the sum of the utility of all travel and activity episodes in that pattern, but components of the utility functions, such as travel times, may be stochastic. In each decision stage, individuals act as if they maximized the expected utility of the remainder of the day. The DDCM-model presented allows for a detailed treatment of timing decision consistent with other choice dimensions, respects time-space constraints, and enables the inclusion of explicitly modeled uncertainties in, for example, travel time.

In a case study, a model for daily planning of activity and travel on workdays is estimated where individuals can perform any number of trips that each is a combination of one of 1240 locations, four modes, and six activities. Simulation results indicate that the model within sample accurately replicates timing decisions, trip lengths, and the distributions of the number of trips, tours, and trips per tour.

Key words: Activity based model, travel demand, dynamic discrete choice model

1. Introduction

Travel demand models have evolved from highly aggregated trip-based models, through to tour-based models and subsequently onto activity based models that focus on the interdependent choice of full daily activity-travel patterns at an individual or household level. As demand for travel is derived from the demand for activity participation, demand for activities should be a determinant if, when, and where trips are being performed. Modelling the interdependent choice of an activity-travel pattern may be especially important when forecasting the effects that policy changes, such as congestion charges, are having on today's traffic planning as how people react to such policies depend on how they trade-off between, e.g., arrival time to work, cost and mode of transport.

To achieve a model of the interdependent choice of a full daily activity-travel pattern, it is common to assume that individuals (or households) preferences for such a pattern can be described by a utility function including all activity and travel episodes during the day. An early example is Adler and Ben-Akiva (1979), who assumed that individuals behaved as if they choose the utility maximizing travel pattern among the set of all feasible travel patterns. To implement a model based on this assumption in 1979 Adler and Ben-Akiva (1979) imposed two additional restrictions. Firstly, the choice sets was limited to observed travel patterns; and secondly, they did not model the timing of trips. One of the significant challenges with implementing a model for the choice of a full activity-travel pattern is the immense size of the choice set, especially when considering the timing of trips. The large size of the choice set has led to the development of models that do not require choice set formation. The Household Activity Pattern Problem (HAPP) formulates the choice of all household members daily travel patterns including time-of-day, mode and location choices as a constrained mixed-integer optimization problem (Recker 2001, Recker, Duan, and Wang 2008, Kang and Recker 2013, Yuan 2014). The problem is solvable, but computationally very demanding, and with only 19 locations the computation time is reported to an average of 614 s/household (Kang and Recker 2013). The multistate-supernetwork model treats the choice of an activity-travel pattern as a shortest-path problem in an abstract network (Arentze and Timmermans 2004b), where states keep track of, for example, remaining mandatory activities and thus capture time-space constraints (Liao, Arentze, and Timmermans 2013, Liao 2016). Similarly to HAPP, the computation time is still a limiting factor. Liao (2016) reports that evaluating the choice of a daily travel pattern takes 8 s when an individual has two fixed activities to perform in a day and six possible locations.

Starchild (Recker, McNally, and Root 1986a,b) developed a random process to construct a small choice set of alternative activity-travel patterns that were compared based on their full utilities. Somewhat similarly, MATSim models the choice of route, mode, timing and location for a fixed number of activities by randomly perturbing and exchanging alternatives in small a choice set while iterating with a traffic simulator, and continue iterating until the process has stabilized and the random process no longer improve the choice sets (Lefebvre and Balmer 2007, Grether et al. 2009, Balmer, Raney, and Nagel 2005, Horni, Nagel, and Axhausen 2012). AMOS (Kitamura et al. 1996) set out to mimic how individuals adapt their travel patterns in the event of changes to the environment and evaluate whether the perturbations improved the full daily utility. Similarly, AURORA focuses on how individuals reschedule a given set of activities by adjusting their activity-travel pattern until further adjustments do not motivate the search cost (Timmermans, Arentze, and Joh 2001, Joh, Arentze, and Timmermans 2003, 2005).

The standard practice in the field of travel demand modelling is based on random utility maximization and, more specifically, nested logit. Bowman and Ben-Akiva (2001) developed a nested logit structure for the choice of all trips performed during a day. The model treats tours and activities sequentially based on their importance to the individual. The model consists of five nests: 1) the choice of activity pattern, including the number of tours carried out during the day; 2) the choice of time of day for the primary tour and all its trips; 3) the mode and destination for the primary tour; 4) the time of day for the secondary tours; and 5) the mode and destination of the secondary tours.

The nested logit structure ensures that higher order decisions, such as the choice of activity pattern, includes the individual specific information regarding all available tour-combinations that the pattern includes. (Vovsha and Bradley 2004) extended the model to allow for greater detail in time-of-day choice, and in a recent implementation, time-of-day is modeled at a 30-minute resolution, where individuals consider a sampled subset of possible 30-minute intervals for each trip (Bradley, Bowman, and Griesenbeck 2010). In a discrete choice framework, it is possible to use the expected (maximum) utility from a day-path as input to higher order models and for cost appraisal (Geurs et al. 2010). The expected utility could also be used to get disaggregated measures of accessibility (Dong et al. 2006).

An alternative to consider the choice of a full daily travel pattern as a single joint choice has been to model a sequential choice of, for example, whether or not to perform a trip, departure time, mode of transport, destination and purpose. The probability to make a new choice is then modelled conditionally on previous choices, and a sequence of choices will produce a daily travel pattern. This is the approach taken in, for example, Albatross (Timmermans, Arentze, and Joh 2001) and CEMDAP (Bhat et al. 2004). In such sequential models, the full history can influence the probability in any choice situation, ensuring an interdependence between trips. Habib (2011) developed a discrete-continuous random utility model for weekend traveling whereby agents decided on mode, destination, and activity based on the utility of the combination. Agents in the model of Habib (2011) also considered how their decisions influenced future opportunities by including a utility component for the value of future time. They model the value of future time as a time-of-day dependent composite good, which is parameterized and estimated.

Some work on activity planning has been explicitly formulating Markov chains to analyze the sequential dependence of different activity purposes. Allahviranloo and Recker (2013) estimates transition probabilities from one activity to another based on socio-demographics and previously performed activities, formulated as a Markov chain. Susilo and Axhausen (2014) used a Markov chain to model day-to-day dependencies in activities. Hasan and Ukkusuri (2018) used a Markov chain to model sequential choices of activities and location in order to infer missing information in geolocation data gathered from social media.

In this paper, we model the choice of a daily activity-travel pattern as a Markov decision process (MDP) which is further formulated as a dynamic discrete choice model (DDCM) to allow for estimation and simulation, thus extending the work presented in Karlström (2005). The DDCM approach taken in this paper closely follows the standard nested logit practice, but introduces time explicitly, respecting the fact that time has a direction and that decisions in real-life actually can be made sequentially in time, given the information available at each point in time when a decision is made. Compared to the existing models of nested logit type, this model allows for greater detail and consistency in the time dimension. Notably, the DDCM approach allows for an explicit introduction of travel time uncertainty thus allowing forecasts of the system-wide effects of more reliable travel times. Travel time uncertainty has been observed to impact departure time decisions and has been extensively studied using scheduling models following Small (1982). On the other hand, most scheduling models considers a single trip in isolation (as in, for example, Fosgerau and Karlström 2010), or possibly a chain of two trips (Jenelius, Mattsson, and Levinson 2011). Similarly to nested logit based models, the DDCM model presented in this paper can theoretically and consistently be translated into a tool for cost-benefit analysis. It can also be used to obtain time-of-day dependent accessibility measures taking into account the presence of mandatory activities, time-space constraints, and travel time uncertainty, as illustrated by Jonsson et al. (2014).

This paper differs from and extends upon the work in Karlström (2005) and Jonsson et al. (2014) in the following ways. We focus on the choice of a daily activity-travel pattern, rather than a model of between day planning, and present a specification of an MDP which allows for incorporation of, for example, time-space constraints in mandatory activities and dependencies on previous mode and activities. We show how time can be modeled as a continuous variable using interpolation between grid-points in time-space, thus consistently allowing the model to incorporate travel time uncertainty. The proposed specification avoids loops between states, which is part of the reason why the case study model can be estimated and used for simulation even with a large number of locations (1240). Previous specifications and implementations have only allowed for the evaluation of small example problems (with ≈ 20 locations) and still required minutes for evaluating the choice probabilities for a single individual.

Finding the optimal decision rule in a DDCM requires evaluating every possible state-action combination. This process can be computationally very demanding when the number of such combinations is large. It may not seem behaviourally realistic that individuals actually solve this complicated problem. As noted by Rust (1988), an adaptation and learning algorithm may provide a better explanation of how people may find the optimal decision rule in an MDP and thus act *as if* they were utility maximizers. It is possible to find an approximate solution to the optimal decision rule in an MDP using reinforcement learning which some authors have argued represents a good description of how people plan their days (see, e.g., Arentze and Timmermans 2004a). MDP's of activity-scheduling have been solved using reinforcement learning by Vanhulsel et al. (2009) to model the sequential planning of activities. Sidney (2018) proposed in his thesis how the decision rules in an MDP for activity scheduling can be estimated using inverse reinforcement learning. Karlström, Waddell, and Fox (2009) also investigated how a Boltzmann machine can be used to solve an MDP of the DDCM type proposed in this paper.

The paper is structured as follows. In Section 2 of this paper, we formulate the choice of a daily activity-travel pattern as a Markov Decision Process (MDP). Following the random utility maximizing approach, we assume that individuals maximize the expected achieved utility from a path through activity-location-time-space throughout one day. In the presence of stochasticity, a utility-maximizing agent will derive a decision policy outlining how to act conditionally on the outcome of the stochastic process. Such a policy could be interpreted as a contingency plan, specifying which travel pattern to choose for each possible combination of outcomes of the stochastic process. This is in contrast to available models where utility-maximization of an activity-travel pattern has been modelled, e.g., Starchild, MATSim, HAPP and the multistate-supernetwork. One can formulate the choice of an activity-travel pattern as a possible policy where the decisions are fixed before the day start, but the optimal policy will in general not be a fixed activity-travel pattern.

In Section 3, we follow the work by Rust (1987) to formulate the MDP of Section 2 as a Dynamic discrete choice model (DDCM). The DDCM gives state dependent choice probabilities that are a function not only of the state but also of the expected future utility conditional of the chosen action. We discuss how the expected value function can be approximated using linear interpolation when modelling time as a continuous variable. With the obtained expected value functions it is possible to calculate choice probabilities that can be used for simulation and maximum likelihood estimation. Following Fosgerau, Frejinger, and Karlström (2013), we also show how the model in a special case degenerates into an MNL model over sequences of actions and thus can be estimated using sampling of alternatives. Estimates obtained using sampling of alternatives are in contrast to direct maximum likelihood estimates not sensitive to approximations used when calculating the value function.

In section 4, we present a case study where a model over a working day is specified and estimated on a travel survey. The case study presented in this paper focuses on the special case that can be estimated using sampling of alternatives and thereby excludes travel time uncertainty. By focusing on this special case, we can assess the effects of the expected value function approximation on aggregate predictions. In a separate paper, this case study has been extended with a mixed-logit specification (Zimmermann et al. 2018). Section 5 discusses the results, and section 6 concludes.

2. Activity travel planning as a markov decision process

In this section, a Markov Decision Process (MDP) is formulated to model the choice of a daily activity-travel pattern. The activity-travel pattern can subject to time-space constraints have any number of trips that each includes the choice of purpose, duration, and mode of transport.

