

Principal Component Analysis for Fault Detection and Diagnosis. Experience with a pilot plant

Thamara Villegas
Dept. of Electronic and Circuits
Simón Bolívar University
Caracas 1081-A, Venezuela
e-mail:tvillega@usb.ve

María Jesús Fuente
Dept. of Engineering and Control
Valladolid University
Valladolid 47011, Spain
e-mail:maria@autom.uva.es

Miguel Rodríguez
Dept. of Industrial Technology
Simón Bolívar University
Caracas 1081-A, Venezuela
e-mail:mirodriguez@usb.ve

Abstract—This paper describes the application of Principal Component Analysis (PCA) for fault detection and diagnosis (FDD) in a real plant. PCA is a linear dimensionality reduction technique. In order to diagnosis the faults, the PCA approach includes one PCA model for each system behavior, i.e., a PCA model for normal operation conditions and a PCA model for each faulty situation. Data set is generated in closed loop. The method of fault detection and diagnosis is based on the definition of threshold minimum. These are calculated by the Q statistics and levels of significance. The PCA models outputs (in this case the Q statistics) are compared with their thresholds minimum, with and without faults. The only one that does not violate it threshold says us the actual system situation, i.e., identify the fault. Finally, this technique is applied to a two tanks system, and can be demonstrated that it is possible to detect and identify faults.

Index Terms—Fault detection, Fault diagnosis, Principal Component Analysis (PCA).

I. INTRODUCTION

Due to increasing interest in safety and consistent product quality, the real time monitoring of the process performance has become a key issue for both productivity and safety enhancement. Modern process systems are using enormous amount of data, coming from many variables, that are monitored and recorded continuously day to day. It is key for the operation of these systems, a rapid fault detection that responses effectively to events that mislead the process and deteriorate the system reliability. Fault Detection and Diagnosis (FDD) schemes consider problems during the plant operation caused by uncertainty, disturbances, faults and incomplete knowledge of the process model. Therefore, a fault has to be understood as an unexpected change of the system functionality.

Several approaches to fault detection and diagnosis have been presented in the literature, based on models or knowledge, using different techniques as observer schemes, parity equations, neural networks, etc [1]. Also, multivariate statistical approaches have been investigated to deal with the problem of fault detection, ([2], [3] and [4]). This approach can be more efficient due to the dimensionality reduction techniques used, such as principal component analysis (PCA) or partial least squares (PLS).

The objective of this paper is to implement a fault detection and diagnosis scheme, based on Principal Component Analysis (PCA). Through different techniques, multivariate statistic analysis reduces dimensionality of the data monitored,

allowing highlight data key variables and characteristics involving decorrelation in principal values, variables correlation or maximizing differences between diverse data groups. PCA is a linear dimensionality reduction technique, which ignores the nonlinearities that may exist in the process data. Industrial processes are inherently nonlinear; therefore, in some cases nonlinear methods for monitoring may result in better performance compared to the linear methods [5].

Even though the idea of PCA appeared in 1889, PCA research and applications are still in study. MacGregor et al [6] referred to several industrial applications of PCA for process monitoring. Raich and Cinar [7] developed a diagnostic method based on angle discriminant when using PCA for fault detection. Kramer [8] has generalized PCA to the nonlinear case by using an autoassociated neural network. Dong and McAvoy [9] have developed a nonlinear PCA approach based on principal curves and neural networks that produce independent principal component.

The PCA scheme, used in this paper, contains a PCA model for each system's behavior, i.e., a PCA model is calculated for normal and different faulty conditions. This approach is applied to a two-tanks real plant, a nonlinear and closed loop system that works at different operating points. Fault detection and diagnosis is made by evaluating the Q statistic for each PCA model with the actual data collected from the plant.

The paper is organized as follows: section 2 briefly describes principal component analysis theory; section 3, the methodology on PCA scheme for fault detection and diagnosis is presented; section 4 considers the experimental design, the system conditions and summarize the results; finally, the conclusions are given in the last section.

