Midterm Exam

Statistics 139

Tuesday, October 11

NAME ____________________________ **HUID** ____________________________

- The exam covers material from Lectures 1-10 and Problem Sets 1-4.
- The exam consists of 4 problems and 11 pages. The exam is worth 100 points and the point value for each question is displayed.
- The last two pages includes output from R's `pnorm` and `qt` commands.
- Before submitting the exam, read and sign the statement below confirming that you have not cheated on this exam.
- Be sure to read the questions carefully. Some parts of a problem statement may ask for more than one calculation.
- Some parts of a question may require the answer to an earlier part. If you cannot solve the earlier part, you can still receive partial credit for the later parts; make up a reasonable answer for the earlier part to use in subsequent parts of the problem.
- Show your work and explain your reasoning; the final answer is not as important as the process by which you arrived at that answer. We can more easily give partial credit if you have written out your steps clearly.
- You are allowed a calculator (graphing is fine) and two double-sided pages of your own notes as reference during the exam.

By signing below, I confirm that I have not cheated while taking this exam: I have not used any unauthorized resources nor copied another student's responses.

Signature: ____________________________

Problem Scoring		
Problem	Point Value	Points Scored
1	30	
2	30	
3	18	
4	22	
Total	100	

Problem 1: Short Answer ***(30 points total)***

(a) Let $Y_1, ..., Y_n$ be a random sample from the $N(\mu_Y, \sigma_Y^2)$ distribution, and let $\bar{Y}$ and S_Y^2 be the usual sample mean and sample variance for these observations (assume $n > 2$). The statistic

$$T = \frac{\bar{Y} - \mu_Y}{S_Y/\sqrt{n}}$$

will have variance (no justification needed):

(A) $\text{Var}(T) < 1$
(B) $\text{Var}(T) = 1$
(C) $\text{Var}(T) > 1$
(D) Cannot be determined.

(b) Imagine you built the following multiple regression model:

$$Y = \beta_0 + \beta_1(X_1) + \beta_2(X_2)$$

where Y = (time spent studying for the final exam) of individuals using on a constant, X_1 = a binary variable which takes on the value 1 for athletes and is 0 for non-athletes, and X_2 = a binary variable which takes on the value 1 for non-athletes and is 0 for athletes. If athletes spend more time studying for the final exam, then you would expect (select one, and briefly explain your answer):

(A) the coefficient for X_1 to have a positive sign, and for X_2 to have a negative sign.
(B) both coefficients to be the same distance from the constant, one above and the other below.
(C) β_1 to estimate the mean of Y for athletes, β_2 to estimate the mean of Y for non-athletes.
(D) none of the OLS estimators to exist because there is perfect multicollinearity.

(c) An analyst runs a regression model in `R` (Model 1) of `lm(Y ~ X)` and gets an R^2 value of 0.4 (both Y and X are quantitative). They then notice the residuals from this model are left-skewed, and runs two more regression models: Model 2 is `lm(log(Y) ~ X)` which ends up with an R^2 value of 0.5, and Model 3 is `lm(Y^2 ~ X)` which ends up with an R^2 value of 0.3.

i. Which model is likely to be most reliable for inferences describing the relationship between Y and X? Explain in 3 or fewer sentences.

ii. Provide an alternative method for handling the situation above. Explain in 1 or 2 sentences.

(d) A simulation was performed (with 5000 iterations under each condition) to compare 3 methods to test whether there is a difference in means between 2 groups (X and Y). The first group ($n_1 = 50$) was independently sampled from a $X \sim N(\mu = 5, \sigma^2 = 5^2)$ and the second group ($n_2 = 30$) was first independently sampled from a $Y \sim N(\mu = 5, \sigma^2 = 5^2)$ (an *effect size* of 0), and then second independently sampled from a $Y \sim N(\mu = 8, \sigma^2 = 5^2)$ (an *effect size* of 3). The 3 analysis methods considered were:

i. Two-sample t-test (unpooled)
ii. t test from a simple linear regression with a binary predictor
iii. Wilcoxon Rank Sum Test

The rejection rates under each combination of true mean for Y and analysis approach (2x3) are shown below.

```
##           test1  test2  test3
## effect=0 0.0526 0.0488 0.0500
## effect=3 0.7174 0.6982 0.7218
```

i. Which approach was which? Clearly indicate which analysis approach is labeled as `test1`, `test2`, and `test3` in the output, and explain your choice in a few sentences.

ii. What were the estimated Type I error rates and power for each test?

iii. Which approach is best for this setting? Explain in 2-3 sentences.