A sequence of actions forming a path between states where a state s_k may define the location l and time of day t among other things represents the daily activity-travel patter. The set of all states is denoted by S and the set of states available at a specific point in time t for S_t. We will let time t be a continuous state variable but assume that decisions are taken at discrete points in time. The index k is used to denote the order of the state s_k in the sequence of states that are traversed during the day. In each state, an individual can choose an action $a_k \in C(s_k)$ where $C(s_k) \subset C$ defines the subset of discrete actions which are feasible in the specific state s_k. An action may define, e.g., type and duration of an activity or destination and mode of transport of a trip. If the environment is stochastic, the state s_{k+1} reached when choosing a_k in state s_k may be uncertain and given by some probability density function $q(s_{k+1}|a_k, s_k)$. Daily variations in travel times may cause such uncertainty. It may also be that a friend suddenly calls, that a meeting is canceled or that a family member is expected to call and ask for a lift. One could have a state which represents the need to perform an activity on a specific day and allow the stochastic process q to model how this need evolves over a day, resembling how need-based models such as Arentze, Ettema, and Timmermans (2011) captures how the need to perform activities evolve between days over a week. We assume that individuals are aware of the stochasticity introduced by q and take it into account when making decisions. The agents preferences for taking a decision a_k in a specific state s_k and reaching the state s_{k+1} is represented by the one-stage utility function $u(s_k, a_k, s_{k+1})$. The utility function is dependent on factors such as the travel time (which is dependent on s_k and s_{k+1}), travel cost, the time spent performing different activities and the time-of-day when they take place. Together the Markov decision process consists of: the state space S; decision space C and constrained choice set $C(s_k)$; transition probabilities q; and one stage utility functions u. An activity-travel pattern can be defined by the sequences $\mathbf{s}$ and $\mathbf{a}$ of actions and states traversed during a day respectively. We will assume that an individuals preference for an activity-travel pattern can be represented by some utility function $U(\mathbf{s}, \mathbf{a})$:

$$U(\mathbf{s}, \mathbf{a}) = \sum_{k=0}^{K} u(s_k, a_k, s_{k+1}).$$

A rational agent that starts in a state s would behave according to a policy π, determining the action $a_k = \pi(s_k)$ to take when in state s_k, that maximizes the expected future utility of a day. The expected future utility conditional on a state is the *value function* in that state and is defined by:

$$V(s) = \max_{\pi} E_{\mathbf{s}} \left\{ \sum_{k=0}^{K} u(s_k, a_k, s_{k+1}) \middle| s_0 = s, a_k = \pi(s_k) \right\} \tag{1}$$

where the expectation $E_{\mathbf{s}}$ is with respect to the stochasticity of s_k given the decision rule $a_k = \pi(s_k)$ and K is the maximum number of decision stages in a day. Observe that since individuals are assumed to consider the expected value of one-stage utilities, one can without loss of generality exchange $u(s_k, a_k, s_{k+1})$ for $u(s_k, a_k) = E_{s_{k+1}}[u(s_k, a_k, s_{k+1})]$, and we will in the future use this expected one-stage utility to simplify notation. However, in the context of activity scheduling with stochastic travel time, the one-stage utility will in general be dependent on the stochastic process.

The transition probabilities q, the one-stage utility function u and choice-set $C(s_k)$ are assumed to be Markovian: conditional on s_k and a_k they are independent of the history. The state should, therefore, include all information necessary to formulate the choice set and one-stage utility functions. The Markov assumption is not a problem in theory, as the state could be defined to include all previous history in a finite horizon model. However, due to the curse of dimensionality, it is in practice important to limit the part of the history which is stored in a state in order to reduce the size of the state space S and obtain a tractable model. For example, a state variable can represent the mode of transport used on a previous trip, but it would be untractable to let a state variable represent the total number of minutes spent traveling with each mode. It is possible to introduce a state variable related to the type of activities that have been performed during a day, but it would not be feasible to remember *where* they were performed. The fact that the state space exclude certain factor, such as previous locations, does necessarily mean that future choices are independent of these factors. Where an individual has been will influence where they are and how much time they have left for other activities, which will impact their future destination choices.

As discussed above, an MDP is defined by a tuple (S, C, q, u) where: S is the state space; C is the set of decision for which there exist a function $C(s)$ specifying the set of available actions in a specific state; q is the transition probabilities defining the probability to reach a new state reached when taking a specific action in a state; and u is the one-stage utility function defining the immediate reward obtained from a specific state-action pair. We will in the following describe how respectively part of the MDP is defined, starting with the definition of action a_k and the set of all actions C.

2.1. Actions

The decision variables which defines an action a_k are: destination $\tilde{d} \in L$; mode of transport $\tilde{m} \in M$; and purpose $\tilde{p} \in P$. Here L, M, and P define the set of locations, modes and purposes respectively. The destination variable d stores the index of the location among N_L possible locations, so $d \in L = \{1, 2, \ldots, N_L\}$. The set of modes can contain, e.g., car, public transport, walk, and bike, or potentially any combination of such modes that can be used to travel between a specific origin-destination pair. Besides these modes, the set will also contain an option for staying in the current location. The number of modelled modes is denoted N_M and so the set of modes is given by: $\tilde{m} \in M = \{m_{\text{stay}}, m_1, \ldots, m_{N_m}\}$, where m_{stay} denotes the option to stay and continue with the current activity. The purpose is one out of N_{act} activities ($p \in P = \{p_1, \ldots, p_{N_P}\}$), where p_{home} is an important special case. To summarise, an action a_k can be written as:

$$a_k = \begin{pmatrix} \tilde{d} \\ \tilde{m} \\ \tilde{p} \end{pmatrix} = \begin{pmatrix} \text{destination} \\ \text{mode-of-transport} \\ \text{purpose} \end{pmatrix} \in \begin{Bmatrix} \{1, 2, \ldots, N_L\}, \\ \{m_{\text{stay}}, m_1, \ldots, m_{N_m}\}, \\ \{p_1, \ldots, p_{N_P}\} \end{Bmatrix} = \begin{Bmatrix} L \\ M \\ P \end{Bmatrix} \tag{2}$$

The set of all possible actions C is given by all possible combinations of L, M and P and the total number of alternatives is denoted by N_C.

2.2. States

A state s_k must firstly consist of the current location $l \in L$ and time-of-day t which is modelled as a continuous variable between 0 and T, i.e., $t \in [0,T]$. A number of state variables can then be defined to represent the part of the history which may influence future choices. A state variable $p \in P$ will be used to define the purpose of the action leading to the current state. The state variable $\tau \in \{0,1,2,\ldots,N_{\tau,p}\}$ stores the number of times the individual has chosen to continue the current activity p, where $N_{\tau,p}$ is the maximum memory for purpose p. The state variable τ allows for activities to have a maximum duration and for the marginal utility of performing an activity to be dependent on the previous duration. The maximum value of $N_{\tau,p}$ for any purpose p is denoted N_τ. The state variable $m \in M$ is used to allow for interdependence among the mode choice between subsequent trips. A state variable h stores the relevant history related to previously performed activities in the form of an index. This variable could, in principle, store the number of times each activity has been performed during the day. It could also store information about the need to perform different activities. Let the maximum number of such different histories and need combinations that are modeled be N_h. Then $h \in \{1,2\ldots,N_h\}$. Finally, we let a state vector $\epsilon_k \in \mathbb{R}^{N_C}$ represent the non-modeled random attributes of the available actions. In total, the state s_k is given by:

$$s_k = \begin{pmatrix} t \\ l \\ p \\ \tau \\ m \\ h \\ \epsilon_k \end{pmatrix} = \begin{pmatrix} \text{time-of-day} \\ \text{location} \\ \text{purpose of previous action} \\ \text{time spent on current purpose } p \\ \text{previous/main mode of transport} \\ \text{activity history} \\ \text{random state vector} \end{pmatrix} \in \begin{Bmatrix} [0,T] \\ \{1,2,\ldots,N_L\} \\ \{p_1,\ldots,p_{N_P}\} \\ \{0,N_\tau\} \\ \{m_{\text{stay}},m_1,\ldots,m_{N_m}\} \\ \{1,2\ldots,N_h\} \\ \mathbb{R}^{N_C} \end{Bmatrix} \tag{3}$$

In the future, we will also use the notation $x_k = (t,l,p,\tau,m,h)$ to denote the observable part of the state space, so that $s_k = (x_k,\epsilon_k)$.

2.3. Conditional choice sets

Defining the conditional choice set on the state s_k involves defining the set of purposes, destinations and modes that are available in a specific state s_k. We assume that the conditional choice set is independent of ϵ and use $C(x_k)$ as notation. With a few limitations, an individual can in each time step decide between: either staying ($m = m_{\text{stay}}$) at the same location l and perform the previous activity p for a while longer; or travel with a mode $\tilde{m} \in M(m)$ to a new destination $\tilde{d} \in L$ and start a new activity $\tilde{p} \in P$. The choice set is restricted in the following ways: Firstly, the type of locations that are available in a specific location may be limited. We denote the subset of locations where an agent can perform activity p with $L_{\text{act}}(p) \subset L$. Secondly, the number of times an agent can perform each activity in a day may be limited, so the activity history h may influence the set of available activities, and this subset of activities is denoted $P(h)$. Thirdly, the maximum duration of an activity may be limited, limiting the maximum value of τ. Finally, when $\tau = 0$, the agent travelled in the previous action (a_{k-1}) but have yet to spend time performing an activity and will therefore stay at the current location and perform the activity for at least some minimum duration, so $\tilde{m} = m_{\text{stay}}$ and $\tilde{d} = l$. The limit of τ for activity p is denoted $\tau_{\max}(p)$. Finally, the set of alternatives including travelling is denoted by C_{travel} and the alternative to stay in an activity is denoted by C_{stay}. These sets and the conditional choice set conditional $C(x_k)$ are given by:

$$C_{\text{travel}}(h,m) = \left\{ \begin{array}{c|l} \tilde{d} & \tilde{d} \in L \\ \tilde{m} & \tilde{m} \in M(m) \\ \tilde{p} & \tilde{p} \in P(h), \tilde{d} \in L_{\text{act}}(\tilde{p}) \end{array} \right\}, \; C_{\text{stay}}(l,p) = \left\{ \begin{array}{l} l \\ m_{\text{stay}} \\ p \end{array} \right\} \tag{4a}$$

$$C(x_k) = \begin{cases} C_{\text{travel}}(h,m) \cup C_{\text{stay}}(l,p) & \text{if } \; 0 < \tau < \tau_{\max}(p) \\ C_{\text{travel}}(h,m) & \text{if } \quad \tau = \tau_{\max}(p) \\ \qquad\qquad C_{\text{stay}}(l,p) & \text{if } \quad \tau = 0 \end{cases} \tag{4b}$$

Observe that the constraints on the choice set presented above by no means are an exhaustive list of the type of constraints that could be modeled by the conditional choice set. There are, for example, no constraints specified on when to perform activities. Any constraint could be modeled either through restrictions on the choice set or through the one-stay utility functions. We have chosen to leave time-of-day related constraints to the specification of the one-stage utility function.

2.4. Conditional state transitions

The state s_{k+1} reached when taking action a_k in state s_k is the outcome of a random process described by distribution $q(s_{k+1}|s_k, a_k)$. To distinguish between the state variables in state s_{k+1} and s_k we will use the notation $i^{(k)}$ and $i^{(k+1)}$ for each element in the state vector. The transitions of the state variables for location l, purpose p, time spent on activity τ and previous mode m are determined directly from the state-action pair. The new location is simply the destination of the action, so $l^{(k+1)} = \tilde{d}$. The state variable for purpose is likewise defined as the purpose of the previous action, so $p^{(k+1)} = \tilde{p}$. The counter $\tau^{(k)}$ is reset to zero whenever traveling is performed and increases with one every time the choice is to stay until its maximum value $N_{\tau,p}$ is reached, so $\tau^{(k+1)}$ is given by (5d). The new mode state variable $m^{(k+1)}$ is assumed to be given by its previous value $m^{(k)}$, the mode of transport $\tilde{m}$ and the chosen purpose $\tilde{p}$ and the mapping is denoted $f_m(m^{(k)}, \tilde{p}, \tilde{m})$.