II. PRINCIPAL COMPONENT ANALYSIS

Principal Component Analysis (PCA) is a technique developed to reduce data dimensionality by extrapolating correlated variables in sets of new uncorrelated variables, keeping variance of the original data. It determines a set of orthogonal vectors called loading vectors, ordered by the amount of variance explained in the loading vectors direction. Given a training set of n observations and m process variables, with mean zero and unit variance, stacked into a matrix $X \in R^{n \times m}$, the loading vectors are calculated by singular value decomposition (SVD) of the covariance matrix S ([2], [5], [10], [12], [13]).

$$S = \frac{1}{n-1} X^T X = V \Lambda V^T \quad (1)$$

where the diagonal matrix $\Lambda \in R^{m \times m}$ contains the non-negative eigenvalues of decreasing magnitude ($\lambda_1 \geq \lambda_2 \geq \dots \lambda_m \geq 0$) and the i^{th} eigenvalue equals the square of the i^{th} singular value (i.e. $\lambda_i = \sigma_i^2$) [2].

PCA can handle high dimensional, noisy and correlated data by projecting the data onto a lower dimensional subspace which contains most of the variance of original data. Selecting the column of the loading matrix $P \in R^{m \times a}$ to correspond to the loading vector associated with a singular values, the projections of the observations in X into the lower-dimensional space are contained in the score matrix.

$$T = XP \implies \hat{X} = TP^T = \sum_{i=1}^a t_i p_i^T \quad (2)$$

where t_i is a score vector (orthogonal) which contains information about relationship between samples and p_i is a loading vector (orthonormal) which contains information about relationship between variables. Projection into principal components space reduces the original set of variables to a latent variables.

Generally, the a principal components should explain the variability of a process through its data X , therefore the difference between X and \hat{X} is the residual matrix E that captures the variations associated with $n - a$ singular values, so:

$$X = TP^T + E = \sum_{i=1}^a t_i p_i^T + E = \hat{X} + E \quad (3)$$

It is very important to choose the number of principal components, a , because TP^T represents the principal sources of variability in the process, and E represents the variability corresponding to process noise. There are different techniques to determine the a principal components: percent variance test [10], scree test [11], parallel analysis and prediction residual sum of squares statistic [5]. Percent variance test was used to elaborate this paper.

A. Process statistics with PCA

On-line monitoring of measurement variables can be carry out with the help of the Hotelling's T^2 and Q statistics, this last, also known as the squared prediction error (SPE). These two statistics can be used to detect faults for multivariate process data.

The T^2 statistic for the lower-dimensional space can be calculated for each new observation x by:

$$T^2 = x^T P (\Sigma_a)^{-2} P^T x \quad (4)$$

where Σ_a contains the non-negative real eigenvalues corresponding to the a principal components and P contains the loading vectors associated with the a columns of V .

The upper confidence limit for T^2 is obtained using the F -distribution:

$$T_{a,n,\alpha}^2 = \frac{a(n-1)}{n-a} F_{a,n-a,\alpha} \quad (5)$$

where n is the number of samples in the data, a is the number of principal components and α is the level of significance.

This statistic can be interpreted as measuring the systematic variations of the process, and a violation of the threshold would indicate that the systematic variations are out of control [5].

The portion of the measurement space corresponding to the lowest $m - a$ eigenvalues can be monitored using the squared prediction error (SPE) or Q statistic.

$$Q = x^T (I - PP^T) x \quad (6)$$

where I is the identity matrix.

The upper confidence limit for the Q can be computed from its approximate distribution:

$$Q_\alpha = \theta_1 \left(\frac{h_0 c_\alpha \sqrt{2\theta_2}}{\theta_1} + 1 + \frac{\theta_2 h_0 (h_0 - 1)}{\theta_1^2} \right)^{\frac{1}{h_0}} \quad (7)$$

with:

$$\theta_i = \sum_{j=a+1}^m \lambda_j^i \quad h_0 = 1 - \frac{2\theta_1 \theta_3}{3\theta_2^2}$$

where c_α is the value of the normal distribution with α the level of significance.