Problem 2: Q Guide ***(30 points total)***

Data were extracted from the Committee on Undergraduate Education Guide for courses (The *Q* Guide) from the Computer Science and Statistics offerings for the 2021-2022 academic year. The number of students enrolled in the course (`enrolled`) and the average course rating (`rating`) were measured for each course, along with the department (`dept` with option of either `cs` or `stat`) and an indicator for whether the course offering was for the `spring` term. A snippet of the data is provided below (first 3 rows):

```
##   enrolled rating dept spring
## 1      686   3.76   cs      0
## 2      199   3.92   cs      0
## 3       45   4.88   cs      0
```

(a) A linear regression model was fit to predict `rating` from `enrolled` (on the log scale). Part of the summary output is provided below.

```
summary(modelA <- lm(rating~log(enrolled),data=cue))
```

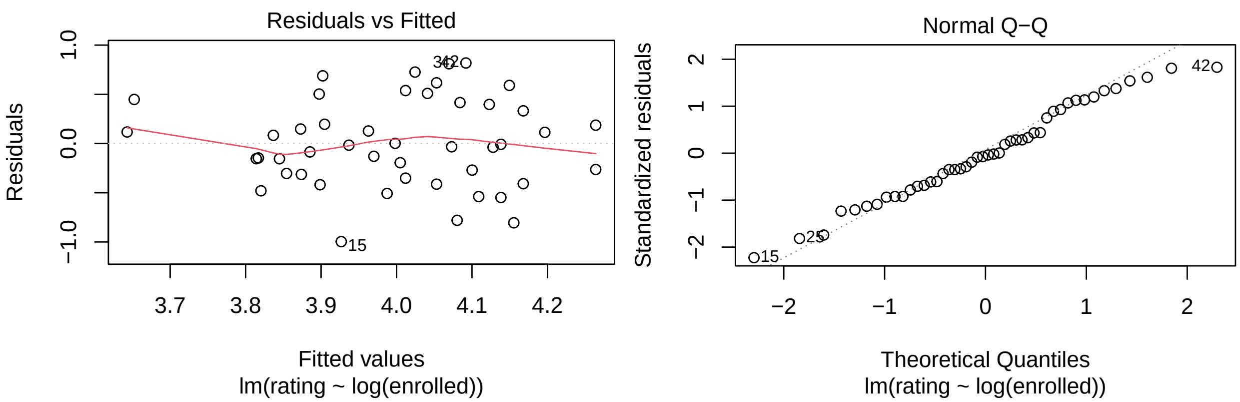
```
##
## Call:
## lm(formula = rating ~ log(enrolled), data = cue)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.9967 -0.3114 -0.0249  0.3808  0.8181
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)     4.6656     0.3205   14.56   <2e-16 ***
## log(enrolled)  -0.1566     0.0737   -2.12    0.039 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.454 on 44 degrees of freedom
## Multiple R-squared:  0.0929, Adjusted R-squared:  0.0723
## F-statistic: 4.51 on 1 and 44 DF,  p-value: 0.0394
```

i. Interpret the slope estimate for this model in context of the problem.

ii. Formally test whether there is an association between `rating` and `enrolled` based on this model.

(b) What are *all* of the assumptions for the linear regression model above? Use the plots below to comment on the ones that apply. Be specific as to which plot you are using for each.

```r
plot(modelA,which=1:2)
```



(c) A linear regression model was fit to predict `rating` from `enrolled` (on the log scale) and `dept` and the summary output is provided below.

```r
summary(modelB <- lm(rating~log(enrolled)+dept,data=cue))$coef
```

```
##                Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)     4.67745    0.35679 13.10971 1.279e-16
## log(enrolled) -0.15836    0.07785 -2.03406 4.814e-02
## deptstat      -0.01168    0.14647 -0.07973 9.368e-01
```

i. Interpret the coefficient (both estimate and significance) for `dept` in this model **in context of the problem**.

ii. Use the models above to comment on whether there is much of a difference in the average enrollment (on the log scale) in Computer Science courses and Statistics courses. Provide 1-2 sentences of justification.