The stochasticity of the process is gathered in the transition of the state variables for time t and activity history h and ϵ, so $q(s_{k+1}|s_k, a_k) = q(t^{(k+1)}, h^{(k+1)}, \epsilon^{(k+1)}|s_k, a_k)$. We assume that the evolution of the need to perform activities ξ may be dependent on the duration that expires between two time steps, but that the distribution of t is independent on that of h; that the observable state variables x_{k+1} in stage $k+1$ provide sufficient statistic to determine ϵ_{k+1} which thus is conditionally independent on the action; and that the stochasticity in the time dimension only stems from uncertainty in travel times, and thus depends on the departure time, mode of transport, origin, and destination. The distribution of $t^{(k+1)}$ is then given by (5a). The evolution of the activity history h is assumed to be dependent only on the time of day, the previous value of h and the purpose of the action $\tilde{p}$, and is thus given by (5f). Finally, the transition probability q factors as:

$$\begin{aligned}q(t^{(k+1)}, h^{(k+1)}, \epsilon^{(k+1)}|s_k, a_k) =& q_\epsilon(\epsilon^{(k+1)}|x^{(k+1)}) \\ &\cdot q_h(h^{(k+1)}|t^{(k+1)}, t^{(k)}, h^{(k)}, \tilde{p}) \\ &\cdot q_t(t^{(k+1)}|t^{(k)}, l^{(k)}, \tilde{m}, \tilde{d}).\end{aligned}$$

In summary, the state transitions are given by:

$$t^{(k+1)} \sim q_t(\cdot|t^{(k)}, l^{(k)}, \tilde{m}, \tilde{d}) \tag{5a}$$

$$l^{(k+1)} = \tilde{d} \tag{5b}$$

$$p^{(k+1)} = \tilde{p} \tag{5c}$$

$$\tau^{(k+1)} = \begin{cases} \min\left(\tau^{(k)} + 1, N_{\tau,p}\right) & \text{if } \tilde{m} = m_{\text{stay}} \\ 0 & \text{else} \end{cases} \tag{5d}$$

$$m^{(k+1)} = f_m(m^{(k)}, \tilde{p}, \tilde{m}) \tag{5e}$$

$$h^{(k+1)} \sim q_h(\cdot|t^{(k+1)}, t^{(k)}, h^{(k)}, \tilde{p}) \tag{5f}$$

$$\epsilon^{(k+1)} \sim q_\epsilon(\epsilon^{(k+1)}|x^{(k+1)}) \tag{5g}$$

2.5. One-stage utility functions

The one-stage utility $u(a_k, x_k)$ of taking an action a_k in the state s_k is assumed to be additatively separable as:

$$u(s_k, a_k) = u(x_k, a_k) + \epsilon_k(a_k).$$

The observable component of the utility, $u(x_k, a_k)$ is further decomposed into four components. Firstly, the expected (dis)utility of travelling $u_{\text{travel}}(t, l, m, \tilde{d}, \tilde{m})$ which is dependent on the origin l, destination $\tilde{d}$, departure time t, mode of transport $\tilde{m}$ and the mode state variable m. Secondly, an interaction component $u_{\text{change}}(p, \tilde{p})$ to model interdependence between the previous purpose p and the new purpose $\tilde{p}$. Thirdly, a utility component obtained from starting an activity $u_{\text{start}}(l, t, h, \tilde{p})$ which captures the availability of services related to purpose $\tilde{p}$ at a specific location l, time-of-day t related preferences for when to start the activity and how the need to perform an activity is dependent on the activity history h. Constraints on when a certain activity can be started in a specific location can be implemented by setting $u_{\text{start}}(l, t, h, p) = -\infty$ if a specific activity p is unavailable at location l at time t. Fourthly, the utility derived from participating in an activity $u_{\text{act.}}(t,p,\tau)$ dependent on the time of day t, the type of activity p and the number of time steps it has been performed τ. The one-stage utility can finally be written as:

$$u(s_k,a_k) = \mu_{x_k}\cdot\epsilon_k(a_k) + \begin{cases} u_{\text{act.}}(t,p,\tau) & \text{if } \tilde{m}=m_{\text{stay}},\ \tau>0 \\ u_{\text{act.}}(t,p,\tau)+u_{\text{start}}(l,t,h,p), & \text{if } \tilde{m}=m_{\text{stay}},\ \tau=0 \\ u_{\text{travel}}(t,l,m,\tilde{d},\tilde{m})+u_{\text{change}}(p,\tilde{p}) & \text{if } \tilde{m}\neq m_{\text{stay}} \end{cases} \tag{6}$$

3. Dynamic discrete choice model specification

In section 2 we defined an MDP consisting of a state space (3), an action space (2,4), transition rules (5), and utility functions (6). We further assumed that individuals act as if they seek a policy which maximize the value function defined by (1). Following Rust (1987), we have assumed that the random state variable ϵ_k is conditionally independent on the previous state and action and enters the one-stage utility additatively. Observe that value function $V(s)$ in (1) can be defined recursively through Bellman's equation as (Bellman 1957, Rust 1987):

$$V(x_k,\epsilon_k) = \max_{a_k}\{u(x_k,a_k)+\epsilon_k(a_k)+EV(x_k,a_k)\} \tag{1}$$

where $EV(x_k,a_k)$ is the expected value of the value function of the state reached when taking action a_k in state (x_k,ϵ_k). If $EV(x_k,a_k)$ is known for each state-action pair, the principle of optimality states that the optimal policy π is obtained by conditionally on a state s_k choose the action a_k which maximize the one-stage problem in (1). $EV(x_k,a_k)$ is given by:

$$\begin{aligned} EV(x_k,a_k) &= E_{t^{(k+1)},h^{(k+1)},\epsilon_{k+1}}\left[V(x_{k+1},\epsilon_{k+1})\mid x_k,a_k\right] \\ &= \int_{t'}\left(\sum_{j=1}^{N_h} q_h(h_j|t',t,h,\tilde{p})\cdot\bar{V}(x_{k+1})\right)dq_t(t'|t,l,\tilde{m},\tilde{d})). \end{aligned} \tag{2}$$

where in turn $\bar{V}(x_k) = E_{\epsilon_k}\left[V(x_k,\epsilon_k)\right]$. The *expected value function* $\bar{V}(x_k)$ will be used to denote the expected value of the value function *before* the state variables ϵ_k has been observed. We will finally assume that $q_\epsilon(\cdot|x_k,a_k)$ is given by the Gumbel distribution translated to obtain a zero mean. When $\epsilon_k(a_k)$ is i.i.d Gumbel distributed (with zero mean), $\bar{V}$ is given by the log-sum:

$$\bar{V}(x_k) = \log\left(\sum_{a_k\in C(x_k)} e^{u(x_k,a_k)+EV(x_k,a_k)}\right). \tag{3}$$

With i.i.d Gumbel distributed ϵ_k, the probability that an action a_k will be the utility maximizing alternative in a state x_k when ϵ_k is unobserved is simply given by the Multinomial Logit Model (MNL):

$$P(a_k|x_k) = \frac{e^{u(x_k,a_k)+EV(x_k,a_k)}}{\sum_{\tilde{a_k}\in C(x_k)} e^{u(x_k,\tilde{a_k})+EV(x_k,\tilde{a_k})}} \tag{4}$$

Allowing for correlation between alternatives available in a state is also possible by instead assuming that ϵ follows a generalized extreme value distribution (as illustrated for the recursive logit specification by Mai, Fosgerau, and Frejinger 2015, Mai 2016).

3.1. Solving the value function

Finding the optimal policy and calculating the choice probabilities using (4) requires computing the expected value function in each state x_k. The equation system consisting of (2) and (3) has a solution under regularity conditions outlined in Rust (1988). In infinite horizon models, e.g., the recursive logit model for route choice (Fosgerau, Frejinger, and Karlström 2013), it is possible to revisit the same state twice, and the value function in future states s_{k+1} is thus dependent on the value function in the current state s_k. As the problem described here has a finite horizon, and every action moves forward in time, it is possible to specify the model so that no such loops exist. The expected value function $\bar{V}$ can, therefore, be calculated by starting at the terminal time step T and moving backward in time. This solution method is often called backward induction. As time is a continuous variable, there is an infinite number of states, so backward induction cannot be used to calculate the expected value function in each state.

3.1.1. Approximation of the value function The expected value function is approximated in a number of discrete time steps. When it's value is needed between these time steps some approximating function is used. The time steps at which the expected value function is calculated will below be denoted t_j and the distance between time steps is Δt, so that $t_{j+1} - t_j = \Delta t$. There are thus a total of $N_T = T/\Delta t$ time steps where the expected value function is calculated. As the expected value function at the discrete time steps are obtained by using interpolated approximations, they are themselves approximations, and $\tilde{V}(x)$ will be used to denote these approximated expected value functions. For the notations below, let x_{k,t_j} specify that $t^{(k)} = t_j$. The values between these time steps is obtained by the function $g(\tilde{V}, x_k)$, and these approximate values are used when calculating an approximative solution to $EV(x_k, a_k)$:

$$\widetilde{EV}(x_k, a_k) = \int_{t'} \left(\sum_{j=1}^{N_h} q_h(h_j | t', t, h, \tilde{p}) \cdot g(\tilde{V}, x_{k+1}) \right) dq_t(t' | t, l, \tilde{m}, \tilde{d})) \tag{5}$$

If linear interpolation is used for approximation, g is obtained by the expected value function at the time steps $j-1$ and j where $t_{j-1} \le t^{(k)} \le t_j$ as:

$$g_{\text{linear}}(\tilde{V}, x_k) = \alpha_1 \tilde{V}(x_{k,t_j}) + \alpha_2 \tilde{V}(x_{k,t_{j+1}}) \tag{6}$$

where $\alpha_1 = \frac{t_{j+1} - t}{\Delta t}$ and $\alpha_2 = \frac{t - t_j}{\Delta t}$. With this approximation, we can give an example of how to account for stochastic travel time in the dynamic discrete choice specification.

Example: Uniform travel time Assume that there is no transition in h so that q_h can be neglected in (5). Further assume that the travel time between location $l^{(k)}$ and destination $\tilde{d}$ is described by a uniform random variable as:

$$q_{t,\text{U}}(t' | t, l, \tilde{m}, \tilde{d}) = \begin{cases} \frac{1}{b-a} & \text{if } a \le t' - t \le b \\ 0 & \text{else} \end{cases}$$

for some a and b such that $b > a \geq 0$. Let n_a and n_b denote the index of the time step before $a + t^{(k)}$ and after $b + t^{(k)}$ so that $t_{n_a} \leq a + t^{(k)} \leq t_{n_a+1}$ and $t_{n_b-1} \leq b + t^{(k)} \leq t_{n_b}$ respectively. Then, with the linear approximation function in (6):

$$\begin{aligned}\widetilde{EV}(x_k, a_k) &= \int_{t'} g_{\text{linear}}(\bar{V}, x_{k+1,t'}) dq_{t,\text{U}}(t'|t, l, \tilde{m}, \tilde{d})) \\ &= \sum_{j=n_a}^{n_b} \beta_j \cdot \bar{V}(x_{k+1,t_j})\end{aligned}$$

where:

$$\beta_j = \frac{1}{b-a} \cdot \begin{cases} 1 & \text{if } n_a + 2 \leq j \leq n_b - 2 \\ 0.5 + \frac{t_{n_a+1} - t^{(k)} - a}{\Delta t} & \text{if } j = n_a + 1 \\ \frac{t^{(k)} - a - t_{n_a}}{\Delta t} & \text{if } j = n_a \\ 0.5 + \frac{t^{(k)} - b - t_{n_b - 1}}{\Delta t} & \text{if } j = n_b - 1 \\ \frac{t_{n_b} - t^{(k)} - b}{\Delta t} & \text{if } j = n_b \end{cases}$$

3.1.2. Backward induction With the general model specification above in mind, it is possible to calculate the expected value function using backward induction. As discussed before, this calculates the expected value function in each state x_k by moving backward in time, starting at time $t = T$ and ending in $t = 0$. Starting the algorithm requires that the value of $\tilde{V}$ at time $t = T$ is defined. Let the terminal value of a state x_T be given by some function $J_T(x_T)\, \forall x_T \in X_T$, where X_T is the set of feasible states at time T. One can define the value at the end of the day so that it is required that an agent returns home before time T or that a specific set of activities h_T is performed during the day by setting $J_T(x_T) = 0$ if $l = l_{\text{home}}$ and $h = h_T$ and $J_T(x_T) = -\infty$ else. The backward induction solution method can now be described by the pseudo-code in Algorithm 1. Observe that it is important that $\tilde{V}$ is updated in $\tau = 0$ before the log-sum of travelling is calculated, as $\tilde{V}$ at the destination may be dependent on the value of $\tilde{V}$ for $\tau = 0$ at the same time step. Let $Q(x_k, a_k) = Q(t, l, h, \tilde{p}, \tilde{m}, \tilde{d})$ return a vector q such that $EV(x_k, a_k) = q \cdot \tilde{V}$.

3.2. Maximum likelihood estimation

We have so far suppressed the parameters θ from the specification of the utility functions, as well as the dependence of the individual i_n that the model describes. Suppose that we have observed an individual i_n who have made sequences of actions $\mathbf{a}_n = \{a_{0,n}, a_{1,n}, \ldots, a_{K_n,n}\}$ and reached states $\mathbf{x}_n = \{x_{0,n}, x_{1,n}, \ldots, x_{K_n+1,n}\}$. The likelihood for the observation of individual i_n is then given by:

$$L_n(\mathbf{a}_n, \mathbf{x}_n | x_0, \theta) = \prod_{k=0}^{K_n} P_n(a_{k,n} | x_{k,n}, \theta_u) \cdot q(x_{k+1} | a_{k,n}, x_{k,n}, \theta_q)$$

Let N observations form the set of observations $\mathscr{O}_N$. The log-likelihood function for $\mathscr{O}_N$ based on the conditional likelihoods becomes:

$$\bar{\mathscr{L}}\mathscr{L}(\mathscr{O}_N; \theta) = \sum_{n=1}^{N} \log\Big(L_n(\mathbf{a}_n, \mathbf{x}_n | x_{0,n}, \theta)\Big)$$

Algorithm 1 Backward induction algorithm used to calculate the expected value function in all states.

