Time Mixed Effects Model

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Mixed Effects Linear Model for Time

To include the data points from all the years collected in both datasets, we decided to incorporate time effects into a mixed effects model. For this model, we used the variables manually selected based on correlations: `x1.6`, `x3.2`, `x5.1`, `x6.4`, and `x7.3`. This mixed effects model is based on the equation:

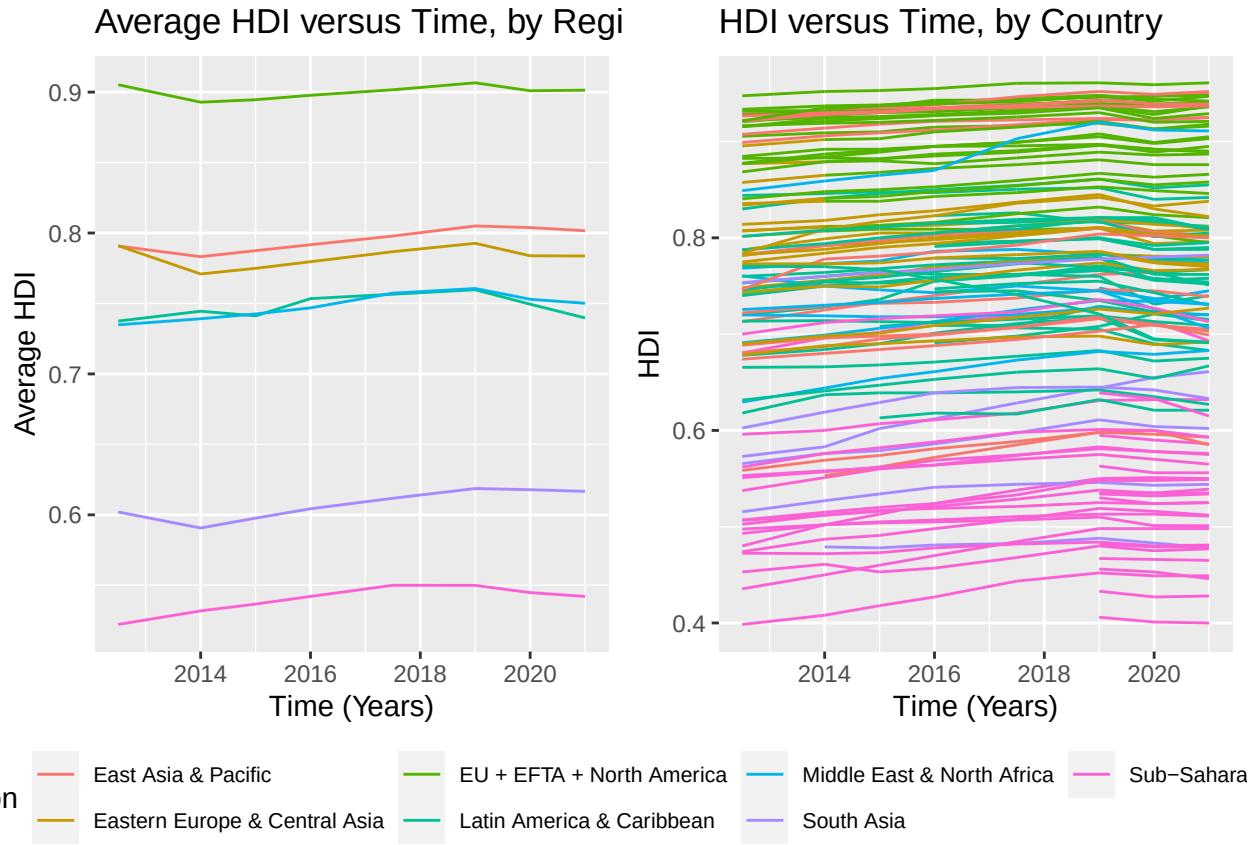
$$HDI_{time,\ country,\ region} = \alpha_{region} + \beta_{region} \times X_{time,\ country,\ region} + a_{country} + b \times X_{time,\ country} + \varepsilon_{time,\ country,\ region}$$

where

$$\alpha_{region} \sim N(\mu_\alpha, \sigma_\alpha^2)$$
$$\beta_{region} \sim N(\mu_\beta, \sigma_\beta^2)$$
$$a_{country} \sim N(\mu_a, \sigma_a^2)$$
$$b_{country} \sim N(\mu_b, \sigma_b^2)$$
$$\varepsilon_{time,\ country,\ region} \sim N(0, \sigma_{HDI}^2)$$

$X_{time,\ country,\ region}$ is the design matrix for the variables `x1.6`, `x3.2`, `x5.1`, `x6.4`, `x7.3`, and intercept, while $X_{time,\ country}$ is the design matrix for the time variable (in years) and intercept.

