MAC 1140 Section 12.3 Systems of Linear Equations: Determinants

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Objectives

Definition

If a, b, c and d are four real numbers, the symbol

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

is called a **2 by 2 determinant**. Its value is the number $ad - bc$, that is

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Example 1: Evaluate

$$\begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix}$$

Example 2: Evaluate

$$\begin{vmatrix} 8 & -3 \\ 4 & 2 \end{vmatrix}$$

Evaluate a 2 by 2 Determinant

Example 3: Evaluate

$$\begin{vmatrix} -3 & -1 \\ 4 & 2 \end{vmatrix}$$

Cramer's Rule for Two Equations Containing Two Variables

The solution to the system of equations

$$\begin{cases} ax + by = s \\ cx + dy = t \end{cases}$$

is given by

$$x = \frac{\begin{vmatrix} s & b \\ t & c \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y = \frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

Provided that

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$$

An example to show the steps of using Cramer's Rule

Step 1: Obtain 2 by 2 determinant from system of equations

$$\begin{cases} x + 3y = 5 \\ 2x - 3y = -8 \end{cases}$$

We obtain:

$$D = \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}, D_x = \begin{vmatrix} 5 & 3 \\ -8 & -3 \end{vmatrix}, D_y = \begin{vmatrix} 1 & 5 \\ 2 & -8 \end{vmatrix}$$

Step 2: Evaluate each determinant
We obtain:

$$D = \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = (1)(-3) - (2)(3) = -9$$

$$D_x = \begin{vmatrix} 5 & 3 \\ -8 & -3 \end{vmatrix} = (5)(-3) - (-8)(3) = 9$$

$$D_y = \begin{vmatrix} 1 & 5 \\ 2 & -8 \end{vmatrix} = (1)(-8) - (2)(5) = -18$$

Step 3: Evaluate the variable x, y using the determinants

$$x = \frac{D_x}{D} = \frac{9}{-9} = -1$$

$$y = \frac{D_y}{D} = \frac{-18}{-9} = 2$$

Solving a system of Linear equations using Cramer's Rule

Example 1:

$$\begin{cases} 3x - 2y = 4 \\ 6x + y = 13 \end{cases}$$

Solution steps:

1. Obtain the determinants of D, D_x, D_y
2. Evaluate the determinants D, D_x, D_y
3. Using the determinants to compute $x = D_x/D$ and $y = D_y/D$

Example 2:
$$\begin{cases} x + y = 8 \\ x - y = 4 \end{cases}$$

Solution steps:

1. Obtain the determinants of D, D_x, D_y
2. Evaluate the determinants D, D_x, D_y
3. Using the determinants to compute $x = D_x/D$ and $y = D_y/D$