$\tilde{V} = \text{zeros}(N_{\text{T}}, N_L, N_P, N_\tau, N_M, N_h) - \infty$
for $x_T \in X_T$ **do**
    $\tilde{V}[x_T] = J_T[x_T]$
end for

for $k \in \{N_{\text{T}} - 1, N_{\text{T}} - 2, \ldots, 0\}$ **do**
    for $m \in M, h \in \{1, 2, \ldots, N_h\}$,$l \in L, p \in p \in P(h)$ s.t $d \in L_{\text{act}}(P)$ **do**

        $v_{\text{stay}} = [\,]$
        for $\tau \in \{0, 1, 2, \ldots, \tau_{max}(p)\}$ **do**
            $u = u_{\text{act}}(t_k, p, \tau) + (\tau == 0) \cdot u_{\text{start}}(l, t_k, h, p)$
            $EV = Q_{\text{stay}}(t_k, l, m, h, p, \tau, h) \cdot \tilde{V}[:]$
            $v_{\text{stay}}[\tau] = \exp(u + EV)$
        end for
        $\tilde{V}[t_k, l, p, \tau = 0, m, h] = \log\left(v_{\text{stay}}[0]\right)$

        $v_{\text{travel}} = 0$
        for $\tilde{d} \in L, \tilde{m} \in M(m), \tilde{p} \in P(h)$ s.t $d \in L_{\text{act}}(P)$ **do**
            $u = u_{\text{travel}}(t_k, l, m, \tilde{d}, \tilde{m}) + u_{\text{change}}(p, \tilde{p})$
            $EV = Q_{\text{travel}}(t_k, l, m, p, h; \tilde{d}, \tilde{m}, \tilde{p}) \cdot \tilde{V}[:]$
            $v_{\text{travel}} = v_{\text{travel}} + \exp(u + EV)$
        end for

        for $\tau \in \{0, 1, \ldots, \tau_{max}(p)\}$ **do**
            $\tilde{V}[t_k, l, p, \tau, m, h] = \log\left((\tau < \tau_{max}(p)) \cdot v_{\text{stay}}[\tau] + (\tau > 0) \cdot v_{\text{travel}}\right)$
        end for

    end for
end for

If it is possible to compute $L_n(\mathbf{a}_n, \mathbf{x}_n | x_{0,n}, \theta)$ (and its gradients), any standard optimization algorithm can to find the parameter vector θ. Below we will discuss two ways in which the likelihood function can be obtained, starting with the nested fixed point method (NFXP), which is the standard method for estimation of dynamic discrete choice models.

3.2.1. Estimation using NFXP NFXP relies on directly using (4) to calculate the conditional choice probabilities $P_n(a_{k,n}|x_{k,n},\theta)$, and is the standard method for estimating dynamic discrete choice models since Rust (1987). To obtain the choice probabilities from, (4) one must first calculate the value of EV in all states. As EV depends on the parameter values, the value of EV (and its gradients) must be updated when evaluating the likelihood-function for a new parameter vector. Algorithm 1 must thus be used to obtain $\bar{V}$ for all individuals for each trial parameter vector in an optimization algorithm. Using Algorithm 1 as an inner algorithm within the optimization algorithm is what constitutes the nested fixed-point problem in the NFXP algorithm.

3.2.2. Special case: Estimation using sampling of alternatives The fact that EV must be calculated using Algorithm 1 for each trial parameter vector makes the NFXP algorithm computationally demanding and further means that any parameter estimates obtained may contain bias due to the approximation used when computing EV. As a special case, the probability for a sequence of actions according to a DDCM as in (4) degenerates into an MNL model over sequences of actions (Fosgerau, Frejinger, and Karlström 2013). Under these conditions, it is possible to estimate a DDCM on a subset of sampled action sequences and obtain consistent but inefficient estimates (McFadden 1978). When using sampling of alternatives estimation, the approximation used for calculating EV will not influence the parameter estimates, but these approximations will still be required when using the model for simulation. Parameters obtained using sampling of alternatives could thus be used to evaluate the aggregate bias introduced by the approximation.

The equivalence between an MNL model over sequences and a DDCM occurs when: 1) each state-action (x_k, a_k) pair deterministically gives a new state x_{k+1} which thus can be described by a mapping $x_{k+1} = f(x_k, a_k)$; and 2) ϵ is i.i.d Gumbel distributed (Fosgerau, Frejinger, and Karlström 2013). Under these restrictions EV from (2) becomes: $EV(x_k, a_k) = \bar{V}(f(x_k, a_k))$. Thus:

$$P(a_k|x_k) = \frac{e^{u(x_k,a_k)+\bar{V}(f(x_k,a_k))}}{\sum_{a'_k \in C(x_k)} e^{u(x_k,a'_k)+\bar{V}(f(x_k,a'_k))}} \tag{7}$$

$$= e^{\frac{1}{\mu}\left(u(a_k,x_k)+\bar{V}(x_{k+1})-\bar{V}(x_k)\right)} \tag{8}$$

where (3) together with (7) gives (8). Now consider a day-path, i.e., a sequence of decisions $\mathbf{a} = (a_0, ..., a_{K-1})$ which starts in a state x_0 and traverse the states $\mathbf{x} = (x_1, ..., x_K)$, where the time at the terminal state $t^{(K)} = T$. The likelihood for this sequence of decisions and states is, according to (8) given by:

$$P(\mathbf{a},\mathbf{x}|x_0) = \prod_{i=0}^{T-1} e^{u(a_i,x_i)+\bar{V}(x_{i+1})-\bar{V}(x_i)}$$
$$= e^{\sum_{k=0}^{T-1} u(a_k,x_k)+\bar{V}(x_K)-\bar{V}(x_0)}$$

Assume that the initial state is known and that there is a single state which is feasible at time T denoted by x_T, and use the notation $u(\mathbf{a}) = \sum_{k=0}^{T-1} u(a_k, x_k)$. Then the probability of a path $\mathbf{a}$ among the set $\mathscr{A}(x_0, x_T)$ of all action sequences that leads from x_0 to x_T is given by:

$$P(\mathbf{a}, \mathbf{x}|x_0) = \frac{e^{u(\mathbf{a})}}{\sum\limits_{\mathbf{a}' \in \mathscr{A}(x_0)} e^{u(\mathbf{a}')}}.$$

Estimation on a subset of the choice set involves selecting a choice set $\tilde{\mathbb{C}}_n \subset \mathbb{C}_n$ and estimating using the conditional choice probability $P_n(\mathbf{a}_n|\tilde{\mathbb{C}}_n)$ instead of the $P_n(\mathbf{a}_n|\mathbb{C}_n)$. Maximum likelihood estimation on a choice set $\tilde{\mathbb{C}}$ gives consistent estimates if the correction term $\log(\bar{q}_n(\tilde{\mathbb{C}}_n|j))$ is added to each alternative and $\bar{q}_n(\tilde{\mathbb{C}}_n|j)$ satisfies the positive conditioning property, i.e., if $j \in \tilde{\mathbb{C}}_n$ and $\bar{q}_n(\tilde{\mathbb{C}}_n|i) > 0$ for some i, then $\bar{q}_n(\tilde{\mathbb{C}}_n|j) > 0$. This holds if $\tilde{\mathbb{C}}_n$ is sampled from the universal choice set $\mathbb{C}_n$ and all alternatives in $\mathbb{C}_n$ have a non-zero probability of being sampled (McFadden 1978).

One way of constructing a choice set consists of drawing R alternatives with replacement from the choice set $\mathbb{C}_n$ consisting of J_n alternatives, and then adding the observed choice to the choice set. The outcome of such a protocol is $(k_{n1}, k_{n2}, \ldots, k_{nJ})$ where k_{nj} is the number of times alternative j appears in the choice set, so that $\sum_{j=1}^{J} k_{nj} = R + 1$, since the observed alternative j is added once extra to the choice set. Let $q_n(i)$ denote the probability that alternative $j \in \mathbb{C}_n$ is sampled. Under this sampling protocol, the estimates are consistent if the correction term $\log(\frac{k_{n\mathbf{a}_n}}{q_n(\mathbf{a}_n)})$ is added to each alternative (Frejinger, Bierlaire, and Ben-Akiva 2009). After adding the correction term, the likelihood function to use for estimation an individual n becomes:

$$\tilde{P}_n(\mathbf{a}) = \frac{e^{u(\mathbf{a}) + \log(\frac{k_{n\mathbf{a}_n}}{q_n(\mathbf{a}_n)})}}{\sum\limits_{\mathbf{a}^* \in \mathbb{C}_n} e^{u(\mathbf{a}^*) + \log(\frac{k_{n\mathbf{a}_n^*}}{q_n(\mathbf{a}_n^*)})}}$$

We propose using the true model with some previously obtained parameter values for sampling alternatives to the choice set. Using the true model with wrong parameters to generate a choice set first requires that the expected value function is calculated once using Algorithm 1 for all individuals. If the expected value function is known, it is simple to calculate choice probabilities and simulate daily travel patterns by starting in an initial state in the morning and simulating new actions until reaching the terminal time T. For all individuals, a choice set is generated with the following steps:

1. Set parameter values $\hat{\theta}$ to be used when generating choice sets
2. Calculate expected value function $\bar{V}_n(\hat{\theta})$ with parameters $\hat{\theta}$ using Algorithm 1
3. Create a choice set using the expected value function $\bar{V}_n(\hat{\theta})$:

   (a) Add the observed daily travel pattern $(\mathbf{a}_n, \mathbf{x}_n)$ and calculate it's choice probabilities using (4) for each state-action pair.

(b) Simulating and adding K_n daily travel patterns to the choice set by starting at state $x_{0,n}$ in the morning and ending at time T using the value function $\bar{V}_n(\hat{\theta})$ and parameters $\hat{\theta}$. In each state, choice probabilities used for simulation are calculated using (4).

Simulating an alternative daily travel pattern will typically be much less computationally demanding than calculating the value function, so generating large choice sets (with thousands of alternatives) have a small impact on the total estimation time. Observe that the parameters $\hat{\theta}$ used for sampling will influence the efficiency of the algorithm. It is important to choose these parameters so that the choice sets obtained contains sufficient variation while still resembling observed behavior.

4. Case study

This section presents a case study where a model is specified and estimated based on the method proposed in Section 2. We have limited this case study to the special case when there is no uncertainty in the transition of the state variables in x_k and when ϵ is i.i.d. Gumbel distributed. As discussed in 3.2.2, this special case can be estimated using sampling of alternative sequences of actions. Limiting the case study to this special case has the benefit that the parameter estimates obtained are independent on the potentially biasing approximations made when calculating the expected value function. These parameter values can, therefore, be used to evaluate the bias introduced by the approximations of the value function. The parameter estimates are used to generate a number of statistics, such as the departure times of trips and aggregate travel times.

We model individuals who work with fairly regular working hours on a day when they go to work. We assume that work is a mandatory activity as well as dropping off children at school or picking them up after school in case such activities are observed. People can have fixed working schedule, in which case the starting time for work is fixed, or flexible, in which case we assume that they are allowed to arrive between 6 am and 10 am. Children can be dropped off between 6 : 30 am and 12 : 00 am and picked up between 12 am and 6 : 30 pm. In the case study, an individual is defined by an observed sequence of actions $\mathbf{a}^n$ together with the following attributes:

$$i^n = \begin{pmatrix} l^n_{\text{home}} \\ l^n_{\text{work}} \\ l^n_{\text{child}} \\ \delta_{car} \\ \delta_{ptcard} \\ \delta_{flex} \\ t^n_{\text{work}} \\ \\ P^n_{\text{mandatory}} \\ \\ \end{pmatrix} = \begin{pmatrix} \text{home location} \\ \text{work location} \\ \text{child location} \\ \text{car ownership dummy} \\ \text{pt card dummy} \\ \text{flexible working hours dummy} \\ \text{work duration} \\ \\ \text{list of mandatory activities} \\ \\ \end{pmatrix} \in \begin{Bmatrix} \{1, 2, \ldots, N_L\} \\ \{1, 2, \ldots, N_L\} \\ \{1, 2, \ldots, N_L\} \\ \{0, 1\} \\ \{0, 1\} \\ \{0, 1\} \\ [0, T] \\ \begin{Bmatrix} [p_{\text{work}}], [p_{\text{child}}, p_{\text{work}}] \\ [p_{\text{work}}, p_{\text{child}}] \\ [p_{\text{child}}, p_{\text{work}}, p_{\text{child}}] \end{Bmatrix} \end{Bmatrix}$$

4.1. Model specification

Below we discuss how the MDP is specified in the case study. This involves defining the actions, states, conditional choice sets, state transitions and one-stage utility functions.

