The Symbolic Equation of Gravitational Dynamic Model for the MTM of dVRK

The joint torque caused by gravitational force $\boldsymbol{\tau}_g$ can be represented as a linear form of dynamic parameters related to the gravitational force ${}^g\boldsymbol{\beta}$

$$\boldsymbol{\tau}_g = {}^g\boldsymbol{Y}(\boldsymbol{q}){}^g\boldsymbol{\beta} \tag{1}$$

where ${}^g\boldsymbol{Y}(\boldsymbol{q})$ and ${}^g\boldsymbol{\beta}$ are the regressor matrix and dynamic parameters vector for the gravitational force.

$$ {}^g\boldsymbol{Y} = \begin{bmatrix} {}^gY^1 \\ {}^gY^2 \\ {}^gY^3 \\ {}^gY^4 \\ {}^gY^5 \\ {}^gY^6 \\ {}^gY^7 \end{bmatrix}^T $$

where ${}^gY^i$ is the i^{th} row of the ${}^g\boldsymbol{Y}(\boldsymbol{q})$. The explicit form of ${}^g\boldsymbol{Y}(\boldsymbol{q})$ are represented as following:

$$ {}^gY^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T $$

$$ {}^gY^2 = \begin{bmatrix} s_2 \\ c_2 \\ c_2c_3 - s_2s_3 \\ -c_2s_3 - c_3s_2 \\ c_2c_3c_4 - c_4s_2s_3 \\ s_2s_3s_4 - c_2c_3s_4 \\ c_4s_2s_3s_5 - c_3c_5s_2 - c_2c_3c_4s_5 - c_2c_5s_3 \\ c_2c_3c_4c_5 - c_3s_2s_5 - c_2s_3s_5 - c_4c_5s_2s_3 \\ c_2c_3s_4s_6 + c_2c_6s_3s_5 + c_3c_6s_2s_5 - s_2s_3s_4s_6 + c_4c_5c_6s_2s_3 - c_2c_3c_4c_5c_6 \\ c_2c_3c_6s_4 - c_6s_2s_3s_4 - c_2s_3s_5s_6 - c_3s_2s_5s_6 - c_4c_5s_2s_3s_6 + c_2c_3c_4c_5s_6 \end{bmatrix}^T $$

$$
{}^{g}Y^{3} = \begin{bmatrix} 0 \\ 0 \\ c_2c_3 - s_2s_3 \\ -c_2s_3 - c_3s_2 \\ c_2c_3c_4 - c_4s_2s_3 \\ s_2s_3s_4 - c_2c_3s_4 \\ c_4s_2s_3s_5 - c_3c_5s_2 - c_2c_3c_4s_5 - c_2c_5s_3 \\ c_2c_3c_4c_5 - c_3s_2s_5 - c_2s_3s_5 - c_4c_5s_2s_3 \\ c_2c_3s_4s_6 + c_2c_6s_3s_5 + c_3c_6s_2s_5 - s_2s_3s_4s_6 + c_4c_5c_6s_2s_3 - c_2c_3c_4c_5c_6 \\ c_2c_3c_6s_4 - c_6s_2s_3s_4 - c_2s_3s_5s_6 - c_3s_2s_5s_6 - c_4c_5s_2s_3s_6 + c_2c_3c_4c_5s_6 \end{bmatrix}^T
$$

$$
{}^{g}Y^{4} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -s_2 + q3s_4 \\ -s_2 + q3c_4 \\ s_2 + q3s_4s_5 \\ -s_2 + q3c_5s_4 \\ s_2 + q3c_4s_6 + c_5c_6s_4 \\ s_2 + q3c_4c_6 - c_5s_4s_6 \end{bmatrix}^T
$$

$$
{}^{g}Y^{5} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -c_2c_3s_5 - s_2s_3s_5 + c_2c_4c_5s_3 + c_3c_4c_5s_2 \\ -c_5s_2s_3 - c_2c_3c_5 + c_2c_4s_3s_5 + c_3c_4s_2s_5 \\ c_5c_6s_2s_3 - c_2c_3c_5c_6 + c_2c_4c_6s_3s_5 + c_3c_4c_6s_2s_5 \\ -c_5s_2s_3s_6 - c_2c_3c_5s_6 + c_2c_4s_3s_5s_6 + c_3c_4s_2s_5s_6 \end{bmatrix}^T
$$

$$
{}^{g}Y^{6} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ c_2c_6s_3s_4 + c_3c_6s_2s_4 + c_2c_3s_5s_6 - s_2s_3s_5s_6 + c_2c_4c_5s_3s_6 + c_3c_4c_5s_2s_6 \\ c_2c_3c_6s_5 - c_2s_3s_4s_6 - c_3s_2s_4s_6 - c_6s_2s_3s_5 + c_2c_4c_5c_6s_3 + c_3c_4c_5c_6s_2 \end{bmatrix}^T
$$

$$
{}^{g}Y^{7} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{T}
$$

where c_i and s_i represent the $cos(q_i)$ and $sin(q_i)$ and q_i is the joint angle of Joint i.

The dynamic parameters vector for the gravitational force, ${}^{g}\boldsymbol{\beta}$, can be shown as

$$
{}^{g}\boldsymbol{\beta} = \begin{bmatrix} l_{arm}(m_2 + m_3 + m_4 + m_5 + m_6)g + cm_{2x}m_2g - cm^{'}_{4x}m^{'}_4g \\ cm_{2y}m_2g - cm^{'}_{4y}m^{'}_4g \\ cm_{3x}m_3g - cm^{'}_{5y}m^{'}_5g + l_{forearm}(m_3 + m_4 + m_5 + m_6)g + l^{'}_{forearm}(m^{'}_4 + m^{'}_5)g \\ cm_{3z}m_3g + cm_{4y}m_4g + cm^{'}_{5x}m^{'}_5g + h(m_4 + m_5 + m_6)g \\ cm_{4x}m_4g \\ cm_{5y}m_5g - cm_{4z}m_4g \\ cm_{5z}m_5g + cm_{6y}m_6g \\ cm_{5x}m_5g \\ cm_{6z}m_6g \\ cm_{6x}m_6g \end{bmatrix}
$$