This equation above assumes independence between random effects. Though this assumption is reasonable for the random effects of Rule of Law variables between regions, it is not as reasonable for the random effects of the time variable between countries. This can be seen from the chart below since the HDI of different countries mostly follow similar trajectories over times, and therefore are associated. There is no closed form solution when we allow correlation between random effects, so the actual mixed effects model will not exactly follow the equation above.



The coefficients from the fixed effects are as follows:

	Estimate	Std. Error	df	t value	Pr(>|t|)
(Intercept)	0.6925175	0.0519539	6.051174	13.3294513	0.0000103
years_from_2000	0.0019365	0.0002481	125.613090	7.8055328	0.0000000
x1.6	0.0044611	0.0108274	5.867850	0.4120229	0.6949501
x3.2	0.0039985	0.0046819	6.165585	0.8540228	0.4250250
x5.1	-0.0065979	0.0106958	5.189926	-0.6168717	0.5633750
x6.4	-0.0088892	0.0082408	4.211854	-1.0786782	0.3385920
x7.3	0.0172646	0.0141687	6.099454	1.2185040	0.2680548

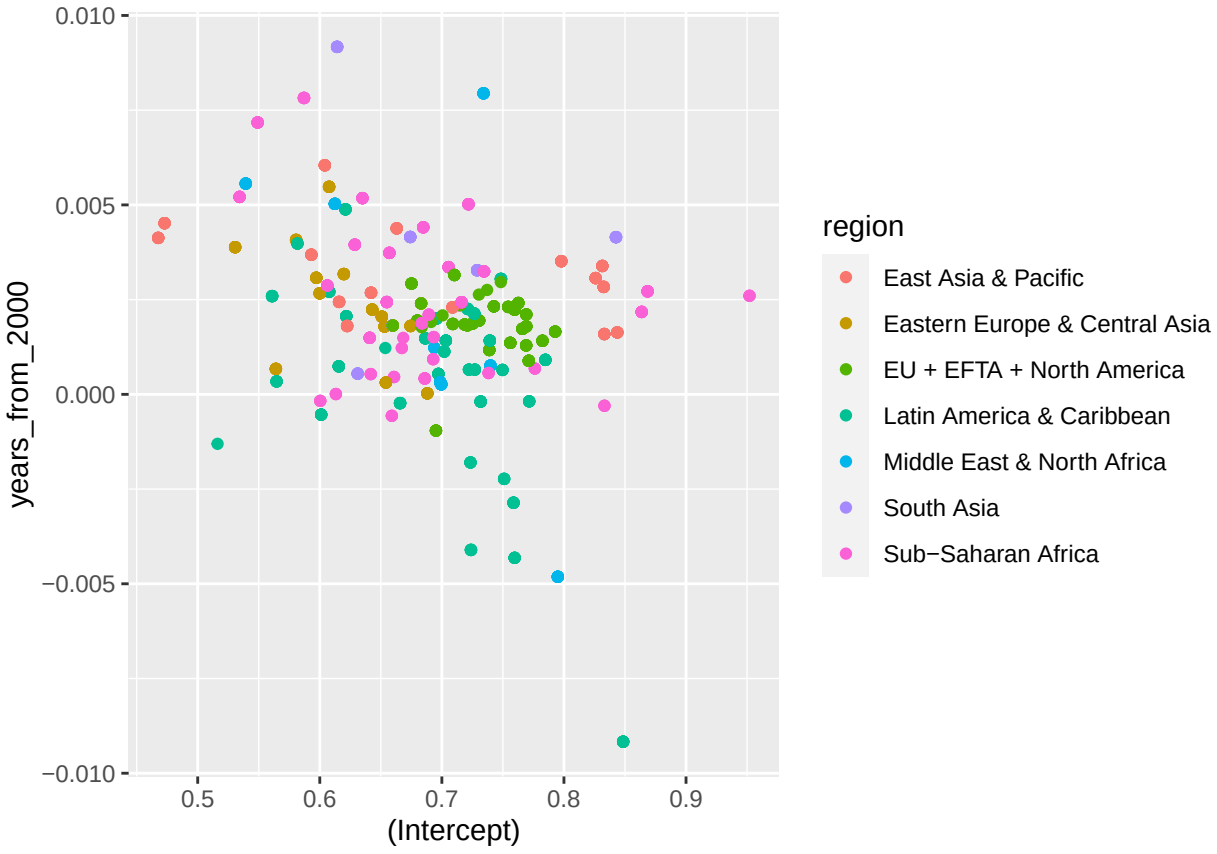
The only fixed variable that was significant (both unadjusted and adjusted) in this model was the `years_from_2000` variable. The other variables were very far from significance.