4.1.1. Actions Remember that an action consisted of the choice of a location, activity and mode. We have used the so called EMME zonal system of the region of Stockholm, which is standard to use in models in the area and for which official statistics are available. The Stockholm region consist of 1240 such zones, so $N_L = 1240$. The modes modelled are car, public transport, bike and walk. Activities are divided into home, work, child (pick up or drop off), social, recreational, grocery shopping and other.

$$a_k = \begin{pmatrix} \tilde{d} \\ \tilde{m} \\ \tilde{p} \end{pmatrix} \in \begin{Bmatrix} \{1, 2, \ldots, 1240\}, \\ \{m_{\text{stay}}, m_{\text{car}}, m_{\text{pt}}, m_{\text{walk}}, m_{\text{bike}}\}, \\ \{p_{\text{travel}}, p_{\text{end}}, p_{\text{home}}, p_{\text{work}}, p_{\text{child}}, p_{\text{rec.}}, p_{\text{social}}, p_{\text{shop}}, p_{\text{other}}\} \end{Bmatrix}$$

4.1.2. States We let a day start at 5 am and end at 11 pm, and let t define the number of 10 minute intervals since 5 am so that $T = 108$. We will for the case study let the marginal utility of activity participation be constant and use τ only to remember if the activity has been started ($\tau = 1$) or not ($\tau = 0$), so $N_\tau = \max N_{\tau,p} = 1$. The mode history will keep track of whether car is used as the mode of transport on a tour. The mode history can thus be described by three values: $m_{\text{stay}}, m_{\text{car}}$ and m_{pt}. The activity history stores the number of completed activities in the list of mandatory activities where the number of mandatory activities for an individual n is $N^n_{\text{mandatory}} \leq 3$.

$$s_k = \begin{pmatrix} t \\ l \\ p \\ \tau \\ m \\ h \\ \epsilon \end{pmatrix} \in \begin{Bmatrix} [0, 108] \\ \{1, 2, \ldots, 1240\} \\ \{p_{\text{home}}, p_{\text{work}}, p_{\text{child}}, p_{\text{rec.}}, p_{\text{social}}, p_{\text{shop}}, p_{\text{other}}\} \\ \{0, 1\} \\ \{m_{\text{stay}}, m_{\text{car}}, m_{\text{pt}}\} \\ \{0, 1, 2, 3\} \\ \mathbb{R}^{N_C} \end{Bmatrix}$$

4.1.3. Conditional choice-set The choice set $C^n(s_k)$ for an individual n requires the specification of $M^n(m)$, $L^n_{\text{act}}(\tilde{p})$ and $P^n(h)$. The available modes are:

$$M^n(m) = \begin{cases} \{m_{\text{pt}}, m_{\text{walk}}, m_{\text{bike}}\} & \text{if } \delta^n_{car} = 0 \\ \{m_{\text{car}}, m_{\text{pt}}, m_{\text{walk}}, m_{\text{bike}}\} & \text{if } \delta^n_{car} = 1, m = m_{\text{stay}} \\ \{m_{\text{car}}\} & \text{if } m = m_{\text{car}} \\ \{m_{\text{pt}}, m_{\text{walk}}, m_{\text{bike}}\} & \text{if } m = m_{\text{pt}} \end{cases}$$

Let $P_{\text{freetime}} = \{p_{\text{rec.}}, p_{\text{social}}, p_{\text{shop}}, p_{\text{other}}\}$. The locations available for a specific individual n is:

$$L^n_{\text{act}}(\tilde{p}) = \begin{cases} L & \text{if } \tilde{p} \in P_{\text{freetime}} \\ l^n_{\text{home}} & \text{if } \tilde{p} = p_{\text{home}} \\ l^n_{\text{work}} & \text{if } \tilde{p} = p_{\text{work}} \\ l^n_{\text{child}} & \text{if } \tilde{p} = p_{\text{child}} \end{cases}$$

Remember that for each individual we have an ordered list of mandatory activities $P^n_{\text{mandatory}}$ where $P^n_{\text{mandatory}}(i)$ is a set containing the i:th activity in list. The available activities conditional on history can now be defined as:

$$P^n(h) = P_{\text{freetime}} \cup \{p_{\text{home}}\} \cup \begin{cases} P^n_{\text{mandatory}}(h+1) & \text{if } h < N^n_{\text{mandatory}} \\ \emptyset & \text{else} \end{cases}$$

4.1.4. State transitions The unspecified state transition are q_t, q_h and f_m. As m is used to remember whether car was the first trip on a tour away from home it is defined as:

$$m^{(k+1)} = \begin{cases} m_{\text{car}} & \text{if } \tilde{m} = m_{\text{car}} \text{ and } m = m_{\text{stay}} \\ m_{\text{pt}} & \text{if } \tilde{m} \neq m_{\text{car}} \text{ and } m = m_{\text{stay}} \\ m_{\text{stay}} & \text{if } \tilde{p} = p_{\text{home}} \\ m^{(k)} & \text{else} \end{cases}$$

As h denotes the number of finished mandatory activities, it is increased by one whenever a mandatory activity is performed:

$$h^{(k+1)} = \begin{cases} h^{(k)} + 1 & \text{if } \tilde{p} = P^n_{\text{mandatory}}(h^{(k)}) \\ h^{(k)} & \text{else.} \end{cases}$$

Finally, when there is no travel time uncertainty, $t^{(k+1)}$ is simply given by:

$$t^{(k+1)} = t^{(k)} + \begin{cases} TT_{\tilde{m}}(t^{(k)}, l^{(k)}, \tilde{d}) & \text{if } m \neq m_{\text{stay}} \\ t^n_{\text{min}}(\tilde{p}) & \text{if } m = m_{\text{stay}},\ \tau = 0 \\ \Delta_p t & \text{if } m = m_{\text{stay}},\ \tau \geq 1 \end{cases}$$

4.1.5. One-stage utility functions The one-stage utility of a state-action pair was in (6) dividable into four parts: the (dis)utility of travelling $u_{\text{travel}}(t, l, m, \tilde{d}, \tilde{m})$; the utility from activity participation $u_{\text{act.}}(t, p, \tau)$; the utility representing preferences for when to start and where to perform an activity $u_{\text{start}}(l, t, h, p)$; and a utility component for interaction between current and future purpose $u_{\text{change}}(p, \tilde{p})$ which is left out from this case study. Below follows the specification of the utility functions used in the case study, starting with the utility of travelling.

Utility of travelling The utility of travelling is given by:

$$u_{trav}(t, l, m, \tilde{d}, \tilde{m}) = \theta_{\tilde{m}} + \theta_{tt,\tilde{m}} \cdot TT_{\tilde{m}}(t, l, \tilde{d}) + \theta_{\text{cost}} \cdot C_{\tilde{m}}(t, l, \tilde{d}) + \theta_{\text{s.z.,walk}} \delta_{\text{s.z.,walk}} \quad (1)$$

where $\theta_{\tilde{m}}$ are mode specific constants; $\theta_{tt,\tilde{m}}$ are mode specific (dis)utilities for travel time $TT_{\tilde{m}}(t, l, \tilde{d})$ and $C_{\tilde{m}}(t, l, \tilde{d})$ denote the travel time and cost with mode $\tilde{m}$ at time t for a trip from origin l to destination $\tilde{d}$; and $\delta_{\text{s.z.,walk}}$ is a dummy indicating if the trip is done within the same zone with walk. Observe that the mode state m does not influence the one-stage utility in the case-study specification.

Utility of starting activity The utility of starting an activity $u_{\text{start}}(l, t, h, p)$ is here independent of h and given by:

$$u_{\text{start}}(l, t, p) = \begin{cases} u_{\text{size}}(l, p) & \text{if } p \in P_{\text{freetime}} \\ u_{\text{child constraints}}(t) & \text{if } p = p_{\text{child}} \\ u_{\text{work constraints}}(t) + u_{\text{start work}}(t) & \text{if } p = p_{\text{work}} \\ 0 & \text{if } p = p_{\text{home}} \end{cases}$$

where u_{size} models the number of opportunities that exist for each purpose in a specific location; $u_{\text{constraints}}$ models constraints on when activities can be performed; and $u_{\text{start time}}$ models preferences for when to start specific activities. The size utility is given by:

$$u_{p,\text{size}}(l) = \theta_{C,p} + \theta_{p,\text{size}} \log\left(\sum_{s=1}^{s=S_p} x_{p,l,s} e^{\theta_{p,s}}\right)$$

where S_p is the number of size variables for activity p, and the size variables $x_{p,l,s}$ can be, e.g., the number of employees in a specific sector at location l. Since the size utility contain an activity specific constant $\theta_{C,p}$, one of the parameters $\theta_{p,s}$ should be fixed for all activities. This also provides an alternative interpretation of the activity specific constants $\theta_{C,p}$ as scales for the size variables $x_{p,l,s}$. A complete list of size variables included for respectively activity is given in table 1.

Child and work constraints ensure that these activities are started within the opening hours of day-care and schools in the case of child activities and within the acceptable starting hours for work for the individual, so:

$$u_{\text{child constraints}}(t) = \begin{cases} 0 & \text{if } t_{\text{drop off after}} \leq t \leq t_{\text{drop off before}} \\ 0 & \text{if } t_{\text{pick up after}} \leq t \leq t_{\text{pick up before}} \\ -\infty & \text{else} \end{cases}$$

$$u_{\text{work constraints}}(t) = \begin{cases} 0 & \text{if } t^n_{\text{work start after}} \leq t \leq t^n_{\text{work start before}} \\ -\infty & \text{else} \end{cases}$$

The time-of-day dependent utility of starting work is specified at a number of points in time t_j by parameters θ_{work,t_j}. If work starts at a time t between two such time point t_j and t_{j+1}, i.e., if $t_j \leq t \leq t_{j+1}$, linear interpolation is used to obtain the start utility:

$$u_{\text{start work}}(t) = \frac{\theta_{\text{work},j} \cdot (t_{j+1} - t) + \theta_{\text{work},j+1} \cdot (t - t_j)}{t_{j+1} - t_j}$$

Utility of activity participation The utility of activity participation is modelled as:

$$u_{\text{act.}}(t,p,\tau) = \begin{cases} \theta_{t,p} \cdot \Delta_t & \text{if } p \in P_{\text{freetime}} \\ u_{\text{stay home}}(t) & \text{if } p = p_{\text{home}} \\ 0 & \text{else} \end{cases} \tag{2}$$

For the utility of staying at home for another time step, we specify the *marginal* utility of activity participation at time t as given by linearly interpolation between the closest parameters, so:

$$u_{\text{marginal stay home}}(t) = \frac{\theta_{\text{home},t_j} \cdot (t_{j+1} - t_j) + \theta_{\text{home},t_{k+1}} \cdot (t - t_j)}{t_{j+1} - t_j}.$$

The stay-utility of an activity episode of duration Δt_p, when $T_k \leq t$ and $t + \Delta t_p \leq T_{k+1}$, then becomes:

$$u_{\text{stay home}}(t) = \int_t^{t+\Delta t_p} u_{\text{marginal stay home}}(y)\,\mathrm{d}y = \alpha_{t_j}\theta_{\text{home},t_j} + \alpha_{t_{j+1}}\theta_{\text{home},t_{j+1}} \tag{3}$$

where:

$$\alpha_{t_j} = \Delta t_p \frac{t_{j+1} - t - 0.5\Delta t_p}{t_{j+1} - t_j}$$
$$\alpha_{t_{j+1}} = \Delta t_p \frac{t + 0.5\Delta t_p - t_j}{t_{j+1} - t_j}.$$

If $t + \Delta t_p > t_{j+1}$, $u_{\text{marginal stay home}}(y)$ in (3) becomes a stepwise linear function and additional factors $\alpha_{t_{j+2}}$ is added to the utility function.

