Problem Set 1

Due: September 15

Reading:

- Chapter 1. *What is a Proof?*
- Chapter 2. *The Well Ordering Principle* through 2.3; (omit 2.4. *Well Ordered Sets*)
- Chapter 3. *Logical Formulas* through 3.3

These assigned readings do **not** include the Problem sections. (Many of the problems in the text will appear as class or homework problems.)

Reminder:

- Instructions for PSet submission are on the class web page. Remember that each problem should be prefaced with an *activity and effort statement*.
- The class has a Piazza forum. With Piazza you may post questions—both administrative and content related—to the entire class or to just the staff. You are likely to get faster response through Piazza than from direct email to staff.

  You should post a question or comment to Piazza at least once by the end of the second week of the class; after that Piazza use is optional.

Problem 1.
Prove by contradiction that $\sqrt{3} + \sqrt{2}$ is irrational.

Hint: $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

Problem 2.
Use the Well Ordering Principle to prove that any integer greater than or equal to 30 can be represented as the sum of nonnegative integer multiples of 6, 10, and 15.

Hint: Use the template for WOP proofs to ensure partial credit. Verify that integers in the interval [30..35] are sums of nonnegative integer multiples of 6, 10, and 15.

Problem 3.
For $n = 40$, the value of polynomial $p(n) ::= n^2 + n + 41$ is not prime, as noted in Section 1.1 of the course text. But we could have predicted based on general principles that no nonconstant polynomial can generate only prime numbers.

In particular, let $q(n)$ be a polynomial with integer coefficients, and let $c ::= q(0)$ be the constant term of q.

(a) Verify that $q(cm)$ is a multiple of c for all $m \in \mathbb{Z}$.

(b) Show that if q is nonconstant and $c > 1$, then as n ranges over the nonnegative integers $\mathbb{N}$ there are infinitely many $q(n) \in \mathbb{Z}$ that are not primes.

Hint: You may assume the familiar fact that the magnitude of any nonconstant polynomial $q(n)$ grows unboundedly as n grows.

(c) Conclude that for every nonconstant polynomial q there must be an $n \in \mathbb{N}$ such that $q(n)$ is not prime.
Hint: Only one easy case remains.

Problem 4.

Claim. *There are exactly two truth environments (assignments) for the variables M, N, P, Q, R, S that satisfy the following formula:*

$$\underbrace{(\overline{P} \text{ OR } Q)}_{\textit{clause (1)}} \text{ AND } \underbrace{(\overline{Q} \text{ OR } R)}_{\textit{clause (2)}} \text{ AND } \underbrace{(\overline{R} \text{ OR } S)}_{\textit{clause (3)}} \text{ AND } \underbrace{(\overline{S} \text{ OR } P)}_{\textit{clause (4)}} \text{ AND } M \text{ AND } \overline{N}$$

(a) This claim could be proved by truth-table. How many rows would the truth table have?

(b) Instead of a truth-table, prove this claim with an argument by cases according to the truth value of P.