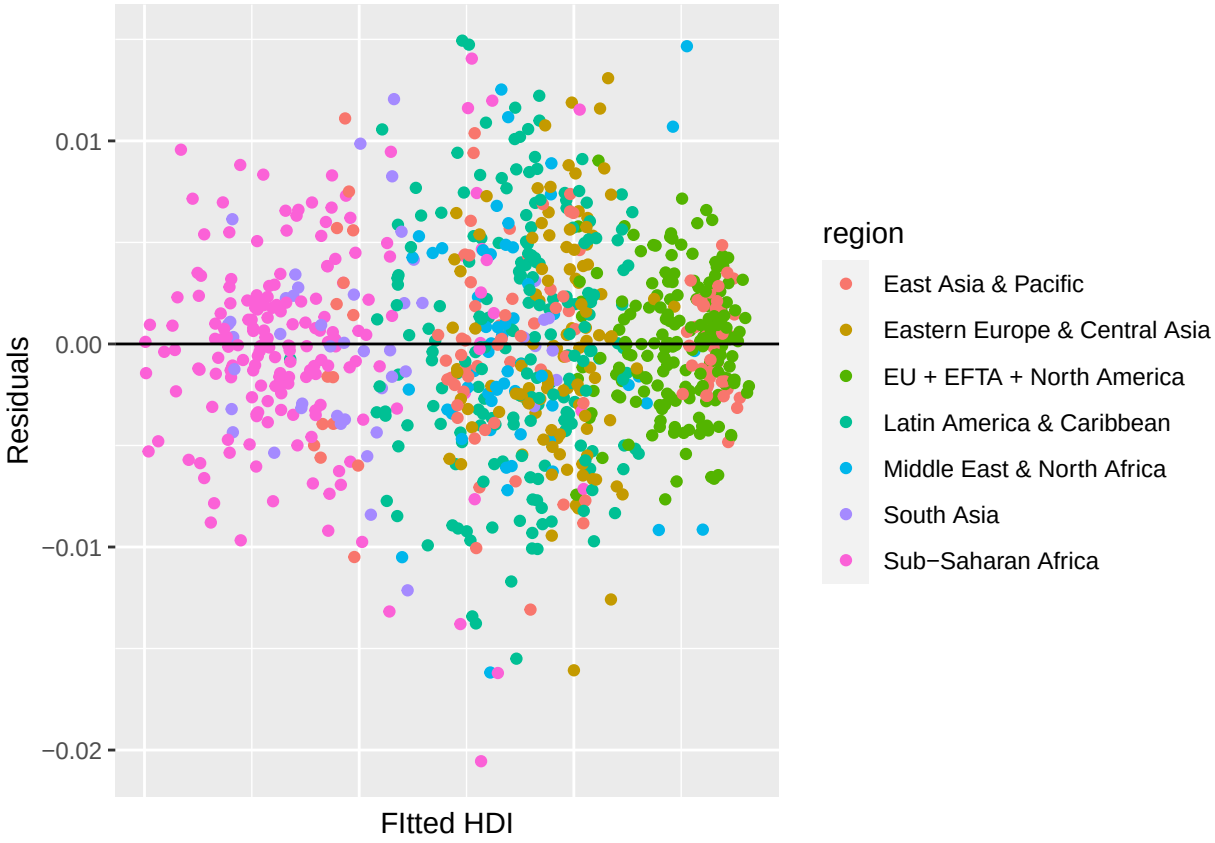
```
## Data: selected_columns
## Models:
## time_mixed_effects_noRegion: hdi ~ years_from_2000 + x1.6 + x3.2 + x5.1 + x6.4 + x7.3 + (1 + years_f
## time_mixed_effects: hdi ~ years_from_2000 + x1.6 + x3.2 + x5.1 + x6.4 + x7.3 + (1 + x1.6 + x3.2 + x5
##                            npar     AIC     BIC logLik deviance  Chisq Df
## time_mixed_effects_noRegion   10 -5356.9 -5308.7 2688.4  -5376.9
## time_mixed_effects            31 -5457.8 -5308.5 2759.9  -5519.8 142.89 21
##                            Pr(>Chisq)
## time_mixed_effects_noRegion
## time_mixed_effects          < 2.2e-16 ***
## ---
```