The Q statistic does not suffer from an over-sensitivity to inaccuracies in the smaller singular values and it is associated with noise measurements. A violation of the threshold would indicate that the random noise has significantly changed, or an usual event has occurred that had produced a change in the covariance structure of the model.

III. THE FDD SCHEME

As statistics properties of signals in a behavior remain the same and from a behavior to another, small variations are significative, Fault Detection and Diagnosis (FDD) scheme proposed in this section is formed by one PCA model per behavior type, i.e., a PCA model for normal operation conditions and a PCA model for each faulty situation.

The PCA method is an optimal technique to capture signals variability. Therefore, model determination depends on signal behavior in the operating point and the variations around this point. A low amount of false alarms guarantee a robust PCA model. PCA models are based on historical data X collected during each behavior mode and Q and T^2 statistics are studied to determine a threshold for each one.

Determining the correct model allows to elaborate a new signals projection for principal components. Obtaining statistic values Q and T^2 within threshold limits if new signals correspond to behavior used for model calculation, and exceeding these threshold limits for a different behavior. The FDD scheme is presented in figure 1.

The actual signals $x_1(k), x_2(k), \dots, x_m(k)$ collected from the plant, and normalized with zero mean and unit variance, define the X matrix, input for each PCA model, then each model

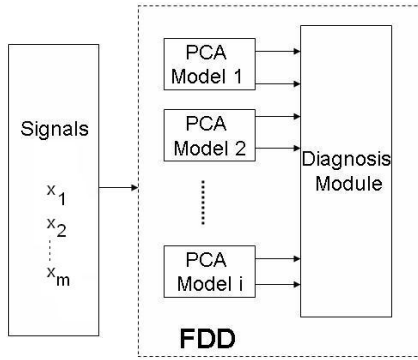


Fig. 1. The FDD Scheme

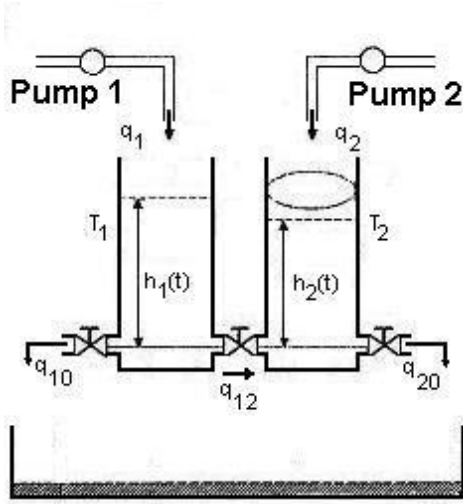


Fig. 2. The Two-tanks system

outputs, the Q and T^2 statistics, provides information of the processed signals to the diagnosis module, that determines system behavior, i.e., identify the normal condition or the corresponding faulty situation.

IV. THE APPLICATION

A. Nonlinear System

The system consists of two cylindrical tanks with the same transversal area and height. The tanks are connected through a cylindrical pipe and have been equipped with level sensors and pumps for flow supply, specifically the levels of the tanks are measured using two level transmitters (Kent-Taylor, series 80), and both tanks have been equipped with a pump (Flojet) for supply flow q_1 and q_2 respectively. The plant objective is to maintain the level of the two tanks at a desired reference, which is reached with two digital PIDs, one for each tank. The connecting pipes are equipped with manually adjustable valves to simulate clogs as illustrated in figure 2. Therefore, the model is represented by:

$$A \frac{dh_1}{dt} = q_1 - q_{10} - q_{12} \quad (8)$$

$$A \frac{dh_2}{dt} = q_2 + q_{12} - q_{20} \quad (9)$$

According to the Torricelli's law, the flows q_{10} , q_{12} and q_{20} are defined by:

$$q_{10} = K_1 \sqrt{2gh_1} \quad (10)$$

$$q_{12} = K_0 \sqrt{2g(h_1 - h_2)} \quad (11)$$

$$q_{20} = K_2 \sqrt{2gh_2} \quad (12)$$

The system's parameters are shown in Table I. The coefficients K_i are parameters obtained experimentally.

