

$$1a) \begin{cases} \cos(z_1) (m + M) \ddot{z}_3 - m d \cos^2(z_1) \dot{z}_1^2 + m d \cos(z_1) \sin(z_1) \ddot{z}_1 - \ddot{z}_2^2 = T_d / r \\ \cos(z_1) (m + M) \ddot{z}_3 - d(m + M) \ddot{z}_2 + g(m + M) \sin(z_1) = 0 \end{cases} \quad \ddot{z}_2 = \cos(z_1) T_d / r$$

$$\begin{cases} (m + M) \ddot{z}_3 - m d \cos(z_1) \dot{z}_1^2 + m d \sin(z_1) \ddot{z}_1 - \ddot{z}_2^2 = T_d / r \\ \cos(z_1) \ddot{z}_3 - d \ddot{z}_2 + g \sin(z_1) = 0 \end{cases} \quad \rightarrow \frac{1}{(M + m \sin^2(z_1))} (-m d \sin(z_1) \ddot{z}_2^2 + g m \cos(z_1) \sin(z_1) + T_d / r)$$

$$m \cos^2(z_1) \ddot{z}_3 - m d \cos(z_1) \dot{z}_1^2 + m g \cos(z_1) \sin(z_1) = 0$$

$$\ddot{z}_3 (m + M - m \cos^2(z_1)) + m d \sin(z_1) \ddot{z}_1^2 - m g \cos(z_1) \sin(z_1) = T_d / r$$

$$\underbrace{(m(1 - \cos^2(z_1)) + M)}_{(M + m \sin^2(z_1))} \ddot{z}_3 (M + \sin^2(z_1) m) \Rightarrow \ddot{z}_3 = \frac{1}{(M + m \sin^2(z_1) m)} (-m d \sin(z_1) \ddot{z}_2^2 + m g \cos(z_1) \sin(z_1) + T_d / r)$$

$$z_u = i_a$$

$$\dot{z}_u = i_a \Rightarrow -u + R_a z_u + L_a \dot{z}_u + k_u \omega_a = 0$$

$$T_d = k_m i_a \quad \omega_a = \frac{V}{r} = \frac{\ddot{z}_3}{r}$$

$$z_u = i_a = \frac{T_d}{k_m}$$

$$\dot{z}_u = \frac{1}{L_a} \left(u - \frac{R_a T_d}{k_m} - \frac{k_u \ddot{z}_3}{r} \right) \quad ?$$

2a)

$$z_{20} = 0 \pm n\pi$$

$$\begin{aligned} \dot{z}_1: & \frac{1}{d(M+m \sin^2(z_{1,0}))} (g(M+m) \sin(z_{1,0}) + \cos(z_{1,0}) z_4 k_m/r) = 0 \\ \dot{z}_2: & \frac{1}{d(M+m \sin^2(z_{1,0}))} (g(M+m) \sin(z_{1,0}) + \cos(z_{1,0}) z_4 k_m/r) = 0 \\ \dot{z}_3: & \frac{1}{(M+m \sin^2(z_{1,0}))} (g m \cos(z_{1,0}) \sin(z_{1,0}) + z_4 k_m/r) = 0 \end{aligned}$$

$$z_{1,0} = \arcsin\left(\frac{z_{2,0}}{g m}\right)$$

$$z_u = (-g m \sin^2(z_{1,0})) \frac{r}{k_m} \quad \text{in (1)}$$

$$\dot{z}_u: \frac{1}{L_a} (u - R_a \dot{z}_{u0} - k_u z_{30}) = 0 \Rightarrow$$

$$\frac{k_u}{L_a r} \left(\frac{u r}{k_u} - z_{30} \right) \Rightarrow z_{30} = \frac{u_0 r}{k_u}$$

$$\Rightarrow z_{u0} = 0$$

$$T_d = u_0$$

$$\sin x \sin x = \cos x \sin x + - \cos x \sin x$$

2b)

$\dot{z}_1 = 0$ säger när roboten står stilla i upprätt läge
 $\dot{z}_3 = 0$ är när hastigheten är noll (förändringen är 0)
 $\dot{z}_u = 0$ när induktansen 0

$$3. \dot{z}_1 = \dot{z}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

konst. grav
 grav-påverkan

$$\dot{z}_2 = \frac{1}{d(M+m \sin^2(z_1))} \left(-\frac{m d}{2} \cdot z_1 \cdot z_2^2 + g(m+M) \cdot z_1 + z_4 k_m \right)$$

$$\Rightarrow \left[(-m d z_2^2 + g(m+M))(d(M+m \sin^2(z_1)) - \dots \right]$$

$$\frac{(-m d z_1 z_2^2 + g(m+M) z_1 + z_4 k_m)(d(M+m \sin^2(z_1)))}{[d(M+m \sin^2(z_1))]^2} \cdot \frac{-m d z_1}{d(M+m \sin^2(z_1))} \cdot z_2$$

$$\left[\frac{(-m d (\pi/2)^2 + g(m+M))(d M)}{(d M)^2}, 0, 0, \frac{k_m}{m d M} \right]$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(-md(n\pi)^2 + g(M+m))}{dM} & 0 & 0 & 0 \\ \frac{(gm - mdn\pi)}{M} & 0 & \frac{k_m}{rM} & 0 \\ 0 & 0 & -\frac{k_u}{rLa} & -\frac{Ra}{La} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{La} \end{bmatrix}$$

$y_* = \theta = z_1$
 $\Delta y = \underbrace{[1 \ 0 \ 0 \ 0]}_C \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$