Problem 3: Battery life *(18 points total)* You have heard that turning off the wireless on a cell phone increases the length of time the battery will last after charging to 100% and then unplugging (*battery life*). You gather some friends and their cell phones and conduct a study. You record the following statistics, with each observation being the measured battery life on the ln(hours) scale:

Group	$\bar{y}$	s_y	n
wifi-on	3.026	0.517	16
wifi-off	3.374	0.721	16

(a) Without seeing graphical representations of the battery life data, why do you think that the battery life is measured in log hours? Answer in one sentence.

(b) You conduct an unpooled two-sample t-test for the null hypothesis of equal means, and calculate a two-sided p-value of 0.1556 with a 95% confidence interval of $(-0.141, 0.838)$. Suppose that the 32 cell phones were **not randomized** to the *wifi-on* and *wifi-off* groups.

i. What estimand is the confidence interval estimating?

iii. Interpret the p-value in context of the problem.

(c) Suppose that you gathered together 32 friends and asked each friend to choose whether to turn the wireless on or off for your study. Most of the friends with Apple iPhones decided to leave wifi on. Most of the friends with other phones (Samsungs, etc.) decided to leave the wifi off. In two or three sentences, describe what you would specifically do in the analysis if you wanted to attempt to improve the inferences from this study.

Problem 4: Variance Estimators ***(22 points total)***

The *sum of squares* of a sample of data is the squared deviation around some number c. Define $g(c)$ as a function with respect to c as:

$$g(c) = \sum_{i=1}^{n} (X_i - c)^2.$$

(a) Show that this function is minimized at the value $c = \bar{X}$.

(b) Use the result in the previous part to justify the use of $n - 1$ in the denominator of the standard sample variance estimator, S^2. Justify with 3 or fewer sentences.

(c) Let $X_1, ..., X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution. Assume μ is known and define:

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2$$

i. Determine the distribution of $\hat{\sigma}^2$.

ii. Use the result in the previous problem to show that $\hat{\sigma}^2$ is unbiased for σ^2.

iii. Which estimator is preferred in practice: $\hat{\sigma}^2$ or the usual sample variance estimate, S^2. Explain in 2-3 sentences.

Table of Distributions

Name	Paramater	PMF or PDF	Mean	Variance
Bernoulli	p	$P(X=1)=p$, $P(X=0)=1-p$	p	$p(1-p)$
Binomial	n, p	$\binom{n}{k} p^k (1-p)^{n-k}; k \in \{0,1,\ldots,n\}$	np	$np(1-p)$
Geometric	p	$p(1-p)^k; k \in \{0,1,2,\ldots\}$	$(1-p)/p$	$(1-p)/p^2$
Negative Binomial	r, p	$\binom{r+k-1}{r-1} p^r (1-p)^k; k \in \{0,1,\ldots\}$	$r(1-p)/p$	$r(1-p)/p^2$
Hyper-geometric	w, b, n	$\binom{w}{k}\binom{b}{n-k} \Big/ \binom{w+b}{n}; k \in \{0,1,\ldots,n\}$	$\mu = \frac{nw}{w+b}$	$\left(\frac{w+b-n}{w+b-1}\right) n \left(\frac{\mu}{n}\right)\left(1-\frac{\mu}{n}\right)$
Multinomial	$p_1, \ldots, p_k$ $\sum p_i = 1$	$\frac{n!}{x_1! x_2! \ldots x_k!} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k}$	$E(X_i) = np_i$	$Var(X_i) = np_i(1-p_i)$ $Cov(X_i, X_j) = -np_i p_j$, for $i \neq j$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}; k \in \{0,1,\ldots\}$	λ	λ
Uniform	$a < b$	$\frac{1}{b-a}; a < x < b$	$(a+b)/2$	$(b-a)^2/12$
Normal	μ, σ^2	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\mu)^2/(2\sigma^2)\right)$	μ	σ^2
Log-Normal	μ, σ^2	$\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-(\log(x)-\mu)^2/(2\sigma^2)\right)$ $; x > 0$	$\theta = \exp(\mu+\sigma^2/2)$	$\theta^2[\exp(\sigma^2)-1]$
Exponential	λ	$\lambda \exp(-\lambda x); x > 0$	$1/\lambda$	$1/\lambda^2$
Gamma	a, λ	$\frac{\lambda^a}{\Gamma(a)} x^{a-1} \exp(-\lambda x)$	a/λ	a/λ^2
Beta	a, b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}; 0 < x < 1$	$\mu = \frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$
Chi-Square	d	$\frac{1}{2^{d/2}\Gamma(d/2)} x^{(d/2)-1} \exp(-x/2);$ $x > 0$	d	$2d$
t	d	$\frac{\Gamma((d+1)/2)}{\sqrt{d\pi}\Gamma(d/2)} (1+x^2/d)^{-(d+1)/2}$	0 if $d > 1$	$d/(d-2)$ if $d > 2$
F	d_1, d_2	$\frac{\Gamma(d_1+d_2)}{\Gamma(d_1)\Gamma(d_2)x} \sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1+d_2}}}$	$d_2/(d_2-2)$ if $d_2 > 2$	$2d_2^2(d_1+d_2-2)/[d_1(d_2-2)^2(d_2-4)]$ if $d_2 > 4$