TABLE I
PARAMETERS OF THE TWO TANKS SYSTEM

Symbol	Value	Parameter
A	$66,5 \text{ cm}^2$	Tank transversal area
H_{max}	$36,3 \text{ cm}$	Maximum level
$q_1 = q_2$	100 ml/sec	Maximum flow
K_0	$0,25$	Flow correction coefficient
K_1	$0,3$	Flow correction coefficient
K_2	$0,35$	Flow correction coefficient

In the system, five operation modes were considered. They are the following:

- Normal behavior (the valves are open and level sensors h_1 and h_2 are correct) (Normal).
- Tank 1 outlet clogging (f_1)
- Tank 2 outlet clogging (f_2)
- Level sensor h_1 fault (f_3)
- Level sensor h_2 fault (f_4)

Five PCA models were determined for the five operation modes, where the inputs for each PCA model are: $[h_1(t), h_2(t), q_1(t), q_2(t)]$, and two outputs, the T^2 and Q statistic, are calculated in each PCA model.

An adequate simulator was constructed with Simulink/Matlab 5.0/6.5 including the controllers, the noise effects and the physics limitation expressed in Table I. A closed loop simulation was made for each case (5 possible situations) with 10,000 samples, that include all variation spectrum of the four inputs considered, taking a sampling time of 1 sec to obtain the statistics threshold.

After determine each one of the PCA models considered, a diagnosis module was included in the simulator. This last module, is a program to evaluate, with the actual data, each operation mode, i.e., comparing the T^2 and Q statistic of each PCA model with its threshold, and determine the system behavior.

B. Numerical Results

In this section is presented the validation of PCA models with simulation and real plant data, taken from a pilot plant located at Valladolid University, Spain. The system to study is non linear, with multiple inputs and outputs. The PCA

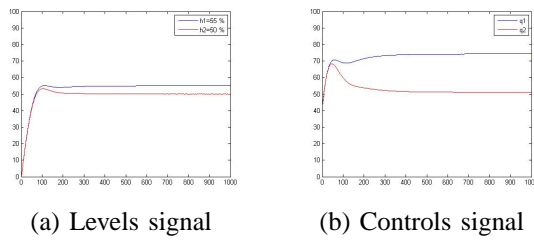


Fig. 3. Normal behavior with simulation data

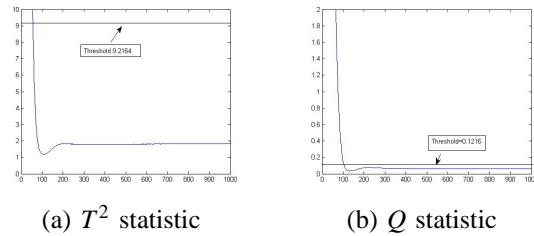


Fig. 4. T^2 and Q statistic of the PCA Model for normal behavior

models use two principal components, according to percent variance test. Normal behavior PCA model with two principal components represents 99.42% of the total variance of the system; case f_1 98.57%, case f_2 98.43%, case f_3 96.35% and case f_4 98.68%.

The T^2 and Q statistics were obtained to each PCA model, the upper confidence limits were computed with simulation data at significance level $\alpha = 1$ (Table II and Table III).

TABLE II

T^2 STATISTIC WITH $\alpha = 1$

n	a	α	F_α	Threshold
10,000	2	1%	4.61	9.2164

TABLE III

Q STATISTIC WITH $\alpha = 1$

Normal	f_1	f_2	f_3	f_4
0.1216	0.3596	0.3994	0.9497	0.3362

Figure 3 shows the control signals q_1 and q_2 and the operating point $h_1=55\%$ and $h_2=50\%$, under normal behavior in a closed loop simulation with 1,000 samples.