4.1.6. Computational details With the current implementation, calculating the value function in all states and thus evaluating the probability of a path once for a single individual takes between $4 - 10\,\text{s}$ when using a single core on an Intel(R) Core(TM) i7-6820HQ CPU @ 2.70GHZ. Almost all (98%) of the computation time is consumed by the function calculating the value function in all end-activity states when summing up the alternative trips that can be started. When excluding working hours, an example individual has 11 h of free time left between 5 am and 11 pm. The value function is evaluated on a 10 minute grid, giving 65 grid points in which it will be evaluated. In each of these grid points, there are a total of 4 modes available for a car owner and 1 240 locations that are both states and destinations. This gives a total of $65 \cdot 4 \cdot 1\,240 \cdot 1\,240 \sim 4 \cdot 10^8$ links. For each of these links, one must calculate the travel time (taking 16% of the time), one-stage utilities (23%), obtain the future expected value function and sum this with the utility (41%) and finally calculating the exponent $e^{u_i + EV_j}$ (20%). The main program is written in C#, but Intel MKL has been used for vector mathematics when applicable and C++ routines have been used for other time-consuming parts. As a comparison, simply performing e^x in MATLAB for a vector x with 10^8 elements on the same computer takes 1 s. Observe that the from-all-to-all destinations operation is currently the computational limiting factor. One possible way to speed up the model evaluation would be to sample locations. If sampling 100 locations, the computation time could potentially decrease to 0.05 s/individual.

With 10 s/observation to calculate EV, 70 variables, 3 300 observations and 100 iterations before convergence, estimating the case study using NFXP would require $10 \cdot 0.4 \cdot 70 \cdot 3\,300 \cdot 100\,\text{s} \approx 1\,000$ days using a single core. The program is parallelized using MPI and could potentially benefit from hundreds of cores reducing the computation time to days. It would be possible to obtain efficient estimates and allow for nested logit or GEV formulations by starting with inefficient estimates obtained using sampling of locations or some alternative approximate or possibly biased estimation technique to find a good enough specification of the model and then finally obtain full-information estimates using a cluster.

4.2. Data

We have estimated the model using the Stockholm travel survey from 2004, where individuals report a full day travel diary. Estimating using full day-paths puts a high demand on the reported diaries since the information for all trips in an observation must be correct in order for it to be usable. Further, travel times as reported in the diaries are rarely the same as the data we have on travel times or the travel times we calculate for the same origin-destination, and sometimes the discrepancy is significant. We have, rather than using reported values, chosen to use level of service attributes obtained from a static traffic assignment model with a nested-logit based travel demand model which is available for the region of interest, The traffic assignment model produces travel times and costs for peak and off-peak periods respectively. It is worth noting that when considering day-paths, changing the travel time of an observed trip will influence the starting time and duration of all remaining actions on the same day. Travel cost with a car is calculated as 1.4 kr/km, travel times with a bike are calculated assuming a speed of 15 km/h and travel times when walking are calculated assuming a speed of 4 km/h.

We have so far limited the model to individuals that go to work on a weekday, leaving us with 5200 observations with sufficient information. Out of these, 3300 behave in a way that is modelled in the case study model. The ~ 2000 observations that are removed at this stage may, for example, be ending their workday with a business trip, and therefore not ending the workday at their work location which is required; have longer than $2h$ breaks in the middle of their workday; work late (later than 8pm) or start early (earlier than 6am); leave a car somewhere or not returning home before 11pm. We have demanded that car should be used for either all or no trips on a tour. As no separation has been made between drivers and passengers, observations with passengers are likely to be removed. Besides individuals reporting that they were passengers, 3% of the observations included a tour with a mix of car and other modes.

4.3. Estimation result

Table 1 gives estimation result for parameters in u_{start}, Table 2 parameters for the utility to travel u_{travel} and Table 3 gives the parameters influencing the utility obtained when participating in an activity u_{act}. Most parameters are significant and have the expected sign. Cost is negative and spending time on activities is preferred to spending time on traveling. Home time is valued higher early in the morning and late in the evening. Spending time on free-time activities is preferable to be spending time at home between 1-4 pm, and there is no significant difference between 5-9pm. Since not all time parameters can be identified, $\theta_{\text{Rec Time}}$ is fixed, and the linear-in-time parameters should only be compared against each other. Although the choice of the parameter to fix does not affect the theoretical properties of the estimates, it can impact the obtained standard deviation. When

$\theta_{\text{PT Time}}$ was fixed rather than $\theta_{\text{Rec Time}}$, the standard deviation of all time parameters was close to 0.006, rather than varying between 0.001 and 0.005. Travel time parameters are significantly smaller than activity duration parameters, so participating in an activity is preferred to traveling. Since time parameters can only be interpreted in relation to each other, it is not possible to calculate a single value of travel time saved. A travel time saving with car that gives one minute extra at home at 6 pm would be valued $(\theta_{\text{tt, car}} - \theta_{\text{5-6pm Time}})/\theta_{\text{cost}} = 7.3\,\text{SEK/minute}$.

The time-specific constants for work hours seem quite large in comparison to the time parameters, but this is mainly due to the scaling of the parameters. When comparing two alternative sequence, one that arrives at work at 6 am and arrive back home at 4 pm and one that arrives at work at 7 am and back home at 5 pm, the difference in utility per minute at work from arriving at the different times will be $(\theta_{\text{work, 6am}} - \theta_{\text{work, 7am}})/60 = -0.007$. This is greater than the difference between $\theta_{\text{home,6pm}}$ and $\theta_{\text{home,7pm}}$ but smaller than the difference between $\theta_{\text{home,5pm}}$ and $\theta_{\text{home,4pm}}$. The difference in the valuation of time spent at home at different times of the day and the difference in the valuation of time spent at work at different times of the day will, therefore, both be of equally important when determining departure time to work.

Interpreting the constants for mode and activities is not entirely straightforward. Firstly, they are all normalized by fixing $\theta_{\text{Home ASC}}$ to zero. Further, the scale of the size parameters is arbitrary and obtained by fixing one of the size parameters to zero.

As estimation is performed using sampling of alternatives efficient estimates are not obtained. Since the number of alternatives in the universal choice set so immense, it would be possible that the obtained estimates were very inefficient or biased. When validating the estimation by estimating using a simulated data set where the true parameters are known, the estimation result was often poor if parameters far away from the true parameters were used for generating choice sets. It is also possible that the approximations used to calculate $\widetilde{EV}$ would cause a bias when the model is used to simulate day-paths, as estimation does not take this approximation into account. To analyze the extent that sampling for estimation and approximation of value functions has on the mode output 1000 day-paths per individual was simulated and their aggregated attributes were compared to the observed data. The resulting differences can be found in Table 4. The simulated data deviates from the observed data by $0.1 - 1.7\%$, but a relative difference over one percent is only observed for attributes of small quantities. For example, the number of other activities are underpredicted by 0.002 times/day, meaning 2%. In absolute terms it translates to errors of 0.05 min/day in travel time, 0.14 SEK/day in travel cost and 0.005 trips/day per mode. We think that this error is small enough to be negligible in any practical applications.

Table 1 Parameters for utility of starting activity, u_{start}

Notation	Parameter		Estimate	Rob. t-test
Parameters for the utility to start work $u_{start\ work(t)}$ at a specific time-of-day. Scaled by setting $\theta_{work,8am} = 0$				
$\theta_{\text{work},6am}$	Work	ASC 6am	1.1	2.9
$\theta_{\text{work},7am}$		ASC 7am	0.68	3.5
$\theta_{\text{work},8am}$		ASC 8am	0	
$\theta_{\text{work},9am}$		ASC 9am	-1.4	-7.9
$\theta_{\text{work},10am}$		ASC 10am	-5.1	-12
Parameters for utility to start free-time activity, $u_{p,size(l)}$. Scaled by fixing one of the size-parameters to zero for each activity type.				
$\theta_{C,\text{shop}}$	Shop	ASC	-6.6	-37
$\theta_{\text{size, shop}}$		LSM Size	0.51	9.2
$\theta_{\text{pop, shop}}$		population	0	
$\theta_{\text{shop, shop}}$		emp. shop	3.4	11
$\theta_{C,\text{social}}$	Social	ASC	-9.2	-43
$\theta_{\text{size, social}}$		LSM Size	0.43	3.8
$\theta_{\text{pop, social}}$		population	0	
$\theta_{C,\text{rec.}}$	Recreative	ASC	-7.7	-47
$\theta_{\text{size, rec}}$		LSM Size	0.084	1.9
$\theta_{\text{pop, rec}}$		population	0	
$\theta_{\text{rest, rec}}$		emp. rest	5.8	8.5
$\theta_{C,\text{other}}$	Other	ASC	-7.3	-34
$\theta_{\text{size, other}}$		LSM Size	0.34	5.4
$\theta_{\text{oe, other}}$		emp other.	0	

Observe that as size-parameters enter the utility as e^{θ}, the t-test cannot be used to determine their significance. The population has been fixed to zero for Rec, Social and Shop whereas 'No employed Other' was fixed for Other.

Table 2 Parameters for utility of travelling, u_{travel}, see (1).

Notation	Parameter		Estimate	Rob. t-test
Parameters common for all modes.				
θ_{cost}		Cost	-0.012	-7.4
Mode specific parameters				
θ_{car}	Car	Constant	-2.7	-26
$\theta_{tt,\text{ car}}$		Time	-0.084	-17
θ_{bike}	Bike	Constant	-4.2	-31
$\theta_{tt,\text{ bike}}$		Time	-0.057	-13
θ_{walk}	Walk	Constant	-1.7	-17
$\theta_{tt,\text{ walk}}$		Time	-0.051	-24
$\theta_{\text{s.z, walk}}$		same zone	-0.53	-4.1
θ_{pt}	PT	Constant	-3.8	-38
$\theta_{tt,\text{ pt}}$		Time	-0.038	-4.9
$\theta_{wait,\text{ pt}}$		Wait time	0.0041	0.43

Table 3 Parameters for utility of participating in an activity, $u_{\text{act.}(t,p,\tau)}$ in (2), and Log-likelihood value for estimates.

Notation	Parameter		Estimate	Rob. t-test
Parameters for utility participating in free-time activities.				
$\theta_{t,\text{shop}}$	Shop	Time	-0.021	-15
$\theta_{t,\text{social}}$	Social	Time	0.00067	0.57
$\theta_{t,\text{rec.}}$	Recreative	Time		
$\theta_{t,\text{other}}$	Other	Time	-0.0086	-5.8
Parameters for marginal time-of-day dependent utility of spending time at home, $u_{marginal\ stay\ home}(t)$.				
$\theta_{\text{home},6am}$	Home	6am	0.041	8.2
$\theta_{\text{home},7am}$		7am	0.043	12
$\theta_{\text{home},8am}$		8am	0.020	6
$\theta_{\text{home},9am}$		9-10am	0.015	2.8
$\theta_{\text{home},1pm}$		1-4pm	-0.011	-9.9
$\theta_{\text{home},5pm}$		5-6pm	0.0036	3.9
$\theta_{\text{home},7pm}$		7-8pm	0.0024	2.5
$\theta_{\text{home},9pm}$		9pm	0.020	13
Log-likelihood goodness of fit based on sampled choice-sets				
LL	-12156.4			

4.4. Simulation result

To test how well the model manages to produce realistic behavior we simulated daily activity schedules for the observed individuals and compared some characteristics of the simulated data with the real data. Since we have parameters for attributes such as travel time, travel cost, activity time, and the number of trips, these quantities should be the same in the simulated sample as in data, and this is also the case as can be seen in Table 4. We are therefore focusing on quantities that we do not directly estimate but that are outcomes of the estimated parameters. Here we report result on activity timing, distribution of trips and tours, and trip length distribution.

Three factors determine the timing of activities: firstly, time constraints on working hours and child-errand times; secondly, preferences for when to arrive at work in the morning; and thirdly, preferences on when to be home. From Figure 1 it seems as if these determinants are enough to predict when people go to work and when they get home. For child-errands, the start and end time for school are likely to play an important role, and this has not been included in the model. Start times for free time activities are quite well reproduced by the model, but slightly shifted towards later hours. This could also be due to the lack of opening hours in the model, or because only home-time utility is time-of-day dependent. Overall the model can reproduce the timing of activities very well.