Useful R output:

```r
# Standard Normal Table
z=seq(0,3.07,0.01)
p=round(pnorm(z),4)
names(p)=z
p
```

```
##      0   0.01   0.02   0.03   0.04   0.05   0.06   0.07   0.08   0.09    0.1   0.11   0.12   0.13
## 0.5000 0.5040 0.5080 0.5120 0.5160 0.5199 0.5239 0.5279 0.5319 0.5359 0.5398 0.5438 0.5478 0.5517
##   0.14   0.15   0.16   0.17   0.18   0.19    0.2   0.21   0.22   0.23   0.24   0.25   0.26   0.27
## 0.5557 0.5596 0.5636 0.5675 0.5714 0.5753 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6064
##   0.28   0.29    0.3   0.31   0.32   0.33   0.34   0.35   0.36   0.37   0.38   0.39    0.4   0.41
## 0.6103 0.6141 0.6179 0.6217 0.6255 0.6293 0.6331 0.6368 0.6406 0.6443 0.6480 0.6517 0.6554 0.6591
##   0.42   0.43   0.44   0.45   0.46   0.47   0.48   0.49    0.5   0.51   0.52   0.53   0.54   0.55
## 0.6628 0.6664 0.6700 0.6736 0.6772 0.6808 0.6844 0.6879 0.6915 0.6950 0.6985 0.7019 0.7054 0.7088
##   0.56   0.57   0.58   0.59    0.6   0.61   0.62   0.63   0.64   0.65   0.66   0.67   0.68   0.69
## 0.7123 0.7157 0.7190 0.7224 0.7257 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.7517 0.7549
##    0.7   0.71   0.72   0.73   0.74   0.75   0.76   0.77   0.78   0.79    0.8   0.81   0.82   0.83
## 0.7580 0.7611 0.7642 0.7673 0.7704 0.7734 0.7764 0.7794 0.7823 0.7852 0.7881 0.7910 0.7939 0.7967
##   0.84   0.85   0.86   0.87   0.88   0.89    0.9   0.91   0.92   0.93   0.94   0.95   0.96   0.97
## 0.7995 0.8023 0.8051 0.8078 0.8106 0.8133 0.8159 0.8186 0.8212 0.8238 0.8264 0.8289 0.8315 0.8340
##   0.98   0.99      1   1.01   1.02   1.03   1.04   1.05   1.06   1.07   1.08   1.09    1.1   1.11
## 0.8365 0.8389 0.8413 0.8438 0.8461 0.8485 0.8508 0.8531 0.8554 0.8577 0.8599 0.8621 0.8643 0.8665
##   1.12   1.13   1.14   1.15   1.16   1.17   1.18   1.19    1.2   1.21   1.22   1.23   1.24   1.25
## 0.8686 0.8708 0.8729 0.8749 0.8770 0.8790 0.8810 0.8830 0.8849 0.8869 0.8888 0.8907 0.8925 0.8944
##   1.26   1.27   1.28   1.29    1.3   1.31   1.32   1.33   1.34   1.35   1.36   1.37   1.38   1.39
## 0.8962 0.8980 0.8997 0.9015 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9131 0.9147 0.9162 0.9177
##    1.4   1.41   1.42   1.43   1.44   1.45   1.46   1.47   1.48   1.49    1.5   1.51   1.52   1.53
## 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 0.9332 0.9345 0.9357 0.9370
##   1.54   1.55   1.56   1.57   1.58   1.59    1.6   1.61   1.62   1.63   1.64   1.65   1.66   1.67
## 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525
##   1.68   1.69    1.7   1.71   1.72   1.73   1.74   1.75   1.76   1.77   1.78   1.79    1.8   1.81
## 0.9535 0.9545 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9608 0.9616 0.9625 0.9633 0.9641 0.9649
##   1.82   1.83   1.84   1.85   1.86   1.87   1.88   1.89    1.9   1.91   1.92   1.93   1.94   1.95
