MAC 1140 Section 13.2 Arithmetic Sequences

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Objectives

1. Determine Whether a Sequence Is Arithmetic
2. Find the nth term of an Arithmetic Sequence

Can you guess the missing term of the sequence?

Example 1:

1. $\{2, 4, 6, 8, 10, 12, 14, \_\_\_, 18, \dots\}$
2. $\{37, 27, 17, 7, -3, \_\_\_, -23 \dots\}$
3. $\{2, 11, 20, 29, 38, 47, 56, \_\_\_, 74, \dots\}$

Definition

An **arithmetic sequence** may be defined recursively as

$$a_1 = a, a_n - a_{n-1} = d$$

or as

$$a_1 = a, a_n = a_{n-1} + d$$

where a and d are real numbers.
The number a is the first term.
The number d is called the **common difference**.

Find a_1 and d of arithmetic sequence

Example 2

1. $\{2, 4, 6, 8, 10, 12, 14, 16, 18, \dots\}$
2. $\{37, 27, 17, 7, -3, -13, -23 \dots\}$
3. $\{2, 11, 20, 29, 38, 47, 56, 65, 74, \dots\}$

Determine Whether a Sequence Is Arithmetic

Example 3

Determine whether the sequence $\{s_n\} = \{3n + 5\}$ is arithmetic.

Solution steps:

1. Set n=1 to find the first term s_1.
2. Use $d = s_n - s_{n-1}$ to find the common difference d
3. If s_1 and d are constant real numbers, then $\{s_n\}$ is arithmetic sequence.

Theorem: nth Term of an Arithmetic Sequence

For an arithmetic sequence $\{a_n\}$ whose first term is a_1 and whose common difference is d, the nth term is determined by the formula

$$a_n = a_1 + (n-1)d$$

Find a particular term of an Arithmetic Sequence

the nth term formula of an arithmetic sequence

$$a_n = a_1 + (n-1)d$$

Example 4, Given the first term of arithmetic sequence $a_1 = 5$ and common difference $d = 2$

1. Find a_{101}
2. Find a_{21}

Find a particular term of an Arithmetic Sequence

the nth term formula of an arithmetic sequence

$$a_n = a_1 + (n-1)d$$

Example 5, Suppose $\{a_n\}$ is an arithmetic sequence with $a_4 = 5$ and common difference $a_{10} = 17$.Find the term a_{24}

Solution steps:

1. Write a_4 and a_{10} in terms of a_1 and d using the formula.
2. Solve the linear system with two variables a_1 and d .
3. Use the results of a_1 and d with the formula to compute a_{20}

Find a particular term of an Arithmetic Sequence

the nth term formula of an arithmetic sequence

$$a_n = a_1 + (n-1)d$$

Example 5, Suppose $\{a_n\}$ is an arithmetic sequence with $a_4 = 5$ and common difference $a_{10} = 17$.Find the term a_{24}

Step1: Write a_4 and a_{10} in terms of a_1 and d using the formula.

$$\begin{cases} a_4 = a_1 + (4-1)d = 5 \\ a_{10} = a_1 + (10-1)d = 17 \end{cases}$$

Find a particular term of an Arithmetic Sequence

the nth term formula of an arithmetic sequence

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Step2: Solve the linear system with two variables a_1 and d.

$$d = 2, a_1 = -1$$

Find a particular term of an Arithmetic Sequence

Example 5, Suppose $\{a_n\}$ is an arithmetic sequence with $a_4 = 5$ and common difference $a_{10} = 17$.Find the term a_{24}

Solution:

Step1: Write a_4 and a_{10} in terms of a_1 and d using the formula.

$$\begin{cases} a_4 = a_1 + (4-1)d = 5 \\ a_{10} = a_1 + (10-1)d = 17 \end{cases}$$

Step2: Solve the linear system with two variables a_1 and d.

$$d = 2, a_1 = -1$$

Step3: Use the results of a_1 and d with the formula to compute a_{24}

$$a_{24} = a_1 + (24-1)d = -1 + 23 \times 2 = 45$$