Supplement of Learning Deep Nets for Gravitational Dynamics with Unknown Disturbance through Physical Knowledge Distillation: Initial Feasibility Study

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1 Prove

The motivation to use non-minimal representation is to solve the limitation that (a)the minimal representation cannot guarantee the continuity of the functional mapping $\mathcal{G}$ in joint space, which might lead to unstable performance. On the contrary, (b) the trigonometric representation, a non-minimal representation, can guarantee the continuity, which effectively solved the limitation.

We prove the statements (a)(b) as following:

Definition. *A functional mapping $\mathcal{F}$ is continuous w.r.t. input $\boldsymbol{x}$ if*

$$\lim_{\boldsymbol{\delta}\to\mathbf{0}} \mathcal{F}(\boldsymbol{x}-\boldsymbol{\delta}) = \mathcal{F}(\boldsymbol{x}+\boldsymbol{\delta})$$

for any $\boldsymbol{x}$ in the input space $\mathcal{X}$[1].

Lemma. *if two functional mapping $\mathcal{F}(x)$ and $\mathcal{G}(x)$ are continuous, we can derive that $\mathcal{G}(\mathcal{F}(x))$ is continuous.[2]*

Prove Statement (a):

For the minimal representation, the joint position is represented as $\boldsymbol{q} = \begin{bmatrix} q_1 & q_2 & \dots & q_n \end{bmatrix}^T \in \mathbb{R}^n$, where the position of the i^{th} joint is $q_i = \theta_i \in [0, 2\pi)$ for a revolute joint and $q_i = d_i \in \mathbb{R}$ for a prismatic joint without considering joint limits. However, the minimal representation $\boldsymbol{q}$ is discontinuous *w.r.t.* joint space when revolute joints are included. Since when a revolute joint reach the singularity of the representation, we can easily find that

$$\lim_{\boldsymbol{\delta}\to\mathbf{0}^+} q_i(x_i+\boldsymbol{\delta}) = 0 \neq \lim_{\boldsymbol{\delta}\to\mathbf{0}^+} q_i(x_i-\boldsymbol{\delta}) = 2\pi.$$

Therefore, if $\boldsymbol{q}$ is discontinuous in joint space, we cannot guarantee a functional mapping $\mathcal{G}(\boldsymbol{q})$ is continuous in joint space using the minimal representation.

Prove Statement (b):

trigonometric representation can be formulated as

$$\boldsymbol{z} = \Phi(\boldsymbol{q}) = [\Phi(q_1), \Phi(q_2), \ldots, \Phi(q_n)]^T \in \mathbb{R}^{n+k},$$

where k is the number of revolute joints for a robot; The function for the trigonometric transformation Φ is defined as

$$\Phi(x) = \begin{cases} [\sin(x), \cos(x)]^T & \text{for a revolute joint} \\ x & \text{for a prismatic joint} \end{cases}.$$

When q_i reaches the singularity of the representation for a revolute joint, we can easily prove that

$$\lim_{\boldsymbol{\delta} \to \boldsymbol{0}^-} \Phi(q_i(x_i - \boldsymbol{\delta})) = \Phi(\begin{bmatrix} sin(0) & cos(0) \end{bmatrix}^T) = \lim_{\boldsymbol{\delta} \to \boldsymbol{0}^-} \Phi(q_i(x_i + \boldsymbol{\delta})) = \Phi(\begin{bmatrix} sin(2\pi) & cos(2\pi) \end{bmatrix}^T) \tag{1}$$

$$\lim_{\boldsymbol{\delta} \to \boldsymbol{0}^+} \Phi(q_i(x_i - \boldsymbol{\delta})) = \Phi(\begin{bmatrix} sin(2\pi) & cos(2\pi) \end{bmatrix}^T) = \lim_{\boldsymbol{\delta} \to \boldsymbol{0}^+} \Phi(q_i(x_i + \boldsymbol{\delta})) = \Phi(\begin{bmatrix} sin(0) & cos(0) \end{bmatrix}^T) \tag{2}$$

$$\left.\begin{matrix} (A) \\ (B) \end{matrix}\right\} \implies \lim_{\boldsymbol{\delta} \to \boldsymbol{0}} \Phi(q_i(x_i - \boldsymbol{\delta})) = \Phi(q_i(x_i + \boldsymbol{\delta}))$$

On the other hand, when q_i do not reach the singularity, the equation $\lim_{\boldsymbol{\delta} \to \boldsymbol{0}} \Phi(q_i(x_i - \boldsymbol{\delta})) = \lim_{\boldsymbol{\delta} \to \boldsymbol{0}} \Phi(q_i(x_i + \boldsymbol{\delta}))$ is still valid. Therefore, trigonometric representation $\Phi(\boldsymbol{q})$ is continuous in joint space. Based on the lemma and the continuity of trigonometric representation, given a continuous mapping $\mathcal{G}(\cdot)$, we can infer $\mathcal{G}(\Phi(\boldsymbol{q}))$ is continuous in joint space.

References

[1] Wikipedia contributors, "Continuous function," 2020. [Online]. Available: https://en.wikipedia.org/wiki/Continuous_function

[2] StackExchange contributors, "If f,g are continuous functions, then fg is continuous," Mathematics Stack Exchange, 2020. [Online]. Available: https://math.stackexchange.com/q/2494413