The T^2 and Q statistics for normal behavior are showed in the figure 4. Both of them are below its threshold after the settling time.

Evaluating all the PCA models calculated with these and real plant signals, the statistic T^2 no satisfy the limit threshold in all the cases, therefore only the Q statistic is used for detection and diagnosis. The different models answers are observed in figure 5 for simulation with 1,000 samples. The PCA model for normal behavior produces a Q statistic below its threshold after the settling time, i.e., the time that the system takes in order to get the stationary state, and the other fault

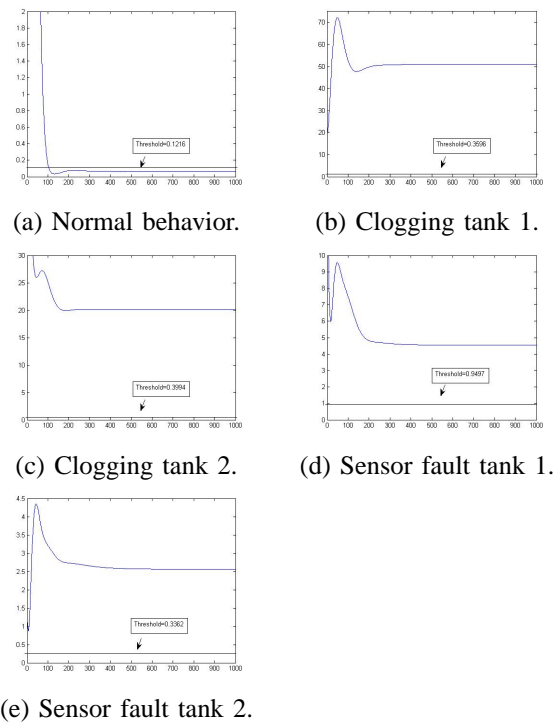


Fig. 5. The Output of the PCA Models for normal behavior with their thresholds.

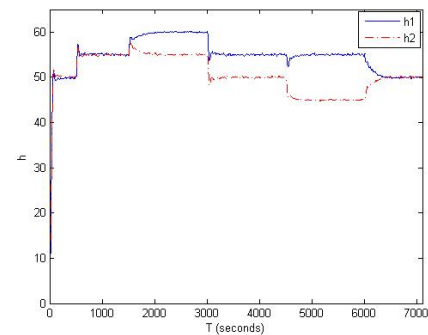
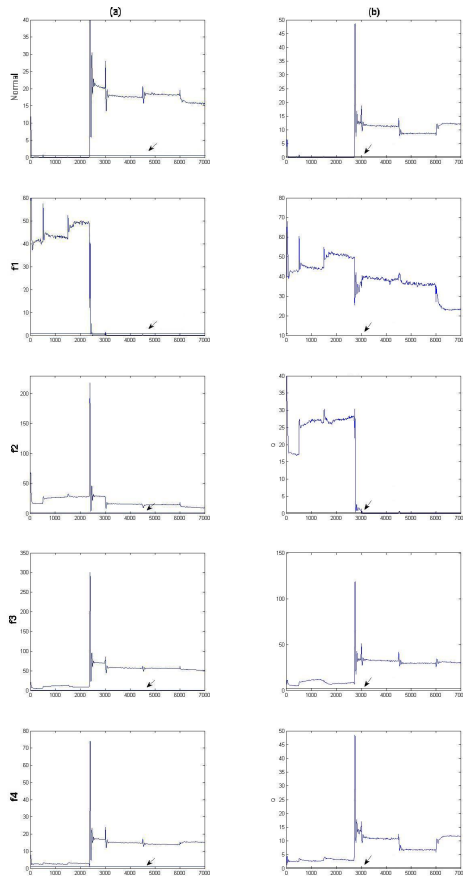


Fig. 6. Different operation points.

behavior models produce a statistic Q above their thresholds, therefore, normal behavior is detected, because is the only system behavior that does not violate its threshold.