## 0.9656 0.9664 0.9671 0.9678 0.9686 0.9693 0.9699 0.9706 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744
##   1.96   1.97   1.98   1.99      2   2.01   2.02   2.03   2.04   2.05   2.06   2.07   2.08   2.09
## 0.9750 0.9756 0.9761 0.9767 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817
##    2.1   2.11   2.12   2.13   2.14   2.15   2.16   2.17   2.18   2.19    2.2   2.21   2.22   2.23
## 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9854 0.9857 0.9861 0.9864 0.9868 0.9871
##   2.24   2.25   2.26   2.27   2.28   2.29    2.3   2.31   2.32   2.33   2.34   2.35   2.36   2.37
## 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911
##   2.38   2.39    2.4   2.41   2.42   2.43   2.44   2.45   2.46   2.47   2.48   2.49    2.5   2.51
## 0.9913 0.9916 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9936 0.9938 0.9940
##   2.52   2.53   2.54   2.55   2.56   2.57   2.58   2.59    2.6   2.61   2.62   2.63   2.64   2.65
## 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960
##   2.66   2.67   2.68   2.69    2.7   2.71   2.72   2.73   2.74   2.75   2.76   2.77   2.78   2.79
## 0.9961 0.9962 0.9963 0.9964 0.9965 0.9966 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974
##    2.8   2.81   2.82   2.83   2.84   2.85   2.86   2.87   2.88   2.89    2.9   2.91   2.92   2.93
## 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9981 0.9981 0.9982 0.9982 0.9983
##   2.94   2.95   2.96   2.97   2.98   2.99      3   3.01   3.02   3.03   3.04   3.05   3.06   3.07
## 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 0.9987 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9989
```

```r
# t Table
df=c(1:30,10*4:20,Inf)
p = c(0.6,0.7,0.75,0.8,0.9,0.95,0.975,0.99,0.995,0.999)
grid = expand.grid(p,df)
t=round(qt(p=grid[,1],df=grid[,2]),4)
as.data.frame(matrix(t,ncol=length(p),dimnames=list(df,p),byrow=T))
```

```
##        0.6    0.7   0.75    0.8   0.9  0.95  0.975   0.99  0.995   0.999
## 1   0.3249 0.7265 1.0000 1.3764 3.078 6.314 12.706 31.820 63.657 318.309
## 2   0.2887 0.6172 0.8165 1.0607 1.886 2.920  4.303  6.965  9.925  22.327
## 3   0.2767 0.5844 0.7649 0.9785 1.638 2.353  3.182  4.541  5.841  10.214
## 4   0.2707 0.5686 0.7407 0.9410 1.533 2.132  2.776  3.747  4.604   7.173
## 5   0.2672 0.5594 0.7267 0.9195 1.476 2.015  2.571  3.365  4.032   5.893
## 6   0.2648 0.5534 0.7176 0.9057 1.440 1.943  2.447  3.143  3.707   5.208
## 7   0.2632 0.5491 0.7111 0.8960 1.415 1.895  2.365  2.998  3.499   4.785
## 8   0.2619 0.5459 0.7064 0.8889 1.397 1.859  2.306  2.897  3.355   4.501
## 9   0.2610 0.5435 0.7027 0.8834 1.383 1.833  2.262  2.821  3.250   4.297
## 10  0.2602 0.5415 0.6998 0.8791 1.372 1.812  2.228  2.764  3.169   4.144
## 11  0.2596 0.5399 0.6974 0.8755 1.363 1.796  2.201  2.718  3.106   4.025
## 12  0.2590 