The distribution for the number of trips and tours in a day is determined by a vast number of factors, but the total number should be correct as tour and trip constants are estimated. The tour

Table 4 Average simulated and observed statistics.

	Attribute	Observed	Simulated	Obs-Sim	% difference
$\theta_{\text{home},6am}$	Home 6am	87.03	87.00	0.024	-0.028%
$\theta_{\text{home},6am}$	Home 7am	45.21	45.15	0.069	-0.152%
$\theta_{\text{home},8am}$	Home 8am	19.61	19.55	0.069	-0.354%
$\theta_{\text{home},9am}$	Home 9am	3.65	3.62	0.026	-0.726%
$\theta_{\text{home},1pm}$	Home 1pm	11.39	11.49	-0.097	0.844%
$\theta_{\text{home},5pm}$	Home 5pm	58.58	58.62	-0.038	0.065%
$\theta_{\text{home},7pm}$	Home 7pm	102.37	102.41	-0.039	0.038%
$\theta_{\text{home},9pm}$	Home 9pm	146.86	146.73	0.130	-0.089%
$\theta_{\text{home},9pm}$	Home 9pm	146.86	146.73	0.130	-0.089%
$\theta_{tt,\text{car}}$	Car Time	18.69	18.74	-0.049	0.263%
θ_{car}	Car Constant	0.99	0.99	-0.003	0.287%
$\theta_{tt,\text{pt}}$	PT Time	29.96	29.97	-0.004	0.014%
θ_{pt}	PT Constant	1.06	1.06	-0.001	0.077%
$\theta_{wait,\text{ pt}}$	PT Wait time	22.49	22.50	-0.008	0.034%
$\theta_{tt,\text{bike}}$	Walk Time	9.60	9.63	-0.030	0.315%
$\theta_{\text{s.z, walk}}$	Walk same zone	0.06	0.07	-0.001	1.645%
θ_{walk}	Walk Constant	0.35	0.36	-0.002	0.672%
$\theta_{tt,\text{bike}}$	Bike Time	5.28	5.25	0.033	-0.623%
θ_{bike}	Bike Constant	0.24	0.24	-0.001	0.501%
θ_{cost}	Cost	49.13	49.27	-0.140	0.285%
$\theta_{\text{work},6am}$	Work ASC 6am	0.01	0.01	-0.000	1.658%
$\theta_{\text{work},7am}$	Work ASC 7am	0.05	0.06	-0.000	0.900%
$\theta_{\text{work},9am}$	Work ASC 9am	0.15	0.15	0.001	-0.867%
$\theta_{\text{work},10am}$	Work ASC 10am	0.02	0.02	-0.000	0.164%
$\theta_{t,\text{shop}}$	Shop Time	7.20	7.25	-0.057	0.79%
$\theta_{t,\text{social}}$	Social Time	2.82	2.77	0.047	-1.7%
$\theta_{t,\text{other}}$	Other Time	5.29	5.41	-0.120	2.2%
$\theta_{C,\text{social}}$	Social ASC	0.03	0.02	0.001	-2.3%
$\theta_{C,\text{rec.}}$	Rec. ASC	0.12	0.12	-0.000	0.20%
$\theta_{C,\text{other}}$	Other ASC	0.09	0.09	-0.002	2.6%
$\theta_{C,\text{shop}}$	Shop ASC	0.19	0.19	-0.002	1.32%

For each individual, 1000 alternatives are sampled. That the difference is very small indicates that the linear approximation of $\widetilde{EV}$ works well and that enough alternatives are sampled to the choice set.

constant is equal to the home constant, as each additional tour will include an additional trip home. A specific trip constant would not be possible to identify given the constants we already have for modes and activities, but these constants will together ensure that the number of trips is correct. However, we do not have any constants governing the distribution of the number of trips per day, the number of tours per day or the number of trips per tour. The overall structure of the model will, therefore, determine these distributions, and seems to be enough to give good predictions, as Figure 2 show.

The length of trips by mode will mainly be determined by the utility of time and money for respective mode and by network characteristics, and this gives a good distribution of travel times, as can be seen in Figure 3. Since a large share of the trips are made to and from work, where the location is fixed and the trip is mandatory, the model is guaranteed to reproduce a large share of

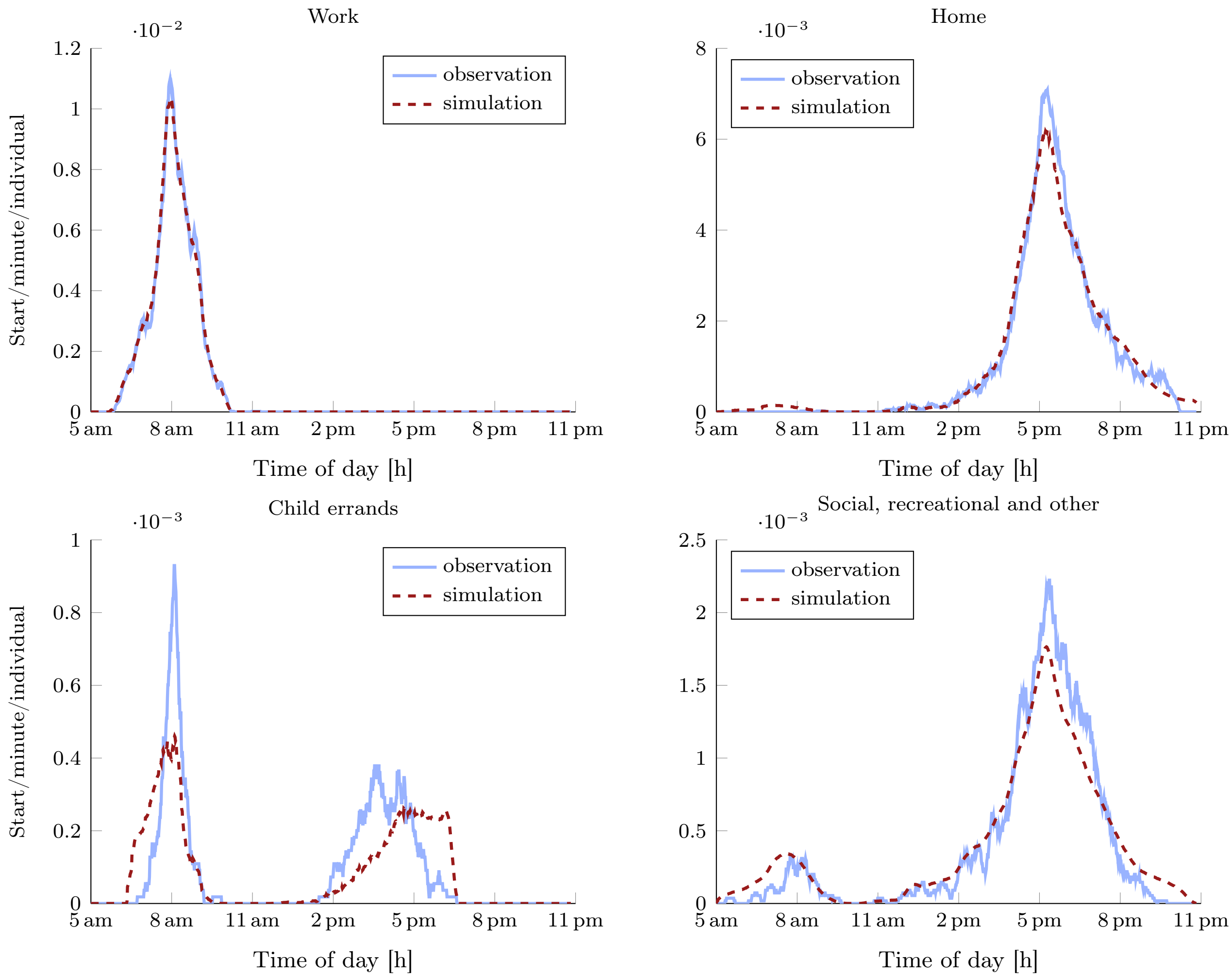

Figure 1 **Time of day when activities are started for simulated and observed data respectively .**

Note. The plot has been obtained by first constructing a histogram with bin-size of one minute, and then average each time step over the closest 20 minutes.

the trips well. However, the observed mode for the trip to work will only be the chosen mode in some of the simulated observations, and these restrictions will not give the distribution directly.

The "same zone" dummy parameter $\theta_{\text{s.z, walk}}$ for walking is negative, which seems counter-intuitive as one would expect walking to be the preferred mean of transport for shorter distances. From Figure 3 it is clear that for same-zone trips (trips with zero travel time) walking also the preferred mean of transport. The reason for this is that the mode-specific constant θ_{walk} is the largest of the constants, even after adding $\theta_{\text{s.z, walk}}$, and when the travel time is small, it will have the highest utility. The parameters for travel time are almost the same for car, walk, and bike but significantly smaller for PT. Since the travel time is longer for bike and walk, they will be less frequently used for longer trips. PT has the smallest time coefficient and trips with longer travel time are therefore more common with public transport. The lower speed of bike and walk in comparison with the motorized modes will make them less common for longer trips, as can be seen in Figure 3. The lower alternative specific constants and the fact that more locations will become available with the same travel time with PT and car also make bike and walk less common.

5. Discussion

5.1. Comparison with nested logit model

As the DDCM travel demand model presented in this paper closely resembles the nested-logit approach of Bowman and Ben-Akiva (2001) type models, we think it is worth comparing the two modeling approaches in some more detail and highlighting what we see as the main advantages of the proposed approach. Firstly, the DDCM model is formulated in a way that allows for the modeling of decision making under uncertainty, such as stochastic travel time or in the need to perform activities. Secondly, the natural inclusion of time in the DDCM model allows for a finer discretization of the time dimension, and in the case study of Section 4 we discretized time into 10-minute intervals. Principally, it is possible to have a very fine discretization of the time dimension in Bowman Ben-Akiva type models, but in practice, the number of time steps is limited for computational reasons. Bradley, Bowman, and Griesenbeck (2010) discretized time into 30-minute intervals and only allowed a sampled subset of these discretized time periods actually to be chosen for each individual. Further, in Bradley, Bowman, and Griesenbeck (2010), individuals choose at which of the sampled time steps

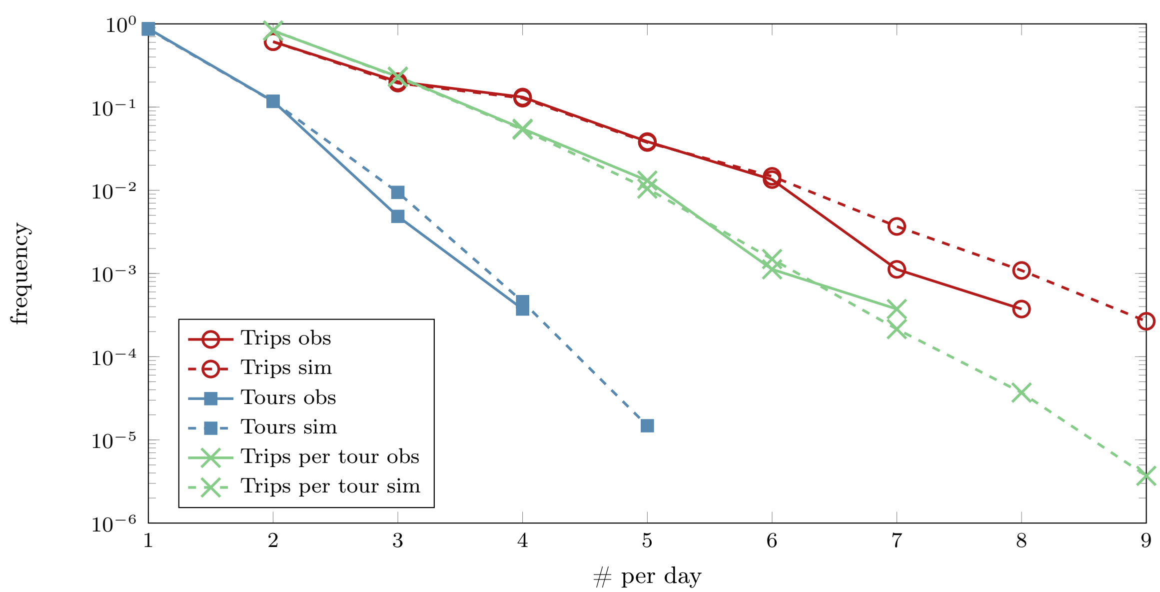

Figure 2 Distribution of number of trips, tours and trips per tour

Note. Comparsion between simulated and observed distributions of number of trips and tours, where a tour is a number of trips starting and ending at home. While parameters for trips and tours should guarantee that the total number of trips and tours are the same, no parameters directly give the distribution, nor the number of trips per tour.