These PCA models were validated with real plant data and signals with changes around the operating point (Fig. 6) assuring PCA scheme robustness for system's non-linearities.

Figure 7 presents the statistic Q for two real plant behaviors, evaluated during 7,000 samples. Column (a) shows five models evaluating normal behavior during first 2,350 samples and then with a 50 % clogging in tank 1 outlet. For each behavior, only a Q statistic below its threshold is generated. During first 2,350 samples the PCA model, calculated by normal operation, generates a Q statistic below its threshold, meanwhile the output of the other models are above their respective thresholds. This



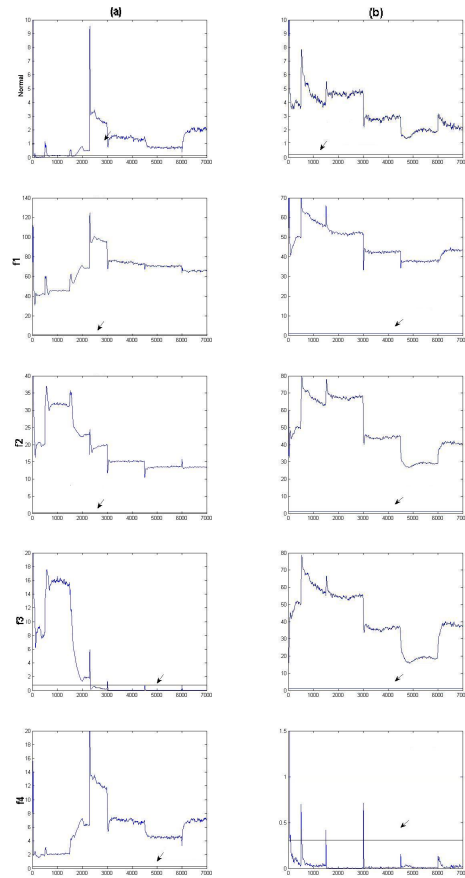
(a) Outlet tank 1 (b) Outlet tank 2

Fig. 7. PCA models outputs for clogging tanks.

conclude that the plant is under control. For sampling above 2,350 samples, the Q statistic changes, generating $f1$ a PCA model with its Q statistic below the threshold and the other models produce a peak in that sampling readjusting later the Q statistics to above the threshold value, i.e., after sampling 2,350 the fault $f1$ is identify.

Column (b) of the same Figure 7 shows five models evaluating normal behavior during first 2,700 samples and then with a 50 % clogging in tank 2 outlet. During first 2,700 samples the PCA model for normal operation generates a Q statistic below its threshold, meanwhile the output of the other models are above their respective thresholds, i.e., a normal behavior is detected. For sampling above 2,700 samples, the Q statistic changes, generating $f2$ a PCA model with a Q below its threshold and the other models produce a peak in that sampling time readjusting later their respective Q statistics above the threshold value, i.e., a fault $f2$ is identify.

For the following faults also were used 7,000 samples. Column (a) in Figure 8 shows five models evaluating normal behavior during first 2,300 samples and then with a tank 1 level sensor fault of 15 %. During first 2,300 samples, the PCA model, defined for the normal behavior, generates a statistic Q below its threshold, meanwhile the other models output



(a) Tank 1 (b) Tank 2

Fig. 8. PCA models outputs for sensor faults.

are above their respective threshold, i.e., the normal situation is detected. For sampling above 2,300 samples, Q changes, generating $f3$ a PCA model with its statistic Q below the threshold and the other output of the models produce a peak in that sampling time, readjusting later their Q statistics to above threshold value. as before, the situation $f3$, fault in the level sensor of tank1 is identify. Column (b) of the same Figure 8 shows a tank 2 level sensor fault of 15 % presented in all the samples. The PCA model, defined for the $f4$ situation, generates a statistic Q below its threshold, with some short time peaks, considered as false alarms due to the set-point changes. The Other PCA models generate an output Q , above their respective thresholds, i.e., a fault $f4$ is identify.