0.5386 0.6955 0.8726 1.356 1.782  2.179  2.681  3.054   3.930
## 13  0.2586 0.5375 0.6938 0.8702 1.350 1.771  2.160  2.650  3.012   3.852
## 14  0.2582 0.5366 0.6924 0.8681 1.345 1.761  2.145  2.624  2.977   3.787
## 15  0.2579 0.5357 0.6912 0.8662 1.341 1.753  2.131  2.603  2.947   3.733
## 16  0.2576 0.5350 0.6901 0.8647 1.337 1.746  2.120  2.583  2.921   3.686
## 17  0.2573 0.5344 0.6892 0.8633 1.333 1.740  2.110  2.567  2.898   3.646
## 18  0.2571 0.5338 0.6884 0.8620 1.330 1.734  2.101  2.552  2.878   3.611
## 19  0.2569 0.5333 0.6876 0.8610 1.328 1.729  2.093  2.539  2.861   3.579
## 20  0.2567 0.5329 0.6870 0.8600 1.325 1.725  2.086  2.528  2.845   3.552
## 21  0.2566 0.5325 0.6864 0.8591 1.323 1.721  2.080  2.518  2.831   3.527
## 22  0.2564 0.5321 0.6858 0.8583 1.321 1.717  2.074  2.508  2.819   3.505
## 23  0.2563 0.5317 0.6853 0.8575 1.319 1.714  2.069  2.500  2.807   3.485
## 24  0.2562 0.5314 0.6848 0.8569 1.318 1.711  2.064  2.492  2.797   3.467
## 25  0.2561 0.5312 0.6844 0.8562 1.316 1.708  2.059  2.485  2.787   3.450
## 26  0.2560 0.5309 0.6840 0.8557 1.315 1.706  2.055  2.479  2.779   3.435
## 27  0.2559 0.5306 0.6837 0.8551 1.314 1.703  2.052  2.473  2.771   3.421
## 28  0.2558 0.5304 0.6834 0.8546 1.312 1.701  2.048  2.467  2.763   3.408
## 29  0.2557 0.5302 0.6830 0.8542 1.311 1.699  2.045  2.462  2.756   3.396
## 30  0.2556 0.5300 0.6828 0.8538 1.310 1.697  2.042  2.457  2.750   3.385
## 40  0.2550 0.5286 0.6807 0.8507 1.303 1.684  2.021  2.423  2.704   3.307
## 50  0.2547 0.5278 0.6794 0.8489 1.299 1.676  2.009  2.403  2.678   3.261
## 60  0.2545 0.5272 0.6786 0.8477 1.296 1.671  2.000  2.390  2.660   3.232
## 70  0.2543 0.5268 0.6780 0.8468 1.294 1.667  1.994  2.381  2.648   3.211
## 80  0.2542 0.5265 0.6776 0.8461 1.292 1.664  1.990  2.374  2.639   3.195
## 90  0.2541 0.5263 0.6772 0.8456 1.291 1.662  1.987  2.369  2.632   3.183
## 100 0.2540 0.5261 0.6770 0.8452 1.290 1.660  1.984  2.364  2.626   3.174
## 110 0.2540 0.5259 0.6767 0.8449 1.289 1.659  1.982  2.361  2.621   3.166
## 120 0.2539 0.5258 0.6765 0.8446 1.289 1.658  1.980  2.358  2.617   3.159
## 130 0.2539 0.5257 0.6764 0.8444 1.288 1.657  1.978  2.355  2.614   3.154
## 140 0.2538 0.5256 0.6762 0.8442 1.288 1.656  1.977  2.353  2.611   3.150
## 150 0.2538 0.5255 0.6761 0.8440 1.287 1.655  1.976  2.352  2.609   3.146
## 160 0.2538 0.5254 0.6760 0.8439 1.287 1.654  1.975  2.350  2.607   3.142
## 170 0.2537 0.5254 0.6759 0.8437 1.287 1.654  1.974  2.349  2.605   3.139
## 180 0.2537 0.5253 0.6759 0.8436 1.286 1.653  1.973  2.347  2.603   3.136
## 190 0.2537 0.5253 0.6758 0.8435 1.286 1.653  1.972  2.346  2.602   3.134
## 200 0.2537 0.5252 0.6757 0.8434 1.286 1.653  1.972  2.345  2.601   3.131
## Inf 0.2533 0.5244 0.6745 0.8416 1.282 1.645  1.960  2.326  2.576   3.090
```