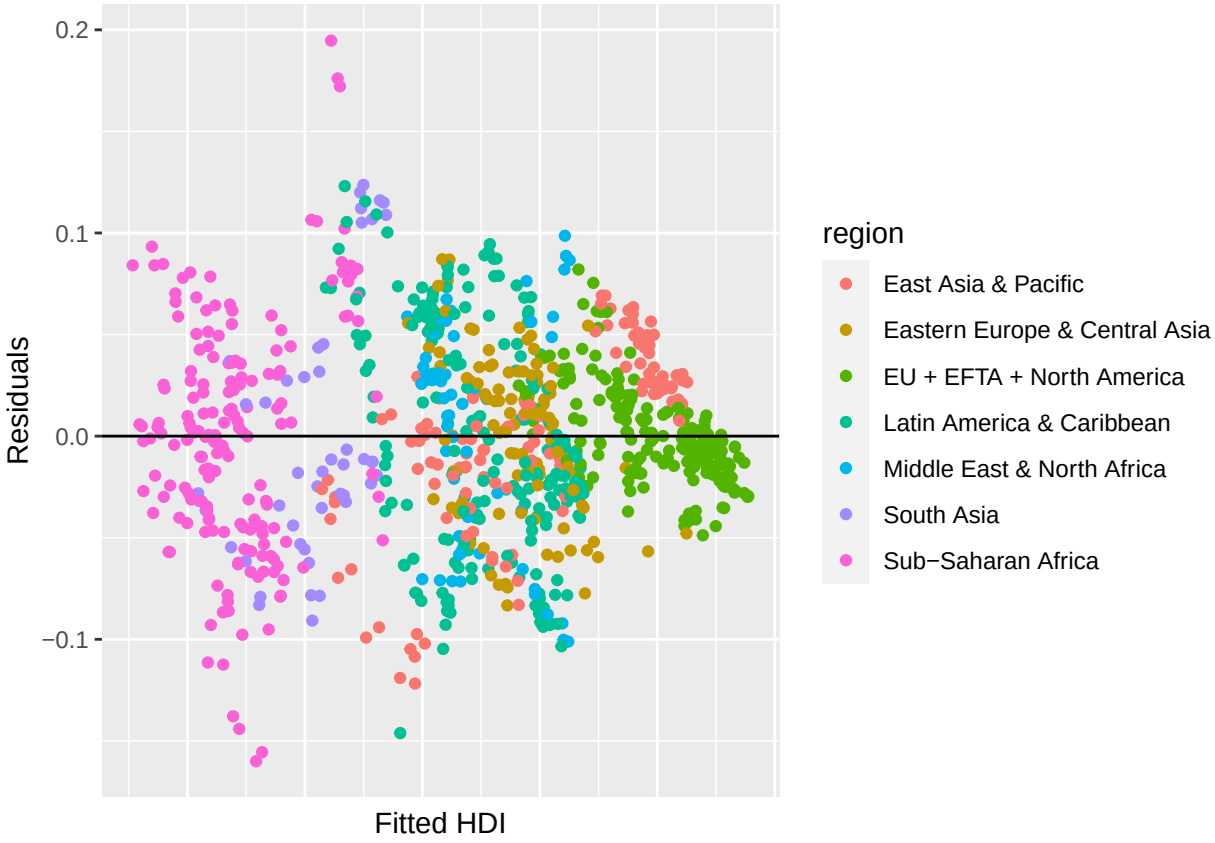
```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Data: selected_columns
## Models:
## time_mixed_effects_noYear: hdi ~ years_from_2000 + x1.6 + x3.2 + x5.1 + x6.4 + x7.3 + (1 + x1.6 + x3
## time_mixed_effects: hdi ~ years_from_2000 + x1.6 + x3.2 + x5.1 + x6.4 + x7.3 + (1 + x1.6 + x3.2 + x5
##                           npar     AIC     BIC logLik deviance  Chisq Df
## time_mixed_effects_noYear   30 -5086.1 -4941.6 2573.1  -5146.1
## time_mixed_effects          31 -5457.8 -5308.5 2759.9  -5519.8 373.69  1
##                           Pr(>Chisq)
## time_mixed_effects_noYear
## time_mixed_effects         < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We also analyzed the importance of including random effects of the variables `x1.6`, `x3.2`, `x5.1`, `x6.4`, and `x7.3` based on region, and the random effects of the time variable based on country using an extra-sum-of-squared F test. These F-tests




0.01
0.00
−0.01
−0.02
Residuals
FItted HDI
region
East Asia & Pacific
Eastern Europe & Central Asia
EU + EFTA + North America
Latin America & Caribbean
Middle East & North Africa
South Asia
Sub−Saharan Africa


0.2
0.1
0.0
−0.1
Residuals
Fitted HDI
region
East Asia & Pacific
Eastern Europe & Central Asia
EU + EFTA + North America
Latin America & Caribbean
Middle East & North Africa
South Asia
Sub−Saharan Africa