PCA models were evaluated with real plant data, in both unchanging and changing operating point, to get closer to real processes. Responses with different faults sizes were evaluated, obtaining correct detection and diagnosis for the five behaviors defined.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper uses an approach to face the fault detection and identification in processes using Principal Components

Analysis (PCA). In order to diagnosis the faults different PCA models are generated, one for each situation (normal conditions and different faulty situations). The detection and diagnosis are based on the definition of a threshold minimum, which are calculated by the Q statistics and levels of significance. The PCA models outputs (in this case the Q statistics) are compared with theirs thresholds minimum, with and without faults. The only one that does not violate it threshold says us the actual system situation, i.e., identify the fault.

The technique has been proved in a real plant, the two communicated tanks system showing good results because all the faults considered, faults in the level sensors and clogging in the tanks, are clearly detected and identify. This PCA scheme based on behavior models has been proved with changes around the operating point assuring PCA scheme robustness for system's non-linearities.

B. Future Works

Developing this study was found that bigger size faults require bigger detection thresholds. A more exhaustive analysis of relation between fault sizes and detection thresholds limits could lead to new methodology to automatic fault size detection.

VI. ACKNOWLEDGMENT

This work was supported by the national research agency of Spain (CICYT) through the project DPI2006-15716-C02-02, and by the research and development office of the USB through the project S1-IN-CAI-007-09.

REFERENCES

- [1] J. Chen and R.J. Patton. *Robust Model-Based Diagnosis for Dynamics Systems*, Kluber. Academic Publisher; 1999.
- [2] J.E. Jackson and G.S. Mudholkar, "Control procedures for residuals associated with principal component analysis", *Technometrics*, vol. 21, 1979, pp. 341-349.
- [3] T. Kourti, Process Analysis and Abnormal Situation Detection: From Theory to Practice, *IEEE Control System Magazine*, 2002, pp 10-25.
- [4] B.M. Wise and N.B. Gallagher, The Process Chemometrics Approach to Process Monitoring and Fault Detection, *Journal of Process Control*, vol. 6, 1996, pp 329-348.
- [5] C. Chatfield and A.J. Collins, *Introduction to Multivariate Analysis*, Chapman and Hall/CRC; 2000.
- [6] J.F. MacGregor, T. Kourti and P. Nomikos, "Analysis, monitoring and fault diagnosis of industrial processes using multivariate statistical projection methods", *In Proceedings of IFAC Congress*, vol. M, 1996, pp. 145-150.
- [7] A.C. Raich and A. Cinar, "Process disturbance diagnosis by stadistical distance and angle measures", *In Proceedings of IFAC Congress*, vol. M, 1996, pp. 283-288.
- [8] M. A. Kramer, "Nonlinear principal component analysis using autoassociative neural networks", *AIChE J.*, vol. 37(2), 1991, pp. 233-243.
- [9] D. Dong and T.J. McAvoy, "Nonlinear principal component analysis-Based on Principal Curses and Neural Networks", *Computers chem. Engng.*, vol. 2, 1996, pp. 65-78.
- [10] A.A. Afifi and V. Clark. *Computer-Aided Multivariate Analysis*, Third Ed. Chapman-Hall/CRC; 1996.
- [11] R.B. Castell, "The Scree test for the number of factors", *Journal Multivariate Behavioral Research*, vol. 1, 1966, pp. 245-276.
- [12] L.H. Chiang, E.L. Russell and R.D. Braatz. *Fault detection and diagnosis in industrial systems*. Springer-Verlag; 2001.
- [13] R.A. Johnson and D.W. Wichern, *Applied Multivariate Statistical Analysis*. Second Ed. Prentice-Hall; 1998.