to end each activity, rather than the duration of each activity as in the DDCM approach in this paper, made possible by the fact that time is seen as continuous. The possible duration of an activity will thus depend on the origin and destination of the trip leading to the destination. For example, if the duration of a trip would be 29 minutes, the duration of an activity can be 1 minute, 31 minutes, and so on, whereas if the trip is 31 minutes the duration can be 29, 59, and so on. That may have an unintended effect on the joint destination-duration choice. All in all, we believe that the finer discretization of the time dimension and the potential to include planning under uncertainty in the DDCM approach makes it more suitable for evaluating policies which influence: timing of trips; the behavior of people during sharp peaks in travel times; and how people react to changes in travel time uncertainty and reliability of different transport modes. As choices are made sequentially in

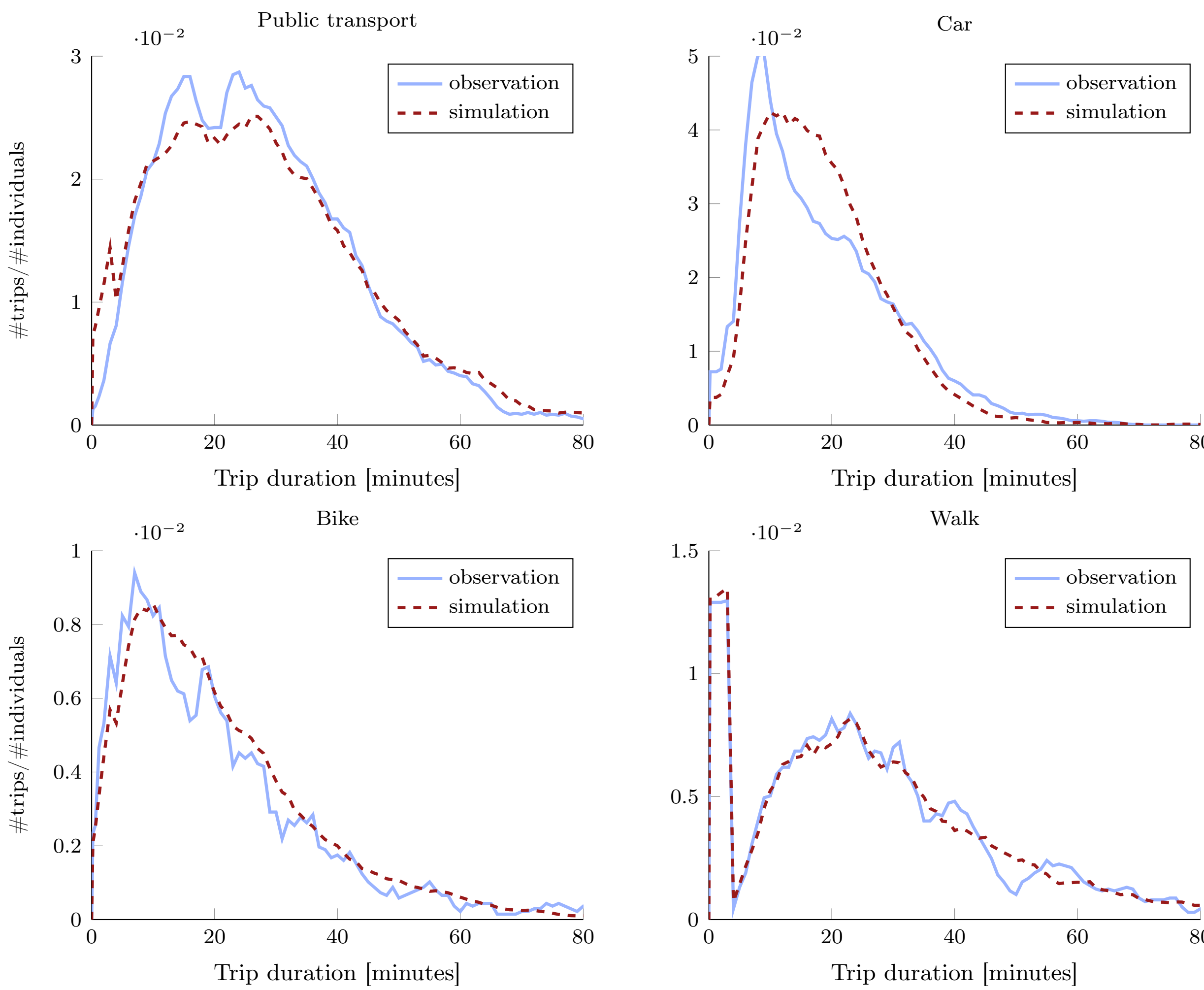

Figure 3 Distribution of trip lengths of respectively mode for observed choices (solid) and simulated (dashed).
Note. The graph has been obtained by first constructing a histogram with bin-size of one minute, and then average each time step over the closest 5 minutes.

time, the DDCM model can also be used to model how people adapt to unexpected disruptions in the transportation system.

5.2. Correlation between alternatives

It is common practice in route choice modeling to add a size attribute to each link to take correlation among paths that overlap into account, e.g., using Path-Size Logit (Ben-Akiva and Bierlaire 1999). For their link-based route choice model, Fosgerau, Frejinger, and Karlström (2013) obtains a size coefficient from an approximation of the expected flow on each link obtained using non-estimated parameters. Calculating a size-attribute of the type proposed in Fosgerau, Frejinger, and Karlström (2013) would be possible in the DDCM models discussed here. However, it is not as easy to define overlapping of paths in the dynamic activity-travel network presented here as in the car network as the network is dynamic. If two paths are identical besides that the start time for all activities in one path is 10 min after the start time in the other path, there can be practically no overlapping as defined by the Path-Size Logit although the two paths would be very similar.

In trip-generation models, it is common to have nests for mode choice, location choice and activity choice, as in, e.g., Bowman and Ben-Akiva (2001). It is worth noting that the expected value function in 7 might pick up some of the correlation in the observable ϵ that is usually captured by introducing nests in a trip or tour-based model. Since a trip including walking, public transport and biking all reach the same state, except for the arrival time, EV in the reached state is correlated for the three alternatives.

It is possible to change the assumption on the error term to allow for nests in each choice situation using the Nested Recursive Logit model described in Mai, Fosgerau, and Frejinger (2015). Guevara and Ben-Akiva (2013b) recently showed that Multivariate Extreme Value (MEV) models such as Nested Logit can be estimated using sampling of alternatives. A Nested RL-model is however not the same as an MEV model, so the transferability of the result is uncertain. If nests are introduced within the network, as in Mai, Fosgerau, and Frejinger (2015), it would further require that the value function was approximated in all states. An alternative would be to introduce nests over paths, for example nesting alternatives that include specific activities or modes, moving the model in the direction of Bowman and Ben-Akiva (2001). The computation time is however likely to grow linearly with the number of nests. Given that the model already is time demanding, creating nests for all combinations of modes and activities would not be computationally feasible, but nesting over certain activities or modes would.

Recursive Logit models have recently been developed extending the MNL case discussed in Fosgerau, Frejinger, and Karlström (2013) to cover Nested Logit (Mai, Fosgerau, and Frejinger 2015), MEV (Mai 2016) and Mixed Logit specifications (Mai, Bastin, and Frejinger 2016). They have also been applied in a number of scenarios, e.g., in Zimmermann, Mai, and Frejinger (2017) application to route choice for bikes, possibly with the largest network so far in a RL model. The number of links in these models are between 7 000-40 000. From the discussion in Section 4.1.6 it is clear that the model presented here is around 2 000 times bigger, so although the models are very similar the estimation techniques used for RL-models cannot be directly transferred to this context.

Another issue is the correlation in preferences over time. Individuals' variances in preferences for, e.g., mode or activities are likely to be consistent over time and therefore to some extent be the same throughout the day. Including nests on a trip level would not capture this correlation. A possible solution would be to introduce mixed parameters for, e.g., activities and modes, that would be the same for an individual for the full day. Our estimation approach is based on sampling of alternatives and recent research by Guevara and Ben-Akiva (2013a) shows that the same method gives consistent estimates for mixed logit models. Mixing parameters has been explored in an extension of the case study (Zimmermann et al. 2018).

5.3. Future work: Sampling of locations

In the case study presented in this paper we estimated the DDCM model using sampling of alternative daily travel patterns. Doing so allows for consistent estimates only in the special cases when either all error terms area i.i.d. Gumbel or potentially when they are distributed to create nests over sets of full daily travel patterns (using Guevara and Ben-Akiva 2013b). Sampling of sequences will not work in a model including travel time uncertainty or different distributions of the error structure within the day, and estimating the model for such cases will, therefore, be extremely time-consuming (see discussion in Section 4.1.6). By using sampling of alternative sequences it is possible to estimate the model with very few restrictions on the full choice set, and in the case study, the model was estimated with 1240 alternative possible locations for each trip. All individuals consider all possible locations for each new action. Locations are therefore both state variables and alternative actions, so the computation time increases quadratically with the number of locations. The curse of dimensionality connected to the number of zones is sometimes solved by sampling a subset of the zones through some auxiliary model (see, e.g., (Liao, Arentze, and Timmermans 2013)), or by approximating the log-sums through importance sampling (similar to what Bradley, Bowman, and Griesenbeck 2010 do in a nested logit framework). Rust (1997) shows how randomization can be used to approximate EV in dynamic discrete choice models, and it would be one possible way to decrease computation time. In order to speed up the model in the future, we believe approximating the model by sampling of locations may be necessary both for simulation and estimation. Work in this direction has also been started in Saleem, Västberg, and Karlström (2018), who use sampling of alternatives when simulating daily travel patterns in order to allow for the model to feed demand to a micro-simulator.

Approximately solving the problem may also be possible using reinforcement learning. (Vanhulsel et al. 2009) showed the feasibility when considering only scheduling of activities. It is theoretically also possible to simulate from the MNL-version preseneted in the case study using Metropolis-Hastings algorithm (Danalet 2015).

6. Conclusions

During the last decades, many activity-based models have been presented in the literature. However, there is still a lack of random-utility based models for which time integrates consistently in all choice dimensions. A natural approach would be to introduce time explicitly in the models, respecting that time has a direction and that it is possible to make decisions sequentially taking into account the available information at that time. It is also natural to respect that people are not completely myopic, but are capable of forward-looking, for instance taking into account the consequences for afternoon activity opportunities when deciding whether to take the car to work in the morning.

The challenge with such a natural extension of the existing state-of-practice modeling framework is the immense combinatorial problem of, at least technically and consistently, considering all possible combinations of activity-location pattern throughout one single day, let alone combinations of days. In this paper, we formulate a dynamic discrete choice model which overcomes this curse of dimensionality using dynamic programming. In the framework, time is respected in the above-mentioned aspects making it dynamically consistent. We also demonstrate that it is indeed possible to estimate the model in a case study. Formulating an activity based dynamic discrete choice travel demand model so that it can be estimated in a reasonable time is the main purpose and the main achievement of this paper. The proposed and thus estimated model is also validated in-sample.

There are a number of immediate extensions of this model that can and will be explored in further research. First, it is natural to extend the model to multiple days. Some activities can be postponed to later days, and there is an interaction between activity patterns during consecutive days. For instance, shopping for food is an activity in which a planning horizon of more than one day is very relevant to consider.

Another limitation of the model proposed in this paper is the IID assumption between daily activity schedules. In a sequential decision context, it may be important to consider fixed effects, in particular recognizing that the same individual is making the decisions throughout one day. A natural extension of this model is therefore to consider a mixed panel logit model, where preferences for, e.g., cost, time, modes and activities are heterogeneous between individuals but constant throughout the day for a single individual. Work in this direction is started in Zimmermann et al. (2018), where the case study model presented in this paper is extended to allow for correlation in the preferences for modes across the day.

The main challenge addressed in this paper was consistent estimation. Another very important aspect is to operationalize the model in implementation. For instance, when using the proposed model in the context of (or in conjunction with) a dynamic traffic assignment (DTA) model, it will be necessary to repeatedly simulate travel schedules for millions of individuals. In another extension, Saleem, Västberg, and Karlström (2018) uses sampling of locations to allow for fast simulation of tens of thousands of individuals and iteration with a traffic assignment model and observed that locations could be sampled without introducing any noticeable bias.

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